২। Sampling:

Sinusoidal: একটি periodic signal যার S আকৃতির shap-

Ex: $y = \sin x / \cos x$.

Analog signal: একটি continuous signal represent করে some other quantity.

Sampling: ~~sam~~ এটি একটি ~~process~~ signal এর value record করার process at given points in time.

Reconstruction: এটি একটি discrete time digital signal কে continuous time analog signal এ রূপান্তর করার process। এটি analog filter ব্যবহার করে যেটি remove করে high-frequency components above the Nyquist frequency.

Nyquist frequency: defined as half of the rate of Discrete signal processing.

Method: Sinusoidal signal frequency = 1 kHz

Amplitude = 5V

Sampled using an ADC at a sampling rate of 10 kHz

Sine ⇒ return an array whose output as same size of x.

$$y = A\sin(\omega t + \phi); \quad \omega = 2\pi f$$

Z-transform:

Digital signal processing এর

used: হচ্ছে Analyze এবং manipulate করতে ব্যবহার করা হয়।

mathematically,

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

z = complex variable

properties: ১। Linearity ২। time shifting → $x(t-t_0)$

৩। convolution ৪। stability.

Inverse Z-T: হচ্ছে Z-transform function থেকে discrete time function converte করে।

used: recover the original discrete time signal from Z-T.

$$x(n) = \frac{1}{2\pi j}\oint_c X(z)\, z^{n-1}\, dz$$

c = close contour in the complex plane.

constelation

Z-plane: complete and display pole-zero diagram.

↳ [Function এ বড় থেকে ছোট মান]

x → হচ্ছে pole

o হচ্ছে zero

Ex-

$b = [0, 1, 1] \quad a = [1, -2; 3]$

$$\therefore \; X(Z) = \frac{0 + Z^{-1} + Z^{-2}}{1 - 2Z^{-1} + 3Z^{-2}}$$

Zero → $Z^{-1} + Z^{-2} = 0$

$\Rightarrow \frac{1}{Z} + \frac{1}{Z^2} = 0$

$\Rightarrow \frac{Z+1}{Z^2} = 0$

$\therefore Z = -1$

$1 - 2Z^{-1} + 3Z^{-2} = 0$

$\Rightarrow 1 - \frac{2}{Z} + \frac{3}{Z^2} = 0$

$= \frac{Z^2 - 2Z + 3}{Z^2} = 0$

$\Rightarrow Z^2 - 2Z + 3 = 0$

সূত্র: $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$; $\sum_{k=n_1}^{n_2} a^n = \frac{a^{n_1} - a^{n_2+1}}{1-a}$

$$\sum_{n=k}^{\infty} a^n = \frac{a^k}{1-a}$$

DFT:

Used: Finite duration signal এর spectrum বের করে

DFT: এটি একটি Function এর equally-spaced এর finite list.

DFT একটি ~~method~~ sequence & N-complex number কে convert করার একটি method.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{2\pi}{N} jkn} \quad \text{for } 0 \le k \le N-1$$

FFT: একটি important mesurement method. এটি একটি signal কে individual spectral convert করে।

↳ **DIT:** N-point sequence $x(n)$ time domain থেকে frequency domain এ convert করে।

DIF: Decomposed into two subsequence as first half and second half of a sequence

$$x(n) = \{1, \underset{\uparrow}{2}, 3, 4, 5\}$$

DTFT: $$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

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* **Filter:** A filters is a circuit that passes a specific range of frequencies.

* frequency response of a filter describe করার জন্য plot the voltage gain $= \frac{V_{out}}{V_{in}}$ in dB.

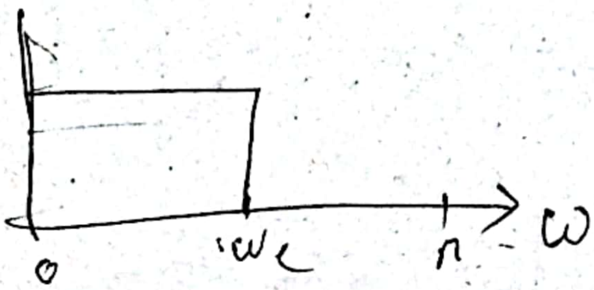
Digital filter: digital signal as input and produces another digital signal as output.

FIR: यह impulse is of finite duration.

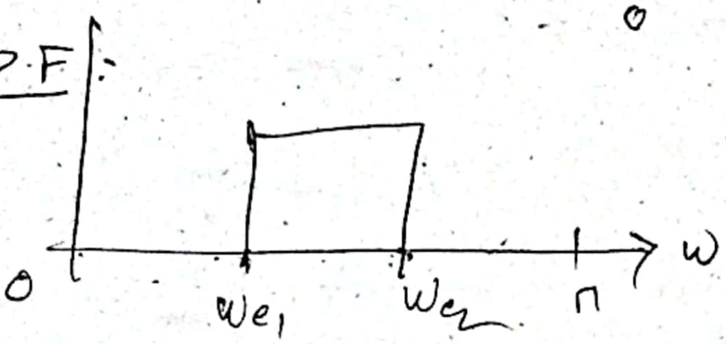
LPF

passes → $w_c \geq w \geq 0$

stop → $\pi \geq w \geq w_c$



Band P.F



$w_{c_2} \geq w \geq w_{c_1}$ → passes

Cutoff frequency: is a break frequency is defined as a boundary in a system's frequency.

Notch Filter: Narrow range of frequency around a central frequency.

FIR	IIR
(i) Finite implulse response	(i) Infinite impulse response
uses difficult	(ii) uses easy
(iii) Control easy	(iii) Control difficult
only zero are present	~~only~~ Both poles & zeros present

ROC: Region of convergence is the range of complex variable Z in the Z-plane.

(i) ROC does not include any pole.

(ii) For Right side signal, ROC will be outside the circle.

(iii) For left sided signal, ROC — inside

(iv) For stability, ROC include of unit circle

(v) For both side, signal ROC is a ring.

2) Low pass filter

With order low pass filter return ~~(N+1)~~ the filter coefficient in length (N+1) vector B.

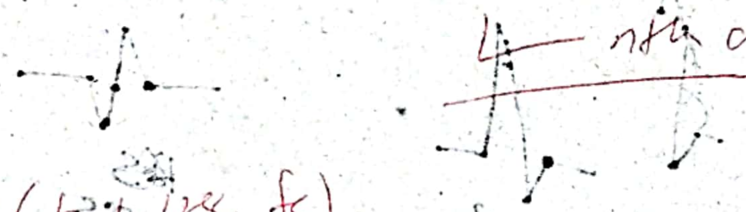
* Wn → The cut off frequency range 0 < Wn < 1

* Normalized gain of the filter at Wn is -6 dB

w = f / (fs/2) ; w = cut off frequency 0 — 1

b = window method : b = fir1 (n, w, 'low')

↳ nth order



[h, w] = freqz (b, n, fs)

h = N-point complex frequency response vector

w = N-point frequency vector in radians / sample

[h, w] = freqz (b, A, N, Fs)

N = 512 → Default = Number of evaluation points

b, a → Transfer function coefficient

freqz → Compute the frequency response of digital filter

* PART - B *

Forier representation of signal: Signal can be represented as superposition of weighted complex sinusoids. This is an alternative expression for LTI system, input output compared to the convolution approach.

Fourier analysis: The study of signals and systems using sinusoidal representation is called Fourier analysis.

Frequency response: Sinusoid input characterizes the behavior of the system. It is obtained in the term of impulse response by using convolution and a complex sinusoid. → It is called frequency response.

***DTFS:** If $x(n)$ is a discrete time signal with fundamental period N then we can represent by DTFS as $\boxed{x(n) = \sum_{k=0}^{N-1} X(k)\, e^{jk\Omega_0 n}}$; $\boxed{X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-jk\Omega_0 n}}$ $x(n) \xleftrightarrow{DTFS} X(k)$

Alternative process of DTFS determination:

Step 1: Expand $x(n)$ in terms of complex sinusoids.

Step 2: Compare outcome of step-2 with each term of the following equation, ~~$x(n) = \sum_{k=0}^{N-1} X(k) e^{jk}$~~

$$x(n) = \sum_{k=0}^{N-1} X(k)\, e^{jk\Omega_0 n}.$$

DTFT: It is used to represent a discrete time non-periodic signal as a superposition of complex sinusoids. DTFT is unlike DTFS that following as

$$\boxed{x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})\, e^{j\Omega n}\, d\Omega \qquad X(e^{j\Omega}) = \sum_{-\infty}^{\infty} x(n)\, e^{-j\Omega n}}$$

DTFT Representation: $x(n)$ non periodic signal
- one period of periodic signal
- $N = 2M+1$ → $X(e^{j\Omega k}) = X(k)(2M+1)$

Condition for existence of DTFT: $X(e^{j\Omega}) = \sum_{-\infty}^{\infty} x(n)\, e^{-j\Omega n}$

if $\sum_{-\infty}^{\infty} |x(n)| < \infty$ or $\sum_{-\infty}^{\infty} |x(n)|^2 < \infty$

* Deduce the DTFT → $x(n) = \lim_{M\to\infty} \hat{x}(n) \Rightarrow \hat{x}(n) = \sum_{-M}^{M} X(k)\, e^{jk\Omega_0 n}$

→ $X(k) = \frac{1}{2M+1} \sum_{-M}^{M} \hat{x}(n)\, e^{-jk\Omega_0 n}$

* Find DTFT? $\Rightarrow \sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1 \\ N, & \alpha = 1 \end{cases}$ $X(e^{j\Omega}) = \sum_{-\infty}^{\infty} x(n)\, e^{-j\Omega n}$

* Find DTFT of $x(n) = \begin{cases} 1 & -M \le n \le M \\ 0 & |n| > M \end{cases}$ → $m = n+M$

* Find DTFT of $x(n) = a^n u(n) \Rightarrow$

→ $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$; $\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha}$

→ $|X(e^{j\Omega})|$; $\arg\{X(e^{j\Omega})\}$

* Find DTFT of $x(n) = 2(3)^n u(-n)$? → $\sum_{k=n_1}^{n_2} a^n = \frac{a^{n_1} - a^{n_2+1}}{1-a}$

* Find inverse of DTFT → ~~$X(k)$~~

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})\, e^{j\Omega n}\, d\Omega$$

* Fourier Representation properties:

① Periodicity: DTFT & DTFS are periodic, since complex sinusoids are 2π-periodic function of frequency. That is discrete time sinusoids whose frequency differ by integer, multiples of 2π are identical.

② Liniarity: All Fourier representation are linear in nature.

$$z(n) = a\,x(n) + b\,y(n) \xleftrightarrow{DTFT} Z(e^{j\Omega}) = a\,X(e^{j\Omega}) + b\,Y(e^{j\Omega})$$

$$z(n) = a\,x(n) + b\,y(n) \xleftrightarrow{DTFS,\Omega} Z(k) = a\,X(k) + b\,Y(k)$$

This property is used to find Fourier representations of signal that are constructed as sums of signal.

(iii) Symmetry:

Fourier representation	Real-valued Time Signal	Imaginary valued time signal
DTFS & DTFT	Magnetude spectrum even func and phase spectrum odd function	magnetude spectrum odd function and phase spectrum even function

Real $X(N-k)$

$$X(k) = X^*(N-k)$$

(iv) Convolution properties:

$$x(t) * h(t) \xleftrightarrow{FT} X(j\omega)\,H(j\omega) \quad ; \quad x(t) \circledast h(t) \xleftrightarrow{FS,\omega_0} T\,X(k)\,H(k)$$

$$x(n) * h(n) \xleftrightarrow{DTFT} X(e^{j\Omega})\,H(e^{j\Omega}) \quad ; \quad x(n) \circledast h(n) \xleftrightarrow{DTFS,\Omega} N\,X(k)\,H(k)$$

* Find Convolution sequence ? $\Rightarrow H(e^{j\Omega}) * X(e^{j\Omega})$

$$H(e^{j\Omega}) = \sum_{0}^{\infty} \left(\tfrac{1}{2} e^{-j\Omega}\right)^n \quad ; \quad X(e^{j\Omega}) = \sum_{0}^{\infty} \left(\tfrac{1}{3} e^{-j\Omega}\right)^n$$

$$A = 3, \; B = -2$$

DFT: FT that are suited for computation on digital computers are called DFT. DFT is similar to DTFT but with some differences. There is connection between DTFS & DFT.

DFT & IDFT: Consider $x(n)$ is N-point sequence, DFT of $x(n)$ is given by

$$DFT \Rightarrow \boxed{X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-jk\Omega n}} \; ; \; \Omega = \frac{2\pi}{N}$$

IDFT: The inverse of DFT is called IDFT. is give by

$$IDFT \Rightarrow \boxed{x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{jk\Omega n}} \; ; \; \Omega = \frac{2\pi}{N}$$

Truncation

Both DFT & IDFT are periodic with Period N.

<u>F.F.T:</u> A FFT is an algorithm that computes the DFT of a sequence or it's inverse (IDFT)

Why it is needed?

(i) The total number of stages is $\log_2 N$

(ii) Computational complexity $N\log_2 N$

(iii) In-place computation is possible.

FFT Algorithm:
- DIT: Input $x(n)$ is devided into subsequence.
- DIF: output $x(k)$ is

* Radix-2 Algorithm: DIT & DIF algorithms are applicable only when the data length N is a power of 2. ie $N = 2^P$, where P = positive integer. e.g, N = 2, 4, 8, 16 ... and hence they are commonly known as Radix-2 algorithms.

* Derive the equation for exponential FS.

$x(t) = \sum_{k=-\infty}^{\infty} x(k) e^{jk\Omega_0 t}$; multiplying by $e^{-jn\Omega_0 t}$; case 1: $n = k$. case-2: $n \neq k$

FS: $\boxed{x(t) = \sum_{k=-\infty}^{\infty} \underbrace{x(k)}_{a_k} e^{jk\Omega_0 t}; \quad \underbrace{x(k)}_{a_k} = \frac{1}{T}\int_{T_1}^{T_1} x(t)\, e^{-jk\Omega_0 t}\, dt}$

* Properties of DFT:

Periodicity: $x(n+N) = x(n)$ for all n

$X(k+N) = X(k)$ for all k

$x(n) \xleftrightarrow{DFT} X(k)$

→ N-point sequence → its DFT

Linearity: $A x_1(n) + B x_2(n) \xrightarrow[N\text{-point}]{DFT} A X_1(k) + B X_2(k)$

2017
* Distinguish between FS & F.T.

F.S	F.T
Gives the harmonic content of a periodic time function.	Gives the frequency information for an aperiodic signal
Discrete frequency spectrum.	Continuous frequency spectrum
Periodic	Non-periodic

DTFS: $x(n) = \sum_{k=0}^{N-1} x(k)\, e^{jk\Omega n}$. $\quad x(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-jk\Omega n}$

FS: $x(t) = \sum_{-\infty}^{+\infty} x(k)\, e^{jk\Omega t}$ $\quad x(k) = \frac{1}{T}\int_{-T_1}^{T_1} x(t)\, e^{-jk\Omega t}$

~~DTF~~ DFT:

$x(k) = \sum_{k=0}^{N-1} x(n)\, e^{-jk\Omega n}$ $\quad x(n) =$

$x(n) = \frac{1}{N}\sum_{k=1}^{N-1} x(k)\, e^{jk\Omega n}$ $\quad$ DFT: $x(k) = \sum_{n=0}^{N-1} x(n)\, e^{-jk\Omega n}$.

DTFT:

$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})\, e^{j\Omega n}\, d\Omega$

$x(e^{j\Omega}) = \sum_{-\infty}^{\infty} x(n)\, e^{-j\Omega n}$

Channel: Noise & interference from the other Signals all can distort the signal in propagation

$$x(n) = \frac{y(n) - \sum_{m=0}^{n-1} x(m)\, h(n-m)}{h(0)}$$