point-like target
spin 1/2 probe

$e^- + N \to e^- + N$
N is point like.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Mott}\\ \text{point like}\\ \text{No target recoil}}} = \frac{\alpha^2}{q^4} E^2 \left(1-\beta^2 \sin^2\frac{\theta}{2}\right) = \frac{\alpha}{q^4} \cos^2\frac{\theta}{2}$$

$\beta \simeq 1.$

target not point like $\to$ $\left.\frac{d\sigma}{d\Omega}\right|_{\text{non point like}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point like}} \times |F(q^2)|^2$

$q^2 \simeq 0$ $\quad F(q^2) \simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle$

$$F(q^2) = \int d^3r\, e^{i\vec{q}\cdot\vec{r}} f(r) \qquad \rho(r) = Z e f(r)$$

$$A = \int d^3r\, f(r)$$

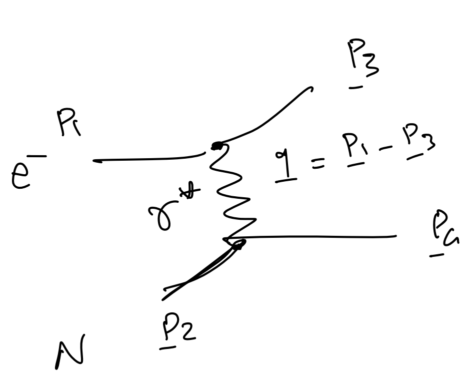
In principle $f(r) = f(\vec{r}) = f(r, \theta, \varphi)$

Typically assume $f(r) = f(r)$ no dependence on θ & φ.

$$F(q^2) = \int \frac{f(r)}{A} e^{i\vec{q}\cdot\vec{r}} d\cos\theta\, d\varphi.\ r^2 dr$$

$$= \frac{2\pi}{A} \int dr\, f(r)\, r^2 \underbrace{\int_{-1}^{1} d\cos\theta\ \underbrace{e^{i\vec{q}\cdot\vec{r}}}_{e^{iqr\cos\theta}}}_{\frac{2\sin(qr)}{qr}}$$

$$= \frac{4\pi}{A} \int dr\, f(r)\, r^2 \frac{\sin(qr)}{qr}$$



Take into account recoil of target.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Point like}\\ \text{with recoil}}} = \frac{\alpha^2}{q^4}\ \underbrace{\cos^2\frac{\theta}{2}}_{\text{Spin of } e^-}\ \frac{E'}{E}$$

Ruth.

Mott cross section.

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}.$$

For non-pointlike target.

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{Mott} \times |F(q^2)|^2$$ Form factor $F(q^2)$

So far not taking into account spin of target

Dirac theory for $S = 1/2$ mass m

intrinsic magnetic moment $\mu_e = g\frac{e\hbar}{4m}$

g : gyromagnetic ratio

electron pointlike $\rightarrow g = 2$ $\mu_e = \frac{e\hbar}{2m}$

proton, neutron spin $= 1/2$ $m = m_N \simeq 1$ GeV.

$\mu_p = g_p \, \mu_N$ $\mu_N = \frac{e\hbar}{4m_N}$ nuclear magneton

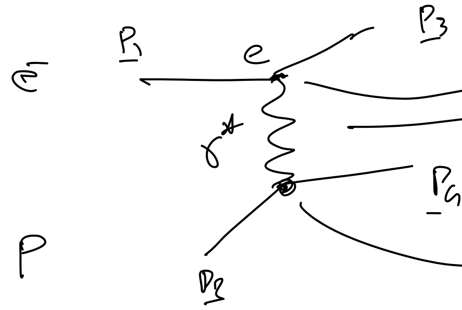
$\mu_n = g_n \, \mu_N$

Expectation: $g_p = 2$. $g_n = 0$. $g_n = 0$

Experimentally $g_p = 2.79$ $g_n = -1.91$ anomalous magnetic moment.

$\Rightarrow$ Indication that p, n are not point like

Consider $e^- + p \rightarrow e^- + p$



$\bar{u}(p_3)\gamma^\mu u(p_1)$ electron current vertex.

$\frac{1}{q^2}$

$\bar{u}(p_4)\left\{F_1(q^2)\gamma^\mu + i\frac{q^\nu \sigma_{\mu\nu}}{2M}K\, F_2(q^2)\right] u(p_2)$

$q = p_1 - p_3$ q^ν is $p_1^\nu - p_3^\nu$ $K = g - 1 = 1.79$ proton.

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ γ_μ : Dirac matrices.

$F_1(q^2)$, $F_2(q^2)$ 2 Form factors.

$F_1(0) = F_2(0) = 1$ $q^2 = 0$.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth.}} = \frac{\alpha^2}{q^4}\underbrace{E^2\cos^2\frac{\theta}{2}}_{\text{spin } e^-} \times \underbrace{\frac{E'}{E}}_{\substack{\text{recoil}\\ \text{of target.}}} \times\left[\left(F_1^2+\frac{\kappa^2Q^2}{4M^2}F_2^2\right) + (F_1+\kappa F_2)^2\frac{Q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

$Q^2 = -q^2 > 0 \quad \forall\, E, E', \theta.$

F_1, F_2 incorporate ignorance about proton structure.

$$Q^2 = 4EE'\sin^2\frac{\theta}{2}$$

Sample different Q^2 by measuring # events at different θ.

$$\frac{E'}{E} = \frac{1}{1+\frac{2E}{M}\sin^2\frac{\theta}{2}}$$

$Q^2 \to 0 \qquad M = m_p$

For $\frac{Q^2}{M^2} \ll 1.$ Suppose $E = 0.5$ GeV.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth}} \simeq \frac{\alpha^2}{q^4}E^2\cos^2\frac{\theta}{2}\,|F_1(q^2)|^2$$

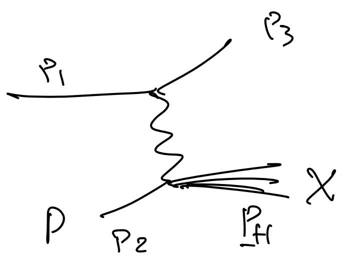
$$F_1(q^2) = 1 - \frac{1}{6}q\langle r^2\rangle.$$

For $\frac{Q^2}{M^2} \gg 1.$

$$\left.\frac{d\sigma}{d\Omega}\right| \simeq (--)\left(\frac{\kappa^2Q^2}{4M^2}|F_2(q)|^2 + (F_1+\kappa F_2)^2\frac{Q^2}{2M^2}\tan^2\frac{\theta}{2}\right)$$

measurement of $F_1(q^2)$ for $Q^2\to 0$

Inelastic limit.



$W^2 = \sum_i p_i^2 = M_X$

Deep Inelastic Scattering.

$$\left.\frac{d\sigma}{d\Omega}\right|_{DIS} = \frac{\alpha^2}{q^4}E^2\cos^2\frac{\theta}{2}\left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right].$$

W_1, W_2 : Structure functions.

Elastic scattering: F_1, F_2 depend only on Q^2

Inelastic scattering: W_1, W_2 depend on Q^2, ν

$$W^2 = \underline{P}_H^2 = (\underline{P}_2 + \underline{q})^2 = M^2 - Q^2 + 2M\nu.$$

proton structure:

McAllister, Hofstadter @ SLAC 1956

↪ Nobel 1961

188 MeV e^- on H_2, He target

measure # events for $\theta \in [35, 138]^\circ$

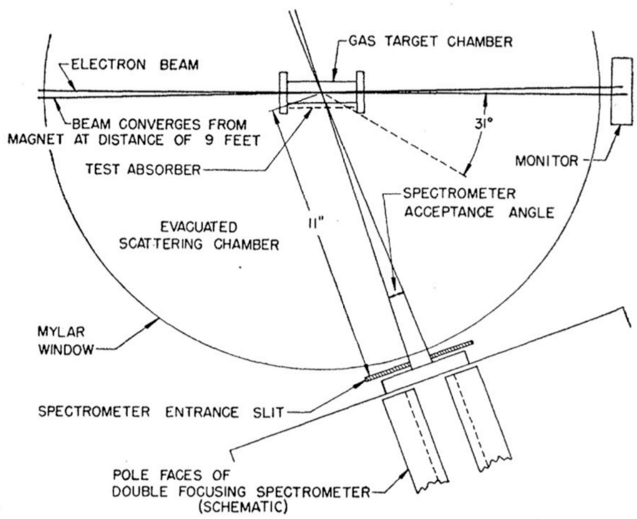
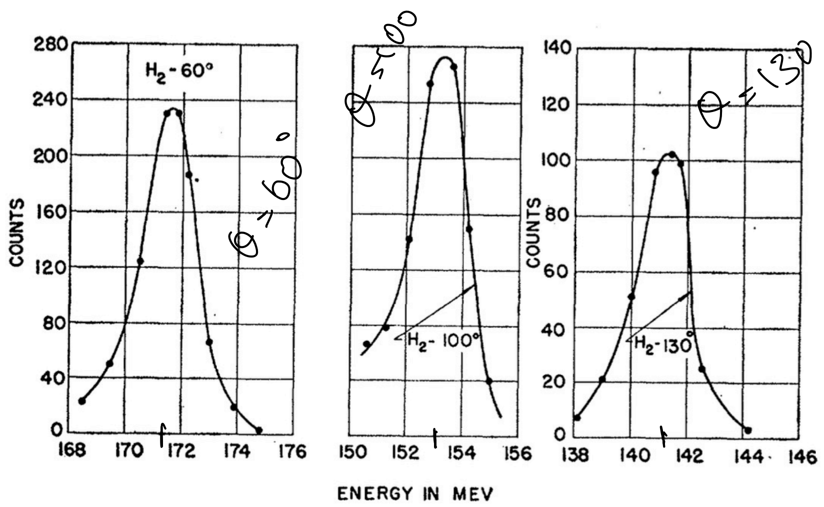


FIG. 2. Arrangement of parts in experiments on electron scattering from a gas target.

Chapter 8 Goldhaber

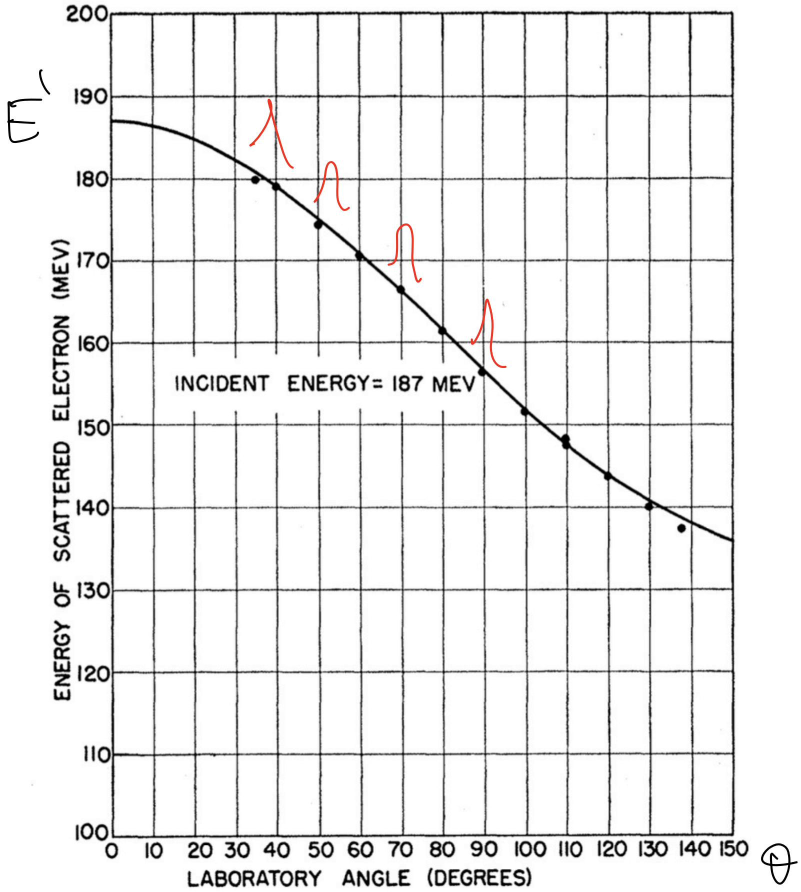


$$E' = \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)} = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

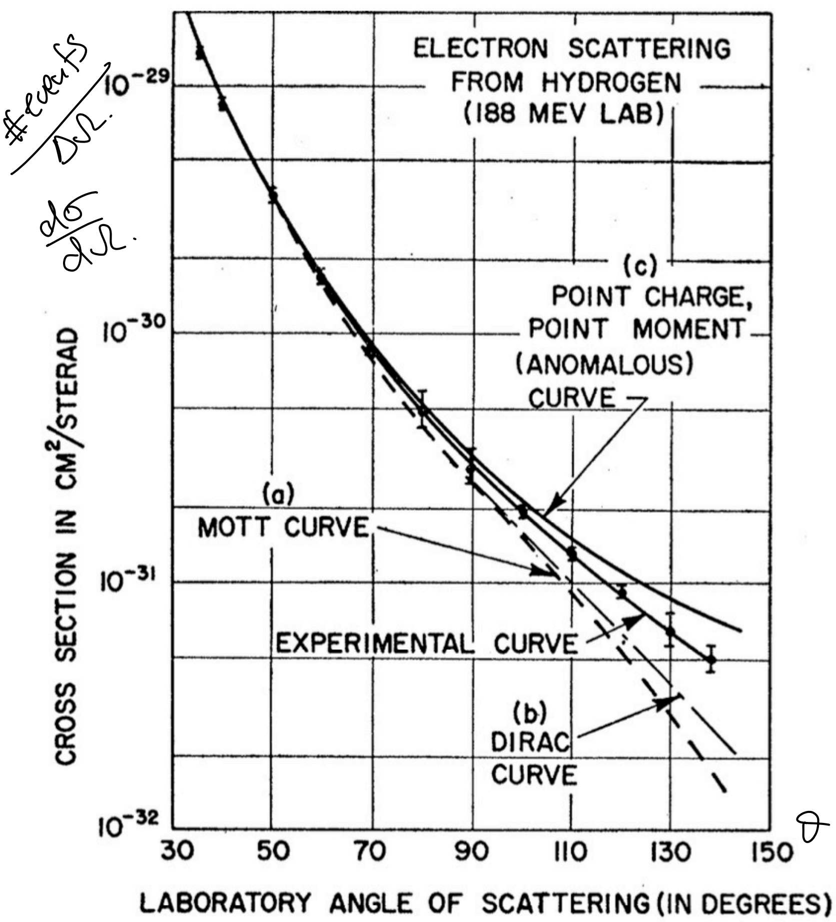
E'

Energy resolution ⊗ $\delta(E')$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{q^4} E^2 \cos^2\frac{\theta}{2} \sim \frac{\alpha^2}{E^2} \frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}$$



$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$



$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{structure}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} \times |F(q^2)|^2$$

$$\simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle .$$

$$\bar{r} = \sqrt{\langle r^2 \rangle} = 0.7 \text{ fm}.$$

FIG. 5. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with the Dirac magnetic moment, curve (c) the theoretical curve for a point proton having the anomalous contribution in addition to the Dirac value of magnetic moment. The theoretical curves (b) and (c) are due to Rosenbluth.[8] The experimental curve falls between curves (b) and (c). This deviation from the theoretical curves represents the effect of a form factor for the proton and indicates structure within the proton, or alternatively, a breakdown of the Coulomb law. The best fit indicates a size of 0.70×10^{-13} cm.

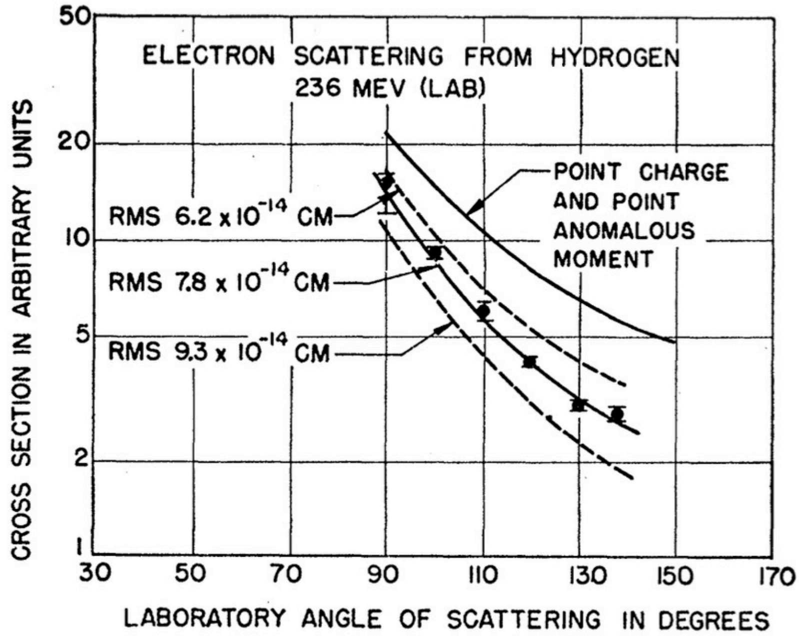


FIG. 6. This figure shows the experimental points at 236 Mev and the attempts to fit the shape of the experimental curve. The best fit lies near 0.78×10^{-13} cm.

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Elastic Scattering of 188-Mev Electrons from the Proton and the Alpha Particle*†‡§‖¶

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The elastic scattering of 188-Mev electrons from gaseous targets of hydrogen and helium has been studied. Elastic profiles have been obtained at laboratory angles between 35° and 138°. The areas under such curves, within energy limits of ±1.5 Mev of the peak, have been measured and the results plotted against angle. In the case of hydrogen, a comparison has been made with the theoretical predictions of the Mott formula for elastic scattering and also with a modified Mott formula (due to Rosenbluth) taking into account both the anomalous magnetic moment of the proton and a finite size effect. The comparison shows that a finite size of the proton will account for the results and the present experiment fixes this size. The root-mean-square radii of charge and magnetic moment are each $(0.74\pm0.24)\times10^{-13}$ cm. In obtaining these results it is assumed that the usual laws of electromagnetic interaction and the Coulomb law are valid at distances less than 10^{-13} cm and that the charge and moment radii are equal. In helium, large effects of the finite size of the alpha-particle are observed and the rms radius of the alpha particle is found to be $(1.6\pm0.1)\times10^{-13}$ cm.

CLASSIFICATION OF COULOMB SCATTERING