Static Quark Model

Gell-Mann 1964 Nobel 1969

Zweig

3 quarks fundamental repres. of SU(3)

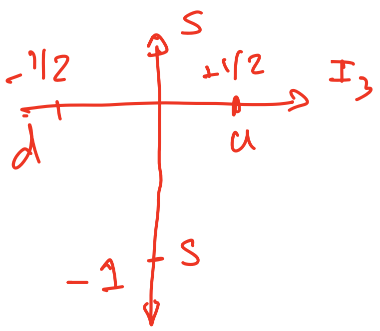
Unitary matrices 3x3 det = 1 tr = $\emptyset$ $U = e^{i\alpha_j T_j}$

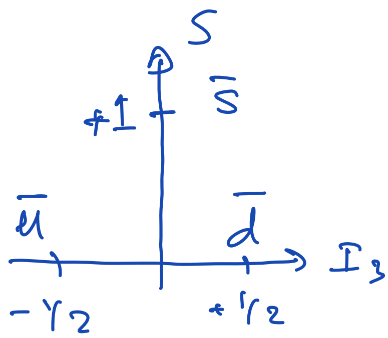
$N^2 - 1 = 8$ generators

$N-1$ diagonal.

SU(2) 3 Pauli matrices I_3, S_z observable.

SU(3) 2 diagonal: I_3, Strangeness

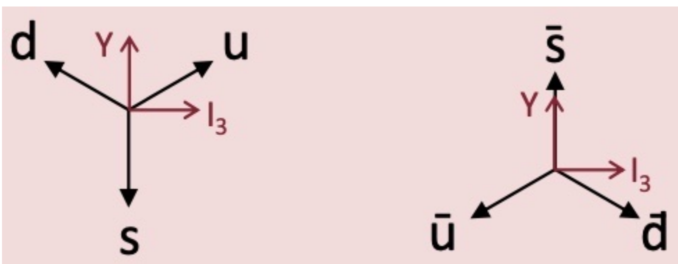




$$Q = I_3 + \frac{B+S}{2}$$

$B = +\frac{1}{3}$ Quarks.

$= -\frac{1}{3}$ antiquarks.



$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ $\bar{q} = (\bar{u}\ \bar{d}\ \bar{s})$

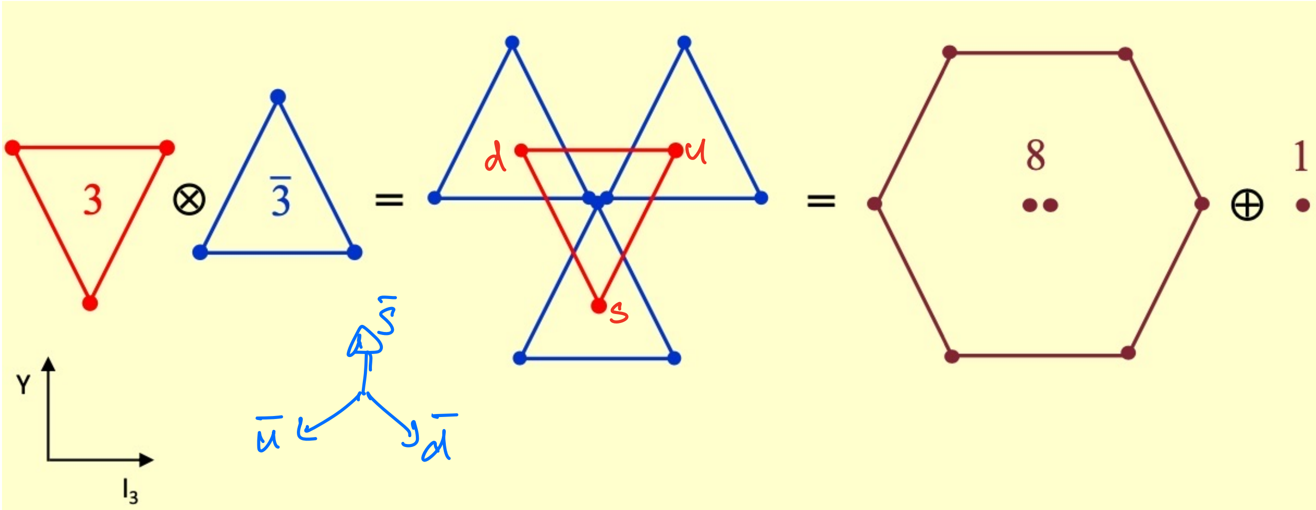
Mesons: $3 \otimes \bar{3} = 8 \oplus 1$ I, I_3, Q

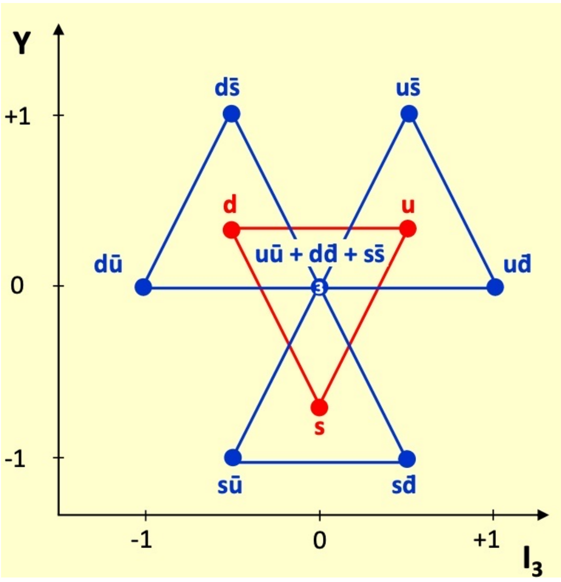
octet singlet

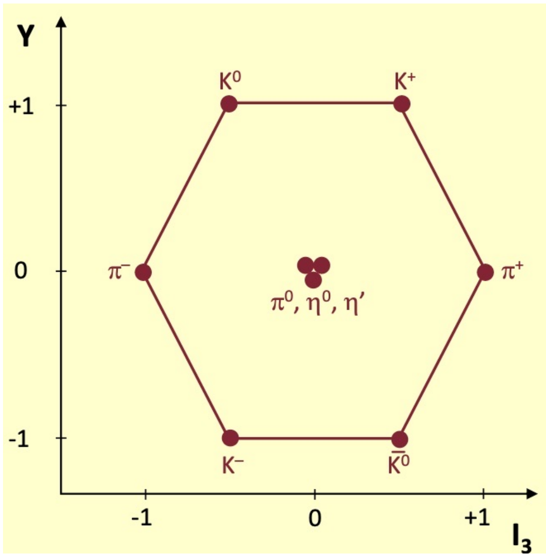
$q, \bar{q}_2$

$S = 0$ $\Uparrow \Downarrow$

$S = 1$ $\Uparrow \Uparrow$







$S = 0.$

$P = (-1)^L \underbrace{P_1 P_2} = (-1)^{L+1}$

$L = 0 \Rightarrow \quad P = (-1)^1 = -1.$

$C = (-1)^{L+S} = (-1)^{0+0} = +1$

$J = L \oplus S = 0$

$J^{PC} = 0^{-+}$

pseudo-scalar mesons

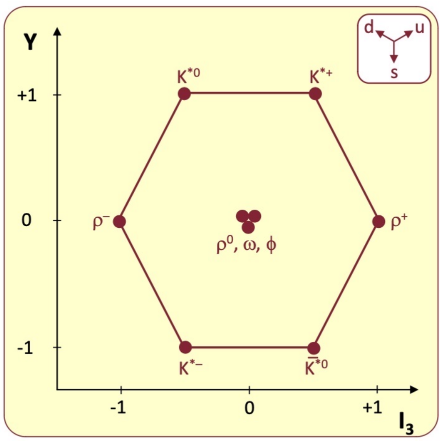
$S = 1.$

$J = L \oplus S = 1$

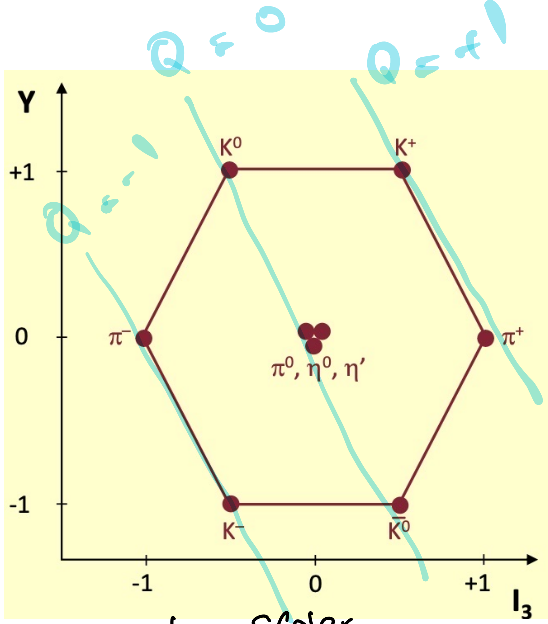
$P = (-1)^{0+1} = -1$

$C = (-1)^{0+1} = -1$

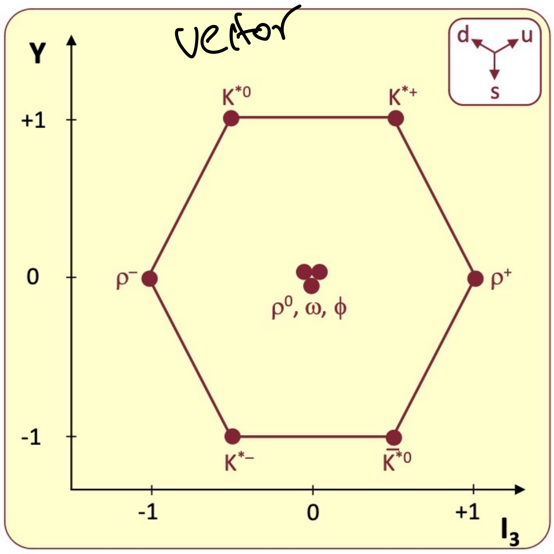
$J^{PC} = 1^{--}$



vector mesons



pseudo-scalar.



L	S	J^{PC}	$^{2s+1}L_J$	I=1 state
0	0	0^{-+}	$^1S_0$	π(140)
	1	1^{--}	$^3S_1$	ρ(770)

π^+ $u\bar{d}$ $\uparrow\downarrow$

ρ^+ $u\bar{d}$ $\uparrow\uparrow$

physical particles/ states: π^0, η^0, η'

graph theory states: $u\bar{u}, d\bar{d}, s\bar{s}$

$$\psi_{8,1} = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad I = 1$$

$$\psi_{8,0} = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \qquad I = 0$$

Octet

$$\psi_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \qquad I = 0$$

singlet

$$f' = \psi_{8,0} \cos\theta_i - \psi_1 \sin\theta_i$$

$$f = \psi_{8,0} \sin\theta_i + \psi_1 \cos\theta_i$$

θ_{ps}: pseudoscalar meson mixing angle

θ_V: vector meson mixing angle.

$$f' = \cos\theta_i \left[\psi_{8,0} - \psi_1 \frac{\sin\theta_i}{\cos\theta_i}\right] =$$

$$= \cos\vartheta_i \left[\frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}) - \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}) \frac{\sin\vartheta}{\cos\theta} \right]$$

$$\text{IF} \quad \frac{\sin\vartheta}{\cos\theta} = \frac{1}{\sqrt{2}} \Rightarrow f' \propto s\bar{s}$$

$$\uparrow$$

$$\theta \simeq 35.3^\circ$$

$n^{2s+1}\ell_J$	J^{PC}	I $= 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	I $= \frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I $= 0$ f'	I $= 0$ f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}[a]	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}[a]	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$[a]	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$[b]	$\phi(2170)$[d]	$\omega(1650)$
1^3D_2	2^{--}		$K_2(1820)$[a]		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$[c]	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$[b]	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$

$$\left.\begin{aligned} \pi^0(140) &\approx \psi^{ps}_{8,1} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \eta(550) &= \psi^{ps}_{8,0}\cos\theta_{ps} - \psi^{ps}_1 \sin\theta_{ps} \\ \eta'(960) &= \psi^{ps}_{8,0}\sin\theta_{ps} + \psi^{ps}_1 \cos\theta_{ps} \end{aligned}\right\} \begin{aligned} &J^P = 0^-, \\ &\theta_{ps} \approx -25^\circ \end{aligned}$$

$$\theta \neq 35^\circ$$

$$\pi^0 \sim u\bar{u}\ d\bar{d}$$

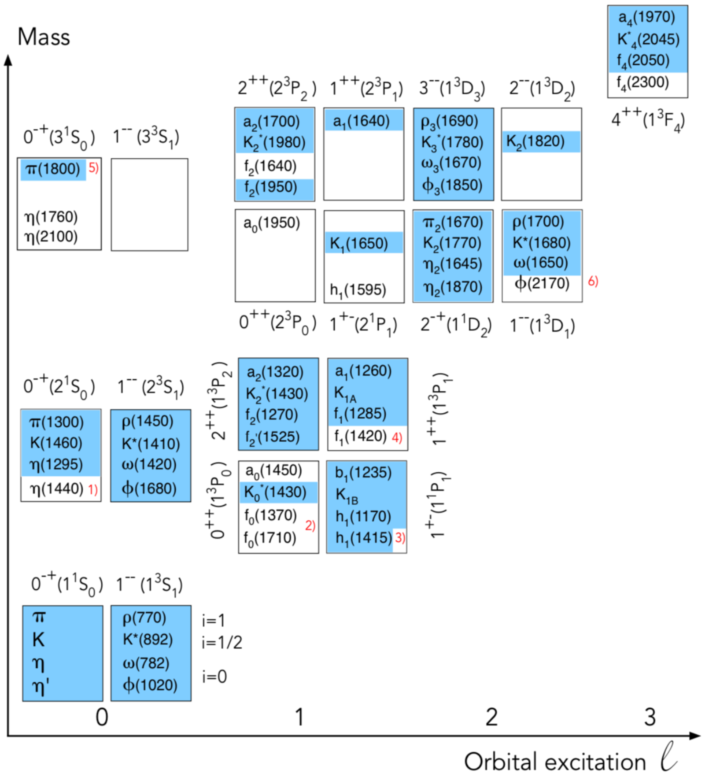
$$\eta, \eta' \sim u\bar{u}, d\bar{d}, s\bar{s}$$

$$\left.\begin{aligned} \rho^0(770) &\approx \psi^{v}_{8,1} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \phi(1020) &= \psi^{v}_{8,0}\cos\theta_{v} - \psi^{v}_1 \sin\theta_{v} \approx s\bar{s} \\ \omega(780) &= \psi^{v}_{8,0}\sin\theta_{v} + \psi^{v}_1 \cos\theta_{v} \approx \\ &\approx (u\bar{u} + d\bar{d})/\sqrt{2} \end{aligned}\right\} \begin{aligned} &J^P = 1^-, \\ &\theta_{vect} \approx 36^\circ \end{aligned}$$

$$\rho^0 \simeq u, d$$

$$\omega \simeq u, d$$

$$\phi \simeq s\bar{s}$$



about 20 multiplets of particles

m_i of multiplets show diff. $\Rightarrow$ SU(3) Flavor not exact

$\Rightarrow$ $m_u \neq m_d \neq m_s$

Production

$$p + p \longrightarrow \pi^+ \pi^- \pi^0 \pi^+ \pi^- \pi^+ \pi^+ + p + p$$

Q $+2$ $\qquad$ 2

B

S

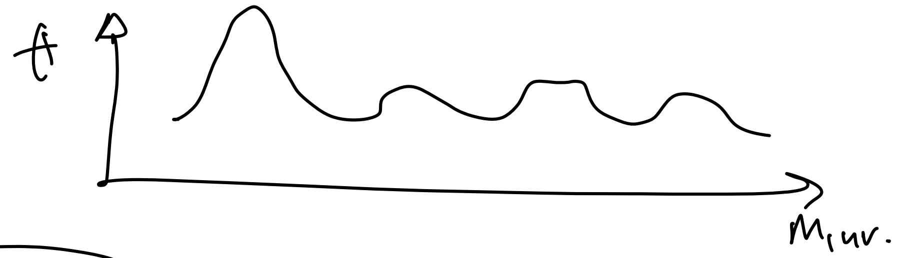
$$\pi^- + p \longrightarrow p + K^+ K^- + \pi^- \pi^0$$

$\qquad K^0 K^- \qquad \pi^+$

$$\sigma \propto |M|^2 \rho(E)$$

$|M|^2 \downarrow H_I$

invariant mass of $\quad \pi^+\pi^- \quad \pi^+\pi^-K^+ \quad \pi^+\pi^0K^-$



Decays

- conservation laws $Q, \vec{P}, E$
- Q-value ≥ 0

$$\rho^0(770) \longrightarrow \pi^+\pi^- \quad m_\rho > 2m_\pi.$$

Q-value $\simeq 0$. $\Rightarrow \Gamma$ small. $\quad K^+K^- \quad m_\rho < 2m_K.$

- EM, strong: strangeness conserved. $\Delta S = \emptyset$
- weak interaction: $\Delta S = 1$ allowed.

$$\Gamma \propto |M|^2 \rho(E)$$

weak: $|M|^2 \sim G_F^2$

Example $\rho^0 \longrightarrow \pi^0\pi^0$?

ρ Decay.

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$\pi\pi$		$\sim$ 100 %	
Γ_2	$K\overline{K}$			
	$\rho(770)^\pm$ decays			
Γ_3	$\pi^\pm\pi^0$		$\sim$ 100 %	
Γ_4	$\pi^\pm\gamma$		(4.5 $\pm$0.5) $\times 10^{-4}$	S=2.2
Γ_5	$\pi^\pm\eta$		< 6 $\times 10^{-3}$	CL=84%
Γ_6	$\pi^\pm\pi^+\pi^-\pi^0$		< 2.0 $\times 10^{-3}$	CL=84%
	$\rho(770)^0$ decays			
Γ_7	$\pi^+\pi^-$		$\sim$ 100 %	
Γ_8	$\pi^+\pi^-\gamma$		(9.9 $\pm$1.6) $\times 10^{-3}$	
Γ_9	$\pi^0\gamma$		(4.7 $\pm$0.8) $\times 10^{-4}$	S=1.7
Γ_{10}	$\eta\gamma$		(3.00$\pm$0.21) $\times 10^{-4}$	
Γ_{11}	$\pi^0\pi^0\gamma$		(4.5 $\pm$0.8) $\times 10^{-5}$	
Γ_{12}	$\mu^+\mu^-$	[a]	(4.55$\pm$0.28) $\times 10^{-5}$	

why not $\rho^0 \to \pi^0\pi^0$

1) $\mathbb{C}$ parity.

$C_{\rho^0} = (-1)^{L+S} = (-1)^{0+1} = -1$

$C_{\pi^0} = +1 \qquad C_{\pi^0\pi^0} = +1$

2) Isospin:

$|\rho^0\rangle = |I=1, I_3=0\rangle$.

$|\pi^0\rangle = |I=1, I_3=0\rangle \qquad = \alpha|2,0\rangle + \beta|0,0\rangle$.

$|\pi^0\pi^0\rangle \qquad 1 \oplus 1 = 0, 2$

$\langle f|H_I|i\rangle = \langle 2,0|H_I|1,0\rangle + \langle 2,0|H_I|0,0\rangle = 0$

$\langle \rho^0|\pi^0\pi^0\rangle = 0$

3) spin statistics.

ρ^0: boson $\qquad \psi_{\rho^0}$ = symmetric

final state: $\qquad$ initial state.

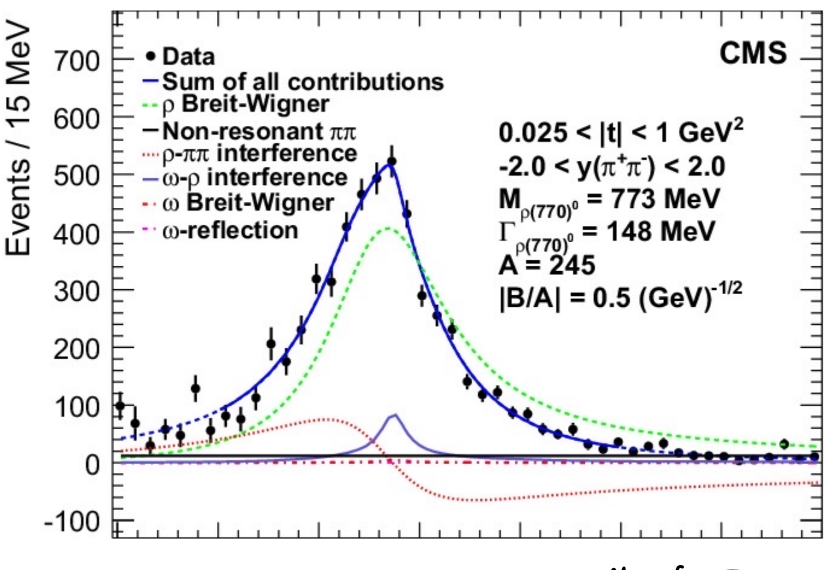
$J_f = L+S = \qquad J_i = 1$

π^0: $S_{\pi^0} = 0$

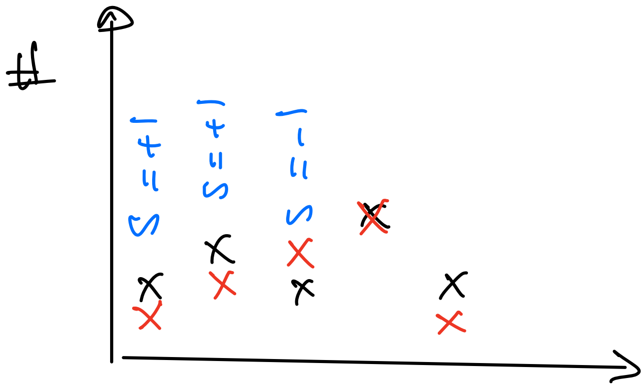
$J_f = 1 \Rightarrow L_f = 1$.

$L=1 \Rightarrow \pi^0\pi^0$ not symmetric

$\Gamma_\rho \simeq 150$ MeV



$M_{\pi^+\pi^-}$

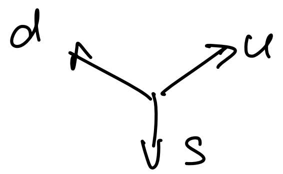


x : total number of events.

$$N = B + S$$

$$S = N - B$$

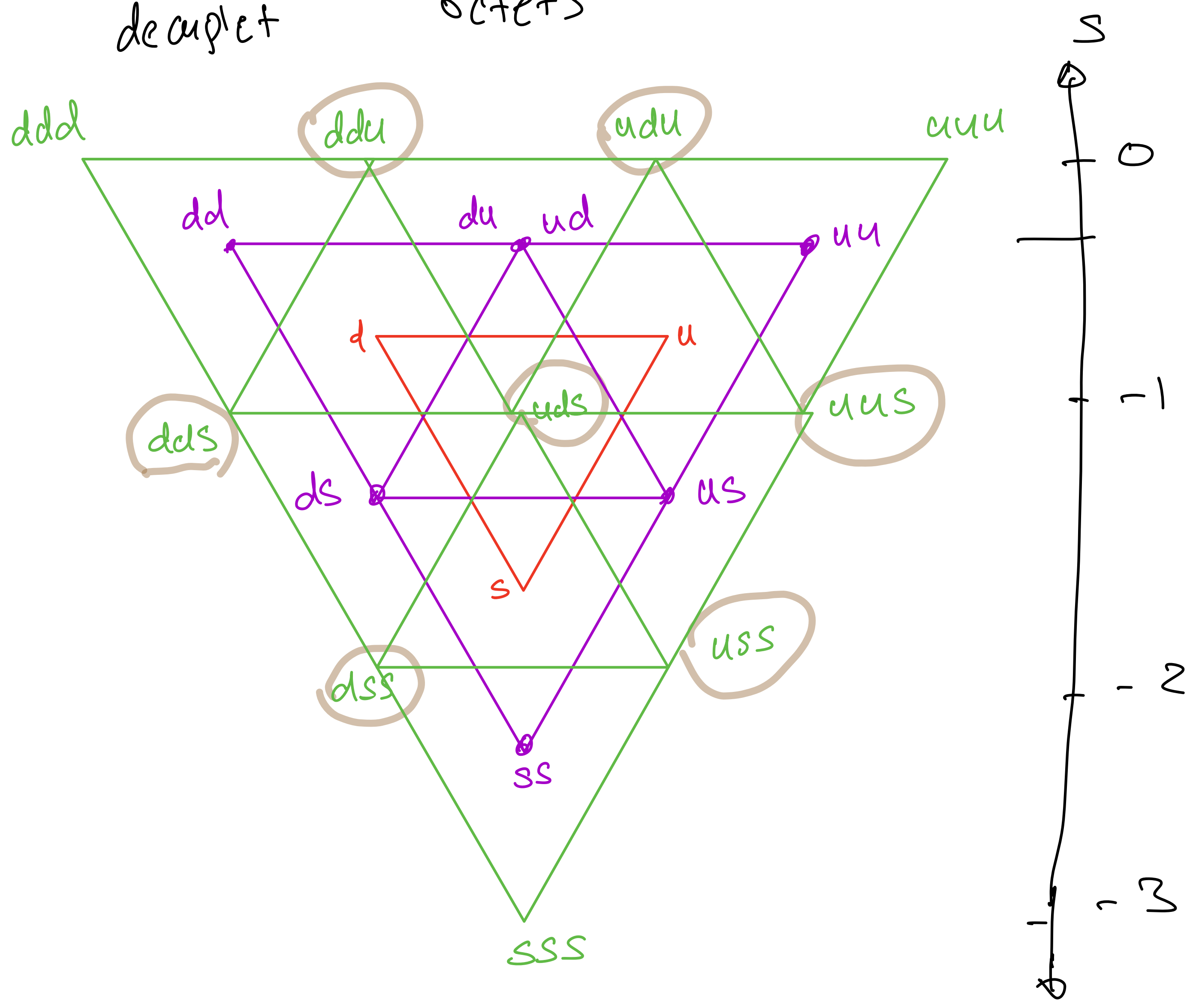
Baryons

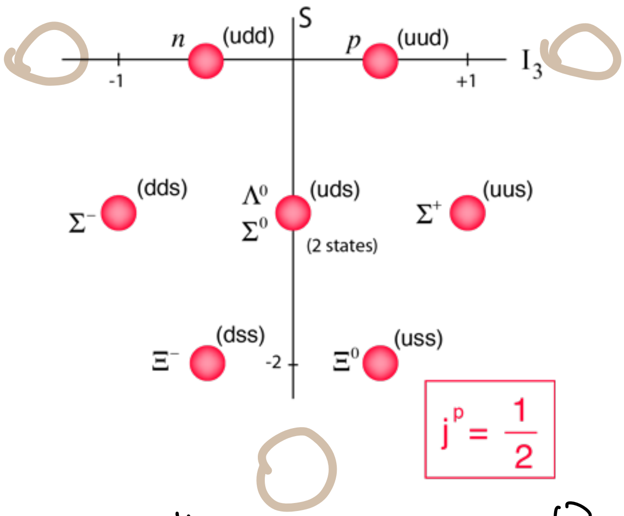


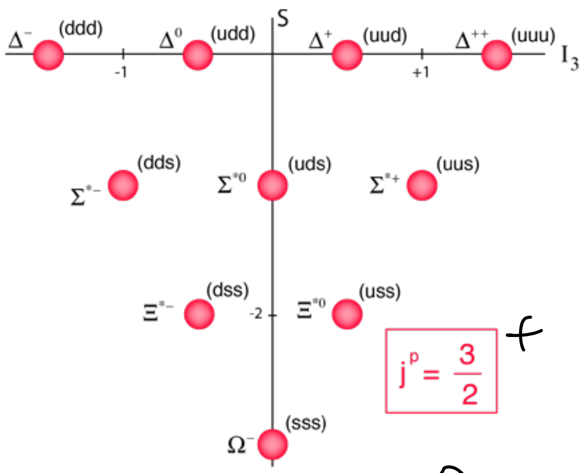
$$B = q_1 q_2 q_3$$

$$= 3 \times 3 \times 3 = (6 \oplus \bar{3}) \otimes 3 = (6 \otimes 3) \oplus (3 \otimes \bar{3})$$

$$= \underbrace{10}_{\text{decuplet}} \oplus \underbrace{8 \oplus 8}_{\text{octets}} \oplus \underbrace{1}_{\text{singlet}}$$







$\Uparrow\Uparrow\Downarrow$

$J^P = \frac{1}{2}^+$

$S = \frac{1}{2}$

$L = 0$

$P = (-1)^L P_1 P_2 P_3 = (-1)^0 = 1$

$\Uparrow\Uparrow\Uparrow$

J^P

$S = 3/2$

$L = 0$

1