$\mathbb{G}$-parity $= \mathbb{C} \times \mathbb{R}_2$ $\qquad \mathbb{R}_2 = e^{i J_2 \pi}$ rotation in isospin space.

$\mathbb{C}\,|\pi^0\rangle = +1\,|\pi^0\rangle$ $\qquad \pi^0 \to \gamma\gamma$

$C_{\pi^0} = (C_\gamma)^2 = (-1)^2 = +1$

$\mathbb{C}\,|\pi^+\rangle = ?$

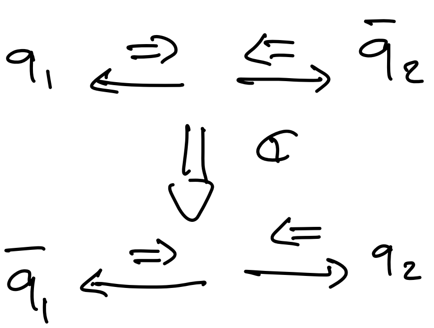
$\mathbb{C}^2\,|\pi^+\rangle = a^2\,|\pi^+\rangle \implies a = \pm 1.$

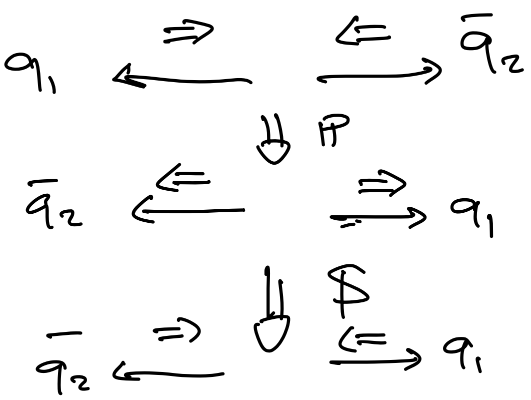
$\mathbb{C}\,|\pi^+\rangle \longrightarrow (-)\,|\pi^-\rangle$
phase

$q\bar{q}$ (neutral state) $\qquad C = (-1)^{L+S}$

$\pi^+ :$ $\cancel{C = (-1)^{L+S} = (-1)^{0+0}} = +1.$ $\qquad \pi^+$: Charged.

Consider a generic state $q_1\bar{q}_2$





$\mathbb{C}(q_1\bar{q}_2) \to q_2\bar{q}_1$ $\qquad (\mathbb{P}+\mathbb{S})(q_1\bar{q}_2) \to q_1\bar{q}_2$

$\mathbb{C} \neq \mathbb{P}+\mathbb{S}$

$\mathbb{G}$ parity $\qquad g = (-1)^{L+S+I}$ $\qquad$ Isospin multiplet $I =$ integer

$I = 1 \qquad -1 \quad 0 \quad +1$

$I = 2 \qquad -2 \quad -1 \quad 0 \quad +1 \quad +2$

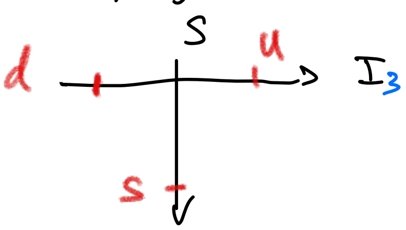
If $I = 1/2$ $\quad -1/2 \quad +1/2$ no G-parity eigenvalue

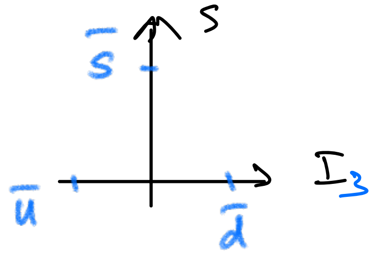
Static Quark model. mesons $q_1 \bar{q}_2$ $\quad 3 \otimes \bar{3} = 1 \oplus 8$

$\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$ Isospin. $\quad s: I = 0.$

Flavor space $\quad u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Strangeness: $u, d = 0$ $\qquad s: S = -1.$



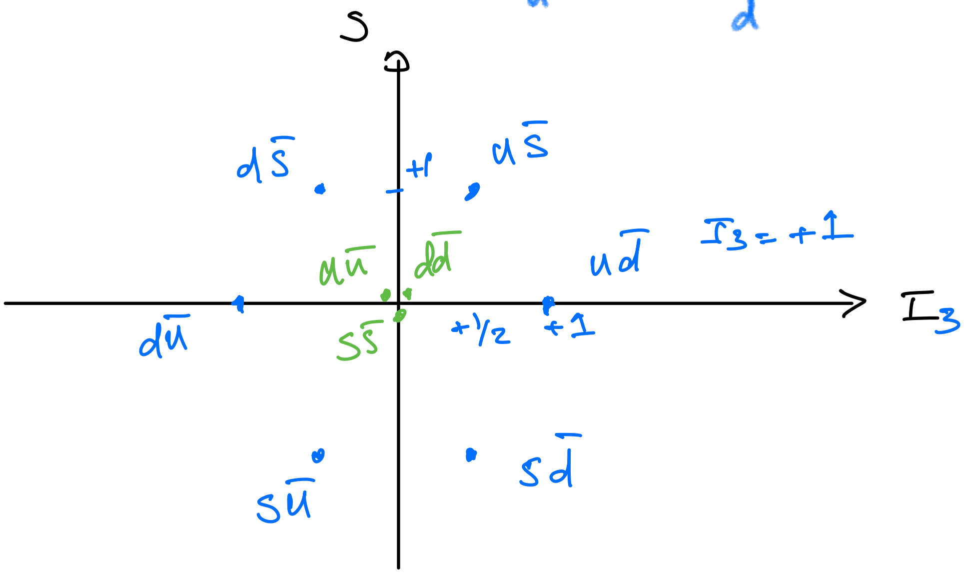


$q_i \bar{q}_j$:

$\psi = \psi_{space} \psi_{spin} \psi_{flav.}$

$q \neq \bar{q} \Rightarrow$ No exclusion principle

$\Rightarrow$ all $q\bar{q}$ combinations are possible.



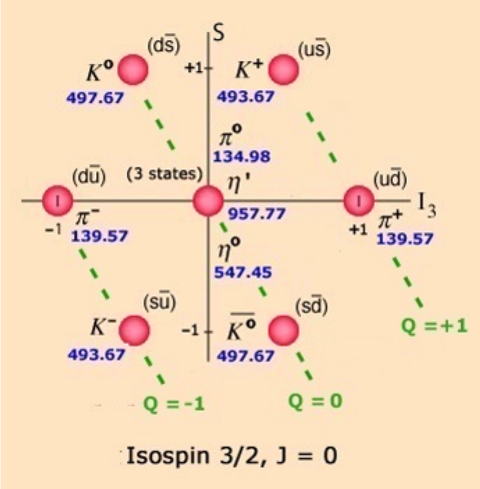
$\psi_{Flavor} = 8$ $\quad$ flavor octet

Also: $\quad u\bar{u},\ d\bar{d},\ s\bar{s}$ $\qquad I_3 = 0, \ S = 0$

singlet $\quad \psi_0 = \frac{1}{\sqrt{3}} \left(u\bar{u} + d\bar{d} + s\bar{s} \right)$

Flavor octet $\left\{ \begin{array}{l} \psi_{8,1} = \frac{1}{\sqrt{2}} \left(u\bar{u} - d\bar{d} \right) \qquad I_3 = 0 \quad I = 1. \\ \psi_{8,0} = \frac{1}{\sqrt{2}} \left(u\bar{u} + d\bar{d} - 2 s\bar{s} \right) \qquad I_3 = 0 \quad I = 0. \end{array} \right.$

Flavor singlet $\quad \psi_{\underset{F,\,I}{1,0}} = \frac{1}{\sqrt{2}} \left(u\bar{u} + d\bar{d} + s\bar{s} \right) \qquad I_3 = 0 \quad I = 0.$



Flavors: u, d, s

static quark model with 3 flavors.

$SU(3)_F$ symm.

Experimentally: $K^\pm, K^0, \bar{K}^0, \pi^\pm$

π^0, η^0, η'

$$\psi_{8,1} = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \longrightarrow \pi^0$$

$$\begin{cases} f' = \psi_{8,0}\cos\theta - \psi_{1,0}\sin\theta \\ f = \psi_{8,0}\sin\theta + \psi_{1,0}\cos\theta \end{cases}$$

mixture of $I_3 = 0$ states.

θ: meson mixing angle.

$q_i \bar{q}_j$ $\quad$ q_i: spin 1/2

$\Uparrow\Uparrow$ $\quad$ $\Downarrow\Uparrow$ $\quad$ Combinations.

$S = 1$ $\quad$ $S = 0$

$L=0$ ground state

$S=0$ $\quad$ $\Uparrow\Downarrow$ $\quad$ $q_i\bar{q}_j$

$P = P_i P_j (-1)^L = -1$

$J = L+S = 0$ $\quad$ $J^P = 0^-$

pseudo-scalar bosons.

$L=0.$

$S=1$ $\quad$ $\Uparrow\Uparrow$

$P = P_i P_j (-1)^L = -1.$

$J = L+S = 1.$ $\quad$ $J^P = 1^-$

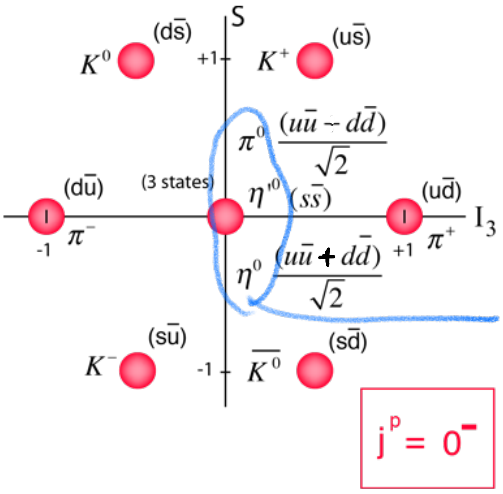
vector mesons.

θ_{ps}: pseudo-scalar meson mixing angle

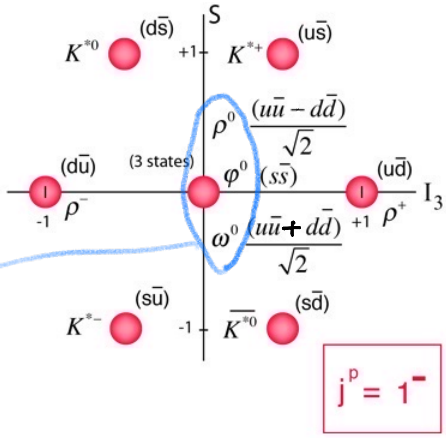
θ_V: vector meson-mixing angle.

pseudo - scalar mesons.

vector mesons.



C eigen states.



$K^0 = d\bar{s} \overset{C}{\Longrightarrow} s\bar{d} = \bar{K}^0$

not all neutral particles can be C eigenstates

$n^{2s+1}\ell_J$	J^{PC}	I $= 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I $=\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I $= 0$ f'	I $= 0$ f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B} [a]	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A} [a]	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$ [a]	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$ [b]	$\phi(2170)$ [d]	$\omega(1650)$
1^3D_2	2^{--}		$K_2(1820)$ [a]		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$ [c]	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$ [b]	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$

$$\left.\begin{aligned}
\pi^0(140) &\approx \psi^{ps}_{8,1} = (u\bar{u}-d\bar{d})/\sqrt{2} \\
\eta(550) &= \psi^{ps}_{8,0}\cos\theta_{ps} - \psi^{ps}_{1}\sin\theta_{ps} \\
\eta'(960) &= \psi^{ps}_{8,0}\sin\theta_{ps} + \psi^{ps}_{1}\cos\theta_{ps}
\end{aligned}\right\}\quad J^P = 0^-,\ \theta_{ps} \approx -25°$$

pseudo-scalar mesons.

$$\rho^0(770) \approx \psi^v_{8,1} = (u\bar{u} - d\bar{d})/\sqrt{2}$$

$$\phi(1020) = \psi^v_{8,0}\cos\theta_v - \psi^v_1 \sin\theta_v \approx s\bar{s}$$

$$\omega(780) = \psi^v_{8,0}\sin\theta_v + \psi^v_1\cos\theta_v \approx$$

$$\approx (u\bar{u} + d\bar{d})/\sqrt{2}$$

$J^P = 1^-$, $\theta_{vect} \approx 36°$

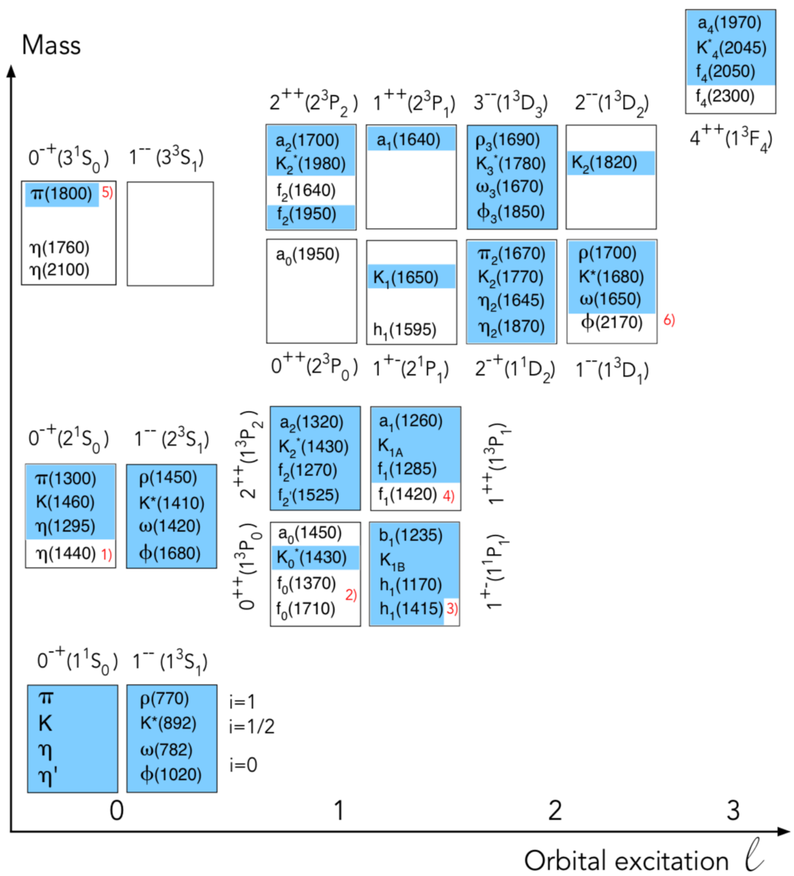
vector mesons.

$\cos\theta \simeq \frac{1}{\sqrt{2}}$

$$\psi_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

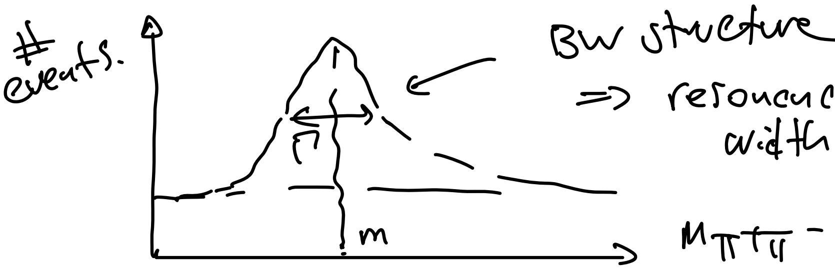
$$\psi_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$\phi(1020) \simeq s\bar{s}$



$p + p \longrightarrow p + p + \pi^+\pi^-\ \pi^+\pi^-\ \pi^0\pi^0\ K^+K^-\ K^0\bar{K}^0\ K^0\pi^+K^-$

invariant mass $\pi^+\pi^0$, $\pi^+\pi^-$, $\pi^+\pi^-\ \pi^+\pi^-$



BW structure

$\Rightarrow$ resonance with mass m width Γ

scattering / decay process: Q, B, L, S, I_3

$\rho^0(770)$ vector meson:

$m_\pi \simeq 140$ MeV

$\rho^0 \to \pi^+\pi^-$

$\pi^0\pi^0 = 0$?

$\cancel{K^+K^-}$

$\cancel{K^0\bar{K}^0}$

$m_\rho - m_K - m_K < 0$ not allowed energetically.

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$\pi\pi$	~ 100 %	
Γ_2	$K\overline{K}$		
	$\rho(770)^{\pm}$ decays		
Γ_3	$\pi^\pm\pi^0$	~ 100 %	
Γ_4	$\pi^\pm\gamma$	$(4.5 \pm 0.5) \times 10^{-4}$	S=2.2
Γ_5	$\pi^\pm\eta$	$< 6 \times 10^{-3}$	CL=84%
Γ_6	$\pi^\pm\pi^+\pi^-\pi^0$	$< 2.0 \times 10^{-3}$	CL=84%
	$\rho(770)^0$ decays		
Γ_7	$\pi^+\pi^-$	~ 100 %	
Γ_8	$\pi^+\pi^-\gamma$	$(9.9 \pm 1.6) \times 10^{-3}$	
Γ_9	$\pi^0\gamma$	$(4.7 \pm 0.8) \times 10^{-4}$	S=1.7
Γ_{10}	$\eta\gamma$	$(3.00 \pm 0.21) \times 10^{-4}$	
Γ_{11}	$\pi^0\pi^0\gamma$	$(4.5 \pm 0.8) \times 10^{-5}$	
Γ_{12}	$\mu^+\mu^-$	[a] $(4.55 \pm 0.28) \times 10^{-5}$	

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why $BF(\rho^0 \to \pi^0\pi^0) = 0$?

1) C parity: $C_{\rho^0} = -1$

$C_{\pi^0} = +1$

$\rho^0 \to \pi^0\,\pi^0.$

C parity $-1 \quad +1 \quad +1$

$C = (-1)^{L+S} = (-1)^{0+1} = -1$

$-1 \longrightarrow (+1)^2$

not possible in strong / EM interaction.

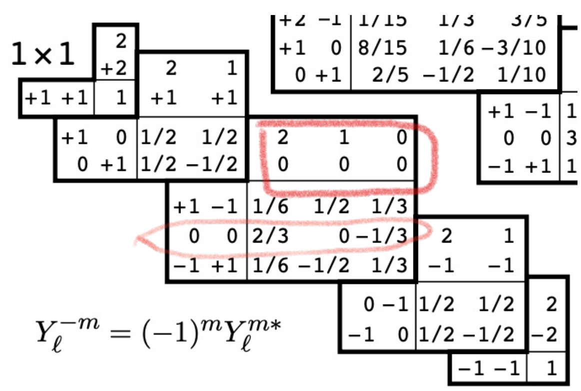
2) Isospin:

$|\rho^0\rangle = |I=1, I_3=0\rangle$

$|\pi^0\rangle = |I=1, I_3=0\rangle.$

$\pi^0\pi^0 = 1 \otimes 1 = 0 \oplus 2$

$|\pi^0\pi^0\rangle = \sqrt{\tfrac{2}{3}}\,|\underset{I}{2}, \underset{I_3}{0}\rangle + \sqrt{\tfrac{1}{3}}\,|0, 0\rangle$



$\langle \pi^0\pi^0 | H_S | \rho^0 \rangle = (\alpha \langle 0,0| + \beta \langle 2,0|) H_S | 1,0 \rangle.$

$= 0$ if H_S conserves isospin

3) spin statistics:

ρ^0: boson. $\Rightarrow$ $\psi_{\rho^0} = \psi_{symm.}$ in ρ^0 rest frame

find state $J_F = L_F + S_F$ $\qquad J_{Init} = +1 \quad \begin{matrix} L=0 \\ S=1 \end{matrix} \; \rho^0$

π^0: $S_{\pi^0} = 0$. $\qquad \pi^0 \leftarrow - - \circ - - \rightarrow \pi^0$

$J_F = 1 \Rightarrow L_F = +1.$

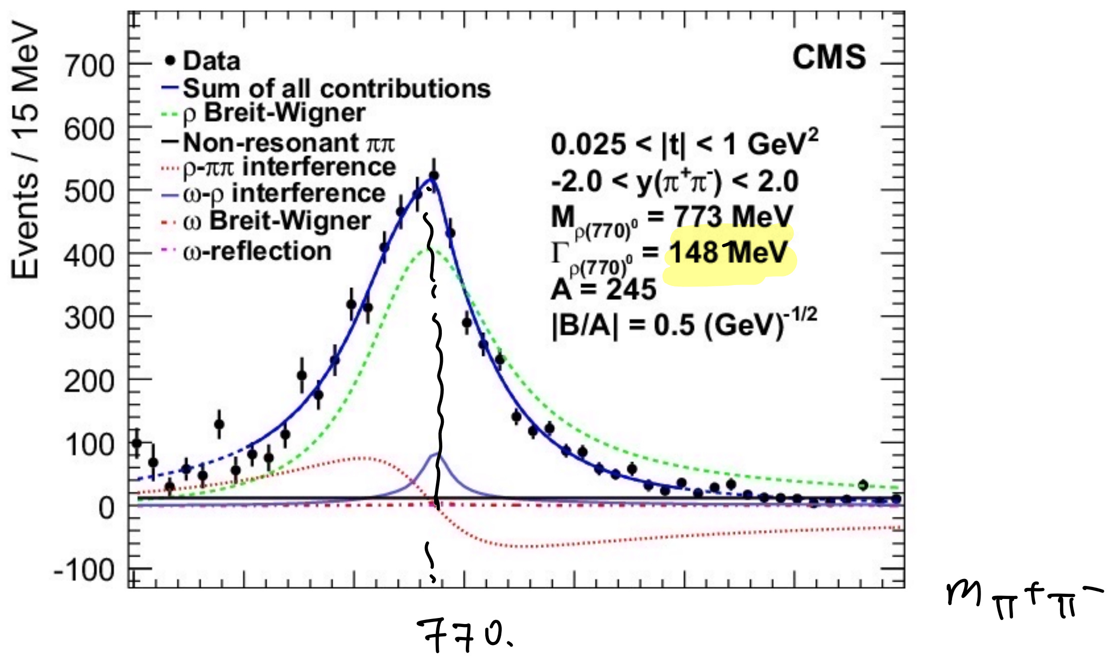
$\pi^0\pi^0$ state. 2 identical bosons.

$\psi_{Final} = \psi_{symm.}$ $\qquad \psi_{Final} = \psi_{space}\psi_{spin}.$

But $L_F \rightarrow$ anti-symm. space wave function.

$\rho^0 \rightarrow \cancel{\pi^0\pi^0}$

$\qquad \pi^+\pi^-$

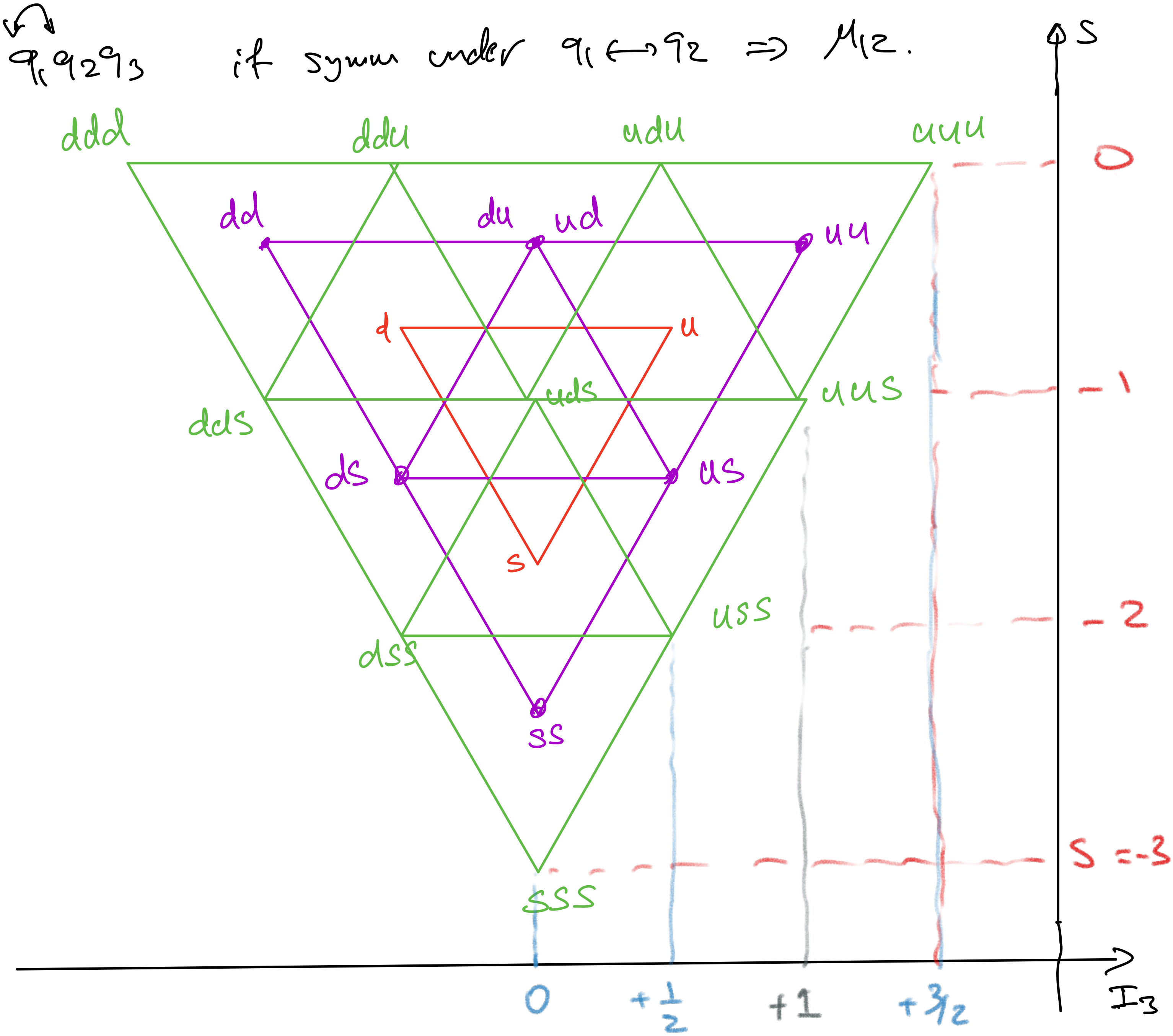


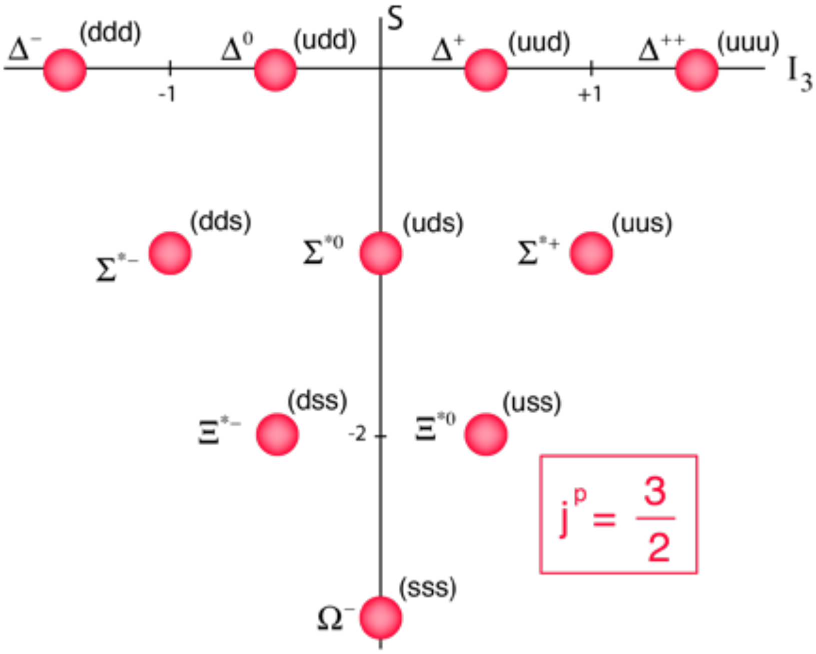
Baryons : $q_1 q_2 q_3$ antibaryons: $\bar{q}_1 \bar{q}_2 \bar{q}_3$

$SU(3)_F$ symmetry.

$$\underset{\text{Flavor}}{B = 3 \otimes 3 \otimes 3} = \underset{\text{singlet}}{1} \oplus \underset{\substack{\text{octet} \\ M_{12}}}{8} \oplus \underset{\substack{\text{octet} \\ M_{23}}}{8} \oplus \underset{\text{decuplet}}{10}$$

$q_1 q_2 q_3$ if symm under $q_1 \leftrightarrow q_2 \Rightarrow M_{12}$.





q_1 q_2 q_3

$\Uparrow\Downarrow$ $\Uparrow\Downarrow$ $\Uparrow\Downarrow$

1st Combination.

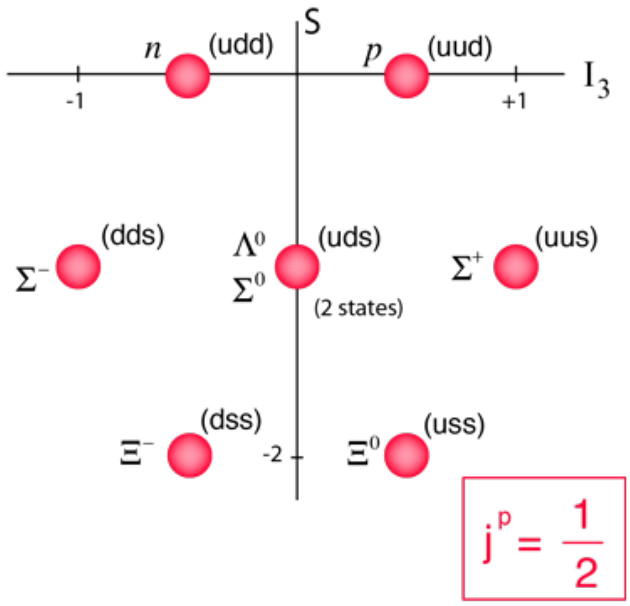
$\Uparrow\Uparrow\Uparrow$ $S = 3/2$.

ground state: $L = 0$.

$\Rightarrow J = 3/2$

$P = P_1 P_2 P_3 (-1)^L$

$= +1$



2nd Combination.

$\Uparrow\Uparrow\Downarrow$

$S = 1/2$

$L=0$ ground state.

$\Rightarrow J = 1/2$

$$\psi_{baryon} = \psi_{space}\,\psi_{spin}\,\psi_{flavor}.$$

Baryons are fermions. $\Rightarrow$ antisymm. wave functio

$L=0$ ground state $\Rightarrow$ $\psi_{spin}\,\psi_{flav} = \psi_{antisymm}$