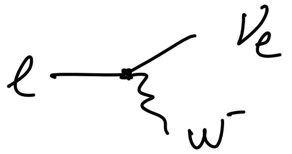
Weak Interactions in Quarks

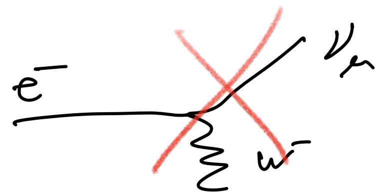


$ig_w \gamma^\mu (1-\gamma^5)$

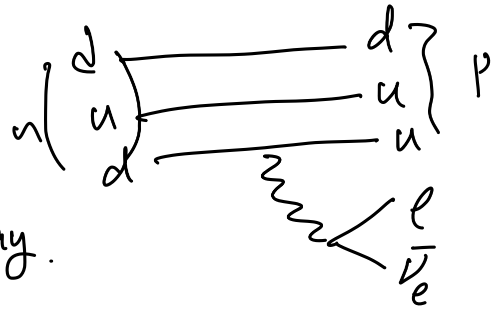
W^- couples to $\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L$ no lepton flavor violation.

$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}_L \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}_L$ e^-_R, ν_R

No such events

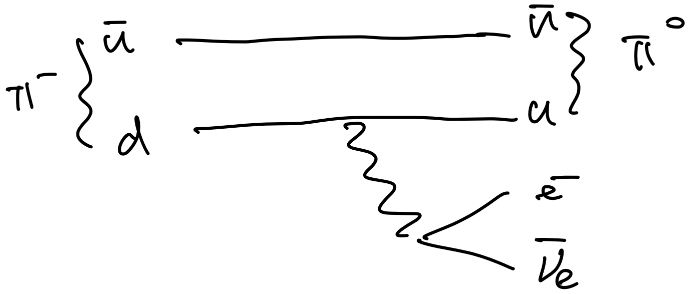


neutron decay: $n \equiv (udd)$



β decay, semileptonic decay.

π^- semileptonic decay:

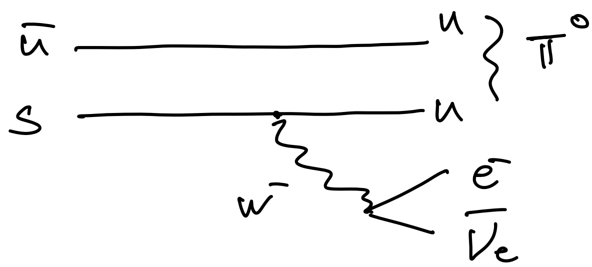


$\pi^- \to \pi^0 e^- \bar{\nu}_e$

$m_{\pi^0} < m_{\pi^-}$

$\Delta S = 0$

K^-:



$g_w \gamma^\mu (1-\gamma^5)$

$\Delta S = 1$ initial state: $S=-1$ final state: $S=0$.

Flavor changing electrically charged weak current

Compare $\Delta S=0$ and $\Delta S=1$ weak decays of leptons.

π^+ DECAY MODES

π^- modes are charge conjugates of the modes below.

For decay limits to particles which are not established, see the section on Searches for Axions and Other Very Light Bosons.

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\mu^+\nu_\mu$	[a] (99.98770 ± 0.00004) %	
Γ_2	$\mu^+\nu_\mu\gamma$	[b] $(2.00 \pm0.25) \times 10^{-4}$	
Γ_3	$e^+\nu_e$	[a] $(1.230 \pm0.004) \times 10^{-4}$	
Γ_4	$e^+\nu_e\gamma$	[b] $(7.39 \pm0.05) \times 10^{-7}$	
Γ_5	$e^+\nu_e\pi^0$	$(1.036 \pm0.006) \times 10^{-8}$	
Γ_6	$e^+\nu_e e^+ e^-$	$(3.2 \pm0.5) \times 10^{-9}$	
Γ_7	$\mu^+\nu_\mu\nu\overline{\nu}$	$< 9 \times 10^{-6}$	90%
Γ_8	$e^+\nu_e\nu\overline{\nu}$	$< 1.6 \times 10^{-7}$	90%
	Lepton Family number (*LF*) or Lepton number (*L*) violating modes		
Γ_9	$\mu^+\overline{\nu}_e$	*L* [c] $< 1.5 \times 10^{-3}$	90%
Γ_{10}	$\mu^+\nu_e$	*LF* [c] $< 8.0 \times 10^{-3}$	90%
Γ_{11}	$\mu^- e^+ e^+ \nu$	*LF* $< 1.6 \times 10^{-6}$	90%

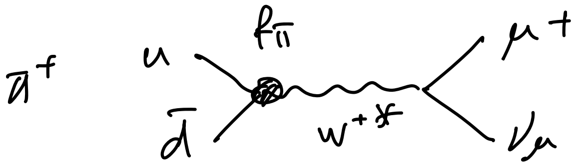
K^+ DECAY MODES

K^+ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Leptonic and semileptonic modes			
$e^+\nu_e$	$(1.582\pm0.007) \times 10^{-5}$		247
$\mu^+\nu_\mu$	(63.56 ± 0.11) %	S=1.2	236
$\pi^0 e^+\nu_e$ Called K^+_{e3}.	(5.07 ± 0.04) %	S=2.1	228
$\pi^0\mu^+\nu_\mu$ Called $K^+_{\mu 3}$.	(3.352 ± 0.033) %	S=1.9	215
$\pi^0\pi^0 e^+\nu_e$	$(2.55 \pm0.04) \times 10^{-5}$	S=1.1	206
$\pi^+\pi^- e^+\nu_e$	$(4.247\pm0.024) \times 10^{-5}$		203
$\pi^+\pi^-\mu^+\nu_\mu$	$(1.4 \pm0.9) \times 10^{-5}$		151
$\pi^0\pi^0\pi^0 e^+\nu_e$	$< 3.5 \times 10^{-6}$	CL=90%	135
Hadronic modes			
$\pi^+\pi^0$	(20.67 ± 0.08) %	S=1.2	205
$\pi^+\pi^0\pi^0$	(1.760 ± 0.023) %	S=1.1	133
$\pi^+\pi^+\pi^-$	(5.583 ± 0.024) %		125

m_π : 135–139 MeV

m_K = 500 MeV

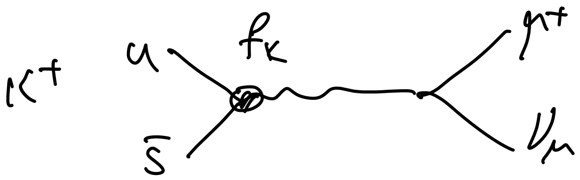
let's take a look at leptonic decays of π^+, K^+



$$\mu \sim f_\pi \frac{g_w^2}{m_W^2} m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$\Gamma(\pi^+ \to \mu^+\nu_\mu) \sim G_{F\pi}^2 f_\pi^2 \frac{m_\ell^2}{m_\pi^3}(m_\pi^2 - m_\ell^2)^2$$

$$\Gamma \propto |M|^2 \rho_{\text{phase space}}$$



$$\Gamma(K^+ \to \mu^+\nu_\mu) \sim G_{FK}^2 f_K^2 \frac{m_\ell^2}{m_K^3}(m_K^2 - m_\ell^2)^2$$

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \frac{G_{FK}^2}{G_{F\pi}^2}\left(\frac{f_K}{f_\pi}\right)^2\left(\frac{m_\pi}{m_K}\right)^3\frac{(m_K^2 - m_\mu^2)^2}{(m_\pi^2 - m_\mu^2)^2}$$

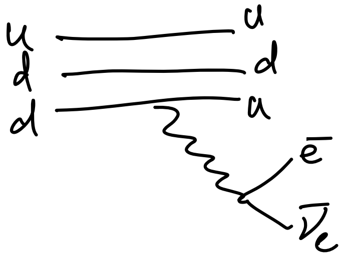
$$\Gamma(K^+ \to \mu^+\nu_\mu) = BF(K^+ \to \mu^+)\,\Gamma^K_{tot} \quad \alpha \quad \frac{\#\, K^+ \to \mu^+ \text{ decays}}{\#\, K^+ \text{ tot}}$$

measured experimentally.

$$\left(\frac{G_{FK}}{G_{F\pi}}\right)^2 \simeq 0.05$$

$n \rightarrow p\, e^- \bar{\nu}_e$

$m_n \approx 1$ GeV

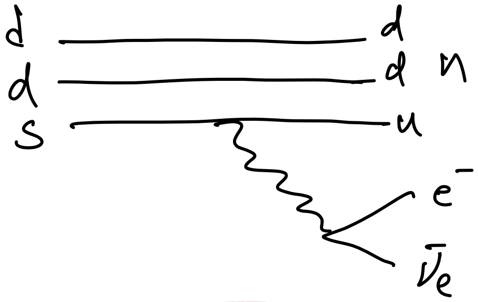


n DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$p e^- \overline{\nu}_e$	100 %	
Γ_2	$p e^- \overline{\nu}_e \gamma$	[a] $(9.2 \pm 0.7) \times 10^{-3}$	
Γ_3	hydrogen-atom $\overline{\nu}_e$	$< 2.7 \times 10^{-3}$	95%

$\Sigma^- \rightarrow n\, e^- \bar{\nu}_e$

$m_\Sigma \sim 1.2$ GeV



Σ^- DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$n \pi^-$	(99.848 ± 0.005) %	
Γ_2	$n \pi^- \gamma$	[a] $(4.6 \pm 0.6) \times 10^{-4}$	
Γ_3	$n e^- \overline{\nu}_e$	$(1.017 \pm 0.034) \times 10^{-3}$	
Γ_4	$n \mu^- \overline{\nu}_\mu$	$(4.5 \pm 0.4) \times 10^{-4}$	
Γ_5	$\Lambda e^- \overline{\nu}_e$	$(5.73 \pm 0.27) \times 10^{-5}$	
Γ_6	$\Sigma^+ X$	$< 1.2 \times 10^{-4}$	90%

$$\left(\frac{G_{F_s}}{G_{F_d}}\right)^2 \sim 0.05$$

$\Rightarrow$ Different behaviour of weak int. with s, d quarks!?

$\neq$

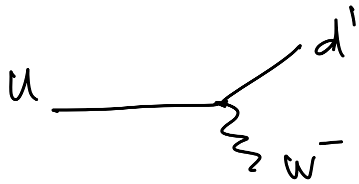
?

Cabibbo Angle

leptons:

W^- couples to $\begin{pmatrix} \ell^- \\ \nu_e \end{pmatrix}_L$

quarks:



$d' = \cos\theta\, d + \sin\theta\, s$

θ: mixing angle between s, d quarks.

nature produces flavor quarks: u, d, s

weak interaction between: u, d' $\quad \begin{pmatrix} u \\ d' \end{pmatrix}$

Flavor eigenstates $\neq$ weak interaction eigenstates.

UNITARY SYMMETRY AND LEPTONIC DECAYS

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(Received 29 April 1963)

We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as "eightfold way,"[1] and the V-A theory for weak interactions.[2,3] Our basic assumptions on J_μ, the weak current of strong interacting particles, are as follows:

(1) J_μ transforms according to the eightfold representation of SU_3. This means that we neglect currents with $\Delta S = -\Delta Q$, or $\Delta I = 3/2$, which should belong to other representations. This limits the scope of the analysis, and we are not able to treat the complex of K^0 leptonic decays, or $\Sigma^+ \to n + e^+ + \nu$ in which $\Delta S = -\Delta Q$ currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of J_μ which is in the eightfold representation.

(2) The vector part of J_μ is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For $\Delta S = 0$, this assumption is equivalent to vector-

To determine θ, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

$$\Gamma(K^+ \to \mu\nu)/\Gamma(\pi^+ \to \mu\nu)$$
$$= \tan^2\theta M_K(1 - M_\mu^{\,2}/M_K^{\,2})^2/M_\pi(1 - M_\mu^{\,2}/M_\pi^{\,2})^2. \quad (3)$$

From the experimental data, we then get[5,6]

$$\theta = 0.257. \quad (4)$$

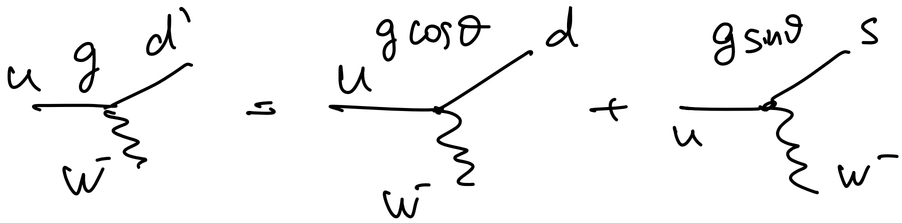
For an independent determination of θ, let us consider $K^+ \to \pi^0 + e^+ + \nu$. The matrix element for this process can be connected to that for $\pi^+ \to \pi^0 + e^+ + \nu$, known from the conserved vector-current hypothesis (2nd assumption). From the rate[6] for $K^+ \to \pi^0 + e^+ + \nu$, we get

$$\theta = 0.26. \quad (5)$$

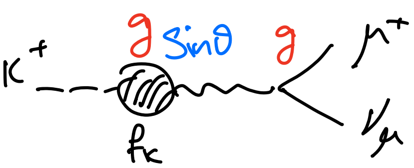
The two determinations coincide within experimental errors; in the following we use $\theta = 0.26$.

We go now to the leptonic decays of the baryons, of the type $A \to B + e + \nu$. The matrix element of

weak interaction vertex



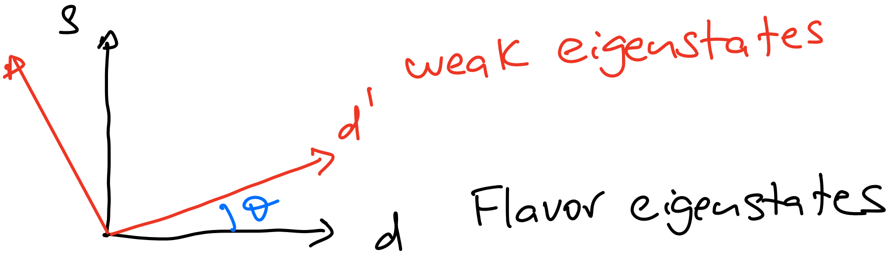
$$\Gamma \propto G_F^{\,2} \cos^2\theta \; m_\mu^{\,2}(m_\pi^{\,2} - m_\mu^{\,2})^2 \frac{1}{m_\pi^{\,3}} \; f_\pi^{\,2}$$



$$\Gamma \propto G_F^{\,2} \sin^2\theta \; m_\mu^{\,2}(m_K^{\,2} - m_\mu^{\,2})^2 \frac{1}{m_K^{\,3}} \; f_K^{\,2}$$

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = (\tan\theta)^2 \times F(m_\mu, m_\pi, m_K, f_\pi, f_K)$$

$\Rightarrow \theta \simeq 0.25 \qquad \theta \approx 13^\circ$



$$\Rightarrow \quad \Gamma(\Delta S = 1) \propto \sin^2\theta . \qquad \text{Cabibbo suppressed decays. } (\Delta S = 1)$$

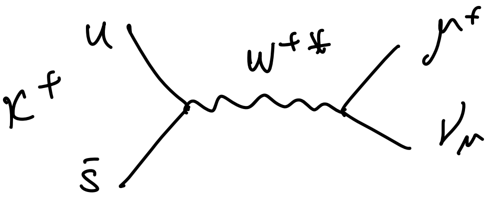
$$\Gamma(\Delta S = 0) \propto \cos^2\theta$$

$$d' = d \cdot \cos\theta + s \cdot \sin\theta$$

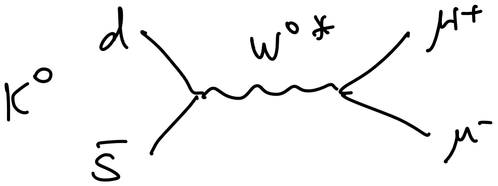
$$s' = -\sin\theta \cdot d + \cos\theta \cdot s .$$

most $\Delta S = 1$ decays explained by Cabibbo angle.

One major problem with leptonic decays.



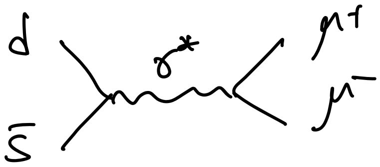
$$\Gamma(K^+ \to \mu^+ \nu_\mu .) = 64\%$$



$$\Gamma(K^0 \to \mu^+ \mu^-) = 10^{-9}$$

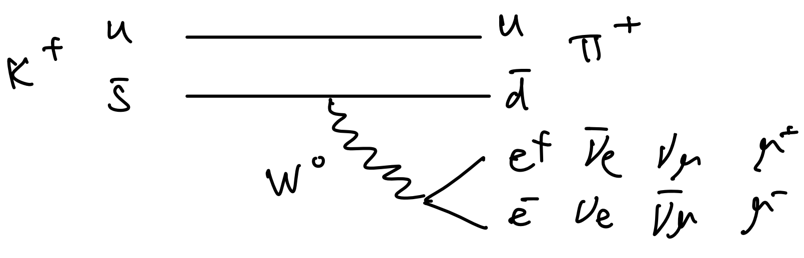
$$\Gamma(K^0 \to \mu^+\mu^-) \ll \Gamma(K^+ \to \mu^+ \nu_\mu)$$

neutral weak current?



$\Delta S \neq 1$ not allowed in QED.

if W^0 exists: $\quad W^0 \to \ell^+ \ell^-, \ \nu_e \bar{\nu}_e$



K^+ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Leptonic and semileptonic modes			
$e^+ \nu_e$	$(1.582 \pm 0.007) \times 10^{-5}$		247
$\mu^+ \nu_\mu$	$(63.56 \pm 0.11)\ \%$	S=1.2	236
$\pi^0 e^+ \nu_e$	$(5.07 \pm 0.04)\ \%$	S=2.1	228
Called K^+_{e3}.			
$\pi^0 \mu^+ \nu_\mu$	$(3.352 \pm 0.033)\ \%$	S=1.9	215
Called $K^+_{\mu3}$.			
$\pi^0 \pi^0 e^+ \nu_e$	$(2.55 \pm 0.04) \times 10^{-5}$	S=1.1	206
$\pi^+ \pi^- e^+ \nu_e$	$(4.247 \pm 0.024) \times 10^{-5}$		203
$\pi^+ \pi^- \mu^+ \nu_\mu$	$(1.4 \pm 0.9) \times 10^{-5}$		151
$\pi^0 \pi^0 \pi^0 e^+ \nu_e$	$< 3.5 \times 10^{-6}$	CL=90%	135
Hadronic modes			
$\pi^+ \pi^0$	$(20.67 \pm 0.08)\ \%$	S=1.2	205
$\pi^+ \pi^0 \pi^0$	$(1.760 \pm 0.023)\ \%$	S=1.1	133
$\pi^+ \pi^+ \pi^-$	$(5.583 \pm 0.024)\ \%$		125

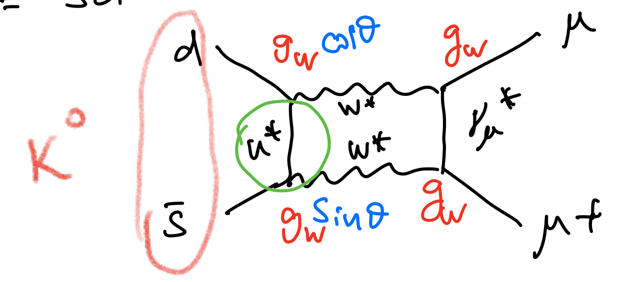
$K^{\pm}$ **Lepton family number (*LF*), Lepton number (*L*), $\Delta S = \Delta Q$ (*SQ*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes**

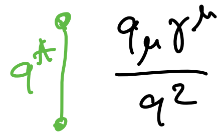
Γ_{35}	$\pi^+\pi^+e^-\overline{\nu}_e$	*SQ*	< 1.3	$\times 10^{-8}$	CL=90%
Γ_{36}	$\pi^+\pi^+\mu^-\overline{\nu}_\mu$	*SQ*	< 3.0	$\times 10^{-6}$	CL=95%
Γ_{37}	$\pi^+e^+e^-$	*S1*	(3.00 ± 0.09)	$\times 10^{-7}$	
Γ_{38}	$\pi^+\mu^+\mu^-$	*S1*	(9.4 ± 0.6)	$\times 10^{-8}$	S=2.6
Γ_{39}	$\pi^+\nu\overline{\nu}$	*S1*	$(1.14\ ^{+0.40}_{-0.33})$	$\times 10^{-10}$	

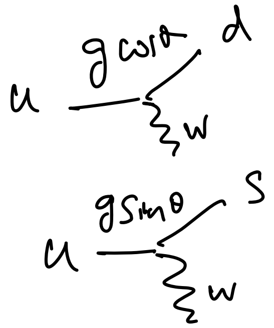
if W^0 does not exist $\Rightarrow$ $K^0 \to \mu^+\mu^- \equiv \emptyset$ at leading order

How to explain $K^0 \to \mu^+\mu^- \sim 10^{-9}$ with charged weak current?

$K^0 = \bar{s}d$







$$\mathcal{M}(K^0 \to \mu^+\mu^-) \sim \underbrace{\frac{g_w^2}{M_w^2}}_{G_F} \; \underbrace{\frac{g_w^2}{M_w^2}}_{G_F} \; \sin\theta \cos\theta$$

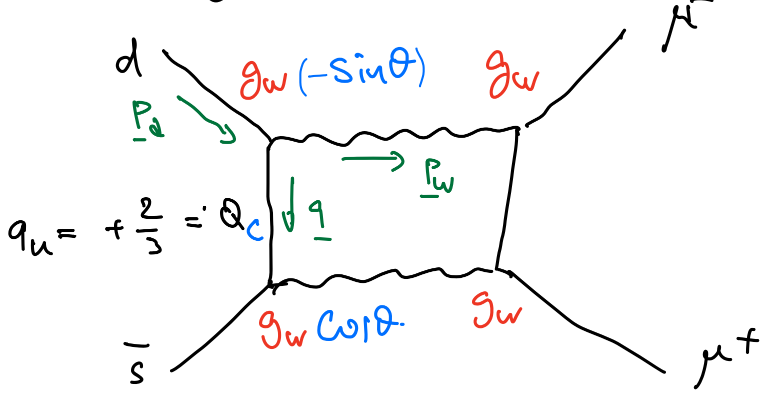
$$\Gamma(K^0 \to \mu^+\mu^-) \sim G_F^4 \sin^2\theta \cdot \cos^2\theta. \qquad \theta \simeq 0.25$$

$$\Gamma^{\text{theor.}}(K^0 \to \mu^+\mu^-) \gg 10^{-9} \text{ (experimental value)}$$

Glashow- Iliopoulos- Maiani (1970) : GIM mechanism.

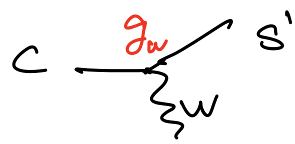
$\Rightarrow$ introduce a new up-like quark. $Q = +\frac{2}{3}$

$\Rightarrow$ Charm quark.



$d' = d \cos\theta + s \sin\theta$

$s' = -\sin\theta \cdot d + s \cos\theta$



$\underline{q} = \underline{P_d} - \underline{P_w}$

$$\mathcal{M}(K^0 \to \mu^+\mu^-) = \mathcal{M}(u\text{-quark}) + \mathcal{M}(c\text{-quark})$$

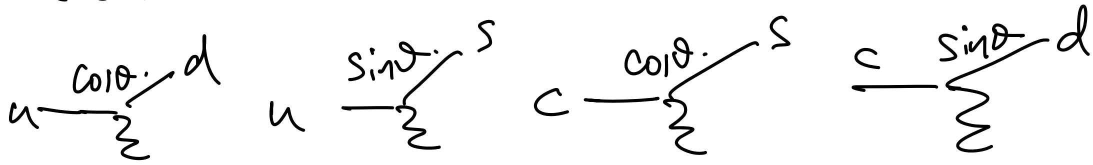
$$\mathcal{M}(u\text{-quark}) \sim (\cos\theta)(\sin\theta)\,\frac{1}{q^2} \qquad q:\ u\text{-quark mom.}$$

$$\mathcal{M}(c\text{-quark}) \sim (-\sin\theta)(\cos\theta)\,\frac{1}{q^2} \qquad q:\ c\text{-quark mom.}$$

$\Rightarrow$ charm quark must have mass $\sim 1-3$ GeV

$\Rightarrow$ predict $K^0 \to \mu^+\mu^- \sim 10^{-9}$

$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$ for weak interaction with charged current



$\Delta S = 1$ $\qquad$ $\Delta C = 1$, $\Delta S = 1$ $\qquad$ $\Delta C = 1$.

Flavor Changing Charged Weak Current