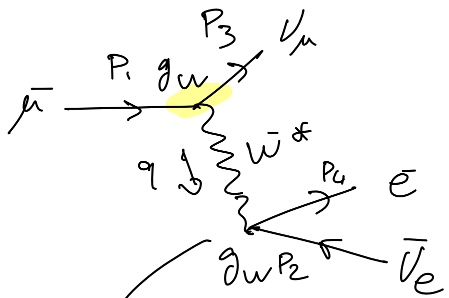
weak interaction

$g_w\, i\gamma^\mu(1-\gamma^5)$

Charged Weak Current

Muon Decay

$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$



$\vec{P}_1 = \vec{P}_3 + \vec{q}$

$$\mathcal{M} \simeq \frac{g_w^2}{q^2 - M_W^2} \times$$

$$\times \left[\bar{u}(P_3)\gamma^\mu(1-\gamma^5)u(P_1)\right] \times$$

$$\times \left[\bar{u}(P_4)\gamma_\mu(1-\gamma^5)u(P_2)\right]$$

$$\frac{1}{q^2 - M_W^2}$$

$q^2 \ll M_W^2$

$q_{max} \simeq m_\mu = 100$ MeV

$M_W = 80$ GeV

$\langle|\mathcal{M}|^2\rangle = \sum \sum$

sum over spins in final state

average over initial state spins.

Final state: ν_μ 1 spin state $\overleftarrow{\longrightarrow}$

e^- 2 spin states $\overleftarrow{\longrightarrow}$ OR $\overrightarrow{\longrightarrow}$ $m_e \neq 0$

$\bar{\nu}_e$ 1 spin state $\overrightarrow{\longrightarrow}$

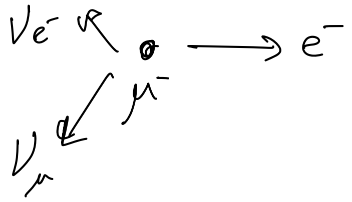
Initial state: μ^- 2 spin states $\overrightarrow{\longrightarrow}$ $\overleftarrow{\longrightarrow}$ $m_\mu \neq 0$.

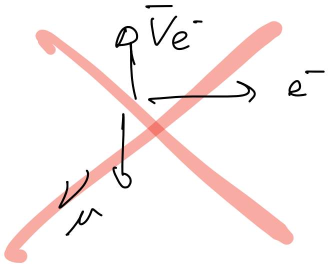
$\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) \propto |\mathcal{M}|^2\, \rho_{\text{phase space}}$

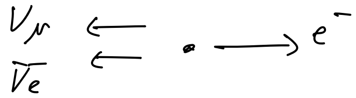
$\Gamma \propto \Big(\underbrace{\frac{g_w^2}{M_W^2}}_{}\Big)^2 (\cdot\,-) \text{ (phase space)}$

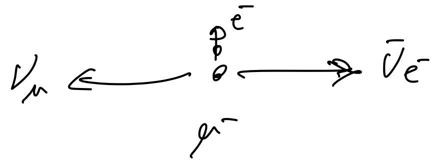
$\Gamma \propto G_F^2 \text{ (phase space)}$

μ^- rest frame.









$E_e \simeq P_{e^-\,max} \simeq \frac{1}{2} m_\mu.$

$P_{e^-} \simeq 0.$

min energy/mom for e^-

$$\rho_{\text{phase space}} = \frac{d^3 P_{\bar{\nu}_e}}{(2\pi)^3} \frac{d^3 P_{\nu_\mu}}{(2\pi)^3} \frac{d^3 P_{e^-}}{(2\pi)^3} \delta^4(\underline{P}_\mu - \underline{P}_{e^-} - \underline{P}_{\bar{\nu}_e} - \underline{P}_{\nu_\mu})$$

$$\Gamma = G_F^2 A$$

$$[\Gamma] = E \qquad [G_F^2] = E^{-4} \Rightarrow [A] = E^5$$

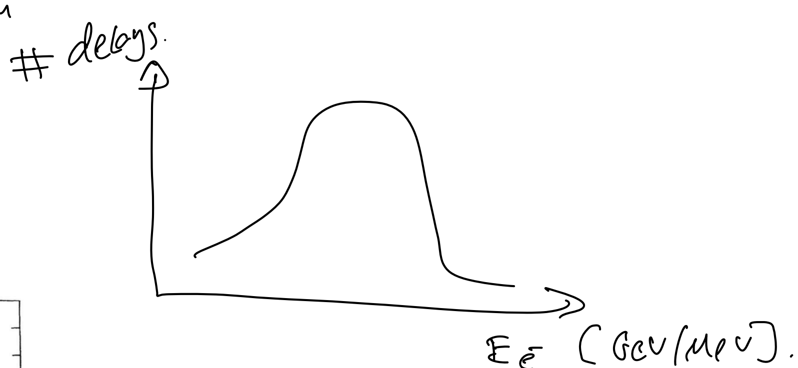
Griffiths 9.2

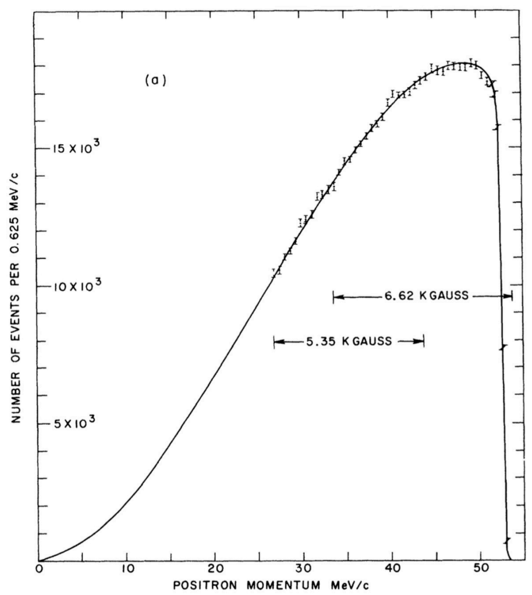
$$\Gamma \propto G_F^2 m_\mu^5$$

From calculation

$$\Gamma = \frac{1}{192\pi^3} G_F^2 m_\mu^5$$

Compute $\frac{d\Gamma}{dE}$





$$\Gamma = \frac{1}{\tau} \propto G_F^2 m_\mu^5$$

measure τ.

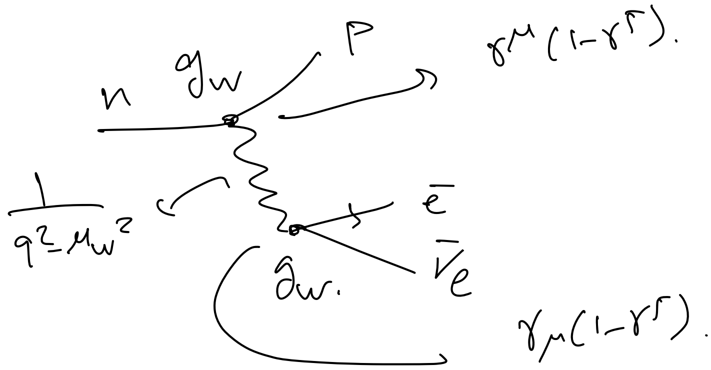
measure m_μ $\Rightarrow$ precesive measurement of $G_F = \frac{g_W^2}{M_W^2}$.

μ MASS	$105.6583755 \pm 0.0000023$ MeV
μ MEAN LIFE τ	$(2.1969811 \pm 0.0000022) \times 10^{-6}$ s
$\tau_{\mu^+}/\tau_{\mu^-}$ MEAN LIFE RATIO	1.00002 ± 0.00008
$(\tau_{\mu^+} - \tau_{\mu^-})/\tau_{\text{average}}$	$(2 \pm 8) \times 10^{-5}$

Neutron Decay

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$.

$n \longrightarrow p\, e^- \bar{\nu}_e$



$q \simeq m_n - m_p - m_e - m_\nu$

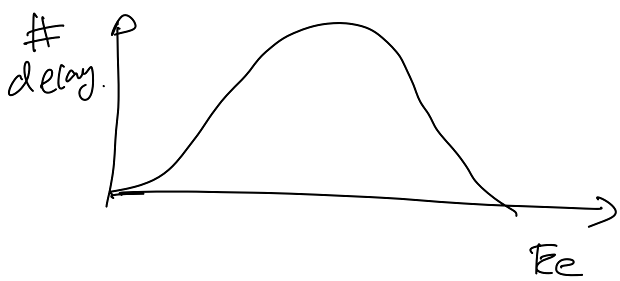
$\simeq 1$ MeV.

$$\Rightarrow \mathcal{M} \simeq \underbrace{\left(\frac{g_W^2}{M_W^2}\right)}_{G_F} \left(\gamma^\mu(1-\gamma^5)\right) \left(\gamma_\mu(1-\gamma^5)\right)$$

$\langle |\mathcal{M}|^2 \rangle$ average over n spins $\Uparrow \Downarrow$.

Sum over $p \Uparrow \Downarrow$ $e^- \Uparrow \Downarrow$ $\bar{\nu}_e \Uparrow$

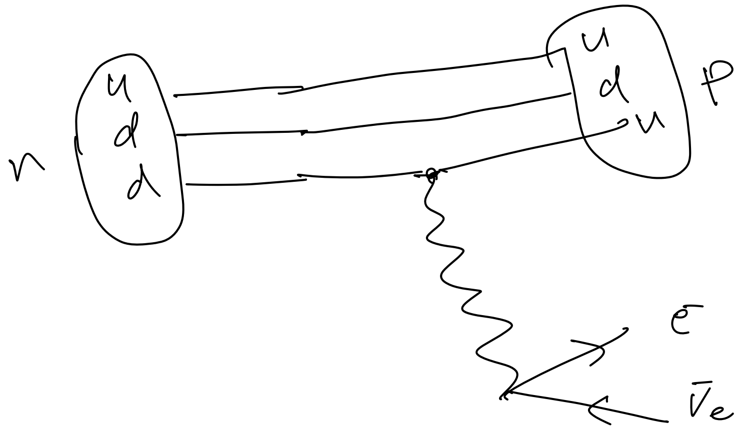
$\frac{d\Gamma}{dE}$



$\Gamma_{tot} = \frac{1}{\tau}$

Exp: $\tau_n = 878.4 \pm 0.5$ s

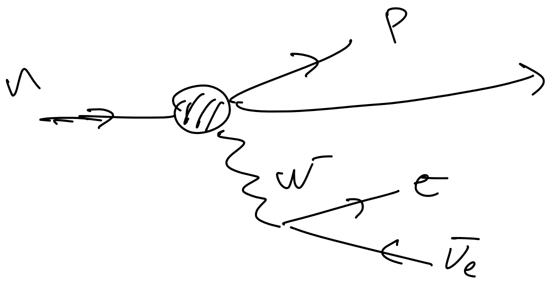
$\Rightarrow \tau_{th} = 1318$ s



weak decay of quark.

- take into account hadron structure.

- do quarks couple to W^- same as leptons?



$\gamma^\mu(C_V + C_A\gamma^5)$ $\quad C_V, C_A$ two arbit. constants.

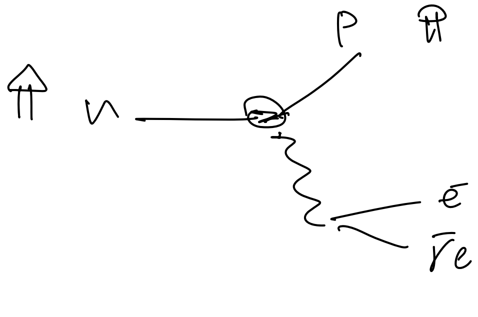
$$\mathcal{M} = \left(\frac{g_W^2}{M_W^2}\right)\left(\bar{u}_p \gamma^\mu (C_V + C_A\gamma^5) u_n\right)\left(\bar{u}_e \gamma_\mu (1-\gamma^5) v_{\bar{\nu}}\right)$$

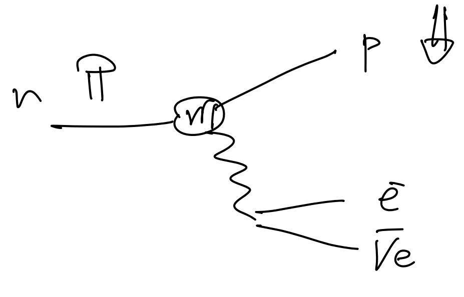
$$\Rightarrow \quad \Gamma = \Gamma(C_V, C_A) \qquad \frac{d\Gamma}{dE} = \frac{d\Gamma}{dE}(E, C_V, C_A)$$

β decays

$${}^A_Z X \to {}^A_{Z-1} Y + e^- + \bar{\nu}_e$$

$${}^A_Z X \to {}^A_{Z+1} Y + e^+ + \nu_e$$





Fermi transitions. $\qquad$ Gamow-Teller Transitions.

C_V $\qquad$ C_V, C_A

one import β decay for C_V.

$${}^{14}O \to {}^{14}N + e^+ + \nu_e$$

Today $\quad C_V = 1.000 \quad$ measured experimentally.

β decays. $|C_A| = 1.267 \quad$ comes from β decays G-T mixture of C_V, C_A.

neutron decays

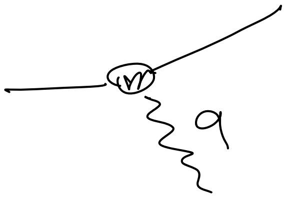
$C_V = 1 \Rightarrow$ CVC: Conserved Vector Current

$C_A \simeq 1 \Rightarrow$ PCAC: partially conserved Axial Current.. (being abandoned)

Determine sign(C_A) from polarized neutron decay.

$$C_A = -1.267.$$

β decay: $\quad \gamma^\mu (1 - 1.267\gamma^5)$

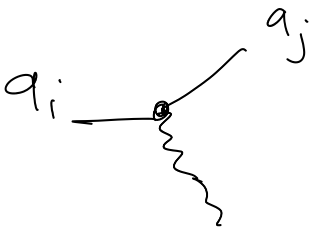


C_V, C_A = functions of q^2

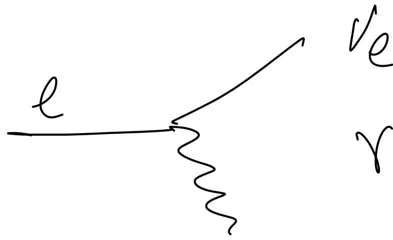
$n \to p e^- \nu_e$	$C_A/C_V = -1.275$	
$\Lambda \to p\pi, n\pi^0$	$= -0.718$	$\Delta S = 1$ weak decay.
$\Sigma^- \to n e^- \bar{\nu}_e$	$+0.340$	
$\Xi^- \to \Lambda e^- \bar{\nu}_e$	-0.25	

at high energy hadrons $\Rightarrow$ free quark interaction

$$C_A = -1.$$



$\gamma^\mu(1-\gamma^5) V_{ij}$



$\gamma^\mu(1-\gamma^5)$

V_{CKM}: Cabibbo-Kobayashi-Maskawa.

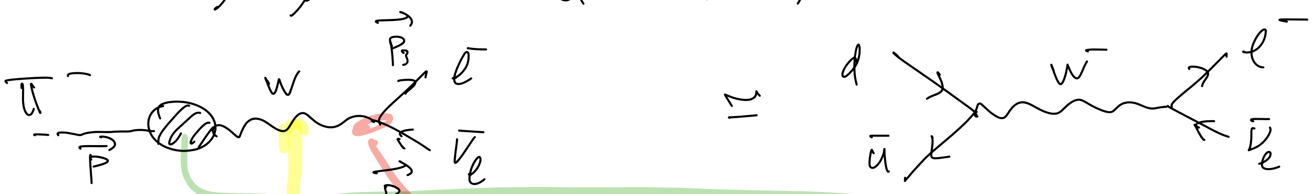
π Decay

$\pi^- \to e^- \bar{\nu}_e$

$\quad\quad \mu^- \bar{\nu}_\mu$

$Q = m_\pi - m_e - m_\nu = m_\pi \simeq 135\ \text{MeV}.$

$Q = m_\pi - m_\mu - m_\nu \simeq 29\ \text{MeV}$



$$\mathcal{M} = \left(\frac{g_w^2}{q^2 - M_W^2}\right)\left(\bar{u}(P_3)\gamma^\mu(1-\gamma^5)v(P_2)\right) F_\mu$$

G_F

$\hookrightarrow$ 4-vector under Lorentz

$\Downarrow$

$\mathcal{M}$ to be scalar.

$q \simeq m_\pi = 135\ \text{MeV}.$

$M_W = 80\ \text{GeV}.$

π spin 0 particle $\Rightarrow$ $F_\mu = f_\pi P_\mu.$

$\hookrightarrow$ pion decay constant.

$\langle|M|^2\rangle$: spin $\pi = 0$. 1 state.

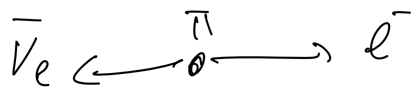
e^- 2 spin states.

$\bar{\nu}$ 1 spin state.

$$\langle|M|^2\rangle = \left(\frac{g_W}{2M_W}\right)^4 f_\pi^2\, m_e^2\,(m_\pi^2 - m_e^2)$$

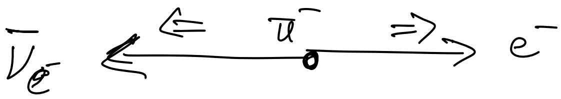
$\Gamma \propto \langle|\mathcal{M}|^2\rangle$ (phase space)

$$\Gamma = \frac{1}{8\pi}\frac{|\vec{P}_2|}{m_\pi^2}\langle|M|^2\rangle$$



$$|\vec{P}_2| = \frac{1}{2m_\pi}(m_\pi^2 - m_e^2)$$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} = 1.283 \times 10^{-4}$$



$J_\pi = 0.$

$J_\pi^{?} = 0.$

For $\vec{J}$ conserv.

$\Rightarrow$ e^- RH.

$$p_e = \frac{1}{2m_\pi}(m_\pi^2 - m_e^2).$$

$m_\pi = 135$ MeV.

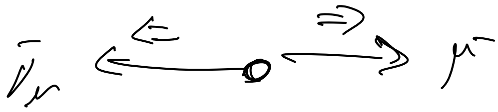
$m_\mu = 106$ MeV.

$m_e = 0.5$ MeV

prob. of having RH e^- $\sim 1-\beta$.

$$\beta_e = \frac{p_e}{E_e} = 1 - 2.6\times10^{-5}$$

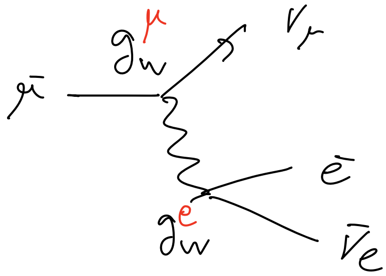
$\beta_\mu = 0.38$



not unlikely.

what if $m_e = m_\mu = 0.$ $\Rightarrow$ π becomes stable.

no lighter hadrons to decay strongly.



How do you know $g_W^e \equiv g_W^\mu$?

$g_W^\tau \equiv g_W^\mu \equiv g_W^e$?

Lepton Universality of weak.