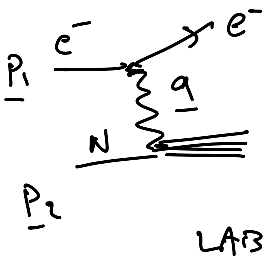
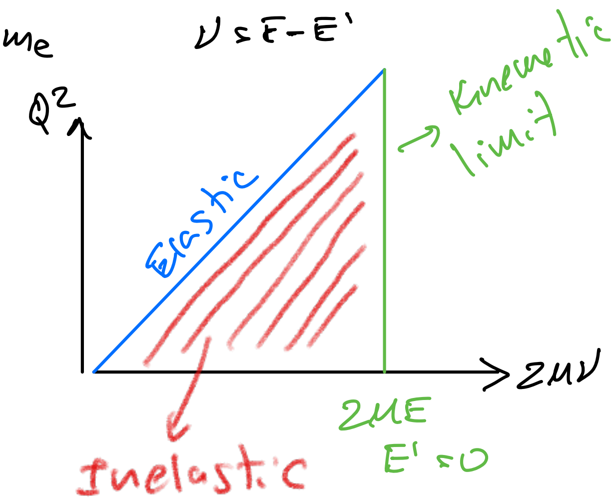


$P_3 = (E', \vec{p}')$ $\qquad$ $q_1 = P_1 - P_3$

$W^2 = |P_H|^2$ $\qquad$ $Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$

LAB frame: $P_2 = (M, 0)$ $\qquad$ $E_e \gg m_e$ $\qquad$ $\nu = E - E'$



$Q^2 = M^2 - W^2 + 2M\nu$

Elastic case: $W^2 \to M^2 \Rightarrow Q^2 = 2M\nu$

Experimentally: E, E', θ

$(E', \theta) \to (Q^2, \nu) \to (x, y)$

$x = \frac{Q^2}{2M\nu}$ Bjorken variable.

$y = \frac{\nu}{E} = \frac{E - E'}{E}$ $\qquad$ $0 \le E' \le E \Rightarrow 0 \le y \le 1$

$x = \frac{W^2 - M^2 + 2M\nu}{2M\nu} = 1 + \frac{M^2 - W^2}{2M\nu}$ $\qquad$ $W^2 \ge M^2$, $\nu \ge 0$

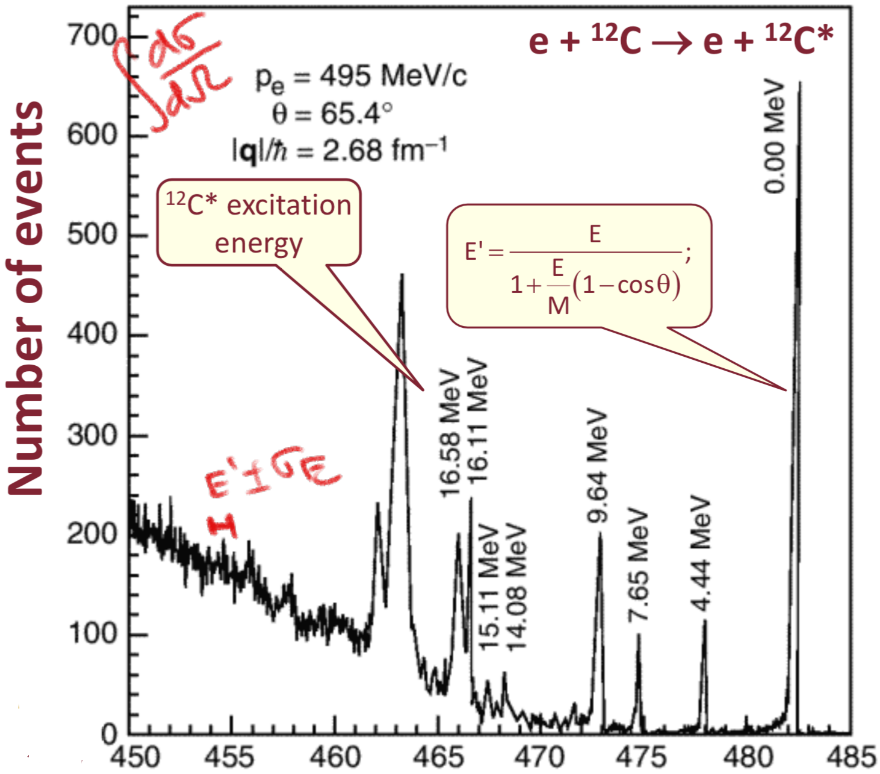
$\Rightarrow \cancel{0 \le} x \le 1$

Elastic: $Q^2 = 2M\nu \Rightarrow x = 1$

$t = q^2 = (P_1 - P_3)^2$

$S = (P_1 + P_2)^2 = M_e^2 + M^2 + 2\, P_1 \cdot P_2 = M_e^2 + M^2 + 2ME$

$= M^2 + 2ME$



Measure E' [MeV] energy of scattered electron.

electron beam:

$E_e = P_e = 495$ MeV

carbon target

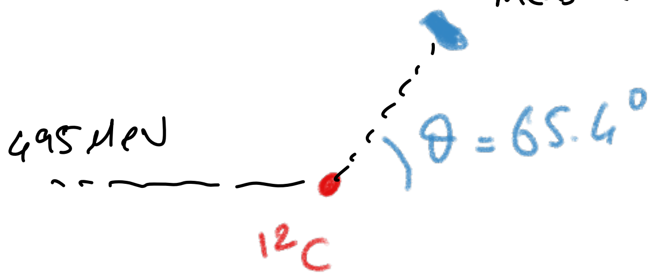
$e^- + {}^{12}C \to e^- + C^{12}$

$E' < E$

$e^- + {}^{12}C \to e^- + {}^{12}C^*$

excitation

$e^- + {}^{12}C \to e^- + X$



$\Gamma = \frac{1}{\tau}$ intrinsic width.

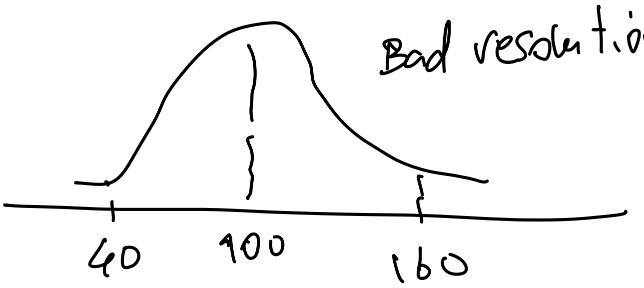
Different width:

Observed width: $BW(E') \otimes \text{Resolution}(E - E')$

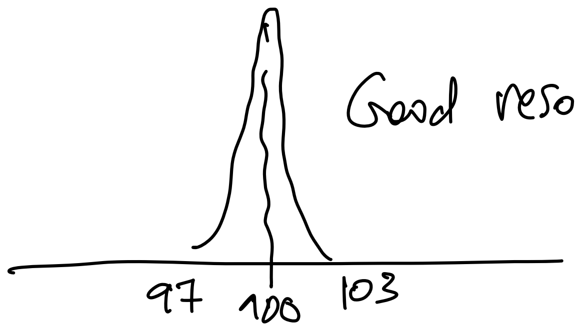
Good detector $\Rightarrow$ narrow Gaussian resolution.

stable particle, $BW \to \delta(E-M)$

$$\int dx' \, \delta(x-x') \, e^{-\frac{(x-x'-\mu)}{\sigma^2}} = e^{-\frac{(x-\mu)}{\sigma^2}}$$



Bad resolution



Good resolution.

$$E' = \frac{E}{1 + 2\frac{E}{M}\left(\sin^2\frac{\theta}{2}\right)}$$

$E = 495$ MeV

$\theta = 65.4°$

$M \simeq 12 m_p = 12$ GeV

$$E' = \frac{1}{1+0.03} E = (1-0.03)E \sim 480 \text{ MeV}$$

1960's @ SLAC: Stanford Linear Accelerator Center.

$e^- + p \to e^- + p$ $\quad p = uud$

$e^- + p^+ \to e^- + \Delta^+(1332)$ hadronic resonance uud

$\to e^- + \pi^+ + n$ $\quad \pi^+ = u\bar{d} \quad \pi^- : \bar{u}d$

$e^- + p + \pi^+ \pi^-$ $\quad \pi^0 : \frac{\bar{u}u + d\bar{d}}{\sqrt{2}}$

$e^- + \Delta(1332) + \pi^0$

$e^- + \Delta(1332)\, \pi^+ \pi^-$

Next step in $e^- + p$ scattering:

so far $\quad \left.\frac{d\sigma}{d\Omega}\right|_{Mott} \sim (\sim) \frac{\alpha^2}{q^4} E'^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$

no recoil

pointlike target AND no spin.

1) spin of target p, n $\quad s = 1/2$ particle

2) structure of target: not-pointlike.

3) recoil of target

$$\frac{|\vec{P}_{out}|}{|\vec{P}_{in}|}$$

$$\frac{d\sigma}{d\Omega} = (\sim) \frac{\alpha^2}{E^4 \sin^4\frac{\theta}{2}} \frac{E'}{E} \underbrace{E'^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)}_{\text{Relelivistic + Spin}}$$

θ°

$65.4^\circ = \theta_0 + \varepsilon$

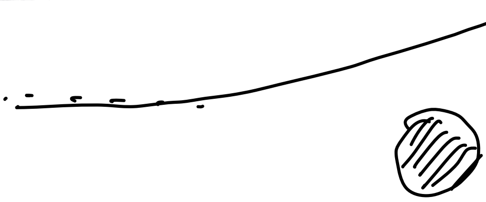
$$\int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} \frac{d\sigma}{d\Omega} d\Omega$$

$$\sigma = 2\pi \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} d\theta \sin\theta \frac{1}{E^4 \sin^4\frac{\theta}{2}} \frac{1}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}} E'^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

Total cross section @ $\theta_0 \pm \varepsilon$

$$\frac{d\sigma}{dE'} = \frac{\#\text{ events}}{\Delta E'}$$

i) structure of target.



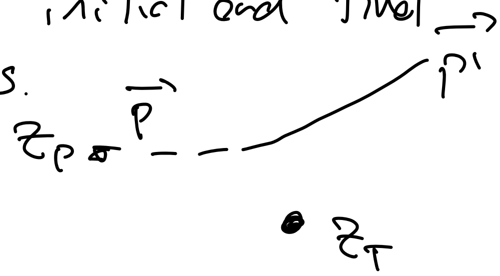
spatial extention.

$$\frac{d\sigma}{d\Omega} \propto |M|^2 \text{ (phase space)}$$

$$M = \langle f | H_I | i \rangle$$

pointlike : $H_I \doteq Z_p Z_T \dfrac{e^2}{r}$

Born approximation: planar wave for initial and final free particles.

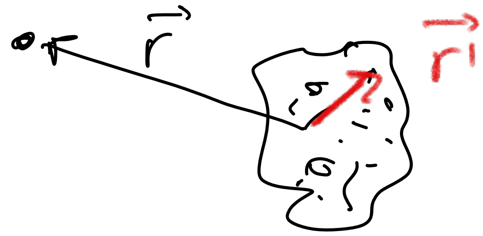


$$\psi_e = \frac{1}{\sqrt{V}} e^{i\vec{p}\cdot\vec{r}}$$

$$M \simeq \int d^3r \, e^{-i\vec{p}\cdot\vec{r}} e^{+i\vec{p}\,'\cdot r} \frac{1}{r} \qquad \vec{q} = \vec{p}\,' - \vec{p}$$

$$= \int d^3r \, \frac{e^{i\vec{q}\cdot r}}{r} \propto \frac{1}{q^2}$$

Extended body:



$\rho(\vec{r}\,')$: charge distribution of target.

$$V(\vec{r}) = \int d^3r' \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$$

pointlike: $\rho(\vec{r}\,') = Z_T e\, \delta(\vec{r}\,')$

Extended body: $\rho(\vec{r}\,') = Z_T e\, f(\vec{r}\,')$

$$M \sim Z_p Z_T e^2 \int d^3r \, e^{i\vec{q}\cdot\vec{r}} \underbrace{\int d^3r' \frac{f(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}}_{V(r)}$$

$$\mathcal{M} \sim \int d^3r \, \frac{e^{i\vec{q}(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|} \underbrace{\int d^3r' \, e^{i\vec{q}\cdot\vec{r}'} f(\vec{r}')}_{=:\ F(q^2)\ \text{Form Factor}}$$

Form factor: Fourier transform of spatial charge distribution.

$$\mathcal{M} \sim \frac{Z_p Z_T e^2}{q^2} F(q^2)$$

$$\Rightarrow \frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 \sim \left(\frac{Z_p Z_T e^2}{q^2}\right)^2 |F(q^2)|^2$$

$$\underbrace{\left.\frac{d\sigma}{d\Omega}\right|}_{e^- \text{ on extended target}} = \underbrace{\left.\frac{d\sigma}{d\Omega}\right|}_{\text{pointlike}} \times |F(q^2)|^2$$

How to measure $F(q^2)$?

Remember $q^2 = -4EE'\sin^2\frac{\theta}{2}$

not practical to scan all q^2

Most accessible values: $q^2 \to 0$

Expand $F(q^2)$ near $q^2 \simeq 0$.

$$F(q^2) = \int d^3r \, e^{+i\vec{q}\vec{r}} f(\vec{r})$$

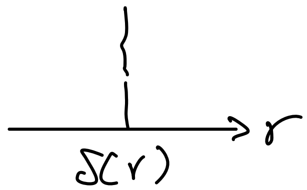
Typically $f(\vec{r}) = f(r) \Rightarrow \int d\varphi \sin\theta d\theta dr \, r^2 e^{i\vec{q}\cdot\vec{r}} f(r)$

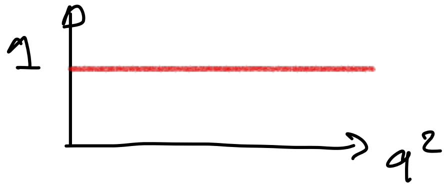
$$\Rightarrow F(q^2) = \frac{4\pi}{A} \int_0^\infty dr \, r^2 \frac{\sin(qr)}{qr} f(r).$$

$$A = \int_0^\infty d^3r \, f(r) = 4\pi \int_0^\infty dr \, r^2 f(r)$$

Examples of $f(r)$:

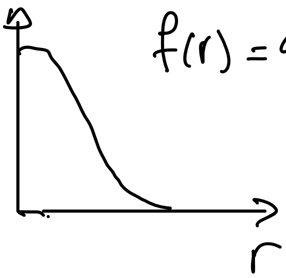
1) point like $\quad f(r) = e \frac{1}{4\pi} \delta(r) \qquad F(q^2) = 1$



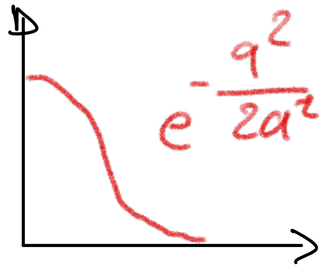


2) Gaussian charge distribution

$$f(r) = a^2 e^{-a^2 \frac{r^2}{2}}$$

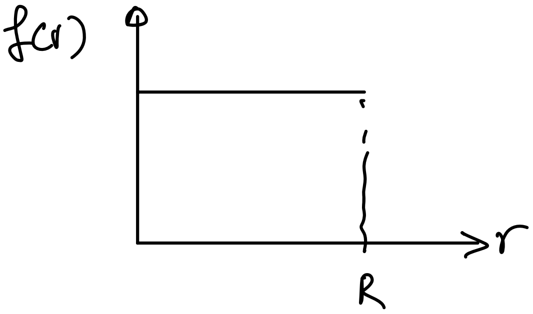


$$e^{-\frac{q^2}{2a^2}}$$

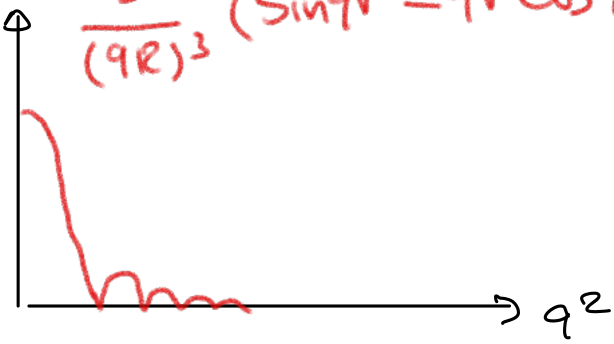


3) Homogenous target

$$f(r) = \frac{1}{\frac{4\pi}{3} R^3} \qquad r \le R$$



$$F(q^2) \qquad \frac{3}{(qR)^3} (\sin qr - qr \cos qr)$$



Measure # events at q^2 and $q^2 \simeq 0 \Rightarrow$ Expand $F(q^2)$ @ $q^2 = 0$.

$$F(q^2) = (\text{Norm}) \int_0^\infty \int_{-1}^{1} \int_0^{2\pi} \underbrace{e^{iqr\cos\theta}}_{1 + iqr\cos\theta - \frac{1}{2}(qr)^2 \cos^2\theta + O(qr)^3} f(r)\, r^2 dr\, d\cos\theta\, d\varphi$$

$$= 1 - \frac{1}{6} q^2 \langle r^2 \rangle \qquad \langle r^2 \rangle = \int_0^\infty f(r)\, r^2\, r^2 dr$$