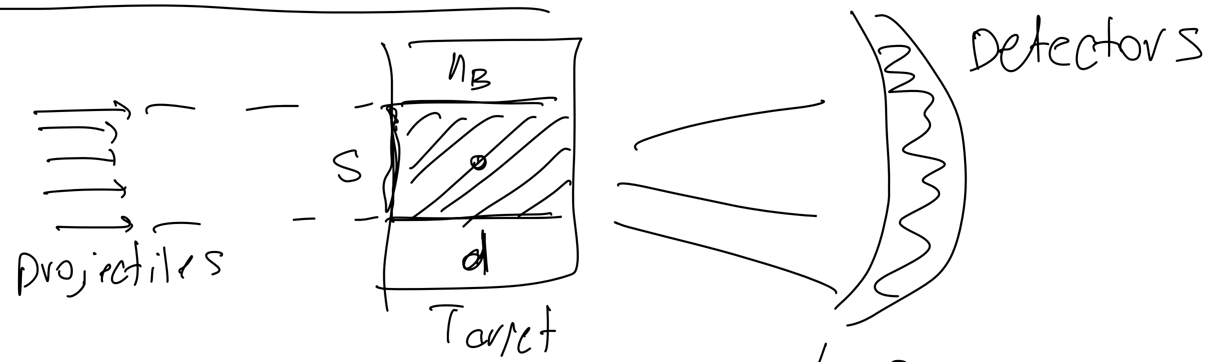
Collision, cross section



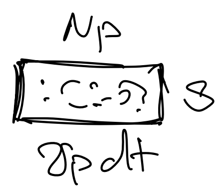
n_B: numerical density = # targets/ cm^3

$$\frac{dN_r}{dt} = \sigma\, n_B \cdot d\, \frac{dN_p}{dt}$$

T^{-1} $\qquad L^{-3}\, L\, T^{-1}$ $\quad \Rightarrow [\sigma] = L^2$

cross section.

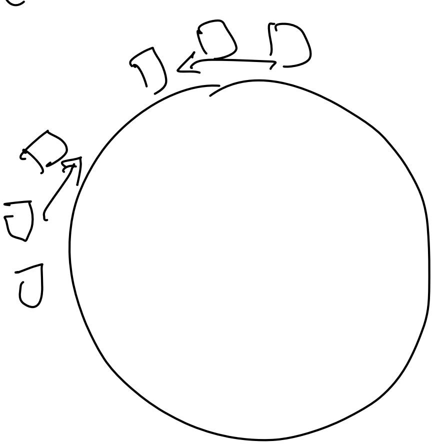
$$= \sigma\, n_B \cdot d\, S\, \frac{dN_p}{dt}\,\frac{1}{S} = \sigma\, N_B\, \Phi_p$$



$n_p \cdot v_p = \Phi_p$

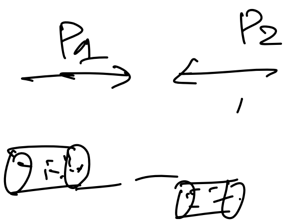
$$\frac{1}{N_B}\frac{dN_r}{dt} = \sigma\, \cancel{A_T}\, \cancel{N_B}\, \Phi_p$$

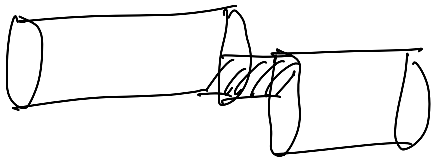
$\Gamma(i \to f)$



Colliding Beams

$$\frac{dN_r}{dt} = \sigma\, N_B\, \Phi_p$$





 S

$$\phi_p = \frac{N_p}{S} f_p$$

$\mathcal{L}$ Instantaneous luminosity

$$\frac{dN_r}{dt} = \sigma \underbrace{N_B \frac{N_p}{S} f_p}_{\mathcal{L}} = \sigma \frac{N_B N_p}{S} f_p$$

$$\left[\frac{dN_r}{dt}\right] = T^{-1}$$

$$[\sigma] = L^2$$

$$[\mathcal{L}] = L^{-2} T^{-1}$$

$$\sigma \propto \Gamma(i \to f) \propto |\mathcal{M}|^2 \qquad \mathcal{M} = \langle f | H_I | i \rangle$$

Rate of events $\sigma \cdot \mathcal{L}$

$\mathcal{L}: cm^{-2}\, t^{-1}$

Total # events $= \sigma \cdot \underline{\mathcal{L}}\ \underline{\Delta t}$

$= \sigma \cdot L$

$1b = 10^{-24}\, cm^2$

$= 10^{-28}\, m^2$

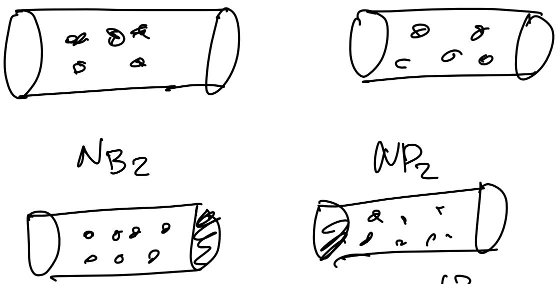
$\mathcal{L}: b^{-1}\, Hz$

Integrated luminosity.

$$L = \int \mathcal{L}\, dt$$

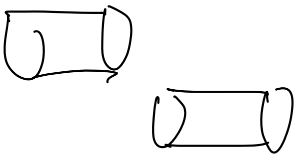
N_{B_1} N_{P_1}

N_{B_2} N_{P_2}



10^2 protons

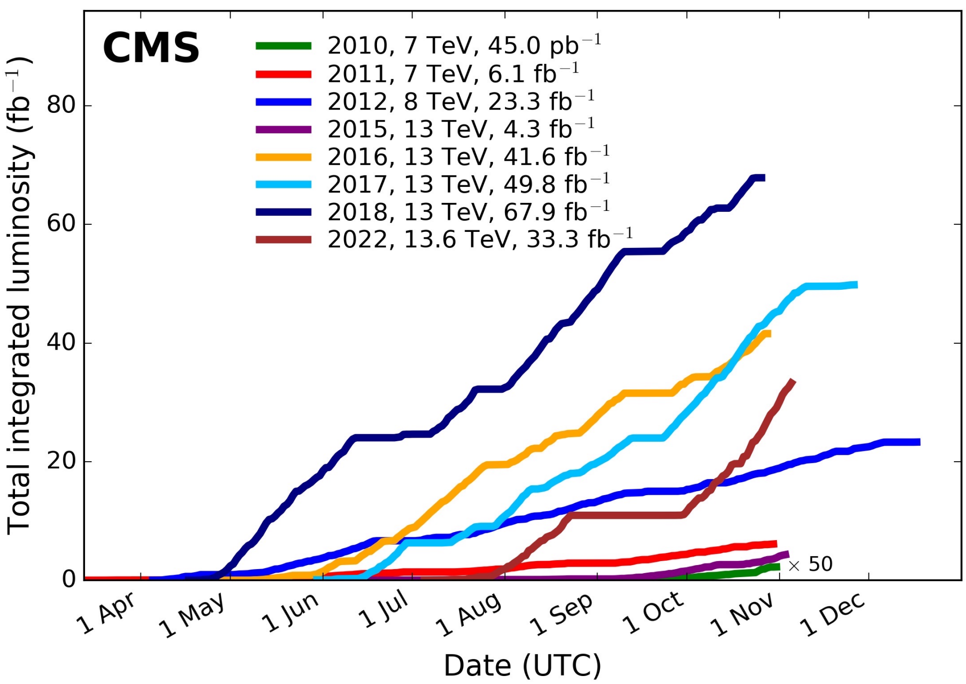
$\mathcal{L}_1 .$ $\quad \dfrac{N_B N_P}{S} f_P$



$$L = \sigma \times \Delta t = \int \mathcal{L}\, dt$$

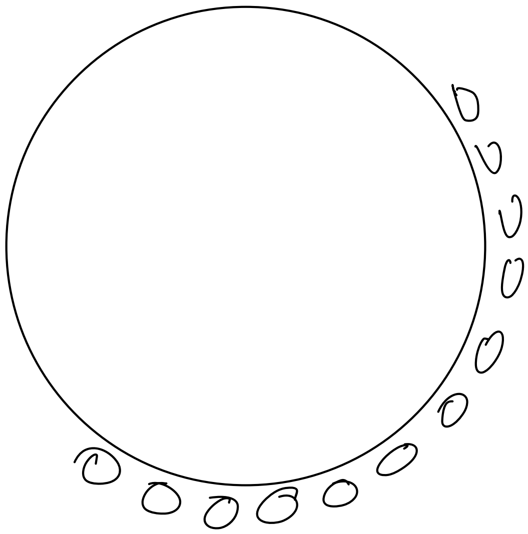
$$\mathcal{L}_1 \Delta t_1 + \mathcal{L}_2 \Delta t_2 = L$$

$$\mathcal{L}_1 \simeq 10^{-3} \mathcal{L}_2$$



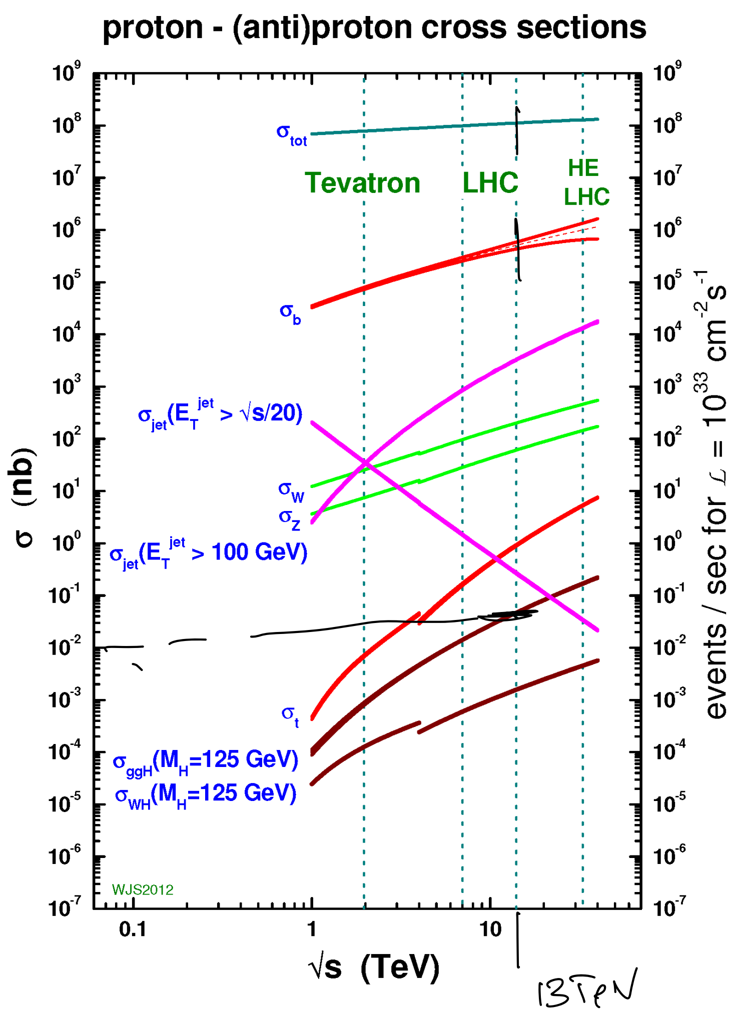
$L_{2022} > L_{2012}.$

$\Delta t_{2022} < \Delta t_{2012} \Rightarrow \mathcal{L}^{inst}_{2022} \gg \mathcal{L}^{int}_{2012}$

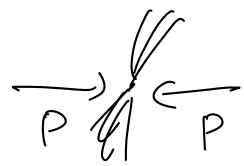


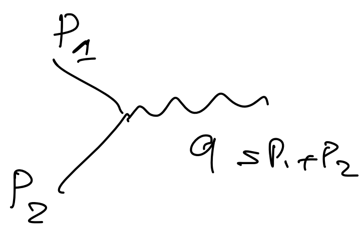
$$N_p = \#\text{bunch} \times \frac{\#\text{proton}}{\text{bunch}}$$

$$\underline{p} = 0.3\, \underline{B(T)}\, R[m]$$



$\sigma(p+p \to X)$





$|M|$

$\sigma(p+p \to H+X) \qquad \sigma \simeq 10^{-2}\,\text{nb}.$

$$N(\text{produced}) = \sigma \times L = N_H$$

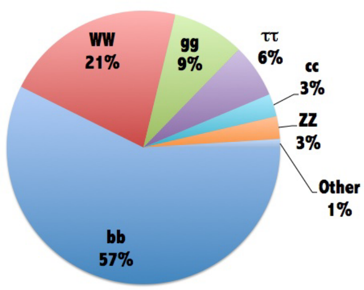
$$p + p \rightarrow H + X$$
$$\hookrightarrow Z^0 \; Z^0$$
$$\hookrightarrow \mu^+\mu^-$$
$$\hookrightarrow \mu^+\mu^-$$

$$p + p \rightarrow [\qquad] \rightarrow \mu^+\mu^- \; \mu^+\mu^-$$
$$|i\rangle \qquad\qquad\qquad |f\rangle$$

$$N(H \rightarrow 4\mu) = \sigma \times L \times BF(H \rightarrow ZZ) \times BF(Z \rightarrow \mu\mu) \times BF(Z \rightarrow \mu\mu)$$

$$BF(H \rightarrow ZZ) = \frac{\Gamma(H \rightarrow ZZ)}{\Gamma(H \rightarrow \text{anything})}$$

Higgs decays at m_H=125GeV



$$BF(H \rightarrow ZZ) = 3\%$$

$$Z \rightarrow \mu\mu \simeq 3\%$$

Z DECAY MODES

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	e^+e^-		(3.363 ±0.004) %	
Γ_2	$\mu^+\mu^-$		(3.366 ±0.007) %	
Γ_3	$\tau^+\tau^-$		(3.370 ±0.008) %	
Γ_4	$\ell^+\ell^-$	[a]	(3.3658±0.0023) %	
Γ_5	invisible		(20.00 ±0.06) %	
Γ_6	hadrons		(69.91 ±0.06) %	
Γ_7	$(u\overline{u}+c\overline{c})/2$		(11.6 ±0.6) %	
Γ_8	$(d\overline{d}+s\overline{s}+b\overline{b})/3$		(15.6 ±0.4) %	
Γ_9	$c\overline{c}$		(12.03 ±0.21) %	
Γ_{10}	$b\overline{b}$		(15.12 ±0.05) %	
Γ_{11}	$b\overline{b}b\overline{b}$		(3.6 ±1.3) $\times 10^{-4}$	
Γ_{12}	ggg		< 1.1 %	CL=95%
Γ_{13}	$\pi^0\gamma$		< 5.2 $\times 10^{-5}$	CL=95%
Γ_{14}	$\eta\gamma$		< 5.1 $\times 10^{-5}$	CL=95%
Γ_{15}	$\omega\gamma$		< 6.5 $\times 10^{-4}$	CL=95%
Γ_{16}	$\eta'(958)\gamma$		< 4.2 $\times 10^{-5}$	CL=95%
Γ_{17}	$\gamma\gamma$		< 5.2 $\times 10^{-5}$	CL=95%
Γ_{18}	$\gamma\gamma\gamma$		< 1.0 $\times 10^{-5}$	CL=95%
Γ_{19}	$\pi^{\pm}W^{\mp}$	[b]	< 7 $\times 10^{-5}$	CL=95%

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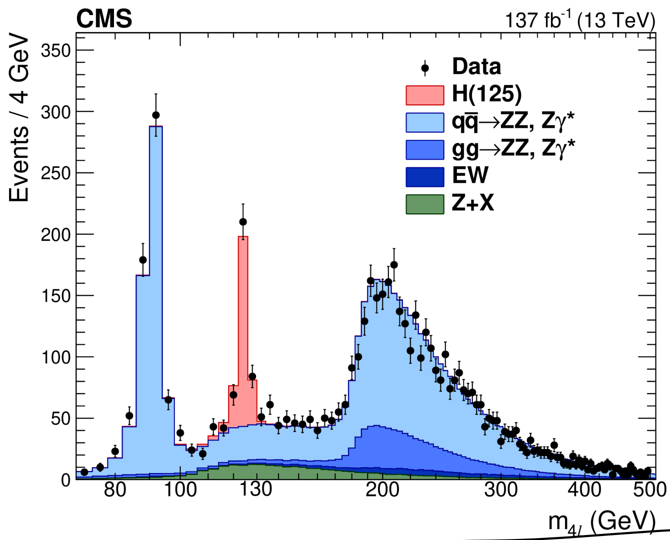
$$N_H \times (3 \times 10^{-2})^3$$
$$\simeq N_H \times 10 \times 10^{-6}$$
$$= N_H \times 10^{-5}$$

No background.

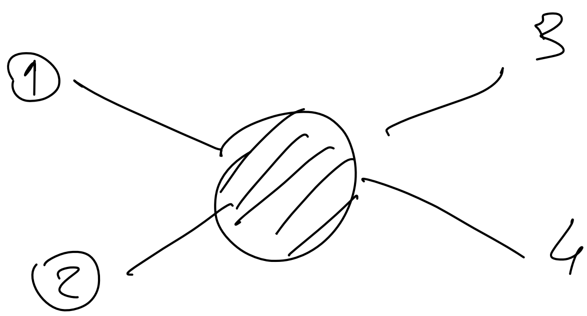
10^5 H $\longrightarrow$ 1 $H \to 4\mu$.

10^6 H $\longrightarrow$ 10 $H \to 4\mu$. 3σ evidence w/ no background.



$$\sqrt{(p_{\mu_1} + p_{\mu_2} + p_{\mu_3} + p_{\mu_4})^2} = m_{inv}(4\ell)$$

$\#$ events $\propto \sigma \cdot L$



$1 + 2 \to 3 + 4 + \dots + n$.

Two-body scattering: $1 + 2 \to 3 + 4$.

$\gamma + e^- \to \gamma + e^-$

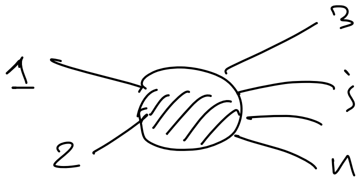
$\alpha + N \to \alpha + N$.

$e^- + p \to e^- + p$

$\pi^- + p \to \pi^0 + n$.

Golden Rule for Scattering

$1+2 \to 3+4+\cdots+n$



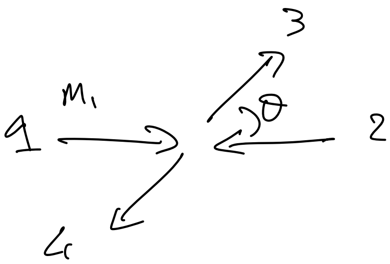
$$\sigma = \frac{S}{4\sqrt{(\underline{p}_1\cdot\underline{p}_2)^2-(m_1 m_2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(\underline{p}_1+\underline{p}_2-\underline{p}_3-\underline{p}_4-\cdots-\underline{p}_n) \times$$

$$\times \prod_{j=3}^{n} \frac{1}{\underbrace{2\sqrt{\vec{p}_j^{\,2}+m_j^2}}_{E_j}} \frac{d^3\vec{p}_j}{(2\pi)^3}$$

$1+2 \to a+a+b+b+b.$

$$S = \frac{1}{N_a!}\,\frac{1}{N_b!} = \frac{1}{2!}\,\frac{1}{3!}$$

$$\overset{m_1}{1} + \overset{m_2}{2} \longrightarrow \overset{m_3}{3} + \overset{m_4}{4}. \qquad S=1$$



Center of Mass

$\vec{p}_1+\vec{p}_2=0 \qquad |\vec{p}_{in}|$

$\vec{p}_3+\vec{p}_4=0. \qquad |\vec{p}_{out}|$

θ

$$(\underline{p}_1\cdot\underline{p}_2)^2-(m_1 m_2)^2 =$$

$$\underline{p}_1=(E_1, \vec{p}_{in}) \qquad \underline{p}_2=(E_2, -\vec{p}_{in})$$

$$\underline{p}_3=(E_3, \vec{p}_{out}) \qquad \underline{p}_4=(E_4, -\vec{p}_{out})$$

$$\to |\vec{p}_{in}|^2 (E_1+E_2)^2$$

$$\sigma = \frac{1}{4|\vec{p}_{in}|(E_1+E_2)} \frac{(2\pi)^4}{(2\pi)^6}\frac{1}{4}\int |\mathcal{M}|^2 \frac{\delta^4(\underline{p}_1+\underline{p}_2-\underline{p}_3-\underline{p}_4)}{\sqrt{\vec{p}_3^{\,2}+m_3^2}\sqrt{\vec{p}_4^{\,2}+m_4^2}}\, d^3\vec{p}_3\, \cancel{d^3\vec{p}_4}$$

$$\delta^4(\cdots) = \delta(E_1+E_2-E_3-E_4)\, \delta^3(\underbrace{\vec{p}_1+\vec{p}_2}_{0}-\vec{p}_3-\vec{p}_4)$$

$$\cancel{\delta(\vec{p}_3+\vec{p}_4)}$$

$$d^3 P_2 = d^3 \vec{P}_{out} = P_{out}^2 \, dP_{out} \, \underline{d\Omega}.$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{16(2\pi)^2} \frac{1}{|\vec{P}_{in}|(E_1+E_2)} \int |M|^2 \underbrace{\frac{\delta(E_1+E_2-E_3-E_4)}{\sqrt{P_{out}^2+m_3^2}\sqrt{P_{out}^2+m_4^2}} P_{out}^2 \, dP_{out}}_{\frac{|\vec{P}_{out}|}{(E_1+E_2)}}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{(E_1+E_2)^2} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} |M|^2$$

LAB Frame

$\underline{P}_1 = (E_1, \vec{P}_{in})$

$\underline{P}_2 = (m_2, \vec{0})$

$$(\underline{P}_1 \cdot \underline{P}_2)^2 - (m_1 m_2)^2$$
$$(E_1 m_2)^2 - (m_1 m_2)^2$$
$$= (E_1^2 - m_1^2)\, m_2^2$$
$$= |\vec{P}_{in}|^2 m_2^2$$

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{(E_1+E_2)^2} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} |M|^2 \quad \text{Center of Mass.}$$

$$\frac{1}{(m_2)^2} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} |M|^2 \quad \text{LAB Frame.}$$

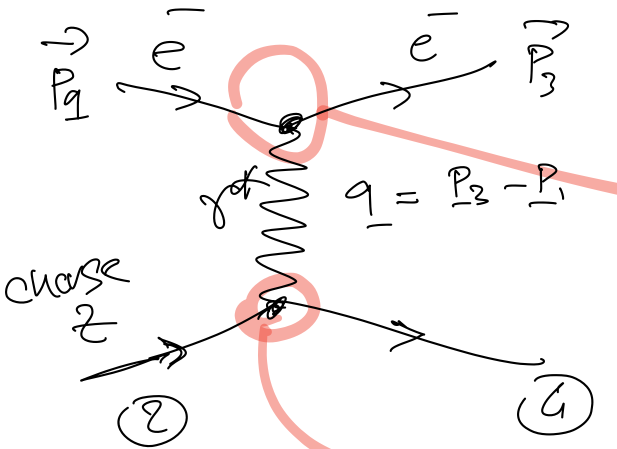
Mott Formula. $1+2 \to 3+4.$

$m_2 >> m_1$.

$e^- + p \to e^- + p$.

$\alpha + N \to \alpha + N$.

$\gamma + e^- \to \gamma + e^-$



$\mathcal{M} \propto$

$(\bar{u})\,\gamma^\mu\sqrt{\alpha}\,(u)\ \frac{-ig_{\mu\nu}}{q^2}$

$(-)\,\gamma^\mu\sqrt{Z\alpha}\,(-)$

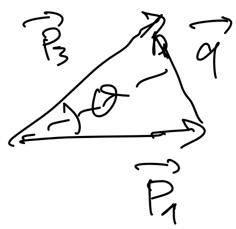
$$|\mathcal{M}|^2 \propto \frac{1}{q^4}$$

$$q = p_3 - p_1 = (E_3 - E_1, \vec{p}_3 - \vec{p}_1)$$

If $m_1 << m_2$ $\quad \vec{p}_4 \simeq 0$ $\quad$ Recoil is negligible

$\vec{p}_2 = 0$ target at rest. $\Rightarrow |\vec{p}_1| \simeq |\vec{p}_3|$

$\vec{p}_4 \simeq 0$ because small recoil. $\quad |\vec{p}_{in}| \simeq |\vec{p}_{out}|$



$$|\vec{q}| = 2|\vec{p}_{in}|\sin\frac{\theta}{2}.$$

$$q^4 = 16\,|p_{in}|^4 \sin^4\frac{\theta}{2}$$

Mott Formula

$$|\mathcal{M}| = \left(\frac{\alpha m_2}{|\vec{p}_{in}|^2 \sin^2\frac{\theta}{2}}\right)^2 \left[m_1^2 + |\vec{p}_{in}|^2 \cos^2\frac{\theta}{2}\right]$$

$m_1 << m_2$.

$m_1 >> |\vec{p}_{in}|$

Rutherford.

$m_1 = \alpha$
$m_2 = M$ nucleus

$m_1 \simeq 3.7$ GeV. $({}^4_2He)$

$|\vec{p}_{in}| \simeq 5-7$ MeV

α decay.

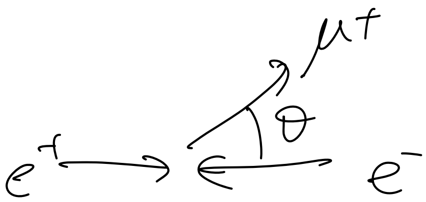
m_2: nucleus mass $\simeq A m_p$.

Rutherford. $\frac{d\sigma}{d\Omega} \propto \cancel{\frac{|\vec{p}_{out}|}{|\vec{p}_{in}|}} \frac{\alpha^2}{|\vec{p}_{in}|^4 \sin^4\frac{\theta}{2}}$.

Mott cross section Formula

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{\cancel{M}^2}\left(\frac{\alpha \cancel{M}}{p_{in}^2 \sin^2\frac{\theta}{2}}\right)^2 \left[m^2 + p_{in}^2 \cos\frac{\theta}{2}\right]$$

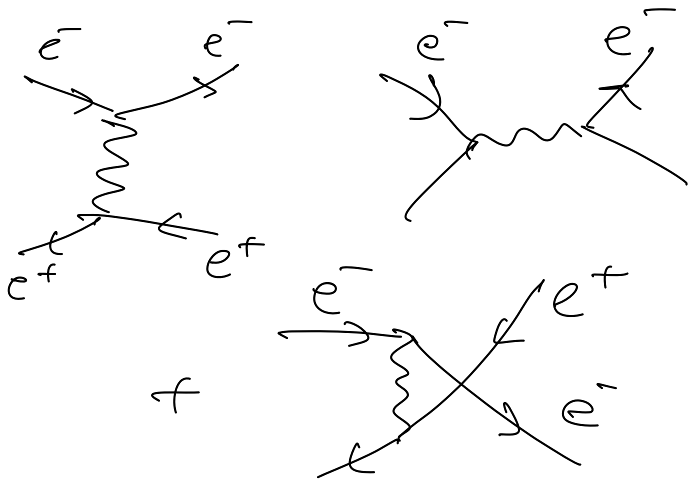
$e^+ + e^- \longrightarrow \mu^+ + \mu^-$

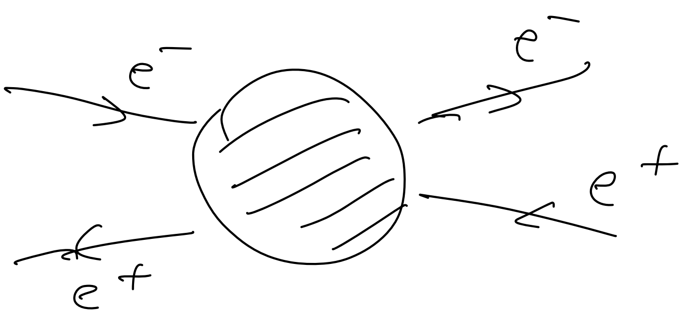


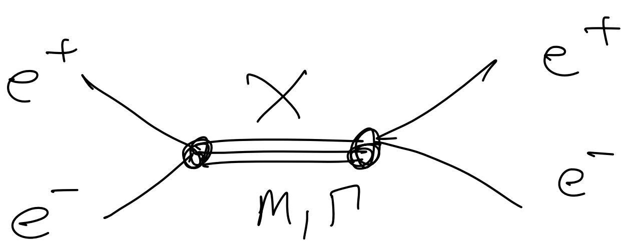
$p+p \longrightarrow - -$

$e^+e^- \longrightarrow \mu^+\mu^-$

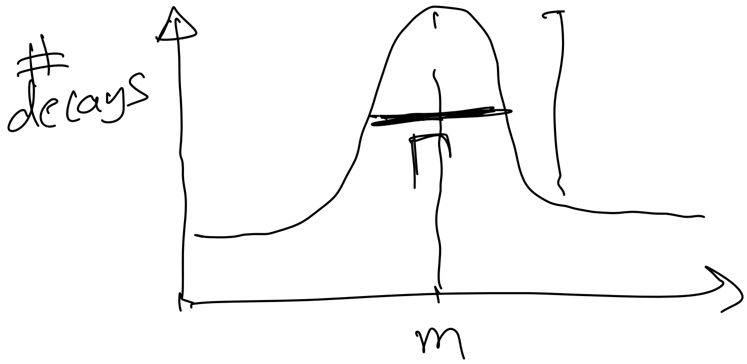
$e^+e^- \longrightarrow e^+e^-$







unstable particle X (m, Γ). $X \to e^+e^-$

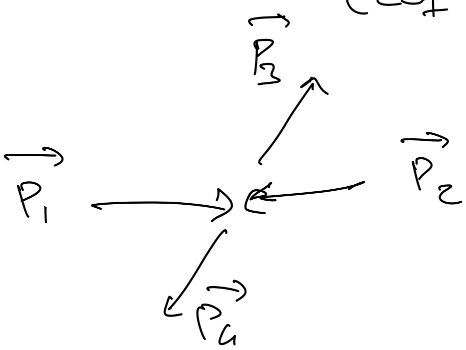


Rest frame of X

Breit-Wigner Distribution

X: mass m spin J
width Γ

$$\sigma(e^+e^- \to X) = \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{4\pi}{|\vec{P}_{in}|^2} \frac{\Gamma^2/4}{(E-E_0)^2 + \frac{\Gamma^2}{4}} \frac{\Gamma_{in}}{\Gamma_{tot}} \frac{\Gamma_{out}}{\Gamma_{tot}}$$

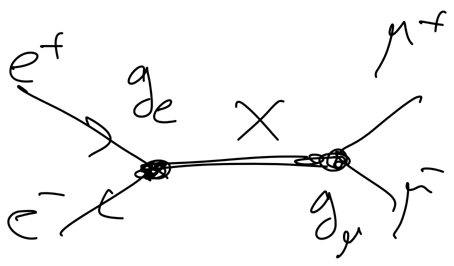


$$|\vec{P}_1| = |\vec{P}_2| = |\vec{P}_{in}|$$

$$e^+e^- \to \mu^+\mu^-$$

$$\Gamma_{in} = \frac{\Gamma_{X \to e^+e^-}}{\Gamma_{tot}}$$

$$\Gamma_{out} = \frac{\Gamma_{X \to \mu^+\mu^-}}{\Gamma_{tot}}$$



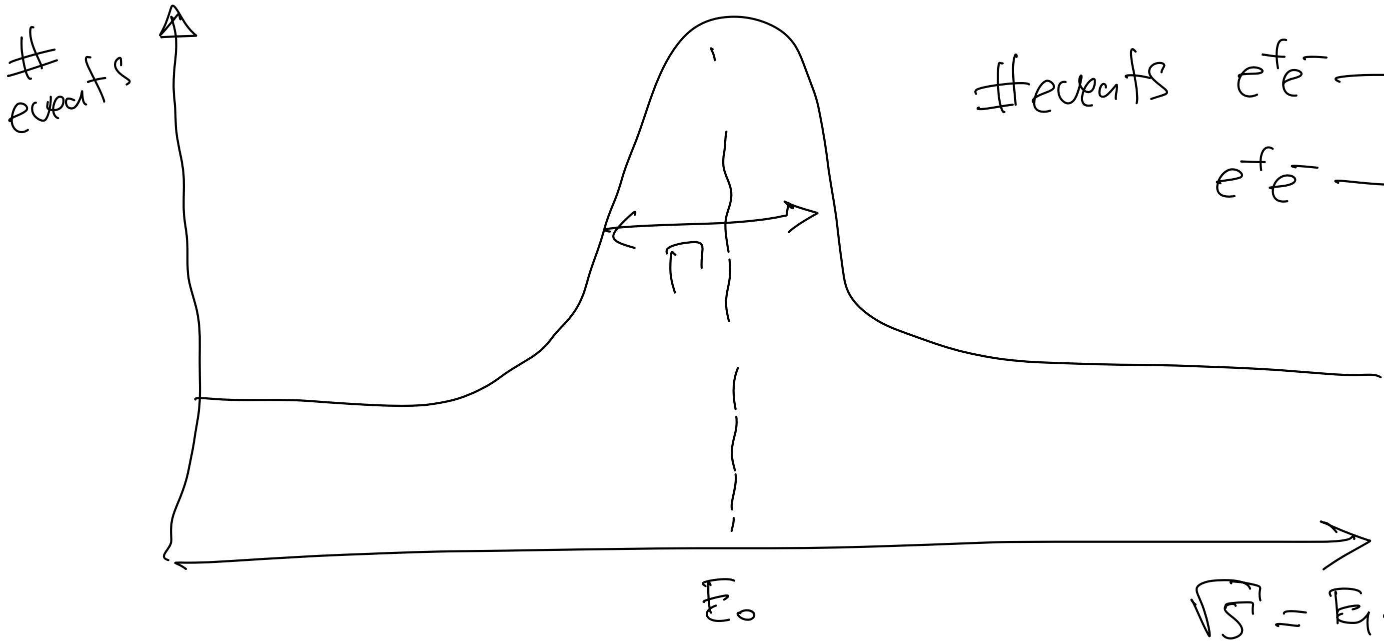
$X \to e^+e^-$

$X \to \mu^+\mu^-$ ✓

$\# \mu^+\mu^- > 0$

$m_X \geq 2m_e$

$2m_\mu$.



#events $e^+e^- \to \mu^+\mu^-$

$e^+e^- \to$ anything.

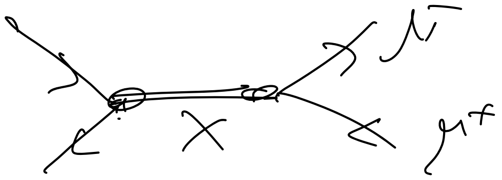
$\underline{P}_1 \longrightarrow \quad \longleftarrow \underline{P}_2$

$$\sqrt{s} = \sqrt{(\underline{P}_1 + \underline{P}_2)^2} =$$

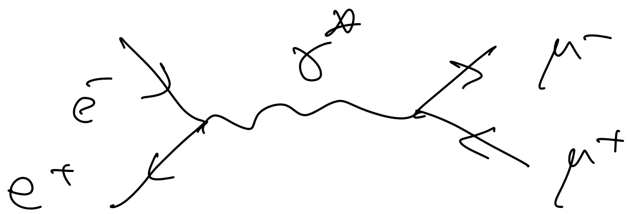
$$= \sqrt{(E_1+E_2)^2 - \underbrace{(\vec{P}_1+\vec{P}_2)^2}_{\sim 0}}$$

$$= E_1 + E_2$$

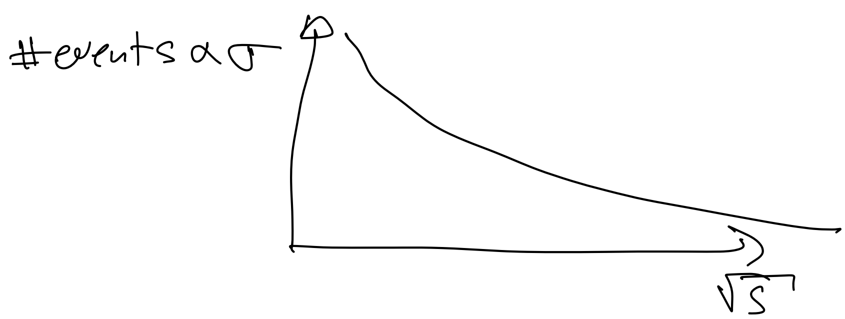
$e^+e^- \longrightarrow X \longrightarrow \mu^+\mu^-$

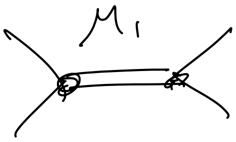


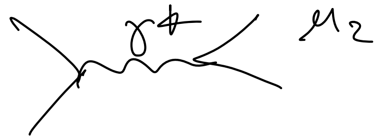
$e^+e^- \longrightarrow \mu^+\mu^-$ without X

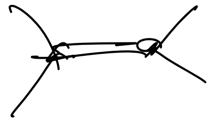
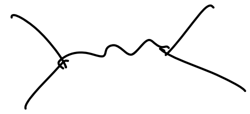


$$\mathcal{M} \propto \frac{1}{q^2} = \frac{1}{s}$$



$e^+e^- \to \mu^+\mu^- =$ 

$+$ 

$\sigma = \Big|$  $\Big|^2 + \Big|$  $\Big|^2$

$+ \; \mathcal{M}_1 \mathcal{M}_2^*$