$\underline{P}$ four-vector $\quad \underline{P} = (E, \vec{p})$ $\quad E^2 = \vec{p}^2 + m^2$

$|\vec{p}| \equiv p$ magnitude of three-vector. $\quad g^{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$

natural units: $\hbar = 1 = c$ $\quad v = \beta c$

$[T] = [L] = E^{-1}$

$\beta = \dfrac{|\vec{p}|}{E}$ $\quad \gamma = \dfrac{E}{m}$ $\quad \beta\gamma = \dfrac{|\vec{p}|}{m}$

$1\ \text{MeV} = 10^6\ \text{eV}$

$\hbar c = 200\ \text{MeV} \times \text{fm}$

$1\ \text{fm} = 10^{-15}\ \text{m}$

$1\ \text{eV} = ?$ Joule

$q = e$ $\quad \Delta E = q\,\Delta V = 1\ \text{eV}$

$\Delta V = 1\,V$

$e = 1.6 \times 10^{-19}\ C$ $\quad \Rightarrow \quad 1\ \text{eV} = 1.6\ 10^{-19}\ J$

1/ Decay

2/ collisions

Decay: $\quad a \to \underbrace{b + c + \cdots + z}_{1 \cdots n \text{ particles}}$

$a \to b$ ✗ En. conservation.

$a \to b + c.$

$a \to b + c + d$

$a \to b + c + \cdots$ $\quad n \geq 2$ bodies.

$E^2 = p^2 + m^2 \Rightarrow$ creation / annihilation operators

$n \to p\, e^- \bar{\nu}_e$ weak

$X^A_Z \to Y^{A-4}_{Z-2} + \alpha$ $\alpha \equiv He^4_2$

$K^0 \to \pi^+ \pi^-$

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$\pi^0 \to \gamma\gamma$

Initial sample of N_0 particles

$$N(t) = N_0 e^{-\Gamma t}$$ # particles at t

$$\frac{N(t)}{N_0} < 1$$ survival probability

$[\Gamma] = T^{-1} = E$ in natural units

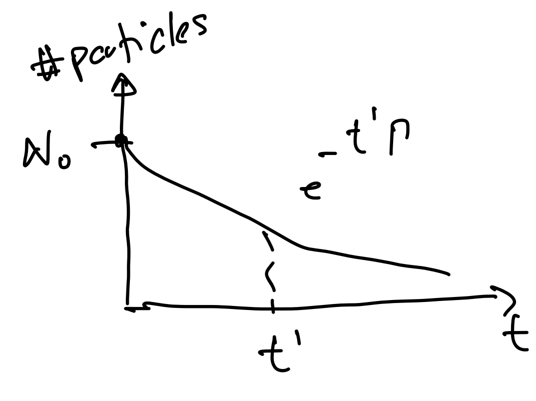
Γ: probability of decay per unit time

$$N(t) = N_0 e^{-t/\tau} \qquad \tau \equiv \Gamma^{-1}$$

τ: mean life time. sec for exp. measurements.

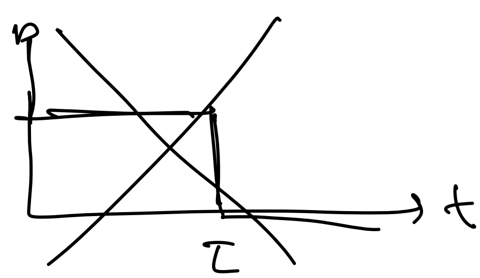
Γ: decay width in MeV

$$\tau \equiv \langle t \rangle \frac{\int_0^\infty e^{-\Gamma t}\, t\, dt}{\int_0^\infty e^{-\Gamma t}\, dt} = \frac{1}{\Gamma}$$



$\tau_\mu = 2.2 \times 10^{-6}$ s

$t = 3 \times 10^{-6}$ s



$a \to b + c$ can it happen? Q-value.

In reference of a: $\vec{P}_a = 0$

$$E_a = E_b + E_c = m_b + K_b + m_c + K_c$$
$$E_a = m_a$$

↳ Definition of kinetic energy in relatistic kin.

Cons. Energy $\quad m_a = m_b + m_c + K_b + K_c$.

In the limit $K_b, K_c \to 0 \Rightarrow m_a \geq m_b + m_c$

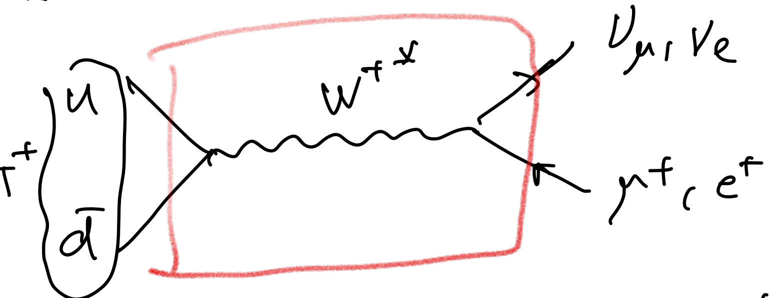
$Q =: m_a - m_b - m_c \geq 0$ for decay to happen.

More in general $\quad a \to b + c + \dots + z$.

$$Q = m_a - m_b - m_c \dots - m_z$$

$\pi^+ \to \mu^+ \nu_\mu$

$\quad\;\; e^+ \nu_e$



$\pi^+ \to \mu^+ + \nu_\mu \quad Q = m_\pi - m_\mu = 140 - 106 \simeq 34$ MeV.

$\pi^+ \to e^+ + \nu_e \quad Q = m_\pi - m_e = 139.5$ MeV

$\pi^0 \to \mu^+ \mu^- \quad Q < 0$

How to estimate Γ, or τ?

$$\Gamma(i \to f) = 2\pi |M|^2 \rho(E)\Big|_{E_f = E_i}$$

Fermi's 2nd Golden Rule.

H_I interaction hamiltonian.

Density of states.

$$M = \langle f | H_I | i \rangle$$

Relativistic Golden Rule

transition: $|i\rangle = |1\rangle$

$1 \longrightarrow 2+3+ \cdots + n.$ $\quad \mathcal{M}_{fi} = |\langle f|H|i\rangle|$ $\quad |f\rangle = |2,3,\ldots n\rangle$

$$\Gamma = \underbrace{S}_{\text{statistical factor}} \frac{1}{2m_1} \int |\mathcal{M}_{fi}|^2 \, \delta^{(4)}(\underline{p}_1 - \underline{p}_2 - \underline{p}_3 - \cdots - \underline{p}_n) \times$$

$$\times \prod_{j=2}^{n} (2\pi) \underbrace{\delta(\underline{p}_j^2 - m_j^2)\,\theta(E_j)}_{\text{j-th particle is on shell}} \frac{d^4 \underline{p}_j}{(2\pi)^4}$$

$a \longrightarrow b+b+ccc$

$$S = \frac{1}{N_b!} \frac{1}{N_c!} = \frac{1}{2!} \frac{1}{3!} = \frac{1}{2} \frac{1}{6} = \frac{1}{12}$$

$K^0 \longrightarrow \pi^0 \pi^0 \pi^0 \qquad S = \frac{1}{3!} = \frac{1}{6}$

$K^0 \longrightarrow \pi^+ \pi^- \qquad S = 1$

$\pi^+ \longrightarrow \mu^+ \nu_\mu \qquad S = 1$

$n \longrightarrow p e^- \bar{\nu}_e \qquad S = 1$

Two-body Decay

$a \longrightarrow b+c \qquad S = 1$

$$\Gamma = \frac{1}{2m_a} \int |\mathcal{M}|^2 (2\pi)^4 \, \delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c) \times$$

$$\times \prod_{j=b,c} (2\pi) \, \delta(\underline{p}_j^2 - m_j^2)\,\theta(E_j) \frac{d^4 p_j}{(2\pi)^4}$$

$$\delta(\underline{p}_j^2 - m_j^2)\,\theta(E_j) = \delta(p$$

$$\hookrightarrow E_j^2 - |\vec{p}_j|^2 - m_j^2 = E_j^2 - (\vec{p}_j^2 + m_j^2)$$

$\delta(f(x)) = \qquad x_0, x_1 : \; f(x_0) = f(x_1) = 0.$

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x)|\big|_{x=x_i}}$$

$$f(E) = E^2 - (P^2 + m^2)$$

$$f'(E) = 2E$$

$$\delta(E^2 - (P^2 + m^2)) =$$

$$E = \pm\sqrt{P^2 + m^2}$$

$$\delta(P_j^2 - m_j^2)\,\theta(E_j) = \delta\left(E_j - \sqrt{p_j^2 + m_j^2}\right) = \frac{\delta(E - E_j)}{2E_j}$$

$$= \frac{\delta(E - E_j)}{2\sqrt{p_j^2 + m_j^2}} \qquad \frac{d^4 P_j}{(2\pi)^4} = \frac{dE_j\, d^3\vec{P}_j}{(2\pi)^4}$$

$$\Gamma_{a\to b+c} = \underbrace{\frac{1}{2m_a}}_{\frac{1}{8(2\pi)^2}} \int |M|^2\, \delta^4(P_a - P_b - P_c)\, \frac{1}{4\sqrt{P_b^2 + m_b^2}\sqrt{P_c^2 + m_c^2}}\, d^3\vec{P}_b\, d^3\vec{P}_c$$

$$\delta^4(P_a - P_b - P_c) = \delta(E_a - E_b - E_c)\,\delta^3(\vec{P}_a - \vec{P}_b - \vec{P}_c)$$

In the rest frame of a: $\vec{P}_b \longleftarrow \cdot \longrightarrow \vec{P}_c$ $\quad P_a = 0.$

$$P_a = 0 \Rightarrow \delta(\vec{P}_b + \vec{P}_c) \Rightarrow \vec{P}_b = -\vec{P}_c$$

$$\delta(E_a - E_b - E_c) = \delta(m_a - E_b - E_c)$$

$$|\vec{P}_b| = |\vec{P}_c| = P$$

$$\Gamma = \frac{1}{32\pi^2}\frac{1}{m_a}\int |M|^2 \frac{\delta(m_a - E_b - E_c)}{\sqrt{p^2 + m_b^2}\sqrt{p^2 + m_c^2}}\, d^3\vec{P}$$

$b \longleftarrow \overset{a}{\cdot} \longrightarrow c$

$$|\vec{P}_b| = |\vec{P}_c| = P$$

$$u = E_b + E_c = \sqrt{p^2 + m_b^2} + \sqrt{p^2 + m_c^2}.$$

$$\frac{du}{dP} = \frac{\cancel{2}P}{\cancel{2}\sqrt{--}} + \frac{\cancel{2}P}{\cancel{2}\sqrt{--}} = \frac{\cancel{2}P\left(\sqrt{--} + \sqrt{--}\right)}{\cancel{2}\sqrt{--}\,\sqrt{--}} = \frac{Pu}{\sqrt{p^2 + m_b^2}\sqrt{p^2 + m_c^2}}.$$

$$\int |\mathcal{M}|^2 \, \delta(m_a - u) \, \frac{1}{u} \frac{du}{dP} \, P \, dP.$$

$$= \int |\mathcal{M}|^2 \, \delta(m_a - u) \, \frac{1}{u} P \, du.$$

$$= |\mathcal{M}|^2 \frac{1}{m_a} P$$

$$b \longleftarrow a \xrightarrow[|\vec{P}|]{} c$$

$$\Gamma = \frac{1}{8\pi} \frac{1}{m_a} \frac{1}{m_a} |\mathcal{M}|^2 \, |\vec{P}|$$

phase space.

$\pi^+ \to e^+ \nu_e$

$\pi^+ \longrightarrow \mu^+ \nu_\mu.$

$$\frac{\Gamma(\pi^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to e^+ \nu_e)} = \frac{|\vec{P}_\mu|}{|\vec{P}_e|} \frac{|\mathcal{M}_{\pi\mu}|^2}{|\mathcal{M}_{\pi e}|^2}$$

$Q(\pi^+ \to \mu^+ \nu_\mu) = 34$ MeV.

$Q(\pi^+ \to e^+ \nu_e) = 139.5$ MeV.

Exercise: $a \longrightarrow b + c + d$