$\rho^0 \to \pi^+\pi^-$
$\pi^0\pi^0$

$I \quad I_3$

$|1,0\rangle \quad |1,0\rangle|1,0\rangle$

$1\times 1 = 0+1+2.$

$$\underset{\pi^0}{|1,0\rangle}\underset{\pi^0}{|1,0\rangle} = \sqrt{\tfrac{2}{3}}\,|2,0\rangle - \sqrt{\tfrac{1}{3}}\,|0,0\rangle.$$

$\pi^0\pi^0$ identical particles.

$$1\times 1 = \underset{S}{0} + \underset{A}{1} + \underset{S}{2}$$

Baryons in static quark model

$3_F = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ $\quad u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$3_F \times 3_F \times 3_F = 1_{F,A} + 8_{F,M_{12}} + 8_{F,M_{23}} + 10_{F,S}$$

$\Rightarrow$ total of 27 states.

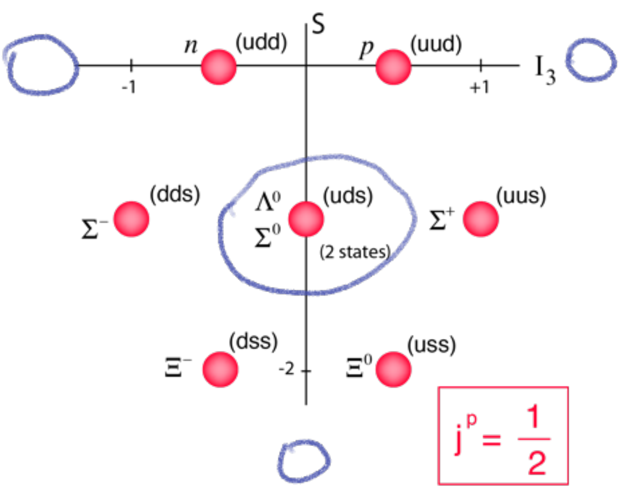
physical states observed are only 18

F,S: symmetric under exchange of any of 3 quarks.

$q_1q_2q_3$ S $\quad 1 \leftrightarrow 2, \quad 2 \leftrightarrow 3, \quad 1 \leftrightarrow 3$

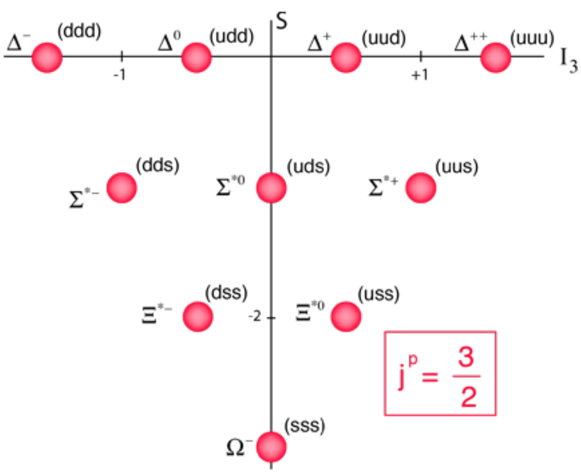
$M_{12} \quad 1 \leftrightarrow 2$

spin: $\Uparrow\Uparrow\Downarrow$ $\qquad L=0.$



$S = 1/2 \Rightarrow J = 1/2 \qquad P = (+1)^3 = +1$

spin: $\Uparrow\Uparrow\Uparrow$ $\qquad L=0.$



$J = 3/2 \qquad P = 1 = (+1)^3$

strangeness

Baryons	qqq	J^P	I	I_3	S	$Q^{(1)}$	mass (MeV)
p, n	uud, udd	½⁺	½	½, -½	0	1, 0	940
Λ	uds	½⁺	0	0	-1	0	1115
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	½⁺	1	1, 0, -1	-1	1, 0, -1	1190
Ξ^0, Ξ^-	uss, dss	½⁺	½	½,-½	-2	1, 0	1320
$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	uuu, uud, udd, ddd	$^3/_2{}^+$	$^3/_2$	$^3/_2, ^1/_2, -^1/_2, -^3/_2$	0	2, 1, 0, -1	1230
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	$^3/_2{}^+$	1	1, 0, -1	-1	1, 0, -1	1385
Ξ^{*0}, Ξ^{*-}	uss, dss	$^3/_2{}^+$	½	½,-½	-2	1, 0	1530
Ω^-	sss	$^3/_2{}^+$	0	0	-3	-1	1670

~150 MeV

150 MeV

1962 observation of Ξ^* but not Ω^-

Gell-Mann and Ne'eman predicted Ω^- with mass of 1680 MeV.

$$M_{\Omega^-} \simeq M_{\Xi^*} + 150\ \text{MeV}.$$

Ω^-: $S=-3$ $\qquad Q = I_3 + \frac{B+S}{2} = 0 + \frac{1-3}{2} = -1$

also predicted life time $\tau \simeq 10^{-10}$ s long (weak not strong delay)

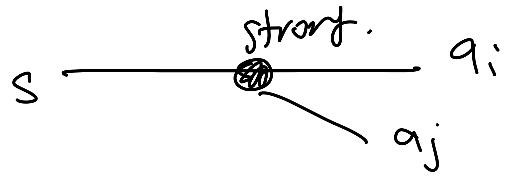
strong

$\Omega^- \longrightarrow X + Y$ $\qquad$ Qvalue $= m_\Omega - m_X - m_Y > 0$

B $\quad$ 1 $\quad$ 1 $\quad$ 0 (meson)

S $\quad$ ~~-3 $\quad$ -2 $\quad$ -1~~

~~-3 $\quad$ -3 $\quad$ 0~~ $\quad$ No other $S=-3$ particle



EM:

$\Omega^- \longrightarrow X^- + \gamma$

S $\quad$ ~~-3 $\quad$ -3~~ $\quad$ 0 $\qquad$ No $X_{S=-3}$

$\Rightarrow$ only weak decays

$\Omega^- \longrightarrow X + Y$

S $\quad$ -3 $\quad$ -2 $\quad$ 0 $\qquad$ $\Delta S = 1$ $\quad$ weak decays.

-3 $\quad$ -1 $\quad$ -1

In principle: $\Omega^- \longrightarrow X + Y$

$S \quad -3 \qquad -2 \qquad -1$

$\Xi^{*-} + \bar{K}^0$ $\qquad \Xi^{*-} = dss$ $\qquad \bar{K}^0 = s\bar{d}$

$S \quad -3 \qquad -2 \qquad -1$

mass $\quad 1680 \qquad 1530 \qquad 500$ MeV

$\Delta S = 0.$ $\qquad$ Q-value < 0

$\Rightarrow$ look at $\Delta S = 1$ decays $\Rightarrow$ weak $\Rightarrow \tau \simeq 10^{-10}$ s.

$\Omega^- \to X + Y$

$S \quad -3 \qquad -2 \qquad 0$

$\Xi^- \quad \pi^0$

$\Xi^0 \quad \pi^-$

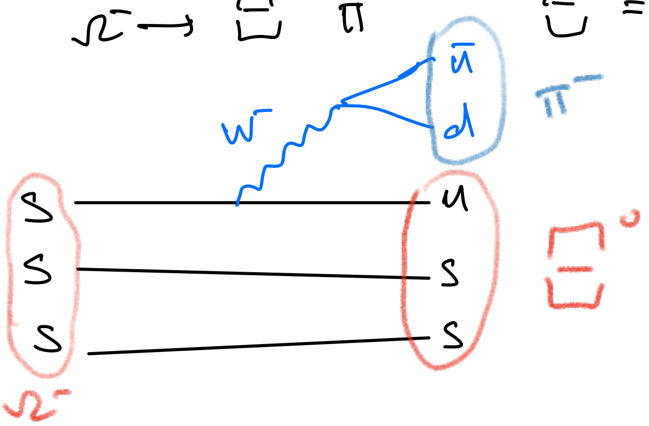
$B \quad 1 \qquad 1 \qquad 0$

mass $\quad 1680 \qquad 1320 \qquad 140 \quad \Rightarrow \quad m_\Omega - m_\Xi - m_\pi = 220$ MeV. ✓

$\Lambda^0 \quad K^-$ $\qquad \Lambda^0 = uds$ $\qquad K^- = s\bar{u}$

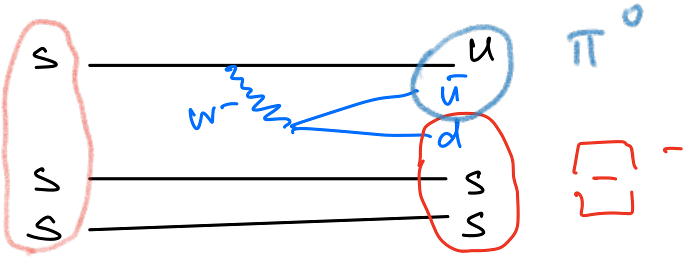
mass $\quad 1680 \qquad 1115 \qquad 500$ $\qquad$ Qvalue $= 65$ MeV. ✓

$\Omega^- \to \Xi^0 \pi^-$ $\qquad \Xi^0 = uss$ $\qquad Q = +\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$



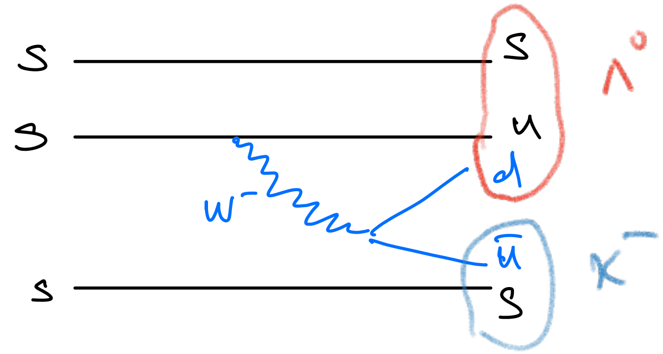
spectator diagram.

$BF(\Omega^- \to \Xi^0 \pi^-) = 24\%$



Internal diagram

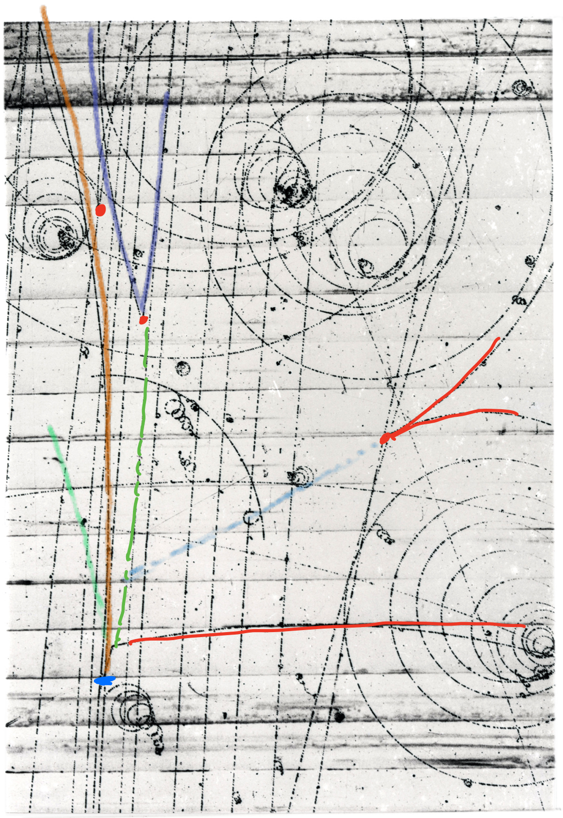
$BF(\Omega^- \to \Xi^- \pi^0) = 9\%$

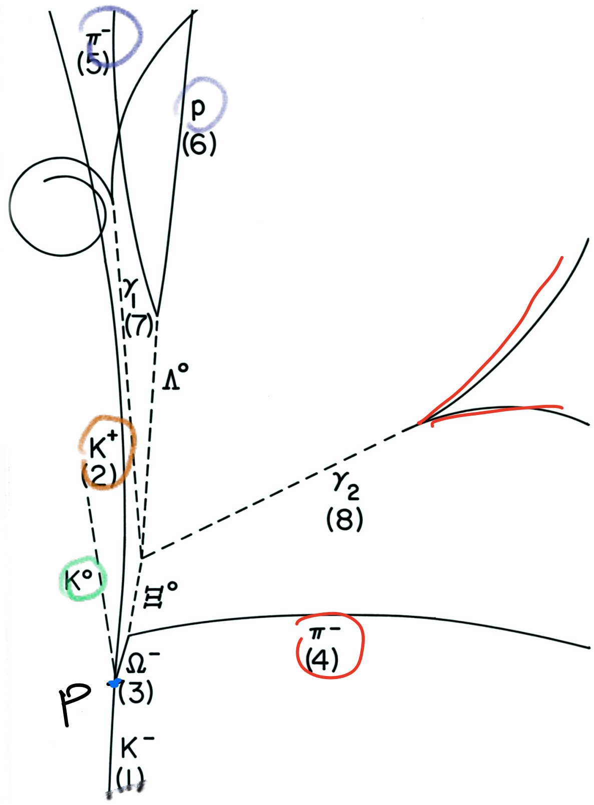


$BF(\Omega^- \to \Lambda K^-) = 68\%$

Experimental proof @ Brookhaven National Lab.
Nick Samios using bubble chambers

	K^-	$+\ p$	$\longrightarrow$	Ω^-	$+\ K^+$	$+\ K^0$
S	-1	0		-3	$+1$	$+1$
B	0	1		1	0	0





$p + p \longrightarrow p + p + K^- + K^+ \implies K^+$ beam.

$K^- + p \longrightarrow \Omega^- + K^+ + K^0$

$\hookrightarrow \Xi^0 + \pi^-$

$\hookrightarrow \Lambda^0 + \pi^0 \hookrightarrow \gamma\gamma$

$\gamma \hookrightarrow e^+e^-$ (conversion)

$\gamma \hookrightarrow e^+e^-$ (conversion)

$\Lambda^0 \hookrightarrow p\pi^-$

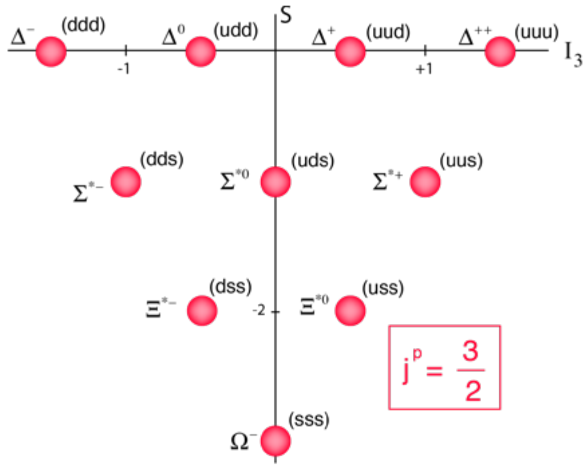
Evidence for $\Omega^- \implies SU(3)_F$ works.

But remember: 18 physical states but 27 potential baryons

$\Delta^{++} = uuu$
$\Uparrow\Uparrow\Uparrow$

$\Delta^{-} = d\ d\ d$
$\Uparrow\ \Uparrow\ \Uparrow$

$\Omega^{-} = s\ s\ s$
$\Uparrow\ \Uparrow\ \Uparrow$



L=0.
S=3/2.

$\psi_{\Delta^{++}} = \psi_{space}\ \psi_{spin}\ \psi_{Flavor}.$

$(-1)^L \quad S=3/2 \quad 1\leftrightarrow 3,\ 1\leftrightarrow 2,\ 2\leftrightarrow 3$

S S S

Δ^{++} fermion because $S=3/2$

$\Rightarrow$ $\psi_{\Delta^{++}}$ must be antisymmetric.

$\psi_{\Delta^{++}} = \psi_{space}\ \psi_{spin}\ \psi_{Flavor}\ \psi_{color}.$

S S S A

Δ^{++} $\Uparrow u_B$ $\Uparrow u_B$ $\Uparrow u_B$

1 color not sufficient.

$\Uparrow u_B$ $\Uparrow u_R$ $\Uparrow u_B$

still symmetric under $1 \leftrightarrow 3$

$\Rightarrow$ at least 3 colors.

$\Uparrow u_B$ $\Uparrow u_R$ $\Uparrow u_G$

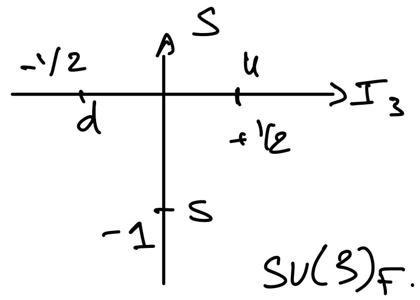
$\psi_{color} = [RBG + BGR + GRB - BRG - RGB - GBR]\ 1/\sqrt{6}$

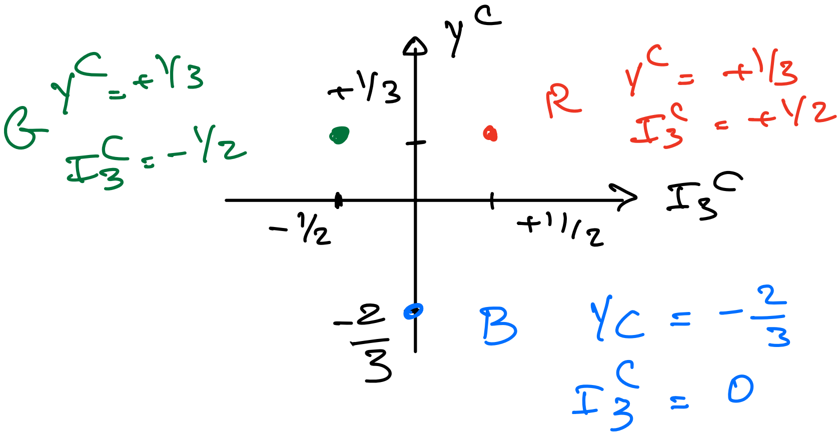
⇒ Introduce $SU(3)_{color}$

8 generators of whic 2 are diagonal

Y^C : color hypercharge

I_3^C : color isospin.



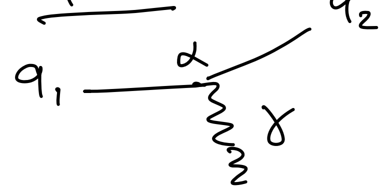


$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

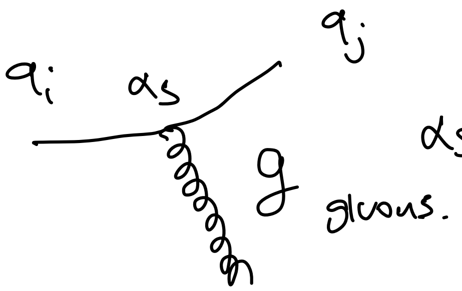
Interaction of color: Quantum Chromo Dynamics (QCD)

Color: new kind of charge for quarks.

QED



$\alpha = \frac{e^2}{4\pi}$



$\alpha_s = \frac{g^2}{4\pi}$

$q = -e$ electron

$q = +e$ positron.

$Q_e(\gamma) = 0$

8 gluons: massless.

$Q_e = 0$

vector bosons $S = 1$

carry color charge.

Hypothesis / Conjucture:

physical particles in nature are colorless (color singlets)

q : 3 colors R, G, B

$\bar{q}$: 3 anticolors $\bar{R}, \bar{B}, \bar{G}$

Experimental challenge: 1) Quarks exist?

2) color exists?

3) all physical states are colorless?

Baryon wave functions

$B = q_1 q_2 q_3$ $\qquad \psi_B = \psi_{space}\, \psi_{spin}\, \psi_{flav.}\, \psi_{color.}$

$\qquad\qquad\qquad\qquad\qquad\qquad S \qquad S \qquad S_F \qquad A$

ground state $L=0 \Rightarrow \psi_{space} \equiv$ symm.

spin: $S = 3/2$ $\Uparrow\Uparrow\Uparrow$ $\qquad \psi^{S=3/2}_{spin}$ symm.

$$3_F \times 3_F \times 3_F = 10_{F,S} + 8_{F,M12} + 8_{F,M23} + 1_{F,A}.$$

$$3_C \times 3_C \times 3_C = 10_{C,S} + 8_{C,M12} + 8_{C,M23} + 1_{C,A}$$

If hypothesis of color singlet $\Rightarrow 1_{C,A}$

Colorless physical states.

$$1_{C,A} = \frac{1}{\sqrt{6}}\left[RGB + GBR + BRG + -GRB - RBG - BGR\right]$$

$$\Rightarrow \psi_{Flavor} = \psi_{F,\,symm.}$$

$\Rightarrow \psi_{F,10S} \Rightarrow$ exists.

Decuplet $\quad \psi_{10\,decup} = \psi_{space}\, \psi^{3/2}_{spin}\, \psi_{F,10,S.}\, \psi_{color}$

$\qquad\qquad\qquad\qquad S \qquad S \qquad S \qquad A$

Flavor singlet: $\psi_1 = \psi_{space}\, \psi_{spin}\, \psi_{F,1,A}\, \psi_{color}$

$\qquad\qquad\qquad\qquad S \qquad S \qquad A \qquad A$

$\Rightarrow 1_{F,A}$ cannot exist.