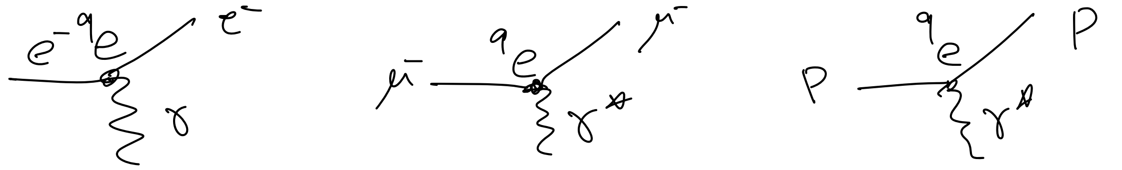
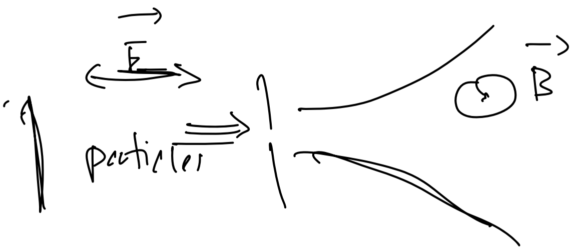
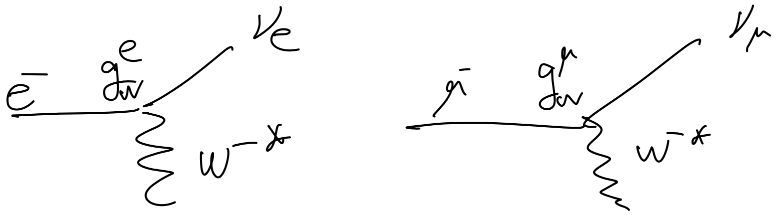
# Lepton Universality in Weak Interactions

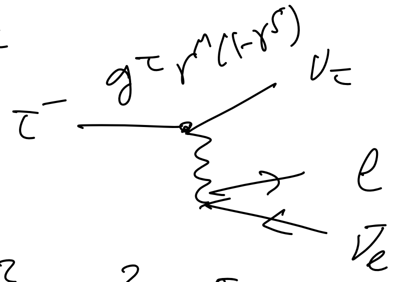



How do we know




$\tau^- \to e^-/\mu^- + \bar{\nu}_{e/\mu} + \nu_\tau$




$\ell = e^-/\mu^-$

$$\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \underbrace{\frac{g_\tau^2}{M_W^2} \frac{g_e^2}{M_W^2}}_{G_F^2} m_\tau^5 \, \rho_e$$

$\rho_e$ → phase space of $\tau^- \to e^- \bar{\nu}_e \nu_\tau$

$$\frac{\Gamma(\tau^- \to \mu^- - -)}{\Gamma(\tau^- \to e^- - -)} = \frac{g_\mu^2}{g_e^2} \frac{\rho_{\tau \to \mu}}{\rho_{\tau \to e}}$$

$$\frac{\Gamma(\tau^- \to \mu^- + X)}{\Gamma_{tot}} = BR(\tau^- \to \mu^- + X)$$

produce $N$ $\tau^-$ leptons.

$$BR = \frac{\#\text{ decays into } \mu^-}{\#\text{ total } \tau^-}$$

$m_\tau = 1.78$ GeV

$m_\mu = 106$ MeV  $m_e = 0.5$ MeV.

| $\tau^-$ **DECAY MODES** | | Fraction $(\Gamma_i/\Gamma)$ | Scale factor/ Confidence level | $p$ (MeV/$c$) |
|---|---|---|---|---|
| **Modes with one charged particle** | | | | |
| particle$^-$ $\geq 0$ neutrals $\geq 0K^0\nu_\tau$ ("1-prong") | | $(85.24 \pm 0.06)$ % | | – |
| particle$^-$ $\geq 0$ neutrals $\geq 0K^0_L\nu_\tau$ | | $(84.58 \pm 0.06)$ % | | – |
| $\mu^-\bar{\nu}_\mu\nu_\tau$ | [g] | $(17.39 \pm 0.04)$ % | | 885 |
| $\mu^-\bar{\nu}_\mu\nu_\tau\gamma$ | [e] | $(3.67 \pm 0.08) \times 10^{-3}$ | | 885 |
| $e^-\bar{\nu}_e\nu_\tau$ | [g] | $(17.82 \pm 0.04)$ % | | 888 |
| $e^-\bar{\nu}_e\nu_\tau\gamma$ | [e] | $(1.83 \pm 0.05)$ % | | 888 |
| $h^- \geq 0K^0_L\ \nu_\tau$ | | $(12.03 \pm 0.05)$ % | | 883 |
| $h^-\nu_\tau$ | | $(11.51 \pm 0.05)$ % | | 883 |
| $\pi^-\nu_\tau$ | [g] | $(10.82 \pm 0.05)$ % | | 883 |
| $K^-\nu_\tau$ | [g] | $(6.96 \pm 0.10) \times 10^{-3}$ | | 820 |
| $h^- \geq 1$ neutrals$\nu_\tau$ | | $(37.01 \pm 0.09)$ % | | – |
| $h^- \geq 1\pi^0\nu_\tau(\text{ex.}K^0)$ | | $(36.51 \pm 0.09)$ % | | – |
| $h^-\pi^0\nu_\tau$ | | $(25.93 \pm 0.09)$ % | | 878 |
| $\pi^-\pi^0\nu_\tau$ | [g] | $(25.49 \pm 0.09)$ % | | 878 |
| $\pi^-\pi^0$ non-$\rho(770)\nu_\tau$ | | $(3.0 \pm 3.2) \times 10^{-3}$ | | 878 |
| $K^-\pi^0\nu_\tau$ | [g] | $(4.33 \pm 0.15) \times 10^{-3}$ | | 814 |
| $h^- \geq 2\pi^0\nu_\tau$ | | $(10.81 \pm 0.09)$ % | | – |
| $h^-2\pi^0\nu_\tau$ | | $(9.48 \pm 0.10)$ % | | 862 |
| $h^-2\pi^0\nu_\tau(\text{ex.}K^0)$ | | $(9.32 \pm 0.10)$ % | | 862 |
| $\pi^-2\pi^0\nu_\tau(\text{ex.}K^0)$ | [g] | $(9.26 \pm 0.10)$ % | | 862 |
| $\pi^-2\pi^0\nu_\tau(\text{ex.}K^0)$, | | $< 9$ | $\times 10^{-3}$ CL=95% | 862 |

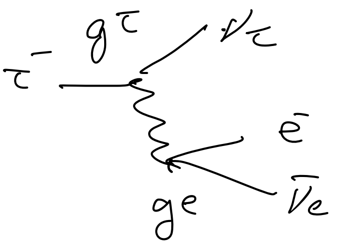
Compute phase space.
measure BR($e, \mu$) $\Rightarrow$ estimate $\dfrac{g_\mu^2}{g_e^2}$

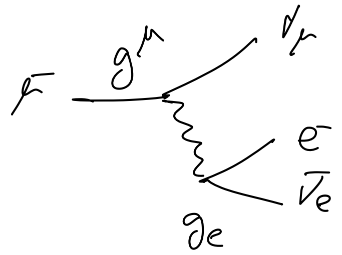
$$\left(\frac{g_\mu}{g_e}\right)^2 \simeq 0.976 \longrightarrow \frac{g_\mu}{g_e} = 1.001 \pm 0.002$$

$\mu \longleftrightarrow e$ are the same for weak int.

what about $\tau \longleftrightarrow \mu$ universality?

BR: $\mu^- \to e^- \bar\nu_e \nu_\mu$.
$\tau^- \to e^- \bar\nu_e \nu_\tau$



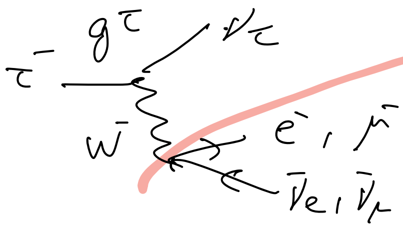




$BR(\mu^- \to e^- \bar\nu_e \nu_\mu) \simeq 100\%$

$BR(\tau^- \to e^- \bar\nu_e \nu_\tau) \simeq 17.8\%$

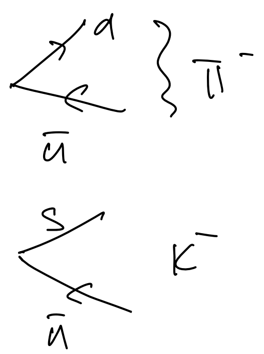
$\Gamma^\mu_{tot} \simeq \Gamma_{\mu\to e}$.

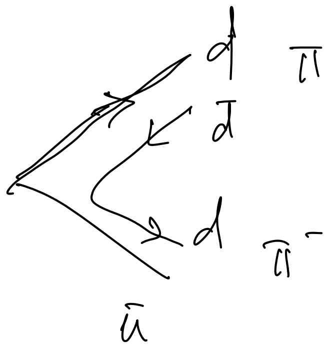
$\tau^- \to \nu_\tau +$



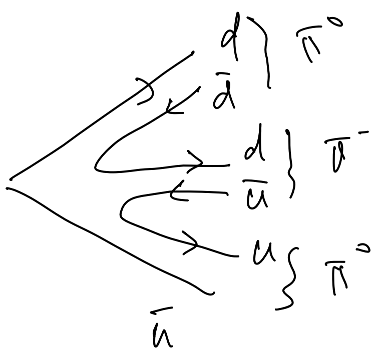

$s$ | $d$
$\bar u$ | $\bar c$

$D^-$
$m \simeq 1.86$ GeV
$> m_\tau$

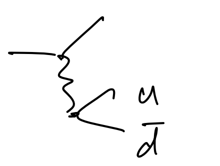


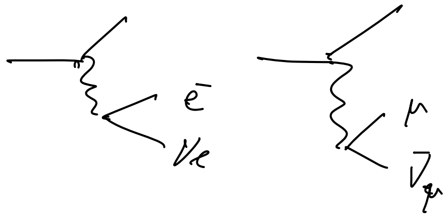




$\tau^- \to \nu_\tau + \pi^- + \pi^0$




$\Gamma_{\tau\to\mu} \cong \Gamma_{\tau\to e} \cong \Gamma_{ud}/3$



$$\Gamma_{tot}^{\tau} \simeq \Gamma_{\tau\to e} + \Gamma_{\tau\to\mu} + \underbrace{(\Gamma_{u\bar{d}})}_{\times 3} \simeq 5\, \Gamma_{\tau\to e}$$

$$\Gamma_{\tau\to e} \simeq \frac{1}{5}\Gamma_{tot} = \frac{1}{5}\frac{1}{\tau_\tau} \qquad \tau_\tau \quad \tau \text{ lifetime.}$$

measure.

$$\frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{1}{\tau^\mu}\,\tau_\tau \; \frac{BR(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{BR(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}$$

measure

$$\frac{\Gamma(\tau \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma_{tot}} = BR(\tau^- \to e^- -) \Rightarrow$$

$$\Gamma(\tau^- \to e^- -) = \Gamma_{tot}^{\tau} \times BR(\tau^- \to e^- - -) = \frac{1}{\tau_\tau} BR(- -)$$

$$\Gamma \simeq \frac{g_\tau^2}{M_W^2}\frac{g_e^2}{M_W^2} m_\tau^5\, \rho_{\tau\to e}.$$

$$= \frac{g_\mu^2}{g_\tau^2}\frac{m_\mu^5}{m_\tau^5}\frac{\rho_{\mu\to e}}{\rho_{\tau\to e}}$$

measured. computed.

$$\frac{g_\mu}{g_\tau} = 1.001 \pm 0.003 \Rightarrow \text{universality of } \tau \leftrightarrow \mu.$$

$\Rightarrow \tau^-$ is a lepton.

$$g_W^e \simeq g_W^\mu \simeq g_W^\tau$$

Universality with hadronic $\tau$ decays.

$$\frac{\Gamma_{\tau\to\mu}}{\Gamma_{\mu\to e}} = \frac{m_\tau^5}{m_\mu^5}.$$

$$\Gamma_{\tau\to\mu} \simeq \frac{1}{5}\Gamma_{tot} = \frac{1}{5}\frac{1}{\tau_\tau}.$$

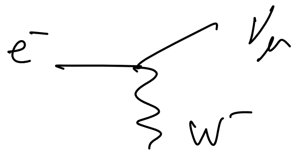
$$\Gamma_{\mu\to e} \simeq \Gamma_{tot} = \frac{1}{\tau_\mu}.$$

$$\tau_\tau = \frac{1}{5}\frac{m_\mu^5}{m_\tau^5}\tau_\mu. = 3.1\times10^{-13}\,s. \quad \text{from PDG values.}$$

$$\tau_\tau^{exp} = 2.9\times10^{-13}\,s.$$

# Weak Interaction in Quarks


but not



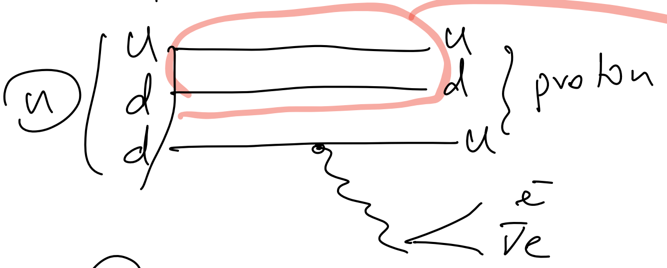

lepton number conservation.

$W^-$ interacts with $\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}$ $\begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$ $\begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$

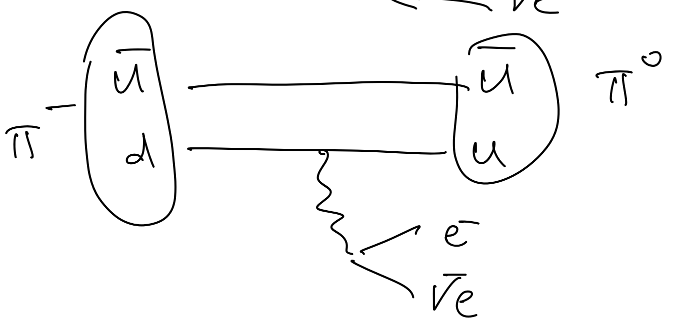
V−A structure not so clear with hadrons.

$\Rightarrow$ corrections from strong interaction.

$n \to p e^- \bar{\nu}_e$




Spectator quark.




$\pi^- \to \pi^0 e^- \bar{\nu}_e$

m 139 135

$Q \simeq 3.5$ MeV.

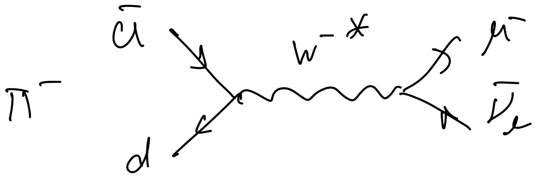
## $\pi^+$ DECAY MODES

$\pi^-$ modes are charge conjugates of the modes below.

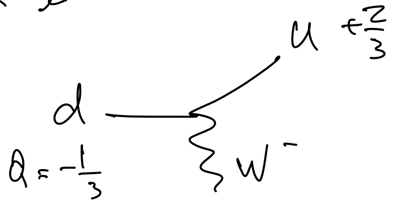
For decay limits to particles which are not established, see the section on Searches for Axions and Other Very Light Bosons.

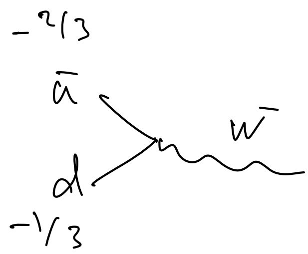
| | Mode | | | Fraction ($\Gamma_i/\Gamma$) | Confidence level |
|---|---|---|---|---|---|
| $\Gamma_1$ | $\mu^+\nu_\mu$ | | [a] | $(99.98770\pm0.00004)$ % | |
| $\Gamma_2$ | $\mu^+\nu_\mu\gamma$ | | [b] | $(2.00 \pm0.25) \times 10^{-4}$ | |
| $\Gamma_3$ | $e^+\nu_e$ | | [a] | $(1.230 \pm0.004) \times 10^{-4}$ | |
| $\Gamma_4$ | $e^+\nu_e\gamma$ | | [b] | $(7.39 \pm0.05) \times 10^{-7}$ | |
| $\Gamma_5$ | $e^+\nu_e\pi^0$ | | | $(1.036 \pm0.006) \times 10^{-8}$ | |
| $\Gamma_6$ | $e^+\nu_e e^+ e^-$ | | | $(3.2 \pm0.5) \times 10^{-9}$ | |
| $\Gamma_7$ | $\mu^+\nu_\mu\nu\bar{\nu}$ | | | $< 9 \times 10^{-6}$ | 90% |
| $\Gamma_8$ | $e^+\nu_e\nu\bar{\nu}$ | | | $< 1.6 \times 10^{-7}$ | 90% |
| | **Lepton Family number (*LF*) or Lepton number (*L*) violating modes** | | | | |
| $\Gamma_9$ | $\mu^+\bar{\nu}_e$ | *L* | [c] | $< 1.5 \times 10^{-3}$ | 90% |
| $\Gamma_{10}$ | $\mu^+\nu_e$ | *LF* | [c] | $< 8.0 \times 10^{-3}$ | 90% |
| $\Gamma_{11}$ | $\mu^- e^+ e^+ \nu$ | *LF* | | $< 1.6 \times 10^{-6}$ | 90% |

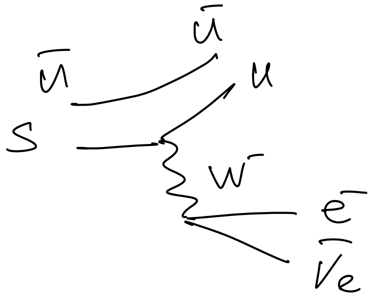
why no $\pi^- \to \pi^0 \mu^- \bar{\nu}_\mu$ $Q < 0$.




$\pi$ decays

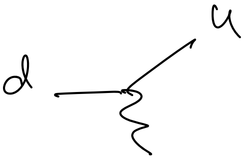



$\longrightarrow t$

$K^- \longrightarrow \pi^0 e^- \bar{\nu}_e$




$\pi^- \rightarrow \pi^0 e^- \bar{\nu}_e$

$\Delta S = 0$

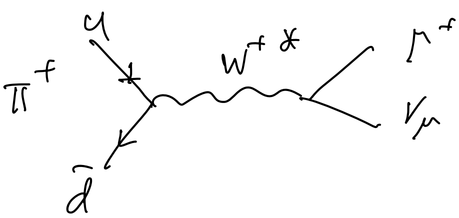

$\Delta S = 1$

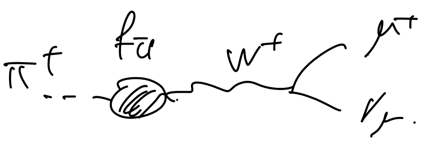
$K^- \rightarrow \pi^0 \; e^- \bar{\nu}_e$ .

$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

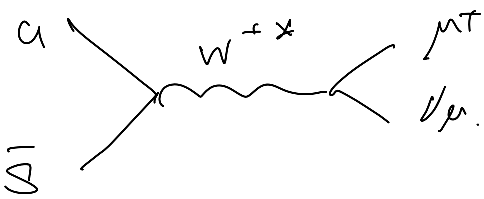
charged current $W^-$ can. changes quark flavor

| $K^+$ DECAY MODES | Fraction ($\Gamma_i/\Gamma$) | Scale factor/ Confidence level | $p$ (MeV/c) |
|---|---|---|---|
| **Leptonic and semileptonic modes** | | | |
| $e^+ \nu_e$ | $(1.582 \pm 0.007) \times 10^{-5}$ | | 247 |
| $\mu^+ \nu_\mu$ | $(63.56 \pm 0.11)$ % | S=1.2 | 236 |
| $\pi^0 e^+ \nu_e$ | $(5.07 \pm 0.04)$ % | S=2.1 | 228 |
| Called $K^+_{e3}$. | | | |
| $\pi^0 \mu^+ \nu_\mu$ | $(3.352 \pm 0.033)$ % | S=1.9 | 215 |
| Called $K^+_{\mu 3}$. | | | |
| $\pi^0 \pi^0 e^+ \nu_e$ | $(2.55 \pm 0.04) \times 10^{-5}$ | S=1.1 | 206 |
| $\pi^+ \pi^- e^+ \nu_e$ | $(4.247 \pm 0.024) \times 10^{-5}$ | | 203 |
| $\pi^+ \pi^- \mu^+ \nu_\mu$ | $(1.4 \pm 0.9) \times 10^{-5}$ | | 151 |
| $\pi^0 \pi^0 \pi^0 e^+ \nu_e$ | $< 3.5 \times 10^{-6}$ | CL=90% | 135 |
| **Hadronic modes** | | | |
| $\pi^+ \pi^0$ | $(20.67 \pm 0.08)$ % | S=1.2 | 205 |
| $\pi^+ \pi^0 \pi^0$ | $(1.760 \pm 0.023)$ % | S=1.1 | 133 |
| $\pi^+ \pi^+ \pi^-$ | $(5.583 \pm 0.024)$ % | | 125 |




$\propto G_F^2 \dfrac{f_\pi^2}{m_\pi^3} m_\mu^2 (m_\pi^2 - m_\mu^2)$



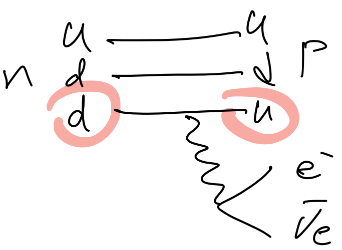


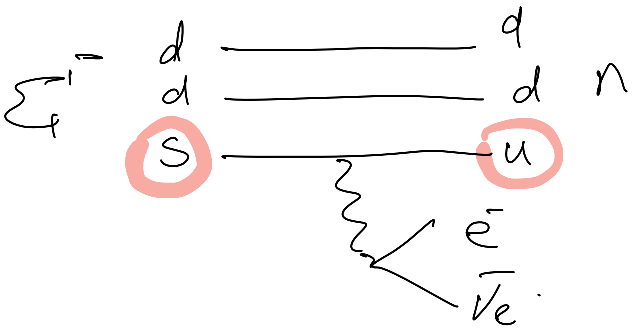
$\propto G_F^2 \dfrac{f_K^2}{m_K^3} m_\mu^2 (m_K^2 - m_\mu^2)$

$$\frac{\Gamma(K^+\to\mu^+\nu_\mu)}{\Gamma(\pi^+\to\mu^+\nu_\mu)} \sim \frac{G_{F_K}^2}{G_{F_\pi}^2}\,\frac{f_\pi^2}{f_K^2}\left(\text{---}\right)$$

$$\frac{BR(K^+\to\mu^+\nu_\mu)}{BR(\pi^+\to\mu^+\nu_\mu)}$$

$$\frac{G_F^2(K^+\to\mu^+\nu_\mu,\ \Delta S=1)}{G_F^2(\pi^+\to\mu^+\nu_\mu,\ \Delta S=0)} = 0.05$$




$\Delta S = 1.$

$n \to p e^- \bar{\nu}_e$

$\Sigma^- \to p e^- \bar{\nu}_e$

$\Gamma \propto G_F^2 \, P_{\text{phase}}$

$\Delta S = 0.$

$\Gamma \propto G_F^2 \, P_{\text{phase space}}$

$$\frac{G_F^2(\Delta S=1)}{G_F^2(\Delta S=0)} \simeq 0.05$$

$(\Delta S=1) \ll \Gamma(\Delta S=0).$

### *n* DECAY MODES

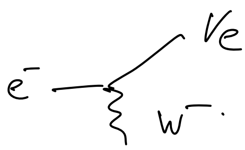
| | Mode | | Fraction ($\Gamma_i/\Gamma$) | Confidence level |
|---|---|---|---|---|
| $\Gamma_1$ | $pe^-\overline{\nu}_e$ | | 100 % | |
| $\Gamma_2$ | $pe^-\overline{\nu}_e\gamma$ | [a] | $(9.2\pm0.7)\times10^{-3}$ | |
| $\Gamma_3$ | hydrogen-atom $\overline{\nu}_e$ | | $< 2.7 \times 10^{-3}$ | 95% |

### $\Sigma^-$ DECAY MODES

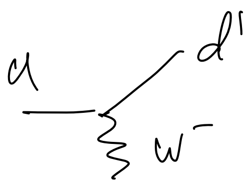
| | Mode | | Fraction ($\Gamma_i/\Gamma$) | Confidence level |
|---|---|---|---|---|
| $\Gamma_1$ | $n\pi^-$ | | $(99.848\pm0.005)$ % | |
| $\Gamma_2$ | $n\pi^-\gamma$ | [a] | $(4.6\pm0.6)\times10^{-4}$ | |
| $\Gamma_3$ | $ne^-\overline{\nu}_e$ | | $(1.017\pm0.034)\times10^{-3}$ | |
| $\Gamma_4$ | $n\mu^-\overline{\nu}_\mu$ | | $(4.5\pm0.4)\times10^{-4}$ | |
| $\Gamma_5$ | $\Lambda e^-\overline{\nu}_e$ | | $(5.73\pm0.27)\times10^{-5}$ | |
| $\Gamma_6$ | $\Sigma^+X$ | | $< 1.2 \times10^{-4}$ | 90% |

## Cabibbo's solution.

$W^-$ couples to $\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}$

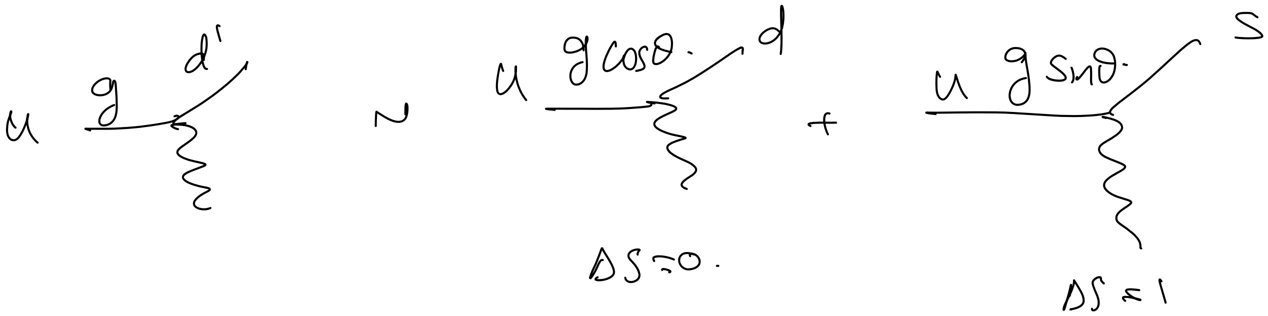



$W^-$ couples to $\begin{pmatrix} u \\ d' \end{pmatrix}$



$d' = \cos\theta \cdot d + \sin\theta \; s.$ mixture of $d, s.$

$s' = -\sin\theta \; d + \cos\theta \; s.$




# UNITARY SYMMETRY AND LEPTONIC DECAYS


Nicola Cabibbo
CERN, Geneva, Switzerland



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We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as "eightfold way,"[1] and the $V$-$A$ theory for weak interactions.[2,3] Our basic assumptions on $J_\mu$, the weak current of strong interacting particles, are as follows:

(1) $J_\mu$ transforms according to the eightfold representation of $SU_3$. This means that we neglect currents with $\Delta S = -\Delta Q$, or $\Delta I = 3/2$, which should belong to other representations. This limits the scope of the analysis, and we are not able to treat the complex of $K^0$ leptonic decays, or $\Sigma^+ \rightarrow n + e^+ + \nu$ in which $\Delta S = -\Delta Q$ currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of $J_\mu$ which is in the eightfold representation.

(2) The vector part of $J_\mu$ is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For $\Delta S = 0$, this assumption is equivalent to vector-

To determine $\theta$, let us compare the rates for $K^+ \rightarrow \mu^+ + \nu$ and $\pi^+ \rightarrow \mu^+ + \nu$; we find

$$\Gamma(K^+ \rightarrow \mu\nu)/\Gamma(\pi^+ \rightarrow \mu\nu) = \tan^2\theta M_K(1 - M_\mu{}^2/M_K{}^2)^2/M_\pi(1 - M_\mu{}^2/M_\pi{}^2)^2. \quad (3)$$

From the experimental data, we then get[5,6]

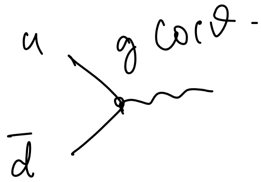
$$\theta = 0.257. \quad (4)$$

For an independent determination of $\theta$, let us consider $K^+ \rightarrow \pi^0 + e^+ + \nu$. The matrix element for this process can be connected to that for $\pi^+ \rightarrow \pi^0 + e^+ + \nu$, known from the conserved vector-current hypothesis (2nd assumption). From the rate[6] for $K^+ \rightarrow \pi^0 + e^+ + \nu$, we get

$$\theta = 0.26. \quad (5)$$

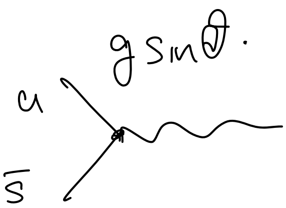
The two determinations coincide within experimental errors; in the following we use $\theta = 0.26$.

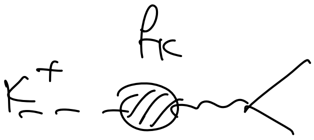
We go now to the leptonic decays of the baryons, of the type $A \rightarrow B + e + \nu$. The matrix element of





$\propto G_F^2 \cos^2\theta \cdot m_\mu^2 (m_\pi^2 - m_\mu^2)$



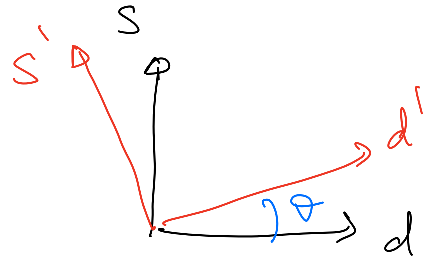

$\propto G_F^2 \sin^2\theta \; m_\mu^2 (m_K^2 - m_\mu^2)$

$$\frac{\Gamma(K^+\to\mu^+\nu_\mu)}{\Gamma(\cancel{K}\pi^+\to\mu^+\nu_\mu)} \simeq \tan^2\theta \times \left( — \quad — \right)$$

$$\theta \simeq 13^\circ$$

$$\cos^2\theta \simeq 0.97$$

$$\sin^2\theta = 0.03$$




$$\tan^2\theta \simeq 0.05$$

$$\Gamma(\Delta S = 1) \propto \sin^2\theta.$$

$$G_{F_K}^2 \simeq \left(\frac{g_W^2}{M_W^2}\right)^2 \sin^2\theta. \qquad \Delta S = 1.$$

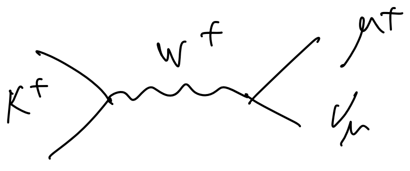
$$G_{F_\pi}^2 \simeq \left(\frac{g_W^2}{M_W^2}\right)^2 \cos^2\theta. \qquad \Delta S = 0.$$

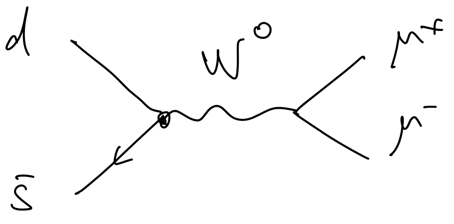
Flavor eigenstates $\neq$ Weak eigenstates
$(d, s)$ $\qquad$ $d', s'$.

$$\Gamma(K^+\to\mu^+\nu_\mu) = 64\%$$

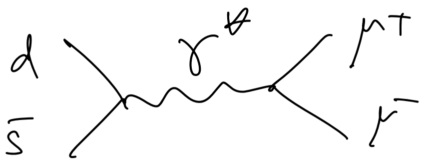
$$\Gamma(K^0\to\mu^+\mu^-) \simeq 7\times10^{-9}$$

| $K^+$ DECAY MODES | Fraction ($\Gamma_i/\Gamma$) | Scale factor/ Confidence level | $p$ (MeV/c) |
|---|---|---|---|
| **Leptonic and semileptonic modes** | | | |
| $e^+\nu_e$ | $(1.582\pm0.007)\times10^{-5}$ | | 247 |
| $\mu^+\nu_\mu$ | $(63.56\pm0.11)$ % | S=1.2 | 236 |
| $\pi^0 e^+\nu_e$ | $(5.07\pm0.04)$ % | S=2.1 | 228 |
| Called $K^+_{e3}$. | | | |
| $\pi^0\mu^+\nu_\mu$ | $(3.352\pm0.033)$ % | S=1.9 | 215 |
| Called $K^+_{\mu3}$. | | | |
| $\pi^0\pi^0 e^+\nu_e$ | $(2.55\pm0.04)\times10^{-5}$ | S=1.1 | 206 |
| $\pi^+\pi^- e^+\nu_e$ | $(4.247\pm0.024)\times10^{-5}$ | | 203 |
| $\pi^+\pi^-\mu^+\nu_\mu$ | $(1.4\pm0.9)\times10^{-5}$ | | 151 |
| $\pi^0\pi^0\pi^0 e^+\nu_e$ | $< 3.5\times10^{-6}$ | CL=90% | 135 |
| **Hadronic modes** | | | |
| $\pi^+\pi^0$ | $(20.67\pm0.08)$ % | S=1.2 | 205 |
| $\pi^+\pi^0\pi^0$ | $(1.760\pm0.023)$ % | S=1.1 | 133 |
| $\pi^+\pi^+\pi^-$ | $(5.583\pm0.024)$ % | | 125 |




$W^0$: neutral current
flavor changing.




not allowed.

if $W^0$ exists.


$K^+ \longrightarrow \pi^+ \nu\nu$