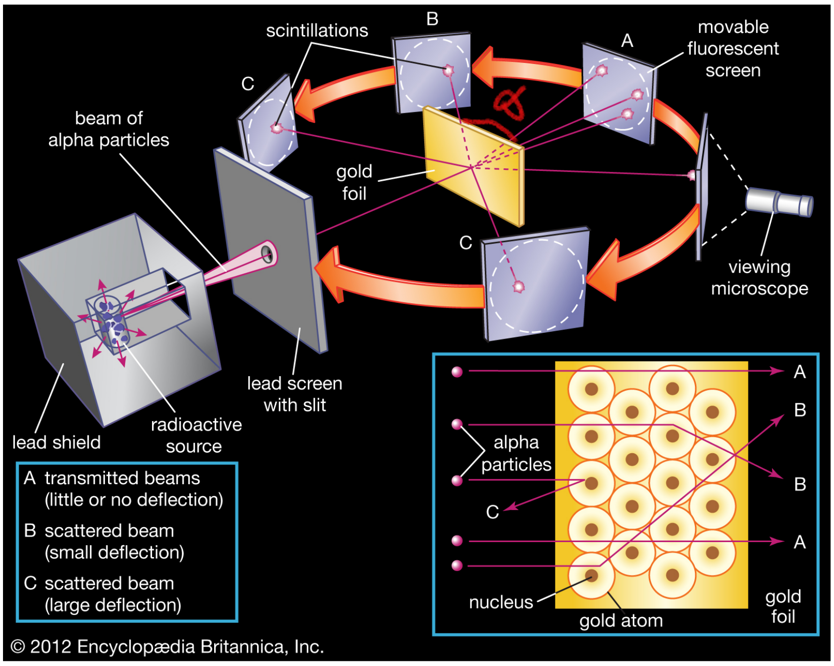
Scattering → nucleon structure → quarks

$1+2 \rightarrow 3+4.$

$\alpha + N \rightarrow \alpha + N.$

$E, \vec{p} \longrightarrow$ ⊗ θ $\rightarrow E', p'$

1910



$$\frac{d\sigma}{d\Omega} \propto \frac{\#\text{events}}{\underbrace{\Delta\Omega}_{\text{detector coverage}}} \text{ vs. } \theta.$$

$$d\Omega = \sin\theta \, d\theta \, d\varphi.$$

$$= 2\pi \sin\theta \, d\theta.$$

$\Rightarrow R_N < 10 \text{ fm}.$

$1 \text{ fm} = 10^{-15} \text{ m}$

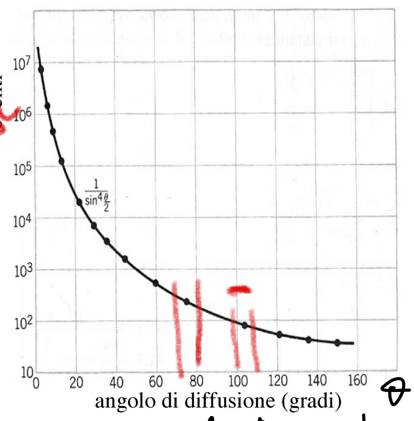
$R_N = r_0 A^{1/3}$ $\quad r_0 \sim 1 \text{ fm}$ $\quad$ nuclear radius

$$\frac{d\sigma}{d\Omega} = \underbrace{(\ldots)(Z_p Z_T \alpha)^2 \frac{1}{q^4} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} E'^2}_{\text{Rutherford}} \underbrace{\left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)}_{\text{Mott}}$$

α: $K \simeq 5$ MeV, $M = 3.7$ GeV $\Rightarrow \frac{|P_{out}|}{|P_{in}|} \simeq 1$ negligible recoil.

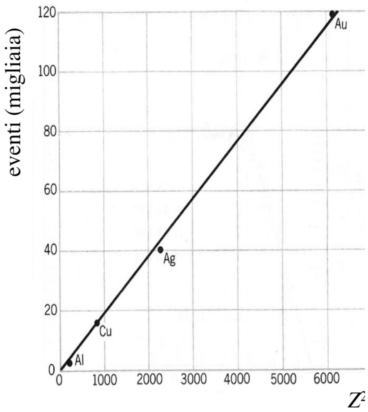
$$\beta \ll 1 = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \sim \frac{1}{(P_{in})^4 \sin^4\frac{\theta}{2}}$$

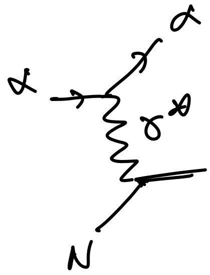
$\#$ events $\propto \frac{d\sigma}{d\Omega}$

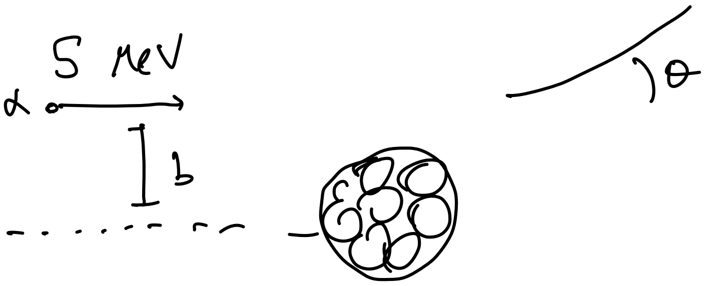


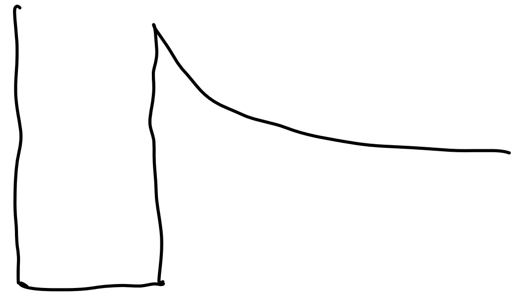
θ

Deflection angle.



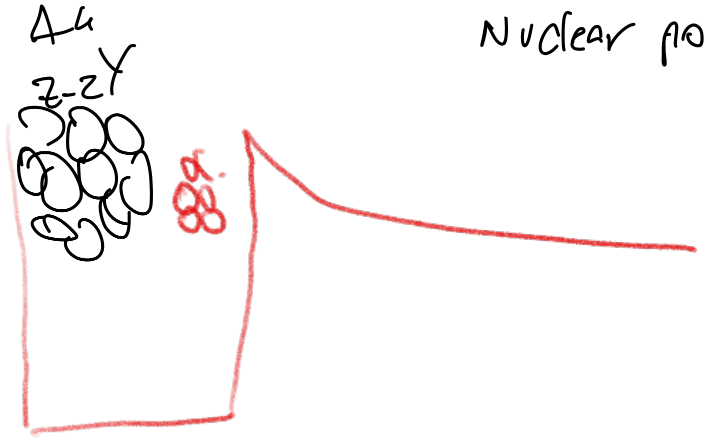






Nuclear potential well.

$^{A}_{Z}X \rightarrow \, ^{A-4}_{Z-2}Y + \alpha.$



Fermi Gas Model for nucleus ⟹ Fermi Energy.

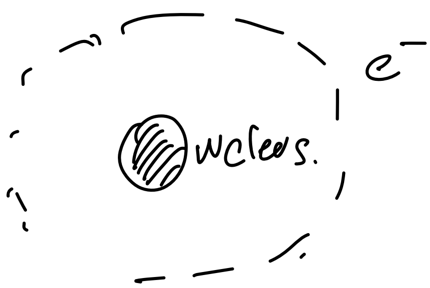
$$E_F = \frac{P_F^2}{2m_N} \qquad P_F \simeq 200 \text{ MeV} \Rightarrow E_F \simeq 20 \text{ MeV}$$

$$\Delta P \cdot \Delta x \simeq 1 \qquad \hbar c = 200 \text{ MeV} \cdot \text{fm} = 1.$$

$$\Rightarrow 200 \text{ MeV} \simeq 1 \text{ fm}^{-1}$$

See nucleus structure ⟹ probes of 20–30 MeV.

Rutherford Model:



$$\frac{d\sigma}{d\Omega} \propto \frac{\alpha^2 (Z_e Z_T)^2}{P_{in}^4 \sin^4\frac{\theta}{2}} E'^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

Pointlike probe on pointlike target

$$\mathcal{M} = \langle f | H_I | i \rangle$$

$$H_I = EM \text{ (Coulomb)}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{meas} \neq \left.\frac{d\sigma}{d\Omega}\right|_{expected} \Rightarrow H_I = ? \text{ or not pointlike}$$

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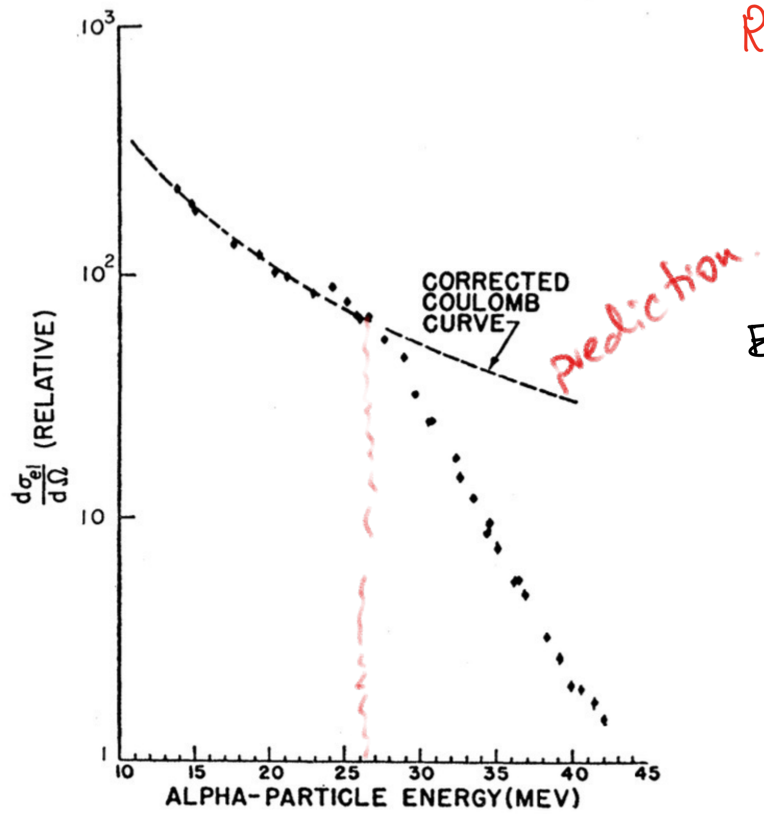
$\alpha + N \rightarrow \alpha + N$



$E \geq 25$ MeV.

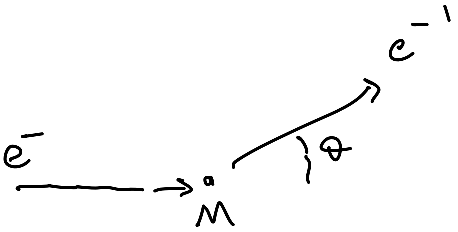
FIG. 5. Differential cross section for the elastic scattering of alpha particles by Pb at 60° as a function of the alpha-particle energy.

$\Rightarrow$ Move to high energy electron beam ~ 1960

$e^- + N \longrightarrow e^- + N$ elastic

$e^- + N^*$ excitation

$e^- + H$ inelastic



observables: E', θ of deflected electron.

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} \sim \alpha^2 \frac{1}{q^4} E'^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$ Mott Formula

Rutherford: pointlike probe (α particle)

Mott: takes into account spin.

ultrarelativistic limit: $E_e \gg m_e \Rightarrow \beta = 1$

$$\Rightarrow \frac{d\sigma}{d\Omega} \propto \frac{\alpha^2}{q^4} E'^2 \underbrace{\left(1 - \sin^2\frac{\theta}{2}\right)}_{\cos^2\frac{\theta}{2}}$$

$\theta = 0$ Max $\frac{d\sigma}{d\Omega}$ $e^- \dashrightarrow \bullet \dashrightarrow e^-$

$\theta = \pi$ $\frac{d\sigma}{d\Omega} \sim \cos^2\left(\frac{\pi}{2}\right) = 0.$ $\xrightarrow{e^-} \bullet$ $\xleftarrow[e^-]{}$

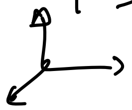
$\overset{\Leftarrow}{\longrightarrow} e^-$ $\bullet$ $e^- \longleftarrow$ $\bullet$

initial state

$\beta = \frac{p}{E}$

helicity : $h = \frac{\vec{P} \cdot \vec{S}}{|\vec{P}| \cdot |\vec{S}|}$ $\overset{\Leftarrow}{\longrightarrow} e^-$ $h = -1$ LAB.

$\beta' > \beta_e$



$\overset{\Leftarrow}{\longleftarrow}$ $h = +1$.

Boosted ref frame

massless particle : $\beta = 1$ in all ref. frames.

γ : $\overset{\vec{S}}{\overset{\Leftarrow}{\longrightarrow}} P$ helicity is Lorentz invariant

$\Rightarrow$ intrinsic property of the particle.

Chirality : $\psi(\vec{r}) = \underbrace{\frac{1}{2}(1+\gamma^5)}_{P_L}\psi + \underbrace{\frac{1}{2}(1-\gamma^5)}_{P_R}\psi$

For ultrarelativistic particles : helicity $\simeq$ Chirality.

e^- : $\beta \simeq 1$ $\overset{\Leftarrow}{\longrightarrow} e^-$ initial $e^- \overset{\Rightarrow}{\longleftarrow} \bullet$ after collision.

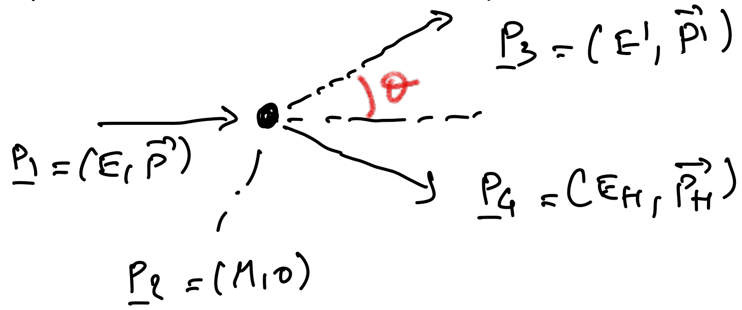
$\Delta J = 1$ $\Rightarrow$ $\theta = \pi$ not possible

in $e^- + N \rightarrow e^- + N$ for ultrarelativistic electrons.

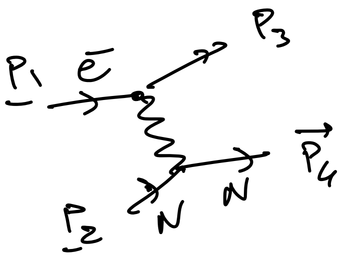
$\frac{d\sigma}{d\Omega} \sim \frac{\alpha^2}{4q^4} E'^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$ includes spin.

But no recoil for target

spin 1/2 relativistic probe on target (no recoil)



$e^- + N \rightarrow e^- + X$



elastic limit: $\underline{P}_4 = (M, \vec{P}_4 \simeq 0)$

Observables: E, E', θ

$$\underline{P}_1 + \underline{P}_2 = \underline{P}_3 + \underline{P}_4 \Rightarrow \underline{P}_4 = \underbrace{\underline{P}_1 - \underline{P}_3}_{\underline{q}} + \underline{P}_2$$

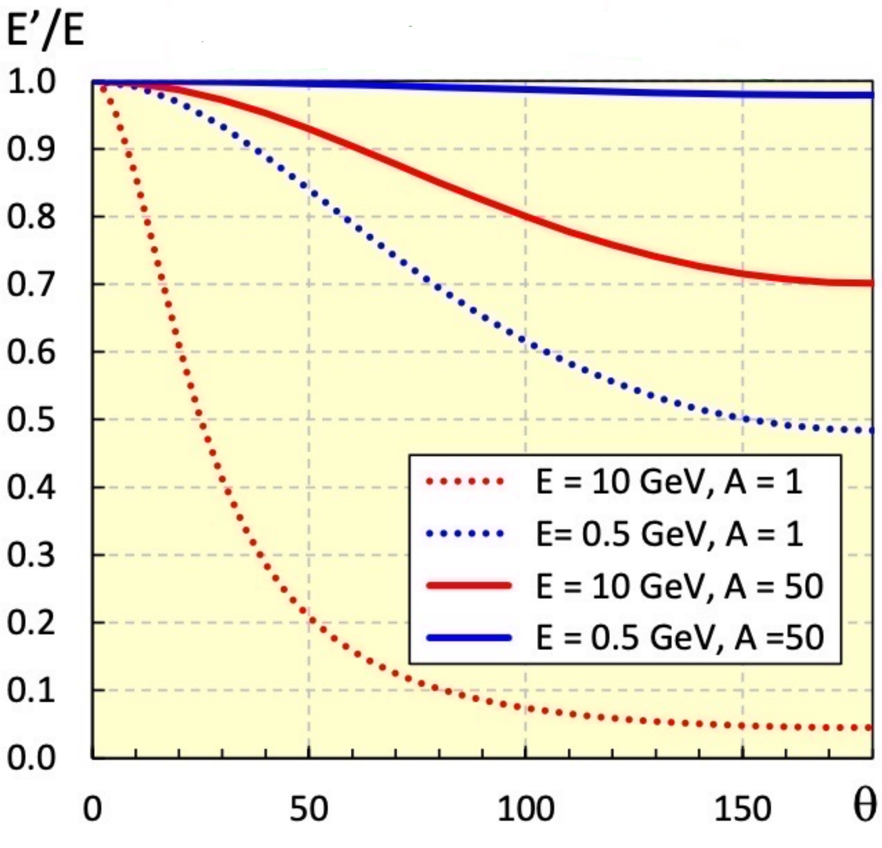
$$|\underline{P}_4|^2 = M^2$$

$$(\underline{P}_1 - \underline{P}_3 + \underline{P}_2)^2 = m_e^2 + M^2 + m_e^2 - 2\,\underline{P}_1 \cdot \underline{P}_3 + 2\,\underline{P}_1 \cdot \underline{P}_2 - 2\,\underline{P}_3 \cdot \underline{P}_2$$

$$E' = \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)} = \frac{E}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}}$$

$E_e \simeq 25$ MeV. $\gg m_e$

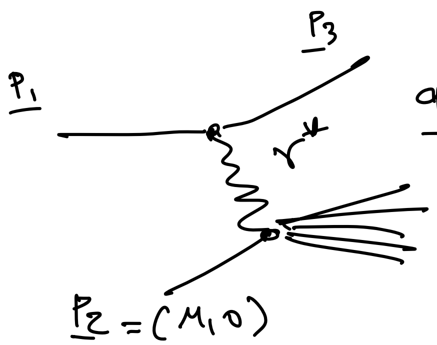
$m_e^2 \simeq 0.$



elastic scattering

Inelastic limit

$e^- + N \to e^- + X$ (more than 1 particle)

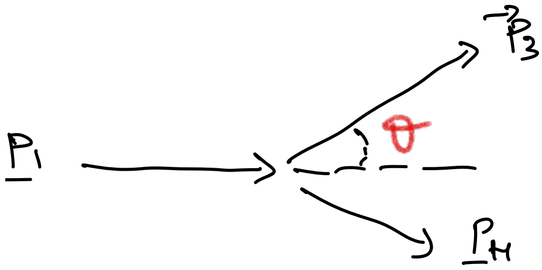


$\underline{P}_3 = (E', \vec{p}')$

$\underline{q} = \underline{P}_1 - \underline{P}_3$

$\underline{P}_4 = \underline{P}_H = \Sigma_i \underline{P}_{H_i}$ (particles in final state)

$\underline{P}_4 = (E_H, \vec{P}_H)$



$$q^2 = 2m_e^2 - 2(EE' - EE'\cos\theta) \simeq -4EE'\sin^2\frac{\theta}{2}$$

$E, E' > 0$ $\quad q^2 < 0$ by definition $\quad$ virtual photon

$Q^2 =: -q^2 = t$ mandelstam variable $\quad t = -(\underline{P}_1 - \underline{P}_3)^2$

$s = (\underline{P}_1 + \underline{P}_2)^2$

$\underline{P}_4 = \underline{P}_2 + \underline{q}$

square both sides.

$|\underline{P}_4|^2 = |\underline{P}_H|^2 = W^2$

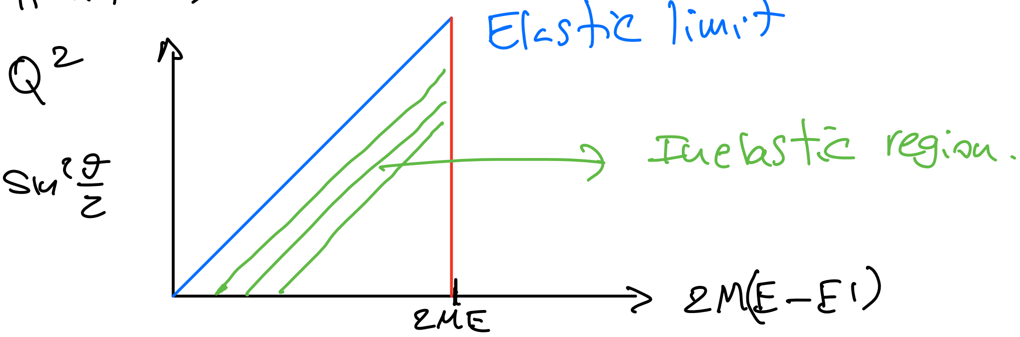
invariant mass of all hadronic particles.

$$(\underline{P}_2 + \underline{q})^2 = M^2 - Q^2 + 2M\underbrace{(E - E')}_{\nu} \qquad \nu =: E - E'$$

$$\boxed{W^2 = M^2 - Q^2 + 2M\nu}$$ general case inelastic.

$e^- + N \to e^- + X$

elastic limit $\Rightarrow$ $W^2 = M^2 \Rightarrow Q^2 = 2M\nu = 2M(E - E')$



$Q^2 = 4EE'\sin^2\frac{\theta}{2}$

inelastic: $W^2 > M^2 \Rightarrow Q^2 = \underbrace{M^2 - W^2}_{<0} + 2M\nu$.