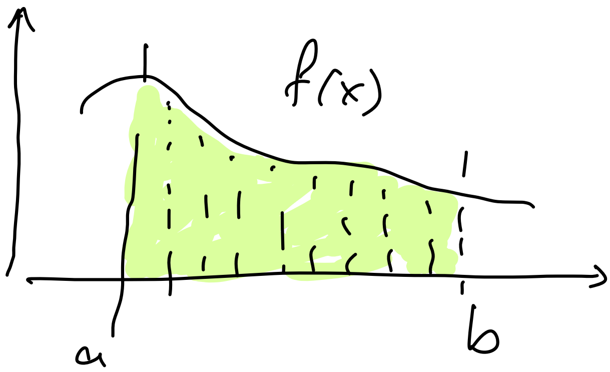
Integrazione con Metodo Monte Carlo

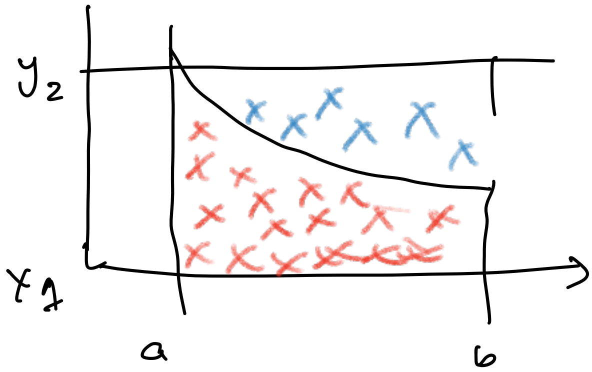
$$I = \int_a^b f(x)\,dx$$



$$f(x) = \frac{d}{dx} F(x)$$

$$I = F(b) - F(a)$$

Metodo hit & miss:



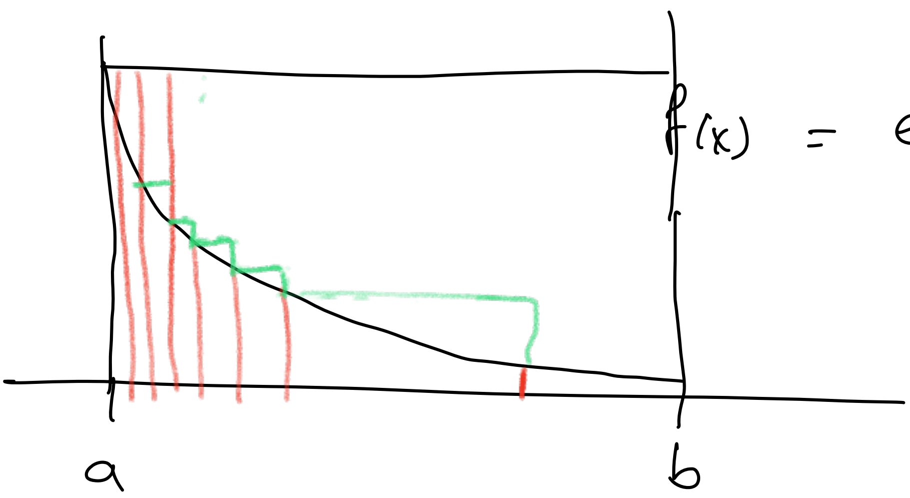
$$I = (y_2 - y_1) \times (b-a) \times \frac{N_{dentro}}{N_{totale}}$$

N_{dentro}: punti rossi sotto la curva.

N_{totale}: punti generati nel rettangolo.

genero (x_i, y_i) 2 num. casuali

$y_i < f(x_i)$ punto dentro



$$f(x) = e^{-\lambda x}$$

Metodo Monte Carlo

$$I = \int_a^b f(x)\,dx = \int_a^b \frac{f(x)}{p(x)}\, p(x)\,dx$$

$p(x)$: funzione a scelta $\qquad S(x) = \frac{f(x)}{p(x)}$

$$\int_a^b p(x)\,dx = 1$$

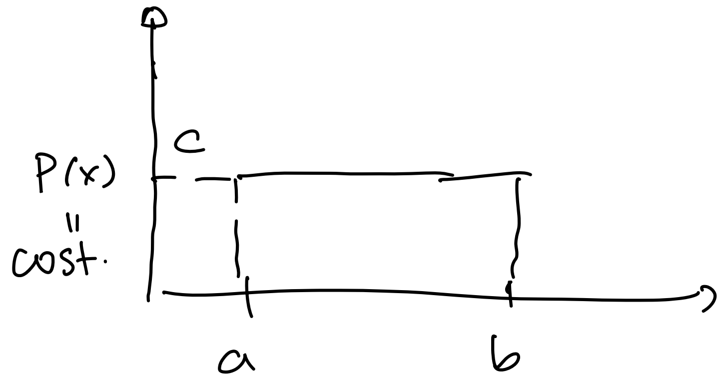
$$I = \int_a^b S(x)\, p(x)\,dx \quad \text{con} \quad \int_a^b p(x)\,dx = 1$$

Valore medio di $S(x)$ con distrib. di prob. $p(x)$

$$I = \langle S(x) \rangle \simeq \frac{1}{N} \sum_i^N S(x_i)$$

x_i: numeri estratti casualmente secondo distribuzione $p(x)$. $x_i \in [a, b]$

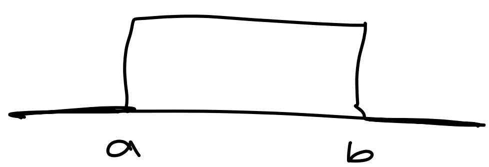
$$S(x_i) = \frac{f(x_i)}{p(x_i)}$$



$$\int_a^b p(x)\,dx = 1 = c \times (b-a)$$

$$\Rightarrow \quad c = \frac{1}{b-a}$$

$$p(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{fuori} \end{cases}$$



$x \in [a,b)$ $\quad x = a + (b-a) * lrand48()/RAND\_MAX$

$\sum_i S(x_i)$

```
sum = 0;
for(i=0; i<Npunti; i++) {
    x = a+(b-a) — —
    sum += f(x) * (b-a)
}
sum = sum / npunti;
```

$$S(x) = \frac{f(x)}{P(x)} = \frac{f(x)}{\frac{1}{b-a}} = (b-a)\, f(x)$$

$$\sum_{i=1}^{N} a x_i = a \sum_{i=1}^{N} x_i$$

```c
#include <stdlib.h>
#include <stdio.h>
#include <math.h>

double myf(double);

int main() {

  double sum = 0;

  double a=0., b=3;

  int npt;
  for(npt=10; npt<=1e6; npt*=10) {
    sum = 0;
    for(int i=0; i<npt; i++) {
      double x = a + (b-a)*lrand48()/RAND_MAX;
      sum += myf(x);
    }
    sum = (b-a)*sum/npt;

    printf("#punti: %8d \t Integral: %.5f\n", npt, sum);
  }

  return 0;
}

double myf(double x) {
  return x*x;
}
```

$f(x) = x^2$

$$\int_0^3 x^2\, dx$$

$$F(x) = \frac{1}{3} x^3$$

$$\int_0^3 x^2 dx = \frac{1}{3} x^3 \Big|_0^3$$

$$= 9$$

```
#punti:       10       Integral: 6.99096
#punti:      100       Integral: 9.49756
#punti:     1000       Integral: 8.83431
#punti:    10000       Integral: 9.00245
#punti:   100000       Integral: 8.98946
#punti:  1000000       Integral: 9.01072
```