Isospin

$p = |1/2, 1/2\rangle$ $\quad n = |1/2, -1/2\rangle$ $\quad$ pn, deuton $= |0,0\rangle$?

$\begin{pmatrix} p \\ n \end{pmatrix}$ doublet $I = 1/2$ $\quad |\pi^{+,0}\rangle$ = triplet $I = 1$

$p+p \to p+p$ elastic

$\xrightarrow{p}$ triplet Inelastic processes

(a)

$p + p \longrightarrow d + \pi^+$ $\quad$ a)

Q	1	1	1	1
B	1	1	2	0
I_3	$+\frac{1}{2}$	$+\frac{1}{2}$	0	+1

$0 \oplus 1 = 1$ $\quad |I, I_3\rangle$

initial $|I, I_3\rangle = |1, +1\rangle$ $\quad$ final state: $|1,1\rangle$

Strong interaction conserves Isospin.

$\langle f | H_I | i \rangle$

$\langle 1,1 | H_I | 1,1 \rangle$

(b)

$p + n \longrightarrow d + \pi^0$

Q	1	0	1	0
B	1	1	2	0
I_3	$+1/2$	$-1/2$	0	0

initial state

$I_3 = 0$ $\quad p+n$ $\quad \frac{1}{2} \otimes \frac{1}{2} = 0 + 1$ initial state.

$|p+n\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle)$

final state: $I_3 = 0$ $\quad d + \pi^0$ $\quad |0,0\rangle \times |1,0\rangle$

$|1,0\rangle$

$|i\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle) \xrightarrow{H_I} |f\rangle = |1,0\rangle$.

$\sigma_a \propto |M_a|^2 \rho(p+p \to d+\pi^+)$

$\sigma_b \propto |M_b|^2 \rho(p+n \to d+\pi^0)$

$\#\text{events} = \sigma \times \frac{dN_p}{dt} d n_B$

$$\frac{\sigma_a}{\sigma_b} = \frac{\#\, p+p \to d+\pi^+}{\#\, p+n \to d+\pi^0} = \frac{|M_a|^2}{|M_b|^2} \frac{\rho(d\pi^+)}{\rho(d\pi^0)}$$

$\frac{\Delta m_{pn}}{m_p} \simeq$ 2 MeV / 1000 MeV. $\qquad \frac{\Delta m_\pi}{m_\pi} \simeq$ 5 MeV / 135 MeV.

$$\Rightarrow \frac{\rho(\pi^+ d)}{\rho(\pi^0 d)} \simeq 1.$$

$f \qquad i$

$M_a = \langle 1,1 | H | 1,1 \rangle \qquad p+p \to d+\pi^+.$

$M_b = \langle 1,0 | H \frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle)$ $\qquad p+n \to d+\pi^0$

H_{strong} conserves isospin. $\Rightarrow$ $M_b = \frac{1}{\sqrt{2}}\langle 1,0|H|1,0\rangle + \frac{1}{\sqrt{2}}\langle 1,0|H|0,0\rangle.$

$$\Rightarrow \frac{\sigma_a}{\sigma_b} \simeq \frac{1}{\left|\frac{1}{\sqrt{2}}\right|^2} \simeq 2$$ Confirmed by $\frac{\#\pi^+ d}{\#\pi^0 d}$

Another experiment $\pi^+ + (p/n) \longrightarrow$

I I_3 $\qquad$ I I_3

$\pi^+ + p \longrightarrow \pi^+ + p$ $\qquad$ (a)

$|3/2, 3/2\rangle$ (1+1) $|1/2, 1/2\rangle$ $\qquad$ 3/2 $\quad$ 3/2

$\pi^+ + p \longrightarrow \pi^{++} + n$ $\qquad$ NO π^{++}

B	0	1		0	1
Q	+1	+1		+2	0
I_3					

$\pi^+ + p \longrightarrow n + \pi^+ \pi^+$

I_3 +1 $\quad$ 1/2 $\quad$ -1/2 $\quad$ +1 $\quad$ +1

$M = \langle f | H | i \rangle.$

$= \langle n\pi\pi | H_S | p\pi\pi \rangle$

$\pi^+ + n \longrightarrow \pi^+ + n$ (d)

$\pi^+ + n \longrightarrow \pi^0 + p$ (g)

$\pi^- + p \longrightarrow \pi^- + p$ (c)

$\pi^- + p \longrightarrow \pi^0 + n$ (j)

$\pi^- + n \longrightarrow \pi^- + n$ (f)

I_3 $\quad -1 \quad -1/2$

$-3/2 \Rightarrow |3/2, -3/2\rangle$

(c) $\quad \pi^- + p \longrightarrow \pi^- + p.$

$I_3 \quad -1 + 1/2 \qquad -1 \quad +1/2.$

for $I_3 = -1/2.$

$|\pi^- + p\rangle = \alpha\,|3/2, -1/2\rangle + \beta\,|1/2, -1/2\rangle$

Table 3.3. *Clebsch–Gordan coefficients in pion–nucleon scattering*

		$I = \frac{3}{2}$				$I = \frac{1}{2}$	
Pion	Nucleon	$I_3 = \frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
π^+	p	1					
π^+	n		$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
π^0	p		$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
π^0	n			$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
π^-	p			$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
π^-	n				1		

$$\mathcal{M} = \left(\langle 3/2, -1/2|\,\alpha + \langle 1/2, -1/2|\,\beta\right)(H_S)\left(\alpha|3/2, -1/2\rangle + b|1/2, -1/2\rangle\right)$$

H_S Conserves Isospin

$$\Rightarrow \quad \mathcal{M}_C = A\langle 3/2|H_S|3/2\rangle + B\langle 1/2|H_S|1/2\rangle$$

$$= \frac{1}{3}\mathcal{M}_{3/2} + \frac{2}{3}\mathcal{M}_{1/2}$$

$$\Rightarrow |\mathcal{M}_C|^2 = \frac{1}{9}|\mathcal{M}_3 + 2\mathcal{M}_1|^2 \qquad \pi^- + p \longrightarrow \pi^- + p.$$

(a) $\pi^+ + p \longrightarrow \pi^+ + p$. $|\mathcal{M}_a|^2 = |\mathcal{M}_3|^2$

$\mathcal{M}_a = \mathcal{M}_{3/2}$

(j) $\pi^- + p \longrightarrow \pi^0 + n$.

$\mathcal{M}_j \sim \frac{\sqrt{2}}{3}\mathcal{M}_{3/2} - \frac{\sqrt{2}}{3}\mathcal{M}_{1/2}$

$\sigma_j \propto |\mathcal{M}_j|^2 \sim \frac{2}{9}|\mathcal{M}_3 - \mathcal{M}_1|^2$

case 1)
if $|\mathcal{M}_1| >> |\mathcal{M}_3| \Rightarrow \sigma_a(\pi^+ + p \to \pi^+ + p) << \sigma(\pi^-)$

$\#(\pi^+ + p) \simeq 0$. $\Rightarrow$ not confirmed by data.

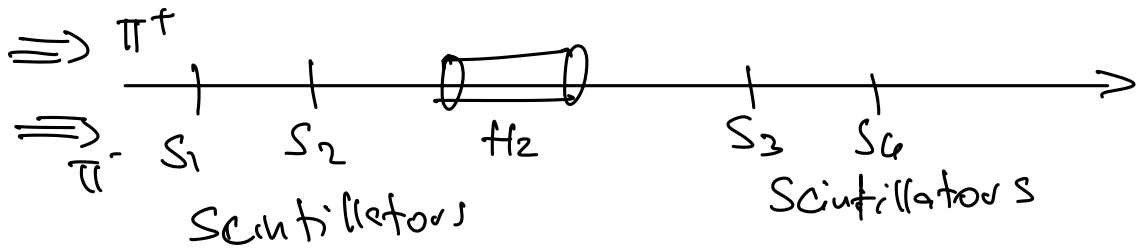
case 2)
$|\mathcal{M}_3| >> |\mathcal{M}_1|$

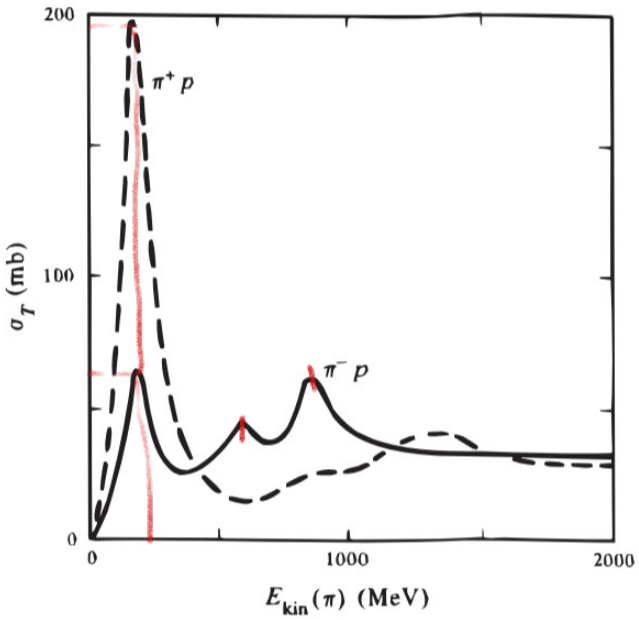
$\mathcal{M}_a \simeq \mathcal{M}_3 \Rightarrow \sigma(\pi^+ + p) \sim |\mathcal{M}_3|^2$

$\mathcal{M}_c \simeq \frac{1}{\sqrt{3}}\mathcal{M}_3 \Rightarrow \sigma_c \simeq \frac{1}{9}|\mathcal{M}_3|^2$.

$\mathcal{M}_j \simeq \frac{\sqrt{2}}{\sqrt{3}}\mathcal{M}_3 \Rightarrow \sigma_j \simeq \frac{2}{9}|\mathcal{M}_3|^2$

$$\frac{\#(\pi^+ + p \to \pi^+ + p)}{\#(\pi^- + p)} = \frac{|\mathcal{M}_3|^2}{\frac{1}{9}|\mathcal{M}_3|^2 + \frac{2}{9}|\mathcal{M}_3|^2} \simeq 3$$





$$E_{K,in} = E_\pi - m_\pi$$

Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm^2.)

	EM	Strong	Weak.
P	✓	✓	✗
C	✓	✓	✗
I		✓	
G		✓	

G parity

$\mathcal{C}|\pi^+\rangle = -|\pi^-\rangle$ $\quad C_{q\bar{q}} = (-1)^{L+S}$

NB: $\pi^+ = u\bar{d} \neq q\bar{q}$

π^{+-} are not eigenstates of $\mathcal{C}$ because Charged.

$\mathcal{C}|\pi^-\rangle = -|\pi^+\rangle$

$\mathcal{C}|\pi^0\rangle = +|\pi^0\rangle$ $\quad C_{\pi^0} = |C_\gamma|^2 = +1$

π^0 is eigenstate of $\mathcal{C}$.

$C_{\pi^0} = (-1)^{L+S} = +1$

R_2: Rotation around I_2 in isospin space.

$R_2 = e^{i\pi I_2}$

$R_2|\pi^+\rangle = (-1)^{I-I_3}|\pi^-\rangle$.

$R_2|I, I_3\rangle = (-1)^{I-I_3}|I, -I_3\rangle$

$R_2|\pi^+\rangle = (-1)^{1-1}|1,-1\rangle = +1|\pi^-\rangle$

$R_2|\pi^-\rangle = (-1)^{1-(-1)}|\pi^+\rangle = +1|\pi^+\rangle$

$R_2|\pi^0\rangle = (-1)^{1-0}|\pi^0\rangle = (-1)|\pi^0\rangle$.

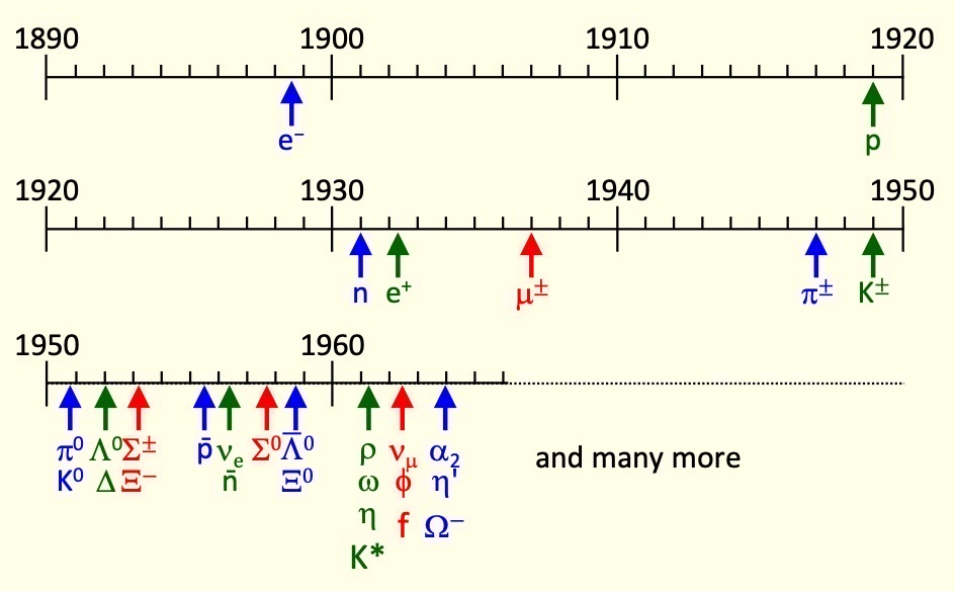
$(\mathbb{I} + R_2)|\pi^+\rangle = (-1)(+1)|\pi^+\rangle. \quad = (-1)|\pi^+\rangle$

$(\mathbb{I} + R_2)|\pi^-\rangle = (-1)(+1)|\pi^-\rangle. \quad = (-1)|\pi^-\rangle.$

$(\mathbb{I} + R_2)|\pi^0\rangle = (+1)(-1)|\pi^0\rangle. \quad = (-1)|\pi^0\rangle.$

$G = \mathbb{I} \times R_2$ Same value on Isospin multiplet

$G = (-1)^{L+S+I}$ for multiplet with L, S, I only conserved in strong interactions.



Strange particles: produced strongly but decay with long lifetime (weak interaction).

$K^{\pm}, K^0, \Lambda$

Gell-Mann–Nishijima Formula for Charge.

$Q \longrightarrow I_3$

nucleus with Z proton + A−Z neutron.

$I_3 = Z \times \left(\frac{1}{2}\right)_p + (A-Z)\left(-\frac{1}{2}\right)_n = Z\frac{1}{2} + Z\frac{1}{2} - \frac{A}{2}$

$$= Z - \frac{A}{2}.$$

$$\Rightarrow \quad Z = I_3 + \frac{A}{2}.$$

$$\downarrow$$

$$Q = I_3 + \frac{B}{2}$$

Gell Mann - Nishijima

π^+: $\quad +1 + \frac{0}{2} = +1$

Strange particles $\Rightarrow$ $\quad Q = I_3 + \frac{B+S}{2}$ $\quad Y$ Hyper Charge

Name	$\pi^{\pm}$	π^0	$K^{\pm}$	K^0	η	p	n	Λ	$\Sigma^{\pm,0}$	Δ
Mass (MeV)	140	135	494	498	548	938	940	1116	1190	1232
Charge	±1	0	±1	0	0	1	0	0	±1,0	2,±1,0
Parity	–	–	–	–	–	+	+	+	+	+
Baryon n.	0	0	0	0	0	1	1	1	1	1
Spin	0	0	0	0	0	½	½	½	½	$^3/_2$

Static Quark Model: classification of known hadrons based on symmetry.

Gell-Mann, Zweig 1961 – 1964

Eightfold Way

Isospin, Spin 1/2 particles SU(2) symmetry.

SU(N) operators:

det = 1

trace = 0

$U = e^{i a_j T_j}$ $\quad j = 1, \ldots$

T_j : generators.

Generators: $N^2 - 1$

$N-1$: are diagonal.

SU(2): 3 generators S_x, S_y, S_z $\quad I_1, I_2, I_3$

diagonal S_z $\quad I_3$

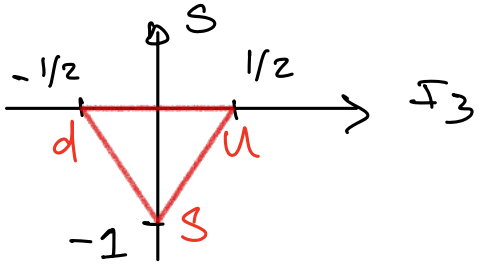
Stetic quark model : => hadrons are multiplets of SU(3)

SU(3) : 8 generators

2 diagonal : I_3, Strangeness.

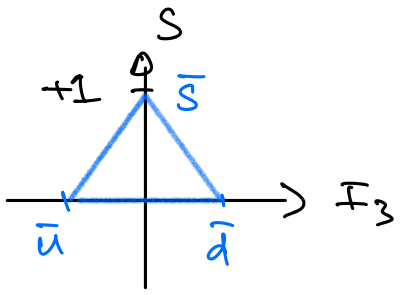
Fundamental repp. of SU(3)

$u = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $d = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $s = \begin{pmatrix}0\\0\\1\end{pmatrix}$



	I_3	S
u	+1/2	0
d	-1/2	0
s	0	-1

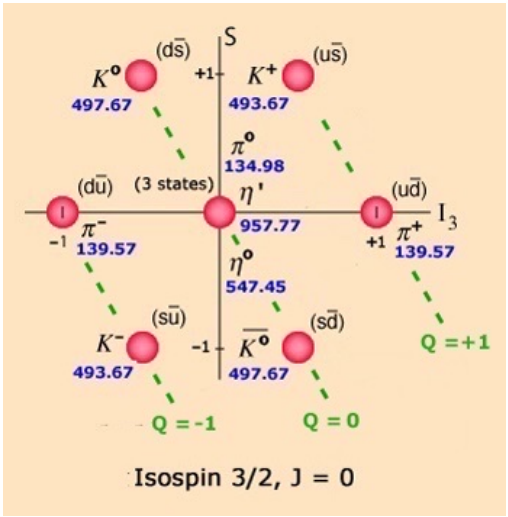
$\bar{u}\,\bar{d}\,\bar{s}$



Baryon number

$B = \frac{1}{3}$ for quarks.

$= -1/3$ for antiquarks.



Fundamental repp:

3 or $\bar{3}$

3	$\bar{3}$
u	$\bar{u}$
d	$\bar{d}$
s	$\bar{s}$

Mesons: $3 \otimes \bar{3} = 1 \oplus 8$

$u \times (\bar{u}, \bar{d}, \bar{s})$

In the middle.

$u\bar{u}, d\bar{d}, s\bar{s}$