point-like target
spin 1/2 probe

$e^- + N \to e^- + N$

N is point like.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Mott} \\ \text{point like} \\ \text{No target recoil}}} = \frac{\alpha^2}{q^4} E^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right) = \frac{\alpha}{q^4} \cos^2\frac{\theta}{2}$$

$\beta \simeq 1.$

target not point like $\rightarrow$ $\left.\frac{d\sigma}{d\Omega}\right|_{\text{NON point like}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point like}} \times |F(q^2)|^2$

$q^2 \simeq 0$ $\qquad F(q^2) \simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle$

$$F(q^2) = \int d^3 r \, e^{i \vec{q} \cdot \vec{r}} f(r) \qquad \rho(r) = Z e f(r)$$

$$A = \int d^3 r \, f(r)$$

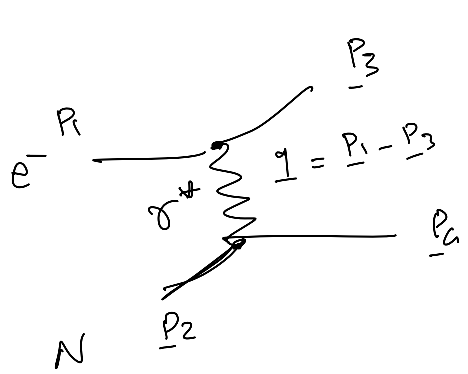
In principle $f(r) = f(\vec{r}) = f(r, \theta, \varphi)$

Typically assume $f(r) = f(r)$ no dependence on θ & φ.

$$F(q^2) = \int \frac{f(r)}{A} e^{i \vec{q} \cdot \vec{r}} \, d\cos\theta \, d\varphi \, r^2 dr$$

$$= \frac{2\pi}{A} \int dr \, f(r) r^2 \underbrace{\int_{-1}^{1} d\cos\theta \, \underbrace{e^{i \vec{q} \cdot \vec{r}}}_{e^{iqr\cos\theta}}}_{\frac{2 \sin(qr)}{qr}}$$

$$= \frac{4\pi}{A} \int dr \, f(r) r^2 \frac{\sin(qr)}{qr}$$



Take into account recoil of target.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Point like} \\ \text{with recoil.}}} = \overset{\text{Ruth.}}{\frac{\alpha^2}{q^4}} \underbrace{\cos^2\frac{\theta}{2}}_{\text{Spin of } e^-} \frac{E'}{E}$$

Mott cross section.

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E}{M} \sin^2\frac{\theta}{2}}.$$

For non-point like target.

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{Mott} \times |F(q^2)|^2$$

Form factor $F(q^2)$

So far not taking into account spin of target

Dirac theory for $S = 1/2$ mass m

intrinsic magnetic moment $\mu_e = g\,\frac{e\hbar}{4m}$

g: gyromagnetic ratio

electron point like $\rightarrow$ $g = 2$ $\quad \mu_e = \frac{e\hbar}{2m}$

proton, neutron spin $= 1/2$ $\quad m = m_N \simeq 1$ GeV.

$\mu_p = g_p\,\mu_N$ $\qquad \mu_N = \frac{e\hbar}{4m_N}$ nuclear magneton

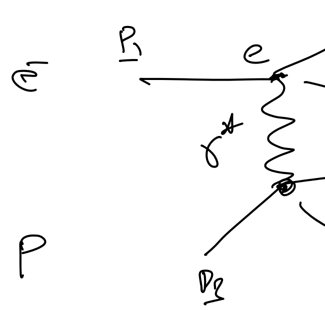
$\mu_n = g_n\,\mu_N$

Expectation: $g_p = 2.$ $\quad g_n = 0.$ $q_n = 0$

Experimentally $g_p = 2.79$ $\quad g_n = -1.91$ $\quad$ anomalous magnetic moment.

$\Rightarrow$ Indication that p, n are not point like

Consider $e^- + p \rightarrow e^- + p$



$\rightarrow$ $\bar{u}(P_3)\,\gamma^\mu\,u(P_1)$ electron current vertex.

$\rightarrow$ $\frac{1}{q^2}$

$\rightarrow$ $\bar{u}(P_4)\left[F_1(q^2)\gamma^\mu + i\,\frac{q^\nu \sigma_{\mu\nu}}{2M} K\, F_2(q^2)\right]u(P_2)$

$q = P_1 - P_3$ $\qquad q^\nu$ is $P_1^\nu - P_3^\nu$ $\qquad K = g - 1 = 1.79$ proton.

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ $\quad \gamma_\mu$: Dirac matrices.

$F_1(q^2)$, $F_2(q^2)$ 2 Form factors.

$F_1(0) = F_2(0) = 1$ $\qquad q^2 = 0.$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth.}} = \frac{\alpha^2}{q^4} \underbrace{E^2 \cos^2\frac{\theta}{2}}_{\text{spin } e^-} \times \underbrace{\frac{E'}{E}}_{\substack{\text{recoil}\\ \text{of target.}}} \times \left[\left(F_1^2 + \frac{\kappa^2 Q^2}{4M^2} F_2^2\right) + (F_1 + \kappa F_2)^2 \frac{Q^2}{2M^2} \tan^2\frac{\theta}{2} \right]$$

$Q^2 = -q^2 > 0 \quad \forall\, E, E', \theta.$

F_1, F_2 incorporate ignorance about proton structure.

$Q^2 = 4EE' \sin^2\frac{\theta}{2}$

Sample different Q^2 by measuring # events at different θ.

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

$Q^2 \to 0$ $\qquad M = m_p$

For $\frac{Q^2}{M^2} \ll 1.$ $\qquad$ Suppose $E = 0.5$ GeV.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth}} \simeq \frac{\alpha^2}{q^4} E^2 \cos^2\frac{\theta}{2}\, |F_1(q^2)|^2$$

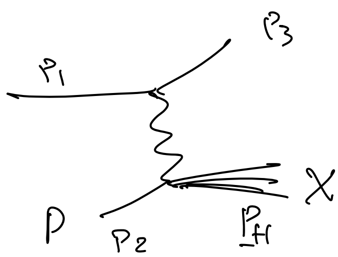
$$F_1(q^2) = 1 - \frac{1}{6} q \langle r^2 \rangle.$$

For $\frac{Q^2}{M^2} \gg 1.$

$$\left.\frac{d\sigma}{d\Omega}\right| \simeq (\,-\,-\,)\left(\frac{\kappa^2 Q^2}{4M^2} |F_2(q)|^2 + (F_1 + \kappa F_2)^2 \frac{Q^2}{2M^2} \tan^2\frac{\theta}{2}\right)$$

measurement of $F_1(q^2)$ for $Q^2 \to 0$

Inelastic limit.



$W^2 = \sum_i p_i^2 = M_X$

Deep Inelastic Scattering.

$$\left.\frac{d\sigma}{d\Omega}\right|_{DIS} = \frac{\alpha^2}{q^4} E^2 \cos^2\frac{\theta}{2} \left[W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2\frac{\theta}{2} \right].$$

W_1, W_2 : Structure functions.

Elastic scattering: F_1, F_2 depend only on Q^2

Inelastic scattering: W_1, W_2 depend on Q^2, ν

$$W^2 = \underline{P}_H^2 = (\underline{P}_2 + \underline{q})^2 = M^2 - Q^2 + 2M\nu.$$

proton structure:

McAllister, Hofstadter @ SLAC 1956

↪ Nobel 1961

188 MeV e^- on H_2, He target

measure # events for $\theta \in [35, 138]^\circ$

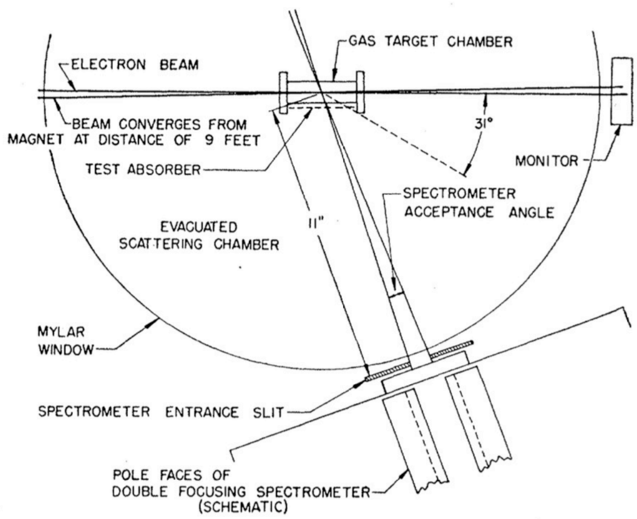
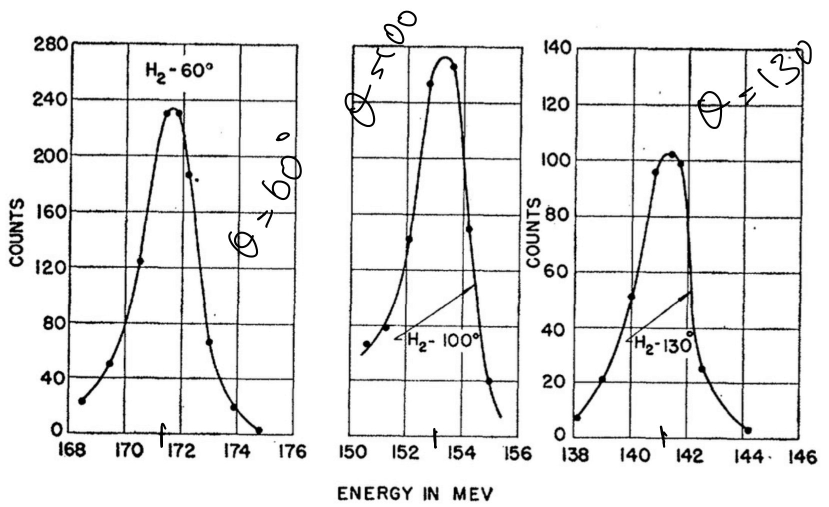


FIG. 2. Arrangement of parts in experiments on electron scattering from a gas target.

Chapter 8 Goldhaber

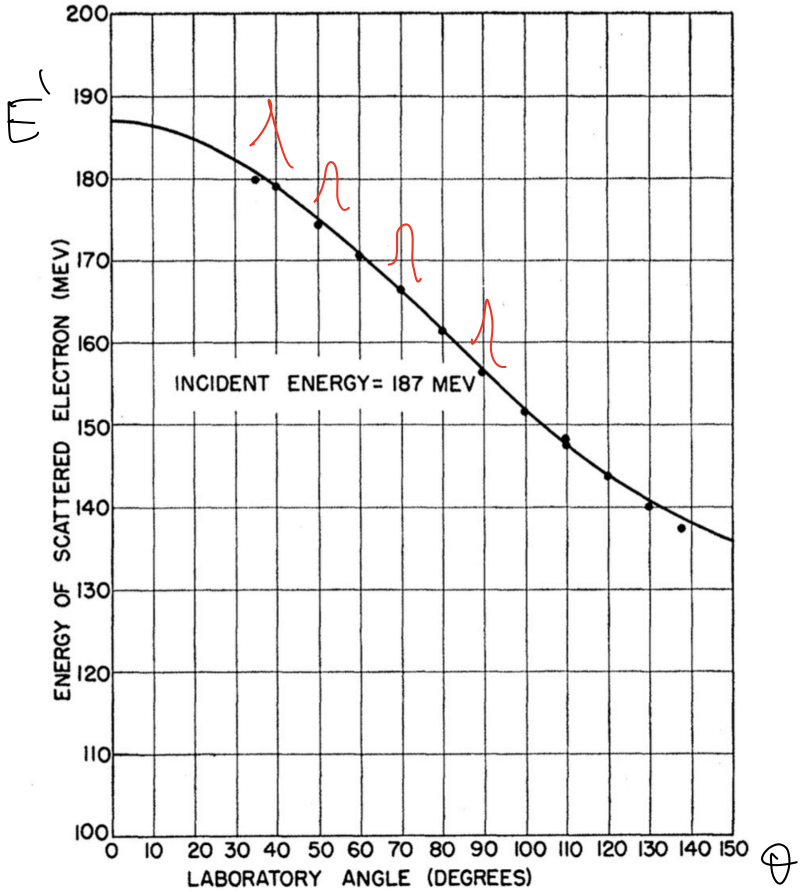


$$E' = \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)} = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

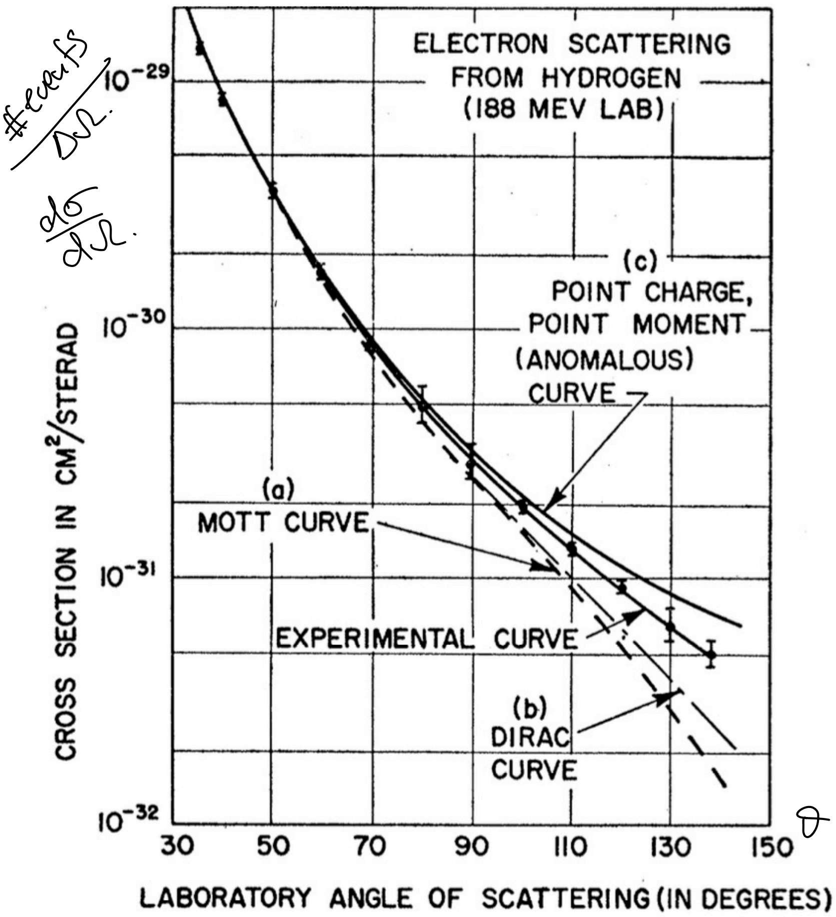
E'

Energy resolution ⊗ $\delta(E')$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{q^4} E^2 \cos^2\frac{\theta}{2} \sim \frac{\alpha^2}{E^2} \frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}$$



$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$



$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{structure}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} \times |F(q^2)|^2$$

$$\simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle .$$

$$\bar{r} = \sqrt{\langle r^2 \rangle} = 0.7 \text{ fm.}$$

FIG. 5. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with the Dirac magnetic moment, curve (c) the theoretical curve for a point proton having the anomalous contribution in addition to the Dirac value of magnetic moment. The theoretical curves (b) and (c) are due to Rosenbluth.[8] The experimental curve falls between curves (b) and (c). This deviation from the theoretical curves represents the effect of a form factor for the proton and indicates structure within the proton, or alternatively, a breakdown of the Coulomb law. The best fit indicates a size of 0.70×10^{-13} cm.

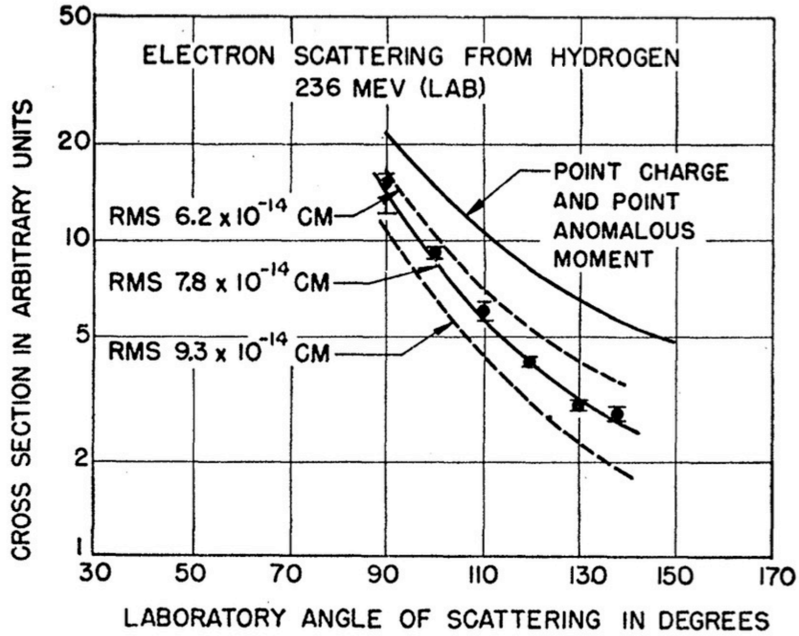


FIG. 6. This figure shows the experimental points at 236 Mev and the attempts to fit the shape of the experimental curve. The best fit lies near 0.78×10^{-13} cm.

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Elastic Scattering of 188-Mev Electrons from the Proton and the Alpha Particle*†‡§‖¶

R. W. McALLISTER AND R. HOFSTADTER
Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California

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The elastic scattering of 188-Mev electrons from gaseous targets of hydrogen and helium has been studied. Elastic profiles have been obtained at laboratory angles between 35° and 138°. The areas under such curves, within energy limits of ±1.5 Mev of the peak, have been measured and the results plotted against angle. In the case of hydrogen, a comparison has been made with the theoretical predictions of the Mott formula for elastic scattering and also with a modified Mott formula (due to Rosenbluth) taking into account both the anomalous magnetic moment of the proton and a finite size effect. The comparison shows that a finite size of the proton will account for the results and the present experiment fixes this size. The root-mean-square radii of charge and magnetic moment are each $(0.74 \pm 0.24) \times 10^{-13}$ cm. In obtaining these results it is assumed that the usual laws of electromagnetic interaction and the Coulomb law are valid at distances less than 10^{-13} cm and that the charge and moment radii are equal. In helium, large effects of the finite size of the alpha-particle are observed and the rms radius of the alpha particle is found to be $(1.6 \pm 0.1) \times 10^{-13}$ cm.