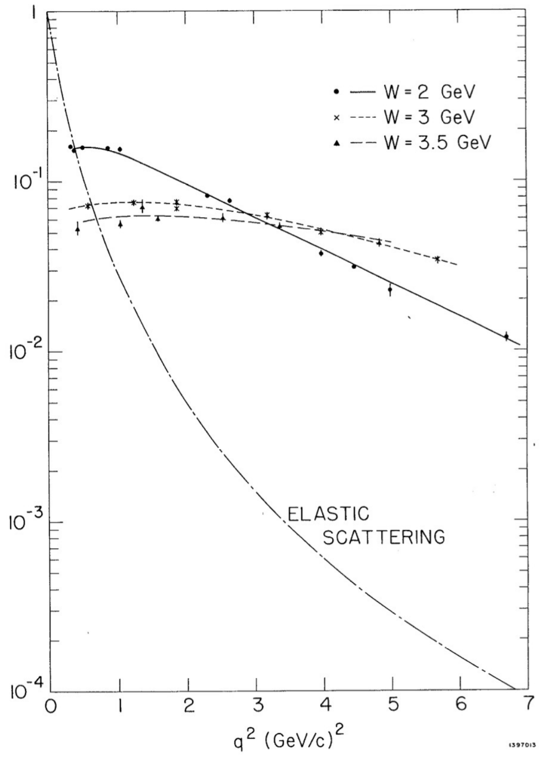
Exams starting 14/6 → 28/6

10/7 → 28/7

Google Calendar / Form by 2/6.



Quark Parton Model

DIS $e^- p \to e^- + X$

SLAC: fixed target.

DESY / HERA: $e^- + p$ beams

higher $\sqrt{s}$, Q^2 accessible.

1)

Flat behaviour of $\sigma_{tot}/\sigma_{Mott}$

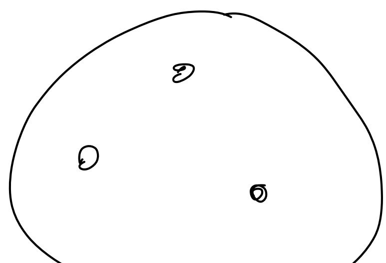
$\Rightarrow$ pointlike interaction!

2) $\frac{\sigma_{tot}}{\sigma_{Mott}} << 1$. $\Rightarrow$ $e^- + q \to e^- + q$. hypothetical elastic scattering.

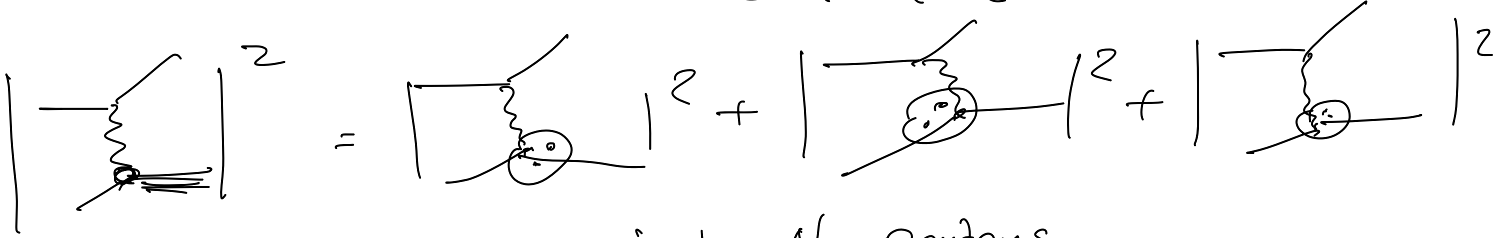
$z_q < 1$

q: hypothetical target inside nucleon

Quark Parton Model: nucleon as a container for partons



DIS $e^- p$ $\equiv$ incoherent sum of elastic scattering of $e^- + q_i$ (partons).



$e^- p \to e^- + X$

$i = 1, \dots N$ partons

incoherent sum

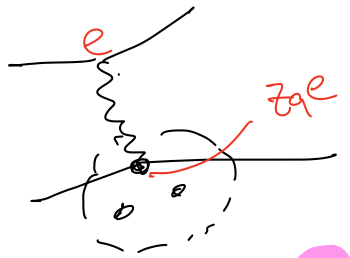
Inelastic e^-p (DIS) $\longrightarrow e^-X$

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{DIS} = \frac{4\alpha^2}{Q^4} E'^2 \left[W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right]$$

Elastic scattering $e^-q \to e^-q$ q: parton $m = xM$ $M = m_p$.

$q^e = z_q e$ electric charge

q: pointlike, spin $1/2$

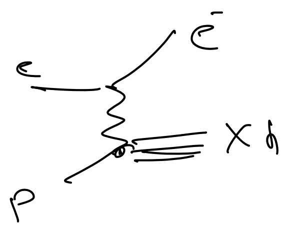


$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 z_q^2}{Q^4} E'^2 \left[\cos^2\frac{\theta}{2} + 2\frac{Q^2}{4m^2}\sin^2\frac{\theta}{2}\right]\delta\left(\nu - \frac{Q^2}{2m}\right)$$

probe e^- has spin $1/2$

target has spin $1/2$

Elastic $e^-q \to e^-q$ $Q^2 = 2m\nu$



$M_X^2 = W^2 = M^2 - Q^2 + 2M\nu$.

elastic limit $\Rightarrow M^2 = W^2$. $\Rightarrow Q^2 = 2M\nu$.

$e^-p \to e^-p$.

$m = xM \Rightarrow x = \frac{m}{M}$

for 1 parton with mass $m = xM$.

$$W_1\big|_x = \frac{Q^2}{4m^2}\,\delta\left(\nu - \frac{Q^2}{2m}\right) z_q^2$$

we donot know distribution of x.

$x \in [0,1]$.

Introduce Parton density Function Pdf.

$f(x)\,dx$ prob. of having parton in $[x, x+dx]$

$$\int_0^1 f(x)\,dx = 1$$

$$W_1(Q^2,\nu) = \int_0^1 dx\, f(x)\, \frac{Q^2}{4m^2}\, z_q^2\, \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$m = xM.$ $\quad$ m: parton $\quad$ M: nucleon/proton.

$$W_1 = \frac{Q^2}{4} z_q^2 \int_0^1 dx\, f(x) \frac{1}{x^2 M^2}\, \delta\left(\nu - \frac{Q^2}{2Mx}\right) =$$

$$= \frac{Q^2}{4M^2} z_q^2 \underbrace{\int_0^1 dx\, \frac{f(x)}{x^2}\, \delta\left(\nu - \frac{Q^2}{2Mx}\right)}_{I}$$

$$I = \int A(x)\, \delta(g(x))\, dx$$

if $g(x = x_0) = 0$. $\Rightarrow$ $I = \dfrac{A(x_0)}{|g'(x_0)|}$.

$$A(x) = \frac{f(x)}{x^2} \qquad g(x) = \nu - \frac{Q^2}{2Mx} \Rightarrow g'(x) = \frac{Q^2}{2Mx^2}$$

$$x_0 \Rightarrow \nu = \frac{Q^2}{2Mx_0} \Rightarrow \boxed{x_0 = \frac{Q^2}{2M\nu}}$$

$$g'(x)\Big|_{x=x_0} = \frac{Q^2}{2M} \frac{1}{x_0^2} = \frac{Q^2}{\cancel{2M}} \frac{(2M\nu)^2}{Q^4} = \frac{2M\nu^2}{Q^2}$$

$$I = \frac{f(x)}{x^2}\Bigg|_{x=x_0} \frac{Q^2}{2M\nu^2} = f(x)\left(\frac{2M\nu}{Q^2}\right)^2 \frac{Q^2}{2M\nu^2} =$$

$$= f(x)\, \frac{1}{Q^2}\, \frac{(2M\nu)^2}{2M\nu^2}$$

$$W_1(Q^2,\nu) = \frac{Q^2}{4M^2} z_q^2\, I$$

$$W_1(Q^2,\nu) = \frac{\cancel{Q^2}}{\cancel{4M^2}} z_q^2\, f(x)\, \frac{1}{\cancel{Q^2}}\, \frac{(\cancel{2M\nu})^2}{2M\cancel{\nu^2}} = z_q^2 \frac{f(x)}{2M}\Bigg|_{x=x_0}.$$

$$W_2\Big|_x = z_q^2\, \delta\left(\nu - \frac{Q^2}{2Mx}\right)$$

$$W_2 = \int_0^1 dx\, f(x)\, z_q^2\, \delta\left(\nu - \frac{Q^2}{2Mx}\right) = z_q^2 f(x) \frac{Q^2}{2M\nu^2} =$$

$$= z_q^2 f(x) \frac{x}{\nu}$$

For one parton

$$W_1 = z_q^2 \frac{f(x)}{2M} \qquad W_2 = z_q^2 f(x) \frac{x}{\nu}.$$

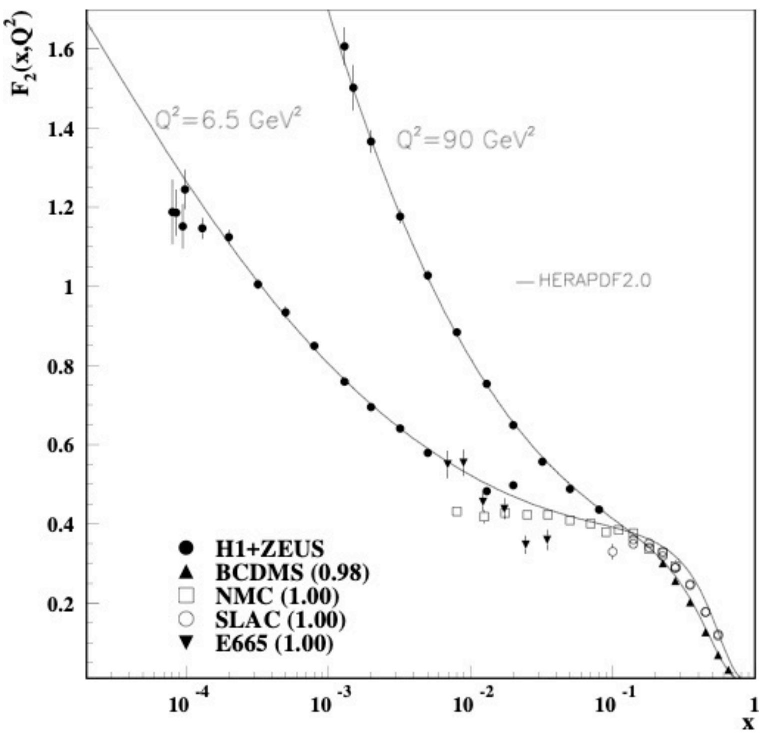
For N partons. $j = 1, \ldots, N$. $\quad m_j, z_{q_j}, f_j(x)$

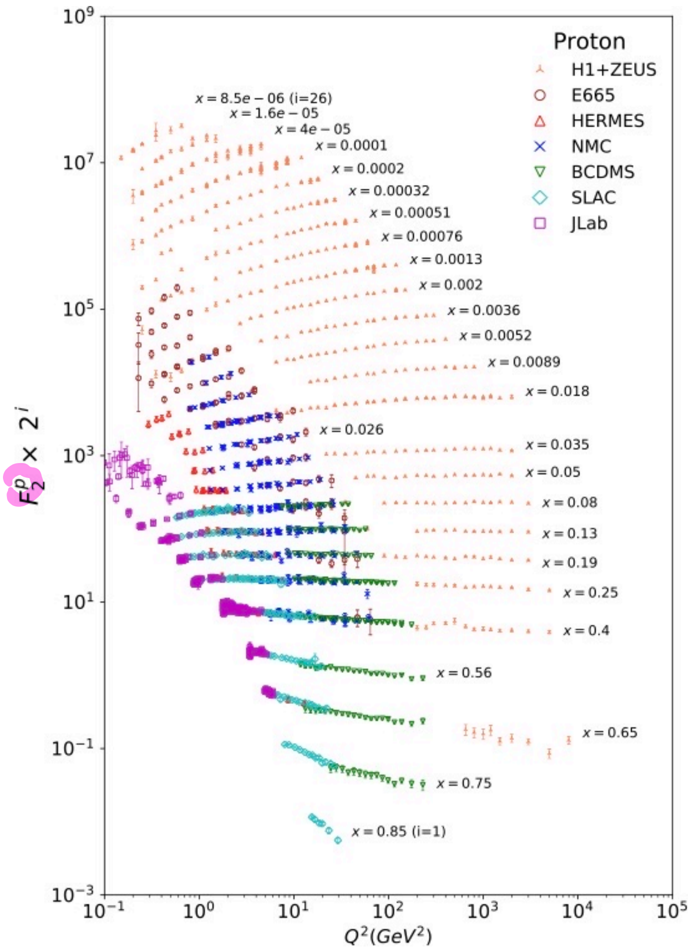
$$W_1 = \sum_j^{\text{partons}} z_{q_j}^2 \frac{f_j(x)}{2M} \qquad W_2 = \sum_j^{\text{part.}} z_{q_j}^2 f_j(x) \frac{x}{\nu}$$

$$F_1(x) \equiv M W_1 = \frac{1}{2} \sum_j z_{q_j}^2 f_j(x)$$

$$F_2(x) \equiv \nu W_2 = \sum_j z_{q_j}^2 f_j(x)\, x$$

$$\boxed{F_2(x) = 2x F_1(x)}$$ Callan-Gross relation.



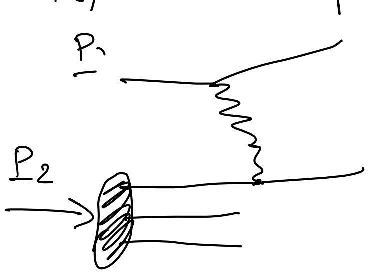


$e^- p \to e^- + X$

Bjorken Scaling.

Bjorken x : property of lepton. $\quad x_B = \frac{Q^2}{2M\nu}$

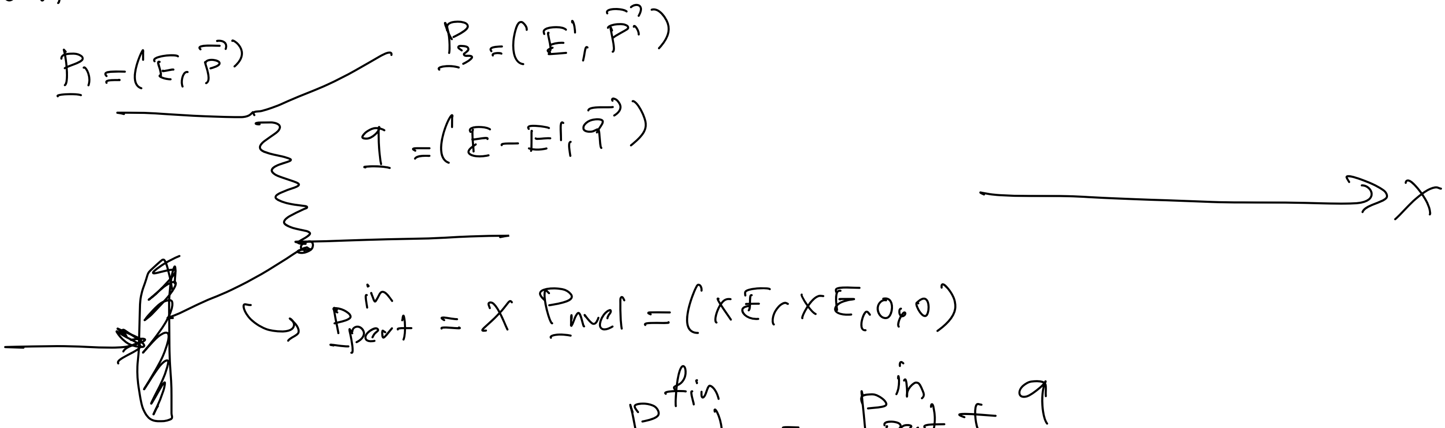
Feynman x : property of partons.



$P_q = x P_2$ initial parton momentum.

$$x_F = \frac{|\vec{P}_{\text{parton}}|}{|\vec{P}_{\text{nucleon}}|} = \frac{|\vec{P}_L|}{|\vec{P}_{\text{nucleon}}|}$$

Ultra relativistic limit: $m_e << E \quad m_p << E$



$\hookrightarrow \underline{P}^{in}_{part} = x\, \underline{P}_{nucl} = (xE, xE, 0, 0)$

$\underline{P}_{nucleon} = (E, E, 0, 0)$

$$\underline{P}^{fin}_{part} = \underline{P}^{in}_{part} + \underline{q}$$

$$|\underline{P}_{part}|^2 = m_{parton}^2 + \underbrace{|\underline{q}|^2}_{-Q^2} + 2\, \underline{P}^{in}_{part} \cdot \underline{q}$$

$$Q^2 = \underbrace{2\, \underline{P}^{in}_{part} \cdot \underline{q}}_{\text{Lorentz-invariant}}$$

in LAB frame $2\, \underline{P}^{in}_{part} \cdot \underline{q} = 2 x_F M (E-E')$
$= 2 x_F M \nu.$

$\Rightarrow \quad Q^2 = 2 x_F M \nu \Rightarrow \quad x_F = \dfrac{Q^2}{2M\nu}$

$\Rightarrow$ high energy limit $\quad x_F = x_B = x = \dfrac{Q^2}{2M\nu}$

Measure $\dfrac{d^2\sigma}{d\Omega dE'} \Longrightarrow$ determine $F_2(x) = 2x F_1(x)$.

How many Partons?

$$F_1(x) = \sum_i z_{q_i}^2 f_i(x)$$

x: fraction of momentum carried by partons

with N protons: $x_i : i = 1, \ldots N \quad x_i \in [0, 1]$

$$\sum_{j=1}^{part.} x_j = 1.$$

3 types of partons:

1) valence quarks.

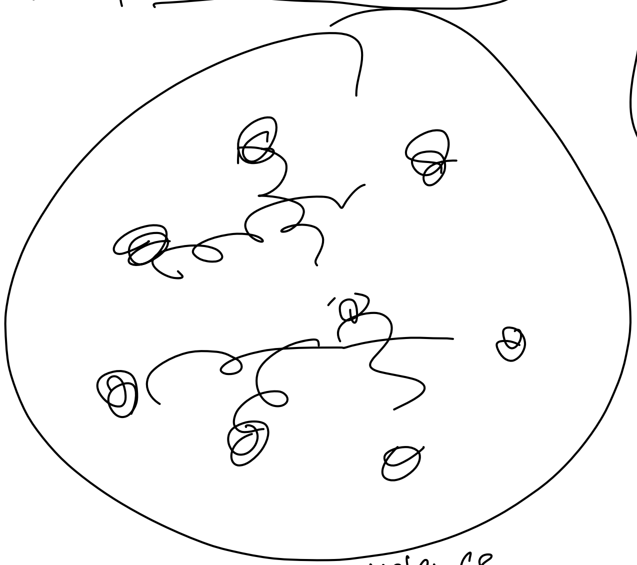
$$p \equiv (uud)$$

proton has 3 valence quarks: u, u, d

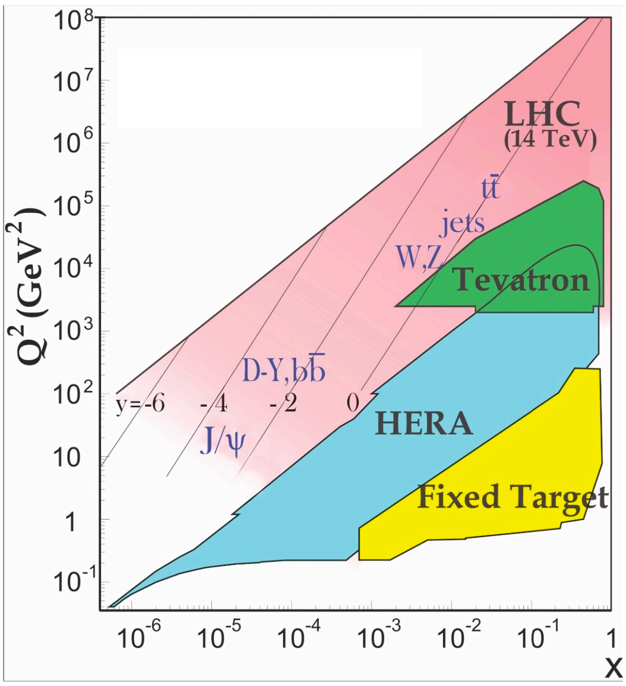
$$p = uud + \underbrace{u\bar{u} + d\bar{d} + s\bar{s}}_{\text{sea quarks antiquark pairs}} + \underbrace{g}_{\text{sea gluons}}$$

sea quarks antiquark pairs.

sea gluons.



Experimentally: $\sum_j^{\text{valence}} x_j = 0.5$



$$y = \frac{E - E'}{E} = 1 - \frac{E'}{E}$$

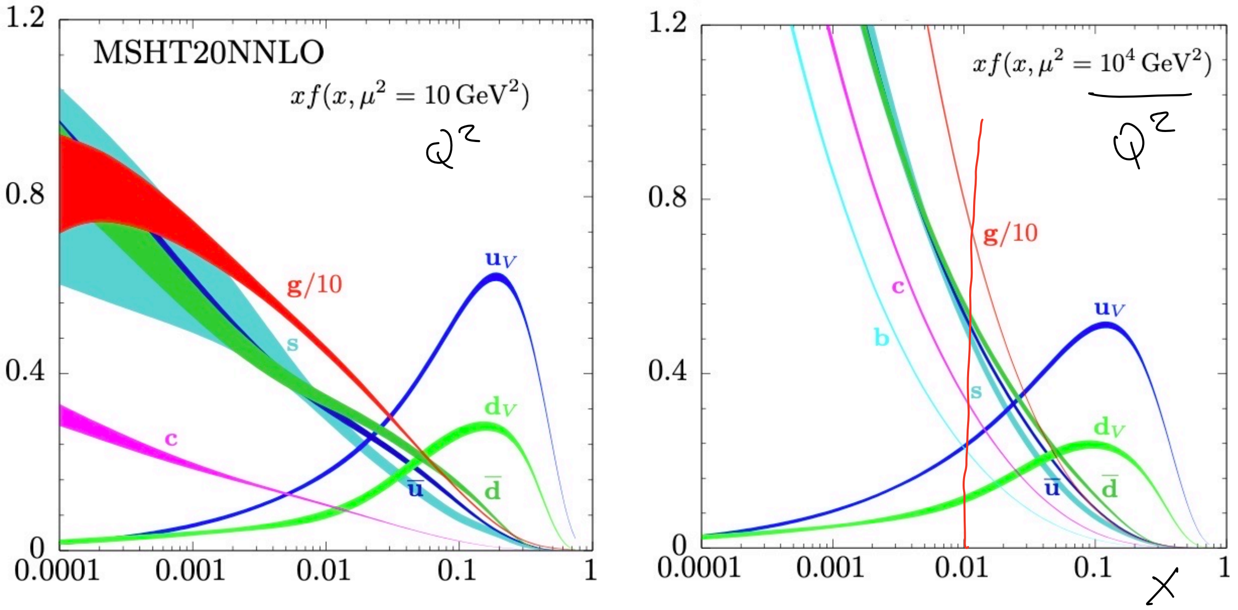
parton density functions:

$f_v(x)$: valence quarks.

$f_s(x)$: sea quarks. $\bar{f}_s(x)$: sea antiquarks. $g(x)$: gluons.

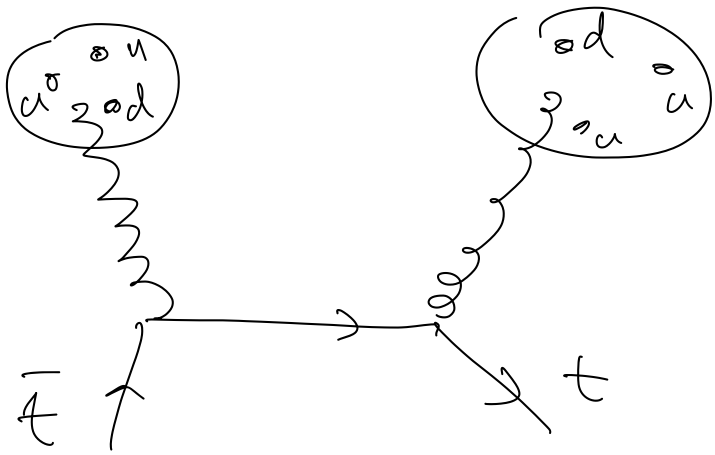
$F_1(x)\ \Sigma_j\ z_{q_j}\ f_j(x).$

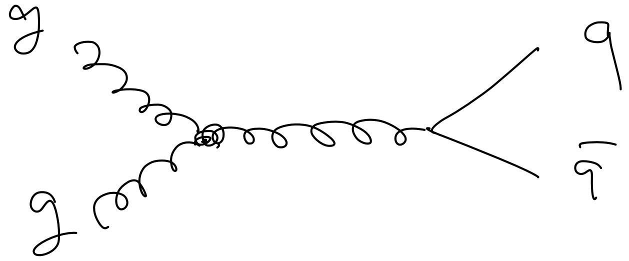
Parton density Function.



$$x = \frac{Q^2}{2M\nu}$$

for $x \to 0 \Rightarrow p + p \longrightarrow g + g$ collision.





Constraints on PDF (parton density functions)

$$\left.\begin{aligned}\int_0^1 dx\left[u_v^p(x) - \bar{u}_s^p(x)\right] &= 2 \\ \int_0^1 dx\left[d^p(x) - \bar{d}^p(x)\right] &= 1\end{aligned}\right\}$$ Baryon Number sum rule.

Momentum sum rule.

$$\int_0^1 dx\, x\left(\left[\sum_i q_i(x) + \bar{q}_i(x)\right] + g(x)\right) = 1$$