Electric field Monte Carlo Method for Simulating Coherent Backscattering of Polarized Light in Optical Active Medium

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Abstract: A rigorous Electric field Monte Carlo method for simulating coherent backscattering of polarized light is presented. The inherent phase difference from multiple scattering between light traveling in a pair of time-reversed paths owing to the vector nature of light is taken into account properly. Simulation results demonstrate that the neglect of this inherent phase difference may result in an appreciable distortion of the enhancement factor and the angular profile of the coherent backscattered light and prevent a correct interpretation of the observed coherent backscattering profile.

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Coherent backscattering (CB) is one intriguing phenomenon of multiple scattering light which manifests itself as a sharp peak of intensity centered at exactly the backscattering direction for an incident coherent beam. This effect is the result of the constructive interference between partial waves traveling along a pair of time-reversed trajectories inside the turbid medium. Laboratory demonstrations of CB were first reported in mid 1980s'. [1–3] Since then, CB has been the subject of active theoretical and experimental research and has seen many applications in remote sensing and particle characterization using electromagnetic scattering in diverse fields such as atmospheric optics, astrophysics, biophysics, and many other areas of science and engineering. The CB enhancement in the exact backscattering direction can be rigorously obtained from the vector theory of radiative transfer of incoherent light as the phase difference between the two partial waves traveling in a pair of time-reversed trajectories is exactly zero. [4] The angular profile of CB of polarized light is much more involved than the vector theory of radiative transfer and the rigorous theory is early in its development. [5] Monte Carlo (MC) simulations have been extensively used to study the enhancement and the angular profile of CB and to compare with experimental observations. [6] Most MC simulations of CB are obtained from Fourier transform of the spatial distribution of the intensity of the incoherent backscattered light. Such a treatment of CB makes one central assumption that the phase difference between partial waves traveling along the forward and reverse paths is given by $\mathbf{q} \cdot \boldsymbol{\rho}$ due to the optical path difference where $\mathbf{q}_b = \mathbf{k}_i + \mathbf{k}_f$, $\mathbf{k}_{i,f}$ is the wave vector of the incident and backscattered light, respectively, and $\boldsymbol{\rho}$ is the radial vector on the surface of the medium pointing from the point of incidence to the point where light escapes. This assumption is however only strictly valid in the exact backscattering direction ($\mathbf{q}_b = 0$) for polarization preserving channels as the two partial waves have in general a nonzero phase difference after multiple scattering in opposite orders. The consequence of this assumption on the simulated CB can be serious.

In this Letter, we present a rigorous Electric field Monte Carlo (EMC) approach for simulating coherent backscattering of polarized light. This work is an extension of our earlier EMC study of polarized light propagation in a turbid medium. [7, 8] Electric fields of partial waves traveling in a pair of time-reversed paths are added coherently to simulate their interference, taking into full account their amplitude and phase difference from multiple scattering when traveling in the pair of paths in opposite orders. Such an extra phase difference than that from their optical path difference is a pure result of the vector nature of light. EMC simulation results manifest the importance of this vector phase difference and point out serious errors may result in interpreting CB based on models neglecting this effect.

The propagation of light in a turbid medium can be formulated as a series of update of the parallel and perpendicular polarized electric components E_j ($j = 1, 2$) with respect to the present scattering plane and a rotation of the local coordinate system spanned by $(\mathbf{m}, \mathbf{n}, \mathbf{s})^T$ where $\mathbf{m}$, $\mathbf{n}$, and $\mathbf{s}$ represent the unit vectors in the directions of parallel polarization, perpendicular polarization, and propagation, respectively. [7, 9] The electric field of light is given by the superposition of its two components $\mathcal{E} = E_1\mathbf{m} + E_2\mathbf{n}$. Denote $\mathbf{E} = (E_1, E_2)^T$. After one scattering event, $\mathbf{E}$ becomes $\mathbf{E}' = S(\mathbf{s}', \mathbf{s})R(\phi)\mathbf{E}$ for the incident electric field propagating in $\mathbf{s}$ direction scattered into $\mathbf{s}'$ direction, where $R(\phi)$ is the rotation matrix,

$$R(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \tag{1}$$

which rotates the reference frame ϕ degrees azimuthally with respect to the axis $\mathbf{s}$ to align the incoming perpendicular polarization delydirection $\mathbf{n}$ to the normal $\mathbf{n}' = \mathbf{s}\times\mathbf{s}'/|\mathbf{s}\times\mathbf{s}'|$ of the present scattering plane, and $S(\mathbf{s}', \mathbf{s})$ is the amplitude scattering matrix. For spherical particles, $S = \mathrm{diag}(S_2(\cos\theta), S_1(\cos\theta))$ is dependent on the scattering angle θ, its size parameter and relative refractive index.

To incorporation of Faraday rotation: The electric field needs first be decomposed into left- and right-circular polarized modes which encounters different phase delays upon propagation. The electric field is decomposed to

$$
\begin{aligned}
\mathbf{E} &= E_{+}\hat{\mathbf{e}}_{+} + E_{-}\hat{\mathbf{e}}_{-} \\
E_{\pm} &= \frac{1}{\sqrt{2}}\left(E_{\parallel} \mp iE_{\perp}\right) \\
\hat{\mathbf{e}}_{\pm} &= \frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}_{\parallel} \pm i\hat{\mathbf{e}}_{\perp}\right).
\end{aligned}
$$

Here $\hat{\mathbf{e}}_{\pm}$ represent right-circularly (anti-clockwise) and left-circularly (clockwise) polarized waves when looking in the direction of propagation. This definition is different from Mishchenko et. al.(2002) [10]. The Faraday rotation of the field yields

$$
\begin{aligned}
\mathbf{E} &= E_{+}\exp(i\delta)\hat{\mathbf{e}}_{+} + E_{-}\exp(-i\delta)\hat{\mathbf{e}}_{-} \\
&= \frac{1}{\sqrt{2}}\left(E_{\parallel} - iE_{\perp}\right)\exp(i\delta)\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}_{\parallel} + i\hat{\mathbf{e}}_{\perp}\right) + \frac{1}{\sqrt{2}}\left(E_{\parallel} + iE_{\perp}\right)\exp(-i\delta)\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}_{\parallel} - i\hat{\mathbf{e}}_{\perp}\right) \\
&= E'_{\parallel}\hat{\mathbf{e}}_{\parallel} + E'_{\perp}\hat{\mathbf{e}}_{\perp}
\end{aligned}
$$

with

$$
\begin{pmatrix} E'_{\parallel} \\ E'_{\perp} \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix}\begin{pmatrix} E_{\parallel} \\ E_{\perp} \end{pmatrix}.
$$

The relation $\mathbf{E}' = S(\mathbf{s}', \mathbf{s})R(\phi)\mathbf{E}$ will be modified to $\mathbf{E}' = S(\mathbf{s}', \mathbf{s})R(\phi)P\mathbf{E}$ with the operator P given by

$$P(l) = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix}$$

where k is the wavenumber in vacuum, $\delta \equiv \frac{\Delta n}{2} kl$, and $\Delta n \equiv n_+ - n_-$. The extra phase delay factor $\exp(i\bar{n}kl)$ with $\bar{n} \equiv (n_+ + n_-)/2$ has been omitted here. The net effect of Faraday rotation produces an extra rotation $\phi \rightarrow \phi + \delta$.

The factor

$$R(\phi)P(l) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} = \begin{pmatrix} \cos\phi\cos\delta - \sin\phi\sin\delta & \sin\phi\cos\delta + \cos\phi\sin\delta \\ -\sin\phi\cos\delta - \cos\phi\sin\delta & \cos\phi\cos\delta - \sin\phi\sin\delta \end{pmatrix}$$

Consider now a beam of light $\mathcal{E}_0$ incident along $\mathbf{s}_0$ enters a semi-infinite turbid medium and encounters a series of scattering at sites $\mathbf{r}_i$ $(i = 1, 2, \ldots, n)$ before escaping the medium in the direction $\mathbf{s}_\mathrm{n}$ (see Fig. 1). Denote the initial reference frame of the incident beam as $(\mathbf{m}_0, \mathbf{n}_0, \mathbf{s}_0)^T$, the local coordinate system as $(\mathbf{m}_i, \mathbf{n}_i, \mathbf{s}_i)^T$ and the amplitude scattering matrix as $S^{(i)}(\mathbf{s}_i, \mathbf{s}_{i-1})$, respectively, at the ith scattering at $\mathbf{r}_i$, ϕ_{i-1} as the azimuthal angle to rotate $\mathbf{n}_{i-1}$ to $\mathbf{n}_i$ about the axis $\mathbf{s}_{i-1}$, and θ_{i-1} as the scattering angle for light being scattered from $\mathbf{s}_{i-1}$ to $\mathbf{s}_i$. The unit vector $\mathbf{n}_i = \mathbf{s}_{i-1} \times \mathbf{s}_i / |\mathbf{s}_{i-1} \times \mathbf{s}_i|$ is the normal of the scattering plane spanned by $\mathbf{s}_{i-1}$ and $\mathbf{s}_i$ at the ith scattering event. The outgoing electric field in the forward path can be written as

$$\mathbf{E}_{\text{out}} = P(|\boldsymbol{r}_{\text{out}} - \boldsymbol{r}_n|)S^{(n)}(\mathbf{s}_n, \mathbf{s}_{n-1})R(\phi_{n-1})P(|\boldsymbol{r}_n - \boldsymbol{r}_{n-1}|)TS^{(1)}(\mathbf{s}_1, \mathbf{s}_0)R(\phi_0)P(|\boldsymbol{r}_1 - \boldsymbol{r}_{\text{in}}|)\mathbf{E}_0, \tag{2}$$

where $T = \prod_{i=2}^{n-1} S^{(i)}(\mathbf{s}_i, \mathbf{s}_{i-1})R(\phi_{i-1})P(|\boldsymbol{r}_i - \boldsymbol{r}_{i-1}|)$ is an ordered product where terms of a larger index i is placed to the left of terms of a smaller index.

The wave traveling in the reverse path is scattered at sites $\mathbf{r}_n$, $\mathbf{r}_{n-1}$, . . ., and $\mathbf{r}_1$, sequentially in the opposite order than that in the forward path, with its propagation direction rotating from $\mathbf{s}_0$ to $-\mathbf{s}_{n-1}$, . . ., to$-\mathbf{s}_1$, and finally escaping in $\mathbf{s}_n$ direction. The electric field in the backward path can be written as

$$\mathbf{E}_{\text{out}}^{\text{rev}} = P(|\boldsymbol{r}_{\text{in}} - \boldsymbol{r}_1|)S^{(1)}(\mathbf{s}_n, -\mathbf{s}_1)R(\phi'_1)T^{\text{rev}}R(\phi'_n)P(|\boldsymbol{r}_{n-1} - \boldsymbol{r}_n|)S^{(n)}(-\mathbf{s}_{n-1}, \mathbf{s}_0)R(\phi_n)P(|\boldsymbol{r}_n - \boldsymbol{r}_{\text{out}}|)\mathbf{E}_0. \tag{3}$$

Here $R(\phi_n)$ is the rotation matrix which aligns $\mathbf{n}_0$ to the normal $\mathbf{n}' = \mathbf{s}_0 \times (-\mathbf{s}_{n-1})/|\mathbf{s}_0 \times (-\mathbf{s}_{n-1})|$ of the scattering plane at the first scattering site, $\mathbf{r}_n$, in the reverse path; $R(\phi'_n)$ aligns $\mathbf{n}'$ to, $-\mathbf{n}_{n-1}$, the normal of the scattering plane at the second scattering site, $\mathbf{r}_{n-1}$, in the reverse path; $T^{\text{rev}} = \prod_{i=2}^{n-1} R(\phi_{i-1})P(|\boldsymbol{r}_{i-1} - \boldsymbol{r}_i|)S^{(i)}(-\mathbf{s}_{i-1}, -\mathbf{s}_i)$ is an ordered product where terms of a larger index i is placed to the right of terms of a smaller index, which represents light being scattered by sites $\mathbf{r}_{n-1}$, $\mathbf{r}_{n-2}$,. . .,$\mathbf{r}_2$ sequentially and the local coordinate system for light being rotated from $(\mathbf{m}_{n-1}, -\mathbf{n}_{n-1}, -\mathbf{s}_{n-1})^T$ to $(\mathbf{m}_1, -\mathbf{n}_1, -\mathbf{s}_1)^T$ after $(n-2)$ scattering events; and $R(\phi'_1)$ is the rotation matrix which aligns $-\mathbf{n}_1$ to the normal $\mathbf{n}'' = -\mathbf{s}_1 \times \mathbf{s}_{\text{out}}/|-\mathbf{s}_1 \times \mathbf{s}_{\text{out}}|$ of the scattering plane at the last scattering site, $\mathbf{r}_1$, in the reverse path. The other quantities involved in the reverse path are the same as those in the forward path. The operator T^{rev} relates to T as $T^{\text{rev}} = QT^TQ$ where $Q = \text{diag}(1, -1)$ owing to the time-reversal symmetry of electromagnetic waves. [11]

In the special case when light is backscattered in the exact backscattering direction ($\mathbf{q}_b = k(\mathbf{s}_0 + \mathbf{s}_n) = 0$), $\mathbf{E}^{\text{rev}}_{\text{out}}$ can be simplified as $R(\phi'_1) = 1$ and $R(\phi'_n) = R(\phi_{n-1})$. If one further rotates the local coordinate systems for $\mathbf{E}_{\text{out}}$ and $\mathbf{E}^{\text{rev}}_{\text{out}}$ to $(\mathbf{m}_0, -\mathbf{n}_0, -\mathbf{s}_0)^T$, the electric field in the forward and the reversed paths can be written as $\mathbf{E}_{\text{out}} = \mathcal{T}\mathbf{E}_0$ and $\mathbf{E}^{\text{rev}}_{\text{out}} = \mathcal{T}^{\text{rev}}\mathbf{E}_0$ where $\mathcal{T} = R(\phi_n)S^{(n)}(\mathbf{s}_n, \mathbf{s}_{n-1})R(\phi_{n-1})TS^{(1)}(\mathbf{s}_1, \mathbf{s}_0)R(\phi_0)$ and $\mathcal{T}^{\text{rev}} = Q\mathcal{T}^TQ$. Inside the polarization preserved channels, the phase difference between the two partial waves $\mathbf{E}_{\text{out}}$ and $\mathbf{E}^{\text{rev}}_{\text{out}}$ is hence zero. It is, however, not true for polarization unpreserved channels even when $\mathbf{q}_b = 0$. The phases (and magnitudes) of $\mathbf{E}_{\text{out}}$ and $\mathbf{E}^{\text{rev}}_{\text{out}}$ are, in general, inherently different due to the vector nature of light.

The EMC method for simulating CB uses Eqs. (2) and (3) to compute the electric field traveling along a pair of time-reversed paths. The intensity of the coherent backscattered light is obtained from $\left|\mathbf{E}_{\text{out}} + \mathbf{E}_{\text{out}}^{\text{rev}} \exp(i\delta)\right|^2$ where $\delta \equiv \mathbf{q}_b \cdot (\mathbf{r}_n - \mathbf{r}_1)$ is the phase delay introduced by the optical path difference. The backscattered light encountering only one scattering is computed separately. A partial photon technique [6] is used to improve the efficiency of the EMC simulation.

Table 1 displays the comparison between the EMC simulated CB enhancement and the rigorous theoretical prediction [12] in the exact backscattering direction for a homogeneous slab consisting of nonabsorptive Rayleigh scatterers. The slab has no refractive index mismatch across its boundary. The thickness of the slab is one, l_t, the transport mean free path. The enhancement factors of CB ζ_{vv}, ζ_{vh}, ζ_{hp}, and ζ_{ho} in the parallel polarized, perpendicular polarized, helicity preserved, and opposite helicity channels are computed, in addition to the enhancement factor ζ_{un} for unpolarized light. A total of 10 millions photons were used in the EMC simulation. Total 8 copies of simulations were performed to estimate the statistical error of the EMC simulated values. Excellent agreement between the EMC and theoretical results is evident. If one adopts the conventional assumption that there is no phase difference between $\mathbf{E}_{\text{out}}$ and $\mathbf{E}_{\text{out}}^{\text{rev}}$ in the polarization unpreserved channels, the enhancement can be obtained from $1 + I_{ms}/(I_{ms} + I_{ss})$ where I_{ms} and I_{ss} are the intensities of the incoherent multiple or single backscattering light in the exact backscattering direction, respectively, yielding $\zeta_{vh} = 2$ and $\zeta_{ho} = 1.406$ which overestimated the enhancement significantly.

Fig. 2 displays the angular profiles of CB enhancement obtained from EMC simulations directly (EMC), 2D Fourier transform of the spatial distribution of the intensity of the incoherent backscattered light in the exact backscattering direction (FT), and the prediction based on diffusion photons (DA) [13] for a slab of Mie scatterers (size parameter 4.20 and relative refractive index 1.19). The thickness of the slab is $20l_t$. The enhancement factors ζ_{ho} and ζ_{vh} from EMC are much smaller than 2 and is also much smaller than the corresponding values predicted by FT. In the "hp" channel, the angular profile from FT is wider than that from EMC; its full width at half maximum (FWHM) is about 19% larger. In the "vv" channel, the angular profile from FT is about 3 times narrower than that from EMC. The disagreement between EMC and FT is much worse in the "ho" and "vh" channels. DA fits well with the EMC result in both "hp" and "vv" channels. The fitted value of l_t however is, $kl_t = 410$, about 10% smaller in the "hp" channel and, $kl_t = 244$, about 50% smaller in the "vv" channel than the input value ($kl_t = 450$).

In conclusion, we have presented a rigorous Electric field Monte Carlo method for simulating coherent backscattering of polarized light, which takes into account properly the phase difference from multiple scattering when traveling in a pair of time-reversed paths in opposite orders resulting purely from the vector nature of light. The neglect of this effect may prevent a correct interpretation of CB from experimental data and lead to inaccurate characterization of the scattering property of the medium. The discrepancy between the experimental and FT data reported for coherent light backscattering from a cold Strontium cloud [14] may just be a manifest of this effect. As CB has emerged as the most reliable approach for probing l_t of a strongly scattering medium, the proposed EMC method shall be very useful in these applications.

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	ζ_{vv}	ζ_{vh}	ζ_{hp}	ζ_{ho}	ζ_{un}
EMC	1.45307 ± 0.00009	1.55327 ± 0.00125	2.00000	1.33742 ± 0.00013	1.46546 ± 0.000
Theory	1.453065	1.553473	2	1.337506	1.465559

Table 1: M. Xu et. al.

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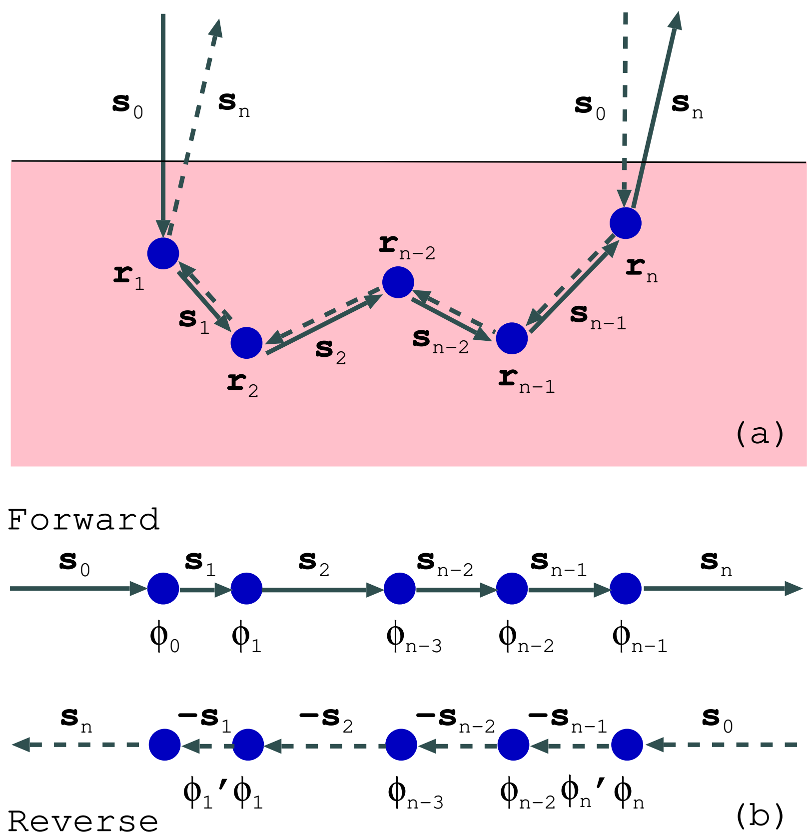
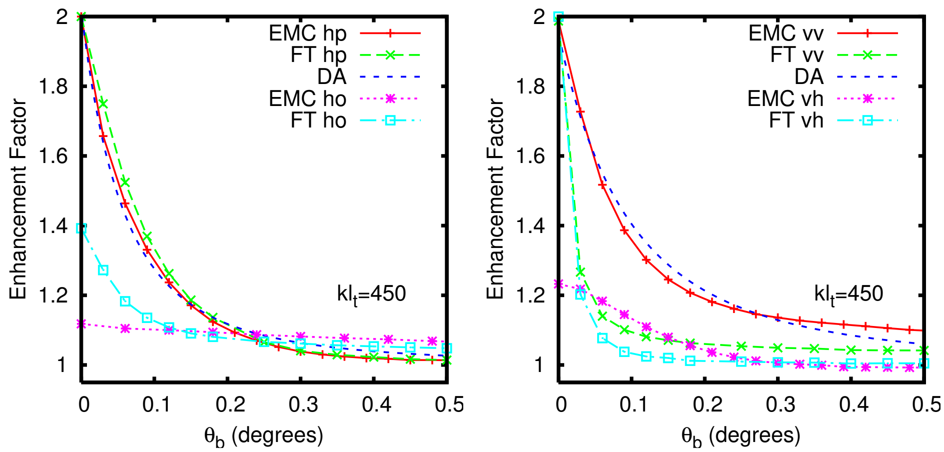


Figure 1: M. Xu et. al.



(a) (b)

Figure 2: M. Xu et. al.