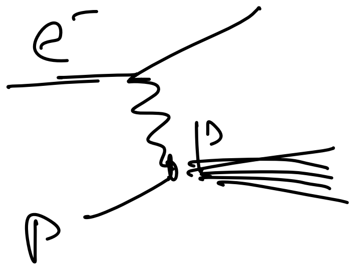


$e^- + p \to e^- + X$ DIS

For N partons:

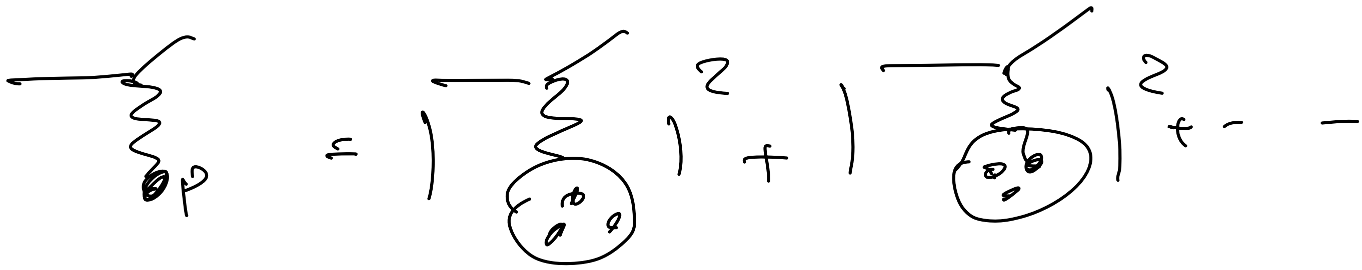
$$W_1(Q^2,\nu) = \sum_j^{\text{partons}} z_{q_j}^2 \frac{f_j(x)}{2M}$$

$$W_2(Q^2,\nu) = \sum_j z_{q_j}^2 f_j(x) \frac{x}{\nu}$$

$f(x)$ parton density function.

$$F_1(x) = M W_1(Q^2,\nu). \quad F_2(x) = \nu W_2(Q^2,\nu)$$

$$F_2(x) = 2x F_1(x)$$ Callan-Gross relation.



elastic scattering on parton.

proton: container of some constituents (parton)

parton: $s = 1/2 \quad m = xM$

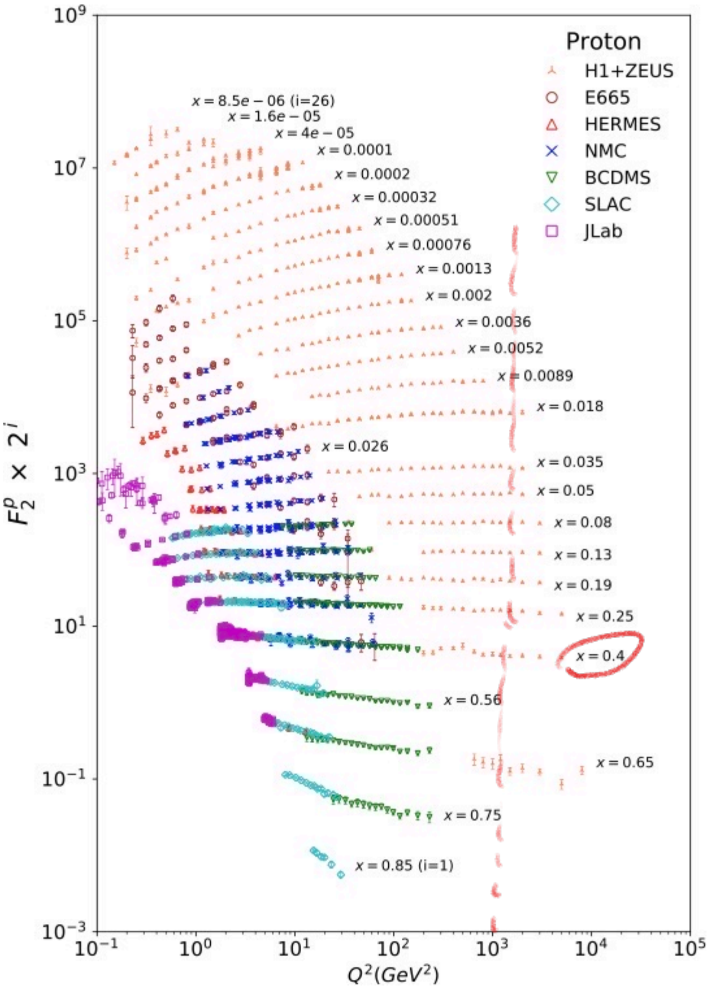
$$x = \frac{Q^2}{2M\nu} \qquad Q^2 = 4EE' \sin^2\frac{\theta}{2} \qquad \nu = E - E'$$

observed/measured quantities with leptons/probe

$$\left.\frac{d\sigma}{d\Omega}\right|_{DIS} \approx \frac{\alpha^2}{Q^4} E'^2 \left[W_2 \sin^2\frac{\theta}{2} + 2W_1 \cos^2\frac{\theta}{2} \right]$$

$$\delta\left(\nu - \frac{Q^2}{2M}\right)$$

parton j has charge $z_{q_j} e < e$



In principle

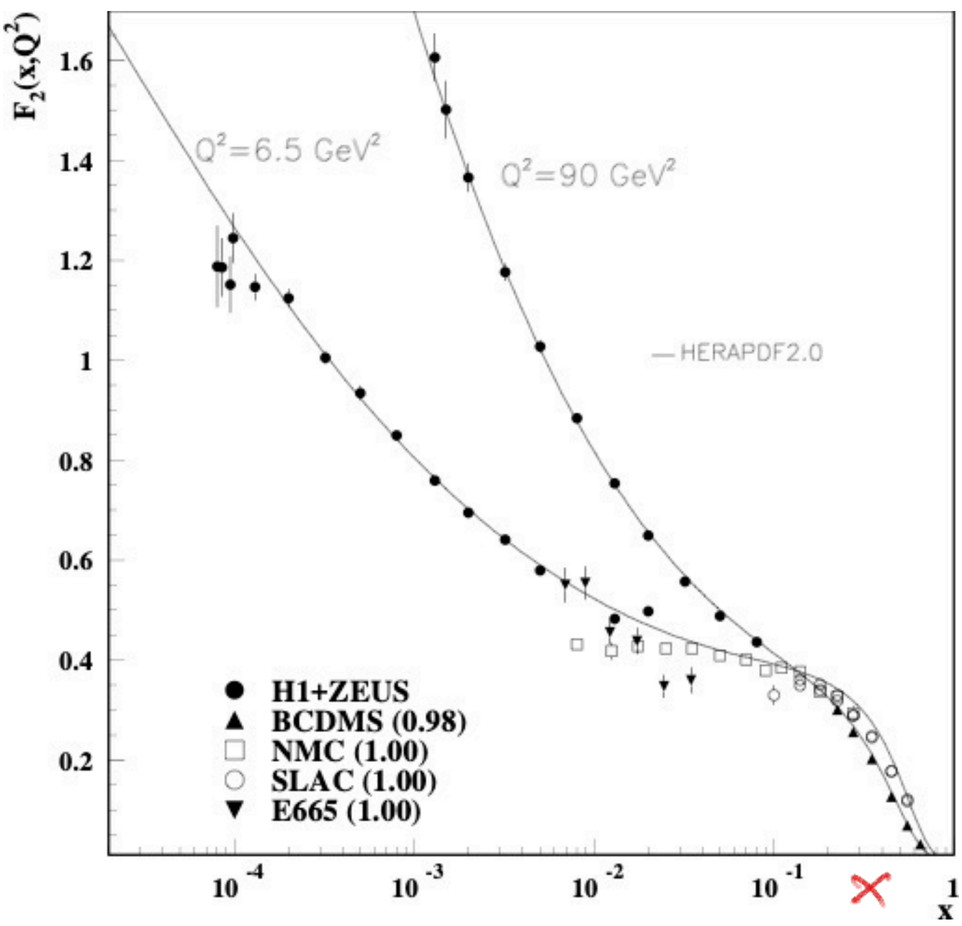
$$F_2 = F_2(Q^2, \nu)$$

$$x = \frac{Q^2}{2M\nu}$$

$\Rightarrow$ F_2 function of x
not Q^2, ν seperately.

$$f(x,y) = x^2 + y^2$$

$$f(x,y) = \frac{x}{y}$$



$$x_B = \frac{Q^2}{2M\nu}$$

Bjorken Scaling
variable x

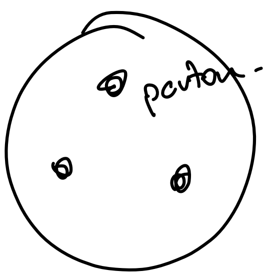
$$\int_0^1 dx\, f(x) = 1$$

$$x \in [0,1]$$

partons: evidence from D.I.S. $\equiv$ incoherent sum
of elastic scattering

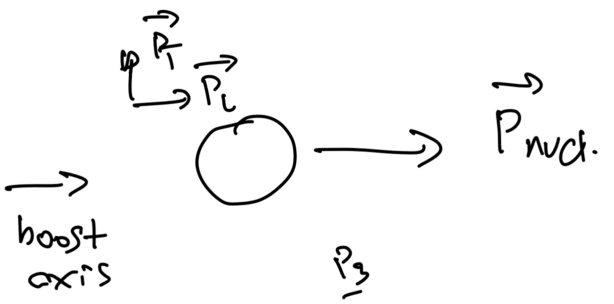
Feynman: x as property of parton.

Quark–parton model



high energy limit. $E \gg m_e, m_p, m_{parton}$.

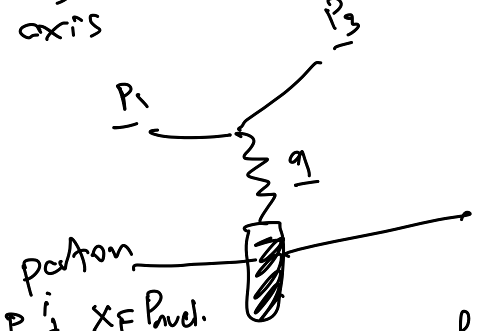
$$x_F = \frac{|\vec{P}_L|}{|\vec{P}_{nucl}|}$$



SLAC experiment: $E_e = 25$ GeV

$$\underline{P}_{part} = x_F \underline{P}_{nucl}$$

x_F: fraction of momentum carried by parton.



$$\underline{P}^f_{part} = \underline{P}^{in}_{part} + \underline{q}$$

$$\underbrace{|\underline{P}^f_{part}|^2}_{m_p^2 \simeq 0} = |\underline{P}^{in}_{part}|^2 + \underline{q}^2 + 2\,\underline{P}^{in}_{part}\cdot\underline{q}$$

high energy limit.

$$q^2 \equiv -Q^2 \quad \Rightarrow \quad Q^2 = 2\,\underline{P}^{in}_{part}\cdot\underline{q} =$$

$$\underline{P}^{LAB}_{nucl} = (M, 0) \qquad \underline{q} = (E-E', \vec{p}_1 - \vec{p}_3) = (\nu, \vec{p}_1 - \vec{p}_3)$$

$$Q^2 = 2x_F M\nu \Rightarrow \underbrace{x_F = \frac{Q^2}{2M\nu}}_{} \simeq x_B \text{ at high energy}$$

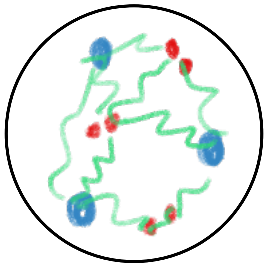
N partons $\quad \sum_i x_i = 1 \qquad \sum_i \underline{P}^i_{part} = \underline{P}_{nucl}$

proton: uud $\qquad$ u, d quarks. $\quad \begin{matrix} +\frac{2}{3} \\ -\frac{1}{3} \end{matrix}\begin{pmatrix} u \\ d \end{pmatrix}$

$$Q_p = +1 = \frac{2}{3} + \frac{2}{3} - \frac{1}{3}$$

proton $u\,u\,d\,d\,\bar{d}\,s\,\bar{s}$

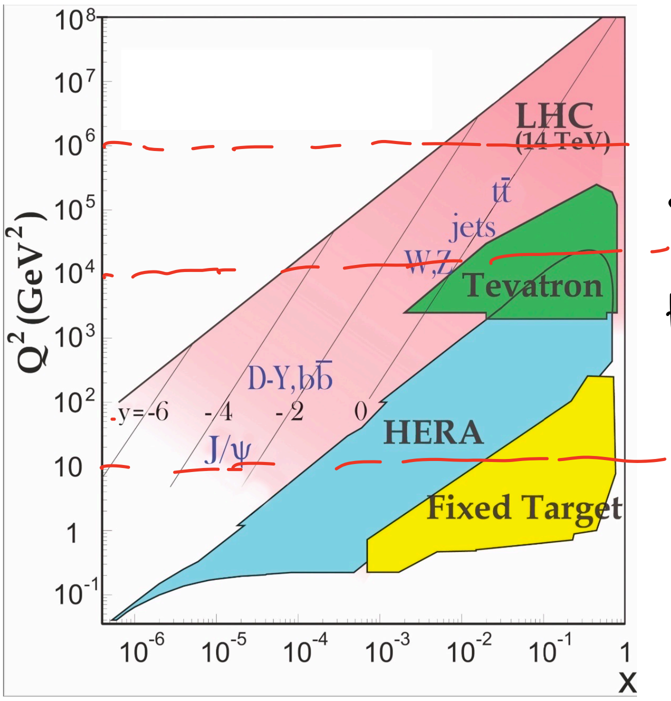
$$\sum x_u \; x_u \; x_d \; x_d \; x_{\bar{d}} \; x_s \; x_{\bar{s}} = 1.$$



Valence quarks: $u\,u\,d$ (electric charge)

Sea quarks/anti-quarks: $s\bar{s} + q_j\bar{q}_j$

gluons: g mediators of strong interaction

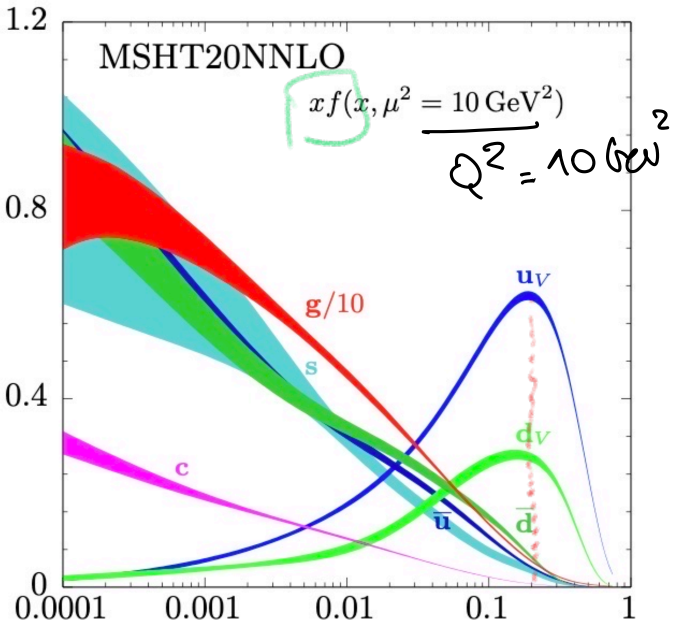


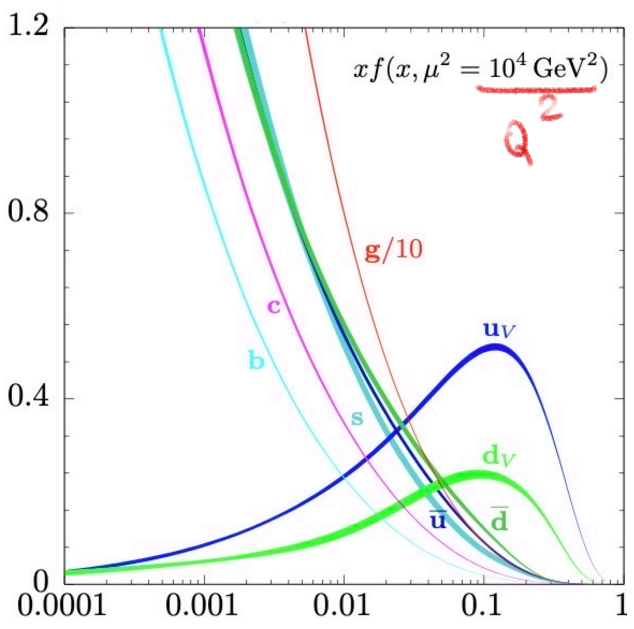
Tevatron: $\xrightarrow{p} \longleftarrow \bar{p}$ $\sqrt{s} = 1.96$ TeV

Hera $e^- + p$(beam).

$\xrightarrow{e^-} \longleftarrow p$

$f(x)$: parton density function.





d_V: $f(x)$ for valence d quark.

u_V: $f(x)$ for valence u quark.

$\bar{u}$: $f(x)$ for sea anti-quark $\bar{u}$

Experimentally: $\sum_i^{\text{Valence}} x_i = 0.5 \Rightarrow$ existence of sea $q\bar{q}$ and gluons.

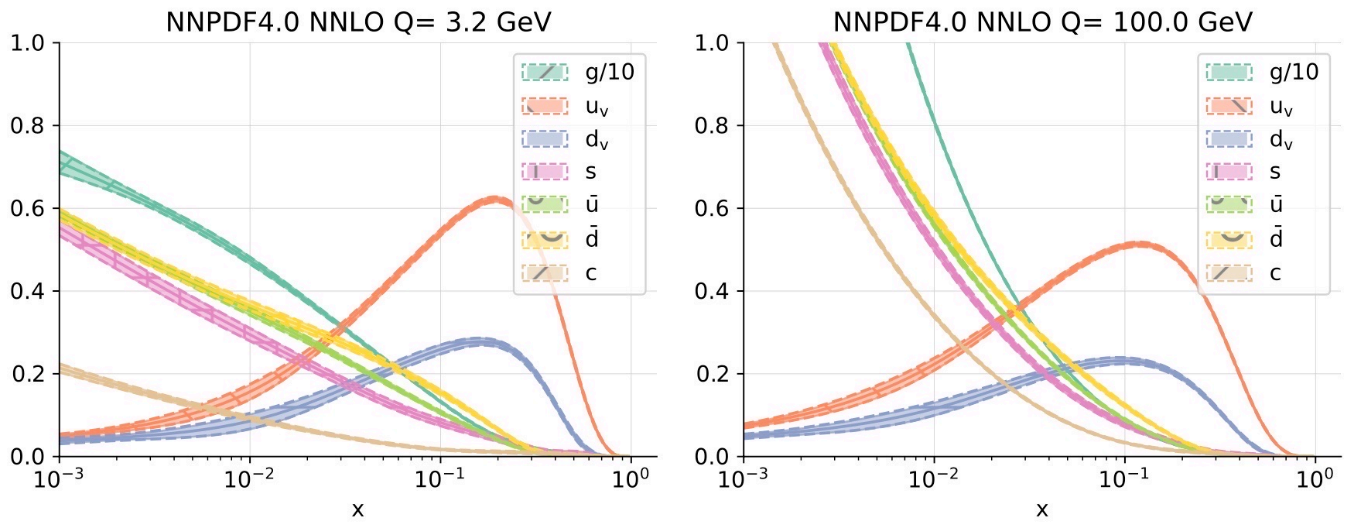
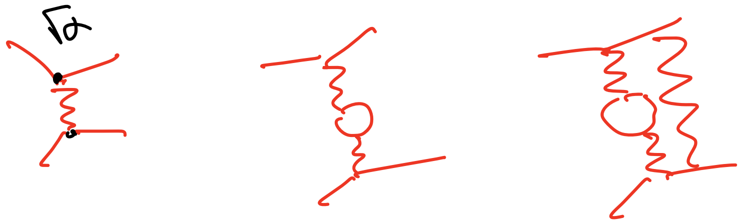


Figure 1.1. The NNPDF4.0 NNLO PDFs at $Q = 3.2$ GeV (left) and $Q = 10^2$ GeV (right).

NNLO: Next-to-Next-to-Leading-order.

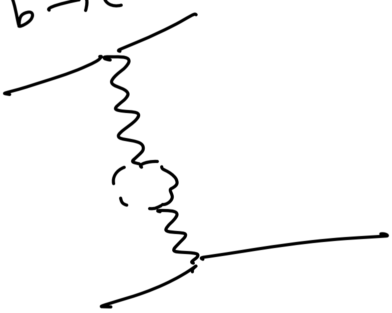
LO:



LO: α. NLO: α^2 NNLO: α^3

N^3LO: Next-Next-Next LO.

$a+b \to c+d$



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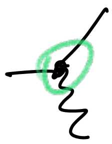
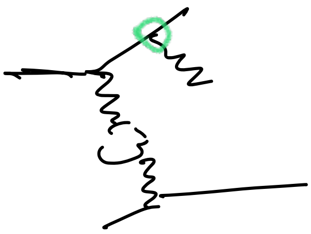
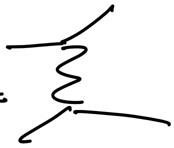
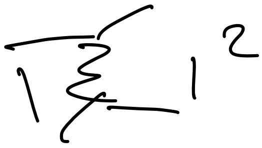
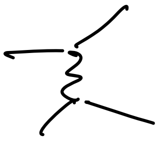
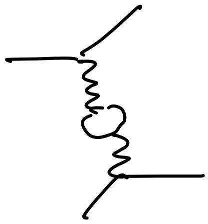
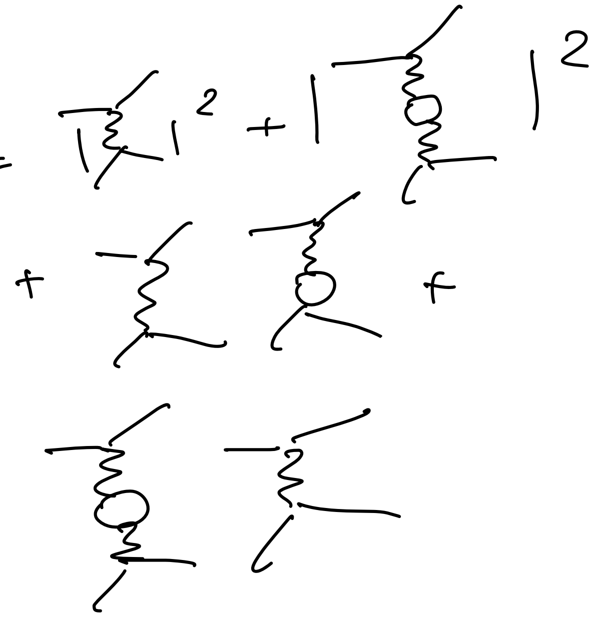


= ?



$a+b \to c+d+\gamma$

$\mathcal{M} =$  $\mathcal{M}^2 =$ 

$\mathcal{M} =$  $+$  $\mathcal{M}^2 =$ 

$$\mathcal{M} = \mathcal{M}(\alpha) + \mathcal{M}(\alpha^2) + \mathcal{M}(\alpha^3)$$

$f(x)$: parton density function.

$$x = \frac{|\underline{P}_{part}|}{|\underline{P}_{nucl}|}$$

$$\sum \underline{P}^{i}_{part} = \underline{P}_{nucl.}$$

$$\int_0^1 dx \Big[\underbrace{\sum f_i(x)}_{\text{partons.}} \Big] = 1.$$

$$\int_0^1 dx \Big[\sum_{\text{flavors}} q_f(x) + \bar{q}_f(x) + g(x) \Big] \underset{\sim}{x} = 1.$$

Momentum sum rule.

$$\int_0^1 dx \left[u^p(x) - \bar{u}^p(x) \right] = 2$$

$$\int_0^1 dx \left[d^p(x) - \bar{d}^p(x) \right] = 1$$

$u^p(x)$ = or $\neq$ $u^n(x)$ $\qquad$ $p = (uud)$

$n = (udd)$

proton density function of u quark in proton. $\qquad$ in neutron.

DGLAP equations. $\Rightarrow$ evolution of pdf

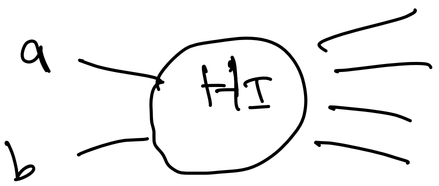
↳ Altarelli - Parisi

DIS $\Rightarrow$ partons (relations) quarks?

1950 – 1960 : $\quad p, n, \pi^{\pm}, K^{\pm}, \pi^0, K^0, \Delta$

Baryons: $q_1 q_2 q_3$

mesons: $q_1 \bar{q}_2$



#events. made of hadrons. leptons.

$$\#\text{events} \propto \sigma \propto |M|^2 \rho(E_f)$$

$$M = \langle f | H_I | i \rangle$$

To study $H_I \Rightarrow$ look for symmetries / selection rules.

symmetry: transformation acts on a state leaves the state unchanged

T: transformation. $\qquad \psi_i = \psi(q, \vec{x}, t)$

↳ charge, quantum #

$$\langle T\rangle = \langle\psi|T|\psi\rangle$$

$$|\psi'\rangle = T|\psi\rangle.$$

$$\langle\psi'| = \langle\psi|T^{+}$$

$$\langle\psi|T^{+}T|\psi\rangle = \langle\psi|\psi\rangle$$

$$\frac{d}{dt}\langle T\rangle = 0 \qquad [T, H] = 0$$

$[T, H] = 0. \Rightarrow$ $T H|\psi\rangle$

$H T|\psi\rangle$

$\psi(q, \vec{x}, t)$

symmetries: 1/ External symm: time reversal, parity, translation. rotation.

Continuous

translation $\vec{x}$

translation t

rotation $\vec{x}$

Discrete.

Time reversal. $T\psi(\vec{x}, t) = \psi(\vec{x}, -t)$

Parity $\vec{x} \to -\vec{x}$ $P\psi(\vec{x}, t) = \psi(\vec{x}, t)$

2/ Internal symmetries: act on internal quantum #.

NOT on $\vec{x}$ or t

Continuous

Isospin

Discrete

charge conjugation.

C: proton $\longrightarrow \bar{p}$

no change of momentum

no change of spin