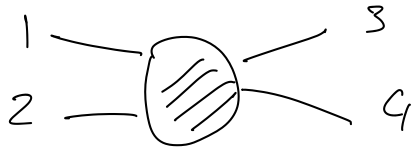
QED to discover quarks, color exist

- $e^{\pm}$ elementary point like.

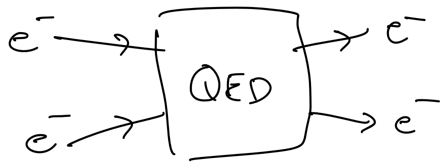


2-body scattering.

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{(E_1+E_2)} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} |M|^2$$ in the Center of mass.

simple QED process

$e^-e^- \longrightarrow e^-e^-$



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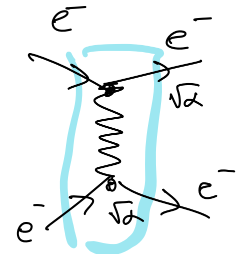

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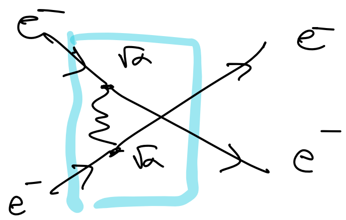
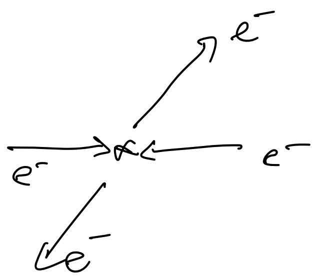
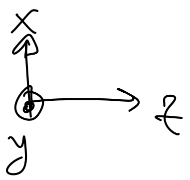


t-Chanel M_1

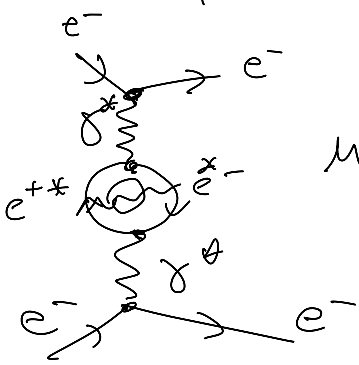
u-Channel. M_2

Möller scattering.





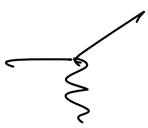
$M_1 \, M_2 \propto \alpha_{EM} \times (\text{—})$

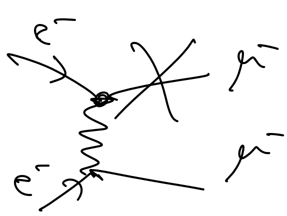


$M_3 \propto \alpha \cdot \alpha = \alpha_{EM}^2$

$\alpha = \frac{1}{137}$

QED vertex

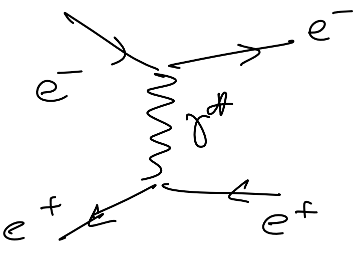




$e^-e^- \cancel{\to} \mu^-\mu^-$

$e^-e^+ \longrightarrow e^-e^+$

Bhabha Scattering



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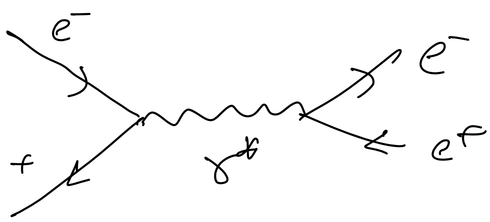
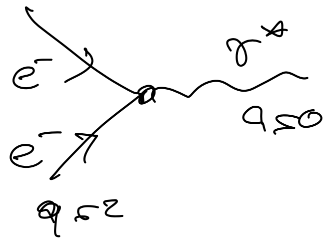


t-Chanel

S-Channel Annihilation.

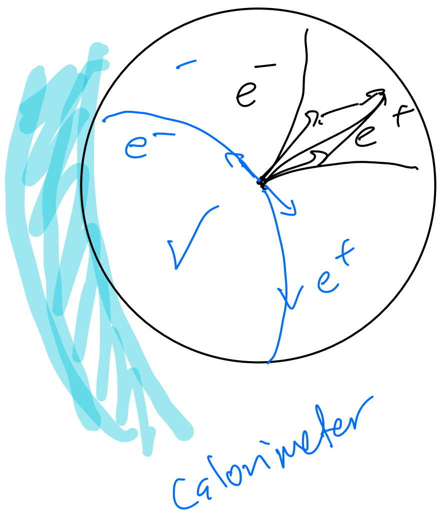
Möller
s-channel

$e^-e^- \to e^-e^-$



$$N = \sigma \times \underbrace{\mathcal{L}_{inst}}_{} \times \Delta t$$

$$\underbrace{N(\text{bhabha scatt}) = N(e^+e^- \to e^+e^-)}_{\text{Count and measure.}} \equiv \underbrace{\sigma_{Bh.}}_{\text{Calculated precisely.}} \times \mathcal{L}_{inst.} \times \Delta t$$



$\vec{B}$ ⊙

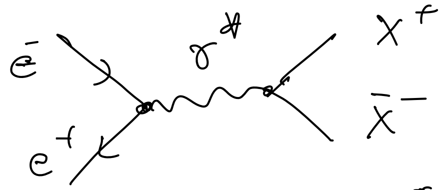
$\to \vec{\gamma}$

Not possible

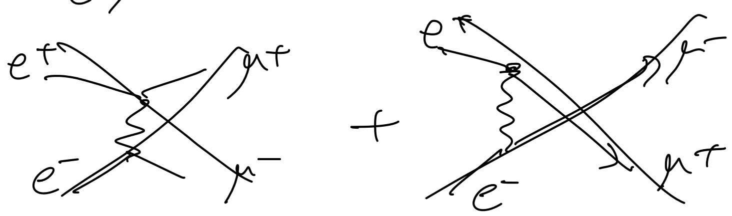
Luminosity Measurement.

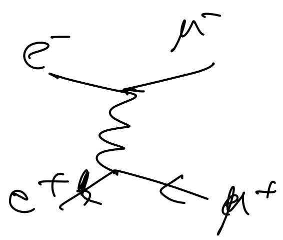
$e^+e^- \to e^+e^-$
$\mu^+\mu^-$

s-channel :

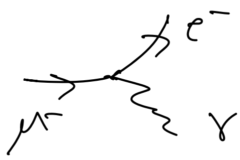


$e^+e^- \to \mu^+\mu^-$





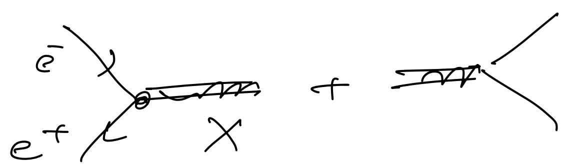
if possible $\Rightarrow$



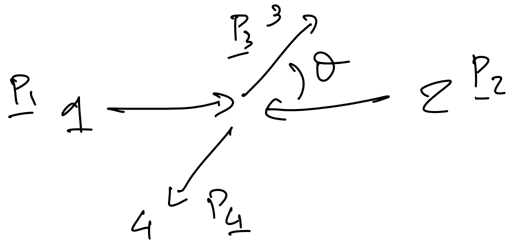
$\mu \to e\gamma$

flavor violating decay.

Meg



$e^+e^- \to \mu^+\mu^-$ $\qquad$ $e^+e^- \to q\bar{q}$



$\vec P_1 + \vec P_2 = 0$ $\qquad$ $\vec P_{in}, \vec P$

$\vec P_3 + \vec P_4 = 0.$ $\qquad$ $\vec P_{out}$

Symmetric beam: $E_1 = E_2$ $\quad$ $\vec P_1 = -\vec P_2$

LAB frame $\equiv$ C.O.M. frame

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{(E_1+E_2)^2}\frac{|\vec P_{out}|}{|\vec P_{in}|}|M|^2$$

Mandelstam Variables: s, t, u $\qquad$ $P_i^2 = m_i^2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = E_i^2 - \vec P_i^2$

$P_1 + P_2 = P_3 + P_4$ $\qquad$ En. Mom. Conservation

$$s = (P_1+P_2)^2 = P_1^2 + P_2^2 + 2P_1\cdot P_2 = m_1^2 + m_2^2 + 2P_1\cdot P_2$$

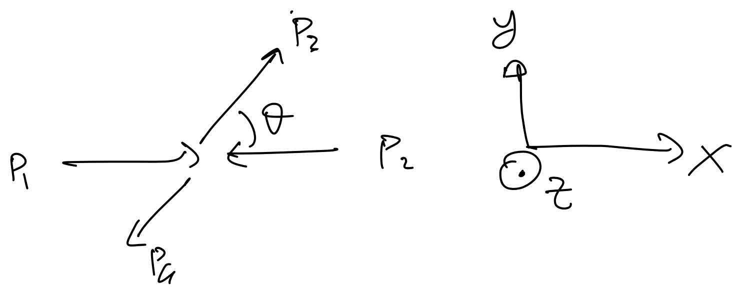
$$t = (P_1-P_3)^2 = m_3^2 + P_1^2 - 2P_1\cdot P_3$$

$$u = (P_1-P_4)^2 = m_4^2 + P_1^2 - 2P_1\cdot P_4$$

$$s+t+u = \underbrace{m_1^2+m_2^2+m_3^2+m_4^2}_{\sum_i m_i^2} + \underbrace{2P_1\cdot P_2 + 2P_1\cdot P_1 - 2P_1\cdot P_3 - 2P_1\cdot P_4}_{2P_1\cdot(\underbrace{P_2+P_1-P_3-P_4}_{=0})}$$

High Energy Regime

$E_i \gg m_i$



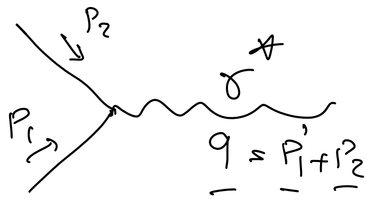
$P_1 = (E_1, P, 0, 0)$ $\qquad$ $P_3 = (E_3, P_{out}\cos\theta, P_{out}\sin\theta, 0)$

$P_2 = (E_2, -P, 0, 0)$ $\qquad$ $P_4 = (E_4, -P_{out}\cos\theta, -P_{out}\sin\theta, 0)$

$s = (P_1+P_2)^2 = (E_1+E_2)^2 = (2E)^2 = 4E^2$ $\qquad$ symm. beams. $E_1 = E_2$.

$\quad = (E_3+E_4)^2$

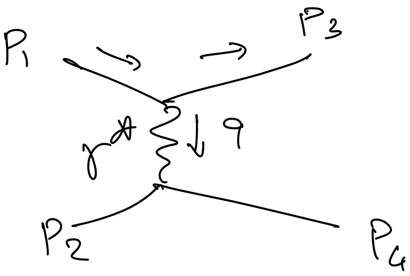
$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 p_2 = m_1^2 + m_2^2 + 2(E_1E_2 - \vec{p}_1\cdot\vec{p}_2)$$
$$= m_1^2 + m_2^2 + 2(E^2 + p^2)$$
$$= (m_1^2 + p^2) + (m_2^2 + p^2) + 2E^2$$
$$= 4E^2$$



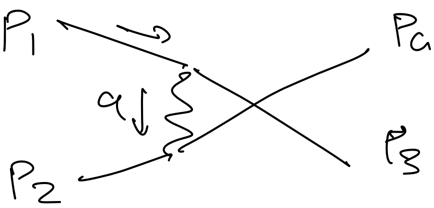
prop $\dfrac{-g_{\mu\nu}}{q^2}$ $\quad \dfrac{1}{s}$

$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \simeq -\frac{1}{2}s(1-\cos\theta)$ high energy regime

$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \simeq \frac{1}{2}s(1+\cos\theta)$



$q = p_1 - p_3$ propagator $\dfrac{-g_{\mu\nu}}{q^2}$ $\quad \dfrac{1}{t}$



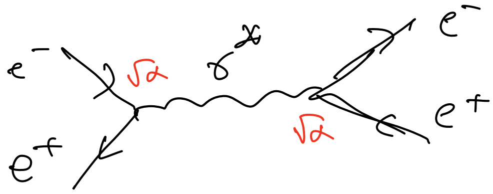
$q = p_1 - p_4$ prop. $\dfrac{-g_{\mu\nu}}{q^2}$ $\quad \dfrac{1}{u}$

Cross section $e^+e^- \to \mu^+\mu^-$ in QED

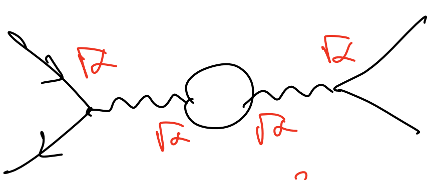
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\underbrace{\frac{1}{(E_1+E_2)^2}}_{s}\frac{|\vec{p}_{out}|}{|\vec{p}_{in}|}|M|^2$$

$E_i \gg m_i$ $\quad p_{out} \simeq p_{in}$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M|^2$$



$M_1 \cdot \alpha_{EM}$



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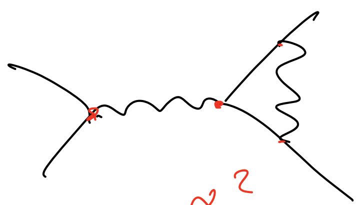
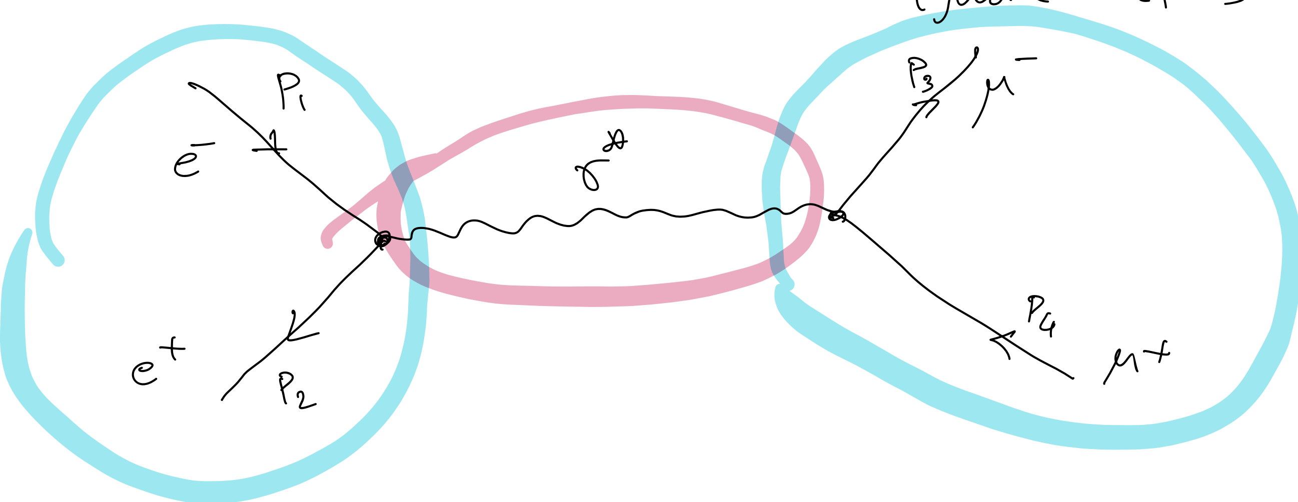


α^2_{EM} $\quad \alpha^2_{EM}$

$$|\mathcal{M}|^2 = |\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3|^2 \simeq |\mathcal{M}_1|^2 + \mathcal{O}(\alpha^2)$$

ignore $\mathcal{M}_2, \mathcal{M}_3$.



$$-i\mathcal{M} = \left[\bar{v}(P_2)\, ie\gamma^\mu\, u(P_1)\right] \times$$

$$\times \frac{-ig_{\mu\nu}}{q^2} \times$$

$$\times \left[\bar{u}(P_3)\, ie\gamma^\nu\, v(P_4)\right]$$

u, v spinors. e^-, e^+

$$-\frac{ig_{\mu\nu}}{s}$$

$$j^\mu_{(e)} = \bar{v}(P_2)\gamma^\mu u(P_1)$$

$$j^\nu_{(\mu)} = \bar{u}(P_3)\gamma^\nu v(P_4)$$

$$\Rightarrow \quad \mathcal{M} = -\frac{e^2}{s}\, \underline{j}_{(e)} \cdot \underline{j}_{(\mu)}$$