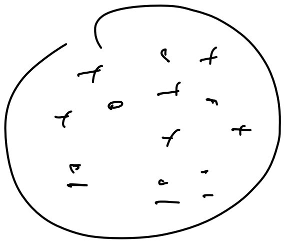
Inelastic Scattering and hadron struct

Atomic Structure $\longrightarrow$ Nucleus.

Atomic scale $1\,\mathring{A}$ 10^{-10} m.

Thompson theory



Rutherford proved: + charge confined in space : nucleus.

scale of nucleus 1 fm 10^{-15} m.

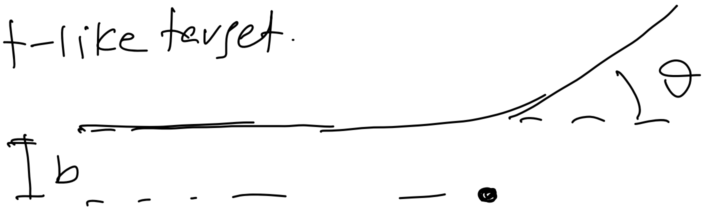
Scattering of $\alpha + Au \rightarrow \alpha + Au$. 1910

1) probe : α particles.
2) heavy target : Au $\Rightarrow$ neglect target recoil.

pure elastic scattering.

α particles here $K = E - m = 5-7$ MeV. from α decay

3) point-like target.



θ. deflection angle.

b

impact parameter

4) measure $\frac{d\sigma}{d\Omega}(\theta)$ differential cross section.

\# reactions vs. θ.

Rutherford: ignore target recoil.

α: mass 3.7 GeV. $K = 7$ MeV highly non relativistic

e^- as probe :
1) ultra relativistic already @ 10 MeV.
2) spin to be accounted (helicity)

Experimental method: 1) predict $d\sigma/d\Omega$.

2) measure $d\sigma/d\Omega$ vs θ.

3) look for deviations.

probe + T $\longrightarrow$ probe + T elastic scattering.

$\longrightarrow$ probe + T^* excitation.

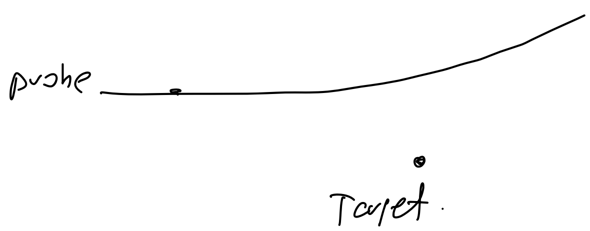
$\longrightarrow$ probe + new particles. $\neq T$ inelastic scattering.

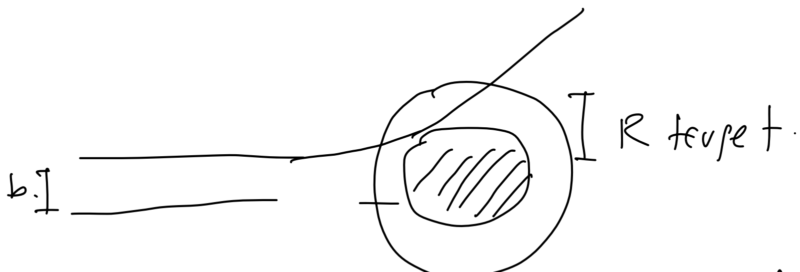
if $\left.\frac{d\sigma}{d\Omega}\right|_{pred.} \neq \left.\frac{d\sigma}{d\Omega}\right|_{meas.}$ $\to$ calculation incorrect

$\to$ structure in the target

$\to$ new interactions.

classical Ruth.





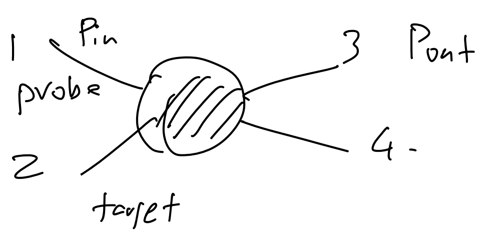
for $b < R_T \Rightarrow$ see effect of target structure.

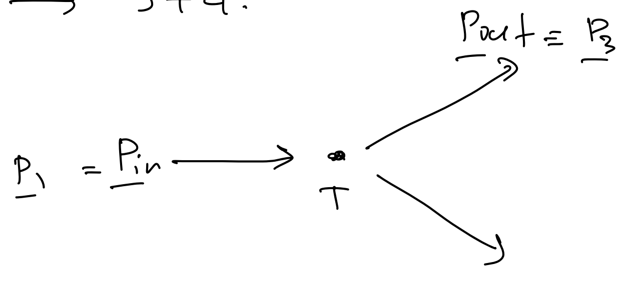
Experimentally: measure 1) # events $\frac{d\sigma}{d\Omega}$.

2) angle θ of deviation.

3) E' energy of the probe after scattering

Lecture 2 of this course: $1+2 \longrightarrow 3+4$.

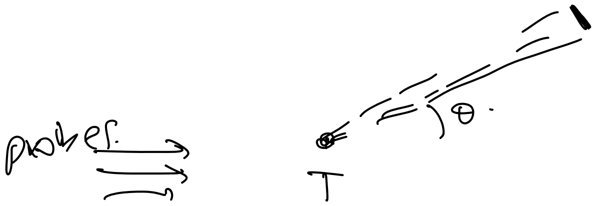




$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{(M_T)^2} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} |M|^2$$

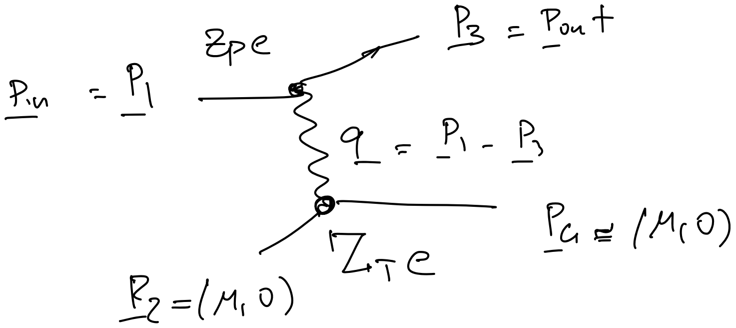
General Mott Formula for 2 body scattering in LAB frame.

$$\frac{\# \text{events}}{\Delta\theta}$$



Rutherford Experiment

$\alpha + Au \rightarrow \alpha + Au.$

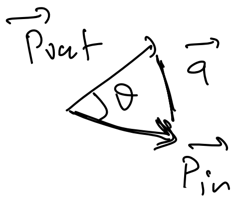


$$M \sim z_p e Z_T e \frac{1}{q^2}$$

$$= \frac{Z_p Z_T \alpha.}{q^2}$$

negligible recoil.

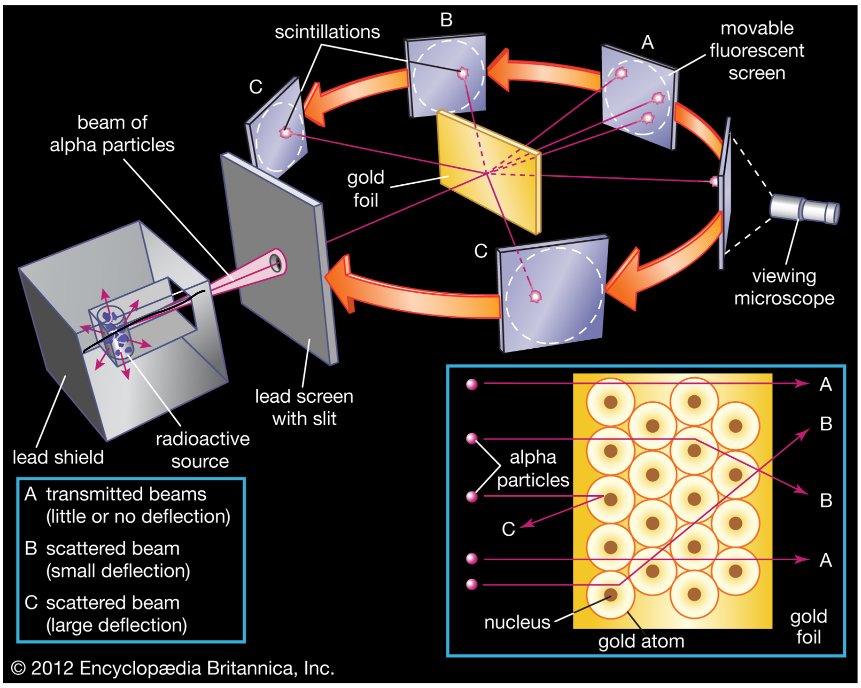
$$\Rightarrow |\vec{P}_1| = |\vec{P}_3| = p.$$

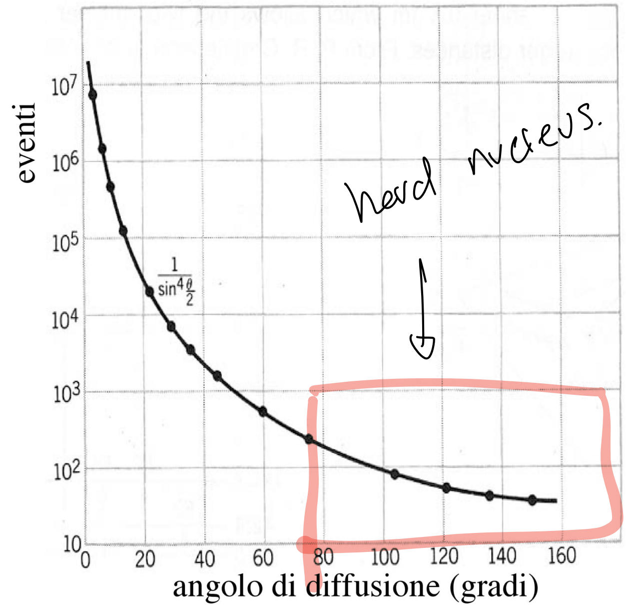


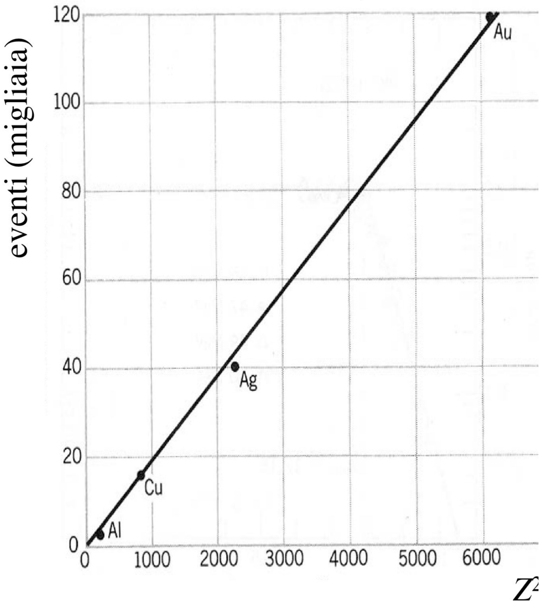
$$\vec{q} \simeq 2p \sin\frac{\theta}{2}.$$

$$\Rightarrow \frac{d\sigma}{d\Omega} \sim (Z_p Z_T \alpha)^2 \frac{1}{q^4}$$

$$\frac{d\sigma}{d\Omega} \sim \alpha^2 (Z_p Z_T)^2 \frac{1}{p^4 \sin^4\frac{\theta}{2}}$$



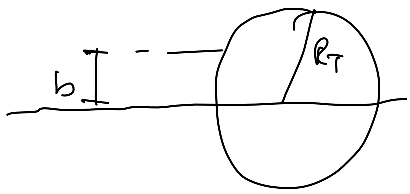


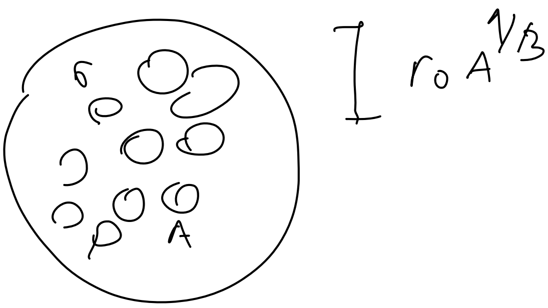


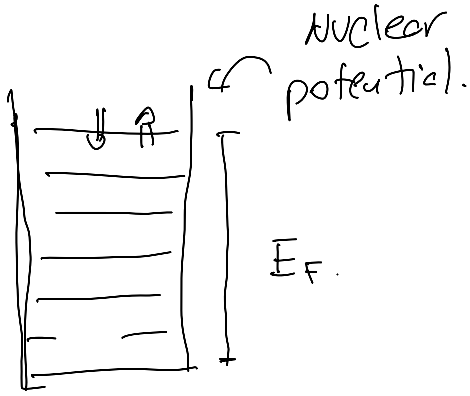
Rutherford: $R_N < 10$ fm.

today we know
$R_N \approx r_0 A^{1/3}$
$r_0 \simeq 1.2$ fm.

Could not prob structure of nucleus:







Assume Fermi gas of nucleons:

Fermi energy: max kin. energy of nucleons in the nucleus.

$$E_F \simeq \frac{P_F^2}{2m_N} \simeq 30 \text{ MeV}. \qquad P_F \simeq 250 \text{ MeV}.$$

To see the structure $\Rightarrow$ accelerate α up to 20-30 MeV.
around 1960's.

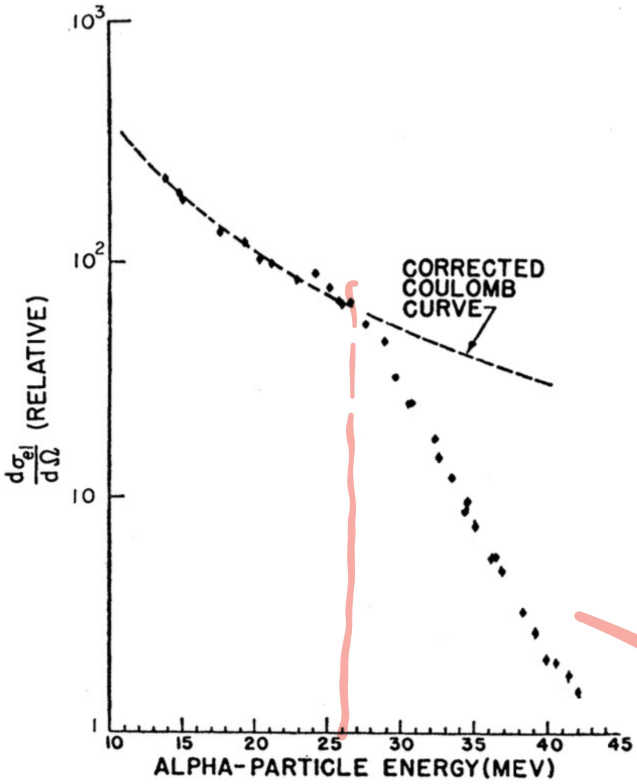
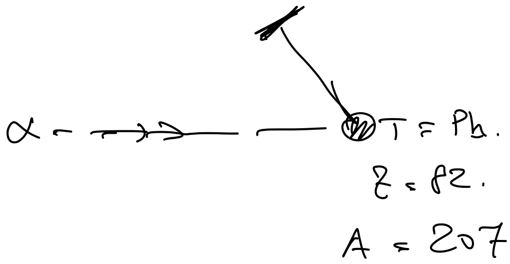


FIG. 5. Differential cross section for the elastic scattering of alpha particles by Pb at 60° as a function of the alpha-particle energy.

Rev. Mod. Phys 33, 190 (1961)

$\alpha + Pb \longrightarrow \alpha + Pb.$

$\theta = 60°.$



Exponential deviation from Coulomb scatt.

For $E \geq 25$ MeV $\Rightarrow$ use e^- which are pointlike.

Study $e^- + N \longrightarrow$ $e^- + N$ elastic

$e^- + N^*$ excitation.

$e^- + H$ Inelastic.

1910 α probes.

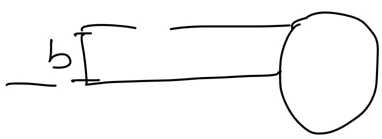
1960 e^- probes.

>1960 - 2000 e/ν probes.

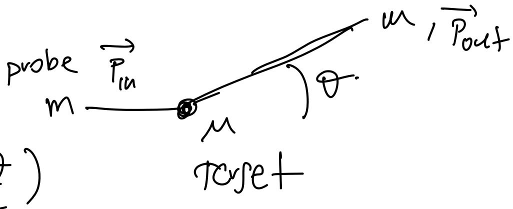
QM: De Broglie wave length of particle of mass M?

$\Delta p \Delta x \simeq 1 \Rightarrow \Delta p \simeq \frac{1}{b}$

200 fm. MeV $= \hbar c = 1.$

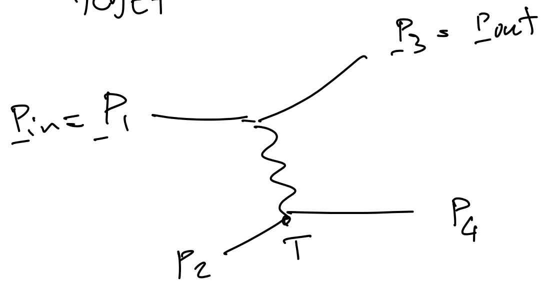


e^- : Spin 1/2 relativistic



$$\frac{d\sigma}{d\Omega} \sim \underbrace{\alpha^2 \frac{1}{q^4}}_{\text{Rutherford.}} \left(m^2 + P_{in}^2 \cos^2\frac{\theta}{2}\right)$$

Mott Formula.



$$m^2 + P^2 \cos^2\frac{\theta}{2} = m^2 + P^2\left(1 - \sin^2\frac{\theta}{2}\right)$$

$$= \underbrace{m^2 + P^2}_{} - P^2 \sin^2\frac{\theta}{2} = E^2 - P^2 \sin^2\frac{\theta}{2} = E^2\left(1 - \left(\frac{P}{E}\right)^2 \sin^2\frac{\theta}{2}\right)$$

$$\beta = \frac{P}{E}$$

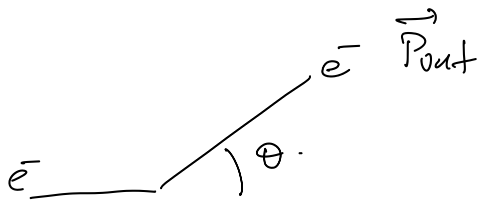
$$E^2\left(1 - \beta^2 \sin^2\frac{\theta}{2}\right).$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} = \left.\frac{d\sigma}{d\Omega}\right|_{Ruth.} \underbrace{\left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)}_{\text{because of } e^- \text{ spin.}}$$

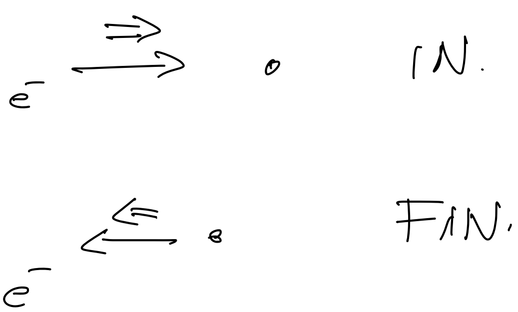
Mott with no recoil in target.

suppose ultrarel. e^- $\Rightarrow$ $\beta \simeq 1$.

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} = \left.\frac{d\sigma}{d\Omega}\right|_{Ruth.} \cos^2\frac{\theta}{2}$$



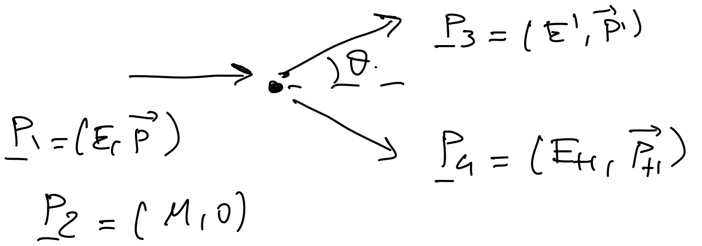
$\theta = \pi \Longrightarrow$



$\beta = 1 \Rightarrow$ helicity conserved. $\Rightarrow$ $\Delta J = 1$. not possible.

NB: Mott with zero recoil of target.

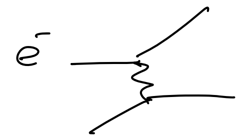
Study $e^- + N \longrightarrow e^- + N/H$



Elastic: $e^- + N \to e^- + N$.

$\underline{P}_4 = (M, \vec{P}_4)$

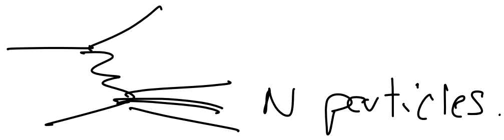
Inelastic:



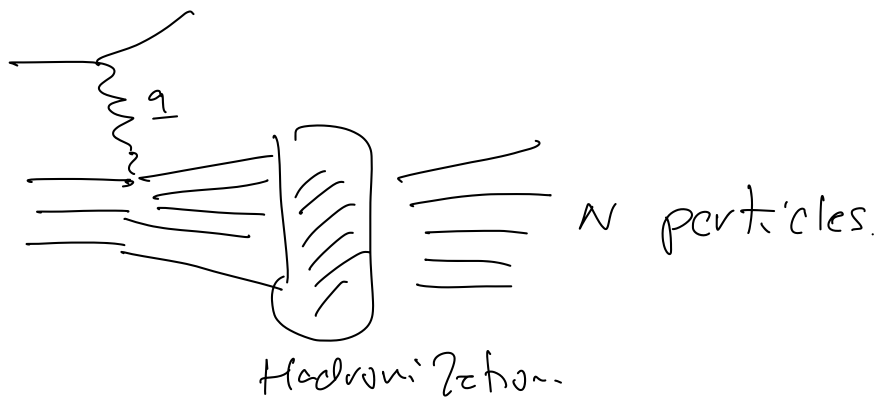
hadronic system.

$$\left\{ \begin{array}{l} E_H = \sum_{i}^{N} E_i \\ \vec{P}_H = \sum_{i}^{N} \vec{P}_i \end{array} \right.$$

$$W = M_H = \left(\sum_i \underline{P}_i \right)^2$$

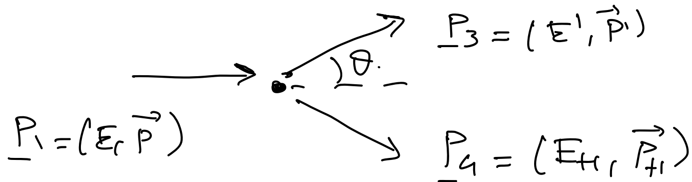


Example: $e^- + p \to e^- + p$.



Elastic case

$$\underline{P}_1 + \underline{P}_2 = \underline{P}_3 + \underline{P}_4.$$



$\underline{q} = \underline{P}_1 - \underline{P}_3$.

4-vector of mom.

transferred to target.

Observable quantities:

θ, E'

$$\underline{P}_4 = \underline{P}_1 - \underline{P}_3 + \underline{P}_2$$

$$\underline{P}_4^2 = M^2$$

$$(\underline{P}_1 - \underline{P}_3 + \underline{P}_2)^2 = m_e^2 + m_e^2 + M^2 + \underbrace{2\underline{P}_1 \cdot \underline{P}_2}_{2EM} - \underbrace{2\underline{P}_1 \cdot \underline{P}_3} - \underbrace{2\underline{P}_3 \cdot \underline{P}_2}_{2E'M}.$$

$$\cancel{M^2} = \cancel{2m_e^2} + \cancel{M^2} + \cancel{2}EM - \cancel{2}E'M - \cancel{2}(EE' - \vec{p}\cdot\vec{p'})$$

$E \gg m_e$ relativistic regime.

$E' \gg m_e$. $|\vec{p}| \simeq E$ $|\vec{p'}| \simeq E'$

$\rightarrow 2m_e^2 \simeq 0.$

$$0 = EM - E'M - (EE' - EE'\cos\theta).$$

$$E' = \frac{E}{1 + \frac{E}{M}(1-\cos\theta)}$$

$1-\cos\theta = 2\sin^2\frac{\theta}{2}.$

Elastic scattering: E', θ are correlated.
not independent.