



$$e^- + p \rightarrow e^- + X \quad \text{DIS}$$

For  $N$  partons:

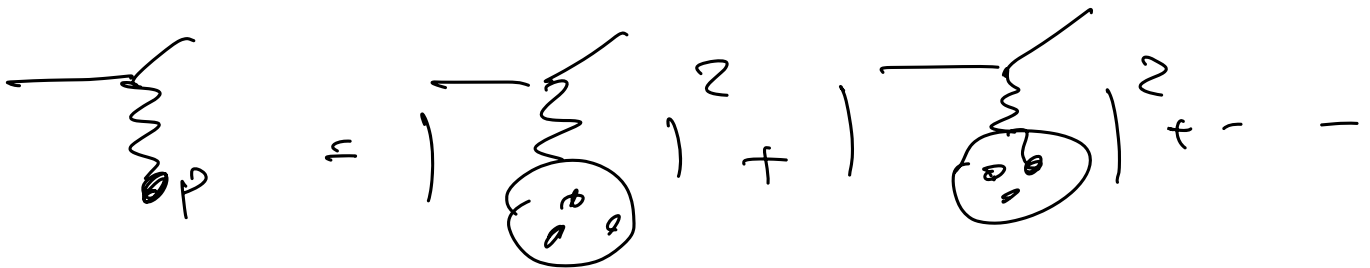
$$W_1(Q^2, \nu) = \sum_j^{\text{partons}} z q_j^2 \frac{f_j(x)}{2M}$$

$$W_2(Q^2, \nu) = \sum_j z q_j^2 f_j(x) \frac{x}{\nu}$$

$f(x)$   
parton  
density  
function.

$$F_1(x) = M W_1(Q^2, \nu). \quad F_2(x) = \nu W_2(Q^2, \nu)$$

$$F_2(x) = 2x F_1(x) \quad \text{Callan-Gross relation.}$$



elastic scattering on parton.

proton: container of some constituents (parton)

$$\text{parton: } S = 1/2 \quad M = XM$$

$$x = \frac{Q^2}{2M\nu}$$

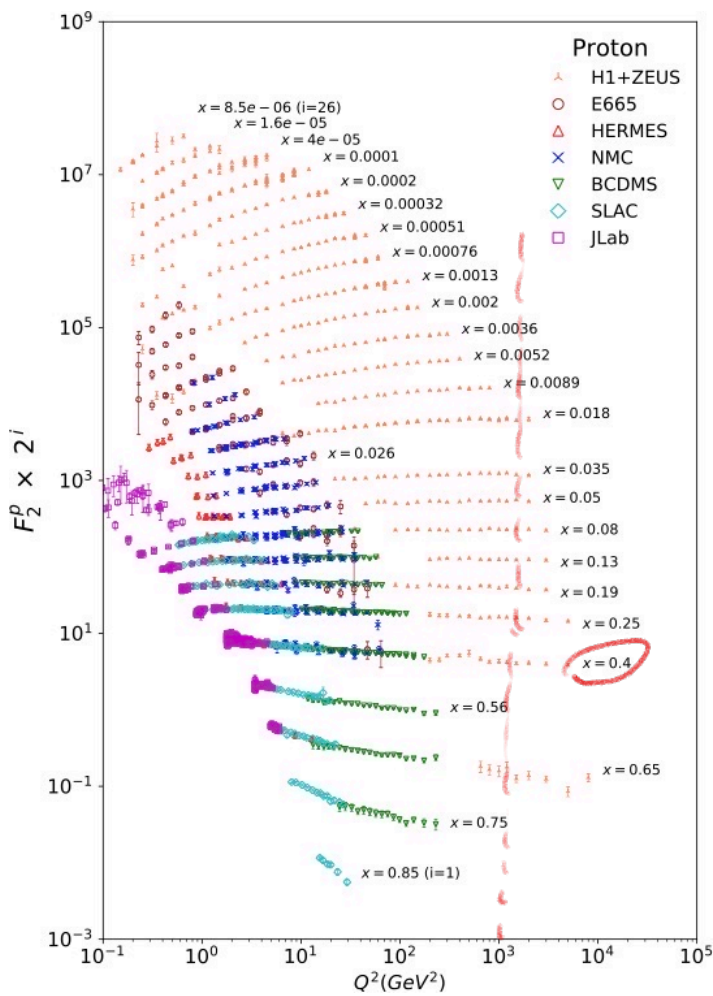
$$Q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad \nu = E - E'$$

observed/measured quantities  
with leptons/probe

$$\left. \frac{d\sigma}{dQ^2} \right|_{\text{DIS}} \sim \frac{\alpha^2}{Q^4} E'^2 \left[ W_2 \sin^2 \frac{\theta}{2} + 2W_1 \cos^2 \frac{\theta}{2} \right]$$

$\delta(\nu - \frac{Q^2}{2M})$

parton  $j$  has charge  $z q_j e < e$



In principle

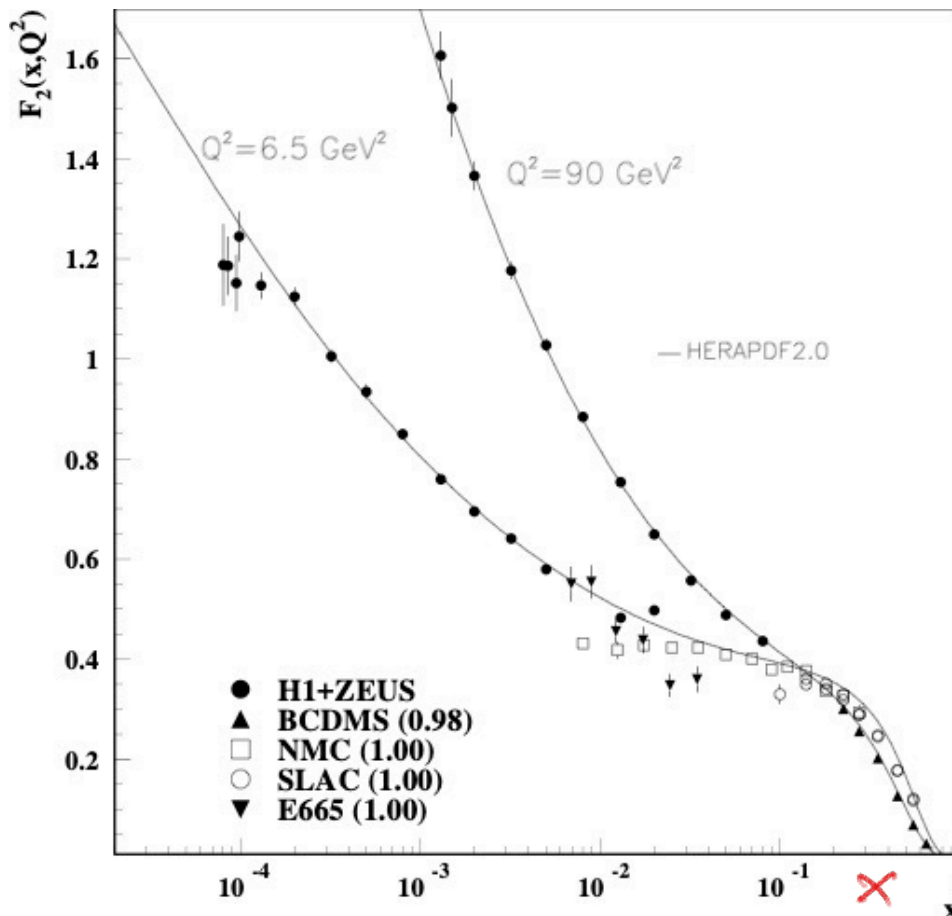
$$F_2 = F_2(Q^2, \nu)$$

$$x = \frac{Q^2}{2M\nu}$$

$\Rightarrow F_2$  function of  $x$   
not  $Q^2, \nu$  separately.

$$f(x, y) = x^2 + y^2$$

$$f(x, y) = \frac{x}{y}$$



$$x_B = \frac{Q^2}{2M\nu}$$

Bjorken Scaling  
variable  $x$

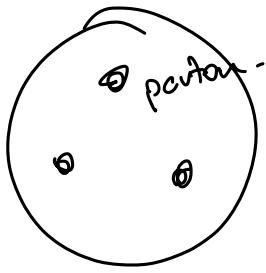
$$\int_0^1 dx f(x) = 1$$

$$x \in [0, 1]$$

perturbative evidence from D. I. S.  $\equiv$  incoherent sum of elastic scattering

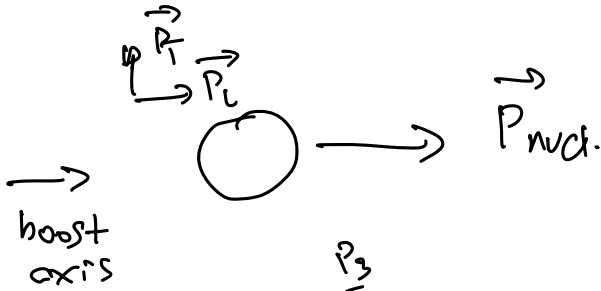
Feynman:  $x$  as property of parton.

## Quark-parton model



high energy limit.  $E \gg m_e, m_p, m_{\text{parton}}$ .

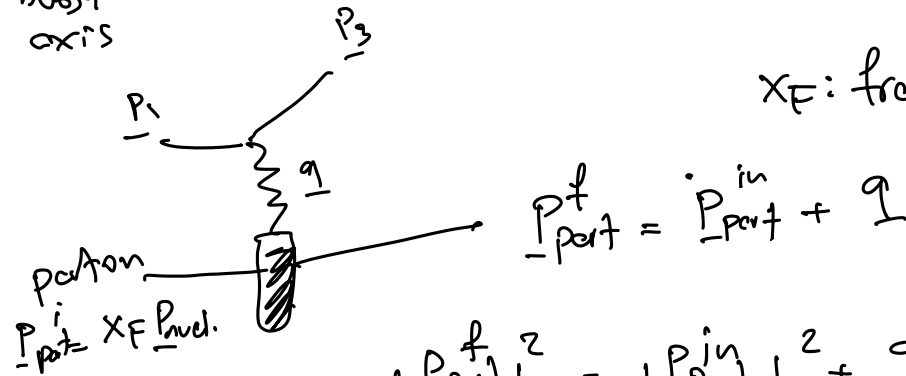
$$x_F = \frac{|\vec{P}_L|}{|\vec{P}_{\text{nucle}}|}$$



SLAC experiment:  $E_e = 25 \text{ GeV}$

$$\vec{P}_{\text{part}} = x_F \vec{P}_{\text{nucle}}$$

$x_F$ : fraction of momentum carried by parton.



$$\vec{P}_{\text{part}}^{\text{f}} = \vec{P}_{\text{part}}^{\text{in}} + \vec{q}$$

$$|\vec{P}_{\text{part}}^{\text{f}}|^2 = |\vec{P}_{\text{part}}^{\text{in}}|^2 + \vec{q}^2 + 2 \vec{P}_{\text{part}}^{\text{in}} \cdot \vec{q}$$

$\underbrace{m_p^2}_{\approx 0} \approx 0$  high energy limit.

$$q^2 = -Q^2 \Rightarrow Q^2 = 2 \vec{P}_{\text{part}}^{\text{in}} \cdot \vec{q} =$$

$$\vec{P}_{\text{nucle}}^{\text{LAB}} = (M, 0) \quad \vec{q} = (E - E', \vec{P}_1 - \vec{P}_3) = (V, \vec{P}_1 - \vec{P}_3)$$

$$\Rightarrow Q^2 = 2 x_F M V \Rightarrow x_F = \frac{Q^2}{2 M V} \approx x_B \text{ at high energy}$$

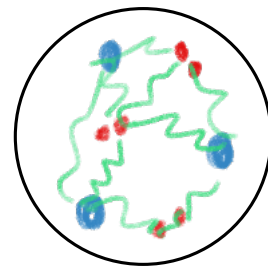
$$N \text{ partons} \quad \sum_i x_i = 1 \quad \sum_i \vec{P}_{\text{part}}^i = \vec{P}_{\text{nucle}}$$

proton: uud and quarks.  $+\frac{2}{3} \begin{pmatrix} u \\ d \end{pmatrix}$

$$Q_p = +1 = \frac{2}{3} + \frac{2}{3} - \frac{1}{3}$$

proton  $uud\bar{d}\bar{d}s\bar{s}$

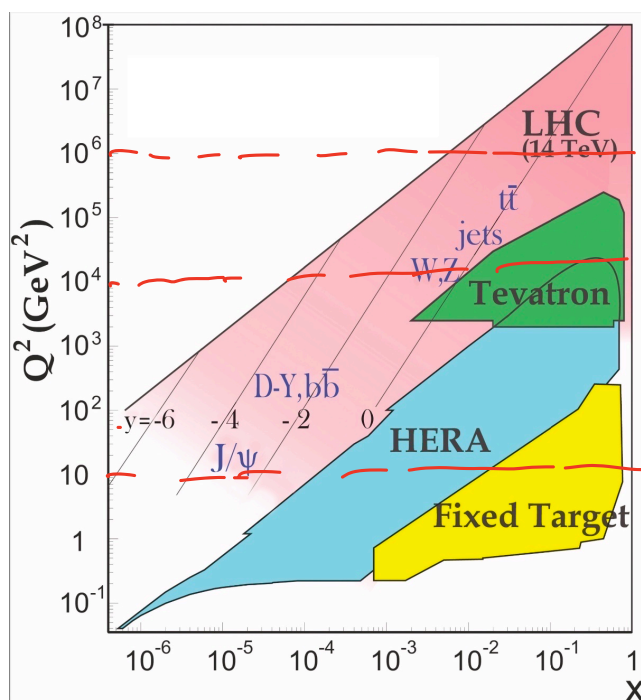
$$\sum x_u x_{\bar{u}} x_d x_{\bar{d}} x_s x_{\bar{s}} = 1.$$



Valence quarks:  $uud$  (electric charge)

Sea quarks/anti-quarks:  $s\bar{s} + q\bar{q}$

gluons:  $g$  mediators of strong interaction

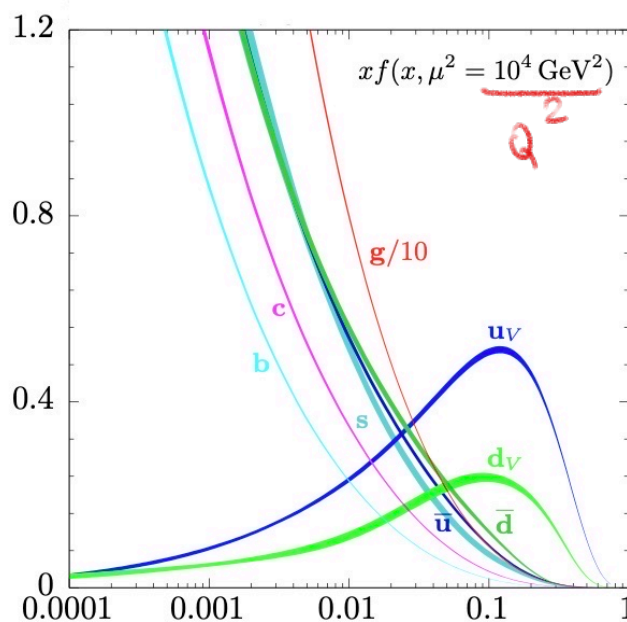
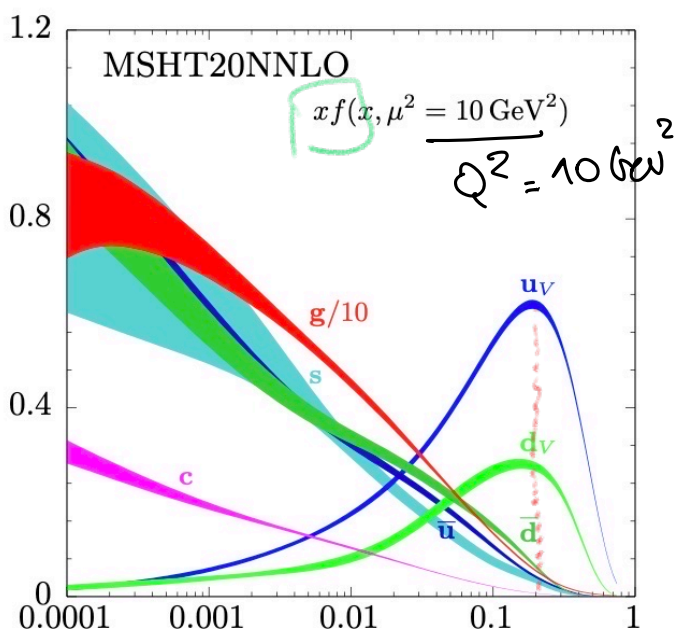


Tevatron:  $\vec{p} \rightarrow \leftarrow p$   $\sqrt{s} = 1.96 \text{ TeV}$

HERA  $e^- + p(\text{beam})$ .

$\vec{e^-} \rightarrow \leftarrow p$

$f(x)$ : parton density function.

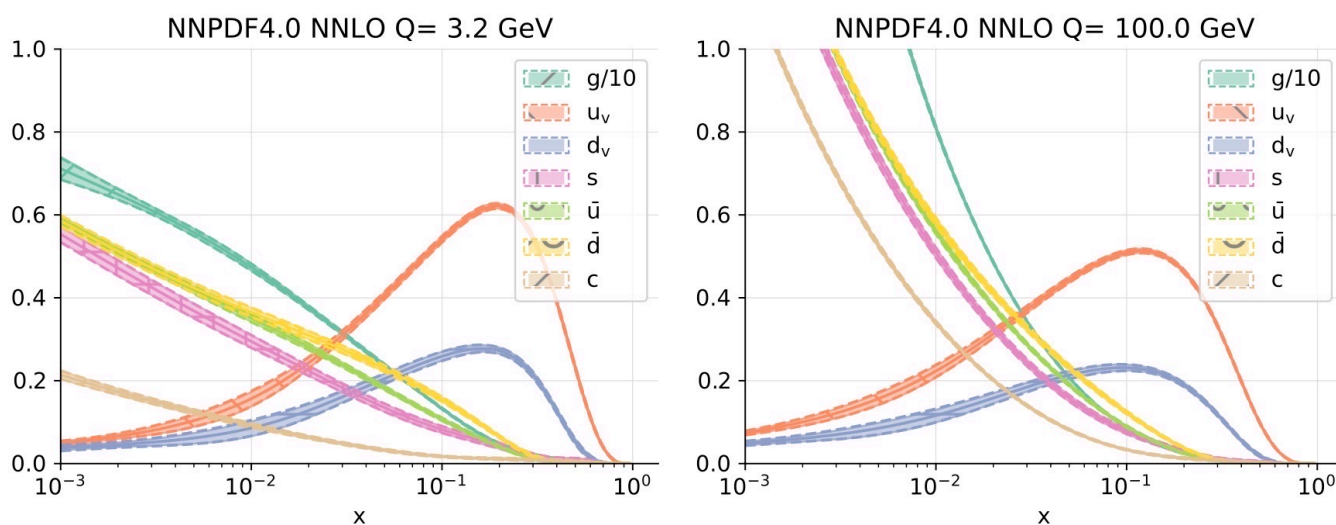


$d_v$ :  $f(x)$  for valence d quark.

$u_v$ :  $f(x)$  for valence u quark.

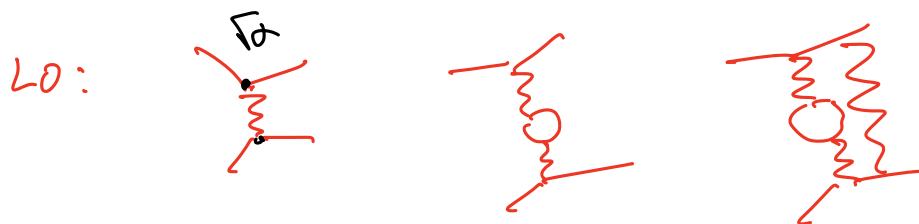
$\bar{u}$ :  $f(x)$  for sea antiquark  $\bar{u}$

Experimentally:  $\sum_i^{\text{Valence}} x_i = 0.5 \Rightarrow$  existence of sea  $\bar{q}$  and gluons.



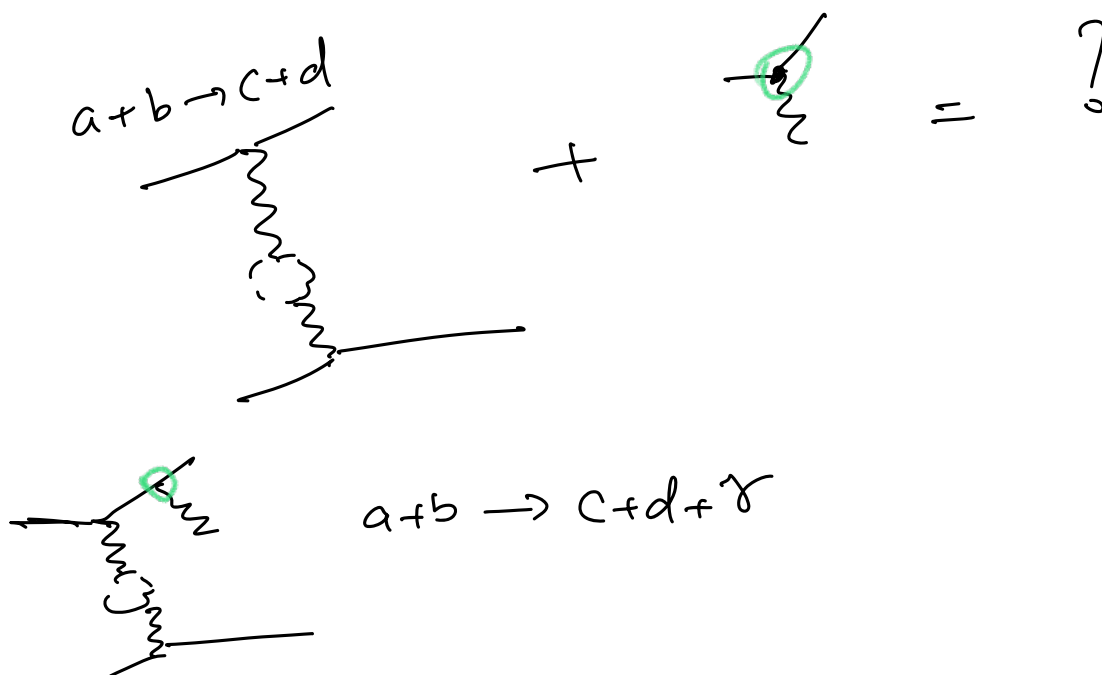
**Figure 1.1.** The NNPDF4.0 NNLO PDFs at  $Q = 3.2$  GeV (left) and  $Q = 10^2$  GeV (right).

NNLO: Next-to-Next-to-Leading-order.



LO:  $\alpha$ , NLO:  $\alpha^2$ , NNLO:  $\alpha^3$

$N^3$ LO: Next-Next-Next LO.



$$\mu = \text{diagram of a single gluon exchange between two quark lines}$$

$$\mu^2 = \left| \text{diagram of a single gluon exchange between two quark lines} \right|^2$$

$$\mu = \text{diagram 1} + \text{diagram 2}$$

$$\mu^2 = \left| \text{diagram 1} \right|^2 + \left| \text{diagram 2} \right|^2 + 2 \left( \text{diagram 1} \cdot \text{diagram 2} \right)$$

$$\mu = \mu(\alpha) + \mu(\alpha^2) + \mu(\alpha^3)$$

$f(x)$ : parton density function.

$$x = \frac{|P_{part}|}{|P_{nucl}|}$$

$$\sum P_{part}^i = P_{nucl}$$

$$\int_0^1 dx \left[ \underbrace{\sum f_i(x)}_{\text{partons.}} \right] = 1.$$

$$\int_0^1 dx \left[ \sum_{\text{flavors}} q_f(x) + \bar{q}_f(x) + g(x) \right] x \approx 1.$$

Momentum sum rule.

$$\int_0^1 dx \left[ u^p(x) - \bar{u}^p(x) \right] = 2$$

2 valence up quarks.  
in total

$$\int_0^1 dx \left[ d^p(x) - \bar{d}^p(x) \right] = 1$$

1 valence down quark.  
in total

$$u^p(x) = \text{or} \neq u^n(x)$$

$$p = (\text{und})$$

$$n = (\text{udd})$$

parton density  
function of  
a quark in proton.  
in neutron.

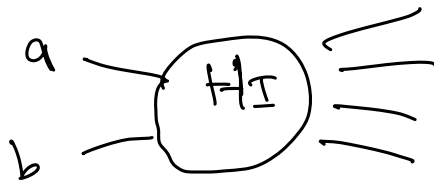
DGLAP equations.  $\Rightarrow$  evolution of pdf  
 $\hookrightarrow$  Altarelli - Parisi

DIS  $\Rightarrow$  partons (relations) quarks?

1950-1960 :  $p, n, \pi^\pm, K^\pm, \pi^0, K^0, \Delta$

Baryons :  $q_1 q_2 q_3$

mesons :  $q_1 \bar{q}_2$



#events. made of hadrons.  
(leptons).

$$\# \text{events} \propto \sigma \propto |M|^2 \rho(E_f)$$

$$M = \langle f | H_I | i \rangle$$

To study  $H_I \Rightarrow$  look for symmetries / selection rules.

symmetry: transformation acts on a state  
leaves the state unchanged

T: transformation.

$$\psi_i = \psi(q, \vec{x}, t)$$

$\hookrightarrow$  charge, quantum #

$$\langle T \rangle = \langle \psi | T | \psi \rangle$$

$$|\psi'\rangle = T|\psi\rangle$$

$$\langle \psi' | = \langle \psi | T^\dagger$$

$$\langle \psi | T^\dagger T | \psi \rangle = \langle \psi | \psi \rangle$$

$$\frac{d}{dt} \langle T \rangle = 0$$

$$[T, H] = 0$$

$$[T, H] = 0 \Rightarrow T H |\psi\rangle = H T |\psi\rangle$$

$$\psi(q, \vec{x}, t)$$

symmetries:

1/ External symm:

time reversal  
parity  
translation.  
rotation.

Continuous  
translation  $\vec{x}$   
translation  $t$   
rotation  $\vec{x}$

Discrete.

$$\text{Time reversal. } T \psi(\vec{x}, t) = \psi(\vec{x}, -t)$$

$$\text{Parity } \vec{x} \rightarrow -\vec{x} \quad P \psi(\vec{x}, t) = \psi(-\vec{x}, t)$$

2/ Internal symmetries. act on internal quantum #.

NOT on  $\vec{x}$  or  $t$

Continuous  
Isospin

Discrete

charge conjugation.

$C$ : proton  $\rightarrow$   $\bar{p}$   
no change of momentum  
no change of spin