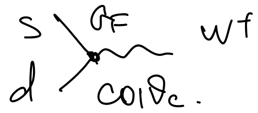
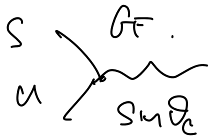
Weak Interactions

Cabibbo angle





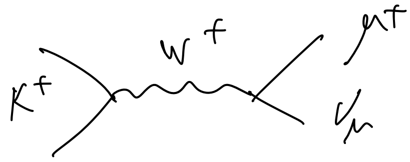
$\Gamma(-) \propto \cos^2\theta_c\, G_F^2$ $\quad \Gamma \propto \sin^2\theta_c\, G_F^2$

BR

$$\frac{\Gamma(\Delta S=1)}{\Gamma(\Delta S=0)} \approx \tan\theta_c^2 \simeq 0.05$$

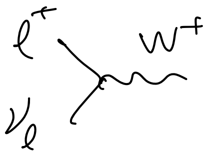
$\Gamma(K^+ \to \mu^+\nu_\mu) = 64\%$

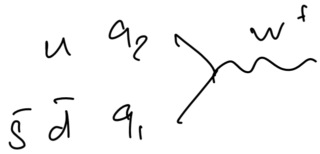
$\Gamma(K^0 \to \mu^+\mu^-) = 7\times10^{-9}$

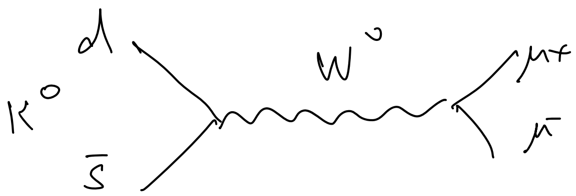


K^+ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Leptonic and semileptonic modes			
$e^+\nu_e$	$(1.582\pm0.007)\times10^{-5}$		247
$\mu^+\nu_\mu$	$(63.56\ \pm0.11\ )\ \%$	S=1.2	236
$\pi^0 e^+\nu_e$ Called K^+_{e3}.	$(5.07\ \pm0.04\ )\ \%$	S=2.1	228
$\pi^0\mu^+\nu_\mu$ Called $K^+_{\mu3}$.	$(3.352\pm0.033)\ \%$	S=1.9	215
$\pi^0\pi^0 e^+\nu_e$	$(2.55\ \pm0.04\ )\times10^{-5}$	S=1.1	206
$\pi^+\pi^- e^+\nu_e$	$(4.247\pm0.024)\times10^{-5}$		203
$\pi^+\pi^-\mu^+\nu_\mu$	$(1.4\ \ \pm0.9\ \ )\times10^{-5}$		151
$\pi^0\pi^0\pi^0 e^+\nu_e$	$< 3.5\ \times10^{-6}$	CL=90%	135
Hadronic modes			
$\pi^+\pi^0$	$(20.67\ \pm0.08\ )\ \%$	S=1.2	205
$\pi^+\pi^0\pi^0$	$(1.760\pm0.023)\ \%$	S=1.1	133
$\pi^+\pi^+\pi^-$	$(5.583\pm0.024)\ \%$		125

So far CC (Charged current) with $W^\pm$







if $\Gamma(K^0 \to \mu^+\mu^-) \ll \Gamma(K^+ \to \mu^+\nu_\mu) \Longrightarrow$ W^0 does not exist.

Flavor-changing neutral current mediated by W^0?

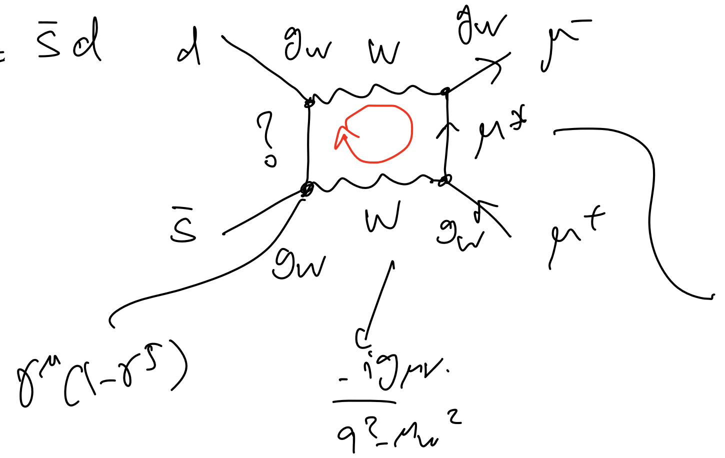
if W^0 does not exist $\Longrightarrow$ $\Gamma(K^0 \to \mu^+\mu^-) = ?\ \emptyset$

not small but $\emptyset$

$\Gamma(K^0 \to \mu^+\mu^-) \neq 0$ experimentally.

tree diagram. or 1st order (minim. # vertices)

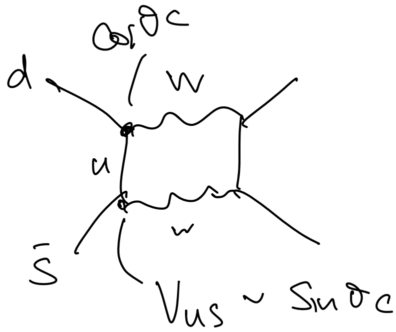
$K^0 = \bar{s}d$



f^* $\frac{i\gamma^\mu q_\mu}{q^2}$

Loop diagram $\geq$ 2nd order.

$$M \sim \underbrace{\left(\frac{g_W^2}{M_W^2}\right)}_{G_F} \underbrace{\left(\frac{g_W^2}{M_W^2}\right)}_{G_F}$$



$$M \sim \frac{g_W^2}{M_W^2} \frac{g_W^2}{M_W^2} \cos\theta_c \sin\theta_c \underbrace{\frac{\not{q}_u}{q_u^2}}_{\text{props.}}$$

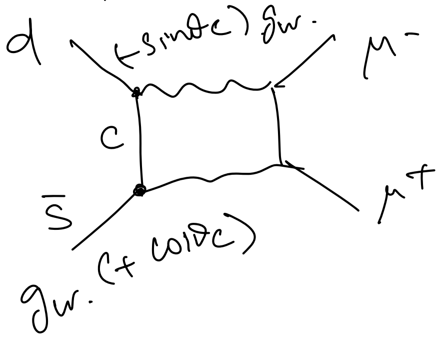
From this diagram $\Rightarrow$ $\Gamma_{th.}(K^0 \to \mu^+\mu^-) > \Gamma_{exp}$ measured.

(GIM mechanism)

To solve this problem: Glashow-Iliopoulos-Maiani 1973.

$\Rightarrow$ existence of a new charm quark.

up-like quark: $q_c = +\frac{2}{3}$. $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$



$d' = d\cos\theta_c + s \cdot \sin\theta_c$.

$s' = -\sin\theta_c \cdot d + \cos\theta_c\, s$.

$W^\pm \to \begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$

$$M_c \sim \frac{g_W^2}{M_W^2} \frac{g_W^2}{M_W^2} (-\sin\theta_c)(\cos\theta_c) \underbrace{\frac{\not{q}_c}{q_c^2}}_{\text{prop.}}$$

if $m_c \simeq 1-3$ GeV. $\Rightarrow$

$M_u + M_c \Rightarrow \Gamma(K^0 \not\to \mu^+\mu^-) \simeq 10^{-9}$ as measured.

if charm exists. ($q_c = +\frac{2}{3}$, $m_c \simeq 1-3$ GeV).

$\Rightarrow$ $K^0 \to \mu^+\mu^-$ does not need flavor-changing W^0.

$\Rightarrow$ proved in 1974 discovery of J/ψ.

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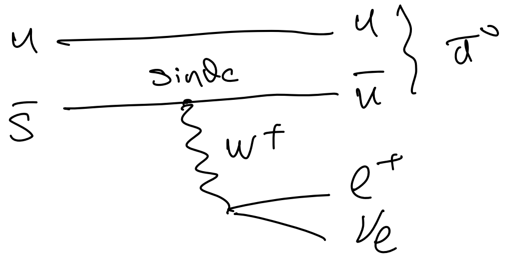
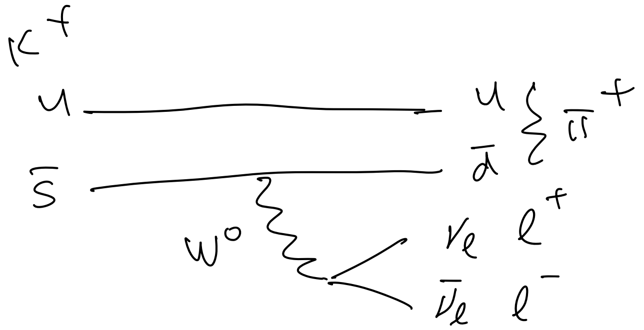
$K^+ \to \pi^0 e^+ \nu_e$





$K^+ \to \pi^+ l\bar{l}$

$\nu_e\bar{\nu}_e$

Lepton family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes

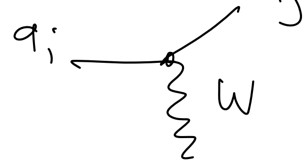
Γ_{35}	$\pi^+\pi^+e^-\bar{\nu}_e$	*SQ*	$<1.3\times10^{-8}$	CL=90%	
Γ_{36}	$\pi^+\pi^+\mu^-\bar{\nu}_\mu$	*SQ*	$<3.0\times10^{-6}$	CL=95%	
Γ_{37}	$\pi^+e^+e^-$	*S1*	$(3.00\ \pm0.09)\times10^{-7}$		
Γ_{38}	$\pi^+\mu^+\mu^-$	*S1*	$(9.4\ \pm0.6)\times10^{-8}$		S=2.6
Γ_{39}	$\pi^+\nu\bar{\nu}$	*S1*	$(1.14\ ^{+0.40}_{-0.33})\times10^{-10}$		

W^0 (flavor changing) suppressed by 10^{-8}

supports GIM mechanism.

$\Rightarrow$ No flavor changing neutral current.

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$$



$g_w \cdot V_{ij}\,\gamma^\mu(1-\gamma^5)$.

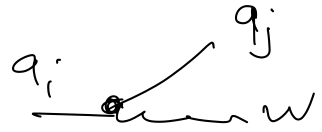
$$\begin{pmatrix} u \\ c \end{pmatrix} \begin{array}{cc} d & s \\ \hline \end{array}\!\!\!\!\!\! \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \underline{V}_{ij}$$

i, j = flavors of quarks.

1973: Kobayashi–Maskawa $\Rightarrow$ proposed a new family.

$$W^\pm \longrightarrow \begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \underbrace{\begin{pmatrix} t \\ b' \end{pmatrix}}_{\text{new}}.$$

$$
\begin{array}{c|ccc|c}
 & d & s & b & \\
V_{CKM} = & V_{ud} & V_{us} & V_{ub} & u \\
 & V_{cd} & V_{cs} & V_{cb} & c \\
 & V_{td} & V_{ts} & V_{tb} & t
\end{array}
$$



$$g_w (V_{CKM})_{ij} \gamma^\mu (1-\gamma^5)$$

3 quark families $\Rightarrow$ to explain $\cancel{CP}$

V_{CKM} is a complex matrix. unitary 3x3.

3 real angles.

Could explain. $\leftarrow$ 1 complex phase.

observed. CP violation

Cabibbo angle.

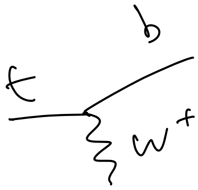
$$
V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$
$$
= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}
$$

$$
V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)
$$

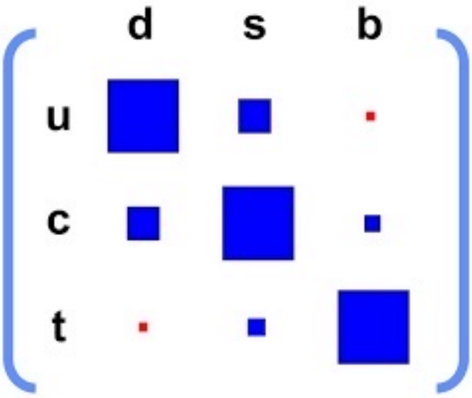
wolfenstein parametriz.

$\lambda \simeq \sin\theta_c$

$$
|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}
$$



$m_t >> m_w + m_b \Rightarrow$ W, b on shell.

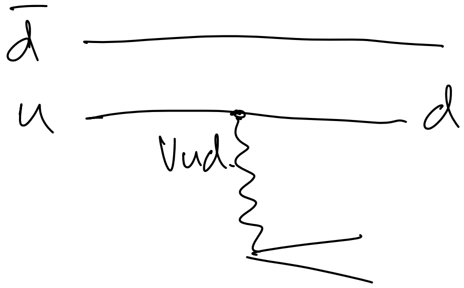


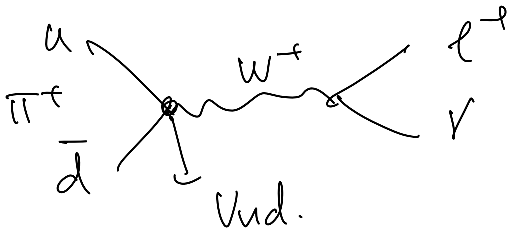
CKM parameters are free param. in Standard Model.

not predicted in SM.
can be only measured.

$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\
 & K \to \pi\ell\nu & \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s &
\end{pmatrix}
$$

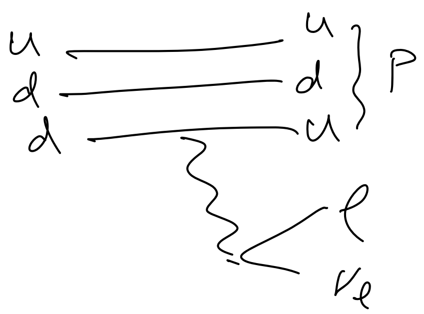
↳ 2nd order process in SM.





$$BF(\pi \to X) \propto |V_{ud}|^2 .$$

$n \to p e^- \nu$

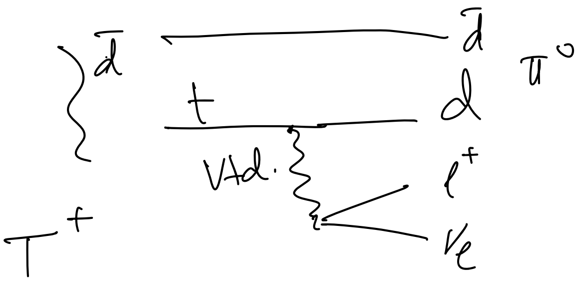


Baryons have larger corrections from QCD.

$$\Gamma \propto |V_{ij}|^2 G_F^2 \text{ (phase space)} \times \text{(QCD correction)}$$

BF or decay rates $\Longrightarrow |V_{ij}|^2$ only.

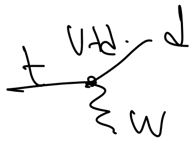
To determine $V_{ij} \neq V_{ij}^*$ $\Rightarrow$ need process with interf.

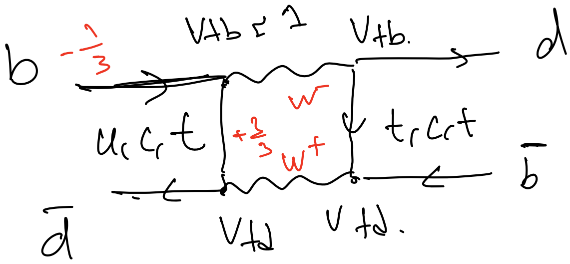


$T^+ \to \pi^0 \ell^+ \nu_e$

$\hookrightarrow$ top meson does not exist

to measure V_{td}





B^0

$$\mathcal{M} \sim \frac{g_w^2}{M_w^2} \frac{g_w^2}{M_w^2} |V_{tb}|^2 |V_{td}|^2$$

$b\bar{d} = \bar{B}^0$ $\qquad M_B \simeq 5.297$ GeV.

$|V_{tb}|^2 \simeq 1.$

$B^0, \bar{B}^0$ oscillation.

Replace d with s. $\Rightarrow$ $\bar{B}_s^0 \longleftrightarrow B_s^0$ $\quad \sim |V_{ts}|^2$

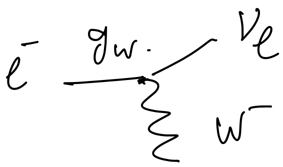
Charged (electrically) weak currents ✓ flavor changing.

Neutral flavor changing neutral current ✗

what about neutral (electrically) flavor-conserving weak current

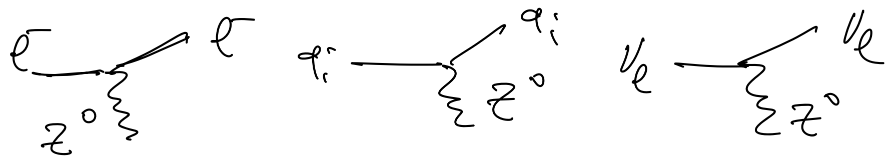
1) $K^0 \to \mu^+\mu^-$ suppressed.
2) $K^+ \to \mu^+\nu_\mu \gg K^0 \to \mu^+\mu^-$.
3) $K^+ \to \pi^+ \nu\bar{\nu}$ suppressed.

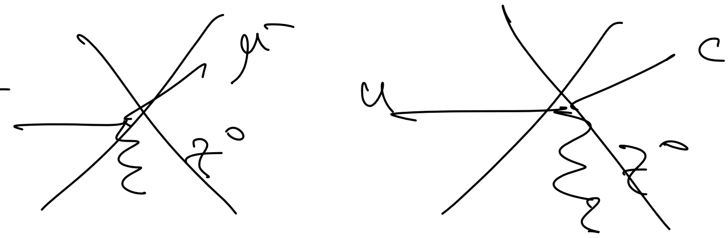
CC



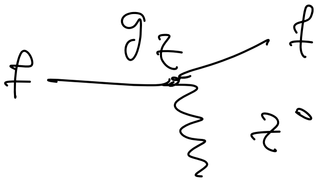
$V_{ij}\, g$

NC:





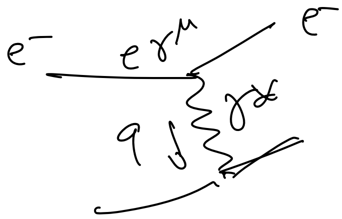
Flavor. cons. Neutral current



$f = \ell, \nu_\ell, q_i$

$i =$ flavors.

1958 Bludman. suggestion of such weak currents.



$$\frac{ie}{q^2} \gamma^\mu$$

$$g_Z \gamma^\mu (1-\gamma^5) \frac{1}{q^2 - M_Z^2}$$

$$\sim \frac{\sqrt{\alpha_W}}{q^2 - M_Z^2} \qquad\qquad \frac{\sqrt{\alpha_{EM}}}{q^2}$$

1) if $\alpha_{EM} > \alpha_{week}$

2) if $q^2 \ll M_Z^2 \Rightarrow$ EM dominates.

2) if $q^2 \simeq M_Z^2$ weak dominates.

Requires high energies.

if at low energy ($q^2 \ll M_Z^2$) $\Rightarrow$ EM dominates.

1961 Glashow: unification of weak and EM. (mixture of γ, Z^0)

1967 Weinberg-Salam: Glashow's theory as a spontaneously broken gauge theory. (similar to QED)

1971 t'Hooft proved that this theory is renormalizable

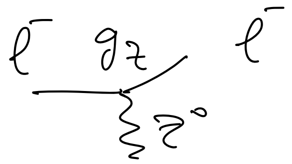
$\Rightarrow$ 1970° search for massive $W^{\pm}$, Z^0
as mediators of weak interaction

Unified Electro-weak theory.

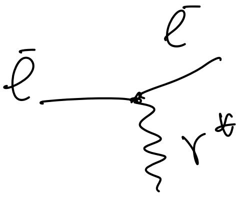
1) Does Z^0 exist ? $\Rightarrow$ process mediated only by Z^0
(indirect proof)

2) produce and observe (invariant mass) $W^{\pm}$, Z^0
(Direct proof)

Indirect proof of Z^0

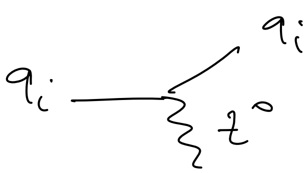


$$\frac{g_Z}{q^2 - M_Z^2}$$



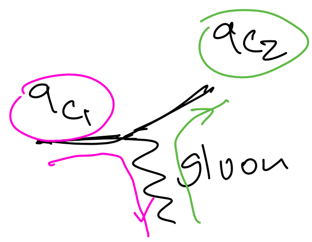
$$\frac{e}{q^2}$$

leptons. $\Rightarrow$ EM dominates.



$$\frac{g_Z}{q^2 - M_Z^2}$$

$$\frac{q_e}{q^2} \times 3$$



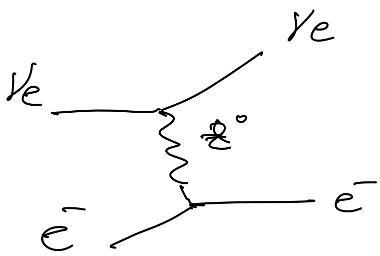
$c_1 \bar{c}_2$

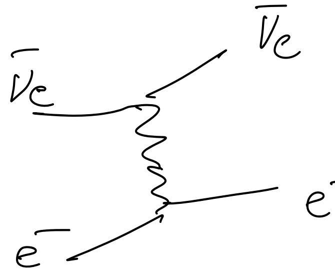
$$\frac{g_s}{q^2} \times N_c \times (F_{color})$$

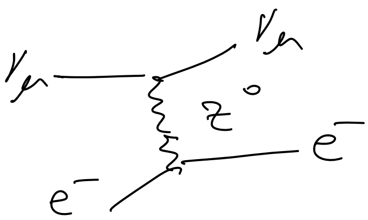
$q^2 \ll M_Z^2$ experimentally. at low energy.

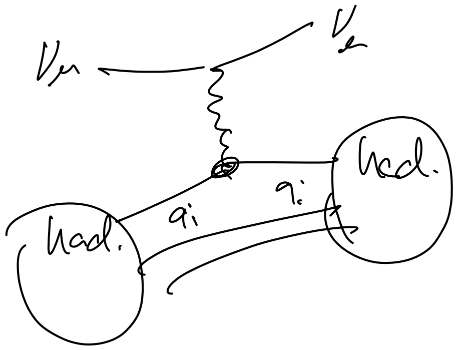
Quarks: Strong dominates.

Neutrinos:









$\Rightarrow$ Needs beam of neutrinos.