Weak Interactions:

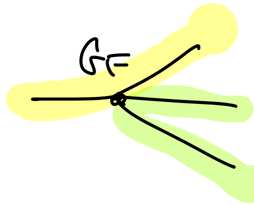
Wu's experiment, violation of $\cancel{P}$
Goldhaber's experiment, violation of $\cancel{C}$ $\Big\}$ $\Rightarrow$ V−A theory

W^{-} $g_w \gamma^\mu (1-\gamma^5)$ Modern weak inter.

$$\mathcal{M}_W = G_F \, \underline{J} \cdot \underline{J} \qquad J = \bar{\psi} \gamma^\mu (1-\gamma^5) \psi$$

$$\bar{\psi} \gamma^\mu \psi \equiv V \qquad \bar{\psi} \gamma^\mu \gamma^5 \psi \equiv A$$

G_F



J_1

J_2

$$\mathcal{M} = G_F \cdot \underline{J_1} \cdot \underline{J_2}$$

Goldhaber experiment: weak interaction produced only ν_L

left-handed neutrino $\xrightarrow{\Leftarrow} \nu$

right-handed anti-neutrino $\xrightarrow{\Rightarrow} \bar{\nu}$

	e^-, μ^-, τ^-	ν
QED	✓	X
QCD	X	X
Weak V−A	✓	$\nu_L, \bar{\nu}_R$

what about $\nu_R, \bar{\nu}_L$

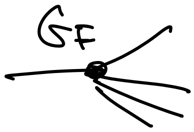
Biggest difference between QED and V−A theory

$ie\gamma^\mu = \sqrt{\alpha}\,\gamma^\mu$

γ^μ

$j_{EM} = ie \bar{\psi} \gamma^\mu \psi$

G_F



$j_w = \bar{\psi} \gamma^\mu (1-\gamma^5) \psi$

$G_F \, \underline{j_w} \cdot \underline{j_w}$

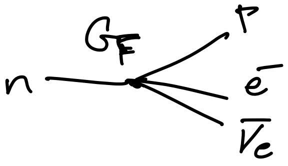
α : pure number in natural units

$$\alpha = \frac{e^2}{4\pi}$$

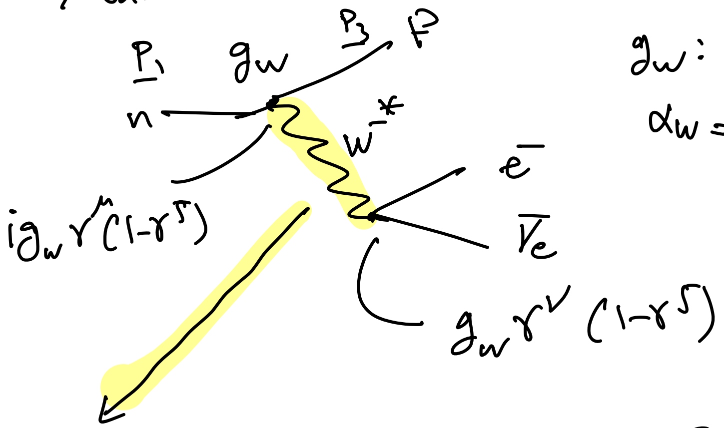
$[G_F] = E^{-2}$

has physical dimensions

Modern Electro-weak theory.



$\mathcal{M}$



g_W : weak charge.

$$\alpha_W = \frac{g_W^2}{4\pi}$$

matrix element of β^- decay.

$$\mathcal{M} = \mathcal{M}(n \to p e^- \bar{\nu}_e)$$

W massive boson:

$m_W = 80$ GeV

$$\frac{-i(g^{\mu\nu} - q^\mu q^\nu / m_W^2)}{q^2 - m_W^2}$$

$q = P_1 - P_3$

consider neutron decay: $n \to p e^- \bar{\nu}_e$ $\quad Q = m_n - m_p - m_e - m_\nu \simeq 1$ MeV.

$$q^2 \simeq (1 \text{ MeV})^2 \ll m_W^2 = (80 \text{ GeV})^2$$

$$\mathcal{M} \sim \underbrace{\frac{g_W^2}{M_W^2}}_{\text{modern theory with W}} \sim \underbrace{G_F}_{\text{V-A theory}}$$

Fermi theory.

$$G_F = \frac{\sqrt{2}}{8} \frac{g_W^2}{m_W^2}$$

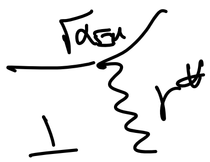
Experimentally $\begin{cases} m_W = 80 \text{ GeV} \\ G_F = 1.16 \times 10^{-5} \text{ GeV}^{-2} \end{cases}$

measured.

$$= g_W = 0.653 \Rightarrow \alpha_W = \frac{g_W^2}{4\pi} = \frac{1}{29.5}$$

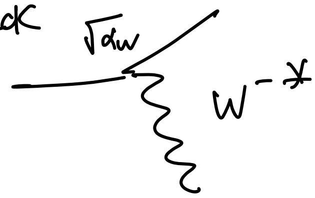
$$\frac{1}{29.5} = \alpha_W > \alpha_{EM} = \frac{e^2}{4\pi} \sim 1/137$$

QED



$\frac{1}{q^2}$

Weak



$\frac{1}{q^2 - m_W^2}$

$\mathcal{M}_{EM} \sim \frac{\alpha_{EM}}{q^2}$ $\qquad$ $\mathcal{M}_W \sim \frac{\alpha_W}{q^2 - m_W^2}$

$q^2 << m_W^2$ $\qquad$ $q \sim 1\,MeV$

$\mathcal{M}_{EM} \sim \frac{\alpha_{EM}}{q^2}$ $\qquad$ $\mathcal{M}_W \sim \frac{\alpha_W}{m_W^2}$

weak interaction amplitude suppressed because of large W mass.

Remember $\alpha_{strong} \sim 0.1$. $\qquad$ $\alpha_{strong} = \frac{g_{strong}^2}{4\pi}$

Relation depends on q^2

How to verify V-A theory of weak interactions.

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$: measure τ_μ and compare to theory prediction.

$\pi^- \to e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$: why μ dominates over e^- and by how much.

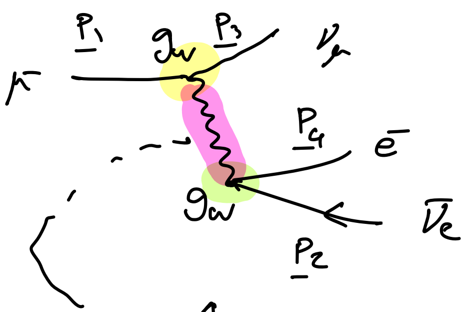
muon decay

Griffiths 9.2 (Goldhaber exercise 6.7)

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$Q = m_\mu - m_e$

$Q_e = 106 - 0.5 \simeq 105\ MeV$



$\sim \frac{1}{q^2 - m_W^2} \sim \frac{1}{m_W^2}$

$\underline{p_1} = \underline{p_3} + \underline{q}$

$\underline{q} = \underline{p_1} - \underline{p_3}$

$$\mathcal{M} = \frac{g_W^2}{\cancel{q^2} - m_W^2} \times [\bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)] \times [\bar{u}(p_4)\gamma_\mu(1-\gamma^5)u(p_2)]$$

$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) \propto |\mathcal{M}|^2 \times (\text{phase space})$$

$$|\mathcal{M}|^2 = \underset{\substack{\text{spins}\\ \text{in final}\\ \text{state}}}{\sum} \; \underset{\substack{\text{average}\\ \text{over spin}\\ \text{initial state.}}}{\overline{\sum}}$$

Final state: ν_μ : 1 spin state

e^- : 2 spin states

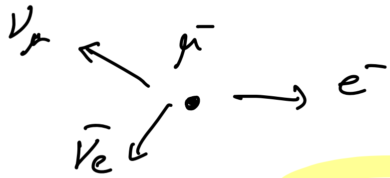
$\bar{\nu}_e$: 1 spin state

Initial state: μ^- : 2 spin states:

$$\Gamma(\mu \to e) \propto \left(\frac{g_W^2}{m_W^2}\right)^2 \underbrace{(\text{- - - -})}_{\hookrightarrow \text{ spinors and } \gamma \text{ matrices}} (\text{phase space})$$

$$\downarrow$$

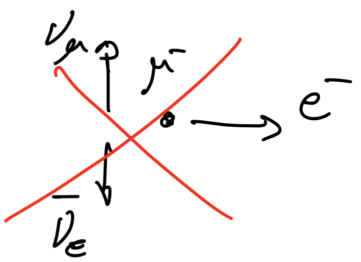
$$\sim G_F^2$$

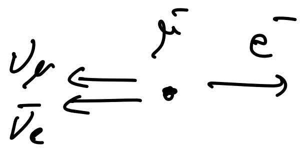


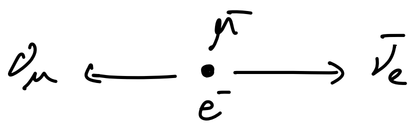
$$\Gamma \sim G_F^2 \times (\text{phase space})$$

$$\rho_{space} = \frac{d^3 p_{\bar{\nu}_e}}{(2\pi)^3} \frac{d^3 p_{\nu_\mu}}{(2\pi)^3} \frac{d^3 p_e}{(2\pi)^3} \delta(\underline{p}_\mu - \underline{p}_e - \underline{p}_{\bar{\nu}_e} - \underline{p}_{\nu_\mu})$$

$$[\rho] = E^5$$







$$E_e^{max} = \frac{1}{2} m_\mu$$

$$E_e \simeq m_e \Rightarrow p_e = 0.$$

$$\Rightarrow p_e^{max} = \frac{1}{2} m_\mu \simeq 53 \text{ MeV}$$

$$\Gamma \propto G_F^2 \rho \qquad [\Gamma] = E \qquad [G_F] = E^{-2} \Rightarrow [\rho] = E^5$$

$\Gamma \propto G_F^2 \, m_\mu^5$ from dimensional analysis

$$\Gamma = \frac{1}{192\pi^3} G_F^2 \, m_\mu^5$$ total width.

$$\tau_\mu = \frac{1}{\Gamma_\mu} \propto m_\mu^{-5}$$

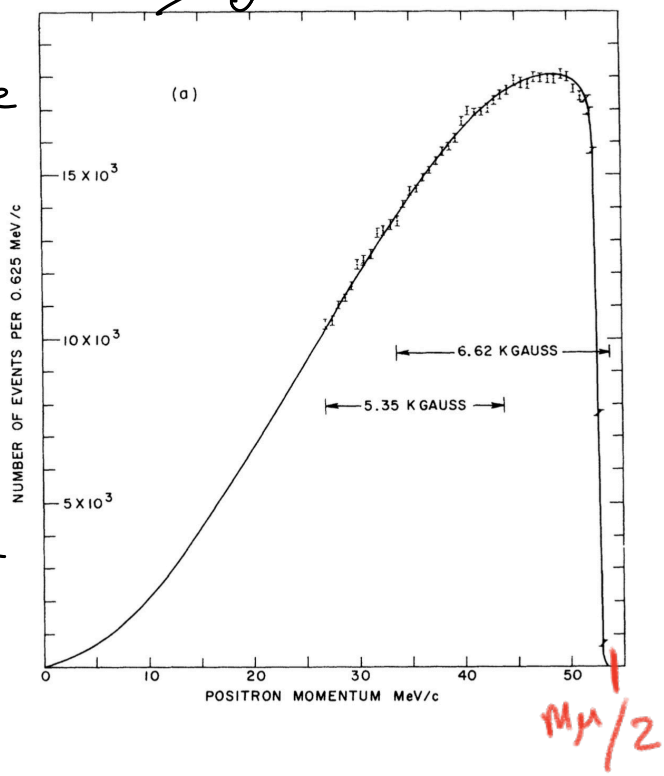
$\frac{d\Gamma}{dE_e}$ — # events / energy bin width

Differential decay rate.

successful prediction of μ^- spectrum.

μ^- decay spectrum.

μ^- decay as function of E_{e^-}

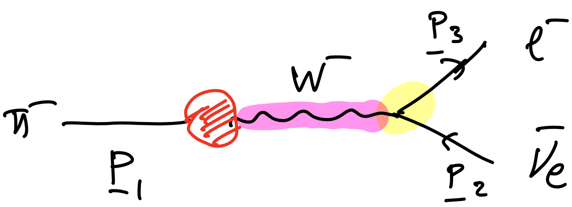


Pion Decay

$\pi^- \to e^- \bar{\nu}_e$ $\quad Q_e = m_\pi - m_e \simeq 135$ MeV

$\quad\ \ \mu^- \bar{\nu}_\mu$ $\quad Q_\mu = m_\pi - m_\mu \simeq 135 - 106 \simeq 29$ MeV

1958 @ CERN: Fidelaro measured $BR(\pi \to e) \neq 0$ but very small.



$$\mathcal{M} = \left(\frac{g_w^2}{q^2 - m_W^2}\right)\left(\bar{u}(p_3)\gamma^\mu(1-\gamma^5)\,v(p_2)\right) F_\mu$$

matrix element must be scalar.

$$|q| \sim m_\pi \ll m_W \Rightarrow \frac{g_w^2}{m_W^2} \sim G_F$$

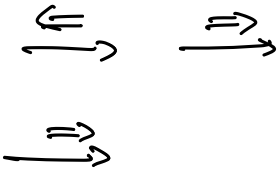
π: spin $= 0$ $\Rightarrow$ $F_\mu = f_\pi \, p_\mu^\pi = f_\pi \, p_1$

f_π: pion decay constant

$\langle |M|^2 \rangle$: spin = 0 $\Rightarrow$ 1 state mi-tial.

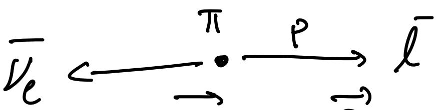
final state: e^- : 2 spin states.

$\bar{\nu}_e$: 1 spin state



$$\langle |M|^2 \rangle = \left(\underbrace{\frac{g_w}{2 m_W}}_{\text{propagator}} \right)^4 f_\pi^2 \, m_e^2 \, (m_\pi^2 - m_e^2)$$

$\Gamma_\pi \propto \langle |M|^2 \rangle \times$ (phase space)



$\vec{P}_\nu = -\vec{P}_e = P_2$

$$m_\pi = E_\nu + E_e$$

$$= P_2 + \sqrt{m_e^2 + P_2^2}.$$

$$\Rightarrow \quad P_2 = \frac{1}{2 m_\pi} (m_\pi^2 - m_e^2) \quad \text{in rest frame of } \pi^-$$

$$\Gamma = \frac{1}{8\pi} \frac{|\vec{P}_2|}{m_\pi^2} \langle |M|^2 \rangle$$

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \underbrace{\frac{m_e^2}{m_\mu^2}}_{\text{matrix element}} \underbrace{\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2}}_{\text{mat. elem} \times \text{phase space}}.$$

$$\frac{m_\pi^2}{m_\pi^2} \frac{(1 - (m_e/m_\pi)^2)^2}{(1 - (m_\mu/m_\pi)^2)^2}.$$

$$\left(\frac{m_e}{m_\mu} \right)^2 = \left(\frac{0.5 \text{ MeV}}{106 \text{ MeV}} \right)^2$$

$\frac{\Gamma_e}{\Gamma_\mu} = 1.283 \times 10^{-4}$ in agreement with experiments.

$J_\pi = 0.$

$J_{\pi z} = 0.$

$J = 0$

$J_z = 0$

Right handed e^-

Right handed muon.

Prob. of right handed e^- $\sim 1 - \beta$ (from WV experiment).

$\beta_e = \frac{P_e}{E_e} \simeq 1 - 2.6 \times 10^{-5}$ $\quad \beta_\mu = 0.38$