

$$(i) \xrightarrow{H_I} (f)$$

1  $N$  ~~for~~ Delays

1/2  $3 - N$  collisions.

$$H_I = H_{EM} + H_W + H_{strong} \quad \text{Standard Model}$$

$$\Gamma(i \rightarrow f) \propto |M_{fi}|^2 \rho(E)$$

$\downarrow$   
 $H_I$

$\downarrow$   
 Kinematics  
 particles

leptons

$$Q_{EM} = -1 \begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} \quad \left. \vphantom{\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}} \right\} \text{SU(2) doublets leptons}$$

$$Q_{quarks} = +\frac{2}{3} \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \quad \left. \vphantom{\begin{pmatrix} u \\ d \end{pmatrix}} \right\} \text{quarks}$$

$\underbrace{\hspace{10em}}_{\text{Fermions } S = \frac{1}{2}}$

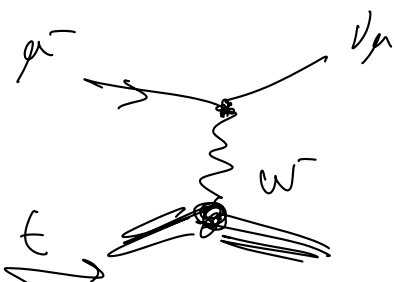
$\uparrow$   
 1st  
 ferm.

$2^{nd}$

$3^{rd}$

$$m_e \approx 0.5 \text{ MeV} \quad m_\mu \approx 106 \text{ MeV} \quad m_\tau \approx 1.8 \text{ GeV}$$

$$m_{uds} \approx 10 \text{ MeV} \quad m_c \approx 1.5 \text{ GeV} \quad m_{top} \approx 170 \text{ GeV}$$



$$\nu_\mu + p \rightarrow \mu^- + X$$

$$\nu_\mu + p \rightarrow e^- + X$$

$= 0$

$$Q_{em} \begin{pmatrix} +1 \\ 0 \end{pmatrix} \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix} \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix} \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix} \text{ anti-leptons}$$

$$+\frac{1}{3} \begin{pmatrix} d \\ \bar{u} \end{pmatrix} \begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix} \begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix} \text{ anti-quarks.}$$

Nature: hadrons not quarks.

free states

Baryons:  $q_1 q_2 q_3$

Mesons:  $q_1 \bar{q}_2$

Pentaquarks:  $q_1 q_2 q_3 q_4 q_5$   
molecule of mesons

for each quark: 3 colors red, green, blue.

anti baryons  $\bar{q}_1 \bar{q}_2 \bar{q}_3$

antimesons  $\bar{q}_1 q_2$

$$\pi^+ \equiv |u\bar{d}\rangle$$

$$Q = +1$$

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$\pi^- \equiv |\bar{u}d\rangle$$

$$Q = -1$$

$$p = |u\bar{u}u\rangle$$

$$Q = 3 \times \frac{2}{3} = \frac{6}{3} = +2$$

$$|uud\rangle$$

$$Q = \frac{4}{3} - \frac{1}{3} = +1.$$

$$p \equiv |uud\rangle$$

$$n \equiv |udd\rangle$$

$$Q = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$

$$B^- = |b\bar{u}\rangle$$

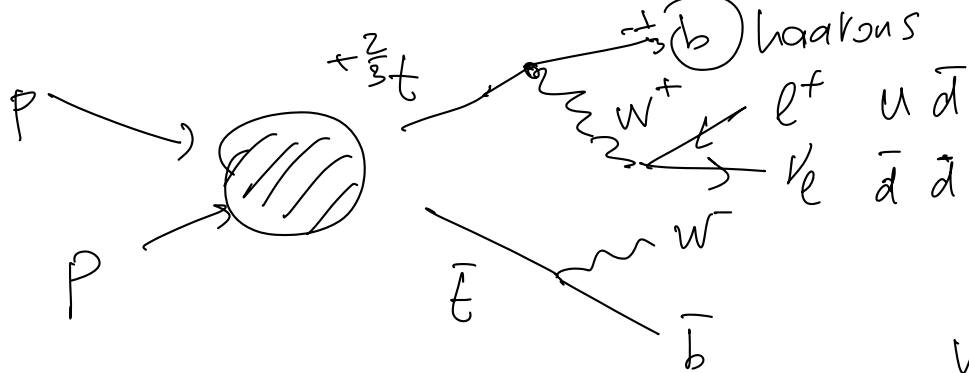
$$-\frac{1}{3}, \bar{u} = -\frac{2}{3}$$

$$\cancel{T^0 = |t\bar{u}\rangle}$$

$$\cancel{\begin{pmatrix} t\bar{q} \\ t q_1 q_2 \end{pmatrix}}$$

$$B^0, B^+, \bar{B}^0$$

no top hadron.



$$W^+ \rightarrow e^+ \nu_e$$

$$\rightarrow \bar{d}^+ \bar{d}^0$$

$$t \rightarrow b W^+$$

170 GeV      5 GeV      80 GeV

$$\pi^\pm, \pi^0, p, n, e^\pm, g$$

Mediators of interactions

Bosons  $S=+1$

- $\gamma$  EM.
- $W^\pm, Z^0$  Weak.
- gluons Strong / QCD

Quantum Chromodynamics

Scalar elementary particle.

H Higgs boson.

$$\Gamma \propto |M_f|^2 \rho(E)$$

Symmetries provide hints about  $H_\pi$

Symmetry:  $T|\psi\rangle = |\psi\rangle$

physical observable:  $\hat{O}$  Hermitian operator

$$\langle \hat{O} \rangle = \langle \psi | \hat{O} | \psi \rangle$$

$$[T, H] = 0 \quad T \text{ is a symmetry}$$

$H$ : Hamiltonian.

$$\frac{d}{dt} \langle O \rangle = 0 \quad \longleftrightarrow \quad [O, H] = 0$$

Transformations symmetry: unitary operators.

$$U^\dagger U = \mathbb{I} \quad \Leftrightarrow \quad U^\dagger = U^{-1}$$

Wigner Theorem: in QM all symm. are (anti-)unitary operators.

Types of symmetries

Continuous External Symmetry

$U(a_1, \dots, a_N)$  operates on  $\vec{x}, t$  of system/state.

Continuous function of  $a_1, \dots, a_N$

Translation of  $\vec{x}$   
 $t$

Rotation of  $\vec{x}$



Conserved quantity

Noether's theorem.  
symmetry  
of the system



Conserved quantity

## Continuous Internal Symm:

$U(a_1, \dots, a_n)$  does not operate on  $\vec{x}, t$  of system.

Isospin.

Rotation in internal space

$$\vec{L} = \vec{r} \times \vec{p} \xrightarrow{QM} \vec{S}, \quad \vec{J} = \vec{S} + \vec{L}$$

classical  
mech

## Discrete external symm:

Parity

unitary operator  $\downarrow$  operates on  $\vec{x}, t$   
finite # of operators.

$$\text{parity: } \vec{x} \rightarrow -\vec{x}$$

$$\text{Time inversion: } t \rightarrow -t$$

Inversion:

$$I\psi = a\psi$$

$$I^2\psi = a^2\psi = \psi.$$

$$a^2 = \pm 1$$

$$I^\dagger = I^{-1}$$

## Discrete Internal Symm:

finite unit. operat. not on  $\vec{x}, t$

Charge Conjugation  $\mathbb{C}$

particle  $\rightarrow$  anti-particle

Unitary Operator.  
 $U(a_1, \dots, a_n) = e^{i \sum a_j T_j} = e^{i a_j T_j}$   $T_j$  : generators.

$$U^\dagger U = \mathbb{1} \quad U \simeq \mathbb{1} + i a_j T_j$$

$$\Rightarrow \boxed{T_j^\dagger = T_j} \text{ hermitian oper.}$$

$$\frac{d\langle U \rangle}{dt} = 0 \Rightarrow \frac{d}{dt} \langle T_j \rangle = 0 \Rightarrow \langle T_j \rangle \text{ conserved.}$$

$$U(\text{trans}) = e^{i \vec{p} \cdot \vec{x}} \quad \vec{p} = \text{const.}$$

$$U(E \text{ trans}) = e^{i E t} \quad E = \text{const.}$$

$$U = e^{i \vec{L} \cdot \vec{\theta}} \quad \vec{L} = \text{const.}$$

$$\vec{\theta} = (\theta_1, \theta_2, \theta_3)$$

$$U = e^{i a G} \quad 1 \text{ generator.}$$

$$\psi = \psi_1 \psi_2$$

$$U \psi = U \psi_1 U \psi_2 = e^{i a G} \psi_1 e^{i a G} \psi_2$$

$$\simeq (1 + i a G) \psi_1 (1 + i a G) \psi_2$$

$$= (1 + i a g_1) \psi_1 (1 + i a g_2) \psi_2$$

$$\simeq e^{i a (g_1 + g_2)} \underbrace{\psi_1 \psi_2}_{\psi}$$

$$g_{12} = g_1 + g_2 \quad \text{Eigenvalue of generators.}$$

additive conservation law.

Inversion  $I$  ,  $\psi = \psi_1 \psi_2$

$$I\psi = I\psi_1 I\psi_2 = a_1 \psi_1 a_2 \psi_2 = (a_1 a_2) \underbrace{\psi_1 \psi_2}_{\psi}$$

$$\Rightarrow a_{12} = a_1 \cdot a_2$$

multiplicative.  
conservation law

	$n \rightarrow$	$p$	$e^-$	$\bar{\nu}_e$
B	1	1	0	0
L	0	0	1	-1
Q	0	1	-1	0

$$Q \rightarrow U_{EM}(1)$$

Parity

$$\gamma: P = -1$$

$$\pi^-: P = -1$$

Conventional parity in  $SM^0$

$$\ell, \nu_e, P = +1$$

$$\text{Quarks } P = +1$$

$$\text{antifermions: } P = -1$$

$$\text{antibosons: same parity as bosons}$$

Charge - Conjugation

C-parity

$$\text{particle} \longleftrightarrow \text{antiparticle}$$

$$e^+ \longleftrightarrow e^-$$

$$q^0 \longleftrightarrow ?$$

$$C\psi = a\psi$$

$$C^2\psi = a^2\psi = \psi \Rightarrow a = \pm 1$$

$$n \rightarrow p e^- \bar{\nu}_e$$

$$n \neq \bar{n}$$

$$\bar{n} \rightarrow \bar{p} e^+ \nu_e$$

neutral particles/states candidate to be eigenstate.

$$\pi^0 \rightarrow \gamma \gamma$$

Rest frame of  $\pi^0$

$$\begin{array}{ccc} \gamma & \pi^0 & \gamma \\ \leftarrow \quad \Rightarrow & \bullet & \leftarrow \quad \Rightarrow \end{array}$$

$$S\pi^0 = 0.$$

$$C \quad \Downarrow$$

$$\begin{array}{ccc} \gamma & & \gamma \\ \leftarrow \quad \Rightarrow & & \leftarrow \quad \Rightarrow \end{array}$$

$$\begin{array}{ccc} \gamma & & \gamma \\ \leftarrow \quad \Rightarrow & \bullet & \leftarrow \quad \Rightarrow \end{array}$$

$$\begin{array}{ccc} \gamma & & \gamma \\ \leftarrow \quad \Rightarrow & \Downarrow C & \leftarrow \quad \Rightarrow \end{array}$$

$$\overline{\pi^0} \equiv \pi^0$$

$$i = \pi^0$$

$$f = \gamma\gamma.$$

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$C_{\pi^0} = C_{\gamma\gamma} = (C_{\gamma})^2 = +1.$$

$$\Rightarrow G_{\pi^0} = +1$$



charge parity of pairs of particles.

boson - antiboson  $C = 1$ .

fermion - antifermion  $C = -1$

$$\psi = \psi_{\text{space}} \psi_{\text{spin}} \psi_{\text{intrinsic}}$$