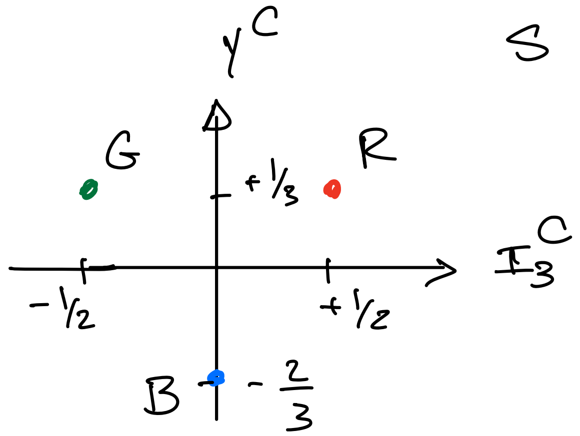
Color

Δ^{++} $u^{\Uparrow} u^{\Uparrow} u^{\Uparrow}$ $\psi = \psi_{space}\, \psi_{spin}\, \psi_{flavor}\, \psi_{color}$

S S S A

$SU(3)_C$



Y^C: color hyperchage

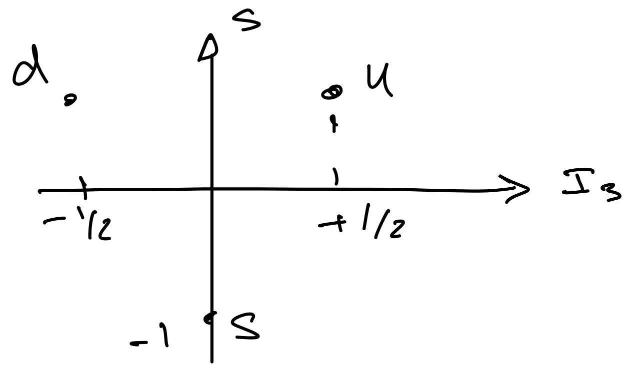
I_3^C: color Isospin

RGB

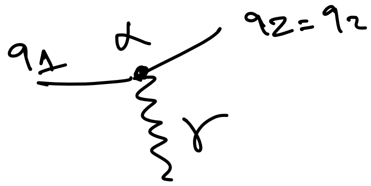
$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

SU(3)



QED



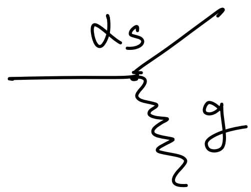
electron $q = -e$

positron $q = +e$

photon massless

$s = 1$

Quantum Chromo Dynamic



α_s: α for strong mediator.

gluons

colors: R, G, B.

anti color: $\bar{R}, \bar{G}, \bar{B}$

massless mediators. gluons

$S = 1$

8 gluons here color

gluon: Carry color and anti-color

Hypothesis/Conjecture: In nature physical particles are colorless. color singlets.

- quarks exist? u, d, s.
- color exists? only 3?
- are all physical particles colorless

} - Build observables.
- Experimental measurement.

Baryon Wave Functions De

$B = q_1 q_2 q_3$ $\quad \psi = \underbrace{\psi_{space}}_{S} \psi_{spin} \psi_{flavor} \psi_{color}$.

$L=0$ ψ_{space} symm.

Spin: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ $\quad S = 3/2$ $\quad \Uparrow\Uparrow\Uparrow \quad \Downarrow\Downarrow\Downarrow$

$\psi_{spin}^{S=3/2}$ symm. $\quad 3_F = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$

$$3_F \times 3_F \times 3_F = \underline{10_{F,S}} + \underline{8_{F,M_{12}} + 8_{F,M_{23}}} + \underbrace{1_{F,A}}_{A}.$$

M_{12}: symm. under exchange of $1 \leftrightarrow 2$.

M_{23}: $\quad 2 \leftrightarrow 3$.

$$3_C \times 3_C \times 3_C = 10_{C,S} + 8_{C,M_{12}} + 8_{C,M_{23}} + 1_{C,A}$$

color confinement $\Longrightarrow$ $1_{C,A}$.
(no colored particles in nature)

$1_{C,A}$: $\quad (RGB + GBR + BRG +$
$(-GRB - RBG - BGR)\frac{1}{\sqrt{6}} = \psi_{color}$.

$$\psi = \underset{S}{\psi_{space}} \times \underset{S}{\psi_{spin}^{3/2}} \times \underset{S_F}{\psi_{flav}} \times \underset{A}{\psi_{color}}.$$

Decuplet.

If physical decuplet exists.

$10_{F,S} \times 1_{C,A}$

$L=0$

$S=3/2$. $\Uparrow\Uparrow\Uparrow$.

Flavor: $\mathbb{1}_{F,A}$. Flavor singlet $= (uds + dsu + sud - dus - sdu - usd)\frac{1}{\sqrt{6}}$

$1\leftrightarrow 2 \quad 1\leftrightarrow 3 \quad 2\leftrightarrow 3$

Color: $\mathbb{1}_{C,A}$.

$$\underset{S}{\psi} = \underset{S}{\psi_{sp.}} \times \underset{S}{\psi_{spin}} \times \underset{A}{\psi_F} \times \underset{A}{\psi_C}.$$

$S \Rightarrow$ Cannot exist

Baryon Octet

$$3^F \times 3^F \times 3^F = 10^F_S + 1^F_A + 8^F_{M12} + 8^F_{M23}$$

Flavor.

$L=0 \Rightarrow \psi_{space}$ symm.

color singlet $\psi_{color} = \mathbb{1}_C$.

$$\underset{A}{\psi} = \underset{S}{\psi_{space}}\ \underset{A}{\psi_{color}}\ \underbrace{\psi_{spin}\ \psi_{Flavor}}_{\text{symmetric}}.$$

Spin: $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \left(\frac{3}{2}\right)_S + \left(\frac{1}{2}\right)_{M12} + \left(\frac{1}{2}\right)_{M23}$

$\Uparrow\Uparrow\Downarrow \xleftrightarrow{12} \Uparrow\Uparrow\Downarrow$ $\quad M_{12}$

$\Uparrow\Downarrow\Downarrow \xleftrightarrow{23} \Uparrow\Downarrow\Downarrow$ $\quad M_{23}$

$M_{13} = M_{12} + M_{23}$ Exercise with clebsch-Gordon Coeff.

$$\underbrace{\psi_{spin}\psi_{Flavor}}_{\text{symm.}} = \psi^{M12}_{S=1/2}\psi_{F,M12} + \psi^{M23}_{S=1/2}\psi_{F,M23} + \psi^{M13}_{S=1/2}\psi_{F,M13}$$

Baryon Octet: $\underbrace{\psi_{space}\,\psi_c}_{A}\ \underbrace{\psi_{spin}\,\psi_{Flav.}}_{S}$

Meson Wave Function

$M = q_1 \bar{q}_2 \qquad 3 \times \bar{3} = 1 + 8$

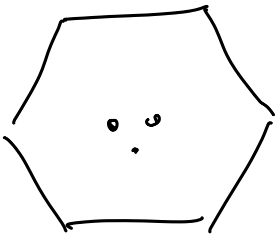
$3_F \times \bar{3}_F = 8_F + 1_F$

$3_C \times \bar{3}_C = 8_C + 1_C$

$$\psi_{Meson} = \underset{\substack{S \\ Y_{00}}}{\psi_{space}}\ \psi_{spin}\ \psi_{Flav}\ \underset{\text{color singlet}}{\psi_{color.}}$$

$S = 0$ pseudoscalar mesons.

$S = 1$ vector mesons.



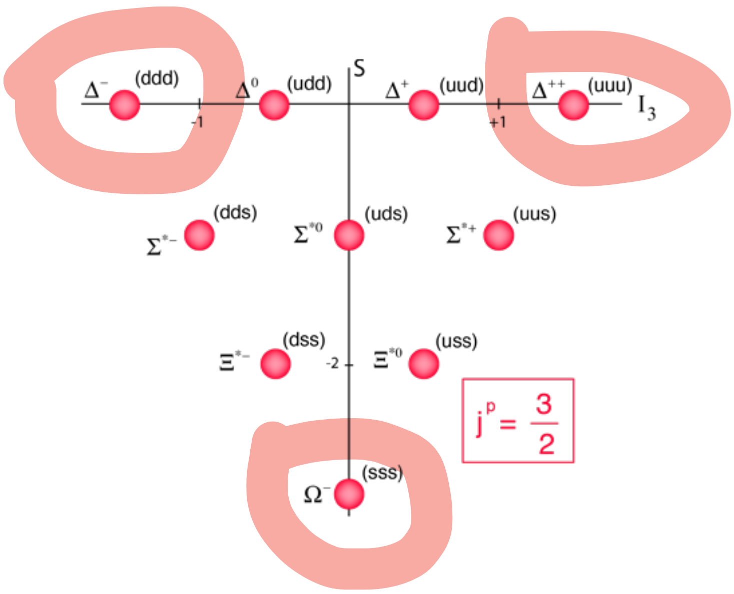
$$\psi^A_{color} = \frac{1}{\sqrt{3}}\left(R\bar{R} + B\bar{B} + G\bar{G}\right)$$

$R + G + B = 0.$

$R + \bar{R} = 0.$

$B + \bar{B} = 0.$

$G + \bar{G} = 0$



Gluons

vector bosons.

$S = 1$. electric charge $q=0$

massless.

Colored.

- gluons carry both color and anti-color

$$3_c \times \bar{3}_c = 1 + 8$$

Octet: $R\bar{G}, R\bar{B}, G\bar{R}, G\bar{B}, B\bar{R}, B\bar{G}, \frac{1}{\sqrt{2}}(R\bar{R} - B\bar{B}),$

$$\frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} - 2B\bar{B})$$

Singlet $\frac{1}{\sqrt{3}}(R\bar{R} + G\bar{G} + B\bar{B})$

γ 

$$\frac{-g^{\mu\nu}}{q^2} \qquad \frac{1}{r} \quad \text{potential.}$$

mom. space $\qquad$ x space

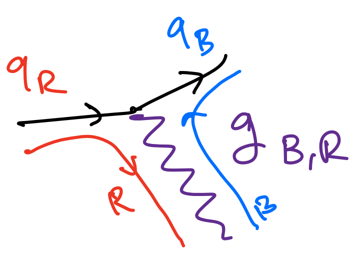
g 

$$\frac{-g^{\mu\nu} + (1-\delta)(p^\mu p^\nu - \ )}{q^2} \qquad \frac{1}{r}$$

Assume colorless gluon does not exist.

$\Rightarrow$ 8 colored gluons.

$\Rightarrow$ no free gluon or quark.



$e^+ + e^- \longrightarrow \qquad q_R \qquad \bar{q}_{\bar{R}}$

$p + p \longrightarrow$

$p + \bar{p} \longrightarrow$

$e^- + p \longrightarrow$

$\nu_e + p \longrightarrow$

Initial color
$=$
0

$$e^+ + e^- \longrightarrow \underbrace{\bar{q}_2 q_R}_{\mu} + \underbrace{q_2 \bar{q}_{\bar{R}}}_{\mu}$$

$$e^+ + e^- \longrightarrow \pi^+ + \pi^-$$

$$u_R \bar{d}_{\bar{R}} + \bar{u}_{\bar{R}} d_R$$

$$\underset{-1}{\bar{d}_{\bar{G}} s_G} \qquad \underset{+1}{d_G \bar{s}_{\bar{G}}} = 0$$

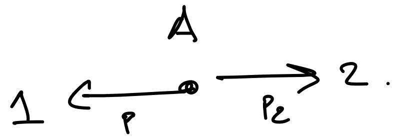
Strangeness 0

strangeness $\cancel{\begin{matrix} s_G & \bar{u}_{\bar{R}} \\ -1 & 0 \end{matrix}}$

$$Q = I_3 + \frac{B + S + C + \text{Beauty}}{2}$$

Decays

$A \to 1 + 2$



Rest frame of A

$$E_1^* = \frac{m_A^2 + m_1^2 - m_2^2}{2m_A} \qquad E_2^* = \frac{m_A^2 + m_2^2 - m_1^2}{2m_A}$$

$$p_1^* = p_2^* = p = \frac{\sqrt{m_A^4 + (m_1^2 - m_2^2)^2 - 2m_A^2(m_1+m_2)^2}}{2m_A}$$

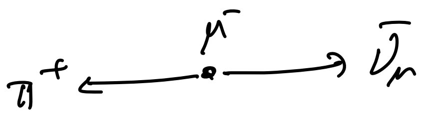
$$p^{*2} = E_1^2 - m_1^2$$

$A \to 1 + 2 + 3$

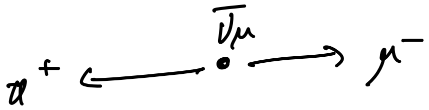
$K^0 \to \pi^+ \mu^- \bar{\nu}_\mu$

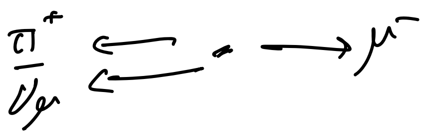
$K^0 \to \pi^+ \pi^-$

$$E_\pi = \frac{m_K^2 + \cancel{m_\pi^2} - \cancel{m_\pi^2}}{2m_K} = \frac{m_K}{2}$$



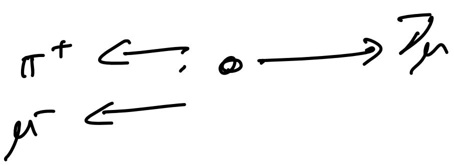
$E_\mu \simeq 0.$





$$E_\mu = \frac{m_K^2 + m_\mu^2 - (m_\pi + m_\nu)^2}{2m_K}$$

$m_\nu \simeq 0.$

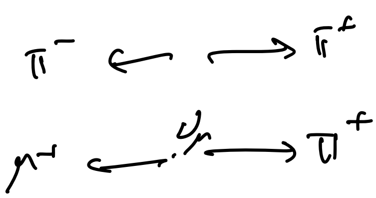


$$\Rightarrow E_\mu^{max} \simeq \frac{m_K^2 + m_\mu^2 - m_\pi^2}{2m_K}$$

$K^0 \to \pi^+ \pi^-$ hadronic decay.

$\pi^+ \mu^- \bar{\nu}_\mu.$ Semi-leptonic decay.

if $\mu^+ \mu^-$ leptonic decay.



similar experimentally.