Symmetry: unitary (anti-unitary operators)

First Nöther's theorem: symm. $\Longleftrightarrow$ some conserved quantity

constant of motion is a physical quantity only for hermitian ops.

External symm: act on $\vec{x}, t$

Internal symm: act on internal quantum numbers:

continuous: $U(a_1, \dots, a_N) = e^{i a_j T_j}$ $\quad$ T_j: generators of U

a_j: parameters.

$T_j = T_j^{+}$ hermition.

$$U(-) = e^{i \vec{P} \cdot \vec{x}}$$
$$U(-) = e^{i E t}$$
$$U(-) = e^{i \vec{L} \cdot \hat{\theta}} \qquad \hat{\theta} = (\theta_1, \theta_2, \theta_3)$$

$$\psi = \psi_1 \psi_2$$

For continuous operators $\Rightarrow$ additive eigenvalues.

1 generator $\quad U(a) = e^{iaG}$ $\qquad a \to 0 \quad U = \mathbb{1} + iaG$

$$U \psi_1 \psi_2 = (U \psi_1)(U \psi_2) = (\mathbb{1} + iaG)\psi_1 (\mathbb{1} + iaG)\psi_2$$

$$G\psi_i = g_i \psi_i$$

$$U \psi_1 \psi_2 = (1 + iag_1)\psi_1 (1 + iag_2)\psi_2 \approx e^{i(g_1 + g_2)a} \psi_1 \psi_2$$

$$g_{(12)} = g_1 + g_2$$

Discrete Transformations:

$\mathbb{P}, \mathbb{C}$

Inversions $\qquad \mathbb{P}: \quad \vec{x} \to -\vec{x}$

$\qquad \vec{P} \to -\vec{P}$

$$\mathbb{P}\mathbb{P}\psi = \mathbb{1}\psi \qquad \mathbb{P}^{-1} = \mathbb{P}^{+} \text{ hermition operator.}$$

$\mathbb{P}^2 \psi = a^2 \psi \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$ eigenvalues of parity

$\mathbb{P}$ hermitian

intrinsic parity: eigenvalue of Parity in the particle rest frame.
Lorentz invariant

parity is conserved in Strong and EM interactions.

For massless particles use Quantum Field Theory.

leptons ($q=-1$), e^-, ν_e (neutrinos) $P=+1$ } $s=1/2$ fermions.

quarks : $P=+1$.

Dirac theory : anti-fermions: $P=-1$.

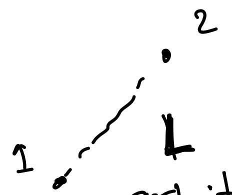
antibosons: same parity as bosons

$f\bar{f}$ pair: intrinsic parity $= -1$

$B\bar{B}$ pair: int. parity $= +1$

parity of γ : -1

$\psi = \psi_1 \psi_2$ $\qquad \mathbb{P}\psi = \underbrace{P_1 P_2}_{\text{intrinsic parity}} \mathbb{P}\psi_{space}$.



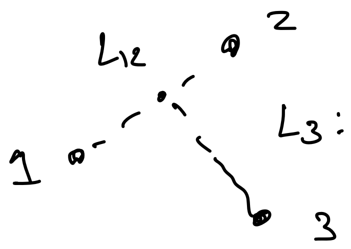
orbital angular mom.

$= P_1 P_2 (-1)^L \psi_{space}$

pair fermion-antifermion: $P = P_1 P_2 (-1)^L = (-1)^{L+1}$

pair boson-antiboson: $P = (+1\,+1)(-1)^L = (-1)^L$

$\psi = \psi_1 \psi_2 \psi_3$ $\qquad \mathbb{P}\psi = P_1 P_2 P_3\ \mathbb{P}\psi_{space}$



ang.
L_3: mom. of 3 wrt center of mass of 12

$P = P_1 P_2 P_3 (-1)^{L_{12}+L_3}$

Example: proton : uud $\quad L=0.$

$$\text{intrinsic } P_{prot} = (+1)(+1)(+1)(-1)^0 = +1$$

neutron: udd

$$P_{neut} = (+1)(+1)(+1)(-1)^0 = +1$$

$$\pi^+ = u\bar{d} \qquad P_{\pi^+} = (+1)(-1)(-1)^{L=0} = -1$$

$$\mathbb{P} \quad \bullet\!\!\longrightarrow \pi^+ \quad \Rightarrow \quad \overset{\pi^+}{\longleftarrow\!\!\bullet}$$

$$\mathbb{P}\,|\pi^+\rangle = -\,|\pi^+\rangle$$

$\mathbb{C}$ parity

Inversion $\Rightarrow$ $\quad \mathbb{C}\,\mathbb{C}\,\psi = a^2\psi \quad \Rightarrow \quad a = \pm 1.$

$\mathbb{C}$: changes particle $\longrightarrow$ anti particle.

$\vec{x}, \vec{p}, \vec{s}$ are unchanged.

all quantum numbers $= (-1) \times$ (quantum number)

$\mathbb{C}$: $q \longrightarrow -q$ electric charge.

$\mathbb{C}\psi = a\,\psi$ $\quad$ only neutral particles can be eigenstates.

n, γ, π^0, ν ?

γ: eigenstate of $\mathbb{C}$

$n \neq \bar{n} \quad , \quad \nu \neq \bar{\nu}$

EM, strong interaction Conserve $\mathbb{C}$

$$C_\gamma = -1$$

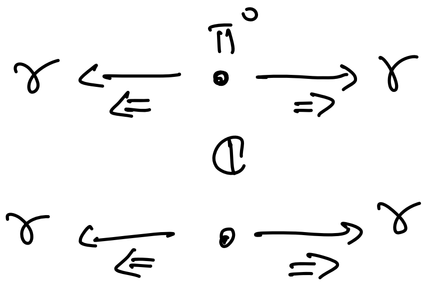
$\pi^+\pi^-$ $\qquad$ $\pi^-\pi^+$

$\mathbb{C}$ $\quad \pi^+\pi^-\pi^0$ $\qquad$ $\pi^-\pi^+\pi^0$

$\pi^+\pi^-\pi^+\pi^-\pi^0$ $\qquad \vdots$

Interesting to study $\mathbb{C}$ acting on state of N particles.

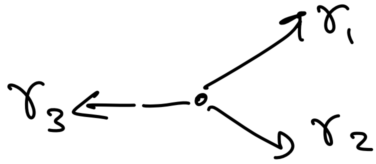
$\pi^0 \to \gamma\gamma$ EM decay observed.



$$\mathbb{C}\,\psi_{\pi^0} = C_\gamma C_\gamma \psi_{\pi^0}$$

$$\Rightarrow (C_\gamma)^2 = C_{\pi^0} \Rightarrow C_{\pi^0} = +1$$

$\pi^0 \xrightarrow[\text{EM.}]{} \gamma\gamma\gamma$



Particle Data Group.

$C_{\pi^0} = +1 \longrightarrow (-1)^3 = -1$

π^0 DECAY MODES

For decay limits to particles which are not established, see the appropriate Search sections (A^0 (axion) and Other Light Boson (X^0) Searches, etc.).

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	2γ		(98.823 ± 0.034) %	S=1.5
Γ_2	$e^+e^-\gamma$		(1.174 ± 0.035) %	S=1.5
Γ_3	γ positronium		$(1.82 \pm0.29) \times 10^{-9}$	
Γ_4	$e^+e^+e^-e^-$		$(3.34 \pm0.16) \times 10^{-5}$	
Γ_5	e^+e^-		$(6.46 \pm0.33) \times 10^{-8}$	
Γ_6	4γ		$< 2 \times 10^{-8}$	CL=90%
Γ_7	$\nu\overline{\nu}$	[a]	$< 2.7 \times 10^{-7}$	CL=90%
Γ_8	$\nu_e\overline{\nu}_e$		$< 1.7 \times 10^{-6}$	CL=90%
Γ_9	$\nu_\mu\overline{\nu}_\mu$		$< 1.6 \times 10^{-6}$	CL=90%
Γ_{10}	$\nu_\tau\overline{\nu}_\tau$		$< 2.1 \times 10^{-6}$	CL=90%
Γ_{11}	$\gamma\nu\overline{\nu}$		$< 6 \times 10^{-4}$	CL=90%
	Charge conjugation (C) or Lepton Family number (LF) violating modes			
Γ_{12}	3γ	C	$< 3.1 \times 10^{-8}$	CL=90%
Γ_{13}	μ^+e^-	LF	$< 3.8 \times 10^{-10}$	CL=90%
Γ_{14}	μ^-e^+	LF	$< 3.4 \times 10^{-9}$	CL=90%
Γ_{15}	$\mu^+e^- + \mu^-e^+$	LF	$< 3.6 \times 10^{-10}$	CL=90%

$\pi^0 \to \gamma\gamma\gamma$ Forbidden by $\mathbb{C}$ conservation in EM.

	$\pi^0 \to$	μ^+	e^-
L_e	0	0	1
L_μ	0	-1	0

Not possible if lepton flavor conserved.

$\mathbb{C}$ parity for $B\overline{B}$ boson - antiboson. $C=+1$

$F\overline{F}$ fermion - $\overline{\text{fermion}}$ $C=-1$

$C_\gamma = -1$

Two examples: $\pi^+\pi^-$ $\quad \mathbb{C}: \pi^+\pi^- \longrightarrow \pi^-\pi^+.$

$$\mathbb{C}\,\psi_{\pi^+\pi^-} = \alpha\,\psi_{\pi^+\pi^-} \qquad \mathbb{P}: \vec{p} \to -\vec{p}$$

$$\pi^+ \longleftarrow \quad \longrightarrow \pi^-$$
$$\mathbb{P} \Downarrow$$
$$\pi^- \longleftarrow \quad \longrightarrow \pi^+$$

$\mathbb{C}$ operation same as $\mathbb{P}$

$$\pi^+ \longleftarrow \quad \longrightarrow \pi^-$$
$$\mathbb{C} \Downarrow$$
$$\pi^- \longleftarrow \quad \longrightarrow \pi^+$$

$$\mathbb{C}\,\psi_{\pi^+\pi^-} \equiv \mathbb{P}\,\psi_{\pi^+\pi^-} = (-1)^L\,\psi_{\pi^+\pi^-}$$

L: angular mom. of $\pi^+\pi^-$ system.

$$C_{\pi^+\pi^-} = C_{\pi^+} C_{\pi^-} (-1)^L$$

π: bosons. $\quad C_{\pi^+} C_{\pi^-} = +1$

$$C_{B\bar{B}} = (-1)^L$$

Example: $e^+ e^-$ $\quad s = 1/2.$ $\quad \mathbb{C}: e^+ \to e^-$

$$e^+ \overset{\Rightarrow}{\longleftarrow} \quad \overset{\Leftarrow}{\longrightarrow} e^- \quad \overset{\mathbb{C}}{\Longrightarrow} \quad \boxed{e^- \overset{\Rightarrow}{\longleftarrow} \quad \overset{\Leftarrow}{\longrightarrow} e^+}$$

$$e^+ \overset{\Rightarrow}{\longleftarrow} \quad \overset{\Leftarrow}{\longrightarrow} e^- \quad \overset{\mathbb{P}}{\Longrightarrow} \quad e^- \overset{\Leftarrow}{\longleftarrow} \quad \overset{\Rightarrow}{\longrightarrow} e^+$$

$$e^+ \overset{\Rightarrow}{\longleftarrow} \quad \overset{\Leftarrow}{\longrightarrow} e^- \quad \overset{\mathbb{S}}{\underset{\text{spin flip.}}{\Longrightarrow}} \quad e^+ \overset{\Leftarrow}{\longleftarrow} \quad \overset{\Rightarrow}{\longrightarrow} e^-$$

$$e^+ \overset{\Rightarrow}{\longleftarrow} \quad \overset{\Leftarrow}{\longrightarrow} e^- \quad \underset{\mathbb{P}+\mathbb{S}}{\Longrightarrow} \quad \boxed{e^- \overset{\Rightarrow}{\longleftarrow} \quad \overset{\Leftarrow}{\longrightarrow} e^+}$$

$$\mathbb{C} \equiv \mathbb{P} + \mathbb{S}$$

$$\mathbb{P}\,\psi_{e^+e^-} = (-1)^L\,\psi_{e^+e^-}.$$

$$\mathbb{S}\,\psi_{e^+e^-} = \mathbb{S}\,\psi_{space}\,\psi_{spin}. = \psi_{space}\,\mathbb{S}\,\psi_{spin}.$$

$$\mathbb{S}\, \psi_{e^+e^-} = (-1)^{S+1}\, \psi_{e^+e^-}$$

Recall.

$$\mathbb{S}\, \psi_{B\bar{B}} = (-1)^{S}\, \psi_{B\bar{B}}$$

$$\mathbb{S}\, \psi_{F\bar{F}} = (-1)^{S+1}\, \psi_{F\bar{F}}.$$

$$\mathbb{C}\, \psi_{e^+e^-} = (\mathbb{P} \cdot \mathbb{S})\, \psi_{e^+e^-} = (-1)^{L} (-1)^{S+1} \underbrace{C_{e^+} C_{e^-}}_{-1 \text{ From Dirac}}$$

$$= (-1)^{L+S} (-1)^{2} = (-1)^{L+S}$$

$$\mathbb{C}\, \psi_{F\bar{F}} = (-1)^{L+S}\, \psi_{F\bar{F}}$$

$$\mathbb{C}\, \psi_{B\bar{B}} = (-1)^{L+S}\, \psi_{B\bar{B}}$$

$$C_{p\bar{p}} = (-1)^{L+S}$$

L: orbital ang. mom.
S: spin of particle p.

$$P_{P_1 P_2} = P_1 P_2 (-1)^{L}$$

apply it to:

$p\bar{p}\ \pi^+\pi^-$

$p\bar{p}\ pp$

$\pi^+\pi^-\pi^0$

$\pi^0\pi^0\pi^0.$

$\pi^+\pi^-\pi^+\pi^-$

Isospin: Continuous internal symmetry

Heisenberg $\quad n \quad m \simeq 940$ MeV. $\quad \frac{\Delta m}{m} \simeq 2/1000$

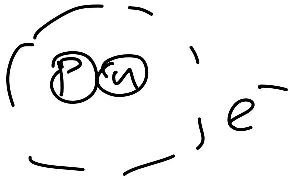
$\quad p \quad \Delta m \simeq 2$ MeV.

nucleon $= \begin{pmatrix} p \\ n \end{pmatrix}$ doublet of isospin. I. Same algebra as $\vec{J}$

$p = |I = 1/2, I_3 = +1/2\rangle \qquad n = |I = 1/2, I_3 = -1/2\rangle$

π^+, π^- have same mass. $m_{\pi^0} \simeq m_{\pi^+}$. $\quad \frac{\Delta m_\pi}{m_\pi} \simeq \frac{5 \text{ MeV}}{140 \text{ MeV}}$

Pion triplet: $\pi^+ = |1, +1\rangle,\ \pi^- |1, -1\rangle \quad \pi^0 = |1, 0\rangle$

deuterium: 

deuton: nucleus of deut.

$\pi^+ + p \rightarrow d + \pi^0$

$nn = ?$

pn

$pp = ?$

$I_{tot} = 0,\ I_3 = 0.$

$p \otimes n = |1/2, 1/2\rangle \otimes |1/2, -1/2\rangle$

pn singlet: $|0,0\rangle$

triplet $|1,0\rangle$

$pp = |1, +1\rangle.$

$pn = |1, 0\rangle.$

$nn = |1, -1\rangle.$

we do not see bound pp or nn $\Rightarrow$ hypothesis pn is isospin singlet.

$p + p \rightarrow$