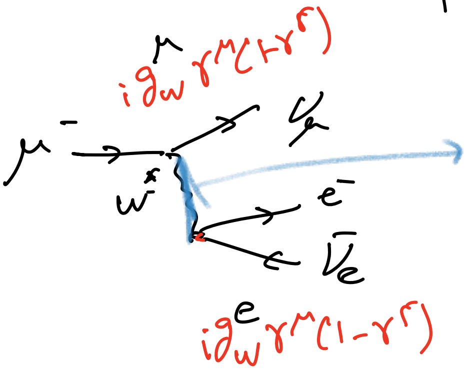
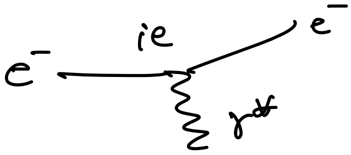
$\Gamma(\mu \to e) \propto G_F^2 \, m_\mu^5$

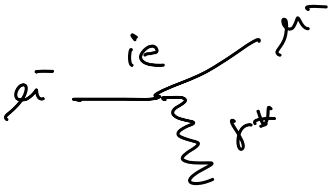


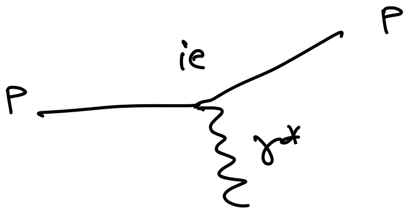
$\frac{1}{m_W^2}$ $\qquad \mathcal{M} \propto \frac{g_w^2}{m_W^2} \sim G_F$

How do we know $g_w^e \equiv g_w^\mu$

Look back at QED!



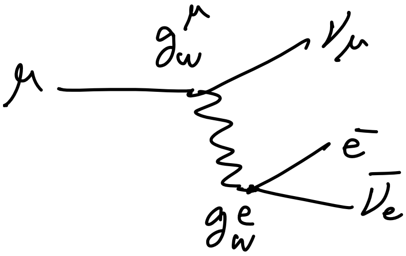




Same charge because experimentally $|Q_e| \equiv |Q_\mu| \equiv |Q_p|$

Remember $\quad P = qBR$

if $\quad g_w^e \neq g_w^\mu$

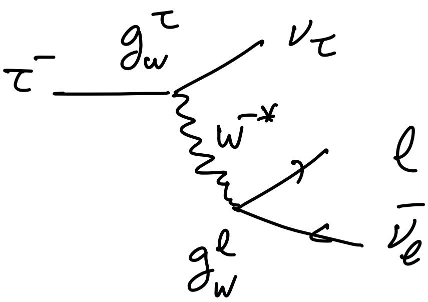


$\mathcal{M} \sim \frac{g_w^e g_w^\mu}{M_W^2}$

$$\Gamma \sim |\mathcal{M}|^2 \sim \underbrace{\left(\frac{g_w^e}{m_W}\right)^2}_{G_F^e} \underbrace{\left(\frac{g_w^\mu}{m_W}\right)^2}_{G_F^\mu} m_\mu^5 \; (\ldots)$$

How to prove $\quad g_w^e \equiv g_w^\mu = g_w^\tau \quad$ Lepton Flavor Universality of weak interactions

τ^- decays:



$\tau^- \to \nu_\tau \, e^- \, \bar{\nu}_e$

$\qquad \nu_\tau \, \mu^- \, \bar{\nu}_\mu$

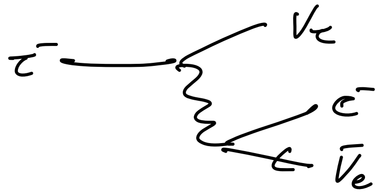
mass τ : 1.78 GeV

m_μ = 106 MeV

m_e = 0.5 MeV

$$P(W) \sim \frac{1}{q^2 - m_W^2} \sim \frac{1}{m_W^2}$$

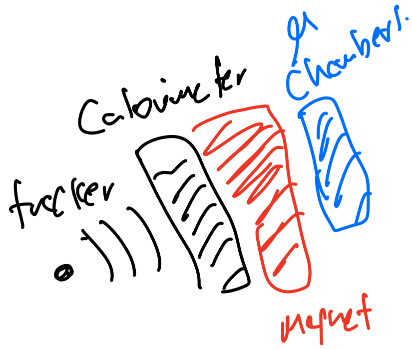
$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$



$|\mathcal{M}|^2 \sim \frac{(g_W^\tau)^2}{m_W^2} \frac{(g_W^e)^2}{m_W^2}$

$$\Gamma_{\tau \to e} = \frac{(g_W^\tau)^2 (g_W^e)^2}{(m_W)^4} m_\tau^5 \, \rho_{\text{phase space}}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$$

$$\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) \doteq \frac{(g_W^\tau)^2 (g_W^\mu)^2}{(m_W)^4} m_\tau^5 \, \rho(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$$



unstable particle: $\Gamma_{tot} = \sum_f \Gamma(i \to f)$

$$BF(\tau^- \to \mu^-) := \frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\Gamma_{tot}}$$

Experimentally measure $BF(\tau^- \to \mu^-) = \dfrac{\#\ \tau^- \to \mu^- \text{ final state events}}{\#\ \text{total } \tau^- \text{ produced}}$

$$\frac{\Gamma(\tau^- \to \mu^-)}{\Gamma(\tau^- \to e^-)} = \frac{BF(\tau^- \to \mu^-)}{BF(\tau^- \to e^-)} \frac{\cancel{\Gamma_{tot}}}{\cancel{\Gamma_{tot}}}$$

Can be measured experimentally.

$$\frac{\Gamma(\tau^- \to \mu^-)}{\Gamma(\tau^- \to e^-)} = \frac{(g_W^\mu)^2}{(g_W^e)^2} \frac{\rho(\tau^- \to \mu^-)}{\rho(\tau^- \to e^-)}$$

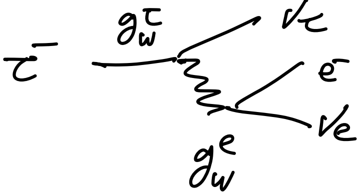
Computed analytically.

$$\Rightarrow \left(\frac{g_W^\mu}{g_W^e}\right)^2 = \frac{BF(\tau^- \to \mu^-)}{BF(\tau^- \to e^-)} \frac{\rho(\tau^- \to e^-)}{\rho(\tau^- \to \mu^-)} = 0.976$$

$$\Rightarrow \frac{g_W^\mu}{g_W^e} \simeq 1.001 \pm 0.002 \Rightarrow g_W^e = g_W^\mu$$

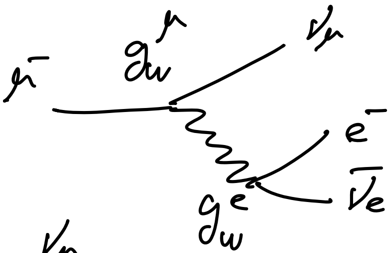
same weak interaction for $e \longleftrightarrow \mu$

How to check g_W^τ

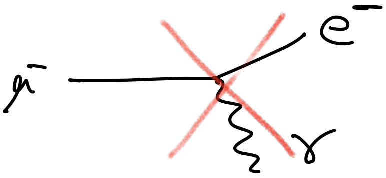


$\tau^- \to \nu_\tau \, e^- \, \bar{\nu}_e$

$\mu^- \to \nu_\mu \, e^- \, \bar{\nu}_e$



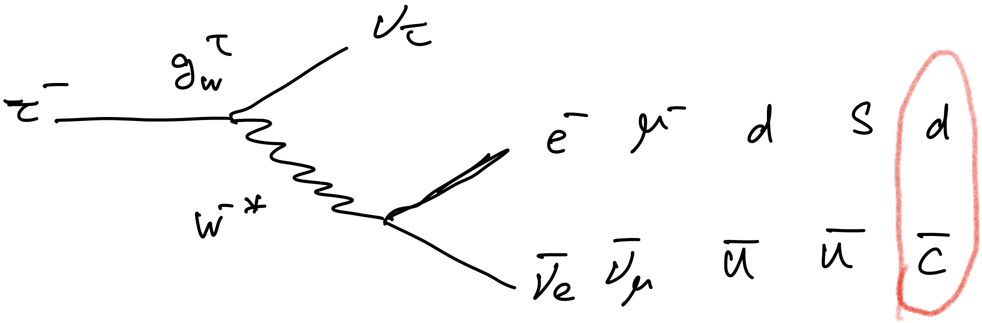
$\Gamma(\mu^- \to e^-) \simeq 100\%$



$$\Gamma(\mu \to e) \sim \frac{(g_W^\mu)^2 (g_W^e)^2}{m_W^4}$$

$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$

$m_\tau = 1.78$ GeV



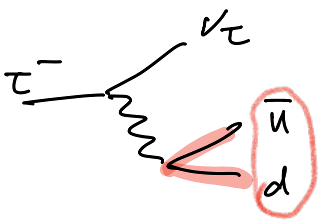
not allowed by kin.

$Q < 0$

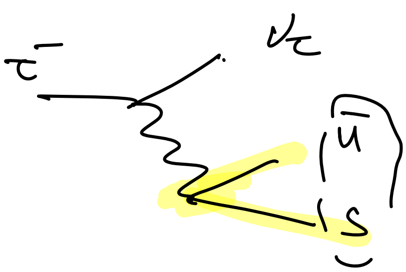
τ^-: has hadronic decays

$\Gamma(\tau^- \to e^-) \sim 17.8\%$

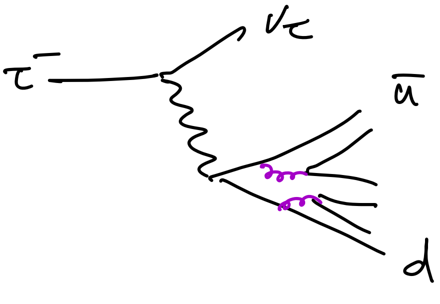
τ^- DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Modes with one charged particle				
particle$^- \geq 0$ neutrals $\geq 0K^0\nu_\tau$ ("1-prong")		(85.24 ± 0.06) %		–
particle$^- \geq 0$ neutrals $\geq 0K^0_L\nu_\tau$		(84.58 ± 0.06) %		–
$\mu^-\bar{\nu}_\mu\nu_\tau$	[g]	(17.39 ± 0.04) %		885
$\mu^-\bar{\nu}_\mu\nu_\tau\gamma$	[e]	$(3.67 \pm 0.08) \times 10^{-3}$		885
$e^-\bar{\nu}_e\nu_\tau$	[g]	(17.82 ± 0.04) %		888
$e^-\bar{\nu}_e\nu_\tau\gamma$	[e]	(1.83 ± 0.05) %		888
$h^- \geq 0K^0_L\ \nu_\tau$		(12.03 ± 0.05) %		883
$h^-\nu_\tau$		(11.51 ± 0.05) %		883
$\pi^-\nu_\tau$	[g]	(10.82 ± 0.05) %		883
$K^-\nu_\tau$	[g]	$(6.96 \pm 0.10) \times 10^{-3}$		820
$h^- \geq 1$ neutralsν_τ		(37.01 ± 0.09) %		–
$h^- \geq 1\pi^0\nu_\tau$(ex.K^0)		(36.51 ± 0.09) %		–
$h^-\pi^0\nu_\tau$		(25.93 ± 0.09) %		878
$\pi^-\pi^0\nu_\tau$	[g]	(25.49 ± 0.09) %		878
$\pi^-\pi^0$non-$\rho(770)\nu_\tau$		$(3.0 \pm 3.2) \times 10^{-3}$		878
$K^-\pi^0\nu_\tau$	[g]	$(4.33 \pm 0.15) \times 10^{-3}$		814
$h^- \geq 2\pi^0\nu_\tau$		(10.81 ± 0.09) %		–
$h^-2\pi^0\nu_\tau$		(9.48 ± 0.10) %		862
$h^-2\pi^0\nu_\tau$(ex.K^0)		(9.32 ± 0.10) %		862
$\pi^-2\pi^0\nu_\tau$(ex.K^0)	[g]	(9.26 ± 0.10) %		862
$\pi^-2\pi^0\nu_\tau$(ex.K^0),		$< 9 \times 10^{-3}$	CL=95%	862



π^- $\times$ colors $N_c = 3$



K^-



$\tau^- \to \nu_\tau +$ hadrons

$$\frac{\Gamma(\mu^- \to e^- \bar\nu_e \nu_\mu)}{\Gamma(\tau^- \to e^- \bar\nu_e \nu_\tau)} = \frac{\overset{\text{meas.}}{BF(\mu^- \to e^- \bar\nu_e \nu_\mu)}}{\underset{\text{meas.}}{BF(\tau^- \to e^- \bar\nu_e \nu_\tau)}} \frac{\Gamma^\mu_{tot}}{\Gamma^\tau_{tot}} = \frac{BF(\mu^- \to e^-)}{BF(\tau^- \to e^-)} \frac{\tau_\tau}{\tau_\mu}$$

$$\Gamma_{tot} = \frac{1}{\tau}$$ experimentally measure $\lambda = \beta\gamma c\tau$

$$= \frac{(g_w^\mu)^2}{(g_w^\tau)^2} \frac{m_\mu^5}{m_\tau^5} \underbrace{\frac{\rho(\mu \to e)}{\rho(\tau \to e)}}_{\text{computed analytically.}}$$

$$\frac{g_w^\mu}{g_w^\tau} = 1.001 \pm 0.003 \Rightarrow g_w^\tau \simeq g_w^\mu \Rightarrow$$ universality of $\tau \leftrightarrow \mu$

$$\Rightarrow g_w^\mu \equiv g_w^\tau \equiv g_w^e \equiv g_w$$

$$\Gamma_\tau^{tot} \simeq \Gamma_{\tau \to e} + \Gamma_{\tau \to \mu} + \Gamma_{ud} \times 3 \Rightarrow \Gamma_\tau \simeq 5\, \Gamma_{\tau \to e}$$

$$\frac{\Gamma_{\tau \to e}}{\Gamma_{\mu \to e}} = \frac{m_\tau^5}{m_\mu^5} \frac{G_F^2}{G_F^2} \frac{\rho_{\tau \to e}}{\rho_{\mu \to e}}$$ $\simeq 1$ $\quad m_\tau, m_\mu >> m_e$

universality

$$\Gamma_{\tau \to e} = \Gamma_{\mu \to e} \frac{m_\tau^5}{m_\mu^5}$$

$$\simeq \frac{1}{5} \Gamma_\tau^{tot} = \frac{1}{5} \frac{1}{\tau_\tau} \Rightarrow \tau_\tau = 5 \times \tau_\mu \times \left(\frac{m_\mu}{m_\tau}\right)^5$$

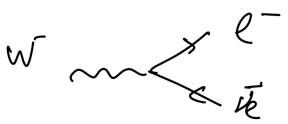
$\tau_\mu \simeq 2.2 \times 10^{-6}$ sec.

$$\frac{m_\mu}{m_\tau} = \frac{106}{1780} \frac{\text{MeV}}{\text{MeV}}$$

$$\Rightarrow \tau_\tau^{exp} = 2.9 \times 10^{-13}\ s$$

$$\tau_\tau^{PDG} = 3.1 \times 10^{-13}\ s$$ Good agreement with many approxim.

$\Rightarrow$ weak interaction for leptons



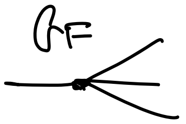
$g_w \gamma^\mu (1-\gamma^5)$

charged weak current

weak interaction of hadrons

go back to β^- decay: $\quad n \to p e^- \bar{\nu}_e$

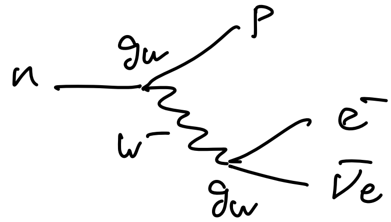
G_F



Fermi

V-A

$G_F \underline{J_{pn}} \, \underline{J_{e\nu}}$



weak propagator

$$\mathcal{M} \sim g_w^2 \, \bar{\psi}_p \gamma^\mu (1-\gamma^5) \psi_n \; \frac{1}{q^2 - m_W^2} \; \bar{\psi}_\nu \gamma^\mu (1-\gamma^5) \psi_e$$

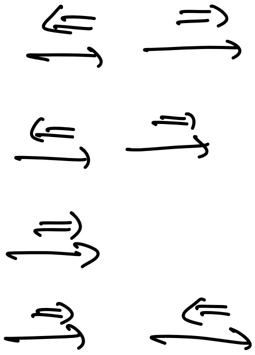
$q \simeq m_n \ll m_W$

initial state $\quad n:$

final state: $\quad p:$

$\bar{\nu}_e:$

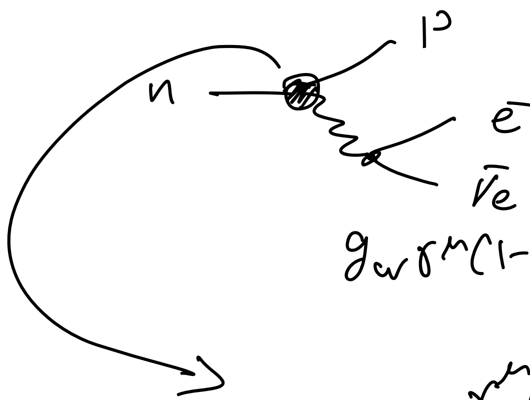
$e^-:$



$\Gamma_{tot}(n \to p e^- \bar{\nu}_e) = \Gamma_{theory} \Rightarrow \frac{1}{\Gamma} = \tau = 1318 \text{ s}$

Experimentally: $\quad \tau_n^{exp} = 878 \text{ s}$

$\Rightarrow$ hadrons behave differently.



$g_w \gamma^\mu (1-\gamma^5)$

$$\mathcal{M} \sim \frac{g_w^2}{m_W^2} \left(\gamma^\mu (c_V + c_A \gamma^5)\right)(\;)\left(\gamma^\mu (1-\gamma^5)\right)(\;)$$

$$\gamma^\mu (c_V + c_A \gamma^5)$$

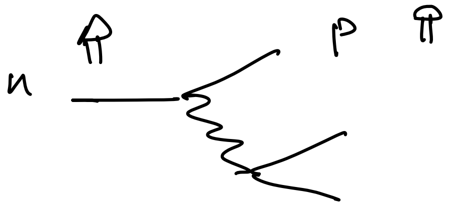
c_V: vector coupling

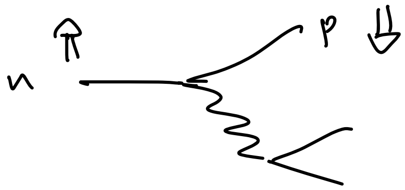
c_A: axial coupling

$${}^A_Z X \to {}^A_{Z+1} Y \, e^- \bar{\nu}_e$$

measure c_V, c_A from β^-, β^+ decays.

$$\to {}^A_{Z-1} Y \, e^+ \nu_e$$





Fermi transitions $\quad\quad\quad\quad$ Gammow-Teller transitions.

C_V $\quad\quad\quad\quad$ C_V, C_A

$$\frac{d\Gamma}{dF}(x \to y e^- \nu_e) = G_F^2 \, F(C_V, C_A).$$

Experimental picture today: $\quad C_V \simeq 1.000$

axial part C_A depends on final states.

neutron decays $\quad |C_A| = 1.267$

polarized neutron decays $\quad C_A = -1.267$

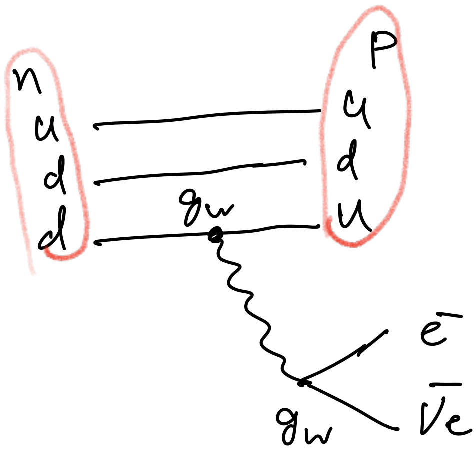
$n \to p e^- \bar{\nu}_e \quad\quad C_A/C_V = -1.276.$

$\Lambda \to p\pi^-, n\pi^0 \quad\quad = -0.718 \quad\quad \Delta S = 1$ decay.

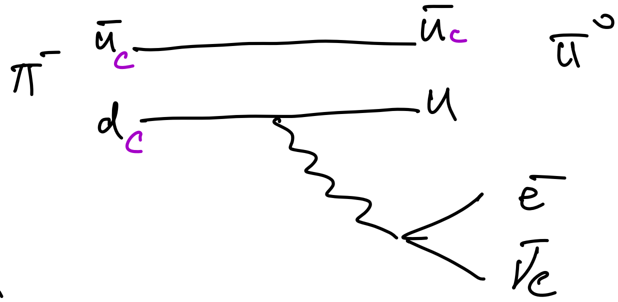
$\Sigma^- \to n e^- \bar{\nu}_e \quad\quad = +0.340$

$\Xi^- \to \Lambda e^- \bar{\nu}_e \quad\quad = -0.25$

$\Rightarrow$ better to consider weak interaction of quarks + strong corrections.



β decay ≡ semileptonic weak decay.

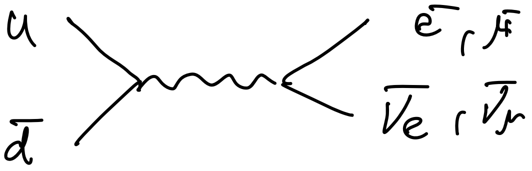


$\pi^- \to \pi^0 e^- \bar{\nu}_e$

Semileptonic

$\pi^- \to e^- \bar{\nu}_e$

$\mu^- \bar{\nu}_\mu$



$m_{\pi^\pm} = 139$ MeV $\qquad m_{\pi^0} = 135$ MeV.

$Q(\pi^- \to \pi^0 e^- \nu_e) = 3.5$ MeV.

π^+ DECAY MODES

π^- modes are charge conjugates of the modes below.

For decay limits to particles which are not established, see the section on Searches for Axions and Other Very Light Bosons.

	Mode			Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\mu^+\nu_\mu$		[a]	(99.98770 ± 0.00004) %	
Γ_2	$\mu^+\nu_\mu\gamma$		[b]	$(2.00 \pm 0.25) \times 10^{-4}$	
Γ_3	$e^+\nu_e$		[a]	$(1.230 \pm 0.004) \times 10^{-4}$	
Γ_4	$e^+\nu_e\gamma$		[b]	$(7.39 \pm 0.05) \times 10^{-7}$	
Γ_5	$e^+\nu_e\pi^0$			$(1.036 \pm 0.006) \times 10^{-8}$	
Γ_6	$e^+\nu_e e^+ e^-$			$(3.2 \pm 0.5) \times 10^{-9}$	
Γ_7	$\mu^+\nu_\mu\nu\overline{\nu}$			$< 9 \times 10^{-6}$	90%
Γ_8	$e^+\nu_e\nu\overline{\nu}$			$< 1.6 \times 10^{-7}$	90%
	Lepton Family number (*LF*) or Lepton number (*L*) violating modes				
Γ_9	$\mu^+\overline{\nu}_e$	*L*	[c]	$< 1.5 \times 10^{-3}$	90%
Γ_{10}	$\mu^+\nu_e$	*LF*	[c]	$< 8.0 \times 10^{-3}$	90%
Γ_{11}	$\mu^- e^+ e^+ \nu$	*LF*		$< 1.6 \times 10^{-6}$	90%

$\beta^+ \qquad {}^{A}_{Z}X \to {}^{A}_{Z-1}Y \; e^+ \nu_e$

$m_X = m(A,Z)$

$= Z m_p + (A-Z) m_n$

$- B(A,Z)$

Binding Energy

NOT POSSIBLE $\quad p \to n \; e^+ \nu_e$

$m_p > m_n + m_e$

$Q = m_X - m_Y - m_e =$

$= -B_X(A,Z) + B_Y(A,Z-1) - m_e$

For A,Z values with $Q>0 \Rightarrow \beta^+$ decay.

See Bethe–Weizsacker formula for nuclear binding energy and Segrè plot.