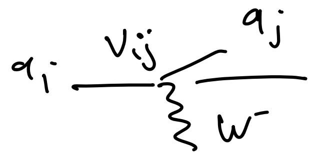
$K^0 \to \mu^+\mu^-$ is suppressed $\neq 0$.

GIM mechanism $\Rightarrow$ existence of up-like c quark



$g_W \gamma^\mu (1-\gamma^5)$

$i, j = u, d, s, c$

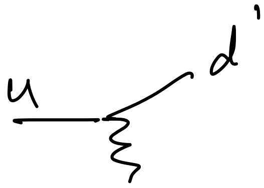
$$(u \;\; c) \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$V_{ud} \simeq \cos\theta$

$V_{us} \simeq \sin\theta$.

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

weak interactions



for leptons $\quad (e^- \;\; \mu^-) \begin{pmatrix} V_{e\nu_e} & \cancel{V_{e\nu_\mu}} \\ \cancel{V_{\mu\nu_e}} & V_{\mu\nu_\mu} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$

$$\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}$$

weak interaction:

$\cancel{C}$ Goldhaber exp. $\quad \nu_L, \; \bar\nu_R$

$\cancel{P}$ Wu experiment

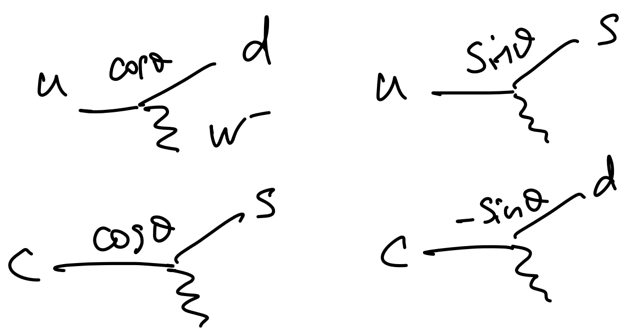
CP a good symm? $\Rightarrow$ No $\quad \cancel{CP}$

1964 in Kaon mesons.

in B mesons 2002

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}_L \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}_L$$

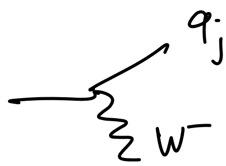
$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix}$$



$\cancel{CP}$ not explained in Kaon decays

1973: Kobayashi-Maskawa proposed new quarks and their interaction

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$



$V_{ij} g_w \gamma^\mu (1-\gamma^5)$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo-Kobayashi-Maskawa matrix for flavor-changing charged weak current

$$\begin{aligned} V_{\mathrm{CKM}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned}$$

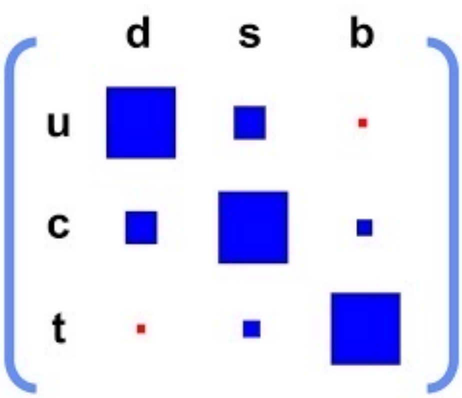
3 real parameters, 1 complex parameter.

$$\begin{array}{cc} V_{ud} & V_{us} \end{array}$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ V_{cd}\ -\lambda & V_{cs}\ 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

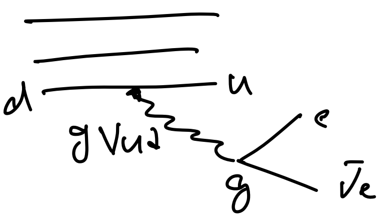
$\rightarrow A, \rho, \eta$

$\cos\theta \simeq 1 - \frac{1}{2}\sin^2\theta$ $\qquad \lambda = \sin\theta_c$



area $\propto |V_{ij}|^2$

$n \to p e^- \bar{\nu}_e$



$$\Gamma(n \to e \bar{\nu} p) \propto |V_{ud}|^2 \underbrace{\left(\frac{g_w^4}{m_w^4}\right)}_{G_F^2} \cdots$$

$$\Gamma(n \to p e^- \bar{\nu}_e) \propto \frac{\#\, n \text{ decays}}{\#\, n \text{ total}}$$

Decay rates measure $|V_{ij}|^2$ not sensitive to complex phase.

Interference between processes to measure a complex phase.

$$\begin{tikzcd} a \arrow[r, "M_1"] \arrow[r, bend right, "M_2"'] & b \end{tikzcd}$$

$$M = M_1 + M_2 = |M_1| + |M_2| e^{i\alpha}$$

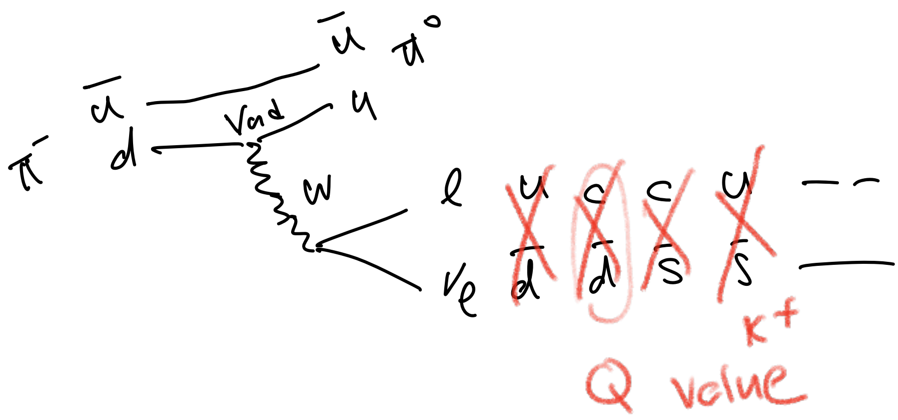
$$M_1 = M_1^* \qquad M_2 = |M_2| e^{i\alpha}$$

$$M_2^* = |M_2| e^{-i\alpha}$$

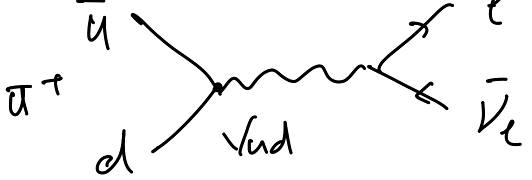
$$|M|^2 = |M_1|^2 + |M_2|^2 + |M_1||M_2| e^{i\alpha} + |M_1||M_2| e^{-i\alpha}$$

$$\Rightarrow \quad \Gamma = \Gamma(|M_1|, |M_2|, \alpha)$$

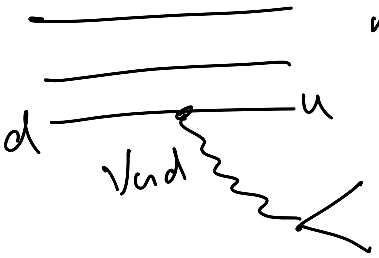
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$



$$\pi^- \to \pi^0 e^- \bar{\nu}_e \propto |V_{ud}|^2$$



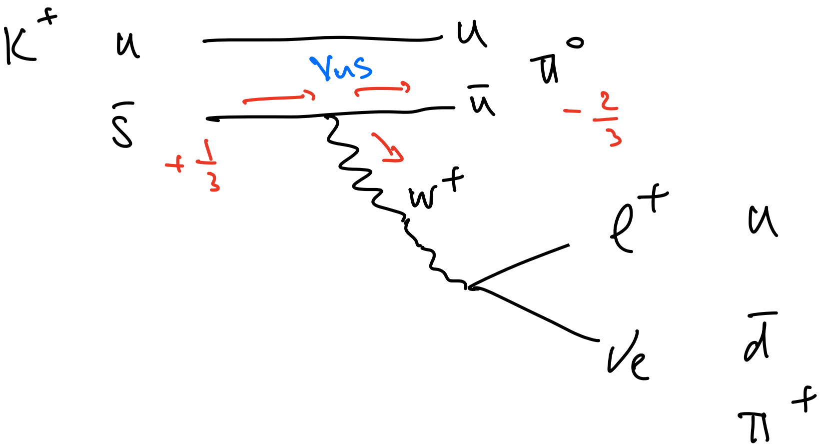
$\pi^- \to \mu^- \bar\nu_\mu \propto |V_{ud}|^2 \times$ (QCD corrections)



$n \to p e^- \bar\nu_e$ $\qquad \Gamma \propto |V_{ud}|^2 \times$ (QCD corrections)

In general larger hadronic corrections in baryons.

$\Rightarrow$ mesons preferred.



$K^+ \to \pi^0 \mu^+ \nu_\mu$

$K^+ \to \pi^0 \pi^+$

$m_\pi \simeq m_\mu \Rightarrow$ phase space similar

$\mathcal{M}(K^+ \to \pi^0 \pi^+) \propto V_{us} V_{ud}$

$\mathcal{M}(K^+ \to \pi^0 \mu \nu_\mu) \propto V_{us}$

$\#(K^+ \to \pi^0 \pi^+) \propto |V_{us}|^2 |V_{ud}|^2$

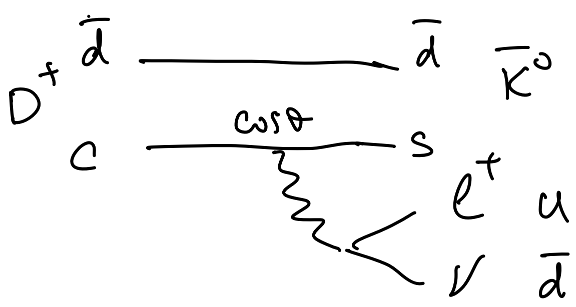
$\#(K^+ \to \pi^0 \mu \nu_\mu) \propto |V_{us}|^2$ $\qquad$ preferred because only V_{us}

You can measure $|V_{ud}|$ with

$$\pi^+ \to \mu^+ \nu_\mu$$
$$\pi^+ \to \pi^0 e^- \bar{\nu}_e$$
$$n \to p e^- \bar{\nu}_e$$

$$\left.\begin{array}{l}\text{leptonic decays}\\ \text{semileptonic (meson, baryon)}\end{array}\right\} \propto |V_{ij}|^2$$

hadronic decays: $\propto |V_{ij}|^2 \, |V_{kl}|^2$



$D \to K \ell \nu \propto |V_{cs}|^2$

$D \to K \pi^+ \quad \propto |V_{cs}|^2 \, |V_{ud}|^2$

$D \to K K^+ \quad \propto |V_{cs}|^2 \, |V_{us}|^2$

$m_D = 1.8$ GeV.

$m_\pi = 0.135$ GeV

$m_K = 0.5$ GeV

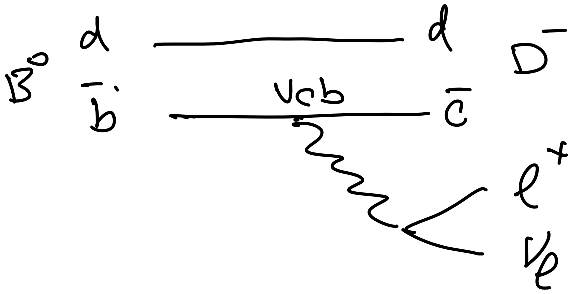
$$Q = m_D - m_K - (m_K / m_\pi)$$

$$\frac{\Gamma(D \to K\pi^+)}{\Gamma(D \to KK^+)} \sim \underbrace{\frac{|V_{ud}|^2}{|V_{us}|^2}} \; \frac{\text{P space}}{\text{P space}}$$

$$\left(\frac{\cos\theta}{\sin\theta}\right)^2 \quad \frac{1}{(\tan\theta)^2}$$

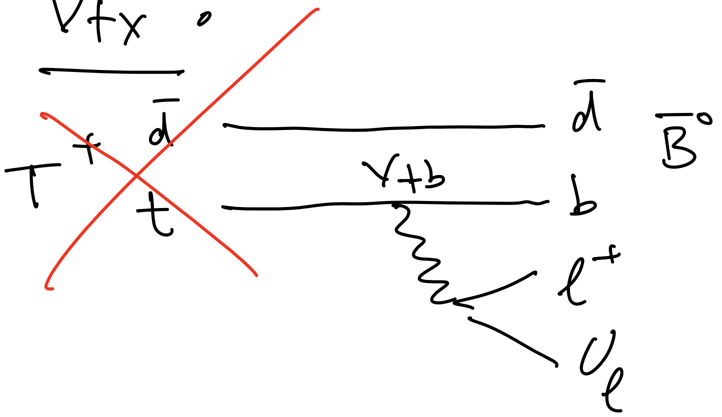
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

V_{xb} requires B mesons: $B^0 = \bar{b}d$ $B^+ = \bar{b}u$



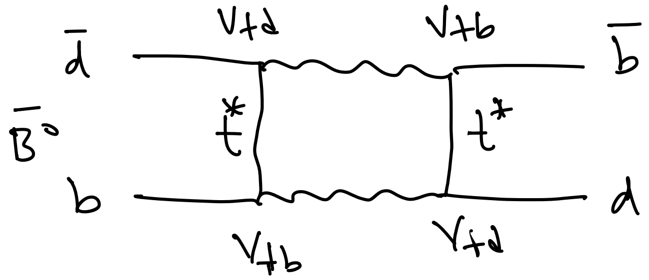
$B^0 \to D^- \ell^+ \nu_e \;\propto\; |V_{cb}|^2$

V_{tx}:



top decays too quickly.

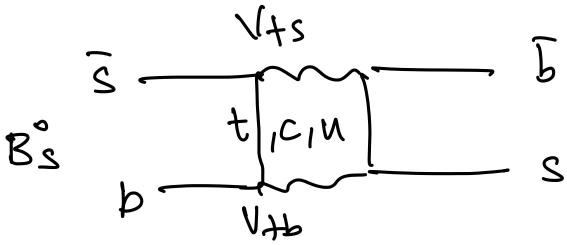
$\Rightarrow$ look at process with virtual top in loops



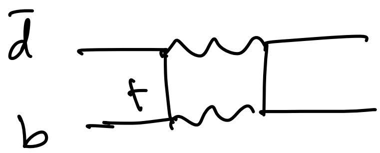
$B^0 \;\propto\; |V_{td}|^2 \, |V_{tb}|^2$

$m = 5.279$ GeV

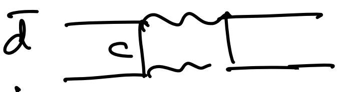
flavor oscillation $B^0 \to \bar{B}^0$ $\Delta b = 2$



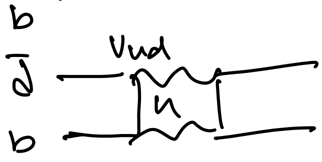
$B_s^0 \to \bar{B}_s^0$



$\propto |V_{td}|^2 \, |V_{tb}|^2 \sim \lambda^3 \cdot 1$

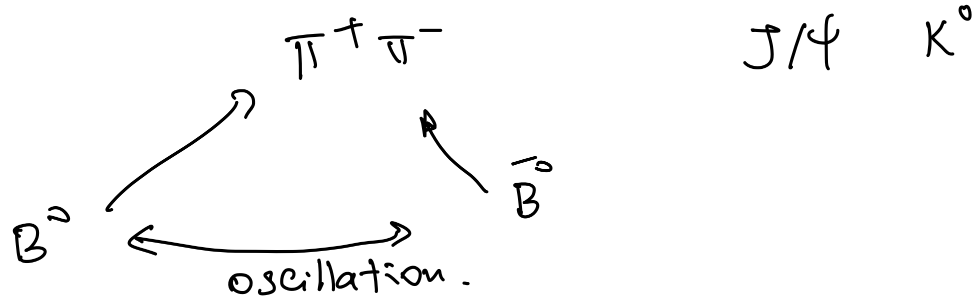


$\propto |V_{cb}|^2 \, |V_{cd}|^2 \sim \lambda^2 \lambda$



$\propto |V_{ud}|^2 \, |V_{ub}|^2 \sim 1 \, \lambda^3$

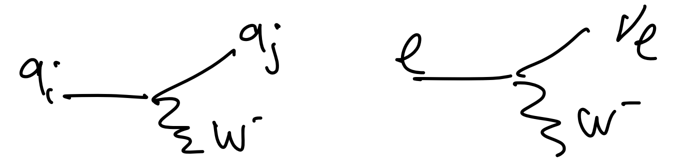
$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



$B^0 \longrightarrow \pi^+\pi^- \quad \mathcal{M}_1$ $\qquad \mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$

$B^0 \to \bar{B}^0 \to \pi^+\pi^- \quad \mathcal{M}_2$

CKM matrix for Flavor changing charged weak current



Neutral weak current?

1) $K^0 \to \mu^+\mu^- \neq 0$ but very small.

2) $K^+ \to \mu^+\nu_\mu \gg K^0 \to \mu^+\mu^-$

3) $K^+ \to \pi^+ \nu\bar{\nu}$ suppressed.

Flavor changing neutral current

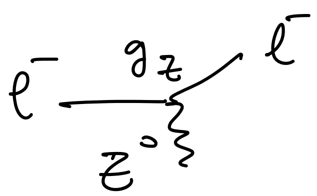
K^+ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Leptonic and semileptonic modes			
$e^+\nu_e$	$(1.582\pm0.007)\times10^{-5}$		247
$\mu^+\nu_\mu$	$(63.56\ \pm0.11\ )$ %	S=1.2	236
$\pi^0 e^+\nu_e$ Called K^+_{e3}.	$(5.07\ \pm0.04\ )$ %	S=2.1	228
$\pi^0\mu^+\nu_\mu$ Called $K^+_{\mu3}$.	(3.352 ± 0.033) %	S=1.9	215
$\pi^0\pi^0 e^+\nu_e$	$(2.55\ \pm0.04\ )\times10^{-5}$	S=1.1	206
$\pi^+\pi^- e^+\nu_e$	$(4.247\pm0.024)\times10^{-5}$		203
$\pi^+\pi^-\mu^+\nu_\mu$	$(1.4\ \pm0.9\ )\times10^{-5}$		151
$\pi^0\pi^0\pi^0 e^+\nu_e$	$< 3.5\times10^{-6}$	CL=90%	135
Hadronic modes			
$\pi^+\pi^0$	$(20.67\ \pm0.08\ )$ %	S=1.2	205
$\pi^+\pi^0\pi^0$	(1.760 ± 0.023) %	S=1.1	133
$\pi^+\pi^+\pi^-$	(5.583 ± 0.024) %		125

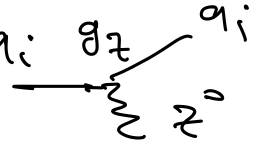
$K^\pm$ **Lepton family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes**

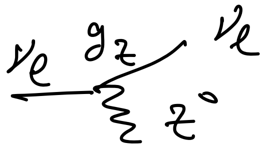
Γ_{35}	$\pi^+\pi^+ e^-\overline{\nu}_e$	*SQ*	$< 1.3\times10^{-8}$	CL=90%	
Γ_{36}	$\pi^+\pi^+\mu^-\overline{\nu}_\mu$	*SQ*	$< 3.0\times10^{-6}$	CL=95%	
Γ_{37}	$\pi^+ e^+ e^-$	*S1*	$(3.00\ \pm0.09\ )\times10^{-7}$		
Γ_{38}	$\pi^+\mu^+\mu^-$	*S1*	$(9.4\ \pm0.6\ )\times10^{-8}$		S=2.6
Γ_{39}	$\pi^+\nu\overline{\nu}$	*S1*	$(1.14\ {}^{+0.40}_{-0.33})\times10^{-10}$		

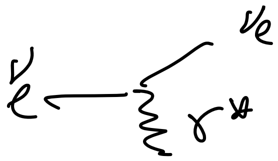
what about flavor conserving weak neutral current?

NC

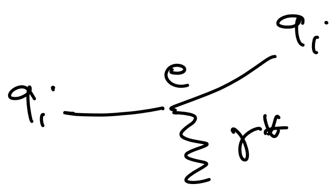




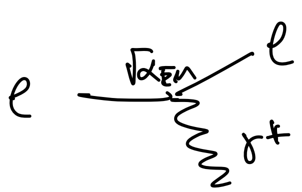


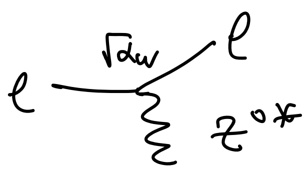


$q_\nu = 0 \Rightarrow$ no QED.



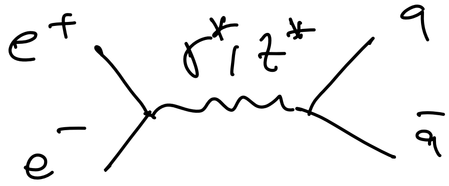
Compare QED to weak neutral current.





$$\mathcal{M} \sim \frac{\sqrt{\alpha_{EM}}}{q^2} \qquad \frac{\sqrt{\alpha_W}}{q^2 - M_Z^2}.$$

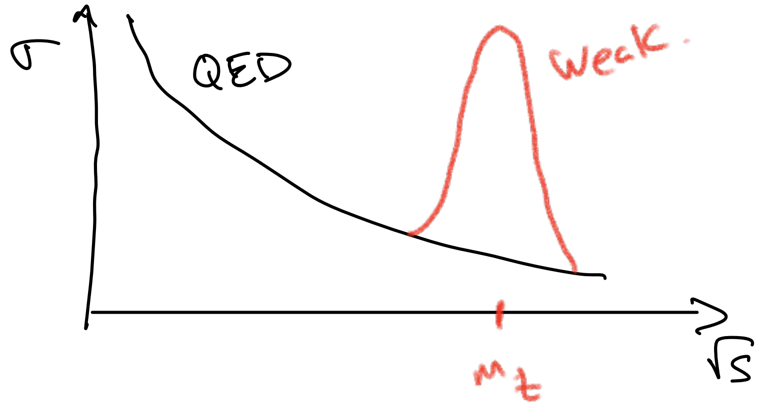
at $q^2 \ll M_Z^2$ $\qquad \dfrac{\sqrt{\alpha_{EM}}}{q^2} \sim \dfrac{\sqrt{\alpha_W}}{M_Z^2}$



$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z$

Direct proof

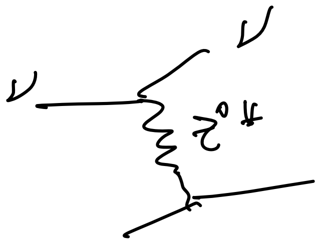
For increasing q^2



At low energy:

leptons, quarks: QED dominates.

neutrinos



Indirect proof

$\nu + (\ldots) \longrightarrow \nu + (\ldots)$