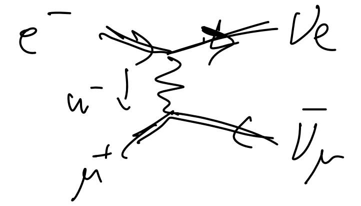
Weak Interactions

- production of $\nu, \bar{\nu}$ → unbalanced energy in detect.
- long lifetime.
- flavor violation $\Delta S = 1$

leptonic process: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ decay

$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$ elastic scatt.



inelastic.

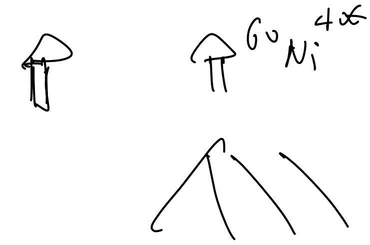
semi-leptonic: $n \to p + e^- + \bar{\nu}_e$

hadronic: $n \to p \pi^-$ ✗ not known. allowed.

$\Lambda \to p \pi^-$ $\Delta S = 1$.

Wu experiment $\cancel{P}$ parity violation

^{60}Co



$^{60}Ni^{4x.}$

e^- preferred.

Goldhaber experiment. ν_e LH. $\Leftarrow \longrightarrow$

helicity $h = \dfrac{\vec{p} \cdot \vec{s}}{|\vec{p}||\vec{s}|}$.

Lee, Yang proposed chiral theory. for weak int

Dirac theory for fermions → accounts for LH, RH. particles.

ψ spinor for a fermion.

$\bar{\psi}$ creates a fermion.

Chiral operator $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$ $\quad \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$ $\quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$ $\quad \sigma^i$ Pauli matrices.

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$(\gamma^5)^2 = \mathbb{1}$ $\qquad (\gamma^5)^2\psi = a\psi = \psi$

$a = \pm 1.$

$P_L = \frac{1}{2}(1-\gamma^5)$

$\psi^{LH} = P_L\psi = \frac{1}{2}(1-\gamma^5)\psi$

$\gamma^5\psi^{LH} = \frac{1}{2}(\gamma^5-(\gamma^5)^2)\psi = -\frac{1}{2}(1-\gamma^5)\psi = -\psi_{LH}$

left handed spinor is eigenstate of γ^5

$\psi^{RH} = \frac{1}{2}(1+\gamma^5)\psi$

$\psi = \psi^{LH} + \psi^{RH}$

$P_L = \frac{1}{2}(1-\gamma^5)$ selects LH chirality particles.

selects RH chirality anti-particles.

$\psi = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$

for $m \to 0$ particle $\quad$ helicity $\to$ chirality

for a fermion with mass m.

$E = \sqrt{p^2+m^2}$

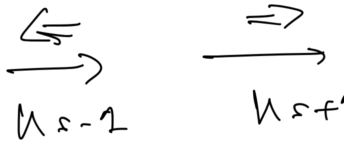
For particle $\psi = \psi^{LH}$ if $E \gg m$.

prob of having $\psi = \psi^{RH}$ $\quad 1-\beta$.

$$\beta = \frac{p}{E} = \frac{p}{\sqrt{p^2+m^2}} = \frac{1}{\sqrt{1+(m/p)^2}}.$$

Massless limit.

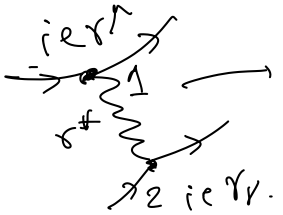
$S=1$, $m=0$. photon.



h is Lorentz invariant

helicity is a good measurement of chirality for $E \gg m$.

QED is a current-current interaction.



$$\frac{-ig_{\mu\nu}}{q^2}$$

$$\mathcal{M} = (\bar{u}\gamma^\mu u)(ie)\frac{-ig_{\mu\nu}}{q^2}(\bar{u}\gamma^\nu u)(ie).$$

$$\mathcal{M} = -\frac{e^2}{q^2} j^\mu g_{\mu\nu} j^\nu = -\frac{e^2}{q^2}\underline{j_1}\cdot\underline{j_2}.$$

$j^\mu = \bar{u}\gamma^\mu u \qquad \bar{\psi}\gamma^\mu\psi$

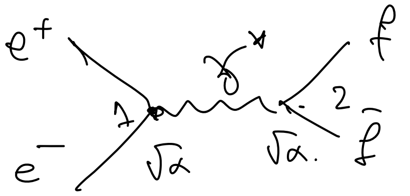
$\underline{j_1}\cdot\underline{j_2} = j_1^0 j_2^0 - \vec{j_1}\cdot\vec{j_2}$

$$\bar{u} = (\circ\ \circ\ \circ\ \circ)\underbrace{\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \vdots & & & \\ \vdots & & & \end{pmatrix}}_{\gamma^\mu}\underbrace{\begin{pmatrix} \\ \\ \end{pmatrix}}_{u} = \underbrace{\begin{pmatrix} \circ \\ \circ \\ \circ \\ \circ \end{pmatrix}}_{\mathcal{J}}$$

$\bar{\psi}\gamma^\mu\psi$ behaves like a 4-vector under Lorentz transf.

$$\underline{P_1}' = \underline{\underline{L}}\cdot\underline{P_1}$$

$$\mathcal{M}_{QED} = -\frac{e^2}{q^2}\underline{j}\cdot\underline{j}$$



$$\mathcal{M} \sim \frac{\alpha}{q^2}\, \underline{j_e} \cdot \underline{j_f}$$

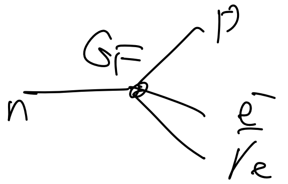
$$\alpha = \frac{e^2}{4\pi}$$

vector $\bar{\psi}\gamma^\mu\psi = j^\mu$

$$P\, j^\mu = (j^0, -\vec{j})$$

QED $$P\mathcal{M} = -\frac{e^2}{q^2}\left(j_1^0 j_2^0 - (-\vec{j}_1)(-\vec{j}_2) \right) = \mathcal{M}.$$

$$P\,\vec{p} = -\vec{p}$$

Fermi theory 

- no mediator
- contact interaction.
- $[G_F] = \text{GeV}^{-2}$

$$\mathcal{M}_{weak} = G_F\, \underline{j_1} \cdot \underline{j_2}$$ how?

current-current theory insted of contact interaction

if $\underline{j_W} = \bar{\psi}\gamma^\mu\psi \Rightarrow$ no violation of parity.

scaler bilinear $\bar{\psi}\psi \Rightarrow$ no $\cancel{P}$

axial vector bilinear $\bar{\psi}\gamma^\mu\gamma^5\psi$

under Lorentz transf as a vector

$$P\, \bar{\psi}\gamma^\mu\gamma^5\psi = \bar{\psi}\gamma^\mu\gamma^5\psi$$

Tensor bilinear $\bar{\psi}\sigma^{\mu\nu}\psi$ $\quad \sigma_{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

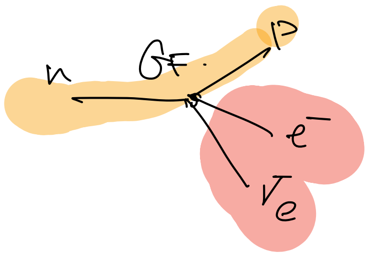
most general current

$$\bar{\psi}\left(c_V \gamma^\mu + c_A \gamma^\mu\gamma^5 + c_T \sigma^{\mu\nu} \right)\psi$$

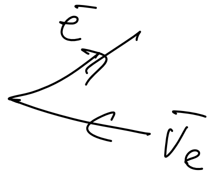
$\hookrightarrow$ coeff. to be measured.

weak current $\bar{\psi}\gamma^\mu(1-\gamma^5)\psi$ V − A Current

$$\underbrace{\bar{\psi}\gamma^\mu\psi}_{V} - \underbrace{\bar{\psi}\gamma^\mu\gamma^5\psi}_{A}.$$



$$\mathcal{M} = G_F\, \underline{j_{e\nu}} \cdot \underline{j_{pn}}$$

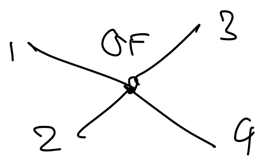


$$\bar{\psi}_\nu \gamma^\mu \psi_e$$

Goldhaber $\Rightarrow$ $\bar{\psi}_\nu = \bar{\psi}_\nu^{LH}$

$$\Rightarrow \bar{\psi}_\nu^{LH}\gamma^\mu\psi_e = \bar{\psi}_\nu\gamma^\mu(1-\gamma^5)\psi_e$$

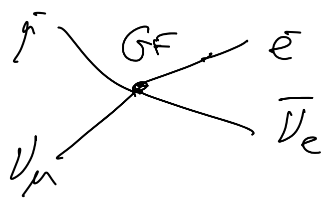
$$\mathcal{M} = G_F(\bar{\psi}_1\gamma^\mu(1-\gamma^5)\psi_2)(\bar{\psi}_3\gamma_\mu(1-\gamma^5)\psi_4)$$



Current − current theory.

How can we verify V−A theory works.

muon decay: $\mu^- \to e^-\bar{\nu}_e\nu_\mu$

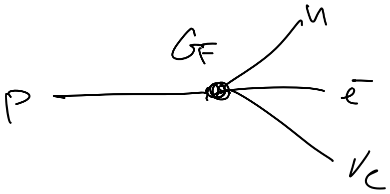


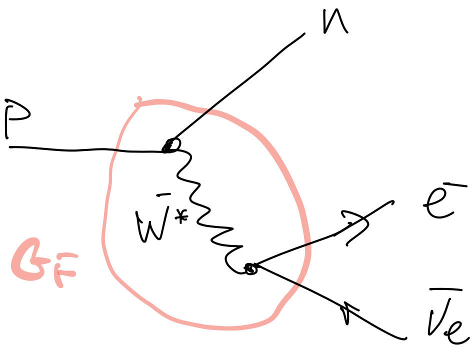
β decay: $n \to p\, e^-\bar{\nu}_e$

π^- : $\pi^\pm \to \ell^\pm \overset{(-)}{\nu_e}$ $\qquad \mu^+\nu_\mu$.

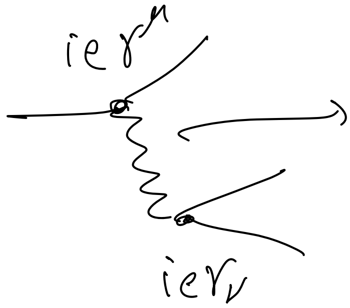
$\qquad e^+\nu_e$

Modern weak interactions.





QED



$$\frac{-ig_{\mu\nu}}{q^2}$$

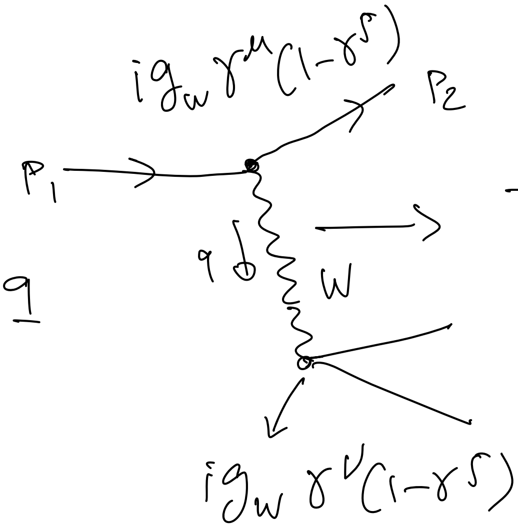
$$\mathcal{M} \sim \frac{e^2}{q^2}(\ldots)$$

$$\alpha = \frac{e^2}{4\pi}$$

$$\mathcal{M} \sim \frac{\alpha}{q^2}$$

QED vertex: $i\sqrt{\alpha}$.

weak interaction:



$\underline{P}_1 = \underline{P}_2 + \underline{q}$

massive propagator.

$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu/M_W^2)}{q^2 - M_W^2}$$

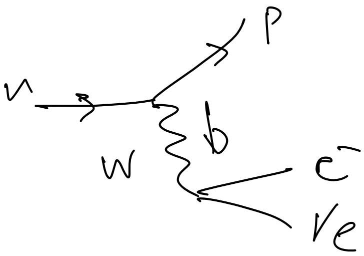
Experimentally $M_W = 80$ GeV.

$n \to p + e^- + \bar{\nu}_e$

$(1+\varepsilon)^n \simeq 1 + n\varepsilon$

$|\vec{q}| \ll M_W$.

$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu/M^2)}{q^2 - M_W^2} \longrightarrow \frac{-ig_{\mu\nu}}{M_W^2}$$



$Q = m_n - m_p - m_e \simeq 1$ MeV.

$q_W \simeq 1$ MeV.

$q^2 - M_W^2 \simeq -M_W^2$

$$G_F \simeq \frac{g_w^2}{M_w^2} \qquad g_w : \text{weak Charge.}$$

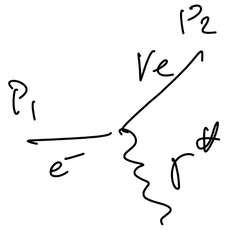
$$\alpha_{EM} = \frac{e^2}{4\pi}. \qquad \alpha_w = \frac{g_w^2}{4\pi}. \qquad G_F = \frac{\sqrt{2}}{8}\frac{g_w^2}{M_w^2}.$$

$M_w \simeq 80$ GeV. measured

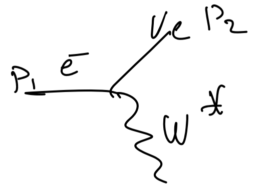
$G_F = 1.16 \times 10^{-5}$ GeV^{-2}. measured.

$$\Rightarrow \quad g_w = 0.653.$$

$$\alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29.5} > \alpha_{EM} = \frac{1}{137}.$$



$\frac{1}{q^2}$



$\frac{1}{q^2 - M_w^2}$.

(if $q^2 \ll M_w^2$

$M_{EM} \simeq \frac{1}{q^2} (-)$ $\qquad M_w \simeq \frac{1}{q^2 M_w^2} (\sim)$

if $|\vec{q}| \ll M_w$

$q \simeq 1$ MeV.

$$M_{EM} \simeq \frac{(\sim)\, e^2}{(1\,\text{MeV})^2}. \qquad M_w \simeq \frac{(-)\, g_w^2}{(80\,\text{GeV})^2}.$$

$$\frac{M_w}{M_{EM}} \simeq \frac{(1\,\text{MeV})^2}{(80\,\text{GeV})^2} \sim (10^{-5})^2$$