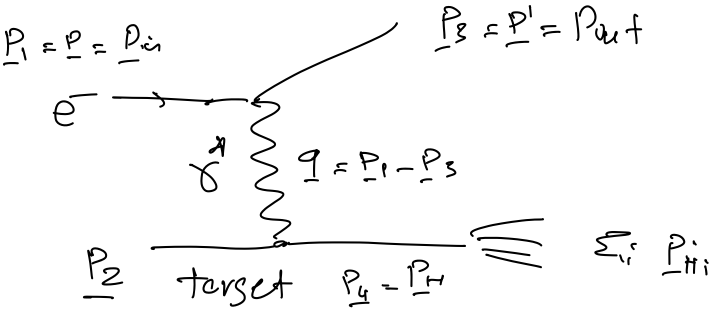
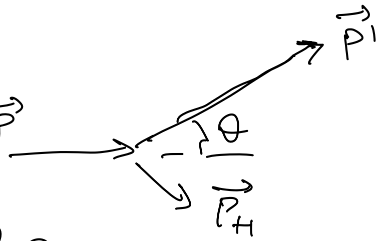


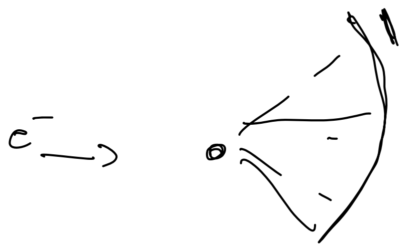
$e^- + N \to e^- + N / N^* / H.$

$P_1 = (E, \vec{P})$ $\quad P_3 = (E', \vec{p}')$

$P_2 = (M, 0)$ $\quad P_4 = (P_H, E_H)$



Elastic scattering.
$$E' = \frac{E}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}}$$



$\frac{d\sigma}{d\Omega} \sim$ # events. as function of θ.

measure E', θ

$q = P_1 - P_3$ transfered 4 momentum.

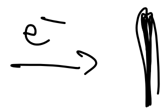
$$q^2 = m_e^2 + m_e^2 - 2(EE' - EE'\cos\theta) \approx -4EE'\sin^2\frac{\theta}{2}.$$

$E, E' >> m_e$ $\qquad E, E' > 0.$ $\qquad \sin^2\frac{\theta}{2} > 0\ \forall\theta.$

$t = Q^2 =: -q^2 \geq 0 \quad \longleftrightarrow \quad \gamma^*$ virtual.

Mandelstam variable.

σ: Lorentz invariant



$\frac{d\sigma}{d\Omega}$: invariant?
No!

$d\Omega = d\cos\theta\, d\varphi.$

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}} = \frac{EM}{M + 2E\sin^2\frac{\theta}{2}} = \frac{EM}{M + 2\frac{EE'}{E'}\sin^2\frac{\theta}{2}}$$

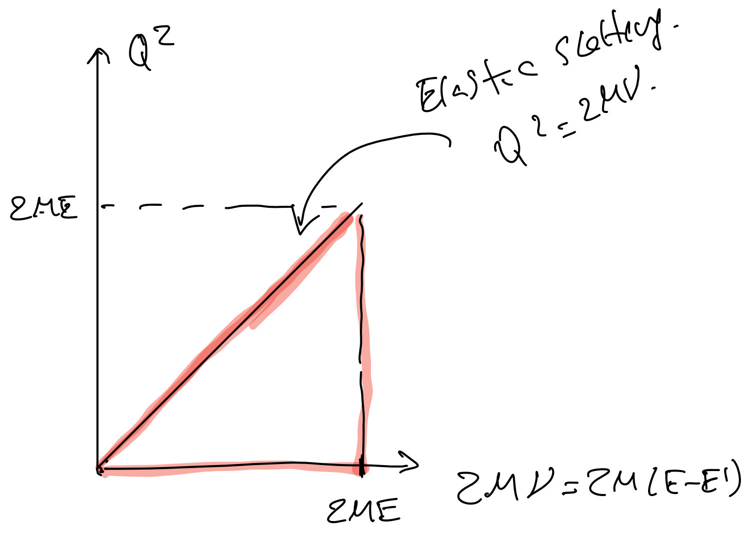
$$E'M + 2EE'\sin^2\frac{\theta}{2} = EM.$$

$$\underbrace{2EE'\sin^2\frac{\theta}{2}}_{\frac{Q^2}{2}} = EM - E'M =$$

$$Q^2 = 2M(E - E')$$
$$= 2M\nu.$$

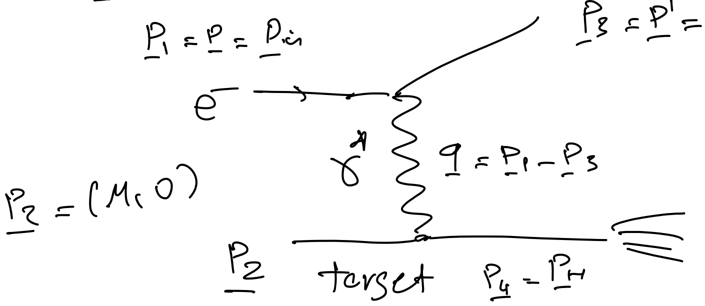
$\nu =: E - E'$

$(E', \theta) \to (Q^2, \nu)$



$E = E' \to Q^2 = 0.$

Inelastic scattering.



ν

$q = (E - E', \vec{p} - \vec{p}')$

$\sum_i p_{Hi} \Big\} \; W^2 = E_H^2 - (\vec{p}_H)^2.$

$$Q^2 = -q^2 = 4EE' \sin^2\frac{\vartheta}{2}.$$

$$\nu = E - E'.$$

$$W^2 = p_H^2 = (p_2 + q)^2 = p_2^2 + q^2 + 2\, p_2 \cdot q =$$

$$= M^2 - Q^2 + 2M(E - E') = M^2 - Q^2 + 2M\nu.$$

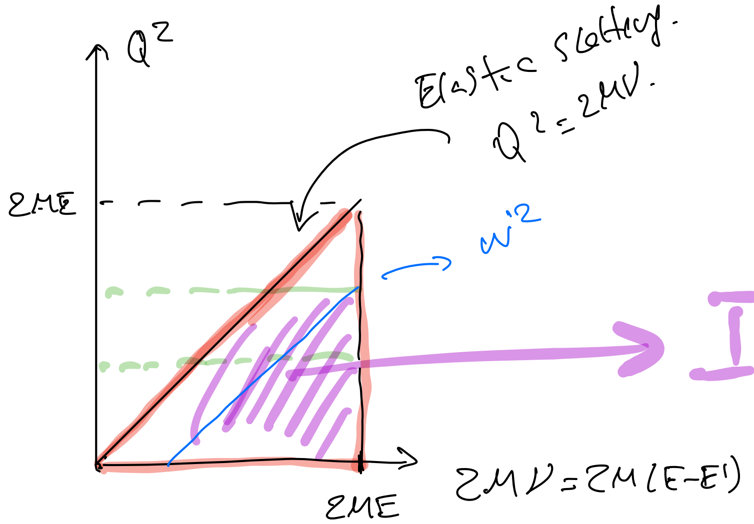
$e^- p \to e^- p.$ elastic $\quad W^2 = m_p^2$

$e^- p \to e^- p \, \pi^0 \qquad W^2 \geq m_p^2$

$\qquad e^- p \, \pi^+ \pi^-$

$$\boxed{W^2 = M^2 - Q^2 + 2M\nu.} \Rightarrow Q^2 = M^2 - W^2 + 2M\nu. \quad \text{Inelastic.}$$

Elastic limit: $W^2 \to M^2 \Rightarrow Q^2 = 2M\nu.$



$$Q^2 \geq 0 \Rightarrow M^2 - W^2 + 2M\nu \geq 0. \Rightarrow W^2 \leq M^2 + 2M\nu.$$

$$s = (p_1 + p_2)^2 = m_e^2 + M^2 + 2\, p_1 \cdot p_2 = m_e^2 + M^2 + 2ME \simeq M^2 + 2ME.$$

Max of $M^2 + 2M\nu$.

$$\text{Max } \nu = \max(E - E') = E.$$

$W^2 \le M^2 + 2M\nu \le M^2 + 2ME = s$

(E', θ), (Q^2, ν).

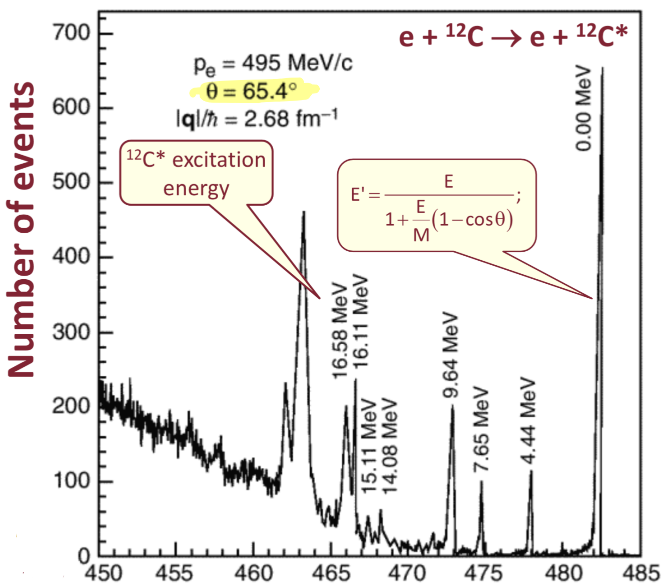
$$x = \frac{Q^2}{2M\nu} = \frac{M^2 - W^2 + 2M\nu}{2M\nu} = 1 + \frac{M^2 - W^2}{2M\nu}.$$

$M^2 - W^2 \le 0. \Rightarrow 0 \le x \le 1.$ Bjorken x

$$y = \frac{E - E'}{E} = \frac{\nu}{E}$$ fraction of energy lost by incoming probe e^-.

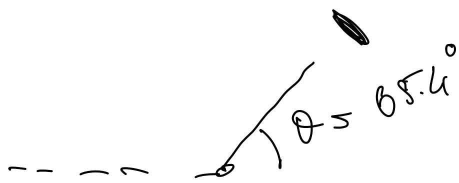
$0 \le y \le 1.$

$(E', \theta) \longrightarrow (Q^2, \nu) \longrightarrow (x, y)$



$e^- + {}^{12}C \longrightarrow e^- + {}^{12}C$

$e^- + {}^{12}C^*$



$\frac{d\sigma}{d\Omega}$ @ 65.4°

E'[MeV]. energy of deflected e^-

$E = 495$ MeV.

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$ for elastic.

$E \doteq 0.5$ GeV.

$M \simeq 12 m_p \simeq 12$ GeV.

$$\simeq \frac{E}{1 + 0.02} \simeq (1 - 0.02)E. \simeq 485 \text{ MeV}$$

1960's at SLAC a series of $e^- + p/n \longrightarrow e^- + X$ experiments.

$e^- p \to e^- \Delta \pi\pi$

$e^- \Delta$

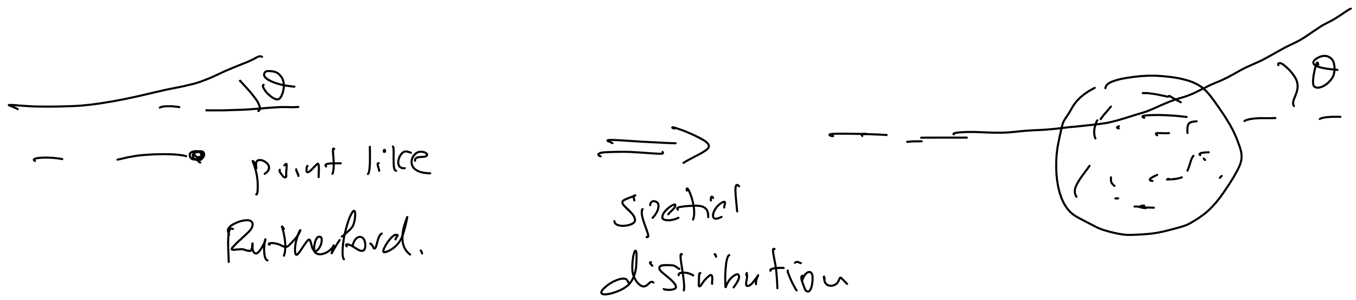
$e^- p \pi\pi$

Rutherford $\left.\frac{d\sigma}{d\Omega}\right|_{Ruth.} \simeq \frac{\alpha}{q^2} E'^2$ relativistic $e^- + T \to e^- + T$.

spin of e^- $\left.\frac{d\sigma}{d\Omega}\right|_{MOtt} = \left.\frac{d\sigma}{d\Omega}\right|_{Ruth.} \times \underbrace{(1 - \beta^2 \sin^2\frac{\theta}{2})}_{\text{Because of spin.}}$

1) Take into account spin of target. spin 1/2 n, p.
$\Rightarrow$ Rosenbluth cross section.

2) what if spatial charge distribution in target



Rutherford. $\sigma \sim |M|^2$.

$M = \langle f | H | i \rangle$.

$H = \frac{q_p q_T}{r}$ $\qquad$ • $Z_T e = q_T$ $\qquad V(\vec{r}) = \frac{Z_T e}{r}$

Born approx: incoming particle. $\underline{\vec{P}_{in} = \vec{p}}$ $\longrightarrow$ $\vec{P}_{out} = \vec{p}'$

$\psi_{in}(x) = \frac{1}{\sqrt{V}} e^{-i\vec{p}\cdot\vec{x}}$ plane wave approx. for free particle.

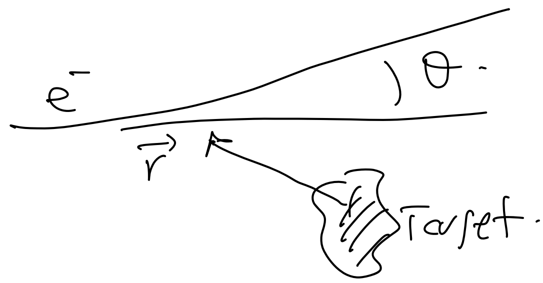
$\psi_{fin}(x) = \frac{1}{\sqrt{V}} e^{-i\vec{p}'\cdot\vec{x}}$

$$M \sim \frac{Z_p Z_T e^2}{\sqrt{V}\sqrt{V}} \int d^3r \, \frac{e^{-i\vec{p}\cdot\vec{r}}}{r} e^{+i\vec{p}'\cdot\vec{r}} = Z_p Z_T \alpha \int d^3r \frac{e^{i\vec{q}\cdot\vec{r}}}{r}$$

$d^3r = r^2 dr \, d\cos\theta \, d\varphi$. $\qquad \vec{q} = \vec{p} - \vec{p}'$

$e^{i\vec{q}\vec{r}} = e^{iqr\cos\theta}$

$$\mathcal{M} \sim \frac{Z_p Z_T \alpha}{q^2}$$



What if not point like target.

$$V(\vec{r}) = \int d^3 r' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

point like $\quad \rho(\vec{r}') = Z_T e\, \delta(r)$

More in general. $\rho(\vec{r}) = Z_T e\, f(\vec{r})$

$$\mathcal{M} \sim Z_p Z_T e^2 \int d^3 r\, \underbrace{e^{-i\vec{p}\cdot\vec{r}}\, e^{+i\vec{p}'\cdot\vec{r}}}_{e^{+i\vec{q}\cdot\vec{r}}} \underbrace{\int d^3 r' \frac{f(\vec{r}')}{|\vec{r}-\vec{r}'|}}_{V(\vec{r})}$$

$$e^{+i\vec{q}\cdot\vec{r}} = e^{+i\vec{q}(\vec{r}-\vec{r}')}\, e^{+i\vec{q}\vec{r}'}$$

$$\simeq \int d^3 r\, \frac{e^{i\vec{q}\cdot(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|} \underbrace{\int d^3 r'\, e^{i\vec{q}\cdot\vec{r}'} f(\vec{r}')}_{\text{Form Factor.}}$$

$\vec{r} \to \vec{R} = \vec{r} - \vec{r}'$

$$\simeq \frac{Z_p Z_T \alpha}{q^2} \times F(q^2)$$

Fourier transform of $f(\vec{r})$

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Ruth} \\ \text{spatiall} \\ \text{distrib}}} = \left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Ruth.} \\ \text{point like}}} \times |F(q^2)|^2$$

$\hookrightarrow \dfrac{\alpha^2}{q^2} E^2$

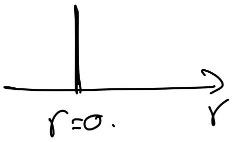
$$\left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{mott} \\ \text{spatial distrib.}}} = \left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Ruth.} \\ \text{point like}}} \times \left(1-\beta^2 \sin^2\frac{\theta}{2}\right) \times |F(q^2)|^2$$

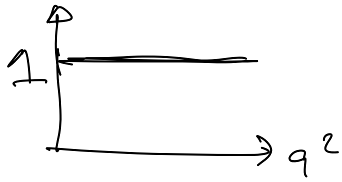
to exp. measure $|F(q^2)|$ $\Rightarrow$ measure $\frac{d\sigma}{d\Omega}$ at different q^2

In principle determine $F(q^2)$. $\Rightarrow$ $f(\vec{r}) = C \int d^3q \, e^{\pm i q r} F(q^2)$

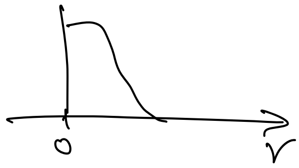
C ↳ Normalization.

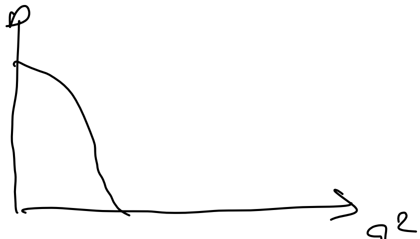
$f(r)$	$F(q^2)$
$\frac{1}{4\pi}\delta(r)$	





$f(r) = a^2 e^{-\frac{a^2 r^2}{2}}$ $\qquad$ $e^{-\frac{q^2}{2a^2}}$



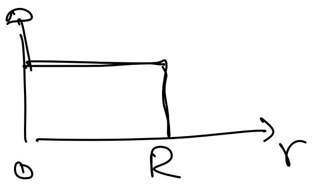


Homogenous.

$f(r) = \frac{1}{\frac{4\pi}{3}R^3}$ $\quad r \le R.$

$\qquad 0 \quad r > R.$

$\frac{3}{(qR)^3}(\sin qr - qr\cos qr)$



$$\frac{d\sigma}{d\Omega}\Big| = \frac{d\sigma}{d\Omega}\Big|_{point} \times |F(q^2)|^2$$

Typically assume $f(\vec{r}) = f(r)$

$$F(q^2) = \frac{4\pi}{A}\int_0^\infty dr\, r^2 \frac{\sin(qr)}{qr} f(r).$$

$$A = \int d^3r \, f(r)$$

Experimental Challenge: no access for all q^2 values.

$q^2 \to 0$ more accessible.

$\Rightarrow$ approximate $F(q^2)$ around $q^2 = 0$.

$$F(q^2) = \underbrace{(\text{Norm.})}_{\text{Normaliz.}} \iiint e^{iqr\cos\theta} f(r)\, r^2 dr\, d\cos\theta\, d\varphi.$$

$$\int dr\, r^2 f(r)\, 2\pi \int_{-1} d\cos\theta \left[1 + iqr\cos\theta - \frac{1}{2}(qr)^2 \cos^2\theta + O(qr^3) \right]$$

$$F(q^2) = (\text{Normaliz.})\, 2\pi \int_0^\infty dr\, f(r)\, r^2 \left[2 + 0 - \frac{1}{6}(qr)^2 \right]$$

$$= (\text{Norm}) \times \left(\underbrace{4\pi \int dr\, f(r)\, r^2} + 0 - \frac{1}{6} 4\pi\, q^2 \int_0^\infty f(r)\, r^4 dr \right) \times \text{Norm}$$

$$=: \langle r^2 \rangle$$

$$= 1 - \frac{1}{6} q^2 \langle r^2 \rangle.$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Structure}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{pointlike}} \times \left(1 - \frac{1}{6} q^2 \langle r^2 \rangle \right).$$

if $\langle r^2 \rangle \neq 0$.

$$R = \frac{\left.\frac{d\sigma}{d\Omega}\right|_{\text{meas.}}}{\left.\frac{d\sigma}{d\Omega}\right|_{\text{pointlike}}} = 1 - \frac{1}{6} q^2 \langle r^2 \rangle.$$

if $R < 1 \Rightarrow$ target has $\langle r^2 \rangle \neq 0$.

$\Rightarrow$ spatial distribution.