$B = q_1 q_2 q_3$ $\qquad \psi_B = \psi_{space}\, \psi_{spin}\, \psi_{flav}\, \psi_{color}$

color singlet $\Rightarrow \psi_{col} = A \quad \mathbb{1}_c$

ground state $\Rightarrow L=0 \Rightarrow \psi_{space} = L_{00} \Rightarrow$ symm.

$\psi_{Bary} = \psi_{Asym} \Rightarrow$ look at $\psi_{spin}\, \psi_{flav}$.

Flavor $3_F \times 3_F \times 3_F = \overset{A}{\mathbb{1}_F} \oplus 8_F^{12} + 8_F^{23} + 10_F^S$

Spin: $S = 3/2 \Rightarrow \psi_{spin}^{S=3/2}$ symm. $\uparrow\uparrow\uparrow$

Decuplet $\quad \psi_F = \psi_S$
$S=3/2 \quad\ \ \psi_S = \psi_S$

$$\Rightarrow \psi^{S=3/2}_{\substack{Baryon\\ decup.}} = \underset{S}{\psi_{space}}\, \underset{S}{\psi_{spin}}\, \underset{S}{\psi_F}\, \underset{A}{\psi_c}$$

$S=3/2$
$\mathbb{1}_F$ Cannot exist

10_F^S with $S=1/2$. $\quad \uparrow\downarrow\uparrow \quad \uparrow\uparrow\downarrow \ldots$

$$\psi^{S=1/2}_{\substack{Baryon\\ 10}} = \underset{S}{\psi_{space}}\, \underset{?}{\psi_{spin}^{S=1/2}}\, \underset{S}{\psi_F}\, \underset{A}{\psi_c}$$

must be symmetric

must be asymmetric because $S=1/2$

$\Downarrow$

NO decuplet with $S = 1/2$

$u_\uparrow u_\uparrow u_\downarrow + u_\uparrow u_\downarrow u_\uparrow$

From $3 \times 8 \times 3 = 27$ states.

10 exist decuplet with $S=3/2$.
10 does not exist $S=1/2$
1 with $S=3/2$ Cannot exist.

$3_F \times 3_F \times 3_F = 1 + 10 + 8_F^{12} + 8_F^{23}$

Spin $= \frac{3}{2}$ or $1/2$.

$$\psi_{\text{Baryon}\atop 8} = \psi_{\text{Asymm}} = \underset{S}{\psi_{\text{space}}}\,\underset{A}{\psi_{\text{color}}}\,\underbrace{\psi_{\text{Flav}}\,\psi_{\text{spin}}}_{\text{Symmetric.}}$$

$S = \frac{3}{2}$ and Flavor $= 8_F^{12}$ or 8_F^{23}.

$\psi_{\text{spin}}^{3/2} = \psi_{\text{symm.}}$ $\Rightarrow$ No octet with $S = 3/2$.

$\Rightarrow$ only possibility to have octet with $S = 1/2$ to satisfy overall wave function asymm.

$$\psi_{\text{spin}}\,\psi_{\text{Flavor}} = \psi_{F,12}^{S=1/2} + \psi_{F,23}^{S=1/2} + \psi_{F,13}^{S=1/2}.$$

From Clebsch-Gordon coeff: $M_{12} + M_{23} \equiv M_{13}$

if $S = 1/2$ and $1 \leftrightarrow 2$ under spin. $\times\ \psi_{F,12}$.

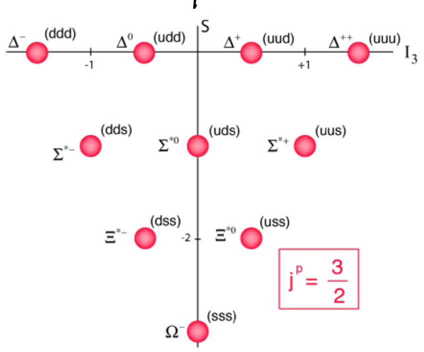
$S = 1/2$ and $2 \leftrightarrow 3$ under spin $\times\ \psi_{F,23}$.

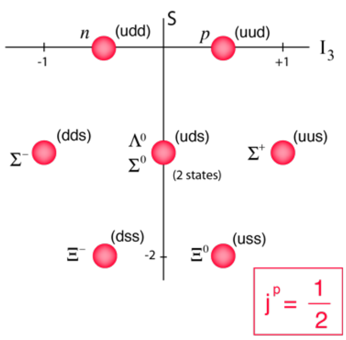
$$\Rightarrow \quad \psi_{M12}^{S=1/2}\,\psi_{F,12} + \psi_{M23}^{S=1/2}\,\psi_{F,23} = \underbrace{\psi_{\text{spin}}\,\psi_{\text{Flav.}}}_{\text{symm.}}$$

$\Rightarrow$ total of 8 states.

$\left.\begin{array}{l}\text{10 states with } S = 3/2. \\ \text{8 states with } S = 1/2.\end{array}\right\}$ physical states observed.

$\Rightarrow$ supports color quantum number.





Mesons:

$M = q_i \bar{q}_j$ $\qquad 3_F \times \bar{3}_F = 1_F + 8_F$

$$3_C \times \bar{3}_C = 1_C + 8_C$$

wave function for mesons.

$$\psi_{Meson} = \psi_{space} \underbrace{\psi_{spin}\,\psi_{flav}}\,\psi_{color}$$

$L=0.$ $\qquad$ Color singlet.

S.

$q_i \leftrightarrow \bar{q}_j \neq 1 \leftrightarrow 2$

↳ not identical particles $\Rightarrow$ no symm under exchange.

3 colors R, G, B for quarks. $\qquad$ 3 anticolors $\bar{R}, \bar{B}, \bar{G}$ for antiquarks.

$R + \bar{R} = 0.$
$B + \bar{B} = 0.$
$G + \bar{G} = 0$
$R + B + G = 0$

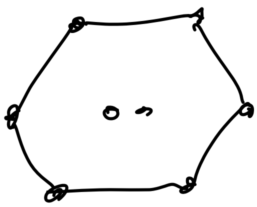
$$3_C \times \bar{3}_C = 1_C + 8_C$$

$$1_C = \frac{1}{\sqrt{3}}\left(R\bar{R} + G\bar{G} + B\bar{B}\right)$$

8_C: $R\bar{B}, R\bar{G}, B\bar{R}, B\bar{G}, G\bar{R}, G\bar{B},$

$\frac{1}{\sqrt{2}}\left(R\bar{R} - B\bar{B}\right),$

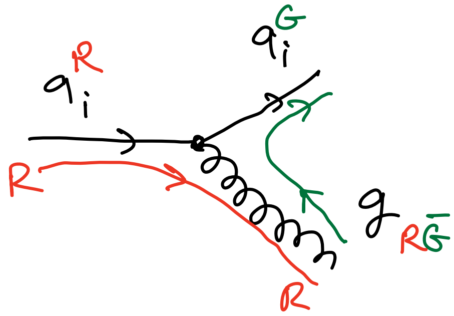
$\frac{1}{\sqrt{6}}\left(R\bar{R} + G\bar{G} - 2B\bar{B}\right)$

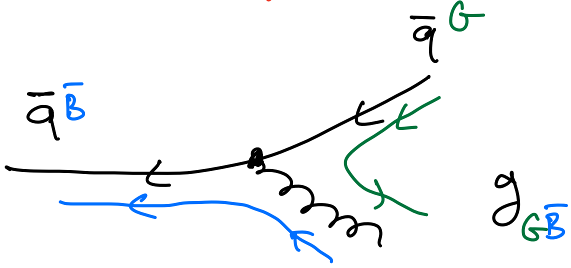


Hypothesis: 8 gluons (colored) and NO colorless gluon.

i = flavor u, d, s, c, b –

gluons do not change FLAVOR of quarks.





Color must be conserved in all interactions

$p + p \longrightarrow p + p$

$p + p + p + \bar{p}$

$p + p + \pi^+ + \pi^-$

$p + p + K^+ + K^-$

C $0 + 0$ $\quad 0 \quad 0 \quad 0 \quad 0$

leptons and photon have no color

$\Uparrow$
$\Downarrow$

No strong (QCD) interaction

$e^+ e^- \longrightarrow \pi^+ \pi^-$

$0 \quad 0 \qquad 0 \quad 0$

$q_i = u, d, s, c$ $\quad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{matrix} +\frac{2}{3} \\ -1/3 \end{matrix}$ Elec. Charge

$e^+ e^- \longrightarrow q_i \ \bar{q}_j$

electric Charge. $+ \quad -$ $\qquad u \quad \bar{d}$

$+\frac{2}{3} \quad +1/3$ $\qquad Q = +1 \neq 0$

$u \quad \bar{c}$ $\qquad Q = 0$

$+\frac{2}{3} \quad -\frac{2}{3}$

charm. Flavor $0 \quad 0 \qquad 0 \quad c = -1$ $\qquad \Delta C \neq 0$

$d \quad \bar{s}$

Strangeness $0 \quad 0 \qquad 0 \quad +1$ $\qquad \Delta S \neq 0.$

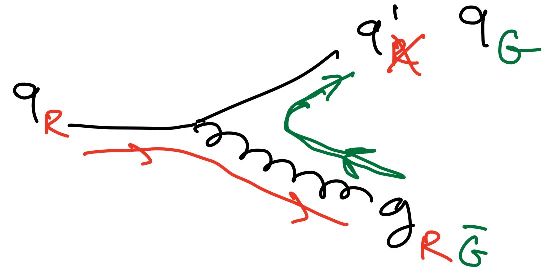
$u \quad \bar{u}$ $\qquad \Delta S = 0, \ \Delta C = 0, \ \Delta Q = 0$

$d \quad \bar{d}$

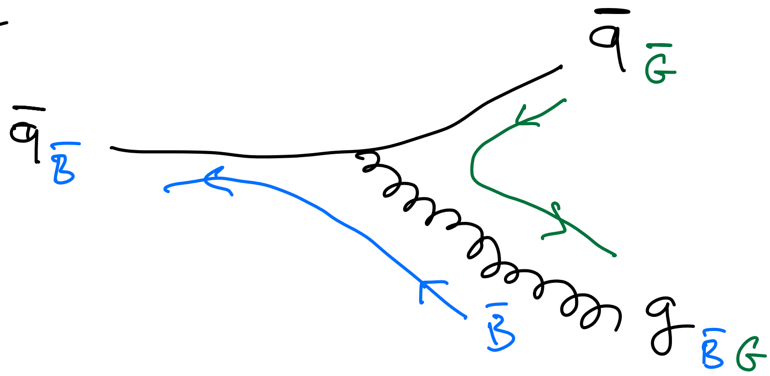
$\cancel{u_R \quad \bar{u}_{\bar{B}}}$ $\qquad R + \bar{B} \neq 0.$

color $\qquad u_R \quad \bar{u}_{\bar{R}}$ $\qquad$ ✓

$q \to q' g$



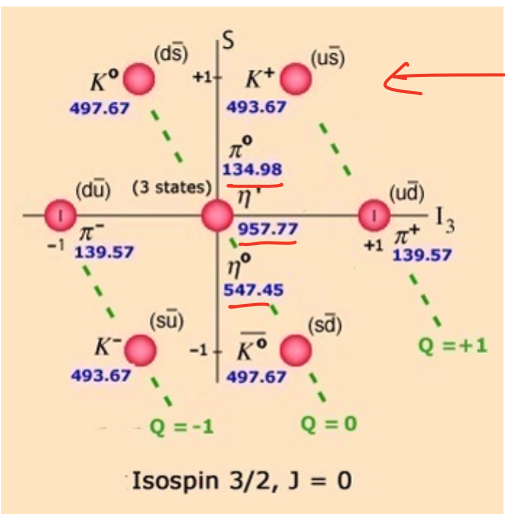
$\bar{q} \to \bar{q}\, g$



$SU(3)_F$: 3 flavors u, d, s and assume $m_u \simeq m_d \simeq m_s$.

$u = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $d = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $s = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$M = 3 \times \bar{3} = 1 + 8 \Rightarrow$ 9 mesons with same mass.



$m \simeq 500$ MeV.

150 MeV

$\simeq 800$ MeV.

$\Downarrow$

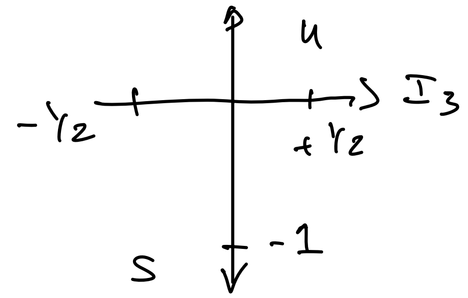
$m_u \simeq m_d \neq m_s$.

Baryons	qqq	J^P	I	I_3	S	$Q^{(1)}$	mass (MeV)
p, n	uud, udd	½⁺	½	½, -½	0	1, 0	940
Λ	uds	½⁺	0	0	-1	0	1115
Σ⁺, Σ⁰, Σ⁻	uus, uds, dds	½⁺	1	1, 0, -1	-1	1, 0, -1	1190
Ξ⁰, Ξ⁻	uss, dss	½⁺	½	½,-½	-2	1, 0	1320
Δ⁺⁺, Δ⁺, Δ⁰, Δ⁻	uuu, uud, udd, ddd	$3/2^+$	$3/2$	$3/2, 1/2, -1/2, -3/2$	0	2, 1, 0, -1	1230
Σ*⁺, Σ*⁰, Σ*⁻	uus, uds, dds	$3/2^+$	1	1, 0, -1	-1	1, 0, -1	1385
Ξ*⁰, Ξ*⁻	uss, dss	$3/2^+$	½	½,-½	-2	1, 0	1530
Ω⁻	sss	$3/2^+$	0	0	-3	-1	1670

$m_s \neq m_u, m_d$.

$SU(3)_F \Rightarrow$ has 2 diagonal generators.

$I_3, S.$



$\begin{matrix}+\frac{1}{2} \\ -1/2\end{matrix}\begin{pmatrix}u\\d\end{pmatrix} = I = 1/2.$ $\quad S: I_3 = I = 0.$

$\begin{pmatrix}u\\d\end{pmatrix}\begin{pmatrix}c\\s\end{pmatrix}$

Charm quark $\Rightarrow$ $SU(4)_F$?

if $m_u = m_d \simeq m_s \simeq m_c$

$N^2-1 = 15$ generators.

$N-1 = 3$ diagonal generators $\Rightarrow$ 3 physical quantities.

$u = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $\quad d = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ $\quad s = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ $\quad c = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

Diag. gener.

I_3	isospin.
S	strangeness.
C	charm quantum number

$$Q = I_3 + \frac{B+S+C}{2}$$

Revisited Gell-Mann Nishijima

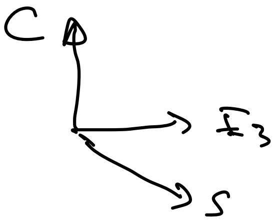
$M = q_i \bar{q}_j = 4 \otimes \bar{4} = \mathbb{1} + \mathbb{15}$

$\begin{pmatrix}u\\d\\s\\c\end{pmatrix}$ Fundam. repp of $SU(4)$ $\qquad \begin{pmatrix}\bar{u}\\\bar{d}\\\bar{s}\\\bar{c}\end{pmatrix}$ Conjugate repp.

Compared to $SU(3)_F$ $1+8+6$ states

↳ $c\bar{u}$ $\quad c\bar{d}$ $\quad c\bar{s}$

$\quad \bar{c}u$ $\quad \bar{c}d$ $\quad \bar{c}s$

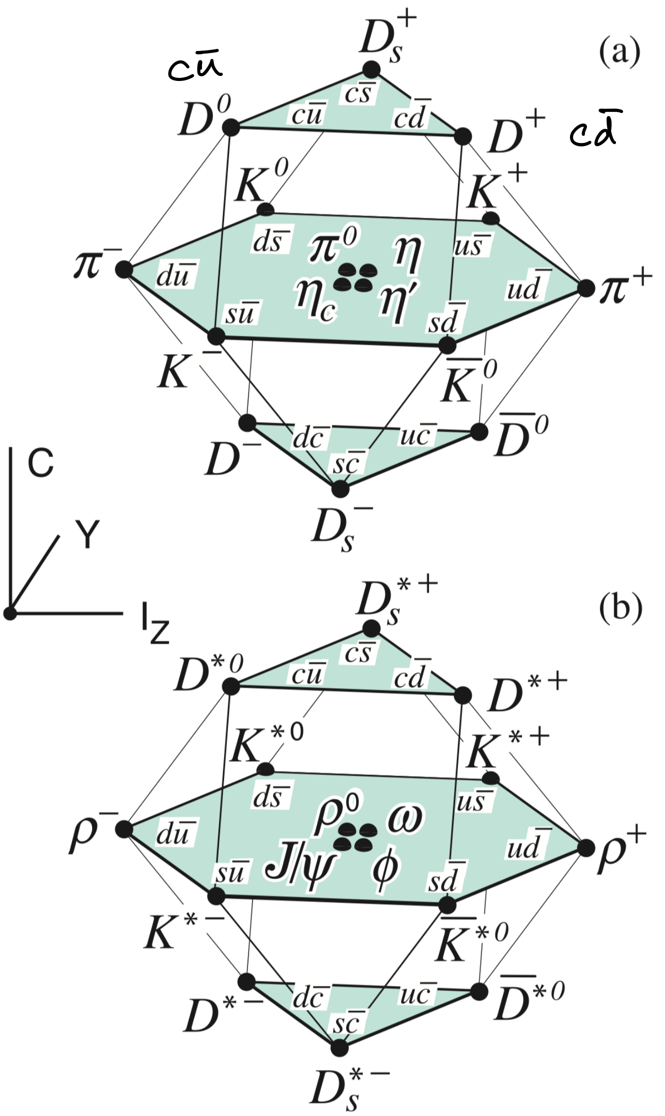


mass:

$D^0 \simeq 1.866$ GeV.

$\Downarrow$

$M_c \neq M_u, M_d, M_s$



c $\quad Q = +\frac{2}{3}$

D meson.

$c\bar{q}_i$

$c\bar{u} \quad Q = 0. \quad D^0$

$c\bar{d} \quad D^+$

$c\bar{s} \quad D_s^+$

$D^0 \neq \bar{D}^0$

$S = 0 \quad P \Downarrow$

$D^* \quad S = 1$

Add beauty quark:

SU(5)F

$5-1 = 4$

diagonal gen.

I_3 isospin.

S strangeness.

C charm

b beauty.

$$Q = I_3 + \frac{B + C + S + b}{2}$$

B mesons: $b\bar{q}_i$

mass: $\simeq 5.279$ GeV $(b\bar{d})$ $\bar{B}^0$

$\Rightarrow M_b \neq M_c \neq m_s \neq M_{u,d}$

$5 \times \bar{5} = 1 + 24$

To test existence of quarks and color $\Rightarrow$ scattering.

$$e^+ e^- \longrightarrow \cancel{u_R \bar{d}_{\bar{B}} \quad \bar{u}_{\bar{R}} d_B}$$

$$\pi^+ = u\bar{d}$$

$$\pi^- = \bar{u} d$$

$$u_B \bar{d}_{\bar{B}} \qquad \bar{u}_{\bar{B}} d_B$$

$$e^+ e^- \longrightarrow \pi^+ \pi^-$$

$$\sqrt{s} \not\geq 2m_\pi$$

$$e^+ e^- \longrightarrow \cancel{u_B \bar{d}_{\bar{B}} \quad \bar{u}_{\bar{B}} s_B}$$

$$\Delta S = 1$$

$$u_B \bar{d} \quad \cancel{\ } \quad \bar{u}_{\bar{B}} b_B$$

$$\Delta B = 1$$