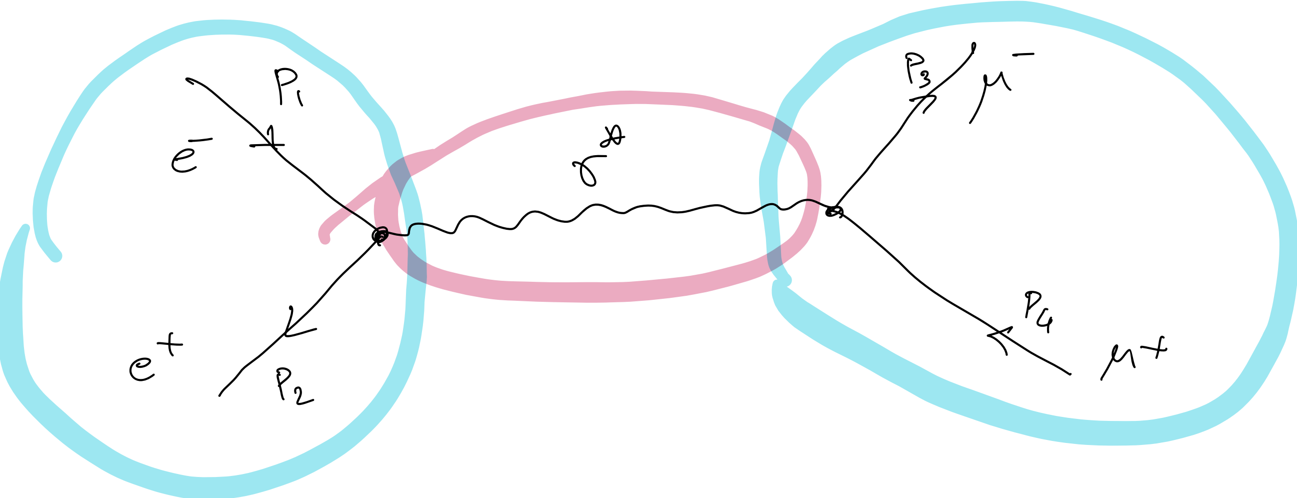
$e^+e^- \longrightarrow \mu^+\mu^-$ cross section



$$-i\mathcal{M} = \left[\bar{v}(p_2)\, ie\gamma^\mu\, u(p_1)\right] \times$$

$$\times \frac{-ig_{\mu\nu}}{q^2} \times$$

$$\times \left[\bar{u}(p_3)\, ie\gamma^\nu\, v(p_4)\right]$$

u, v spinors. e^-, e^+

$$-\frac{ig_{\mu\nu}}{s}$$

$$j^\mu_{(e)} = \bar{v}(p_2)\gamma^\mu u(p_1)$$

$$j^\nu_{(\mu)} = \bar{u}(p_3)\gamma^\nu v(p_4)$$

$$\Rightarrow \quad \mathcal{M} = -\frac{e^2}{s}\, \underline{j}_{(e)} \cdot \underline{j}_{(\mu)}$$

SPIN

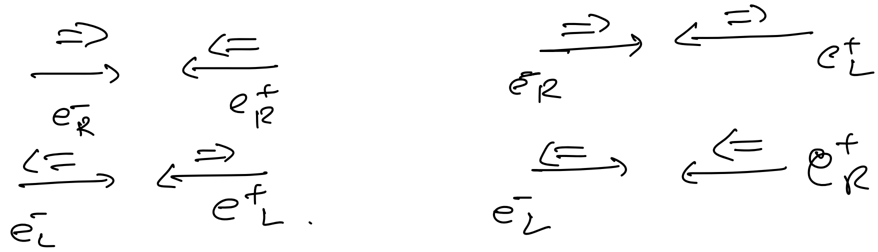
Usually e^+, e^- beams are not polarized.

$\overset{\Leftarrow}{\longrightarrow} e^-$ $\quad \overset{\Rightarrow}{\longrightarrow} e^-$ in the limit $p \gg m_e$

$$\text{helicity} = \frac{\vec{S}\cdot\vec{p}}{|\vec{S}|\cdot|\vec{p}|}$$

for massive particles: h not conserved / invariant

$\overset{\Leftarrow}{\longrightarrow}_{e^-}$ e_L $\qquad \overset{\Rightarrow}{\longrightarrow} e_R$



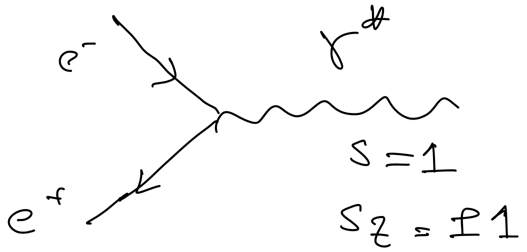
$S_z = 0$ $\qquad\qquad$ $S_z = \pm 1$

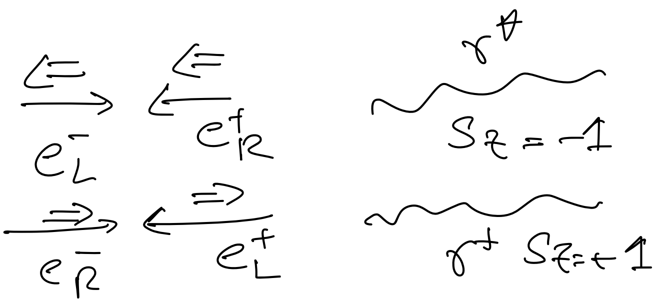
4 initial states.

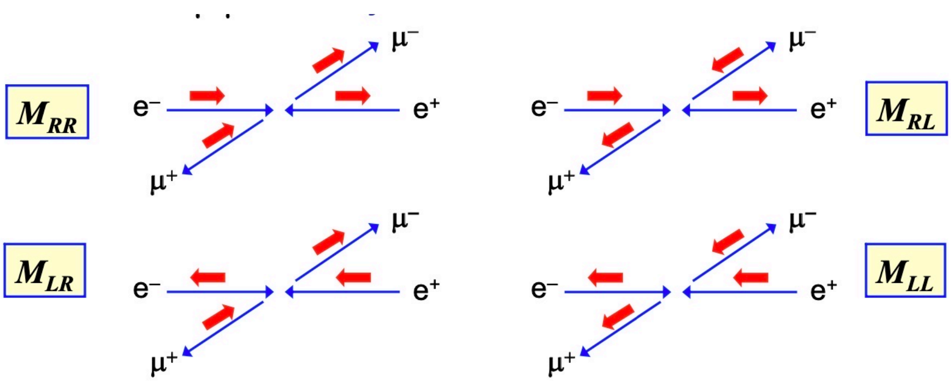
16 final states. $\qquad$ RR $\longrightarrow$ RR, RL, LR, LL

$$\langle|M|\rangle = \frac{1}{4}\sum_{\text{final states}} |M_{i\to f}|$$

Interaction







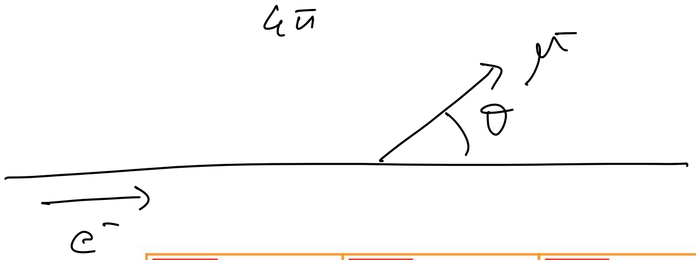
M_{AB}: $\quad$ A: e_A^- $\qquad$ B: μ_B^-

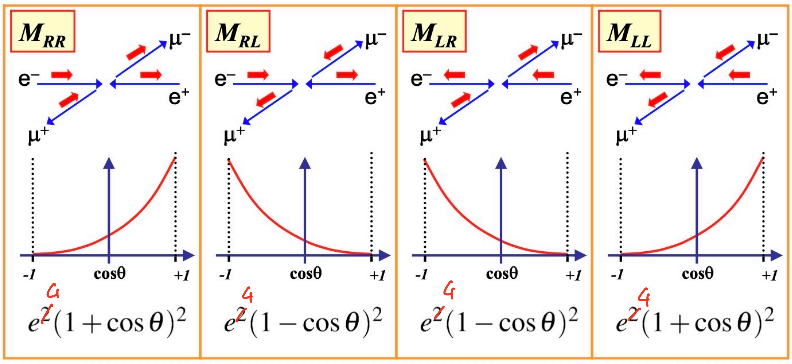
M_{RR}: $\quad e_R^- e_L^+ \longrightarrow \mu_R^- e_L^+$

$$|M_{RR}|^2 = |M_{LL}|^2 = (4\pi\alpha)^2 (1+\cos\theta)^2$$

$$|M_{RL}|^2 = |M_{LR}|^2 = (4\pi\alpha)^2 (1-\cos\theta)^2$$

$$\frac{e^2}{4\pi} = \alpha. \quad \Rightarrow \quad e^2 = 4\pi\alpha$$





$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \underbrace{\frac{1}{4}}_{\text{initial states}} \underbrace{\left(|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 \right)}_{2(1+\cos\theta)^2 + 2(1-\cos\theta)^2} (4\pi\alpha)^2$$

$$= 4 + 4\cos^2\theta.$$

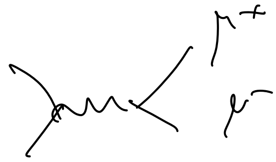
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta) \qquad \text{Differential cross section.}$$

$$\sigma_{tot} = \int \frac{d\sigma}{d\Omega}\, d\Omega. \qquad d\Omega = d\phi\, d\cos\theta.$$

$$= 2\pi\, d\cos\theta.$$

$$\frac{\alpha^2}{4s}\, 2\pi \int_{-1}^{1} (1+\cos^2\theta)\, d\cos\theta$$

$$\sigma_{tot} = \frac{16\pi}{3}\frac{\alpha^2}{s} = \sigma(e^+e^- \rightarrow \mu^+\mu^-)$$



$s =$ GeV^2. $\quad \sigma$ in barn.

$$\frac{16\pi}{3}\alpha^2 = \frac{16\pi}{3}\frac{1}{(137)^2} = 2.23\times 10^{-4}$$

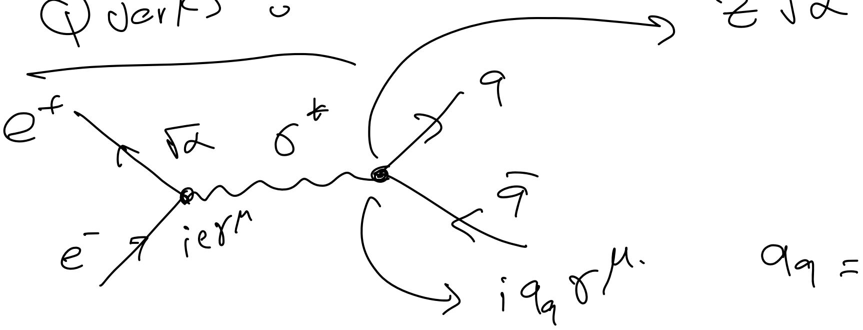
$$\hbar c = 1 = 197 \text{ MeV fm} = 0.197 \text{ GeV fm}.$$

$$1 \text{ fm} = 10^{-15} \text{ m}. \qquad 1 \text{ fm}^2 = 10^{-30} \text{ m} = 10^{-2} \text{ b}.$$

$$\sigma = \frac{86.8 \text{ nb}}{s\,[\text{GeV}^2]} \qquad s = 4E^2$$

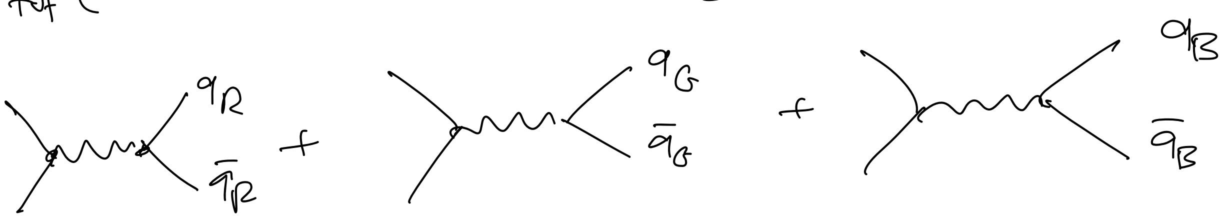
$$= \left(\frac{86.8}{4}\right) \frac{1}{E_{beam}^2\,[\text{GeV}^2]}$$

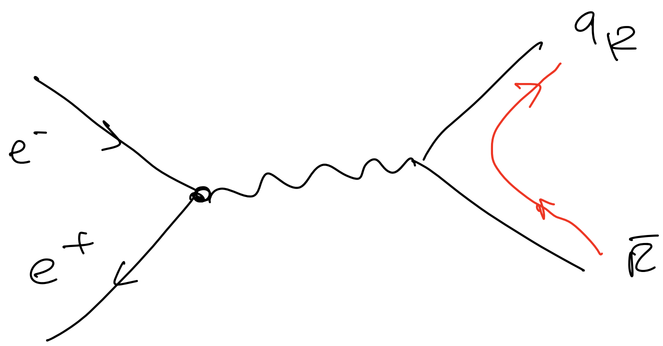
Quarks?



$$q_q = Ze \qquad Z = \begin{cases} +\frac{2}{3} & u, c, t \\ -\frac{1}{3} & d, s, b \\ 1 & \text{leptons.} \end{cases}$$

$$\sigma_{tot}(e^+e^- \to \text{hadrons}) = \frac{16\pi}{3} \frac{\alpha^2}{s} Z_f^2 \times N_C$$





$$R\Big|_{QED} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_{\text{flavors}} Z_f^2 N_C$$

$$R = R(\sqrt{s})$$

$0 < \sqrt{s} < 2m_\mu$: $\qquad e^+e^- \to e^+e^-$

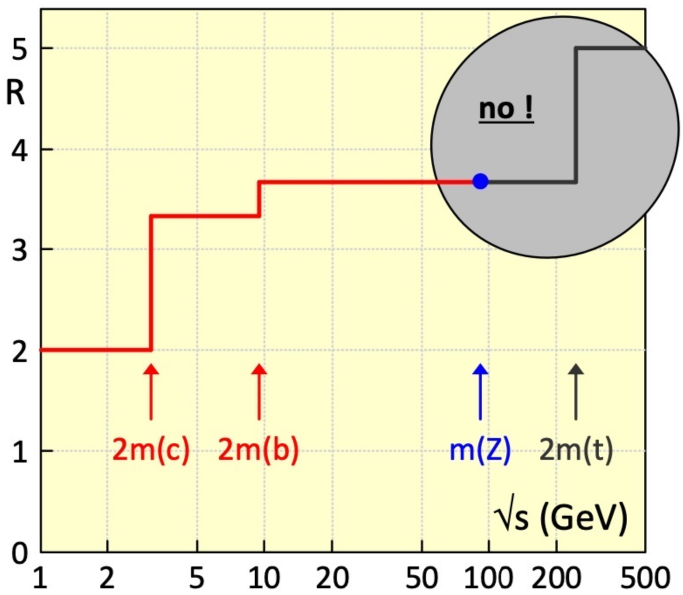
$0 < \sqrt{s} < 2m_c$ only u, d, s

$$R = \sum_{u,d,s} Z_f^2 N_c = \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] 3$$

$$= \frac{6}{9} \times 3 = 2$$

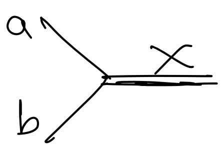
$2m_c < \sqrt{s} < 2m_b$ $\quad R = R_{uds} + (Z_c)^2 N_c = 2 + \frac{4}{9} 3 = 3 + \frac{1}{3}$

$2m_b < \sqrt{s} < 2m_t$ $\quad R = R_{udsc} + \left(-\frac{1}{3}\right)^2 N_c = 3 + \frac{1}{3} + \frac{3}{9} = 3 + \frac{2}{3}$



$\sqrt{s}$

what happens when $\sqrt{s}$ near m_X $\quad X$ resonance.



X: mass m_X
width Γ_{tot}.

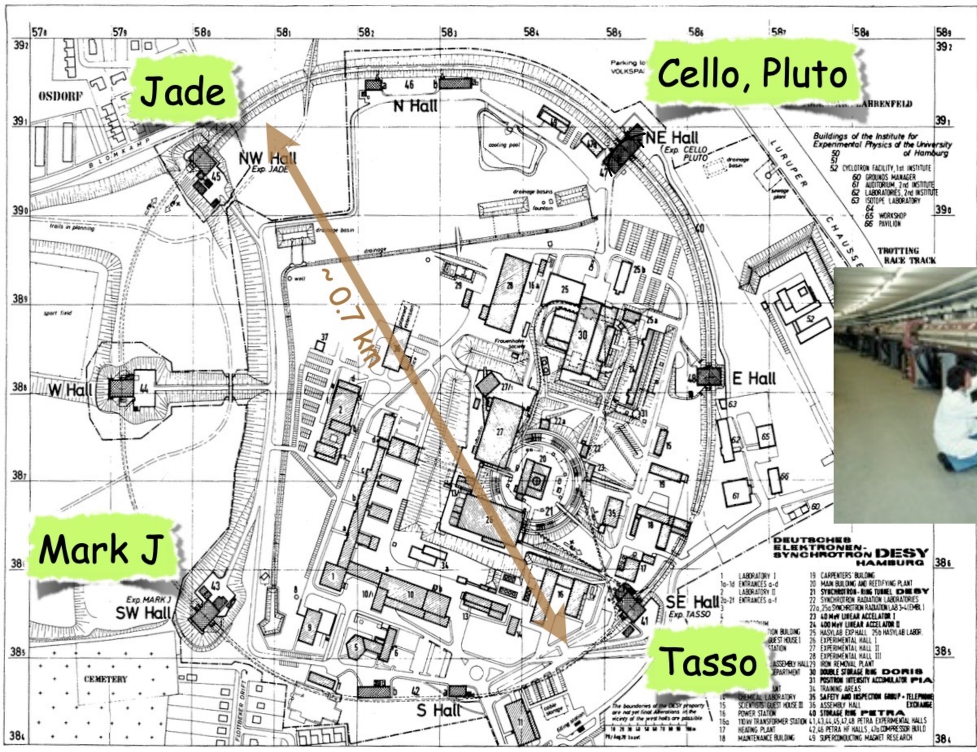
$$\sigma(e^+e^- \to X \to f\bar{f}) = \frac{16\pi}{s} \frac{(2J_X+1)}{(2S_a+1)(2S_b+1)} \left(\frac{\Gamma_{ab}}{\Gamma_{tot}}\right)\left(\frac{\Gamma_{ff}}{\Gamma_{tot}}\right) \times$$

$$\times \frac{\Gamma_{tot}^2}{(m_X - \sqrt{s})^2 + \Gamma_{tot}^2/4}.$$

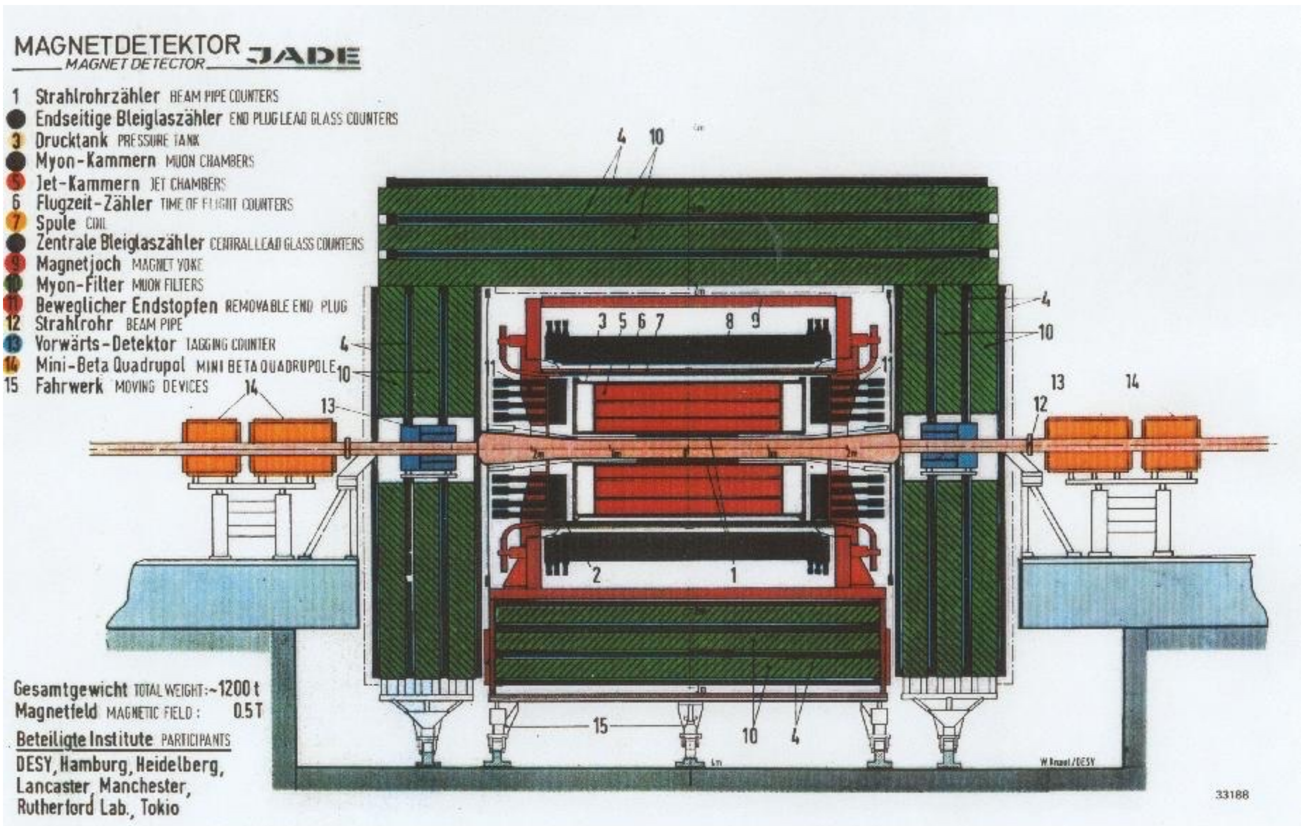
Experimental tests of QED

$$e^+e^- \longrightarrow e^+e^-$$
$$\mu^+\mu^-$$
$$\text{hadrons.}$$

PETRA @ DESY $\qquad$ e^+e^- collider $\quad$ 1978 → 1986

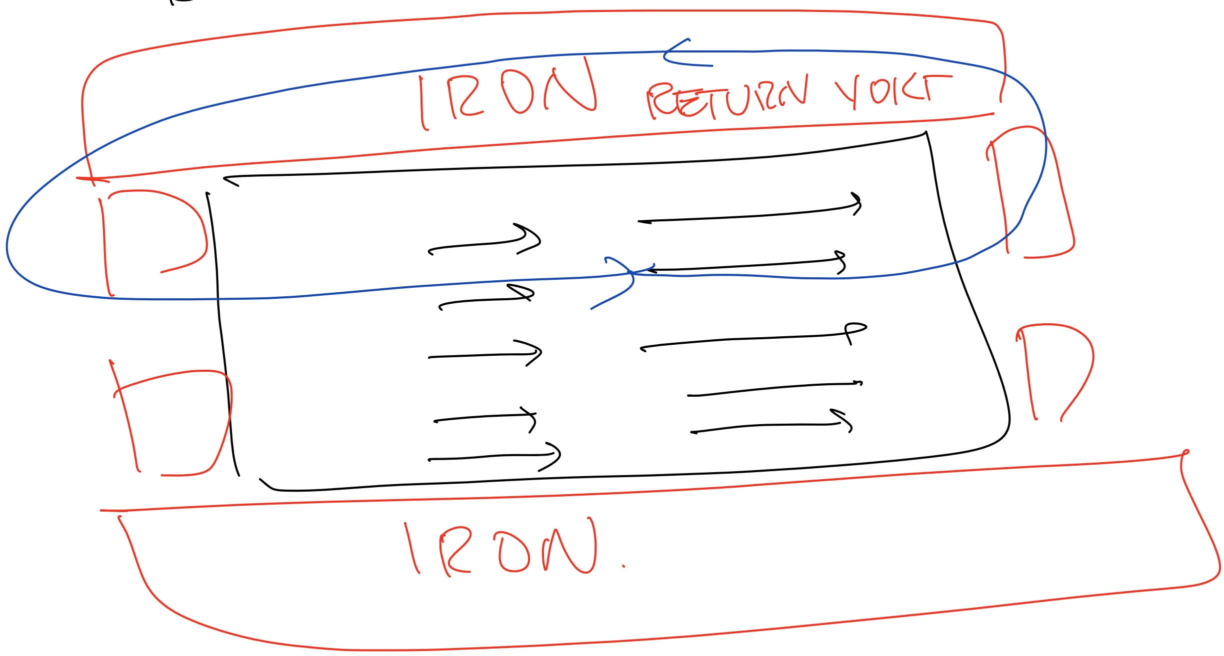


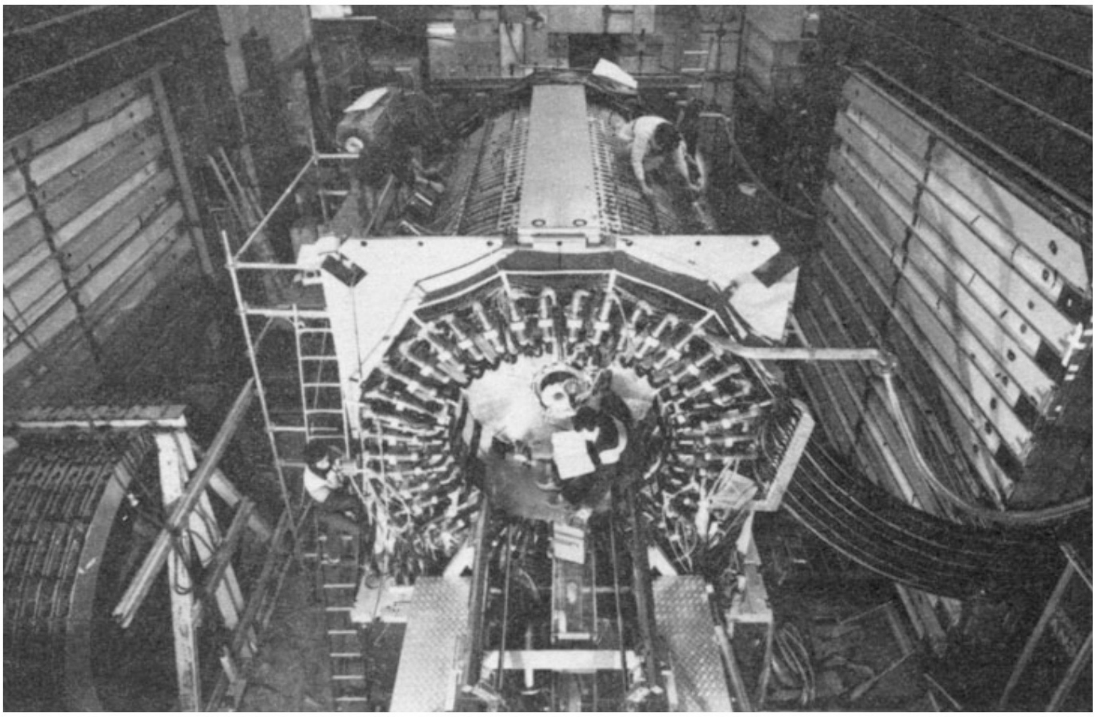
JADE: Japan, Deutsch. and England.



$P = 0.3 \times B[T] \times R[m]$.

$B = 0.5 \Rightarrow p[GeV] = 0.15 \times R[m]$

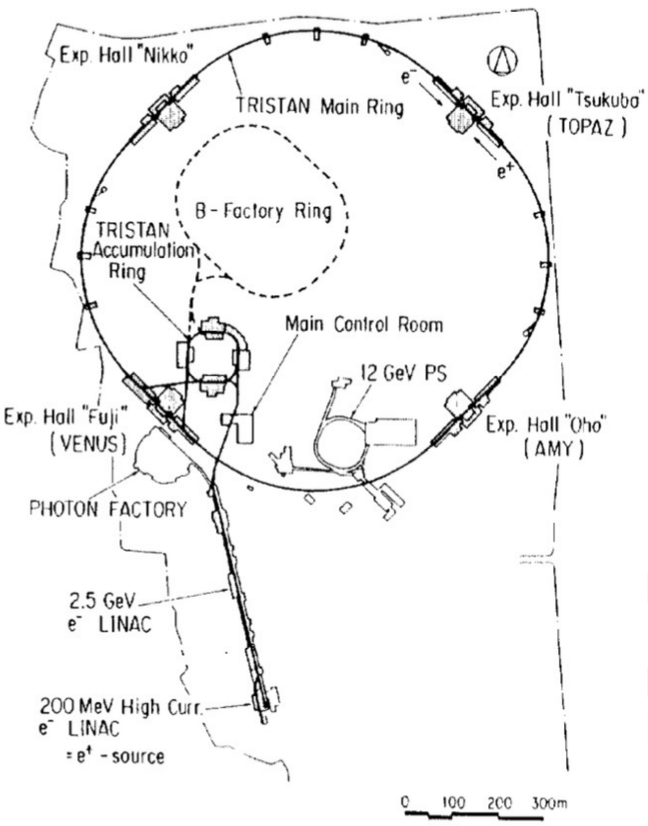




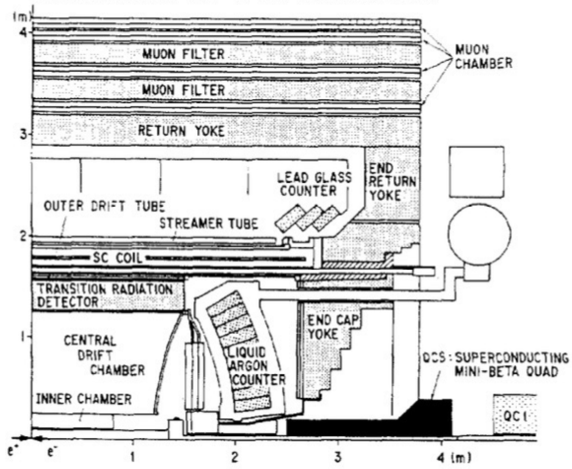
JADE

Experiment

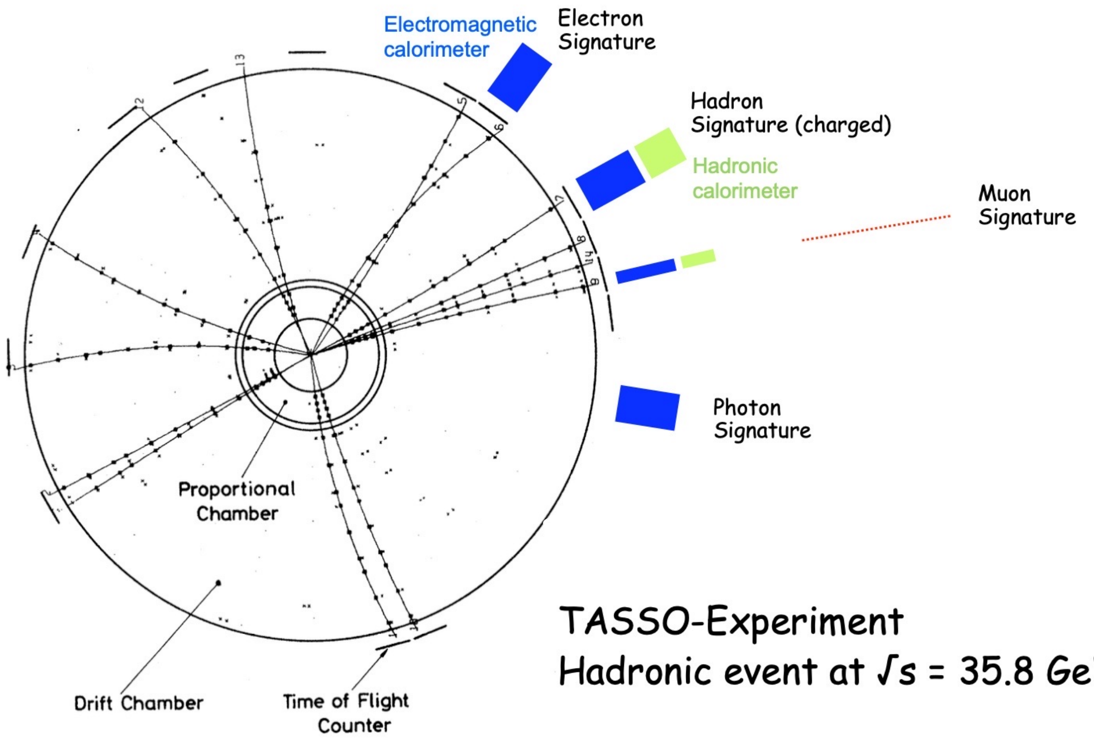
TRISTAN @ JAPAN



Schema of the detector



Max. beam energy: 32 GeV
Injection energy: 8 GeV
Beam lifetime: 5-6 hr.
Peak luminosity: 1.4×10^{31} $cm^{-2}s^{-1}$



TASSO-Experiment
Hadronic event at $\sqrt{s}$ = 35.8 GeV

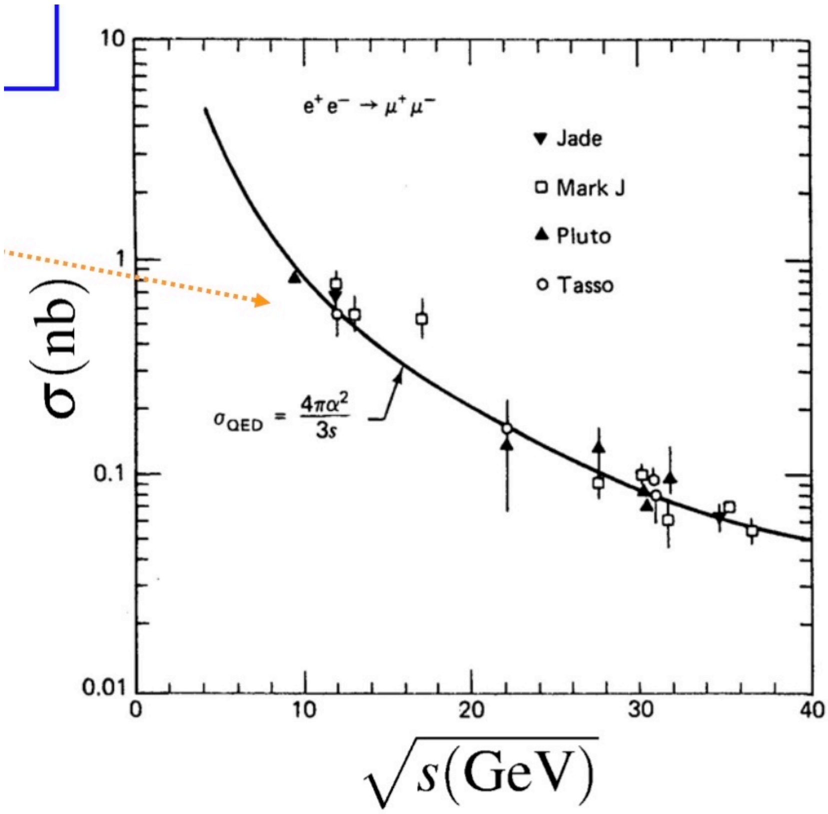
$$e^+e^- \longrightarrow e^+e^-$$

$$\sigma \simeq \frac{86.8\ \text{nb}}{s\,[\text{GeV}^2]}.$$

$\sqrt{s} = 10$ GeV.

$s = 100$ GeV^2

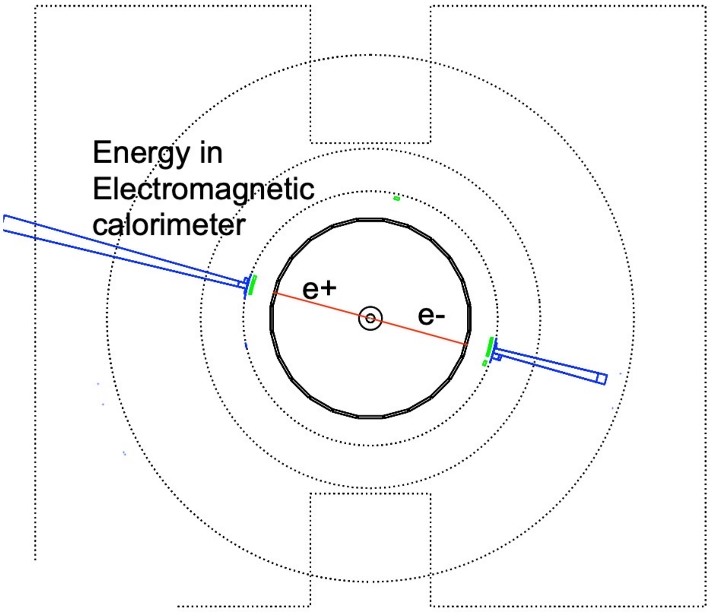
$$\sigma \simeq \frac{\cancel{100x}}{\cancel{100}}\ \text{nb}$$



Total crossection.

$$N = \sigma_{e^+e^-}\, \mathcal{L}_{inst.}\, \Delta t.$$

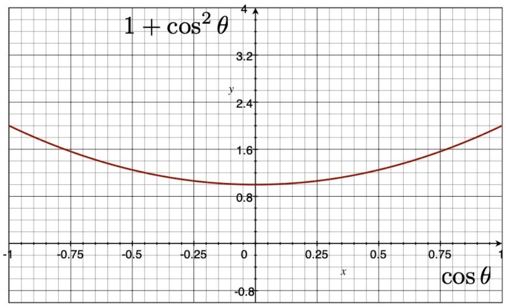
↳ how many times. e^+e^- in the detector



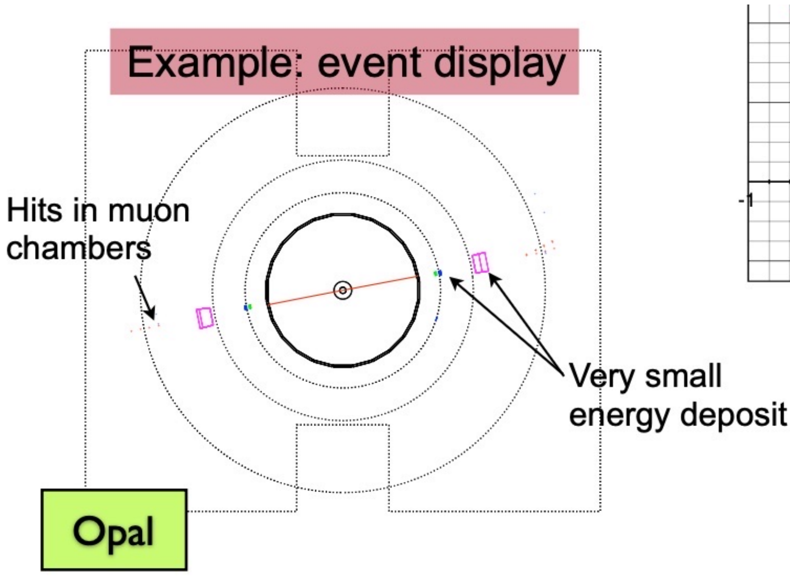
$e^+e^- \to e^+e^-$

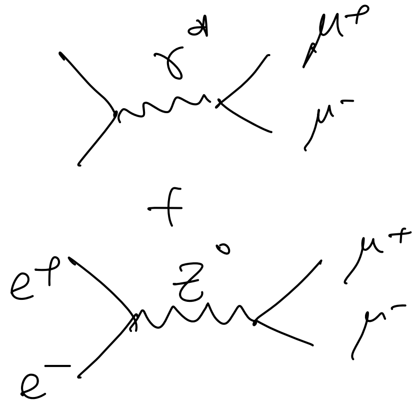
what about $\frac{d\sigma}{d\Omega} = \frac{1}{2\pi}\frac{d\sigma}{d\cos\theta} \propto (1+\cos^2\theta).$

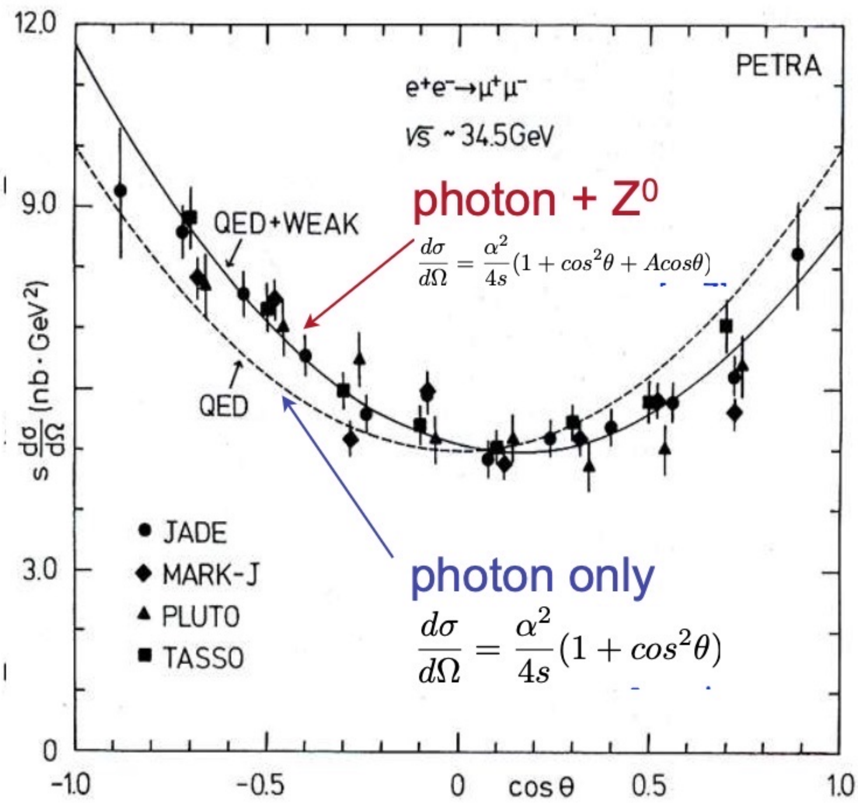
Differential cross section.

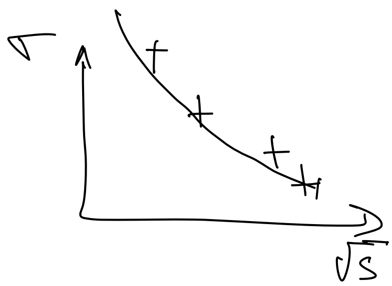


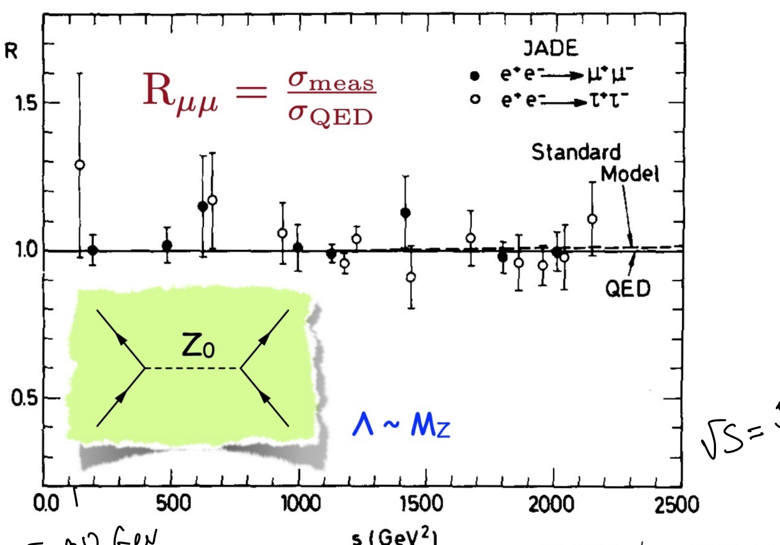
$e^+e^- \rightarrow \mu^+\mu^-$











$\sqrt{s} = 50$ GeV

$\sqrt{s}$ 40 GeV

PETRA √s < 47 GeV

$e^+e^- \longrightarrow e^+e^-$ (Bhabha scattering) σ_1

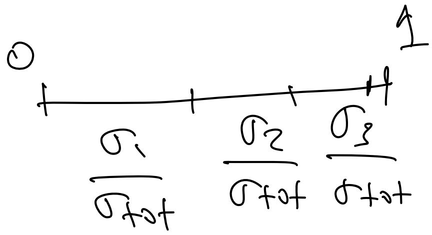
$\mu^+\mu^-$ σ_2

hadrons σ_3

$N_{tot} = (\sigma_{tot}) \times \mathcal{L}_{inst.}\ \Delta t$

$\hookrightarrow \sigma_1 + \sigma_2 + \sigma_3$

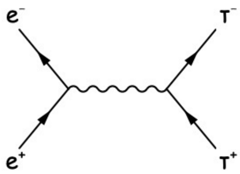
$\frac{\sigma_1}{\sigma_{tot}}$: fraction of e^+e^- events.



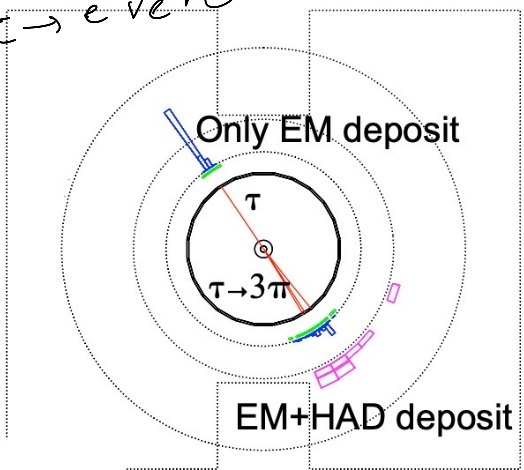
If $\sigma_1 >> \sigma_3$

$m_\tau \simeq 1.78$ GeV

τ decays



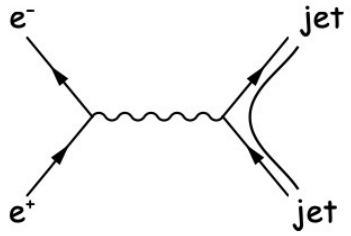
$\tau \to e^- \bar{\nu}_e \nu_\tau$



$\sqrt{s} > 3.6$ GeV

Modes with one charged particle

Γ_1	particle$^- \geq 0$ neutrals $\geq 0K^0\nu_\tau$ ("1-prong")		(85.24 ± 0.06) %	
Γ_2	particle$^- \geq 0$ neutrals $\geq 0K^0_L\nu_\tau$		(84.58 ± 0.06) %	
Γ_3	$\mu^-\overline{\nu}_\mu\nu_\tau$	[a]	(17.39 ± 0.04) %	
Γ_4	$\mu^-\overline{\nu}_\mu\nu_\tau\gamma$	[b]	$(3.67 \pm 0.08) \times 10^{-3}$	
Γ_5	$e^-\overline{\nu}_e\nu_\tau$	[a]	(17.82 ± 0.04) %	
Γ_6	$e^-\overline{\nu}_e\nu_\tau\gamma$	[b]	(1.83 ± 0.05) %	
Γ_7	$h^- \geq 0K^0_L\ \nu_\tau$		(12.03 ± 0.05) %	
Γ_8	$h^-\nu_\tau$		(11.51 ± 0.05) %	
Γ_9	$\pi^-\nu_\tau$	[a]	(10.82 ± 0.05) %	
Γ_{10}	$K^-\nu_\tau$	[a]	$(6.96 \pm 0.10) \times 10^{-3}$	
Γ_{11}	$h^- \geq 1$ neutralsν_τ		(37.01 ± 0.09) %	
Γ_{12}	$h^- \geq 1\pi^0\nu_\tau(\text{ex.}K^0)$		(36.51 ± 0.09) %	
Γ_{13}	$h^-\pi^0\nu_\tau$		(25.93 ± 0.09) %	
Γ_{14}	$\pi^-\pi^0\nu_\tau$	[a]	(25.49 ± 0.09) %	
Γ_{15}	$\pi^-\pi^0\text{non-}\rho(770)\nu_\tau$		$(3.0 \pm 3.2) \times 10^{-3}$	
Γ_{16}	$K^-\pi^0\nu_\tau$	[a]	$(4.33 \pm 0.15) \times 10^{-3}$	
Γ_{17}	$h^- \geq 2\pi^0\nu_\tau$		(10.81 ± 0.09) %	
Γ_{18}	$h^-2\pi^0\nu_\tau$		(9.48 ± 0.10) %	
Γ_{19}	$h^-2\pi^0\nu_\tau(\text{ex.}K^0)$		(9.32 ± 0.10) %	
Γ_{20}	$\pi^-2\pi^0\nu_\tau(\text{ex.}K^0)$	[a]	(9.26 ± 0.10) %	
Γ_{21}	$\pi^-2\pi^0\nu_\tau(\text{ex.}K^0)$, scalar		$< 9 \times 10^{-3}$	CL=95%
Γ_{22}	$\pi^-2\pi^0\nu_\tau(\text{ex.}K^0)$, vector		$< 7 \times 10^{-3}$	CL=95%
Γ_{23}	$K^-2\pi^0\nu_\tau(\text{ex.}K^0)$	[a]	$(6.5 \pm 2.2) \times 10^{-4}$	
Γ_{24}	$h^- \geq 3\pi^0\nu_\tau$		(1.34 ± 0.07) %	
Γ_{25}	$h^- \geq 3\pi^0\nu_\tau(\text{ex. }K^0)$		(1.25 ± 0.07) %	
Γ_{26}	$h^-3\pi^0\nu_\tau$		(1.18 ± 0.07) %	

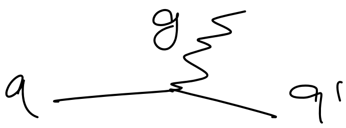


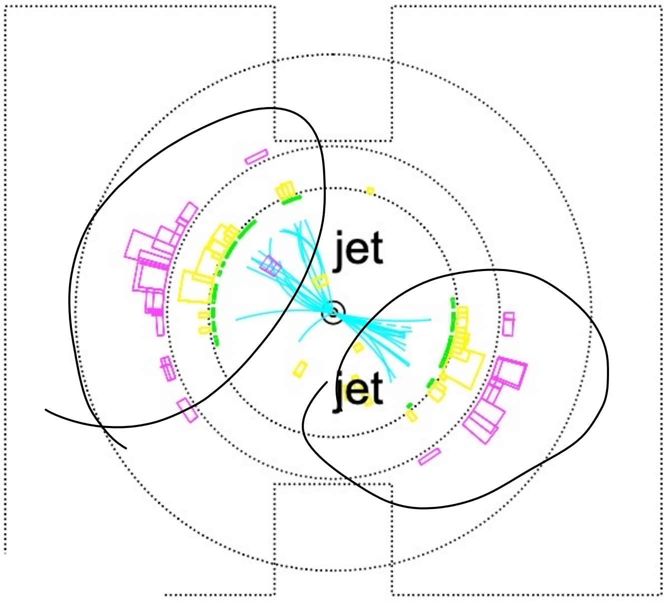
$e^+e^- \longrightarrow q\,\bar{q}$

$q \longrightarrow$ Convert int hadrons.

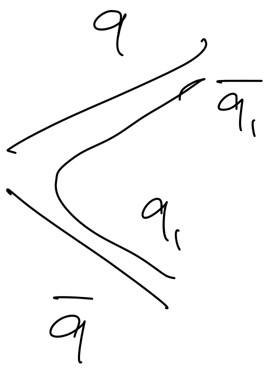
Hadronization.

mesons, baryons.





$\sqrt{s} \simeq 2m_q + \varepsilon$



$q\bar{q}_1$

$\bar{q}q_1$