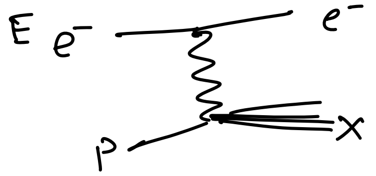
$e^- + p \rightarrow e^- + X$



Deep Inelastic Scattering

So far $e^- + p \rightarrow e^- + p$ $\quad W^2 = M^2 - Q^2 + 2M\nu \quad \nu = E - E'$

$Q^2 >> M^2$ New accelerator @ SLAC 1968 $\quad E \sim 25$ GeV

Detectors can only detect e^- nothing about X.

$$Q^2 = 4EE' \sin^2\frac{\theta}{2}$$ measure E', θ $\quad$ Elastic: $Q^2 = 2M\nu$

Inelastic $(Q^2, \nu), (E', \theta)\ (x, y)$

$$x = \frac{Q^2}{2M\nu} \qquad y = \frac{\nu}{E}$$

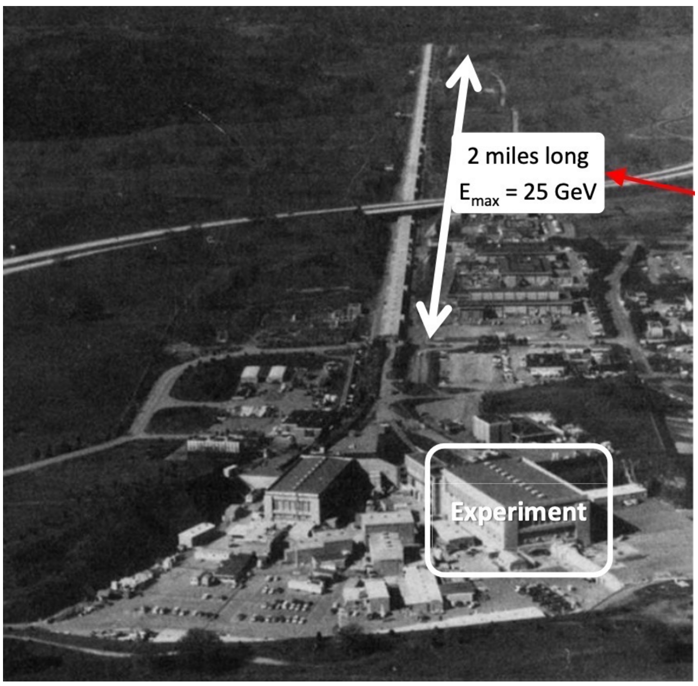
$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{inelastic}} = \left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Mott}\\ \text{recoil}}} \left(W_2(Q^2, \nu) + 2W_1(Q^2, \nu) \tan^2\frac{\theta}{2} \right)$$

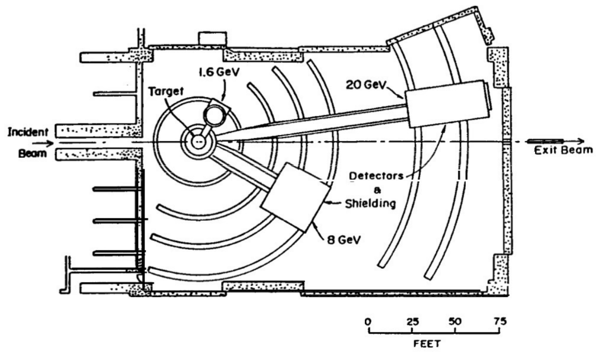
W_1, W_2: functions to be measured.

Experimentally: measure $\frac{d\sigma}{d\Omega}$, or # events @ (E', θ)
(Q^2, ν)

$(E_1', \theta_1), (E_2', \theta_2) \longrightarrow$ give same (Q^2, ν)

Linear accelerator





Dipoles for bending beam.

3 spectrometers:

1.6 GeV. $\theta \simeq 34°$
8 GeV
20 GeV $\Big\}$ $\theta > 12$

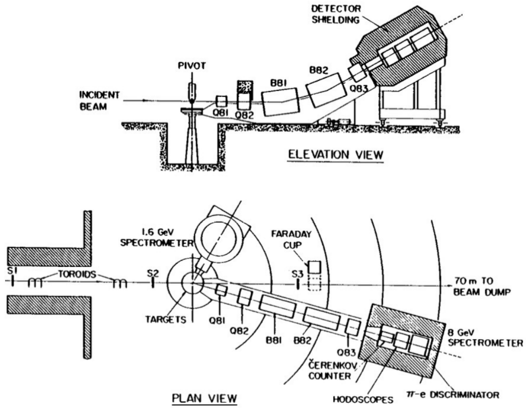


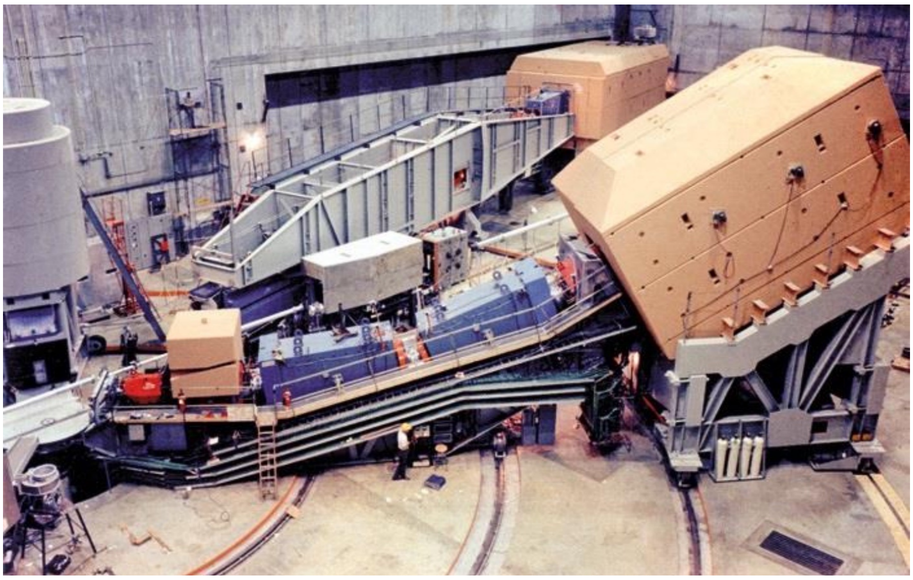
Fig. 2. (a) Plan view of End Station A and the two principal magnetic spectrometers employed for analysis of scattered electrons. (b) Configuration of the 8 GeV spectrometer, employed at scattering angles greater than 12°.

$e^- + p \longrightarrow e^- + p$ elastic σ well known.

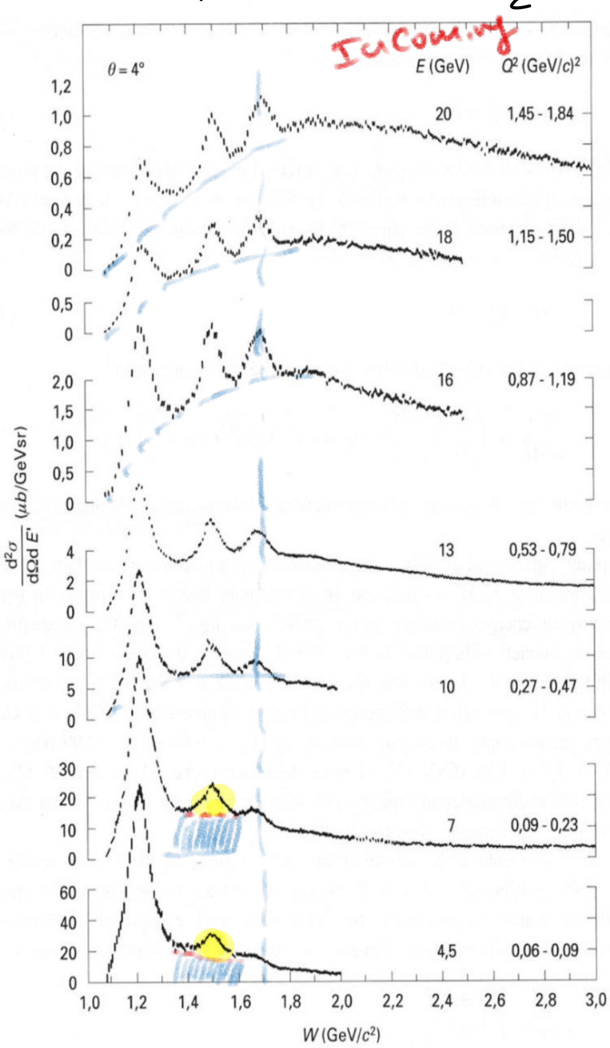
$e^- + X$ inelastic $\quad e^- + p \rightarrow e^- + p + \pi^+ \pi^-$

$$\# \text{events} = N = \underbrace{\mathcal{L}}_{\text{machine parameter}} \cdot \sigma \Rightarrow \mathcal{L}_{\text{machine}} = \frac{N_{\text{elastic}}}{\sigma_{\text{elastic}}}$$

Monitor uniformity of the target



$$Q^2 = 4EE' \sin^2\frac{\theta}{2}$$



$$W^2 = M^2 - Q^2 + 2M\nu .$$

$$\frac{d^2\sigma}{d\Omega dE'} \equiv \frac{\# \text{events.}}{(\text{Couple})(\text{energy bin})}$$

Differential cross section.

$$\frac{\mu b}{\text{GeV sr}}$$

1) Elastic peak decreases with E

2) @ same W
$\sigma \searrow$ with $E \nearrow$

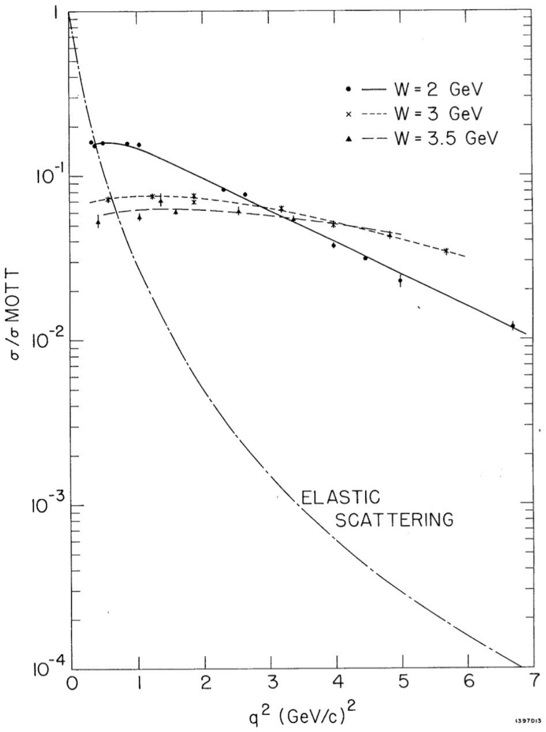
3) @ Fixed E, θ
resonances $\searrow$ with $W \nearrow$

4) For very large W: $\frac{d\sigma}{d\Omega\, dE'} \simeq 1-2\ \mu b/(\mathrm{GeV\ sr})$

$$\frac{\left.\frac{d\sigma}{d\Omega}\right|_{DIS}}{\left.\frac{d\sigma}{d\Omega}\right|_{Mott}} = W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2} = R(Q^2)$$

@ Fixed θ

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} = \frac{\alpha^2}{Q^4} E'^2 \cos^2\frac{\theta}{2}\ \frac{E'}{E}$$ Mott with recoil.



pointlike Dirac proton $s = 1/2$.

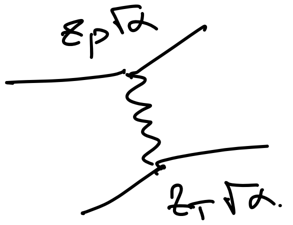
$$\left.\frac{d\sigma}{d\Omega}\right|_{elastic} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left(1 - \frac{q^2}{4M^2}\tan^2\frac{\theta}{2}\right)$$

proton with structure

$\Rightarrow$ Form Factor.

$$\left.\frac{d\sigma}{d\Omega}\right|_{struct} = \left.\frac{d\sigma}{d\Omega}\right|_{point} \times |F(Q^2)|^2$$

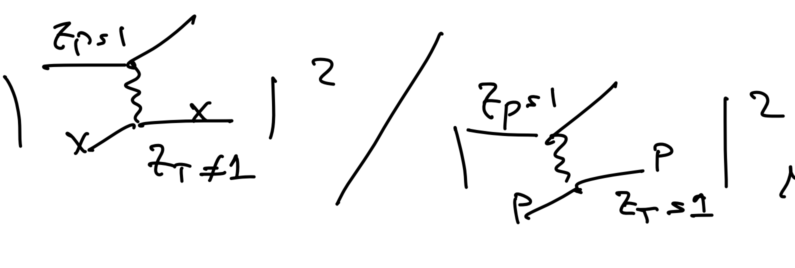
Form factors $F(Q^2=0) = 1$



$$\sigma \propto \frac{(Z_p Z_T \alpha)^2}{Q^4}$$

e^-: $Z_p = 1$ $\qquad \sigma = Z_T^2 \frac{\alpha^2}{Q^4}$

$$\frac{\sigma}{\sigma_{MOTT}} = Z_T^2 < 1 \Rightarrow Z_T < 1$$



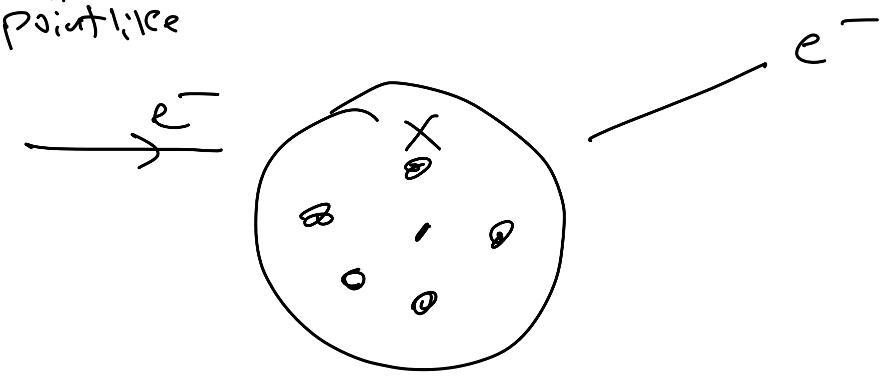
Mott pointlike.

$e^- + X(q=1) \longrightarrow e^- + X(q=1)$. $\quad \sigma \sim \alpha^2 Z_p Z_T = \alpha^2$

$e^- + X(q \neq 1) \longrightarrow e^- + X(q \neq 1) \quad \sigma \sim \alpha^2 Z_T^2$

$$\frac{\sigma}{\sigma_{Mott}} < 1 \Rightarrow Z_T^2 < 1 \Rightarrow Z_T < 1$$

pointlike



Quark-parton Model: proton as container of smaller particles

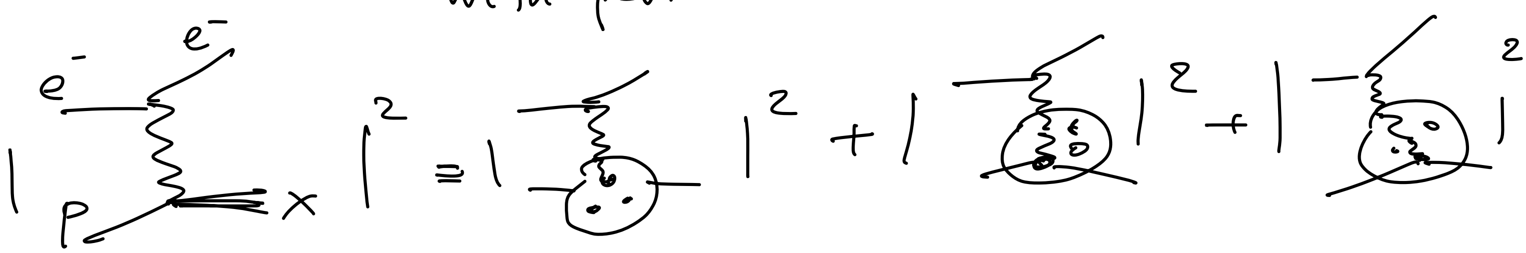
DIS: $\quad e^- + X \longrightarrow e^- + X \qquad X$: spin $= 1/2$, $Z < 1$, $m = ?$

$e^- + p$

$d\sigma$

$$\left.\frac{d\sigma}{d\Omega}\right|_{DIS} = \left.\frac{d\sigma}{d\Omega}\right|_{Mott} \times \left(W_2(Q^2, \nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right)$$

DIS of $e^- + p \equiv$ incoherent superposition of elastic scattering with partons.



Assumption: proton exchanges γ^* with 1 parton.

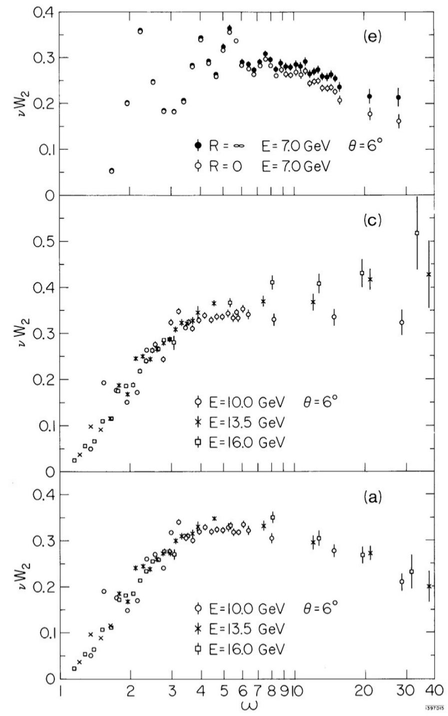


Fig. 2a, 2c, 2e

$F_2(Q^2, \nu) = \nu \times W_2(Q^2, \nu)$

For small angles $\theta \simeq 0$.

$\frac{d\sigma}{d\Omega}$ more sensitive to W_2

W_2 sensitive to x ($1/x$) not to Q^2

$$\omega = \frac{1}{x} = \frac{2M\nu}{Q^2}$$

cross section of $e^- + X$: ~~spin~~ = 1/2, m, pointlike.

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{Q^4} z_X^2 E'^2}_{\text{Mott}} \underbrace{\frac{E'}{E}}_{\text{recoil}} \underbrace{\cos^2\frac{\theta}{2}}_{\substack{S=1/2 \\ \text{probe}}} \left[1 + \underbrace{\frac{Q^2}{2m^2}}_{\substack{S=1/2 \text{ for target} \\ g=2 \\ \text{Dirac Pointlike.} \\ S=1/2 \text{ particle}}} \tan^2\frac{\theta}{2}\right] \delta\left(\nu - \frac{Q^2}{2m}\right)$$

Conservation of energy for partons

M : mass of target $M \simeq m_p$. (proton)

m : mass of X parton

$m \leq M$

$$\frac{d^2\sigma}{d\Omega dE'} = (\,-\,-\,)\, E'^2 \left[\sin^2\frac{\theta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}\right] \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$m = xM$

$$\delta\left(\nu - \frac{Q^2}{2m}\right) \Rightarrow \nu = \frac{Q^2}{2m} \Rightarrow \underline{Q^2 = 2m\nu}$$

elastic scattering $e^- + X$

$$Q^2 = 2xM\nu \Rightarrow x = \frac{Q^2}{2M\nu} \qquad x \in [0, 1]$$

$f(x)$: parton density function,

$f(x)dx$ = prob. of having parton with mass $\in [x, x+dx]$

$$\int_0^1 f(x)dx = 1.$$

$$\frac{d\sigma}{d\Omega}\bigg|_{DIS} = \frac{d\sigma}{d\Omega}\bigg|_{Mott} \times \left(W_2(Q^2,\nu) + \boxed{2W_1(Q^2,\nu)} \tan^2\frac{\theta}{2}\right)$$

$$\frac{d^2\sigma}{d\Omega dE'} = (\,--\,)\, E'^2 \left[\sin^2\frac{\theta}{2} + \boxed{\frac{Q^2}{2m^2}} \sin^2\frac{\theta}{2}\right] \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$$W_1(Q^2,\nu) = \frac{Q^2}{4m^2}\,\delta\left(\nu - \frac{Q^2}{2m}\right) z_q^2$$ for one parton with x.

Take into account all values of x

$$W_1(Q^2,\nu) = \int_0^1 dx\, f(x)\, \frac{Q^2}{4m^2}\, z_q^2\, \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$m = xM$

$$= \int_0^1 dx\, f(x)\, \frac{Q^2}{4M^2x^2}\, z_q^2\, \delta\left(\nu - \frac{Q^2}{2M}\frac{1}{x}\right)$$

$$I = \int A(x)\, \delta(g(x))\, dx = \frac{A(x_0)}{|g'(x_0)|}.$$

$$g(x_0) = 0$$

$$W_1(Q^2,\nu) = \frac{Q^2}{4M^2}\, z_q^2\, f(x)\, \frac{1}{Q^2}\, \frac{(2M\nu)^2}{2M\nu^2} = z_q^2\, \frac{f(x)}{2M}\bigg|_{x=x_0}$$

$$W_2(Q^2,\nu) = z_q^2\, f(x)\, \frac{x}{\nu}$$

For one parton.

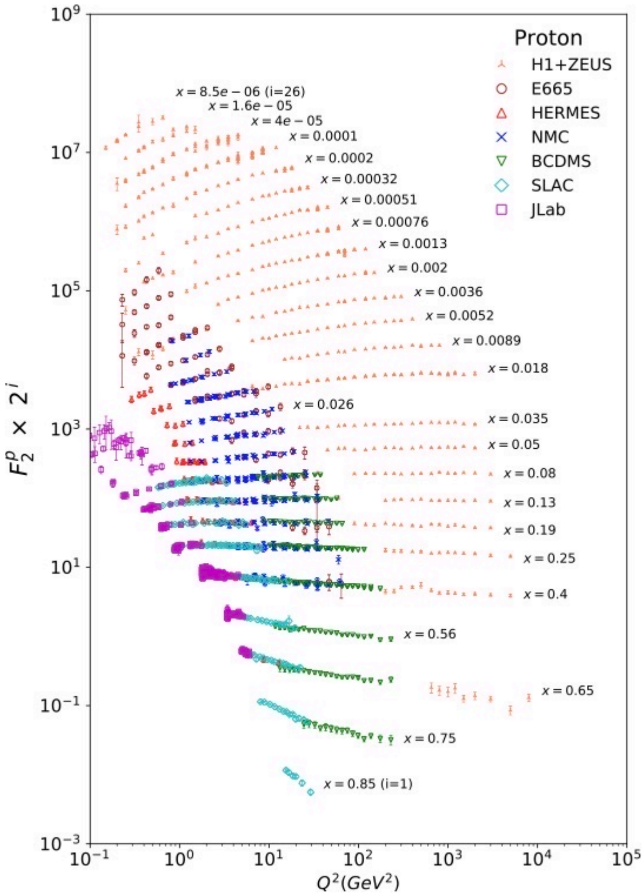
For N partons:

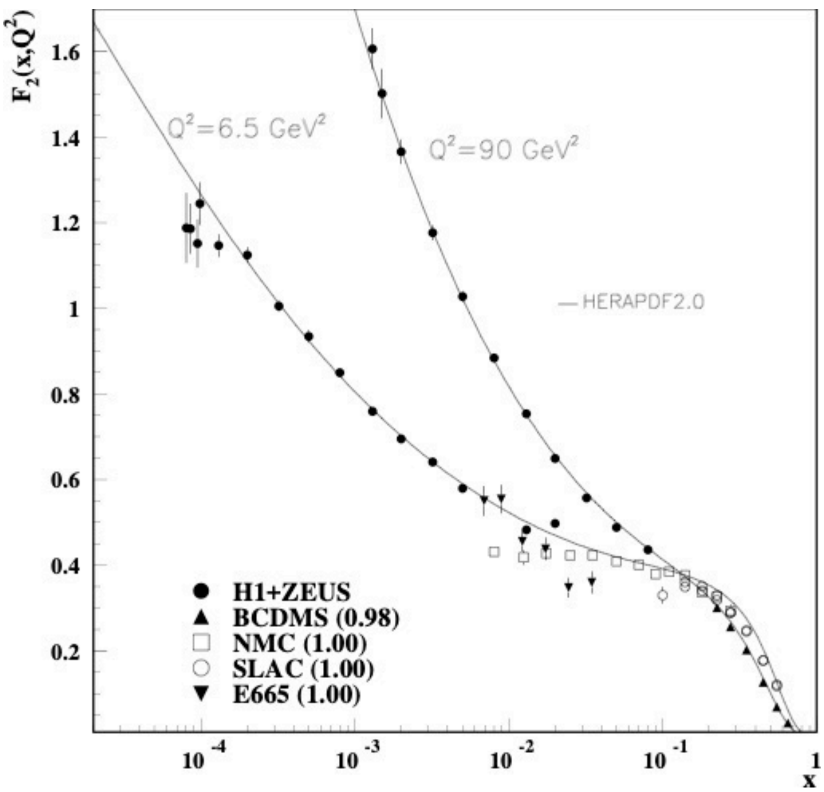
$$W_1(Q^2,\nu) = \sum_j^{\text{partons}} z_{q_j}^2\, \frac{f_j(x)}{2M}$$

$$W_2(Q^2,\nu) = \sum_j z_{q_j}^2\, f_j(x)\, \frac{x}{\nu}$$

$F_1(x) = M W_1(Q^2, \nu)$. $F_2(x) = \nu W_2(Q^2, \nu)$

$F_2(x) = 2x F_1(x)$ Callan- Gross relation.





scaling of F as fuction of x not Q^2