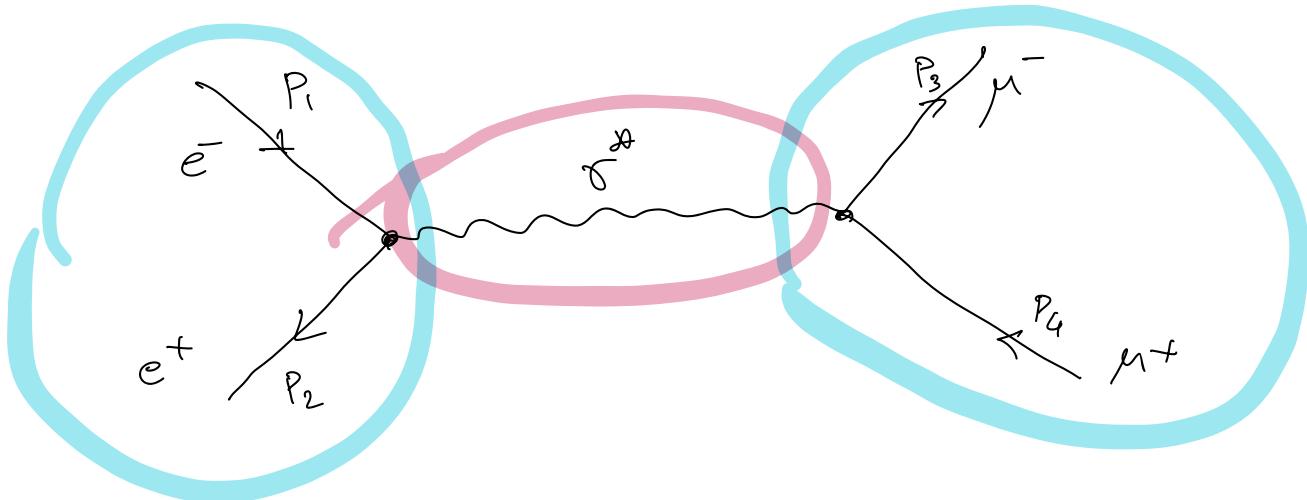


$e^+ e^- \rightarrow \mu^+ \mu^-$  cross section



$$-i\mathcal{M} = [\bar{\nu}(p_2) i\gamma^\mu u(p_1)] X$$

$u, v$  spinors.  $e^- \tau e^+$

$$X \frac{-ig_{\mu\nu}}{q^2} X$$

$$-i \frac{g_{\mu\nu}}{s}$$

$$X \{ \bar{u}(p_3) i\gamma^\nu v(p_4) \}$$

$$j_{(e)}^\mu = \bar{\nu}(p_e) \gamma^\mu u(p_e)$$

$$\Rightarrow \mathcal{M} = -\frac{e^2}{s} \underline{j}_{(e)} \cdot \underline{j}_{(\mu)}$$

$$j_{(\mu)}^\nu = \bar{u}(p_\mu) \gamma^\nu v(p_\mu)$$

### SPIN

Usually  $e^+, e^-$  beams are not polarized.

$$\overleftarrow{e^-} \quad \overrightarrow{e^-} \quad \text{in the limit } P \gg m_e$$

$$\text{helicity} = \frac{\vec{S} \cdot \vec{P}}{|\vec{S}| \cdot |\vec{P}|}$$

for massive particles:  $\vec{h}$  not conserved / divergent

$$\overleftarrow{e^-} e_L \quad \overrightarrow{e^-} e_R$$

$$\begin{array}{c}
 \overleftarrow{\Rightarrow} \quad \overleftarrow{\Leftarrow} \\
 e_R^- \quad e_R^+ \\
 \overleftarrow{\Leftarrow} \quad \overrightarrow{\Rightarrow} \\
 e_L^- \quad e_L^+ \\
 \end{array}
 \quad
 \begin{array}{c}
 \overrightarrow{\Rightarrow} \quad \overleftarrow{\Rightarrow} \\
 e_R^- \quad e_L^+ \\
 \overleftarrow{\Leftarrow} \quad \overleftarrow{\Leftarrow} \\
 e_L^- \quad e_R^+
 \end{array}$$

$$S_Z = 0$$

$$S_Z = \pm 1$$

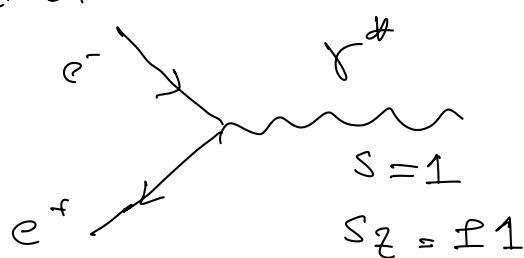
4 initial states.

16 final states.

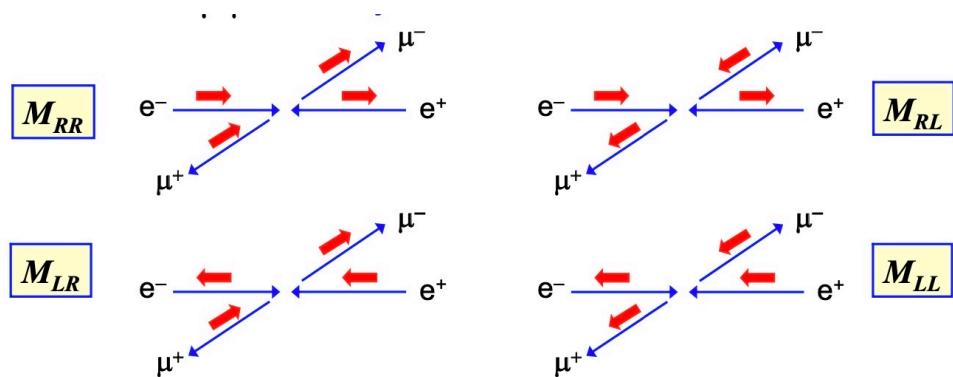
$$RR \rightarrow \begin{array}{l} RR \\ RL \\ LR \\ LL \end{array}$$

$$\langle |M| \rangle = \frac{1}{4} \sum_{\text{final states}} |M_i \rightarrow f|$$

Interaction



$$\begin{array}{c}
 \overleftarrow{\Leftarrow} \quad \overleftarrow{\Leftarrow} \\
 e_L^- \quad e_R^+ \\
 \overrightarrow{\Rightarrow} \quad \overrightarrow{\Rightarrow} \\
 e_R^- \quad e_L^+
 \end{array}
 \quad
 \begin{array}{c}
 \gamma^* \\
 S_Z = -1 \\
 \gamma^* \quad S_Z = +1
 \end{array}$$



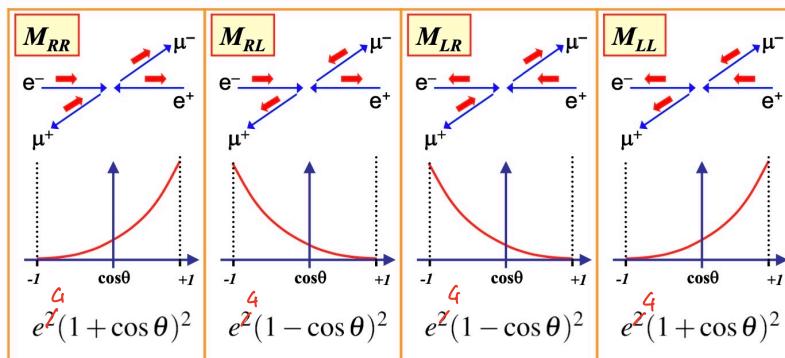
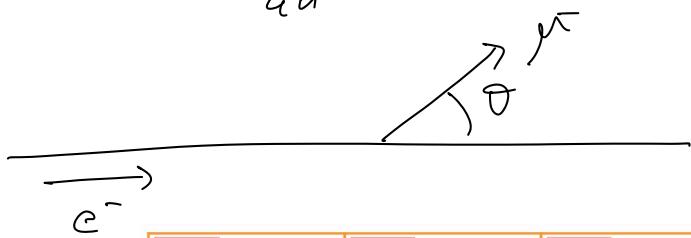
$$M_{AB} : A \ni e_A^- \quad B \ni \mu_B^-$$

$$M_{RR} : e_R^- e_L^+ \rightarrow \mu_R^- e_L^+$$

$$|M_{RR}|^2 = |M_{LL}|^2 = (4\pi\alpha)^2 (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = |M_{LR}|^2 = (4\pi\alpha)^2 (1 - \cos\theta)^2$$

$$\frac{e^2}{4\pi} \propto \alpha \rightarrow e^2 = 4\pi\alpha$$



$$\frac{d\sigma}{d\Omega} = \frac{1}{6\pi v^2 s} \frac{1}{4} \underbrace{\left( |M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 \right)}_{2(1+\cos\theta)^2 + 2(1-\cos\theta)^2} (4\pi\alpha)^2$$

initial states

$$= 4 + 4 \cos^2\theta.$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2\theta) \quad \text{Differential cross section.}$$

$$\sigma_{tot} = \int \frac{d\sigma}{d\Omega} d\Omega.$$

|

$$\frac{\alpha^2}{4s} \cdot 2\pi \int_{-1}^{1} (1 + \cos^2\theta) d\cos\theta$$

$$\sigma_{tot} = \frac{16\pi}{3} \frac{\alpha^2}{s} = \sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

$$S = GeV^2. \quad \tau \quad \text{in barn.}$$

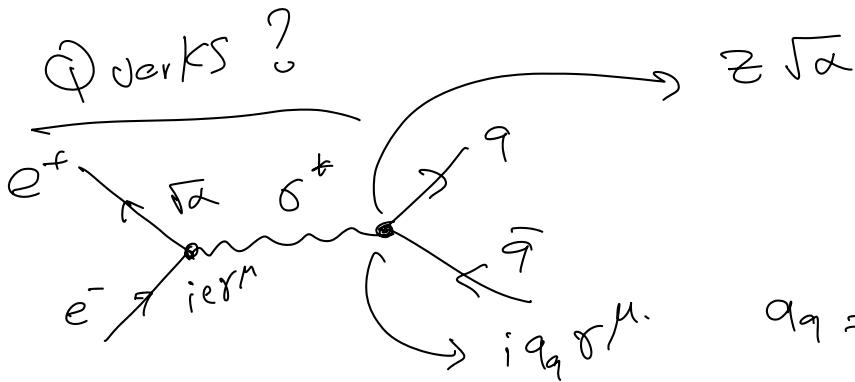
$$\frac{16\pi}{3} \alpha^2 = \frac{16\pi}{3} \frac{1}{(137)^2} = 2.28 \times 10^{-4}$$

$$t \propto = 1 = 197 \text{ MeV fm} = 0.197 \text{ GeV fm.}$$

$$1 \text{ fm} = 10^{-15} \text{ m.} \quad 1 \text{ fm}^2 = 10^{-30} \text{ m}^2 = 10^{-2} \text{ b.}$$

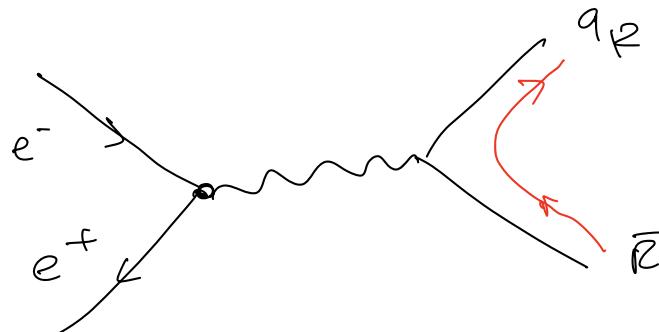
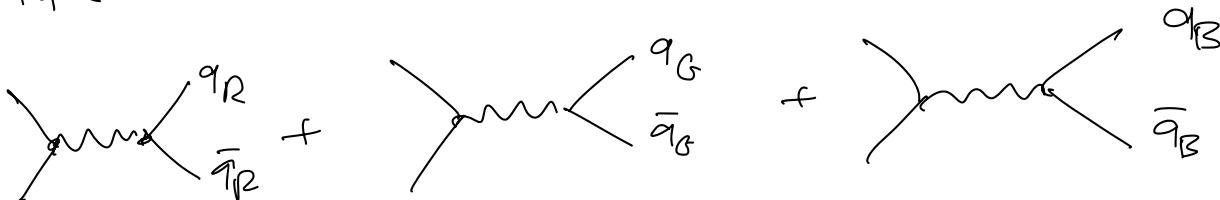
$$\sigma = \frac{86.8 \text{ nb}}{s [\text{GeV}^2]} \quad \rightarrow \leftarrow \quad s = 4E^2$$

$$= \left( \frac{86.8}{4} \right) \frac{1}{\epsilon_{beam}^2 [\text{GeV}^2]}$$



$$\sigma_{tot} (e^+ e^- \rightarrow \text{hadrons}) = \frac{16\pi}{3} \frac{\alpha^2}{s} Z_f^2 \times N_c$$

$$Z = \begin{cases} +\frac{2}{3} & u, c, t \\ -\frac{1}{3} & d, s, b \\ 1 & \text{leptons.} \end{cases}$$



$$R = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma_{QED}} = \sum_{\text{flavors}} Z_f^2 N_c$$

$$R \approx R(\sqrt{s})$$

$$0 < \sqrt{s} < 2m_\mu : \quad e^+ e^- \rightarrow e^+ e^-$$

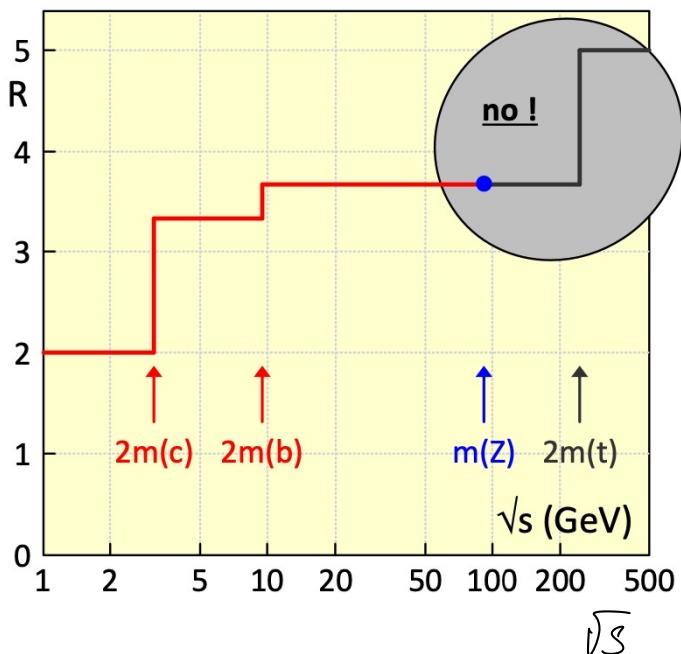
$$0 < \sqrt{s} < 2m_C \quad \text{only } u_{uds}$$

$$R = \sum_{u_{uds}} z_f^2 N_C = \left[ \left( \frac{2}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 \right] 3$$

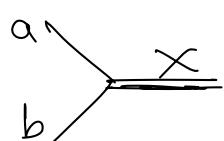
$$= \frac{6}{9} \times 3 = 2$$

$$2m_C < \sqrt{s} < 2m_b \quad R = R_{uds} + (z_C)^2 N_C = 2 + \frac{6}{9} 3 = 3 + \frac{2}{3}$$

$$2m_b < \sqrt{s} < 2m_t \quad R = R_{uds} + \left( -\frac{1}{3} \right)^2 N_C = 3 + \frac{1}{3} + \frac{3}{9} = 3 + \frac{2}{3}$$



what happens when  $\sqrt{s}$  near  $m_X$   $X$  resonance.



$X$ : mass  $m_X$   
width  $\Gamma_{tot}$ .

$$\sigma(e^+e^- \rightarrow X \rightarrow f\bar{f}) = \frac{16\pi}{S} \frac{(2J_{X+1})}{(2s_{a+1})(2s_{b+1})} \left( \frac{\Gamma_{ab}}{\Gamma_{tot}} \right) \left( \frac{\Gamma_{ff}}{\Gamma_{tot}} \right) X$$

$$X \sim \frac{\Gamma_{tot}^2}{(m_X - \sqrt{S})^2 + \Gamma_{tot}^2/4}.$$

# Experimental tests of QED

$$e^+ e^- \rightarrow e^+ e^-$$

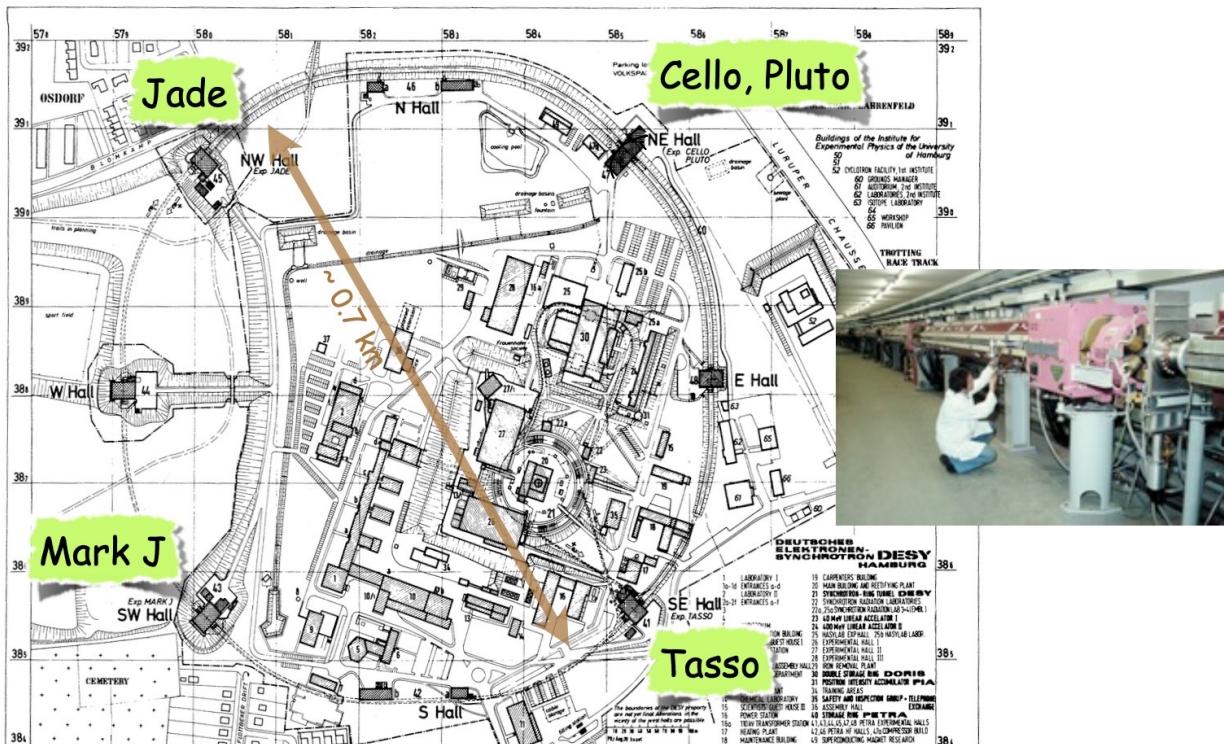
$$\mu^+ \mu^-$$

hadrons.

PETRA @ DESY

$e^+ e^-$  collider

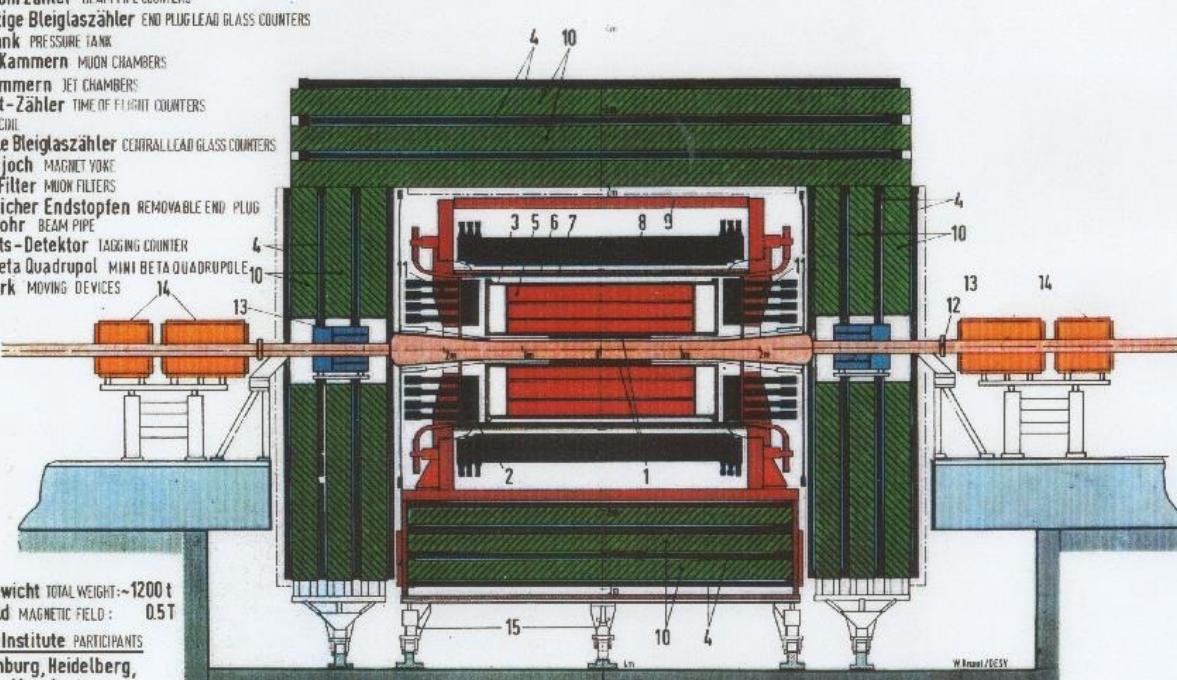
1978 → 1986



JADE : Japan, Deutsch. and England.

## MAGNETDETEKTOR JADE

- 1 Strahlrohrzähler BEAM PIPE COUNTERS
- 2 Endseitige Bleiglaszähler END PLUG LEAD GLASS COUNTERS
- 3 Drucktank PRESSURE TANK
- 4 Myon-Kammern MUON CHAMBERS
- 5 Jet-Kammern JET CHAMBERS
- 6 Flugzeit-Zähler TIME OF FLIGHT COUNTERS
- 7 Spule COIL
- 8 Zentrale Bleiglaszähler CENTRAL LEAD GLASS COUNTERS
- 9 Magnetjoch MAGNET YOKE
- 10 Myon-Filter MUON FILTERS
- 11 Beweglicher Endstopfen REMOVABLE END PLUG
- 12 Strahlrohr BEAM PIPE
- 13 Vorwärts-Detektor TAGGING COUNTER
- 14 Mini-Beta Quadrupol. MINI BETA QUADRUPOLE
- 15 Fahrwerk MOVING DEVICES



Gesamtgewicht TOTAL WEIGHT: ~1200 t

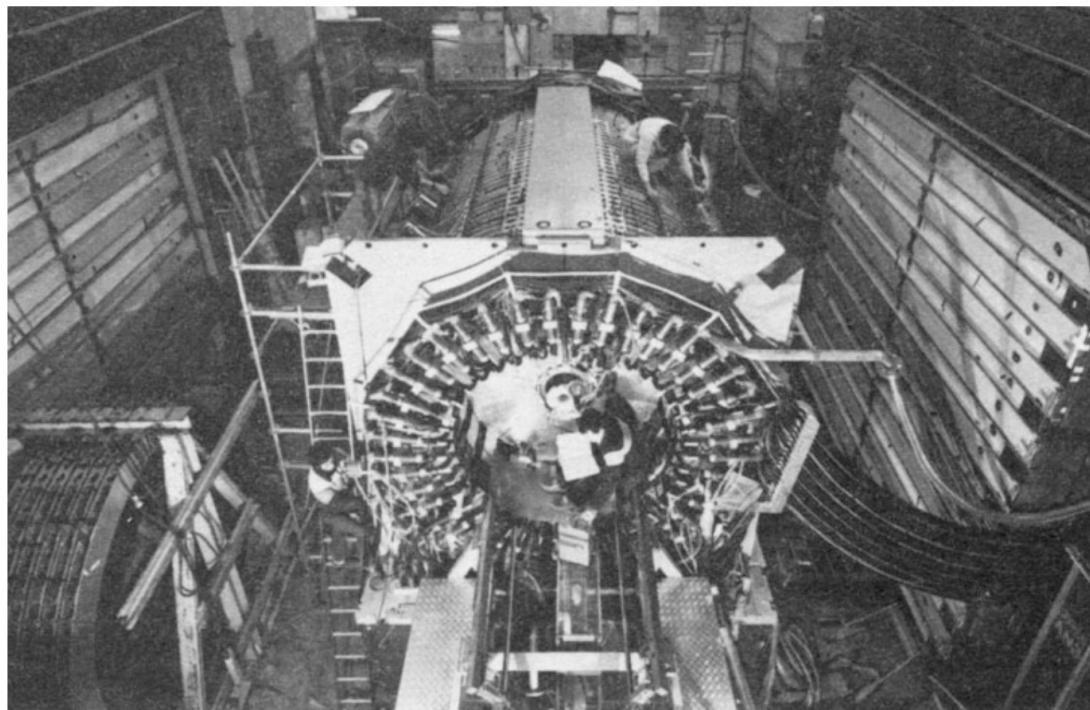
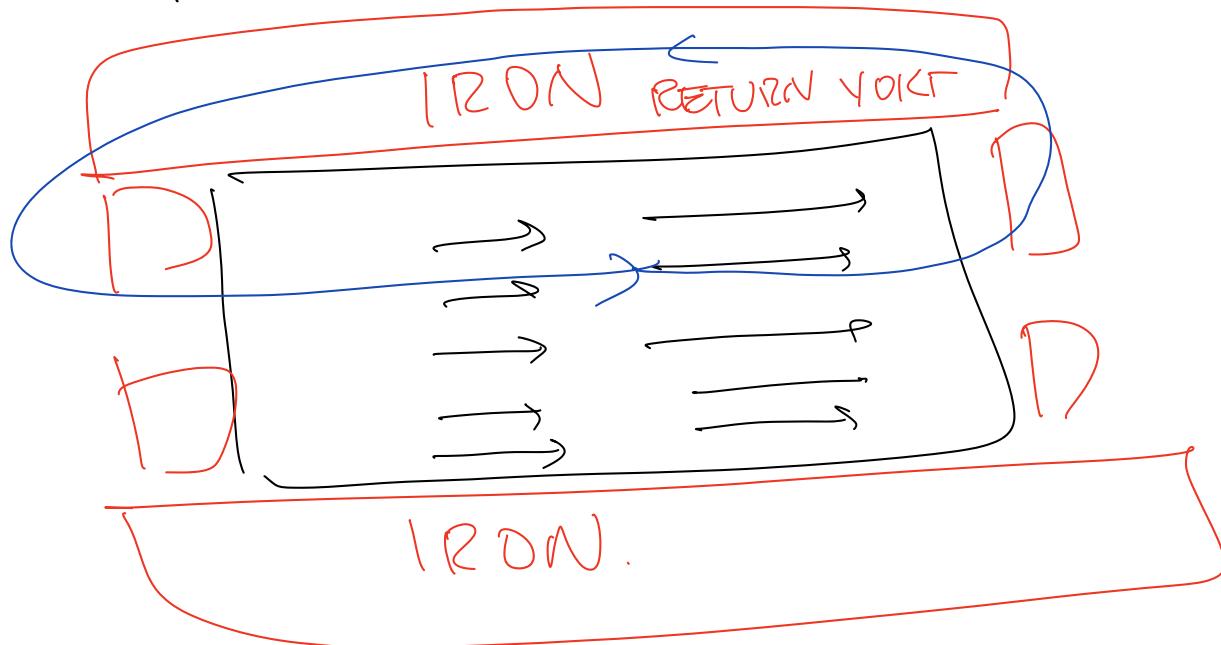
Magnetfeld MAGNETIC FIELD: 0.5 T

Beteiligte Institute PARTICIPANTS

DESY, Hamburg, Heidelberg,  
Lancaster, Manchester,  
Rutherford Lab., Tokio

$$P = \Phi \cdot B \times R[m].$$

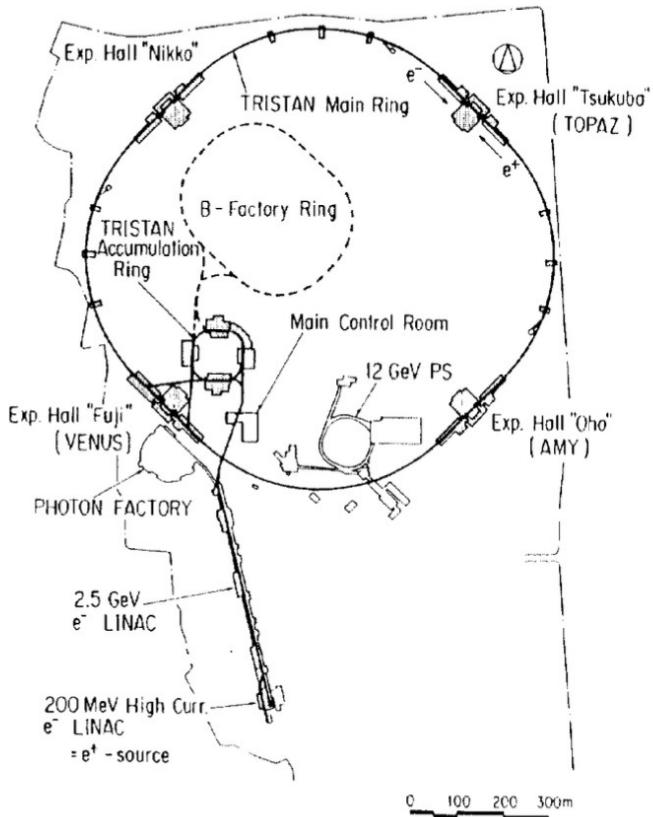
$$B \approx 0.5 \Rightarrow P[\text{GeV}] = 0.15 \times R[m]$$



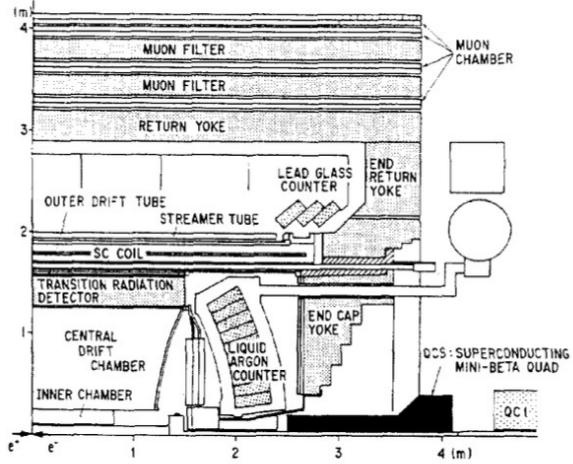
JADE

Experiment

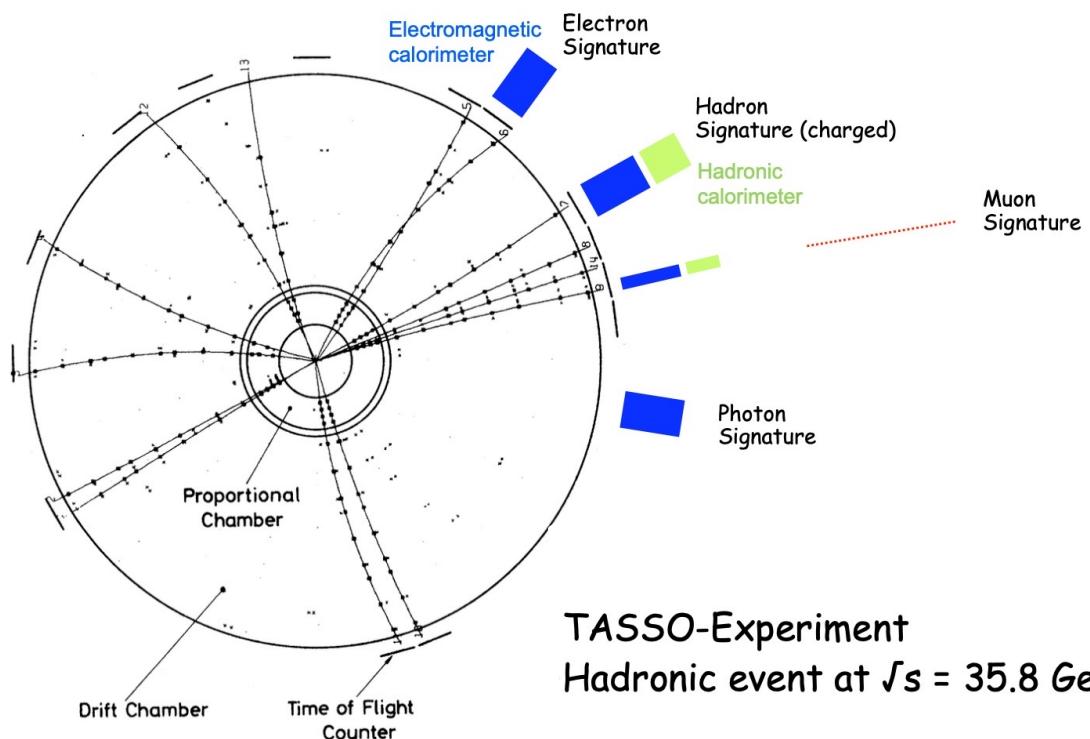
# TRISTAN @ JAPAN



## Schema of the detector



**Max. beam energy: 32 GeV**  
**Injection energy: 8 GeV**  
**Beam lifetime: 5-6 hr.**  
**Peak luminosity:  $1.4 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}$**



**TASSO-Experiment**  
**Hadronic event at  $\sqrt{s} = 35.8 \text{ GeV}$**

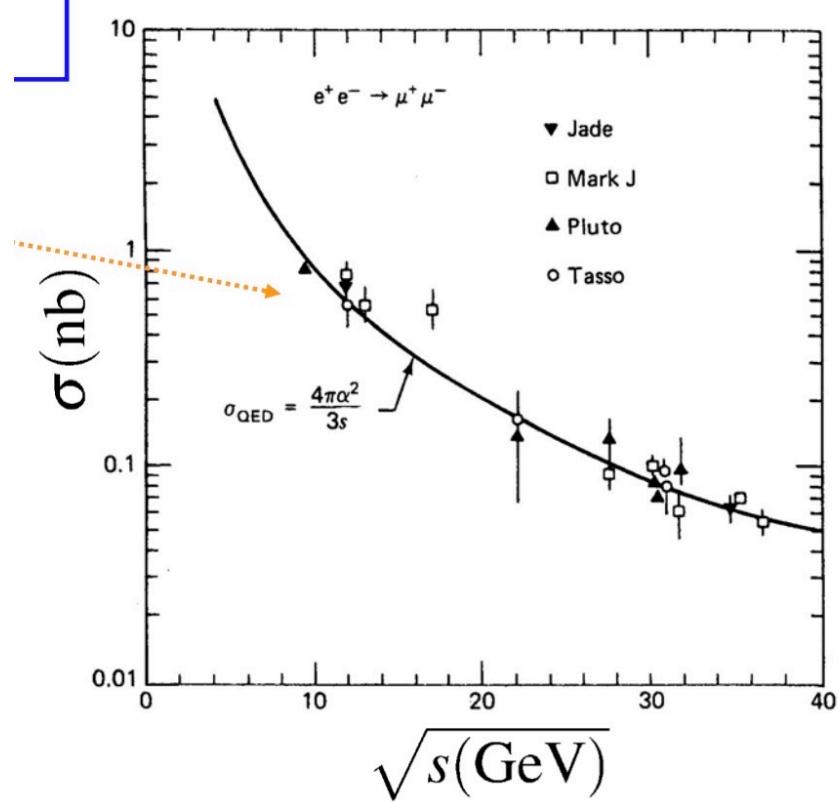
$$e^+ e^- \rightarrow e^+ e^-$$

$$\sigma \approx \frac{86.8 \text{ nb}}{s [\text{GeV}^2]}.$$

$$\sqrt{s} = 10 \text{ GeV}.$$

$$S = 100 \text{ GeV}^2$$

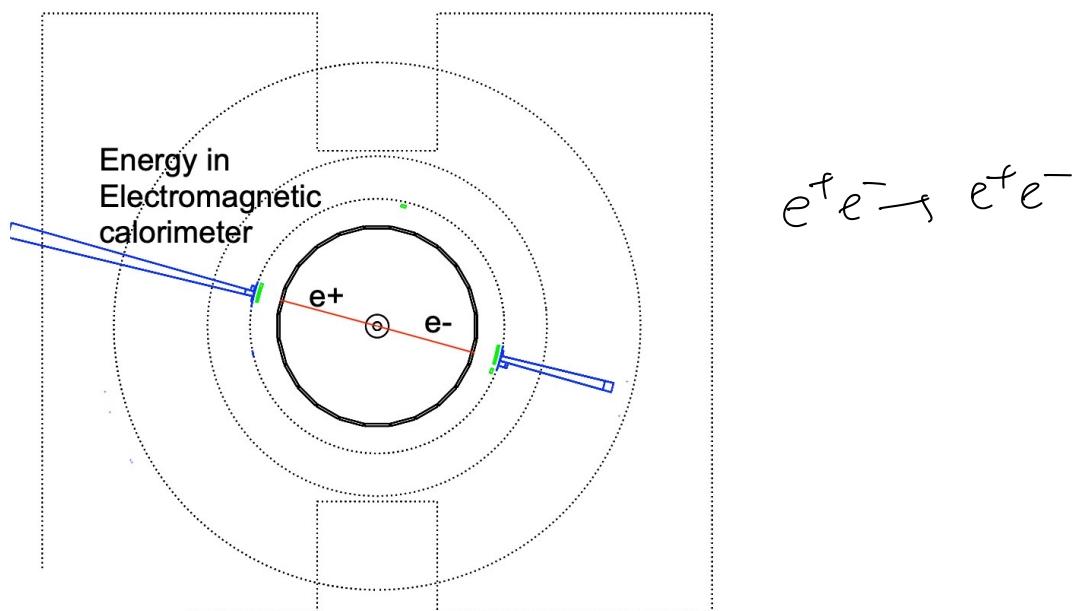
$$\sigma \approx \frac{100 \times 1 \text{ b}}{100}$$



Total cross-section.

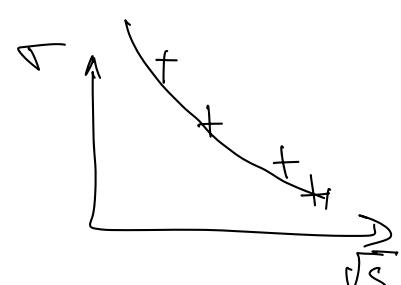
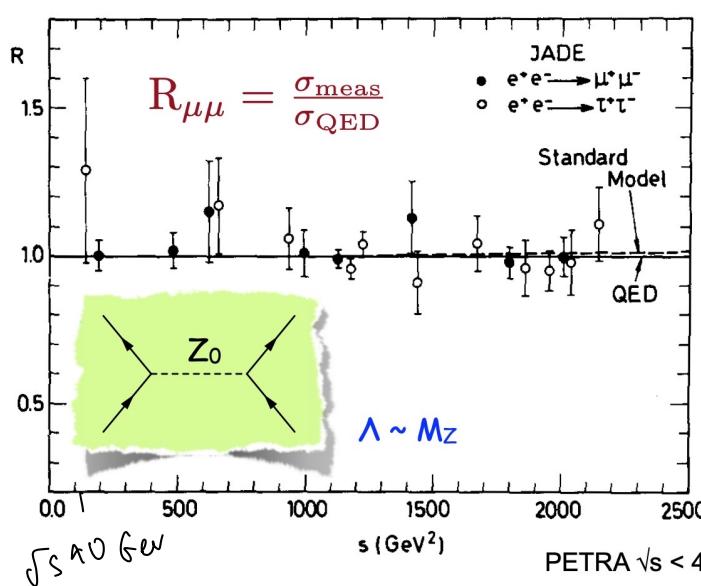
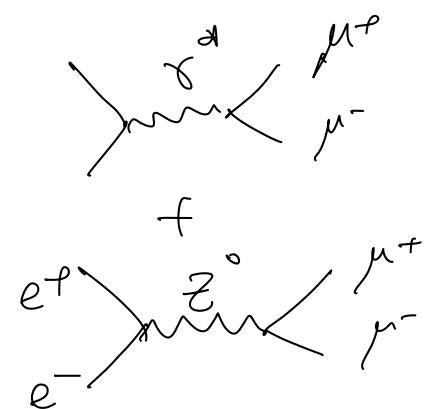
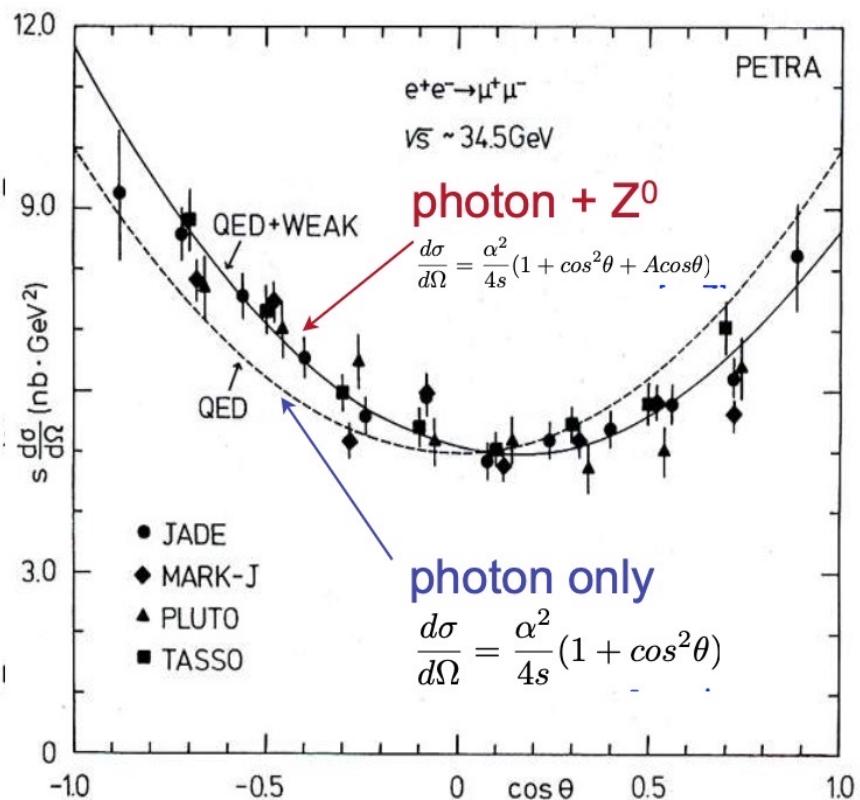
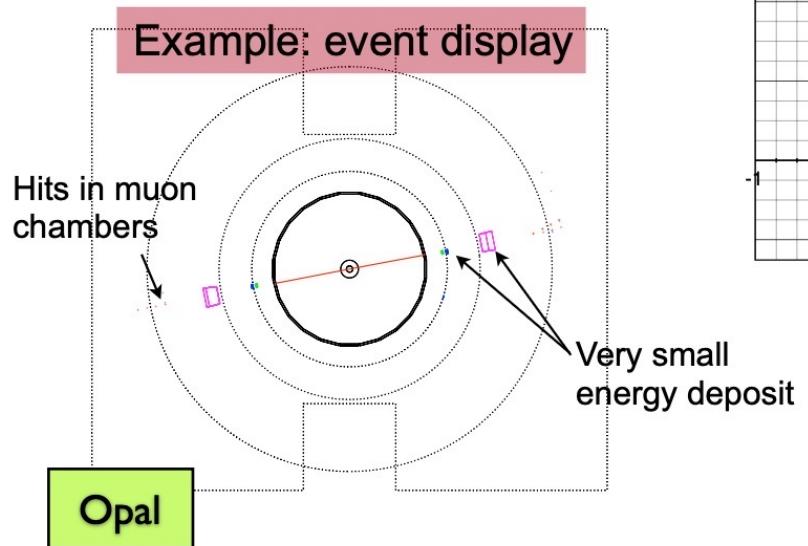
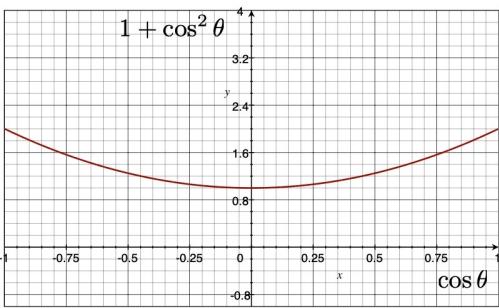
$$N = \sigma_{e^+e^-} L_{\text{inst.}} \Delta t.$$

↳ how many times  $e^+e^-$  in the detector



what about  $\frac{d\sigma}{d\cos\theta} = \frac{1}{2\pi} \frac{d\sigma}{d\cos\theta} \propto (1 + \cos^2\theta)$ .

Differential cross section.



$$e^+ e^- \rightarrow e^+ e^- \text{ (Bhabha scattering)} \quad \sigma_1$$

$$\mu^+ \mu^- \quad \sigma_2$$

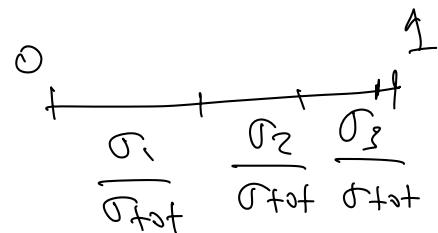
$$\text{hadrons} \quad \sigma_3$$

$$N_{\text{tot}} = (\sigma_{\text{tot}}) \times \text{finst. } \Delta t$$

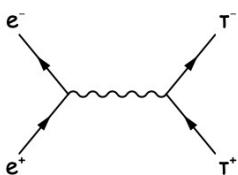
$$\hookrightarrow \sigma_1 + \sigma_2 + \sigma_3$$

$\frac{\sigma_1}{\sigma_{\text{tot}}}$  : fraction of  $e^+ e^-$  events.

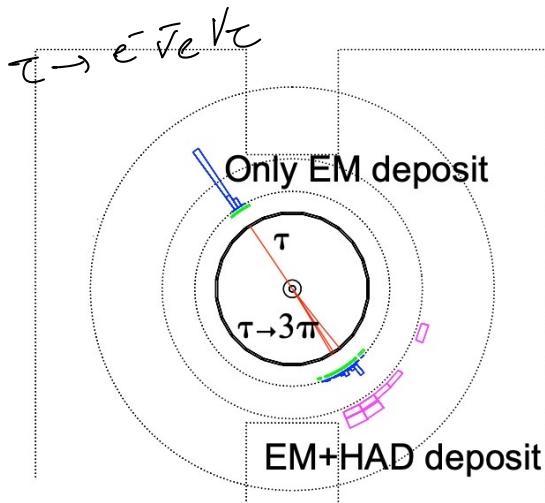
$$\text{If } \sigma_1 \gg \sigma_3$$



$$m_\tau \approx 1.78 \text{ GeV}$$

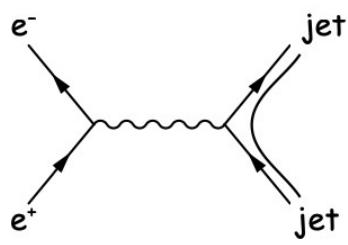


$\tau$  decays



Modes with one charged particle		
$\Gamma_1$	particle $\geq 0$ neutrals $\geq 0 K^0 \nu_\tau$ ("1-prong")	(85.24 ± 0.06) %
$\Gamma_2$	particle $\geq 0$ neutrals $\geq 0 K_L^0 \nu_\tau$	(84.58 ± 0.06) %
$\Gamma_3$	$\mu^- \bar{\nu}_\mu \nu_\tau$	[a] (17.39 ± 0.04) %
$\Gamma_4$	$\mu^- \bar{\nu}_\mu \nu_\tau \gamma$	[b] (3.67 ± 0.08) × 10 <sup>-3</sup>
$\Gamma_5$	$e^- \bar{\nu}_e \nu_\tau$	[a] (17.82 ± 0.04) %
$\Gamma_6$	$e^- \bar{\nu}_e \nu_\tau \gamma$	[b] (1.83 ± 0.05) %
$\Gamma_7$	$h^- \geq 0 K_L^0 \nu_\tau$	(12.03 ± 0.05) %
$\Gamma_8$	$h^- \nu_\tau$	(11.51 ± 0.05) %
$\Gamma_9$	$\pi^- \nu_\tau$	[a] (10.82 ± 0.05) %
$\Gamma_{10}$	$K^- \nu_\tau$	[a] (6.90 ± 0.10) × 10 <sup>-3</sup>
$\Gamma_{11}$	$h^- \geq 1 \text{ neutrals} \nu_\tau$	(37.01 ± 0.09) %
$\Gamma_{12}$	$h^- \geq 1 \pi^0 \nu_\tau \text{ (ex. } K^0)$	(36.51 ± 0.09) %
$\Gamma_{13}$	$h^- \pi^0 \nu_\tau$	(25.93 ± 0.09) %
$\Gamma_{14}$	$\pi^- \pi^0 \nu_\tau$	[a] (25.49 ± 0.09) %
$\Gamma_{15}$	$\pi^- \pi^0 \text{ non-} \rho(770) \nu_\tau$	(3.0 ± 3.2) × 10 <sup>-3</sup>
$\Gamma_{16}$	$K^- \pi^0 \nu_\tau$	[a] (4.33 ± 0.15) × 10 <sup>-3</sup>
$\Gamma_{17}$	$h^- \geq 2 \pi^0 \nu_\tau$	(10.81 ± 0.09) %
$\Gamma_{18}$	$h^- 2 \pi^0 \nu_\tau \text{ (ex. } K^0)$	(9.48 ± 0.10) %
$\Gamma_{19}$	$\pi^- 2 \pi^0 \nu_\tau \text{ (ex. } K^0)$	(9.32 ± 0.10) %
$\Gamma_{20}$	$\pi^- 2 \pi^0 \nu_\tau \text{ (ex. } K^0)$	[a] (9.26 ± 0.10) %
$\Gamma_{21}$	scalar $\pi^- 2 \pi^0 \nu_\tau \text{ (ex. } K^0)$	< 9 × 10 <sup>-3</sup> CL=95%
$\Gamma_{22}$	vector $K^- 2 \pi^0 \nu_\tau \text{ (ex. } K^0)$	< 7 × 10 <sup>-3</sup> CL=95%
$\Gamma_{23}$	vector $K^- 3 \pi^0 \nu_\tau \text{ (ex. } K^0)$	[a] (6.5 ± 2.2) × 10 <sup>-4</sup>
$\Gamma_{24}$	$h^- \geq 3 \pi^0 \nu_\tau$	(1.34 ± 0.07) %
$\Gamma_{25}$	$h^- \geq 3 \pi^0 \nu_\tau \text{ (ex. } K^0)$	(1.25 ± 0.07) %
$\Gamma_{26}$	$h^- 3 \pi^0 \nu_\tau$	(1.18 ± 0.07) %

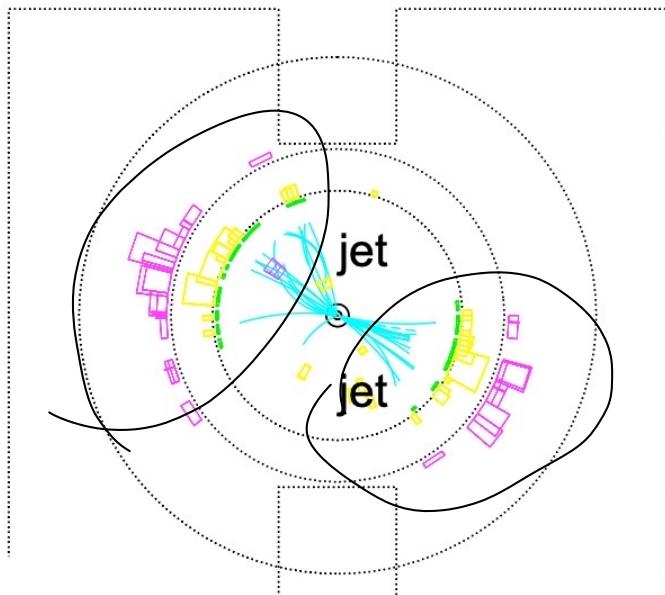
$$\sqrt{s} > 3.6 \text{ GeV}$$



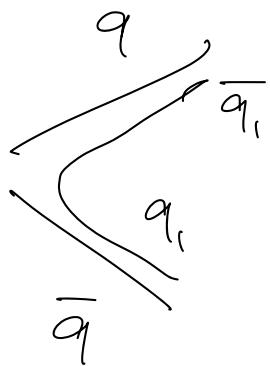
$$e^+e^- \rightarrow q\bar{q}$$

$q \rightarrow$  convert into hadrons.

Hadronization.  
Mesons, baryons.



$$\sqrt{s} \approx 2m_q + \Sigma$$



$$q\bar{q}_1$$

$$\bar{q}q_1$$