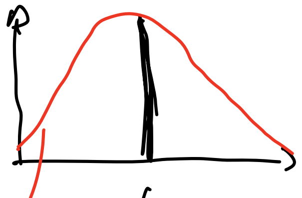
Weak Interactions

$n \to p e^- + X$ in the rest frame of n



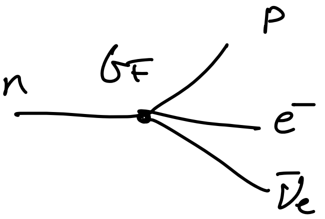
2-body decay.

Experimental evidence

$\Downarrow$

$n \to p e^- \bar{\nu}_e$

Fermi theory



$G_F = 1.16 \times 10^{-5}\ \text{GeV}^{-2}$

β^- decay: $\quad {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$

$Q = M_X(A,Z) - M_Y(A,Z+1) - m_e$

$M_X(A,Z) = A \cdot m_N$ − Binding energy.

$n \to p \pi^-$ $Q < 0$

$n \to \pi^+ \pi^-$
$n \to e^+ e^-$
$\Big\}$ $\cancel{B}$ baryon number violation.

weak int., EM, QCD: conserve B.

	$\mathbb{P}$	$\mathbb{C}$	$\mathbb{CP}$
QED	✓	✓	✓
QCD	✓	✓	✓
weak	✗	✗	✗

Wu experiment $\Rightarrow$ $\mathbb{P}$ violated.

Goldhaber experiment $\Rightarrow$ $\mathbb{C}$ violated.

Cronin–Fitch exp $\Longrightarrow$ $\mathbb{CP}$ violated.

quark flavor violated. $\Lambda\,(uds) \to p\,\pi^-$ $\Delta S = 1$

$\Delta S = 1, \quad \Delta C = 1, \quad \Delta b = 1$

strangeness   charm quark   beauty quark.

e^- ν_e

μ^- ν_μ 1/ how to prove they are different ?

τ^- ν_τ

2/ is $\nu \neq \bar{\nu}$ or not?

$X \to Y + e^- + X$ β^- decay. $n \to p + e^- + X$

Reines- Cowen (1956)

$X + p \to n + e^+$ many events.

$X = \bar{\nu}_e$ no $n + e^-$ events.

no $\mu + n$ events.

Reines- Cowen proved $n \to p + e^- + \bar{\nu}_e$

how to produce ν_μ, or $\bar{\nu}_\mu$

$\mu^- \to e^- \underbrace{\bar{\nu}_e}_{\text{background contamination.}} \nu_\mu$

$\pi^+ \to \mu^+ \nu_\mu$

energetic pions $\xrightarrow{\pi^+}$ μ^+, ν_μ

1962: Lederman- Schwartz, Steinberger AGS @ Brookhaven

1988 Nobel Prize

$p + Be \to \pi^+ + X$

$\hookrightarrow \mu^+ + \nu_\mu$

$\nu_\mu + X \to \mu + X'$

$e + X''$

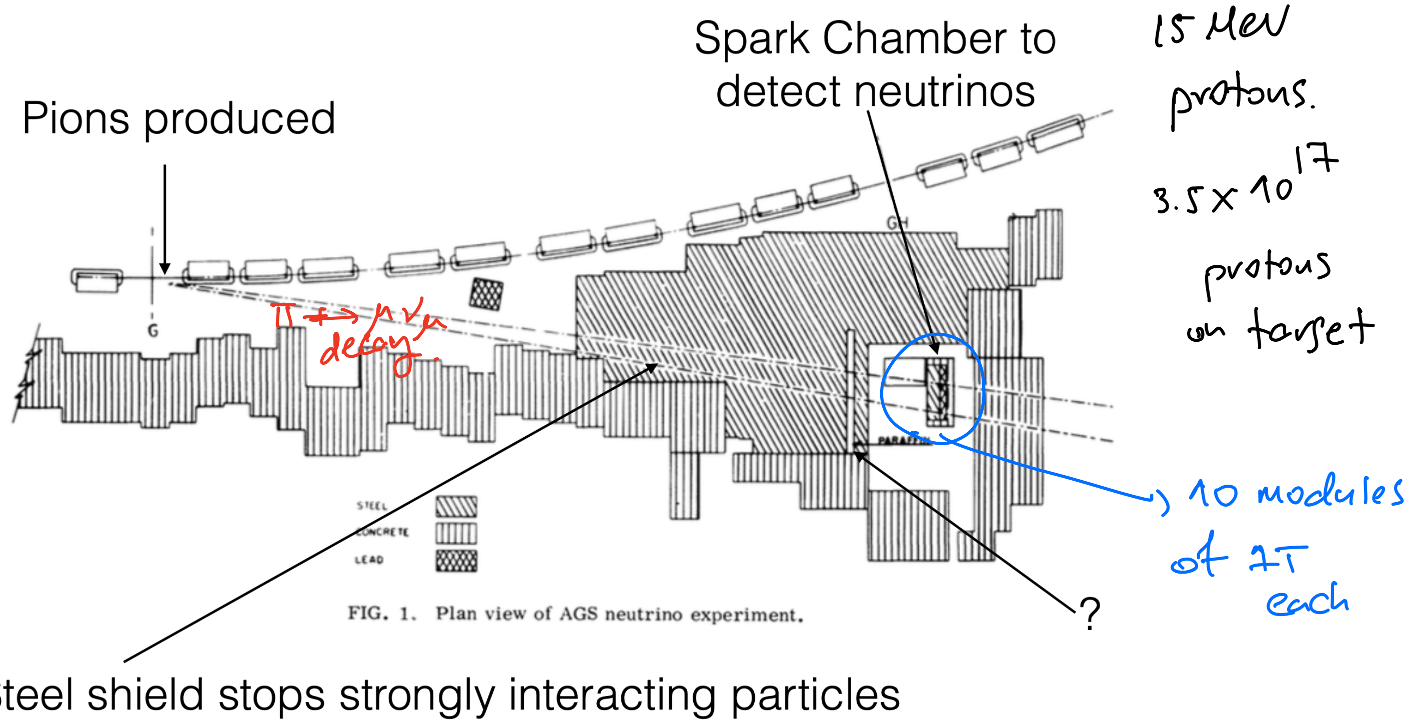


FIG. 1. Plan view of AGS neutrino experiment.

15 MeV protons.

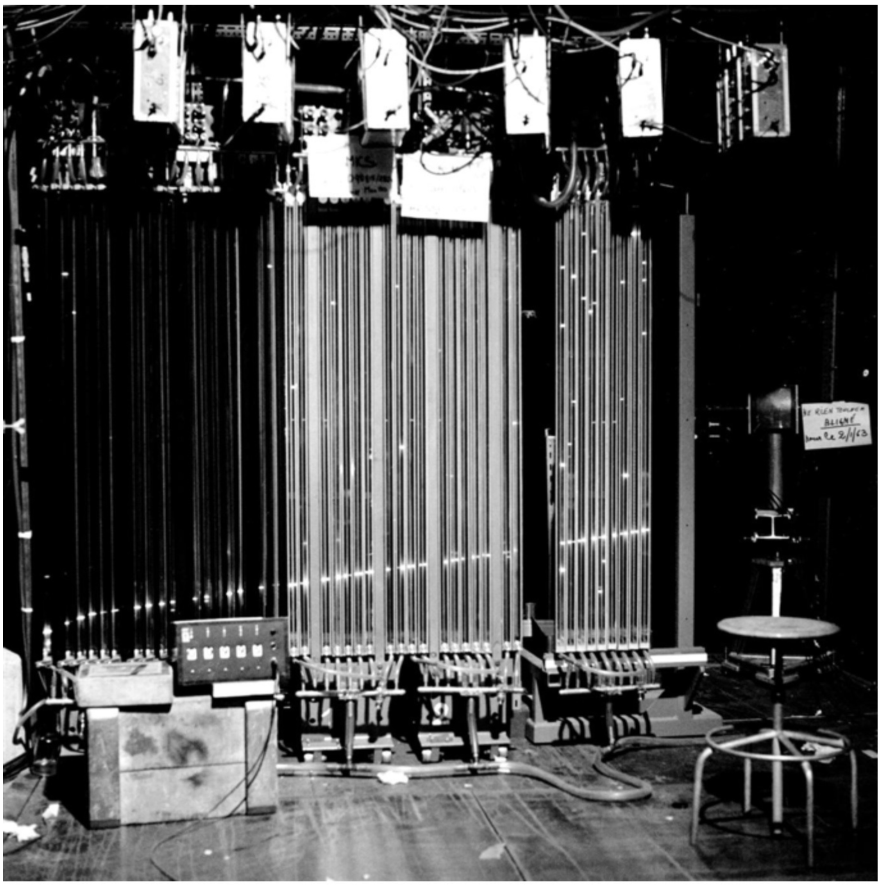
3.5×10^{17} protons on target

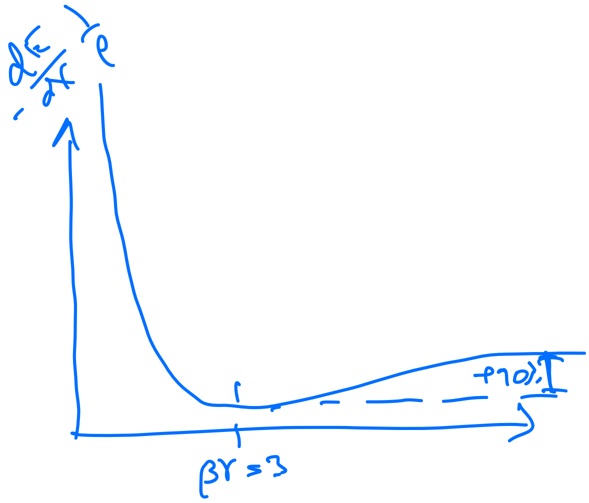
$$p + Be \to \pi^{\pm} + X$$

$$\hookrightarrow \mu^{\pm} + \overset{(-)}{\nu}_\mu$$

$$\hookrightarrow \overset{(-)}{\nu}_\mu + X \longrightarrow \mu^{\mp} + X'$$

$$e^{\mp} + X''$$





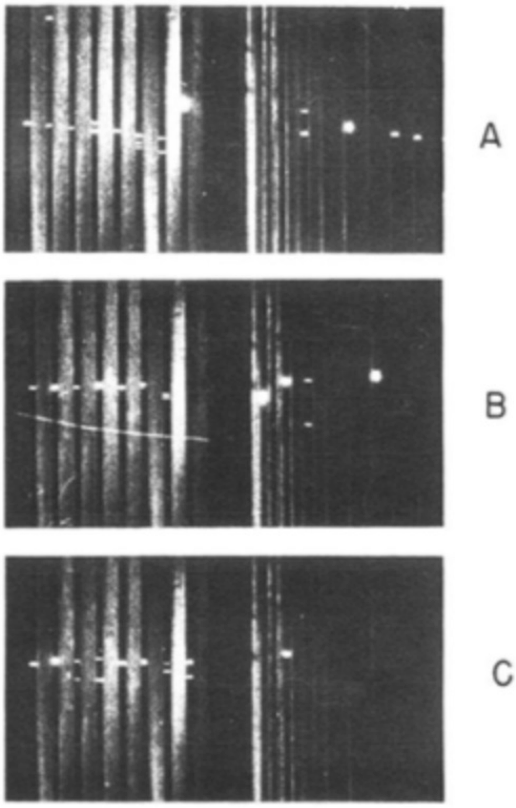


FIG. 8. 400-MeV electrons from the Cosmotron.

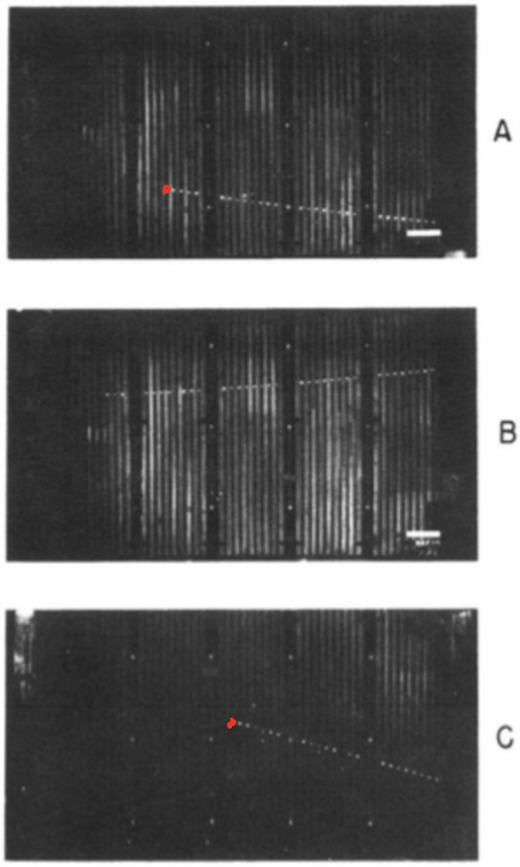


FIG. 5. Single muon events. (A) $p_\mu > 540$ MeV and δ ray indicating direction of motion (neutrino beam incident from left); (B) $p_\mu > 700$ MeV/c; (C) $p_\mu > 440$ with δ ray.

34 events of $\mu^- + X$

6 e^- showers. (In agreement with expectation)

$\Rightarrow$ Experimental evidence $\nu_e \neq \nu_\mu$.

e^-	μ^-	τ^-
ν_e	ν_μ	ν_τ
$L_e = +1$	$L_\mu = +1$.	$L_\tau = +1$.

1998: DONUT experiment @ Fermilab discovered ν_τ

conservation of lepton family number in QED, QCD, weak.

Phenomenology of weak interactions

Semileptonic decays: $n \to p e^- \bar{\nu}_e$

leptonic decays:

$$\pi^+ \to \mu^+ \nu_\mu$$

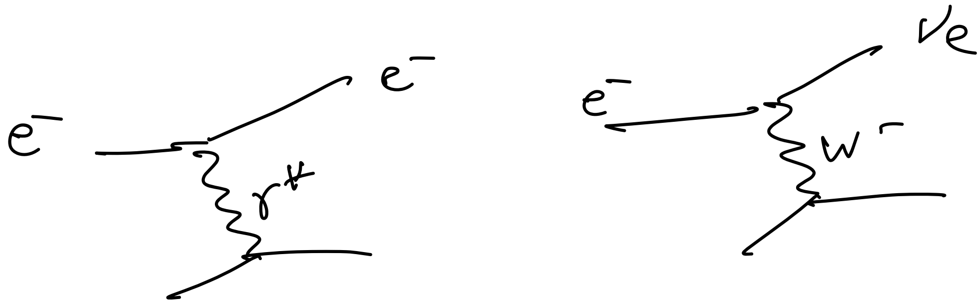
$$\mu^+ \to \bar{\nu}_\mu \; e^+ \; \nu_e$$

	μ^+	$\bar{\nu}_\mu$	e^+	ν_e
L_e	0	0	-1	+1
L_μ	-1	-1		

Hadronic decays: $\Lambda \to p\,\pi^-$ $\qquad \Delta S = 1. \quad \begin{pmatrix} \Delta b = 1 \\ \Delta C = 1 \end{pmatrix}$

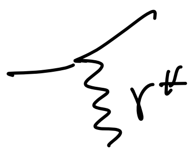
$$K^+ \to \pi^+ \pi^0$$

Experimental signatures:
- unbalance decays or scattering (because of ν)
- long lifetime



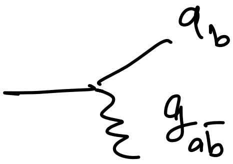
weak. scattering

QED



$\alpha_{EM} = \frac{1}{137}$ $\qquad$ 1 photon massless.
$Q = 0.$
$S = 1$

QCD

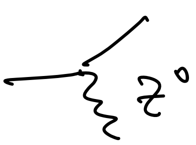


$\alpha_{QCD} \simeq 0.1$ $\qquad$ 8 gluons massless.
$Q = 0.$
colored.
$S = 1$

weak.

$\alpha_{weak} = ?$ $\qquad$ 2 charged vector bosons.

$W^\pm \quad m = 80$ GeV.



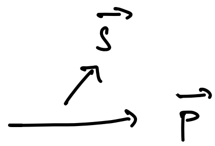
$Z^0 \quad m = 90$ GeV.

$S = 1$

Violation of parity in weak interaction $\Rightarrow$ handedness, helicity, chirality $\Big\}$ important in weak interaction.

any massive particle

$$h = \frac{\vec{p}\cdot\vec{s}}{|\vec{p}||\vec{s}|}$$



not Lorentz invariant

For massless particles: helicity is Lorentz invariant

In high energy limit $E \gg p$, $\beta \to 1$. $\beta = \frac{p}{E}$

$\Rightarrow$ helicity good also for massive particles.

1950's: Lee, Yang proposed chiral theory for weak interaction

Wu (P violation)

Goldhaber (C violation)

$\Rightarrow$ weak interactions depend on chirality of particles.

Fermion Spinor $\psi = \psi^{LH} + \psi^{RH}$

$$\psi^{LH} = \frac{1}{2}(1-\gamma^5)\psi.$$

$$\psi^{RH} = \frac{1}{2}(1+\gamma_5)\psi$$

$$\gamma^5\psi^{LH} = \frac{1}{2}(\gamma^5-(\gamma^5)^2)\psi$$

$$= -\frac{1}{2}(1-\gamma^5)\psi = -\psi^{LH}$$

$$\gamma^5\psi^{RH} = \psi^{RH}.$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

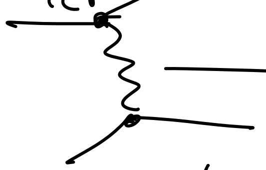
σ^i : Pauli matrices.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(\gamma^5)^2 = \mathbb{1}$$

How to incorporate chirality in weak interaction?

QED: $ie\gamma^\mu$ $\bar{\psi}\gamma^\mu\psi = j^\mu$



$\dfrac{-ig^{\mu\nu}}{q^2}$

$\bar{\psi}\gamma^\nu\psi = j^\nu$

$$\mathcal{M}_{QED} \sim \frac{\alpha}{q^2}\, j^\mu \cdot g_{\mu\nu}\, j^\nu = \frac{\alpha}{q^2}\, \underline{j}\cdot\underline{j}$$

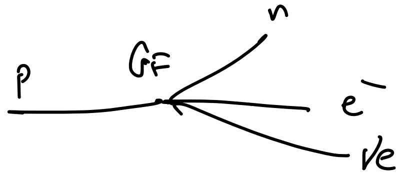
$j^\mu = (\bar{\psi}\gamma^0\psi,\ \bar{\psi}\gamma^i\psi)$ under Lorentz transformation. j^μ is a vector

$\mathbb{P}\, j^\mu = (\bar{\psi}\gamma^0\psi,\ -\bar{\psi}\gamma^i\psi)$ behaves like vector under parity.

$$\mathcal{M}_{QED} = \frac{\alpha}{q^2}\left(j_1^0 j_2^0 - \vec{j}_1\cdot\vec{j}_2\right)$$

$\mathbb{P}\,\mathcal{M}_{QED} = \mathcal{M}_{QED}$ intrinsically QED invariant under parity.

β^- decay:



in similarity with QED.

$$\mathcal{M}_W = G_F \cdot \underline{j}_{pn} \cdot \underline{j}_{e\nu}$$

$\underline{j}_{pn} = \bar{\psi}_n\gamma^\mu\psi_p$ $\qquad \underline{j}_{e\nu} = \bar{\psi}_{\nu_e}\gamma^\nu\psi_e$

if this structure correct $\Rightarrow$ no parity violation.

$\mathbb{P}\,\vec{V} \rightarrow -\vec{V}$

$\mathbb{P}\,S \rightarrow S$

$\mathbb{P}\,\vec{A} \rightarrow \vec{A}$ pseudo-vector (axial)

$\mathbb{P}\,PS \rightarrow -PS$ pseudo-scalar.

$$\mathcal{M}_W = \underline{j} \cdot \underline{j}.$$

$$\mathcal{M} = (\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \quad \text{does not violate } \mathbb{P}.$$

what other current forms:

$\bar{\psi}\psi$ scalar.

$\bar{\psi}\gamma^5\gamma^\mu\psi$ pseudo-vector/axial.

$\bar{\psi}\sigma_{\mu\nu}\psi$ tensor bilinear.

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$

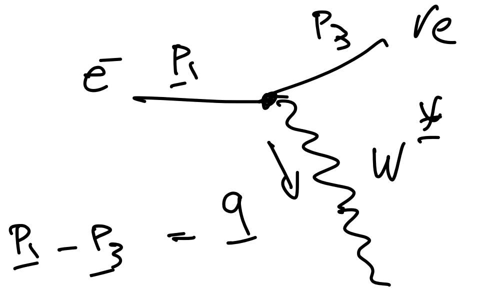
To build a current that violates $\mathbb{P}$arity $\Rightarrow$ Combination of
vector
axial
tensor.

$$j_{weak} = \bar{\psi}\left(C_V\gamma^\mu + C_A\gamma^5\gamma^\mu + C_T\sigma^{\mu\nu}\right)\psi$$

C_V, C_A, C_T: coeff. to be measured

Goldhaber $\Rightarrow$ only LH neutrinos. $\nu_e \overset{\Leftarrow}{\longrightarrow} \vec{p}$

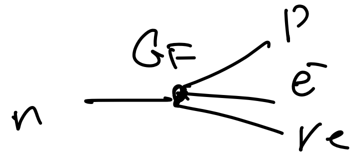
$$j_{weak} = \bar{\psi}\gamma^\mu(1-\gamma^5)\psi \qquad \text{V-A theory}$$



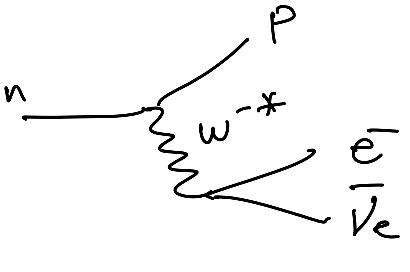
$i g_W \gamma^\mu(1-\gamma^5)$ V-A- theory.

$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu/M_W^2)}{q^2 - M_W^2}$$

$M_W = 80$ GeV.



$\Rightarrow$



$Q = 1$ MeV.