



$$e^- + p \rightarrow e^- + X \quad \text{DIS}$$

For N partons:

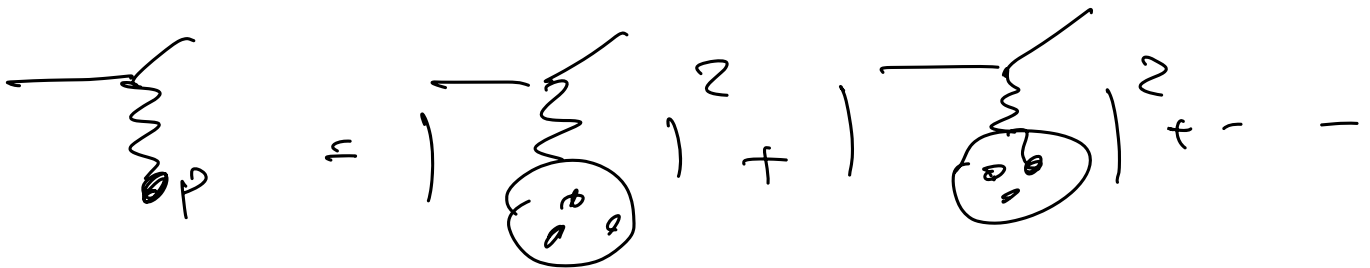
$$W_1(Q^2, \nu) = \sum_j^{\text{partons}} z q_j^2 \frac{f_j(x)}{2M}$$

$$W_2(Q^2, \nu) = \sum_j z q_j^2 f_j(x) \frac{x}{\nu}$$

$f(x)$
parton
density
function.

$$F_1(x) = M W_1(Q^2, \nu). \quad F_2(x) = \nu W_2(Q^2, \nu)$$

$$F_2(x) = 2x F_1(x) \quad \text{Callan-Gross relation.}$$



elastic scattering on parton.

proton: container of some constituents (parton)

$$\text{parton: } S = 1/2 \quad M = XM$$

$$x = \frac{Q^2}{2M\nu}$$

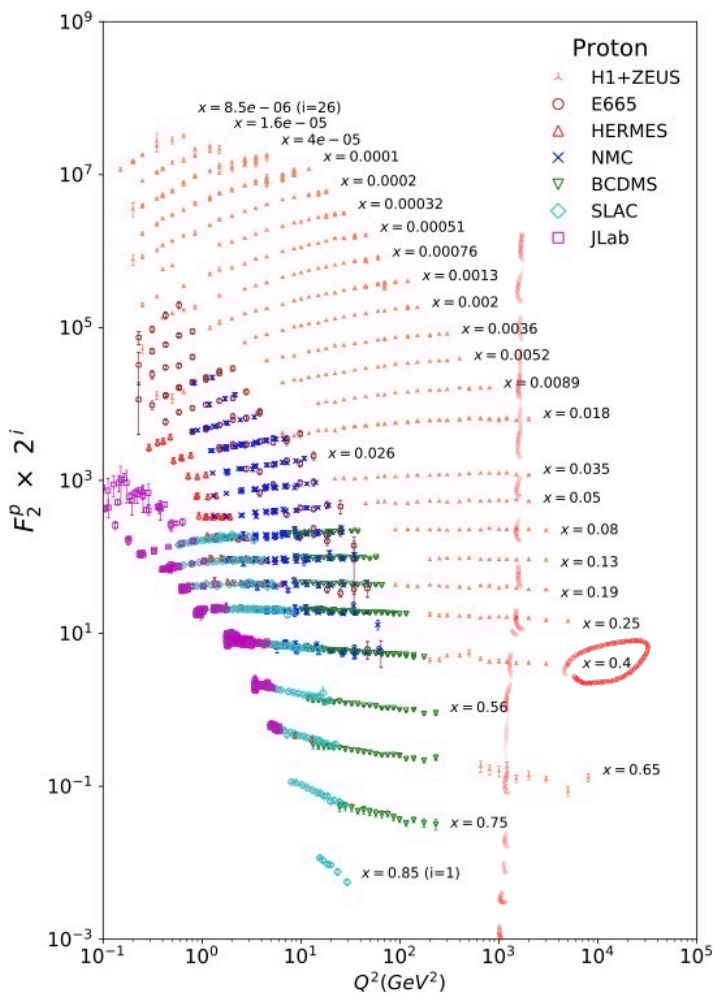
$$Q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad \nu = E - E'$$

observed/measured quantities
with leptons/probe

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{DIS}} \sim \frac{\alpha^2}{Q^4} E'^2 \left[W_2 \sin^2 \frac{\theta}{2} + 2W_1 \cos^2 \frac{\theta}{2} \right]$$

$\delta(\nu - \frac{Q^2}{2M})$

parton j has charge $z q_j e < e$



In principle

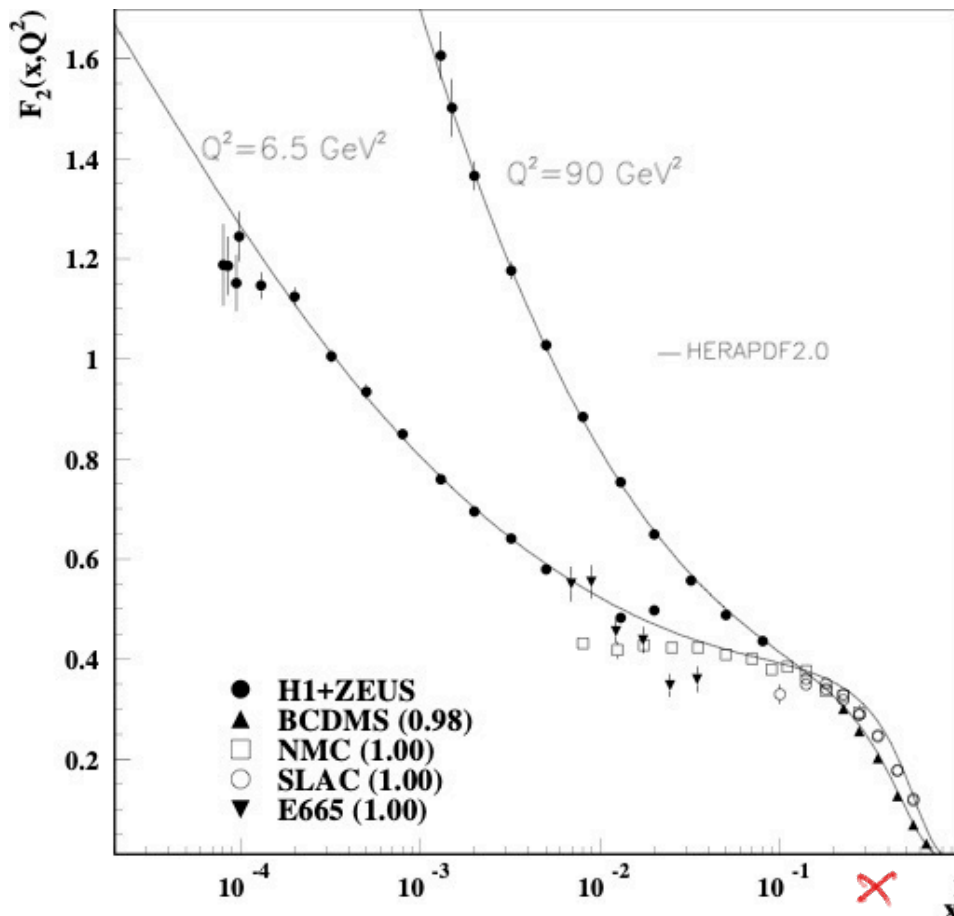
$$F_2 = F_2(Q^2, \nu)$$

$$x = \frac{Q^2}{2M\nu}$$

$\Rightarrow F_2$ function of x
not Q^2, ν separately.

$$f(x, y) = x^2 + y^2$$

$$f(x, y) = \frac{x}{y}$$



$$x_B = \frac{Q^2}{2M\nu}$$

Bjorken Scaling
variable x

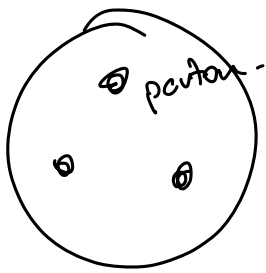
$$\int_0^1 dx f(x) = 1$$

$$x \in [0, 1]$$

perturbative evidence from D. I. S. \equiv incoherent sum of elastic scattering

Feynman: x as property of parton.

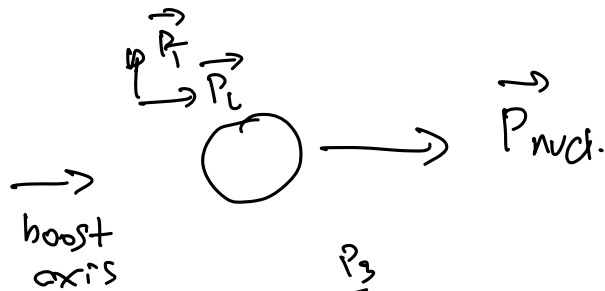
Quark-parton model



high energy limit. $E \gg m_e, m_p, m_{\text{parton}}$.

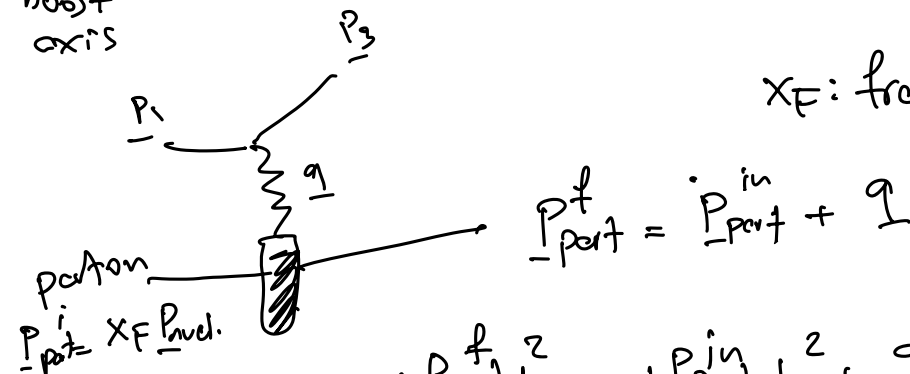
$$x_F = \frac{|\vec{P}_L|}{|\vec{P}_{\text{nucle}}|}$$

SLAC experiment: $E_e = 25 \text{ GeV}$



$$\vec{P}_{\text{part}} = x_F \vec{P}_{\text{nucle}}$$

x_F : fraction of momentum carried by parton.



$$\vec{P}_{\text{part}}^{\text{f}} = \vec{P}_{\text{part}}^{\text{in}} + \vec{q}$$

$$|\vec{P}_{\text{part}}^{\text{f}}|^2 = |\vec{P}_{\text{part}}^{\text{in}}|^2 + \vec{q}^2 + 2 \vec{P}_{\text{part}}^{\text{in}} \cdot \vec{q}$$

$$\underbrace{m_p^2}_{\approx 0} \approx 0 \quad \text{high energy limit.}$$

$$q^2 = -Q^2 \Rightarrow Q^2 = 2 \vec{P}_{\text{part}}^{\text{in}} \cdot \vec{q} =$$

$$\vec{P}_{\text{nucle}}^{\text{LAB}} = (M, 0) \quad \vec{q} = (E - E', \vec{P}_1 - \vec{P}_3) = (V, \vec{P}_1 - \vec{P}_3)$$

$$\Rightarrow Q^2 = 2 x_F M V \Rightarrow x_F = \frac{Q^2}{2 M V} \approx x_B \quad \text{at high energy}$$

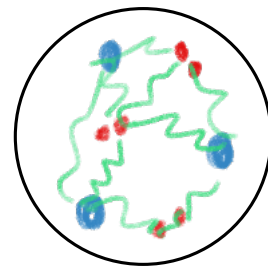
$$N \text{ partons} \quad \sum_i x_i = 1 \quad \sum_i \vec{P}_{\text{part}}^{\text{i}} = \vec{P}_{\text{nucle}}$$

$$\text{proton: } u \text{ and } \bar{u} \text{ quarks. } +\frac{2}{3} \begin{pmatrix} u \\ d \end{pmatrix} -\frac{1}{3}$$

$$Q_p = +1 = \frac{2}{3} + \frac{2}{3} - \frac{1}{3}$$

proton $uud\bar{d}\bar{d}s\bar{s}$

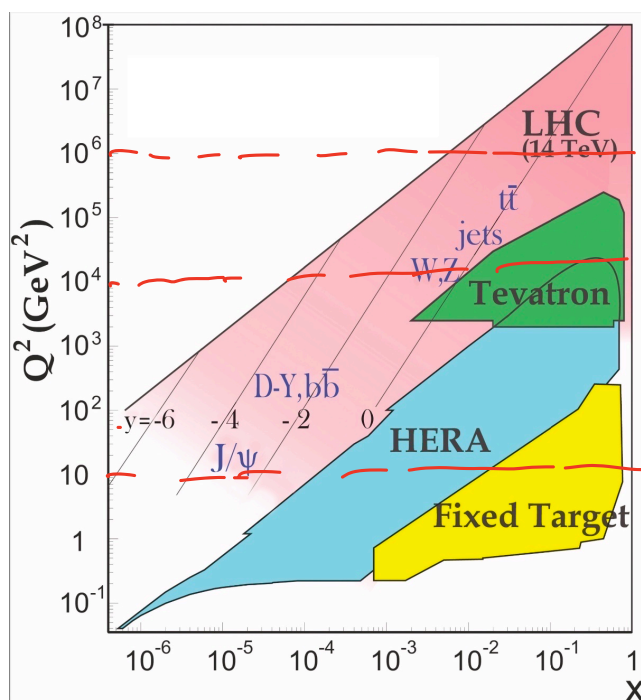
$$\sum x_u x_{\bar{u}} x_d x_{\bar{d}} x_s x_{\bar{s}} = 1.$$



Valence quarks: uud (electric charge)

Sea quarks/anti-quarks: $s\bar{s} + q\bar{q}$

gluons: g mediators of strong interaction

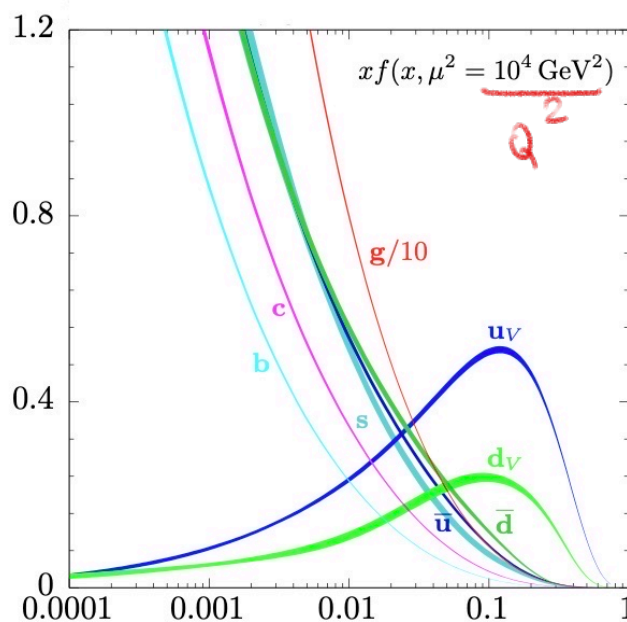
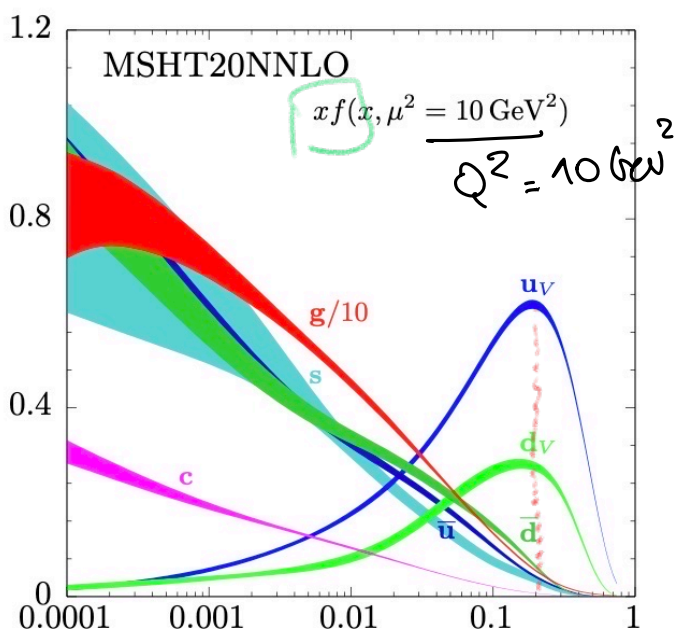


Tevatron: $\vec{p} \rightarrow \leftarrow p$ $\sqrt{s} = 1.96 \text{ TeV}$

HERA $e^- + p(\text{beam})$.

$\vec{e^-} \rightarrow \leftarrow p$

$f(x)$: parton density function.



d_v : $f(x)$ for valence d quark.

u_v : $f(x)$ for valence u quark.

\bar{u} : $f(x)$ for sea anti-quark \bar{u}

Experimentally: $\sum_i^{\text{Valence}} x_i = 0.5 \Rightarrow$ existence of sea \bar{q} and gluons.

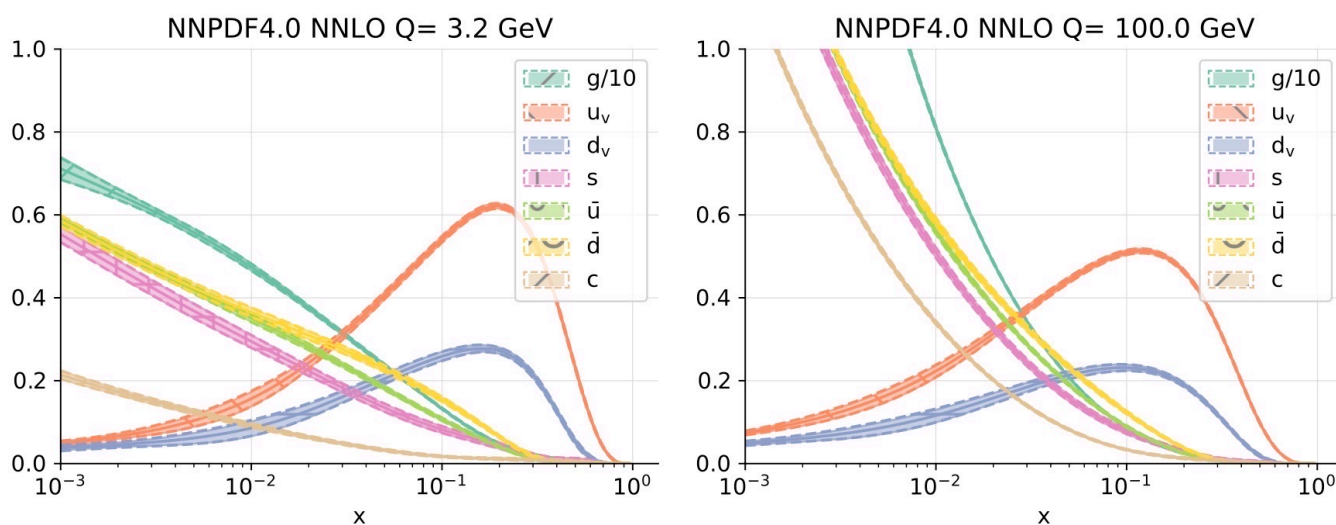
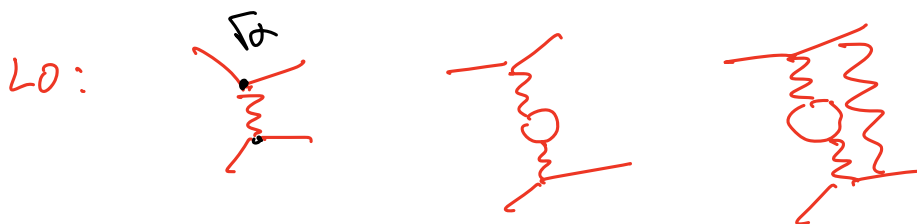


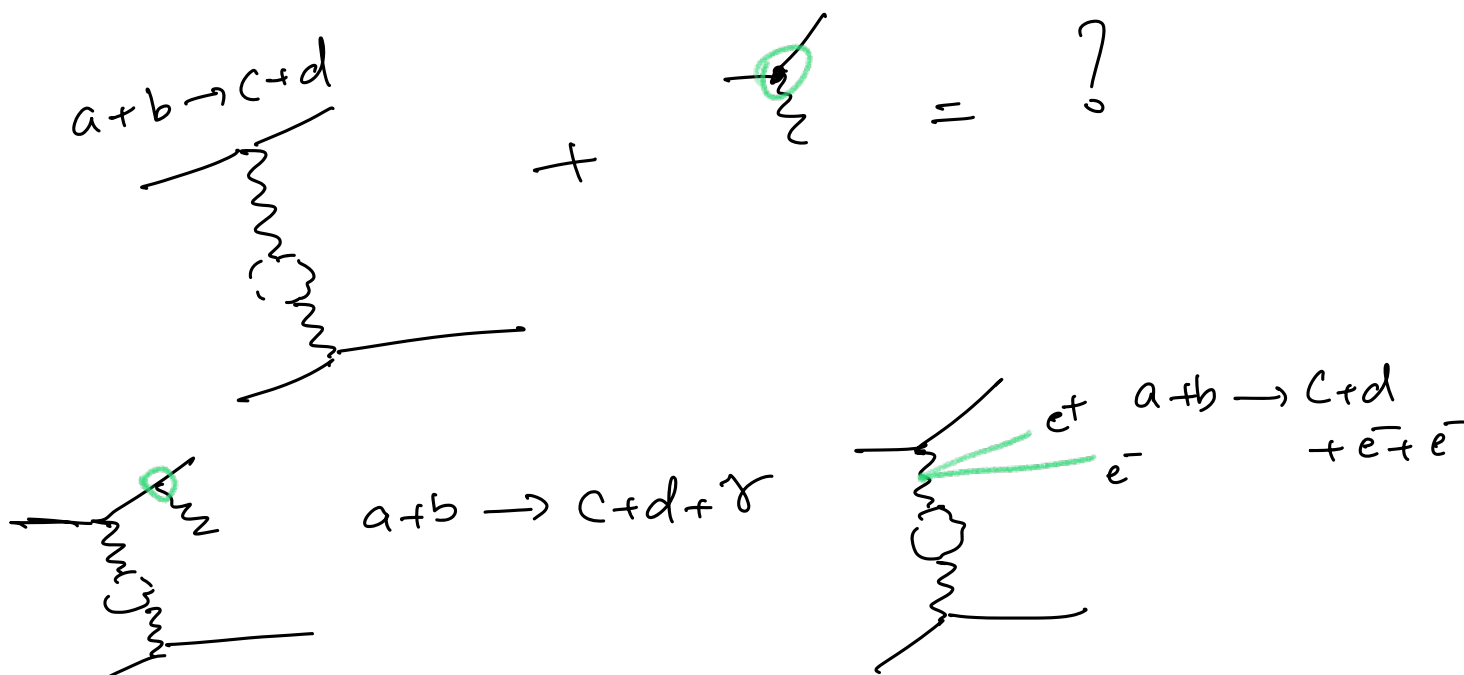
Figure 1.1. The NNPDF4.0 NNLO PDFs at $Q = 3.2$ GeV (left) and $Q = 10^2$ GeV (right).

NNLO: Next-to-Next-to-Leading-order.



LO: α , NLO: α^2 , NNLO: α^3

N^3 LO: Next-Next-Next LO.



$$\mu = \text{[diagram: a single wavy line with two external lines]}$$

$$\mu^2 = \text{[diagram: a wavy line loop with two external lines]}^2$$

$$\mu = \text{[diagram: a single wavy line with two external lines]} + \text{[diagram: a wavy line loop with two external lines]}$$

$$\mu^2 = \text{[diagram: a wavy line loop with two external lines]}^2 + \text{[diagram: a wavy line loop with two external lines]}^2 + \text{[diagram: a wavy line loop with two external lines]} + \text{[diagram: a wavy line loop with two external lines]} + \text{[diagram: a wavy line loop with two external lines]}$$

$$\mu = \mu(\alpha) + \mu(\alpha^2) + \mu(\alpha^3)$$

$f(x)$: parton density function.

$$x = \frac{|P_{part}|}{|P_{nucl}|}$$

$$\sum P_{part}^i = P_{nucl}.$$

$$\int_0^1 dx \left[\underbrace{\sum f_i(x)}_{\text{partons.}} \right] = 1.$$

$$\int_0^1 dx \left[\sum_{\text{flavors}} q_f(x) + \bar{q}_f(x) + g(x) \right] \approx 1.$$

Momentum sum rule.

$$\int_0^1 dx [u^p(x) - \bar{u}^p(x)] = 2$$

$$\int_0^1 dx [d^p(x) - \bar{d}^p(x)] = 1$$

$$u^p(x) = \text{or} \neq u^n(x)$$

$$p = (\text{und})$$

$$n = (\text{udd})$$

parton density
function of
a quark in proton.
in neutron.

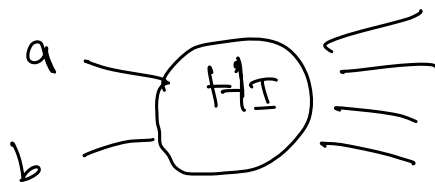
DGLAP equations. \Rightarrow evolution of pdf
 \hookrightarrow Altarelli - Parisi

DIS \Rightarrow partons (relations) quarks?

1950-1960 : $p, n, \pi^\pm, K^\pm, \pi^0, K^0, \Delta$

Baryons : $q_1 q_2 q_3$

mesons : $q_1 \bar{q}_2$



#events. made of hadrons.
(leptons).

$$\# \text{events} \propto \sigma \propto |M|^2 \rho(E_f)$$

$$M = \langle f | H_I | i \rangle$$

To study $H_I \Rightarrow$ look for symmetries / selection rules.

symmetry: transformation acts on a state
leaves the state unchanged

T: transformation.

$$\psi_i = \psi(q, \vec{x}, t)$$

\hookrightarrow charge, quantum #

$$\langle T \rangle = \langle \psi | T | \psi \rangle$$

$$|\psi'\rangle = T|\psi\rangle$$

$$\langle \psi' | = \langle \psi | T^\dagger$$

$$\langle \psi | T^\dagger T | \psi \rangle = \langle \psi | \psi \rangle$$

$$\frac{d}{dt} \langle T \rangle = 0$$

$$[T, H] = 0$$

$$[T, H] = 0 \Rightarrow T H | \psi \rangle = H T | \psi \rangle$$

$$\psi(q, \vec{x}, t)$$

symmetries:

1/ External symm:

time reversal
parity
translation.
rotation.

Continuous

translation \vec{x}

translation t

rotation \vec{x}

Discrete.

Time reversal. $T \psi(\vec{x}, t) = \psi(\vec{x}, -t)$

Parity $\vec{x} \rightarrow -\vec{x}$ $P \psi(\vec{x}, t) = \psi(-\vec{x}, t)$

2/ Internal symmetries. act on internal quantum #.

NOT on \vec{x} or t

Continuous

Isospin

Discrete

charge conjugation.

C : proton \rightarrow \bar{p}
no change of momentum
no change of spin