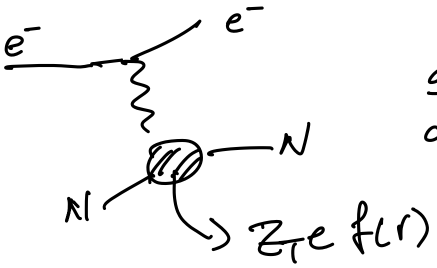
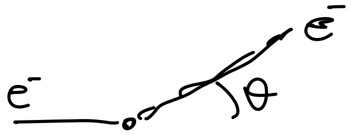



$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{structure}} = (--)\,\underbrace{\underbrace{\underbrace{\frac{\alpha^2}{q^4}E'^2}_{\text{Rutherford.}}\left(1-\beta^2\sin^2\frac{\theta}{2}\right)}_{\text{Mott}}\frac{E'}{E}}_{\text{Includes recoil}}\;\underbrace{|F(Q^2)|^2}_{\text{Form Factor}}$$




$\beta \simeq 1 \qquad 1-\beta^2\sin^2\frac{\theta}{2} \simeq \cos^2\frac{\theta}{2}$ $\qquad \frac{E'}{E} = \frac{1}{1+2\frac{E}{M}\sin^2\frac{\theta}{2}}$

$Q^2 = 4EE'\sin^2\frac{\theta}{2}$ $\qquad Q^2 = -q^2$

Experimentally $Q^2 \simeq 0$ more accessible.

$F(q^2) = 1 - \frac{1}{6}q^2\langle r^2\rangle$ $\qquad f(r)$ : spatial charge distribution.

$$\langle r^2\rangle = \int d^3r\; r^2 f(r)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{struc}} = \underbrace{\left.\frac{d\sigma}{d\Omega}\right|_{\text{point like}}}_{\text{well known.}} |F(q^2)|^2$$

Experimentally $R = \dfrac{\left.\frac{d\sigma}{d\Omega}\right|_{\text{meas.}}}{\left.\frac{d\sigma}{d\Omega}\right|_{\text{pointlike}}} \simeq |F(q^2)|^2 \simeq 1 - \frac{1}{6}q^2\langle r^2\rangle$

$F(0) = 1$ point like.

measure $R < 1 \Rightarrow \langle r^2\rangle \neq 0$

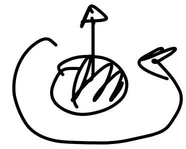
Include spin of target: Dirac target $p, n, e$

$e^-+e^- \to e^-+e^-$ Möller scattering.

$e^-+e^+ \to e^-+e^+$ Bhabha scattering.

Dirac particle $s = 1/2$ $m = m$

magnetic moment $\mu = g \dfrac{e\hbar}{4m}$



electron: $m = m_e$ $g = 2$

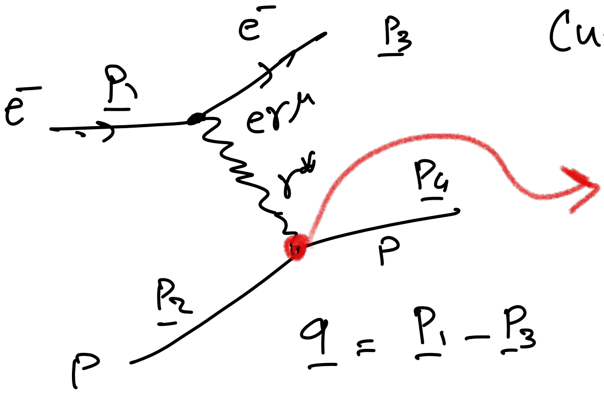
proton: $m = m_p$ $g = 2$ expected $\mu_N = \dfrac{e\hbar}{4m_p}$ nuclear magneton.

neutron: $m = m_n$ $g = 0$

measured $g_p = 2.79$ anomalous magnetic moment

$g_n = -1.91$

$\Rightarrow$ indication of non-elementary particle (not point-like)




$$\underbrace{F_1(q^2)\, e\gamma^\nu}_{\substack{\text{structure}\\ \text{spatial charge distribution.}}} + \underbrace{i\,\frac{\sigma^{\nu\tau} q_\tau}{2M}\, K\, F_2(q^2)}_{\substack{\text{magnetic mom.}\\ \text{spin of target}}}$$

$\underline{q} = \underline{p_1} - \underline{p_3}$

$K = g - 1 = 1.79$ proton

$\sigma^{\nu\tau} = \frac{i}{2}[\gamma^\nu, \gamma^\tau]$ $\gamma^\nu$: Dirac matrices

$q^\nu = \underline{q}$

$F_1(q^2)$, $F_2(q^2)$: Form factors

$F_1(0) = F_2(0) = 1$

$$\mathcal{M} \sim e[\bar{u}\gamma^\mu u]\left[\frac{-i g_{\mu\nu}}{q^2}\right] e\, \bar{u}\left[\gamma^\nu F_1 + \frac{i}{2}\frac{\sigma^{\nu\tau} q_\tau}{2M} K F_2\right] u$$

$$\frac{d\sigma}{d\Omega} \sim |\mathcal{M}|^2 \text{ (phase space)}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth}} = (-)\frac{\alpha^2}{q^4} E'^2 \cos^2\frac{\theta}{2}\,\frac{E'}{E}\left[\left(F_1^2 + \frac{K^2 Q^2}{4M^2} F_2^2\right) + (F_1 + KF_2)^2 \frac{Q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

$$\left(F_1^2 + \frac{K^2 Q^2}{4M^2} F_2^2\right) \times \cos^2\frac{\theta}{2} + (F_1 + KF_2)^2 \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}$$

$\sigma \propto |M|^2 = M M^\dagger \qquad M = a + b \qquad M^\dagger = a^* + b^*$

$|M|^2 = a^2 + b^2 + a^* b + a b^*$

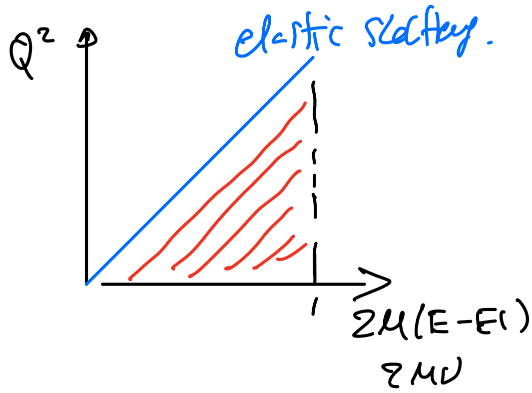
Kinematic limits:

$Q^2 \to 0 \qquad M \simeq m_p \qquad \frac{Q^2}{M^2} \to 0$

Depending on $\frac{Q^2}{M^2}$ magnetic moment matters

For $Q^2 \to 0$ $\quad \frac{d\sigma}{d\Omega}\Big|_{\text{Rosenbluth}} \simeq \frac{d\sigma}{d\Omega}\Big|_{\text{pointlike}} |F_1(q^2)|^2$

spatial distribution $f(r) \longrightarrow F(q^2)$




Elastic $\quad Q^2 = 2M\nu$

Inelastic $\quad W^2 = Q^2 - M^2 + 2M\nu$.

Form factor for protons

McAllister, Hofstadter 1956.
@ SLAC. Nobel prize 1961

$e^- + H_2, He \longrightarrow e^- + X$

$e^-$ beam: 188 MeV

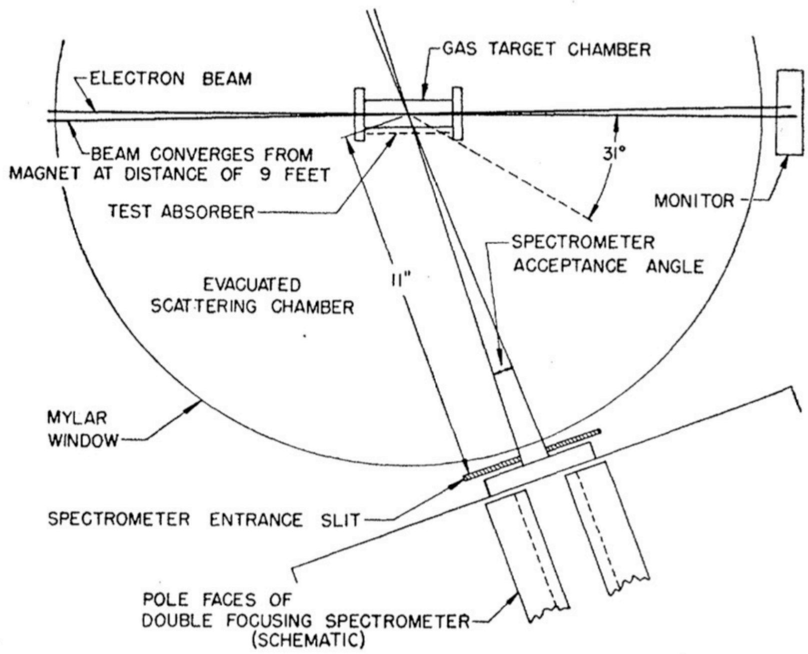



FIG. 2. Arrangement of parts in experiments on electron scattering from a gas target.

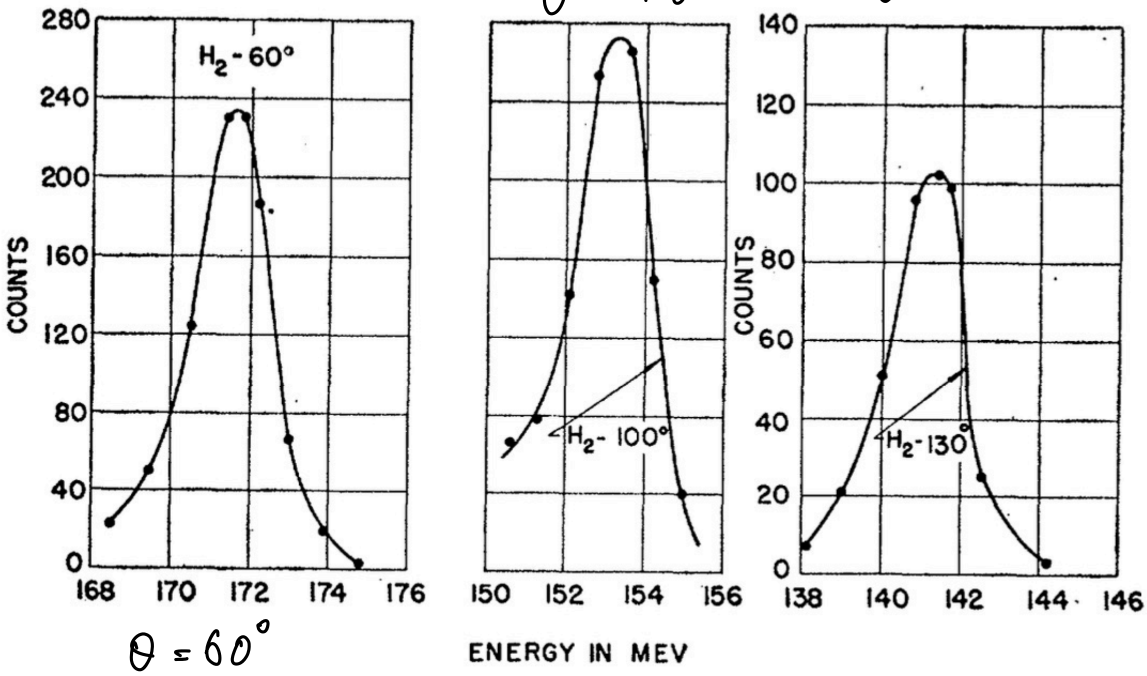
Measure $E'$ for $\theta$.

$\theta \in [35, 138]^\circ$

Goldhaber Ch. 8

$$E' = \frac{E}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}}$$

$\frac{d\sigma}{d\Omega}$




$$\frac{d\sigma}{d\Omega} \propto \cos^2\frac{\theta}{2}$$

$\theta = 60°$

$$\frac{E'}{E} \simeq \frac{1}{1+\frac{E}{M}\left(\frac{1}{2}\right)^2}$$

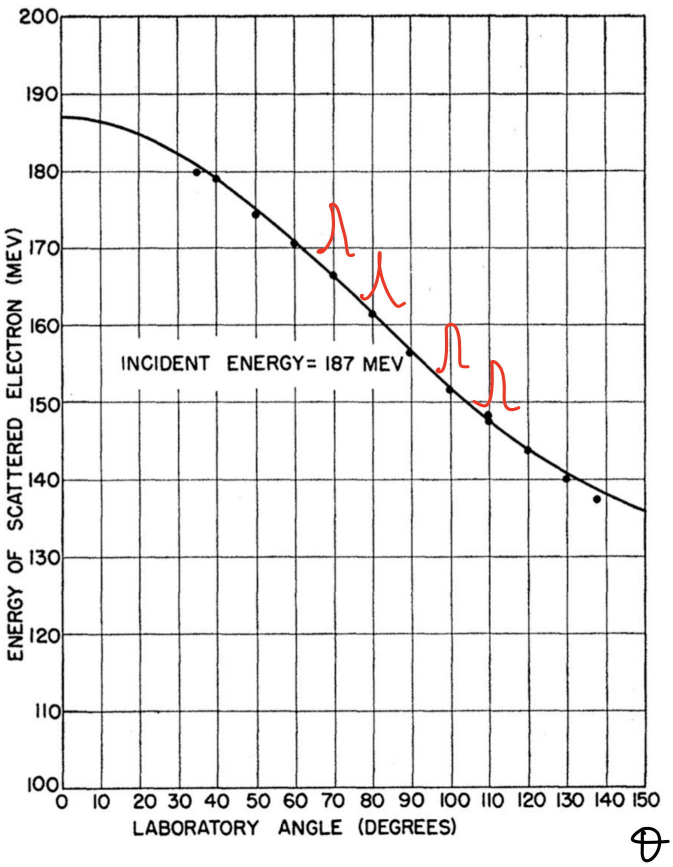
$E = 200$ MeV
$M = 1000$ MeV.

$$= \frac{1}{1+\frac{1}{5}\frac{1}{4}} = \frac{1}{1+\frac{1}{20}} \simeq 1 - \frac{1}{20}$$

$$= 1 - 0.05$$

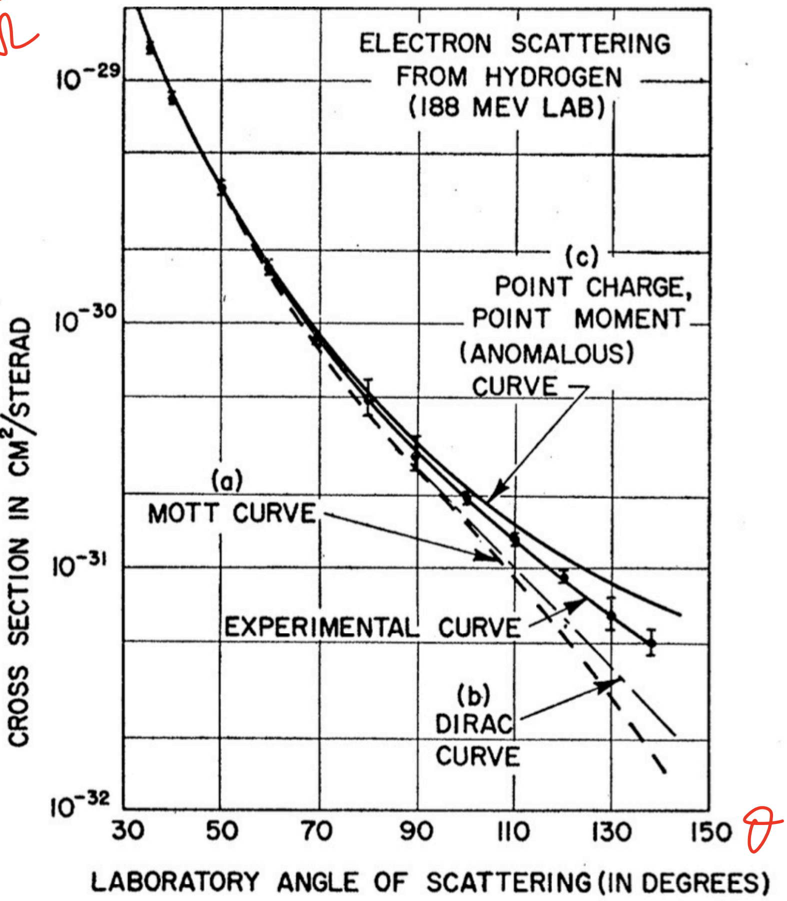
$E = 188$ MeV $\quad E' \simeq 178$ MeV.

$E'$




$$E' = \frac{E}{1+2\frac{E}{M}\sin^2\frac{\theta}{2}}$$

$\frac{d\sigma}{d\Omega}$




$\theta$

FIG. 5. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with the Dirac magnetic moment, curve (c) the theoretical curve for a point proton having the anomalous contribution in addition to the Dirac value of magnetic moment. The theoretical curves (b) and (c) are due to Rosenbluth.[8] The experimental curve falls between curves (b) and (c). This deviation from the theoretical curves represents the effect of a form factor for the proton and indicates structure within the proton, or alternatively, a breakdown of the Coulomb law. The best fit indicates a size of $0.70\times10^{-13}$ cm.

a) proton: no structure
no spin.

b) no structure $F_1 = 0$.
with spin $F_2 \neq 0$
$g = 2$

c) $g \neq 2$ anomalous
no structure.

$$\frac{d\sigma}{d\Omega}\Big|_{struc} \simeq \frac{d\sigma}{d\Omega}\Big|_{point} \times |F(Q^2)|^2$$

$$Q^2 = 4EE' \sin^2\frac{\theta}{2}$$

$$\Rightarrow |F(Q^2)|^2 \neq 0$$

$$F(Q^2) \simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle$$

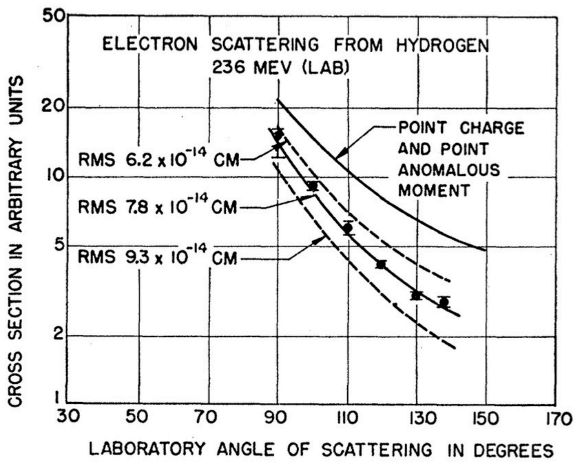



FIG. 6. This figure shows the experimental points at 236 Mev and the attempts to fit the shape of the experimental curve. The best fit lies near $0.78\times10^{-13}$ cm.

$$\text{RMS} \equiv \langle r^2 \rangle$$

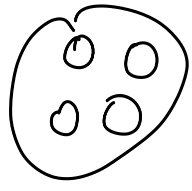
$$(\text{Norm}) \int d^3r \, r^2 f(r)$$

$$\Rightarrow \sqrt{\langle r^2 \rangle} = 0.78 \times 10^{-13} \text{ cm} = 0.78 \text{ fm}$$

$$r_N = r_0 A^{1/3}$$
$$r_0 = 1.2 \text{ fm}$$

$\Rightarrow$


proton.



proton ?

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# Elastic Scattering of 188-Mev Electrons from the Proton and the Alpha Particle*†‡§‖¶

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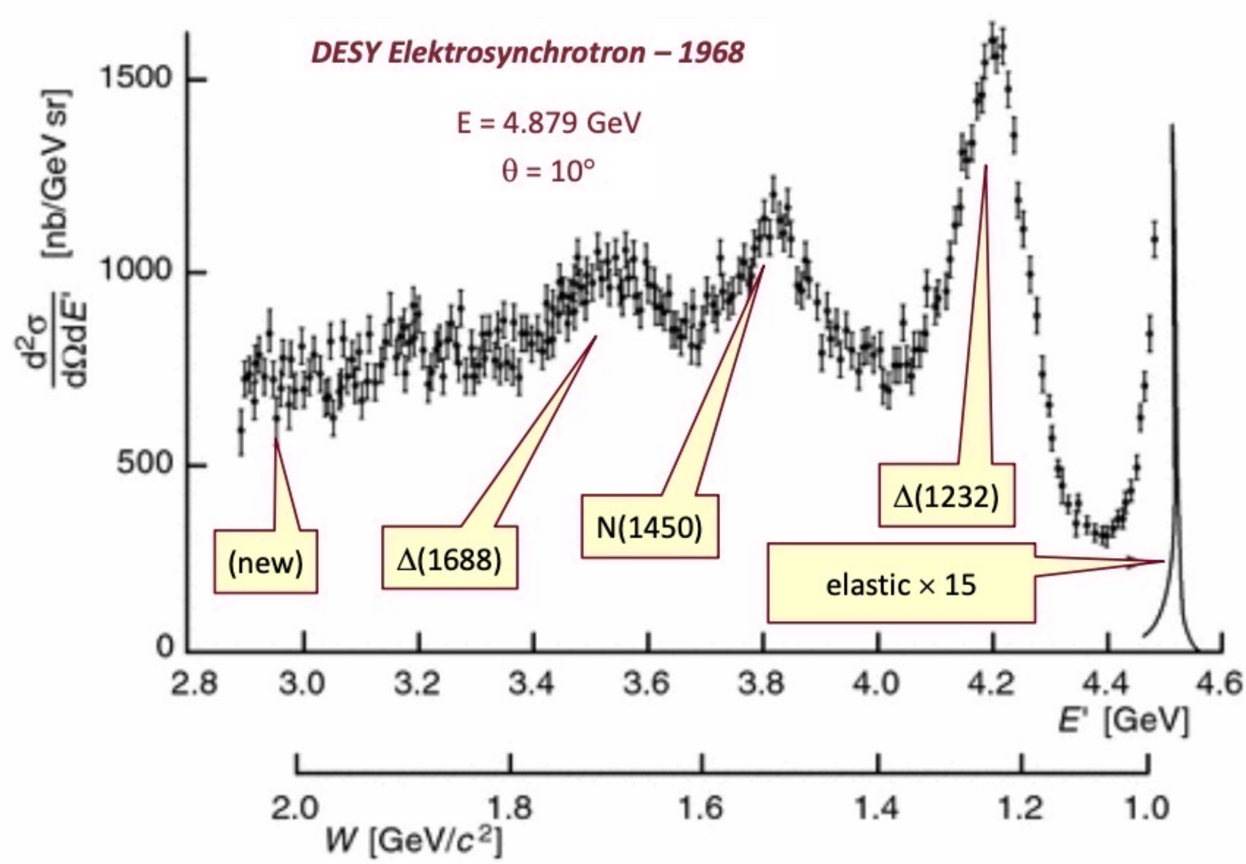
(Received January 25, 1956)

The elastic scattering of 188-Mev electrons from gaseous targets of hydrogen and helium has been studied. Elastic profiles have been obtained at laboratory angles between 35° and 138°. The areas under such curves, within energy limits of ±1.5 Mev of the peak, have been measured and the results plotted against angle. In the case of hydrogen, a comparison has been made with the theoretical predictions of the Mott formula for elastic scattering and also with a modified Mott formula (due to Rosenbluth) taking into account both the anomalous magnetic moment of the proton and a finite size effect. The comparison shows that a finite size of the proton will account for the results and the present experiment fixes this size. The root-mean-square radii of charge and magnetic moment are each $(0.74\pm0.24)\times10^{-13}$ cm. In obtaining these results it is assumed that the usual laws of electromagnetic interaction and the Coulomb law are valid at distances less than $10^{-13}$ cm and that the charge and moment radii are equal. In helium, large effects of the finite size of the alpha-particle are observed and the rms radius of the alpha particle is found to be $(1.6\pm0.1)\times10^{-13}$ cm.

Higher energy to probe proton structure.




$e^- + p \to e^- + p$ $\qquad Q^2 = 2M\nu$.

$\to e^- + X$ inelastic $\qquad W^2 = M^2 - Q^2 + 2M\nu$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Inelastic.}} = \frac{\alpha^2}{q^4} E'^2 \cos^2\frac{\theta}{2}\left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right]$$

$W_{1,2}$ : structure functions

expressing ignorance about proton structure

$E = 5$ GeV $\qquad \beta = \frac{p}{E} \simeq 1$

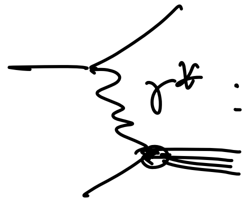
⇒ measure experimentally

$(E', \theta) \to (Q^2, \nu) \to (x, y)$ $\quad \boxed{x = \dfrac{Q^2}{2M\nu}} \quad y = \dfrac{\nu}{E}$

$[\sigma] = L^2 = E^{-2}$ $\quad \left[\dfrac{E'^2}{q^4}\right] = E^{-2}$

$2M W_1(Q^2, \nu) = F_1(x, y)$ $\quad \nu W_2(Q^2, \nu) = F_2(x, y)$

$$\frac{W_2}{W_1} = \left(\frac{Q^2}{\nu^2 + Q^2}\right)(1 + R)$$




: transverse pol.
: long. pol.

$R = \dfrac{\sigma_S}{\sigma_T}$ $\quad \sigma_S$: cross section for longit. pol. $\gamma^*$.

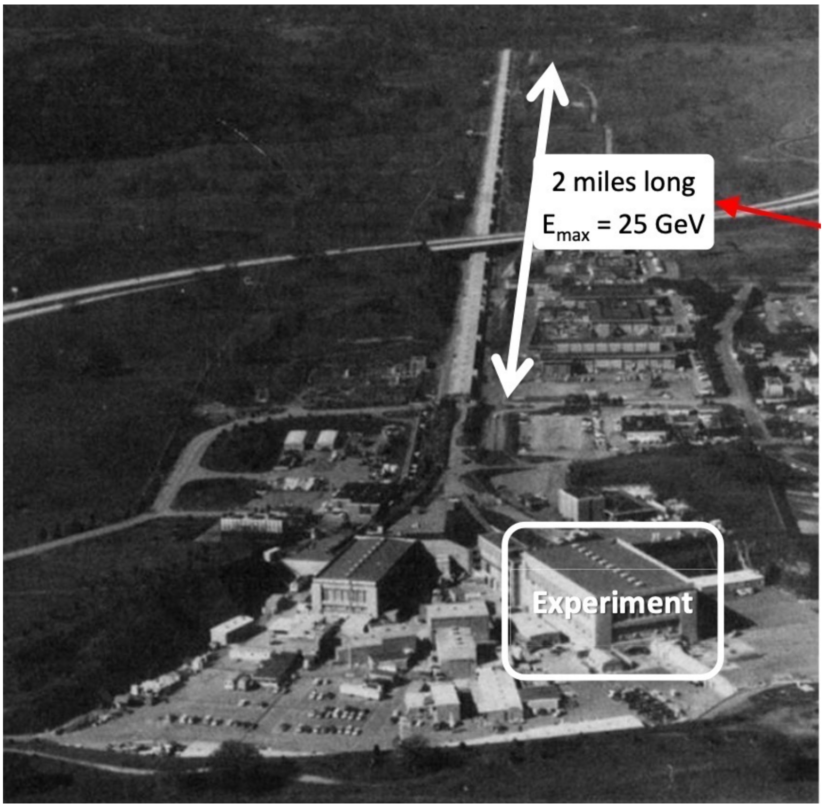
$\sigma_T$: cross section for trans. pol. $\gamma^*$

$\gamma^*$: Virtual photon $m_{\gamma^*} = \sqrt{\underline{q}^2} \neq 0$

$W_2 = \dfrac{1}{\nu} F_2(Q^2, \nu)$ Bjorken suggested. $\quad W_2 = \dfrac{1}{\nu} F_2\left(\dfrac{1}{\boxed{x}}\right)$

$x$: Bjorken scaling variable

$e^- + p \to e^- + X$ @ very high energy for $e^-$




1 fm = 200 $MeV^{-1}$
= 0.2 $GeV^{-1}$

Deep
Inelastic
Scattering