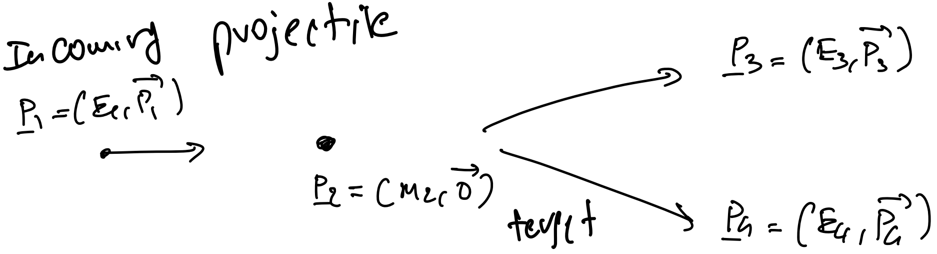


$1+2 \longrightarrow 3+4$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\,\frac{1}{m_2^2}\,\frac{|\vec{P}_{out}|}{|\vec{P}_{in}|}\,|\mathcal{M}|^2$$

Rutherford: $\alpha + N \longrightarrow \alpha + N.$ $\quad m_\alpha/m_N \simeq \frac{4}{79}$

$\gamma + e^- \longrightarrow \gamma + e^-$

$e^- + p \longrightarrow e^- + p.$ $\quad m_e/m_p \simeq 1862$

α particle: $K = 5$ MeV $\qquad E = m + K \qquad E^2 = p^2 + m^2$

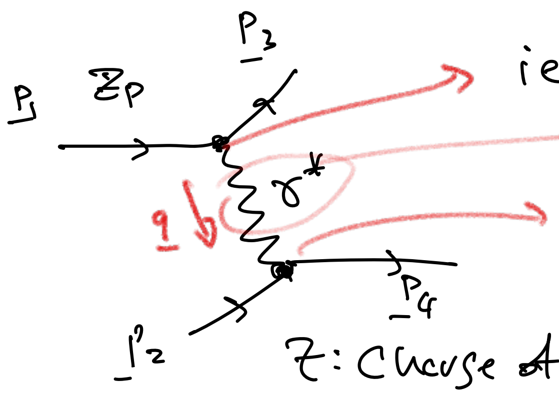
$m = 3.7$ GeV

$K \ll m \Rightarrow K \approx \frac{p^2}{2m}$

electron: $m_e = 0.5$ MeV. $\qquad K = 5$ MeV $\Rightarrow E = 5.5$ MeV.

$E^2 = p^2 + m^2 \Rightarrow p^2 = (5.5)^2 - (0.5)^2 \simeq E^2.$

$\Rightarrow \beta = \frac{p}{E} \sim 1$



$ieZ_p = i\sqrt{\alpha}\, Z_p \qquad \bar{u}\gamma^\mu u$

$$\frac{-ig_{\mu\nu}}{q^2}$$

$ieZ_T = i\sqrt{\alpha}\, Z_T \qquad \bar{u}\gamma^\nu u$

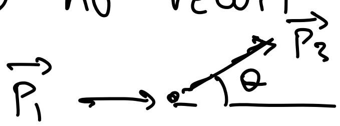
Z: Charge of target

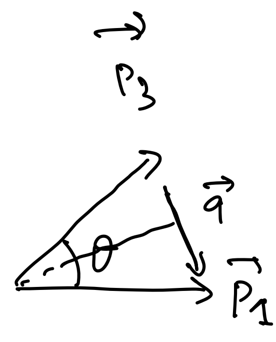
$P_1 = P_3 + q \Rightarrow q = P_1 - P_3 \qquad \mathcal{M} \sim \frac{1}{q^2}$

Rutherford: $\alpha + N \longrightarrow \alpha + N$

$m_p = m_1 = m_\alpha \ll m_T = m_2 = m_N$

$\vec{P}_4 \simeq 0$ no recoil $\Rightarrow |\vec{P}_3| \simeq |\vec{P}_1|$





$$|\vec{q}| = 2|\vec{P}_1|\sin\frac{\theta}{2} \Rightarrow q^2 = 4 P_{in}^2 \sin^2\frac{\theta}{2}$$

$$|\mathcal{M}|^2 = \left(\frac{\alpha m_T}{q^2}\right)^2 \left(m_1^2 + P_{in}^2 \cos^2\frac{\theta}{2}\right)$$

Mott Formula

$m_T \equiv m_2$: mass target $\quad m_1 \equiv m_p$: mass of projectile.

$$\frac{d\sigma}{d\Omega} \sim \frac{1}{\cancel{m_T}^2}\left(\frac{\alpha \cancel{M_T}}{q^2}\right)^2 \left(m_1^2 + P_{in}^2 \sin^2\frac{\theta}{2}\right) \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|}$$

Rutherford: $q^2 = 4 P_{in}^2 \sin^2\frac{\theta}{2}$. $\quad P_{in} \ll m_1$ (non relativistic)

$$P_1 \equiv P_{in} \simeq P_{out} \equiv P_3$$

$$\frac{d\sigma}{d\Omega} \simeq \frac{1}{16\, P_{in}^4 \sin^4\frac{\theta}{2}}\, m_e^2$$

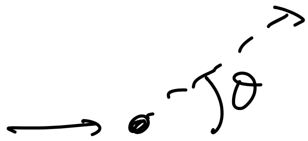
$$m_1^2 + P_{in}^2 \cos^2\frac{\theta}{2} = m_1^2 + P_1^2\left(1 - \sin^2\frac{\theta}{2}\right) =$$

$$= m_1^2 + P_1^2 - P_1^2 \sin^2\frac{\theta}{2} = E_1^2 - P_1^2 \sin^2\frac{\theta}{2}$$

$$= E_1^2\left(1 - \left(\frac{P_1}{E_1}\right)^2 \sin^2\frac{\theta}{2}\right) = E_1^2\left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

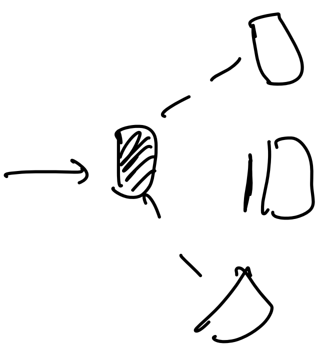
$e^- + N \to e^- + N$ collision.

$$\frac{d\sigma}{d\Omega} \sim \underbrace{(\ldots)\,\frac{1}{q^2}\,\frac{|\vec{P}_{out}|}{|\vec{P}_{in}|^2}\, E_1^2}_{\text{Rutherford}} \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$



$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

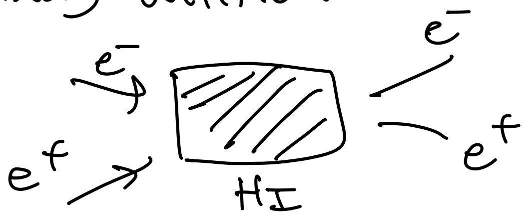
Differential cross section.



$N_{events} \propto \sigma \cdot L$

$$\int \left(\frac{d\sigma}{d\Omega}\right) d\Omega \Rightarrow \sigma_{tot} \sim \frac{1}{q^2}$$

2 body collisions:

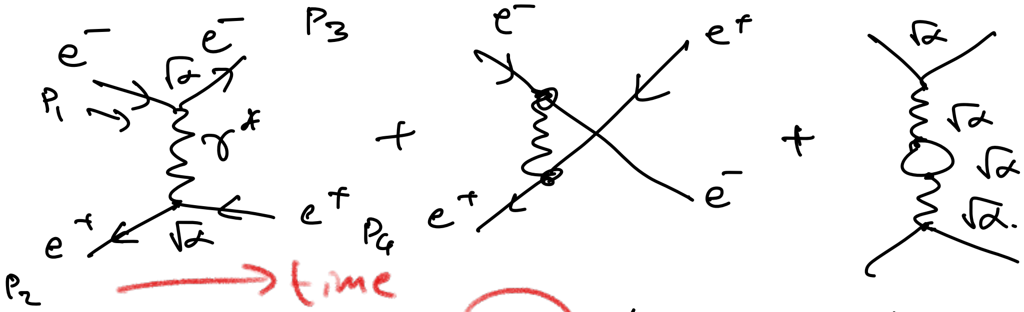


$e^- + e^+ \to e^- + e^+$ elastic scattering.

$\mu^+ + \mu^-$ inelastic scattering.

$e^+e^- \to e^+e^-$

Bhabha scattering



$\underline{P_1} = \underline{P_3} + \underline{q}$

$\mathcal{M}_1 \sim \alpha.$ $\quad \mathcal{M}_2 \sim \alpha$ $\quad \mathcal{M}_3 \sim \alpha^2$

$\alpha = \frac{1}{137}$

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 = a\alpha + b\alpha + \underline{c\alpha^2}$$

$$|\mathcal{M}|^2 \sim \alpha^2 + O(\alpha^4)$$

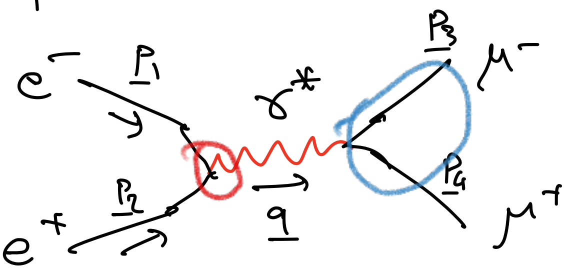
$\uparrow \mathcal{M}_3 \Rightarrow$ ignore $\mathcal{M}_3$

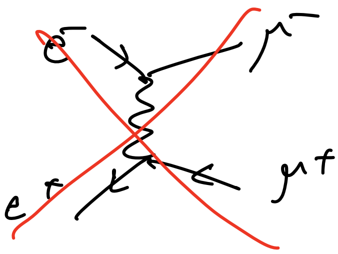
s-channel.

$\mathcal{M}_4$

$$\mathcal{M}(e^+e^- \to e^+e^-) = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_4$$

$$|\mathcal{M}|^2 = \mathcal{M}_1^2 + \mathcal{M}_2 + \mathcal{M}_4^2 + 2\mathcal{M}_1\mathcal{M}_2^* + 2\mathcal{M}_1\mathcal{M}_4^* + 2\mathcal{M}_2\mathcal{M}_4^*$$





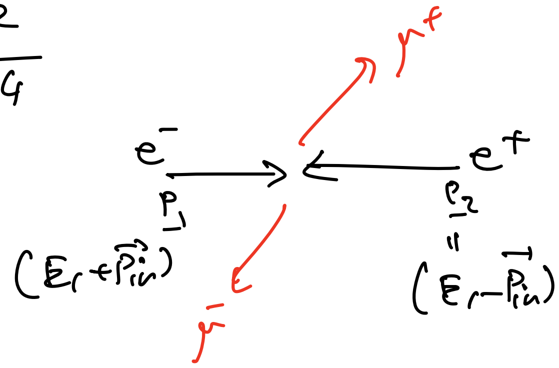
lepton # violation

$\underline{q} = \underline{P_1} + \underline{P_2}$

$$\sigma(e^+e^- \to \mu^+\mu^-) \propto |\mathcal{M}|^2$$

$$\mathcal{M} \sim (\bar{u}\gamma^\mu u) \frac{-ig_{\mu\nu}}{q^2} (\bar{u}\gamma_\nu u) \quad (-ie)(-ie)$$

$$\mathcal{M} \sim \frac{\alpha}{q^2} \Rightarrow \sigma \sim \frac{\alpha^2}{q^4}$$

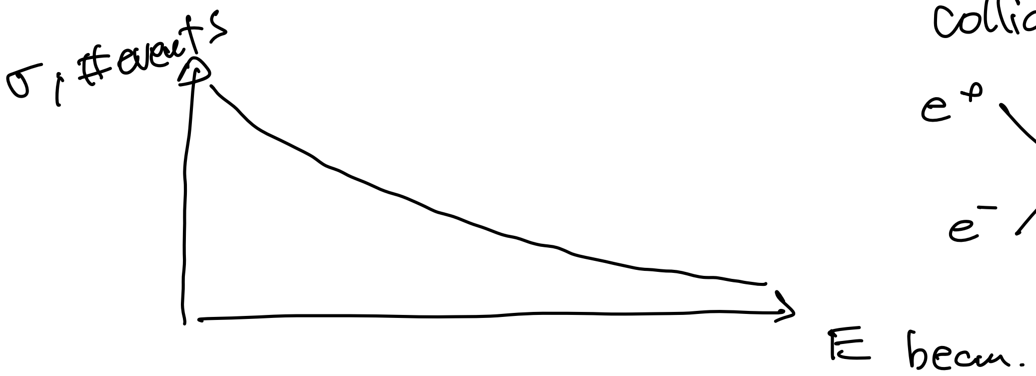


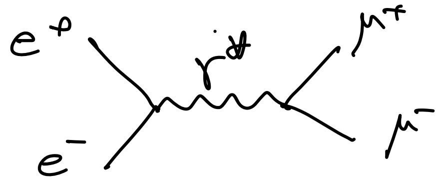
$$\underline{q} = \underline{p}_1 + \underline{p}_2$$

$$= (E_1 + E_2) + \vec{0}$$

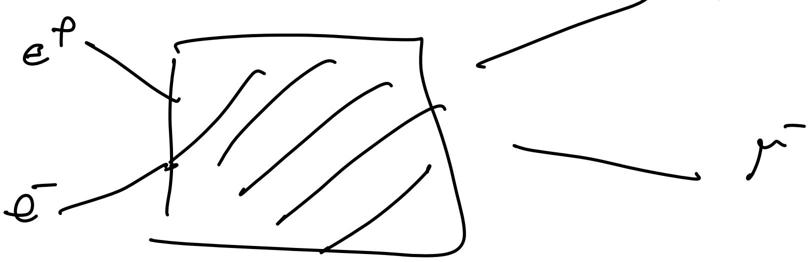
$$q^2 = (E_1 + E_2)^2 = 4E^2$$

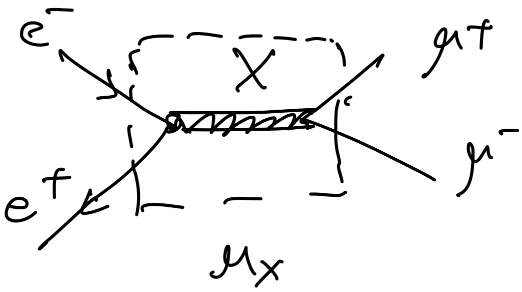
symm. collider. $E_1 = E_2 = E$.





Experimentally



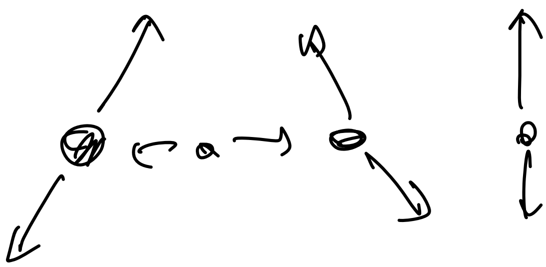


X: mediator with mass m_X

$$\mathcal{M} = \mathcal{M}_{QED} + \mathcal{M}_X$$

$$e^+e^- \to X \to \mu^+\mu^-$$

$$E_1 + E_2 = \sqrt{s} = M_X \to \mu^+\mu^-$$



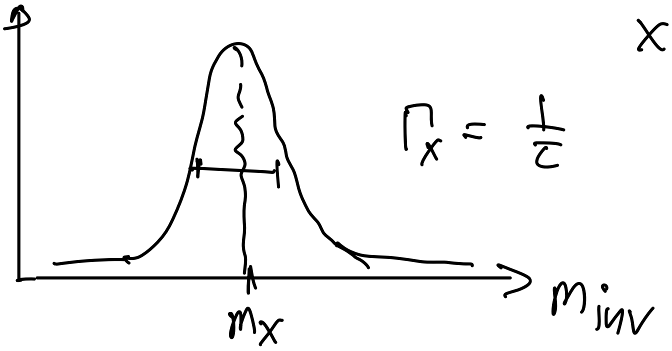
$E_{\mu^+} = E_{\mu^-} = M_X/2$ in rest frame of X

Measure $\underline{p}_{\mu^+}, \underline{p}_{\mu^-}$ $\quad (E_{\mu^+}, \vec{p}_{\mu^+}) \quad (E_{\mu^-}, \vec{p}_{\mu^-})$

$$m_{inv}^2 = m_{\mu\mu}^2 = (\underline{p}_{\mu^+} + \underline{p}_{\mu^-})^2 = (E_{\mu^+} + E_{\mu^-}, \vec{p}_{\mu^+} + \vec{p}_{\mu^-})^2$$

events

$e^+e^- \to X \to \mu^+\mu^-$



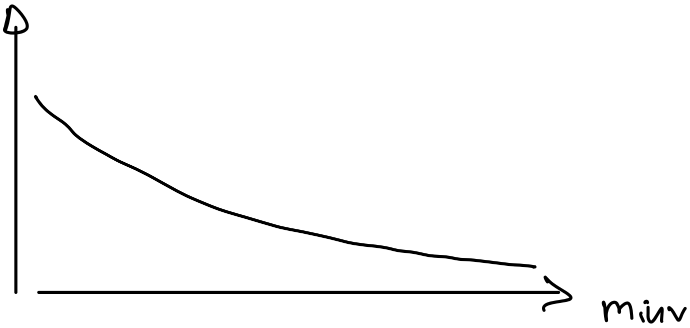
$\Gamma_X = \frac{1}{\tau}$

X: mass m_X
width Γ_X
lifetime τ_X

$$\frac{\Gamma^2/4}{(m-m_X)^2 + \Gamma^2/4}$$

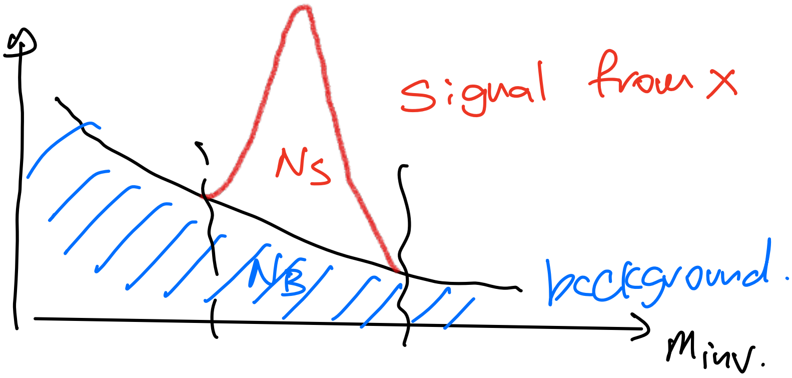
events

$e^+e^- \to \mu^+\mu^-$
QED



events

Reality



Observed $N = N_S + N_B$

$\Rightarrow$ estimate $N_S = N - N_B$

estimate N_B, poisson: $\delta N_B = \sqrt{N_B}$.

$$\frac{N^{obs} - N_B^{est.}}{\sqrt{N_B^{est}}} > 5\sigma \quad \text{observation.}$$

3σ evidence

J of X

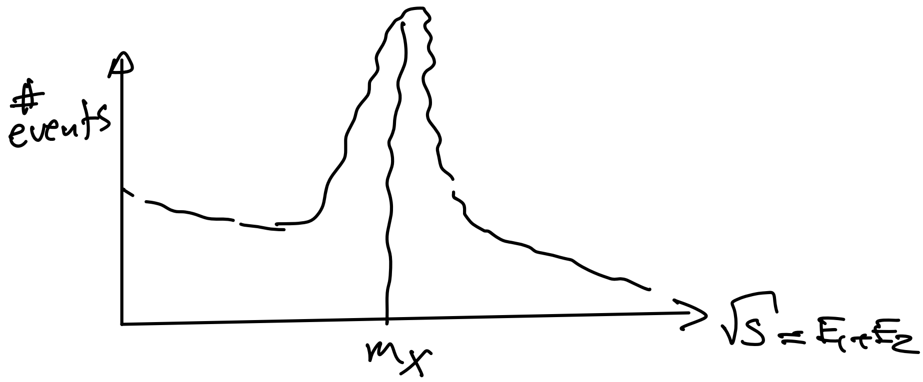
$$\sigma(e^+e^- \to X \to \mu^+\mu^-) = \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{4\pi}{|P_{in}|^2} \underbrace{\frac{\Gamma^2/4}{(E-E_0)^2 + \Gamma^2/4}}_{\text{Breit-Wigner}} \frac{\Gamma_{in}}{\Gamma_{tot}} \frac{\Gamma_{out}}{\Gamma_{tot}}$$

S_1, S_2 are spin of projectiles.

$$\Gamma_{in} = \frac{\Gamma_{X \to e^+e^-}}{\Gamma_X} \qquad \Gamma_{out} = \frac{\Gamma_{X \to \mu^+\mu^-}}{\Gamma_X}$$

$$X \longrightarrow \begin{array}{ll} e^+e^- & \Gamma_1 \\ \mu^+\mu^- & \Gamma_2 \\ \tau^+\tau^- & \Gamma_3 \\ \pi^+\pi^- & \Gamma_4 \end{array}$$

$$\frac{1}{\tau} = \Gamma_X = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4$$



$\sqrt{s} \geq m_X$ to produce X