- Decays
- collisions

$E^2 = p^2 + m^2$ $\quad a^+ \; a^-$

$\underline{P} = (E, \vec{p})$ $\quad g^{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$

$|\underline{P}|^2 = E^2 - m^2$

mass m: $\quad E = p^2 + m^2$

$\beta = \frac{|\vec{p}|}{E}$

$\gamma = \frac{E}{m}$ $\quad \Rightarrow \quad \beta\gamma = \frac{|\vec{p}|}{m}$

collision: $\quad a + b \to c + d$

$p + p \to p + p$ elastic.

$p + p \to p + p + p + \bar{p}$ inelastic scatt.

collision $\quad 1 + 2 \to 3 + 4 + 5 + \cdots + n$

$e^- + \gamma \to e^- + \gamma$ Compton.

Decay: $\quad a \to b + c + \cdots n.$

α decay. $\quad \cancel{a \to b}$ $\quad a \neq b.$

${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + \alpha$

${}^{4}_{2}He$

$a \to b + c$ 2-body decay.

$a \to b + c + d$ 3-body decay.

β decay: $\quad n \to p + e^- + \bar{\nu}_e$

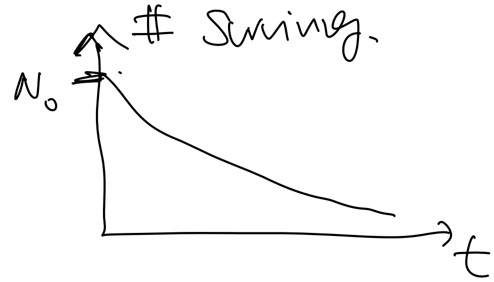
${}^{A}_{Z}X \to {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$

Decays:

$N(t=0) = N_0$ particles.

$N(t) = N_0 e^{-t/\tau}$

$= N_0 e^{-t\Gamma}$



τ: lifetime.

Γ: total width. $\Gamma = \frac{1}{\tau}$.

$\hbar = c = 1$

E: MeV.

L: $[\text{MeV}]^{-1}$

T: $[\text{MeV}]^{-1}$.

c dimensionless $\Rightarrow [L] = [T]$

same units.

$E = \hbar\nu. \Rightarrow [E] = [T]^{-1}$.

$\frac{N(t)}{N_0} = e^{-t/\tau}$ = prob survival @ t.

muon: $\tau = 2.2 \times 10^{-6}$ s.

Create μ @ $t=0$.

$t \simeq 5\tau$

Γ_e $\quad \underset{140\,\text{MeV}}{\pi^+} \to \underset{106\,\text{MeV}}{\mu^+} + \nu_\mu$. # count $\quad Q = 34$ MeV

Γ_μ $\quad \pi^+ \to \underset{0.5\,\text{MeV}}{e^+} + \nu_e$ # count $\quad Q \simeq 140$ MeV

$\frac{1}{\tau} = \Gamma_{tot} = \Gamma_e + \Gamma_\mu$. (MeV)

Branching fraction. $B(a \to i) = \frac{\Gamma_i}{\Gamma_{tot}}$

ratio.

Q value $\quad m_a - m_b - m_c - \cdots - m_n.$

$$a \to b + c$$

$$Q = m_a - m_b - m_c \geq 0 \qquad \text{Decay happens.}$$

In the rest frame of a:

$$E_a = m_a = E_b + E_c = m_b + \underbrace{K_b}_{\text{kinetic energy}} + m_c + K_c$$

at limit $\quad K_b = K_c = 0.$

$$m_a = m_b + m_c.$$

$$N(t) = N_0 \, e^{-\Gamma t} \qquad [\Gamma] = t^{-1}$$

probability of decay / unit time.

$$\Gamma(i \to f) = 2\pi \, |M_{fi}|^2 \, \rho(E)$$

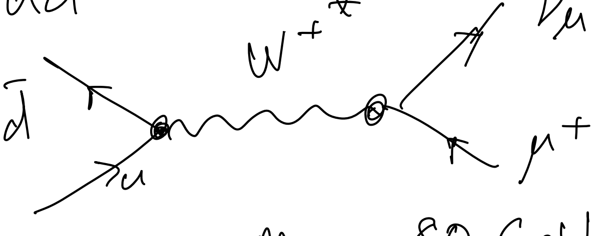
Fermi's Second Golden Rule.

Matrix Element of H_I

phase space.

$$\pi^+ \to \mu^+ + \nu_\mu.$$

$\pi^+ : u\bar{d}$



M_{fi}

$\langle \mu^+ \nu_\mu | H_I | \pi^+ \rangle$

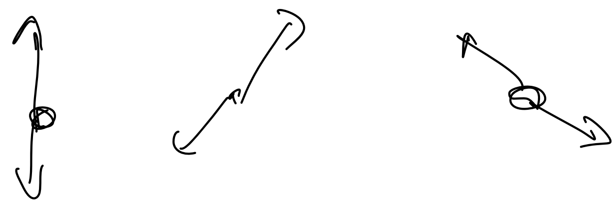
140 MeV $\qquad M_W = 80$ GeV.

$\Delta E \cdot \Delta t \simeq 1 \qquad$ uncertainty principle

$\Delta t \simeq \frac{1}{80\ \text{GeV}}$

$$\nu_\mu \longleftarrow \overset{\pi^+}{\bullet} \longrightarrow \mu^+ \qquad \vec{P}_{\mu^+} = -\vec{P}_{\nu_\mu}$$

$$\vec{P}_{\pi^+} = 0 = \vec{P}_{\mu^+} + \vec{P}_{\nu_\mu}$$



Relativistic Golden Rule

$1 \longrightarrow 2 + 3 + \cdots + n.$ $\qquad |i\rangle = |1\rangle$

$|f\rangle = |2345 \cdots n\rangle$

$$\Gamma = S \frac{1}{2m_1} \int |M_{fi}|^2 \, \delta^{(4)}(\underline{p}_1 - \underline{p}_2 - \underline{p}_3 - \cdots - \underline{p}_n) \times$$

$$\prod_{j=2}^{n} \underbrace{2\pi \delta(\underline{p}_j^2 - m_j^2)\, \theta(E_j)}_{\text{j particle is on shell.}} \frac{d^4 \underline{p}_j}{(2\pi)^4}$$

Statistical.

$d^4 \underline{p}_j = dE_j \, d^3\vec{p}_j$

$$\underline{p}_j^2 = E_j^2 - |\vec{p}_j|^2$$

$$a \longrightarrow b + b + c + c + c$$

$$S = \frac{1}{N_b!} \frac{1}{N_c!}$$

$a \rightarrow b + c$ $\qquad \pi^+ \rightarrow \mu^+ + \nu_\mu. \quad S = 1.$

$n \rightarrow p + e^- + \bar{\nu}_e \quad S = 1.$

$$K^0 \to \pi^0 \pi^0 \pi^0 \qquad S = \frac{1}{3!}$$

Two-body Decay

$$E_j = \sqrt{\vec{p}_j^{\,2} + m_j^2}$$

$$a \to b + c \qquad S = 1.$$

$$\Gamma = \frac{1}{2m_a} \int |M|^2 (2\pi)^4 \delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c) \times \frac{1}{2\sqrt{p_b^2 + m_b^2}} \, \frac{1}{2\sqrt{m_c^2 + \vec{p}_c^{\,2}}} \, \frac{d^3\vec{p}_b}{(2\pi)^3} \, \frac{d^3\vec{p}_c}{(2\pi)^3}$$

$$\delta(\underline{p}_j^2 - m_j^2)\, \theta(E_j) = \delta(E_j^2 - |\vec{p}_j|^2 - m_j^2)\, \theta(E_j)$$

$$= \frac{1}{2\sqrt{\vec{p}_j^{\,2} + m_j^2}} = \frac{1}{2\sqrt{E_j}}$$

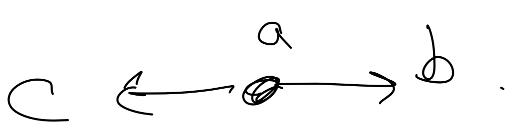
$$\Gamma_{a \to b+c} = \frac{1}{8(2\pi)^2} \frac{1}{m_a} \int |M|^2 \frac{\delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c)}{\sqrt{p_b^2 + m_b^2}\,\sqrt{p_c^2 + m_c^2}} \, d^3\vec{p}_b \, d^3\vec{p}_c$$

$$\delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c) = \delta(E_a - E_b - E_c)\, \delta^3(\vec{p}_a - \vec{p}_b - \vec{p}_c)$$

In the rest frame of a: $E_a = m_a. = E_b + E_c$

$$\vec{p}_a = 0 \implies \vec{p}_b = -\vec{p}_c$$

$$\Gamma = \frac{1}{32\pi^2} \frac{1}{m_a} \int |M|^2 \frac{\delta(m_a - E_b - E_c)}{\sqrt{\vec{p}_b^{\,2} + m_b^2}\,\sqrt{\vec{p}_c^{\,2} + m_c^2}} \, d^3 p_b .$$



$$d^3P_b = P_b^2 \, dP_b \, . \underbrace{\sin\theta \, d\theta \; d\varphi}_{d\Omega.}$$ solid angle.

$$\Gamma = \frac{1}{32\pi^2} \frac{1}{m_a} (4\pi) \int |M|^2 \frac{\delta(m_a - -)}{\sqrt{P_b^2 + m_b^2}\sqrt{P_b^2 + m_c^2}} P_b^2 \, dP_b .$$

$|\vec{P_b}| = |\vec{P_c}|$

$$\Gamma = \frac{1}{8\pi} \frac{1}{m_a} |\vec{P_b}| \, |M|^2$$

$\pi^+ \to \mu^+ \nu_\mu .$ $\qquad \Gamma_\mu \propto \frac{1}{m_\pi} |P_\mu|$

$\pi^+ \to e^+ \nu_e$ $\qquad \Gamma_e \propto \frac{1}{m_\pi} |P_e|$

$$\frac{\Gamma_\mu}{\Gamma_e} \propto \frac{|\vec{P_\mu}|}{|\vec{P_e}|} \frac{|M_{\pi\mu}|^2}{|M_{\pi e}|^2} = ?$$

$Q_\mu = 34$ MeV

$Q_e = 140$ MeV

later after helicity.

$$\frac{\Gamma_\mu}{\Gamma_e} \propto \frac{\#\mu \text{ decays}}{\# e \text{ decays}} .$$