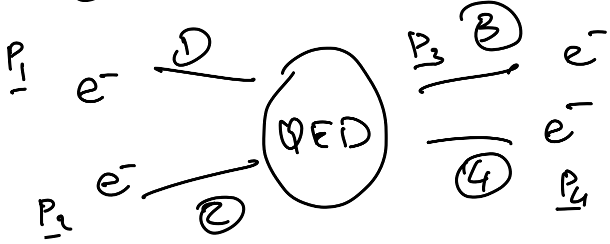
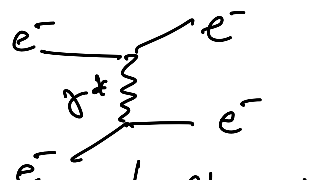
e^+
e^-

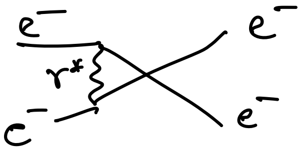
$e^- + e^- \longrightarrow e^- + e^-$ Möller scattering.



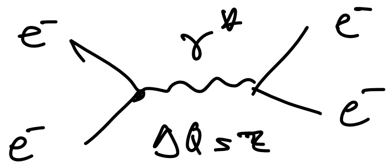


t-channel

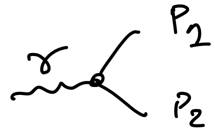
+



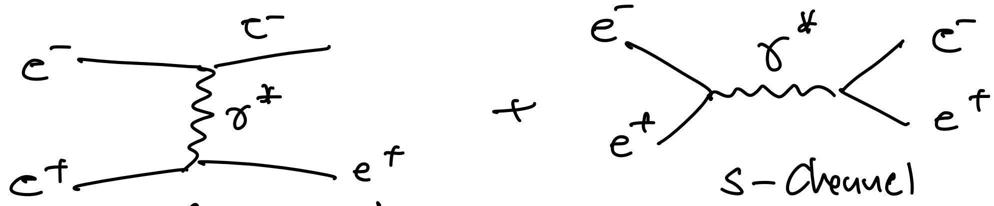
u-channel.



not allowed.

QED: 

$e^- + e^+ \longrightarrow e^- + e^+$



t-channel

s-channel
annihilation diagram.

$$s = (\underline{p}_1 + \underline{p}_2)^2$$
$$t = (\underline{p}_1 - \underline{p}_3)^2 \qquad \underline{p}_3 = \underline{p}_1 + \underline{q}$$
$$u = (p_1 - p_4)^2$$

$\underline{p}_1 + \underline{p}_2 = \underline{p}_3 + \underline{p}_4$
En-mom conservation.

$$s = p_1^2 + p_2^2 + 2\underline{p}_1 \cdot \underline{p}_2 = m_1^2 + m_2^2 + 2\underline{p}_1 \cdot \underline{p}_2$$
$$t = p_1^2 + p_3^2 - 2\underline{p}_1 \cdot p_3 = m_3^2 + p_1^2 - 2\underline{p}_1 \cdot \underline{p}_3 .$$
$$u = p_1^2 + p_4^2 - 2\underline{p}_1 \cdot \underline{p}_4 = m_4^2 + p_1^2 - 2\underline{p}_1 \cdot \underline{p}_4$$
$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 + 2p_1^2 + 2\underline{p}_1 \underline{p}_2 - 2\underline{p}_1 \cdot \underline{p}_3 - 2\underline{p}_1 \cdot \underline{p}_4$$
$$= \sum_i m_i^2 + 2\underline{p}_1 \cdot \underbrace{(\underline{p}_1 + \underline{p}_2 - \underline{p}_3 - \underline{p}_4)}_{=0} = \sum_i m_i^2$$

at high energy limit $E_i \gg m_i \Rightarrow s + t + u = 0$.

$\underline{p}_1 \longrightarrow \longleftarrow \underline{p}_2$

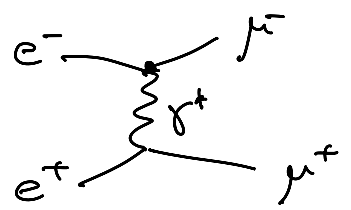
Center of mass reference frame
LAB frame with symm. beams

$$\underline{p}_1 = (E, \vec{p}) \qquad \underline{p}_2 = (E, -\vec{p})$$
$$s = (E + E, 0)^2 = 4E^2 \Rightarrow \sqrt{s} = 2E$$

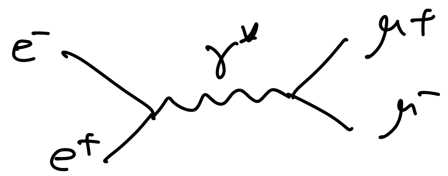
$t = (p_1 - p_3)^2 = 4E^2 \sin^2\frac{\theta}{2}$ (remember from DIS)

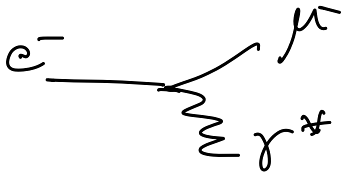
$= S x \sin^2\frac{\theta}{2}$.

$e^+e^- \to \mu^+\mu^-$ in QED.

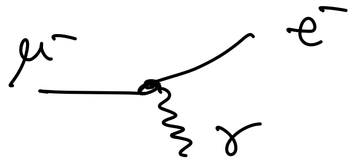


+





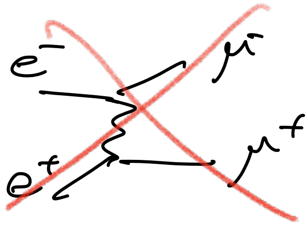
if possible $\Rightarrow$



	μ^-	$\to e^-\gamma$
L_e	0	1
L_μ	1	0

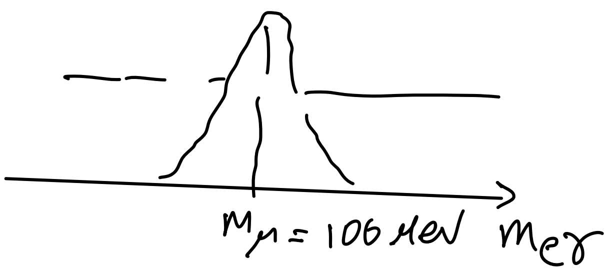
$M_e = 0.5$ MeV $\quad M_\mu = 106$ MeV.

$\Rightarrow$

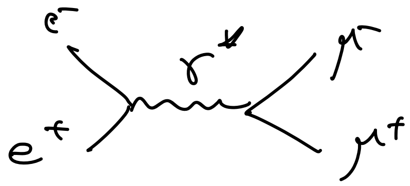


lepton number violation.

$\Rightarrow$ not in QED

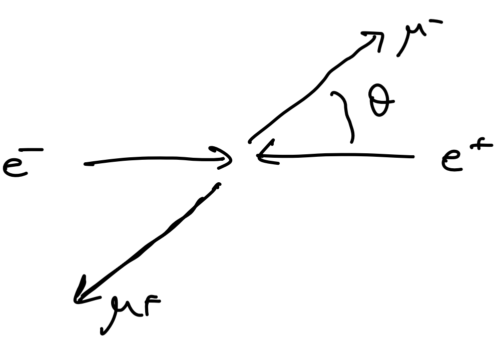


$e^-e^+ \to \mu^-\mu^+$



annihilation diagram.

$\underline{p_3} = (E_{\mu^-}, \vec{p}_{\mu^-}) \qquad \underline{p_4} = (E_{\mu^+}, \vec{p}_{\mu^+})$

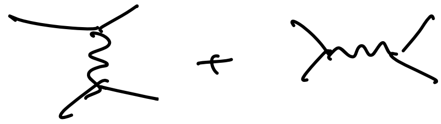


$\vec{p}_{\mu^-} = -\vec{p}_{\mu^+}$

$\sqrt{s} = 2m_\mu$ minimum energy needed

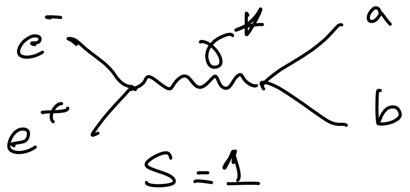
$e^-e^+ \to \mu^+\mu^-$ only if $\sqrt{s} \geq 2m_\mu \Rightarrow 2E \geq 2m_\mu$.

$\Rightarrow E \geq m_\mu$.

$e^+e^- \longrightarrow e^+e^-$ Bhabha 

$\sqrt{s} < 2m_\mu$

$E_e \geq 106$ MeV $\gg m_e$



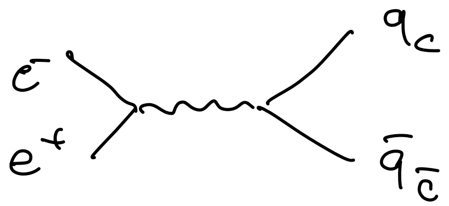
a particle 1

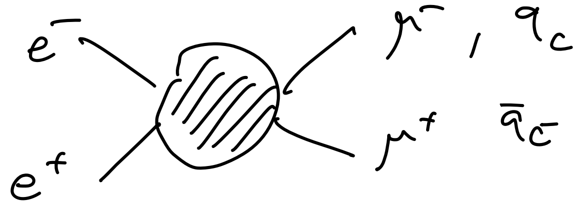
b Particle 2

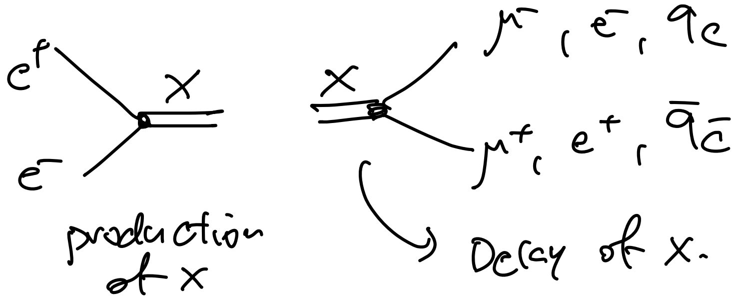
$Q_a + Q_b = 0$

$QN_a = - QN_b$ QN: quantum number.

Final state particle - antiparticle pair.

$\Rightarrow$ possible 

Experimentally 

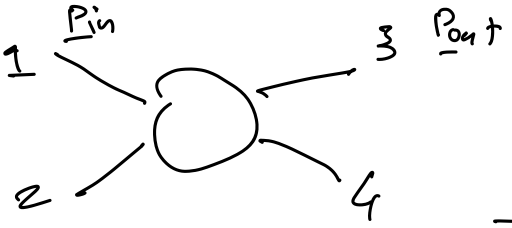
also possible 

$\sqrt{s} \geq m_X$ for production of X.

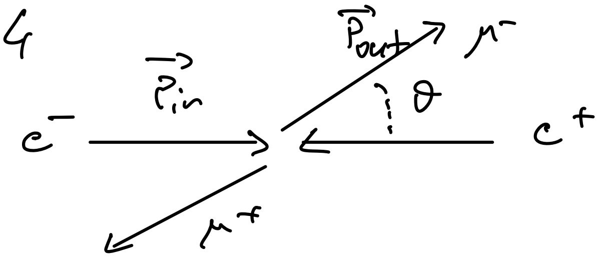
what is the cross section?

$$\sigma(e^+e^- \rightarrow q_c \bar{q}_{\bar{c}})$$

easier to start with $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$

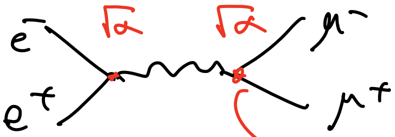


$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{(E_1+E_2)^2} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} |M|^2$$



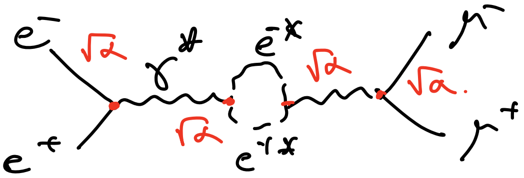
$(E_1+E_2)^2 = s$

if $E \gg m_{e,\mu} \Rightarrow |\vec{P}_{out}| = |\vec{P}_{in}|$

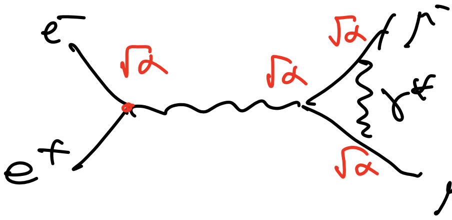
$\mathcal{M}_1 =$ 

$\sim \alpha$ $\quad ie\gamma^\mu$ $\quad \alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}|\mathcal{M}|^2$$

you could also have 

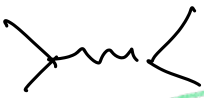
$\sim \alpha^2$ $\quad \mathcal{M}_2$

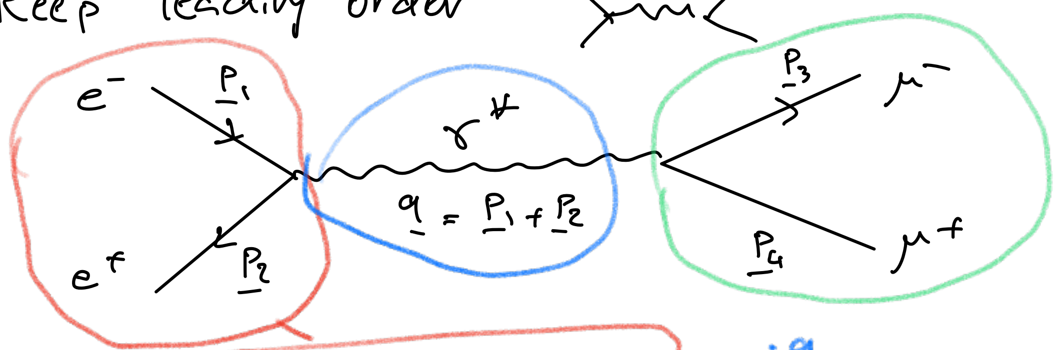


$\sim \alpha^2$ $\quad \mathcal{M}_3.$

$$|\mathcal{M}_{tot}|^2 = |\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3|^2 = \underset{\alpha^2}{|\mathcal{M}_1|^2} + \underset{\alpha^4}{|\mathcal{M}_2|^2} + \underset{\alpha^4}{|\mathcal{M}_3|^2} + \text{interference} + O(\alpha^3) + O(\alpha^4)$$

$$1 + 10^{-4} \qquad 10^{-4} \qquad 10^{-3} \qquad 10^{-4}$$

Keep leading order 



$q^2 = s$

$$-i\mathcal{M} = \boxed{[\bar{v}(P_2)\, ie\gamma^\mu u(P_1)]}\ \frac{-ig_{\mu\nu}}{q^2}\ [\bar{u}(P_3)\, ie\gamma^\nu v(P_4)]$$

Spin configurations

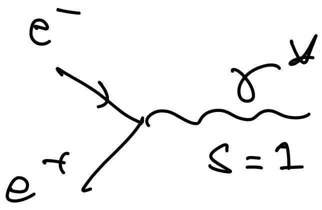
e^-_L $\xrightarrow{\Leftarrow}$ $\xleftarrow{\Leftarrow}$ e^+_R ✓	$\xrightarrow{z}$	$S_z = -1$
e^-_L $\xrightarrow{\Leftarrow}$ $\xleftarrow{\Rightarrow}$ e^+_L ✗		$S_z = 0$
e^-_R $\xrightarrow{\Rightarrow}$ $\xleftarrow{\Leftarrow}$ e^+_R ✗		$S_z = 0$
e^-_R $\xrightarrow{\Rightarrow}$ $\xleftarrow{\Rightarrow}$ e^+_L ✓		$S_z = +1$

Final states: $\mu^-_L \mu^+_L$ $\Rightarrow$ 16 combinations.

$\mu^-_R \mu^+_R$

$\mu^-_L \mu^+_R$

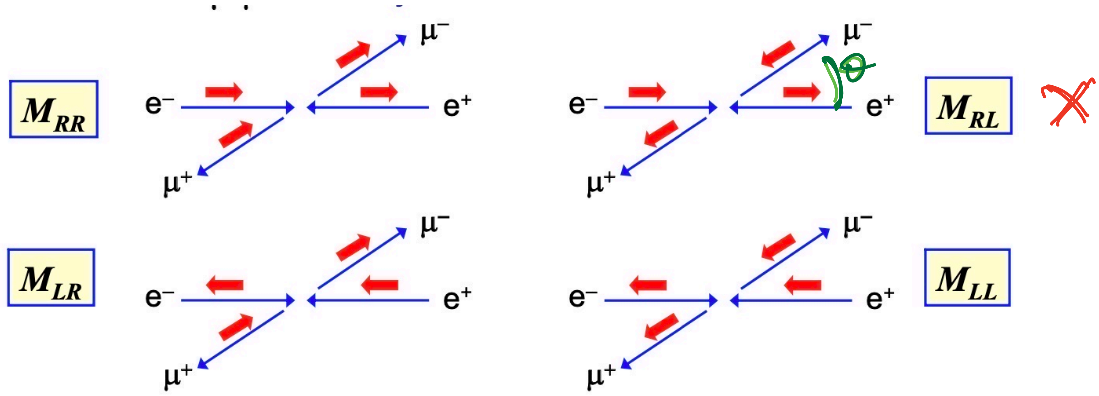
$\mu^-_R \mu^-_L$



$\Rightarrow$ $e^-_L e^+_L$ $(S_z=0)$ not possible.

$e^-_R e^+_R$ $(S_z=0)$

only consider initial states with $S_z = \pm 1$



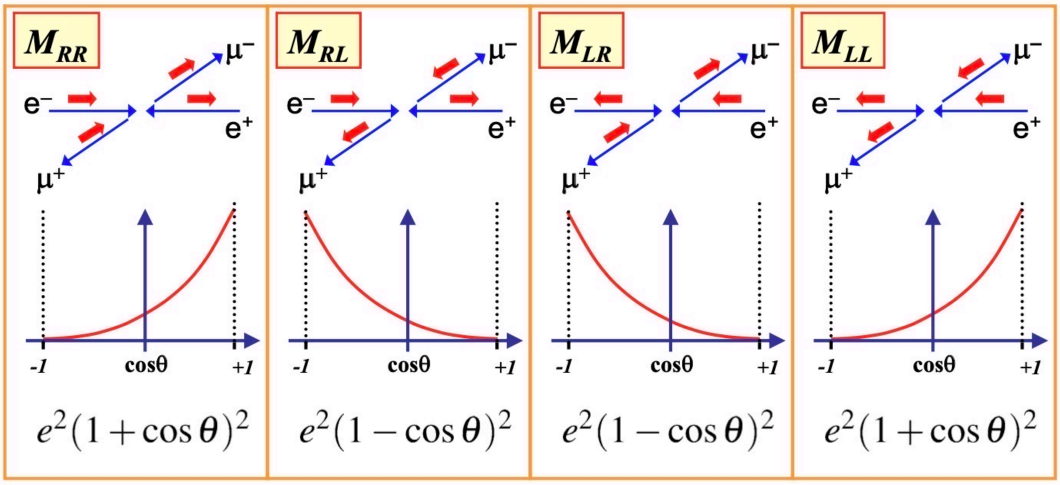
M_{AB}: A: e^-_A B: μ^-_B

$S_z = +1$ $S_z = -1$

$S_z = +1$ $S_z = +1$

$$|M_{\underset{e^-\mu^-}{RR}}|^2 = |M_{LL}|^2 = (4\pi\alpha)^2 (1+\cos\theta)^2.$$

$$|M_{RL}|^2 = |M_{LR}|^2 = (4\pi\alpha)^2 (1-\cos\theta)^2$$



sum over all final states.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\,\frac{1}{s}\,\frac{1}{4}\left[|M_{LL}|^2+|M_{RR}|^2+|M_{RL}|^2+|M_{LR}|^2\right].$$

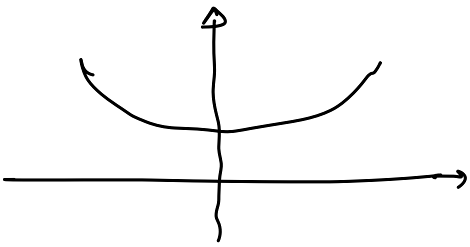
↳ average over initial states.

$$2(1+\cos\theta)^2 + 2(1-\cos\theta)^2 = 4+4\cos^2\theta = 4(1+\cos^2\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\,\frac{1}{s}\,\frac{1}{4}\,4\,(1+\cos^2\theta)$$

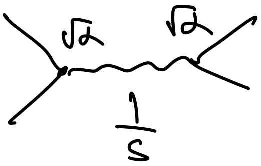
Differential cross section.

Max for $\theta = 0$. $\overset{e^-}{\longrightarrow} \longleftarrow e^+ \Rightarrow \mu^+ \longleftarrow \longrightarrow \mu^-$



Total cross section: $d\Omega = 2\pi \sin\theta\, d\theta$

$$\sigma_{tot} = \int_0^{\pi} \left(\frac{d\sigma}{d\Omega}\right) 2\pi \sin\theta\, d\theta = \frac{16\pi}{3}\,\frac{\alpha^2}{s}$$



$$\frac{16\pi}{3}\alpha^2 = 2.23\times10^{-4}$$

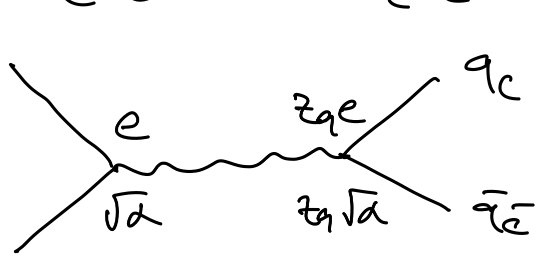
express s in GeV^2 ($\sqrt{s} = 2E$ in GeV) $\quad \hbar c = 197$ fm×MeV.

$$\sigma = \left(\frac{86.8}{4}\right)\frac{1}{E_{beam}^2[\text{GeV}]} = \frac{86.8\ \text{nb}}{s[\text{GeV}^2]} \qquad e^+e^- \to \mu^+\mu^-$$

Consider $e^+e^- \to q_c \bar{q}_{\bar{c}}$

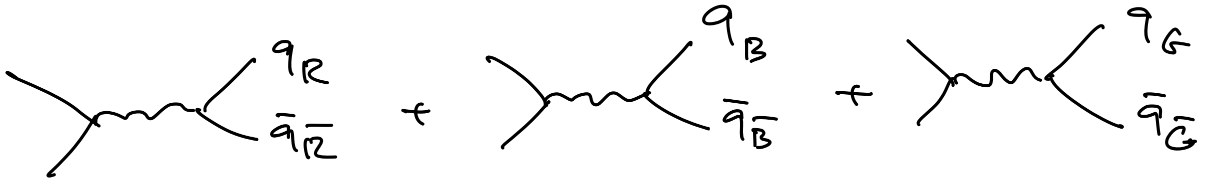
q: spin 1/2.
massless approximation.
1 flavor

$\mathcal{M} \sim$



$$\mathcal{M}^2 \sim z_q^2$$

$$\sigma(e^+e^- \to q_c\bar{q}_c) = \sigma(e^+e^- \to \mu^+\mu^-)\, z_q^2 \times \text{color?}$$



$\equiv \mathcal{M}_{q\bar{q}}$

$$|\mathcal{M}_{q\bar{q}}|^2 = |\mathcal{M}_{q_R\bar{q}_{\bar{R}}}|^2 + |\mathcal{M}_{q_B\bar{q}_{\bar{B}}}|^2 + |\mathcal{M}_{q_G\bar{q}_{\bar{G}}}|^2$$

$$\sigma(e^+e^- \to q\bar{q}) = N_C \times z_q \times \sigma(e^+e^- \to \mu^+\mu^-)$$

For one flavor.

How many flavors? Depends on $\sqrt{s}$.

$$\sigma(e^+e^- \to q\bar{q}) = \sum_{\substack{i \\ \text{flavors}}} N_C\, z_{q_i}^2\, \sigma(e^+e^- \to \mu^+\mu^-)$$

$\hookrightarrow \sqrt{s} \geq m_{q_i}$ mass of flavor i

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{N(e^+e^- \to \text{hadrons})}{N(e^+e^- \to \mu^+\mu^-)} = \sum_i z_{q_i}^2 N_C$$

$\sqrt{s} > m_u, m_d, m_s$

$$R = 3\Big[\underbrace{\left(+\tfrac{2}{3}\right)^2}_{q_u} + \underbrace{\left(-\tfrac{1}{3}\right)^2}_{q_d} + \underbrace{\left(-\tfrac{1}{3}\right)^2}_{q_s}\Big] = \frac{4+1+1}{9}\,3 = 2$$