Isospin

$p = |1/2, 1/2\rangle$ $\quad |n\rangle = |1/2, -1/2\rangle$ $\qquad$ pn, deuton $= |0,0\rangle$?

$\binom{p}{n}$ doublet $I = 1/2$ $\qquad$ $|\pi^{\pm,0}\rangle$ = triplet $I = 1$

p+p → p+p elastic

$\xrightarrow{p}$ target $\quad$ Inelastic processes

(a)

a)

	p + p →		d + π^+	
Q	1	1	1	1
B	1	1	2	0
I_3	$+\frac{1}{2}$	$+\frac{1}{2}$	0	+1

$0 \oplus 1 = 1$ $\quad |I, I_3\rangle$

initial
$|I, I_3\rangle = |1, +1\rangle$ $\qquad$ final state: $|1,1\rangle$

strong interaction conserves Isospin.

$\langle f | H_I | i\rangle$

$\langle 1,1 | H_I | 1,1\rangle$

(b)

	p + n →		d + π^0	
Q	1	0	1	0
B	1	1	2	0
I_3	+1/2	−1/2	0	0

initial state

$I_3 = 0$ $\qquad$ p+n $\quad \frac{1}{2} \otimes \frac{1}{2} = 0 + 1$ $\quad$ initial state.

$|p+n\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle)$

final state: $\quad I_3 = 0$ $\quad$ d+π^0 $\quad |0,0\rangle \times |1,0\rangle$

$|1,0\rangle$

$|i\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle) \xrightarrow{H_I} |f\rangle = |1,0\rangle$.

$\sigma_a \propto |M_a|^2 \, \rho(p+p \to d+\pi^+)$ $\qquad$ $\#\text{events} = \sigma \times \frac{dN_p}{dt} \, d \, n_B$

$\sigma_b \propto |M_b|^2 \, \rho(p+n \to d+\pi^0)$

$$\frac{\sigma_a}{\sigma_b} = \frac{\# \, p+p \to d+\pi^+}{\# \, p+n \to d+\pi^0} = \frac{|M_a|^2}{|M_b|^2} \; \frac{\rho(d\pi^+)}{\rho(d\pi^0)}$$

$\frac{\Delta m_{pn}}{m_p} \simeq 2 \text{ MeV} / 1000 \text{ MeV}.$ $\qquad$ $\frac{\Delta m_\pi}{m_\pi} \simeq 5 \text{ MeV} / 135 \text{ MeV}.$

$$\Rightarrow \frac{\rho(\pi^+ d)}{\rho(\pi^0 d)} \simeq 1.$$

$$M_a = \overset{f}{\langle 1,1|} H \overset{i}{|1,1\rangle} \qquad p+p \to d+\pi^+.$$

$$M_b = \langle 1,0| H \tfrac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle) \qquad p+n \to d+\pi^0$$

H_{strong} conserves isospin. $\Rightarrow$ $M_b = \frac{1}{\sqrt{2}}\langle 1,0|H|1,0\rangle + \cancel{\frac{1}{\sqrt{2}}\langle 1,0|H|0,0\rangle}.$

$$\Rightarrow \frac{\sigma_a}{\sigma_b} \simeq \frac{1}{\left|\frac{1}{\sqrt{2}}\right|^2} \simeq 2 \qquad \text{confirmed by } \frac{\# \pi^+ d}{\# \pi^0 d}$$

Another experiment $\quad \pi^+ + (p/n) \longrightarrow$

I	I_3				I	I_3	
		$\pi^+ + p \longrightarrow \pi^+ + p$					(a)
$\|3/2, 3/2\rangle$		$(1,1)$	$\|1/2, 1/2\rangle$		$3/2$	$3/2$	

	π^+	$+p \longrightarrow$	π^{++}	$+ n$	
B	0	1	0	1	X $\quad$ No π^{++}
Q	+1	+1	+2	0	
I_3					

$$M = \langle f|H|i\rangle.$$

	π^+	$+p \longrightarrow$	n	$+\pi^+$	π^+
I_3	+1	1/2	−1/2	+1	+1

$$= \langle n\pi\pi| H_S | p\pi\rangle$$

$\pi^+ + n \longrightarrow \pi^+ + n$ (d)

$\pi^+ + n \longrightarrow \pi^0 + p$ (g)

$\pi^- + p \longrightarrow \pi^- + p$ (c)

$\pi^- + p \longrightarrow \pi^0 + n$ (j)

$\pi^- + n \longrightarrow \pi^- + n$ (k)

$I_3 \quad -1 \quad -1/2$

$-3/2 \Rightarrow |3/2, -3/2\rangle$

(c) $\pi^- + p \longrightarrow \pi^- + p.$

$I_3 \quad -1 + 1/2 \qquad -1 \quad +1/2.$

tot $I_3 = -1/2.$

$$|\pi^- + p\rangle = \alpha\, |3/2, -1/2\rangle + \beta\, |1/2, -1/2\rangle$$

Table 3.3. *Clebsch–Gordan coefficients in pion–nucleon scattering*

		$I=\frac{3}{2}$				$I=\frac{1}{2}$	
Pion	Nucleon	$I_3=\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
π^+	p	1					
π^+	n		$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
π^0	p		$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
π^0	n			$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
π^-	p			$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
π^-	n				1		

$$\mathcal{M} = \left(\langle 3/2, -1/2|\, \alpha + \langle 1/2, -1/2|\, \beta\right)(H_s)\left(a|3/2, -1/2\rangle + b|1/2, -1/2\rangle\right)$$

H_s Conserves Isospin

$$\Rightarrow \quad \mathcal{M}_c = A\langle 3/2|H_s|3/2\rangle + B\langle 1/2|H_s|1/2\rangle$$

$$= \frac{1}{3}\mathcal{M}_{3/2} + \frac{2}{3}\mathcal{M}_{1/2}$$

$$\Rightarrow |\mathcal{M}_c|^2 = \frac{1}{9}|\mathcal{M}_3 + 2\mathcal{M}_1|^2 \qquad \pi^- + p \to \pi^- + p.$$

(a) $\pi^+ + p \longrightarrow \pi^+ + p.$ $\quad |\mathcal{M}_a|^2 = |\mathcal{M}_3|^2$

$$\mathcal{M}_a = \mathcal{M}_{3/2}$$

(j) $\pi^- + p \longrightarrow \pi^0 + n.$

$$\mathcal{M}_j \sim \frac{\sqrt{2}}{3} \mathcal{M}_{3/2} - \frac{\sqrt{2}}{3} \mathcal{M}_{1/2}$$

$$\sigma_j \propto |\mathcal{M}_j|^2 \sim \frac{2}{9} |\mathcal{M}_3 - \mathcal{M}_1|^2$$

case 1)
if $|\mathcal{M}_1| >> |\mathcal{M}_3| \implies \sigma_a (\pi^+ + p \longrightarrow \pi^+ p) << \sigma(\pi^-)$

$\#(\pi^+ + p) \simeq 0. \implies$ not confirmed by data.

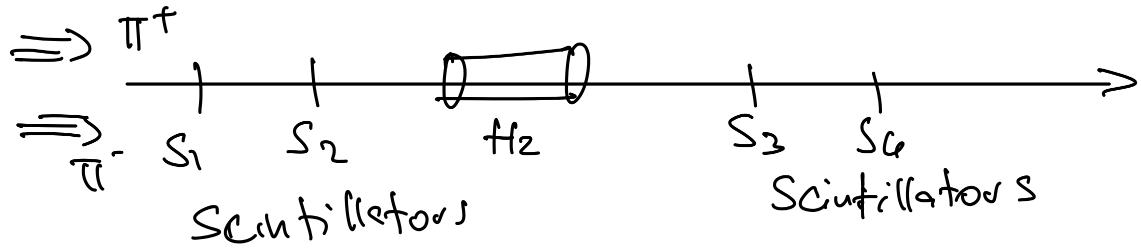
case 2)
$|\mathcal{M}_3| >> |\mathcal{M}_1|$

$$\mathcal{M}_a \simeq \mathcal{M}_3 \implies \sigma(\pi^+ p) \simeq |\mathcal{M}_3|^2$$

$$\mathcal{M}_c \simeq \frac{1}{\sqrt{3}} \mathcal{M}_3 \implies \sigma_c \simeq \frac{1}{9} |\mathcal{M}_3|^2.$$

$$\mathcal{M}_j \simeq \frac{\sqrt{2}}{\sqrt{3}} \mathcal{M}_3 \implies \sigma_j \simeq \frac{2}{9} |\mathcal{M}_3|^2$$

$$\frac{\#(\pi^+ + p \longrightarrow \pi^+ + p)}{\#(\pi^- + p)} = \frac{|\mathcal{M}_3|^2}{\frac{1}{9} |\mathcal{M}_3|^2 + \frac{2}{9} |\mathcal{M}_3|^2} \simeq 3$$



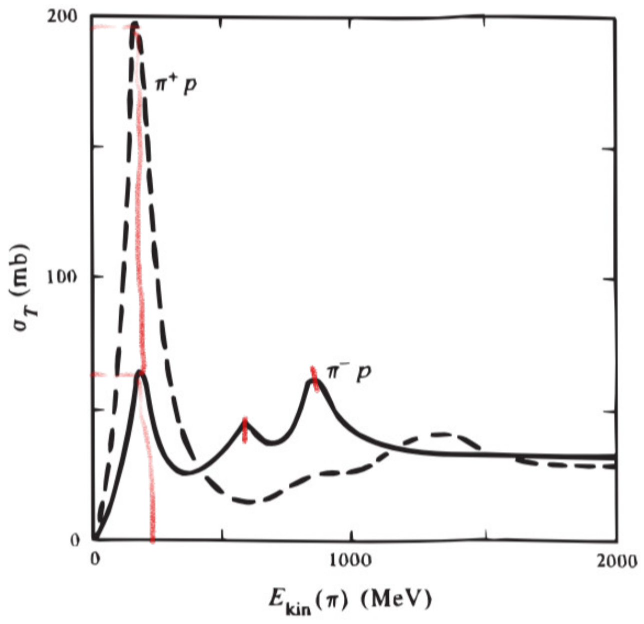


Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm^2.)

$E_{kin} = E_\pi - m_\pi$

	EM	Strong	Weak.
$\mathbb{P}$	✓	✓	✗
$\mathbb{C}$	✓	✓	✗
$\mathbb{I}$		✓	
$\mathbb{G}$		✓	

G parity

$\mathbb{C}\,|\pi^+\rangle = -|\pi^-\rangle$ $\quad C_{q\bar{q}} = (-1)^{L+S}$

$\mathbb{C}\,|\pi^-\rangle = -|\pi^+\rangle$

$\mathbb{C}\,|\pi^0\rangle = +|\pi^0\rangle$ $\quad C_{\pi^0} = (C_\gamma)^2 = +1$

NB: $\pi^+ = u\bar{d} \neq q\bar{q}$

π^{+-} are not eigenstates of $\mathbb{C}$ because charged.

π^0 is eigenstate of $\mathbb{C}$.

$C_{\pi^0} = (-1)^{L+S} = +1$

$\mathbb{R}_2$: Rotation around I_2 in isospin space.

$\mathbb{R}_2 = e^{i\pi I_2}$

$\mathbb{R}_2|\pi^+\rangle = (-1)^{I-I_3}\,|\pi^-\rangle.$

$\mathbb{R}_2|I, I_3\rangle = (-1)^{I-I_3}\,|I, -I_3\rangle$

$\mathbb{R}_2|\pi^+\rangle = (-1)^{1-1}\,|1,-1\rangle = +1\,|\pi^-\rangle$

$$\mathbb{R}_2|\pi^-\rangle = (-1)^{1-(-1)}|\pi^+\rangle = +1|\pi^+\rangle$$

$$\mathbb{R}_2|\pi^0\rangle = (-1)^{1-0}|\pi^0\rangle = (-1)|\pi^0\rangle.$$

$$(\mathbb{C} + \mathbb{R}_2)|\pi^+\rangle = (-1)(+1)|\pi^+\rangle. \quad = (-1)|\pi^+\rangle$$

$$(\mathbb{C} + \mathbb{R}_2)|\pi^-\rangle = (-1)(+1)|\pi^-\rangle. \quad = (-1)|\pi^-\rangle.$$

$$(\mathbb{C} + \mathbb{R}_2)|\pi^0\rangle = (+1)(-1)|\pi^0\rangle. \quad = \underline{(-1)}|\pi^0\rangle.$$

$$G = \mathbb{C} \times \mathbb{R}_2$$

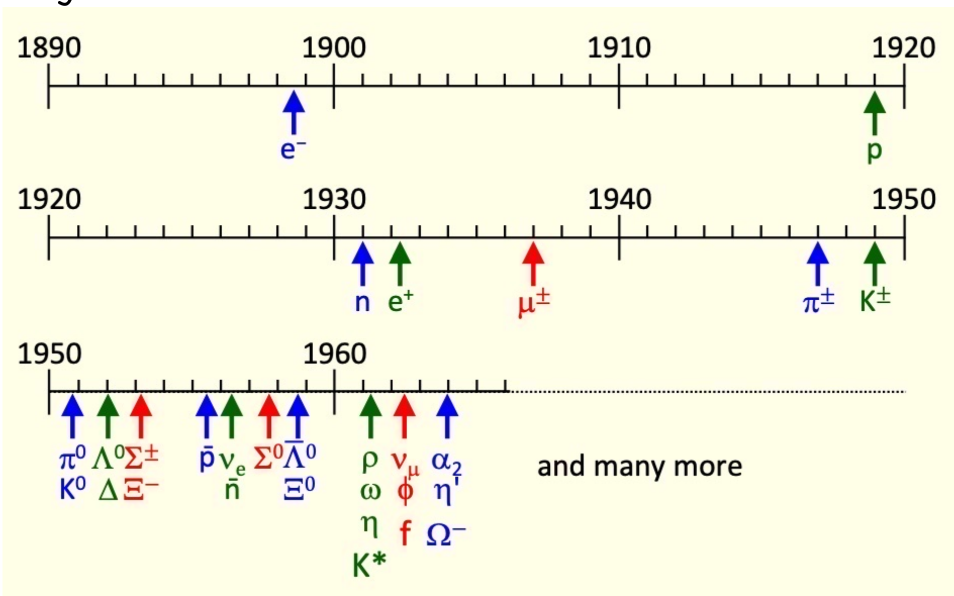
Same value on Isospin multiplet

$$G = (-1)^{L+S+I}$$

for multiplet with L, S, I

the multiplet must be with I = integer

only conserved in strong interactions.



Strange particles: produced strongly but decay with long lifetime (weak interaction).

$K^\pm, K^0, \Lambda$

Gell Mann — Nishijima Formula for Charge.

$$Q \longrightarrow I_3$$

nucleus with Z proton + $A-Z$ neutron.

$$I_3 = \underbrace{Z\times\left(\tfrac{1}{2}\right)}_{p} + \underbrace{(A-Z)\left(-\tfrac{1}{2}\right)}_{n} = Z\tfrac{1}{2} + Z\tfrac{1}{2} - \tfrac{A}{2}$$

$$= Z - \frac{A}{2}.$$

$$\Rightarrow \quad Z = I_3 + \frac{A}{2}.$$

$$\downarrow$$

$$Q = I_3 + \frac{B}{2}$$ Gell Mann - Nishijima

$\pi^+ : \quad +1 + \frac{0}{2} = +1$

strange particles $\Rightarrow$ $$Q = I_3 + \frac{\boxed{B+S}}{2}$$ Y Hyper Charge

Name	$\pi^\pm$	π^0	$K^\pm$	K^0	η	p	n	Λ	$\Sigma^{\pm,0}$	Δ
Mass (MeV)	140	135	494	498	548	938	940	1116	1190	1232
Charge	±1	0	±1	0	0	1	0	0	±1,0	2,±1,0
Parity	–	–	–	–	–	+	+	+	+	+
Baryon n.	0	0	0	0	0	1	1	1	1	1
Spin	0	0	0	0	0	½	½	½	½	$^3/_2$

Static Quark Model: classification of known hadrons based on symmetry.

Gell-Mann, Zweig 1961 – 1964

Eightfold way

Isospin, Spin 1/2 particles SU(2) symmetry.

SU(N) operators: $\quad U = e^{i a_j T_j} \quad j = 1, \ldots$

det = 1 $\quad T_j$: generators.

trace = 0

Generators: $N^2 - 1$

$N-1$: are diagonal.

SU(2): 3 generators $\quad S_x, S_y, S_z \quad I_1, I_2, I_3$

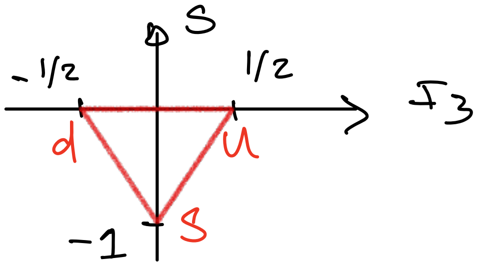
diagonal $\quad S_z \quad I_3$

static quark model : $\Rightarrow$ hadrons are multiplets of SU(3)

SU(3) : 8 generators

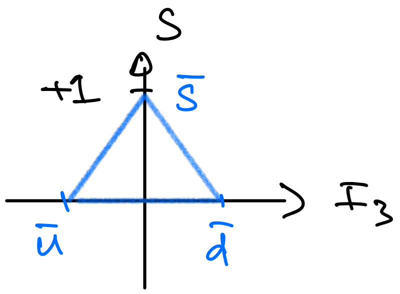
2 diagonal : I_3, Strangeness.

Fundamental repp. of SU(3) $\quad u = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $d = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $s = \begin{pmatrix}0\\0\\1\end{pmatrix}$



	I_3	S
u	+1/2	0
d	-1/2	0
s	0	-1

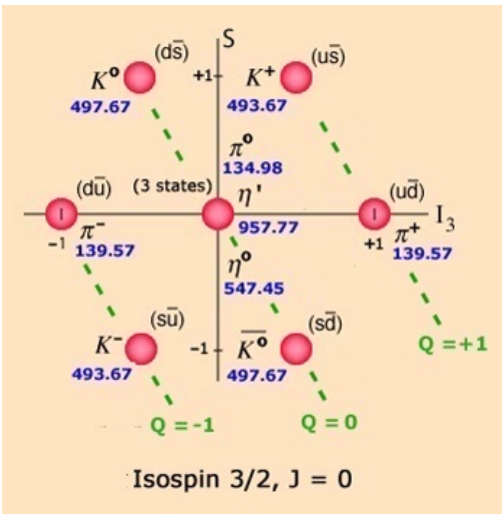
$\bar{u}\ \bar{d}\ \bar{s}$



Baryon number

$B = \frac{1}{3}$ for quarks.

$= -1/3$ for antiquarks.



Fundamental repp:

3	or	$\bar{3}$
u		$\bar{u}$
d		$\bar{d}$
s		$\bar{s}$

Mesons: $3 \otimes \bar{3} = 1 \oplus 8$

$u \times (\bar{u}, \bar{d}, \bar{s})$

In the middle.

$u\bar{u}$, $d\bar{d}$, $s\bar{s}$