- The electromagnetic spectrum is a categorized description of emitted radiations from different elements based on their frequencies/ wavelengths as shown in fig. It includes broad range of radiation such as radio waves; microwaves; infrared, visible, and ultraviolet light; and x and gamma rays etc.

IMPORTANT FORMULAE

where, ΔE is the energy difference between two levels.
N_1 is number of atoms in lower energy level,
N_2 is number of atoms in higher energy level,
k is Boltzmann constant and
T is absolute temperature.

1. Ratio of population of the two states: normal or Boltzmann distribution: $\frac{N_2}{N_1} = e^{-\Delta E/kT}$

Q.3. Find the ratio of population of the two states in a He-Ne laser that produce light of wavelength 6328 Å at 27°C. **S-09**

Ans. Given: 1. Wavelength of light,

$\lambda = 6328 \text{ Å} = 6328 \times 10^{-10}$ m

2. Temperature, $T = 27°C = 300$ K

To find: Ratio of population of the two states, $\frac{N_2}{N_1}$.

Formulae: 1. Energy difference, $\Delta E = \frac{hc}{\lambda}$

2. Ratio of population of the two states,

$$\frac{N_2}{N_1} = e^{-\Delta E/kT}$$

Soln. $\Delta E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6328 \times 10^{-10}}$

$= 3.143 \times 10^{-19}$ J.

$$\frac{\Delta E}{kT} = \frac{3.143 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} = 75.9$$

$$\therefore \frac{N_2}{N_1} = e^{-75.9}$$

$$= 1.1 \times 10^{-33}$$

$$\boxed{\text{Ratio of population of the two states, } \therefore \frac{N_2}{N_1} = 1.1 \times 10^{-33}}$$ Ans.

Q.4. The Ruby Laser has two states at 300 K and 500 K if it emits light of 7000 Å, then calculate relative population.

Ans. Given: 1. Temperature, $T_1 = 300$ K, $T_2 = 500$ K

2. Wavelength of light, $\lambda = 7000$ Å

$= 7000 \times 10^{-10}$ m

To Find: Relative population at T_1 and T_2.

Formulae: 1. Energy difference, $\Delta E = \frac{hc}{\lambda}$

2. Ratio of population of the two states,

$$\frac{N_2}{N_1} = e^{-\Delta E/kT}$$

Soln. 1. Relative population of the two states at $T_1 = 300$K

$$\Delta E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{7000 \times 10^{-10}}$$

$$= 2.84 \times 10^{-19} \text{ J.}$$

$$\frac{\Delta E}{kT} = \frac{2.84 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} = 68.59$$

$$\therefore \frac{N_2}{N_1} = e^{-68.59} = 1.62 \times 10^{-30}$$

2. Relative population of the two states at T_2 = ~~500K~~ 500K

$$\Delta E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{7000 \times 10^{-10}}$$

$$= 2.84 \times 10^{-19} \text{ J}$$

$$\frac{\Delta E}{kT} = \frac{2.84 \times 10^{-19}}{1.38 \times 10^{-23} \times 500} = 41.15$$

$$\therefore \frac{N_2}{N_1} = e^{-41.15} = 1.34 \times 10^{-18}$$

$$\boxed{\begin{array}{l}\text{Relative population of the two states at} \\ T_1 = 300\text{K} = 1.62 \times 10^{-30} \\ \text{Relative population of the two states at} \\ T_1 = 500\text{K} = 1.34 \times 10^{-18}\end{array}}$$ Ans.

Q.5. A three-level laser emits laser light at a Wavelength **5500 Å.**

(i) In the absence of optical pumping, what will be **equilibrium ratio of the population of** the upper level **to that of the lower level? Assume T =** 300K

(ii) At what temperature for the Conditions of **(i) above would the ratio be $\frac{1}{2}$?**

(iii) What conclusion can be drawn on the basis of the results obtained by you in relation to choice of pumping mechanism? **W-03**

Ans. Given: 1. Wavelength of light,

$$\lambda = 5500\ \text{Å} = 5500 \times 10^{-10}\ \text{m}$$

2. Temperature T = 300 K

To Find: 1. Population at T = 300 K

2. Temperature for the conditions of above would the ratio be $\frac{1}{2}$

Formulae: 1. Energy difference, $\Delta E = \frac{hc}{\lambda}$

2. Ratio of population of the two states,

$$\frac{N_2}{N_1} = e^{-\Delta E/kT}$$

3. Temperature, $T = \frac{\Delta E}{K \ln \frac{N_1}{N_2}}$

Soln. 1. Ratio of population of the two states at T = 300K

$$\Delta E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5500 \times 10^{-10}} = 3.62 \times 10^{-19}\ \text{J.}$$

$$\frac{\Delta E}{kT} = \frac{3.62 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} = 87.43$$

$$\frac{N_2}{N_1} = e^{-87.43} = 1.07 \times 10^{-38}$$

2. Temperature for the conditions of (i) above would the ratio be $\frac{1}{2}$, i.e. for $\frac{N_2}{N_1} = \frac{1}{2}$

$$T = \frac{3.62 \times 10^{-19}}{1.38 \times 10^{-23} \times \ln 2} = 3.78 \times 10^{4}\ \text{K}$$

Conclusion: Suitable pumping mechanism is not available to raise the temperature to such a high value.

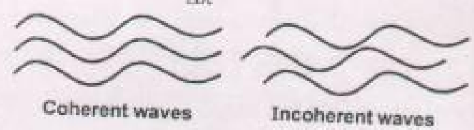
$\therefore$ $\boxed{\begin{array}{l}\text{Relative population of the two states} \\ \text{Temperature } T = 3.78 \times 10^{4}\ \text{K}\end{array}}$ **Ans.**

SPATIAL AND TEMPORAL COHERENCE OF LIGHT WAVE

Q.6. Explain the term: (i) Coherence length (ii) Temporal coherence (iii) Spatial coherence **S-98,01,02,10, W-95,00**

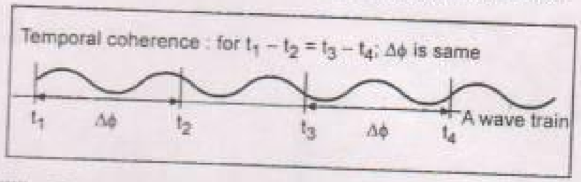
Ans. (i) Coherence : The light waves are emitted from the source in the form of wave trains, which has random phases that last for short duration. Average life time of a wave train is termed as coherence time t_0. A light is said to be coherent if the wave trains bear constant relationship between each other over their existence. The length of the electromagnetic wave over which the spatial coherence is maintained is called coherence length (l_t).

$$l_t = c\,t_0 = c/\Delta\nu = \frac{\lambda_0^2}{\Delta\lambda}$$



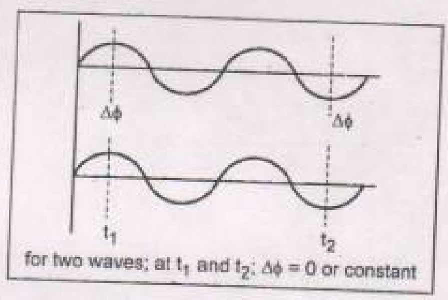
The coherence for a source is specified in terms of two parameters such as Temporal & spatial coherence.

(ii) Temporal coherence: It is used to state the coherence of a single wave. If the wave has same phase for given interval of time, then it is said to be having temporal coherence. If for $(t_2 - t_1) = (t_3 - t_4)$, $\Delta\phi$ remains same then the wave is said to have temporal coherence.



(iii) Spatial coherence: The two waves are said to have a spatial coherence if the path difference between them remains same w.r.t time. Thus if at t_1 and t_2, $\Delta\phi$ remains same then they have spatial coherence. These two features are illustrated in fig. and coherence lengths are also compared for different sources.

The coherence of a wave is measured in terms of its coherence length, 'l' it is given as



$l = c\,t$ where c is velocity of light and t is duration of the pulse.

If $\Delta\nu$ is frequency spread, then $t = \frac{1}{\Delta\nu}$

- Experimentally, the coherent and non-coherent light can be distinguished through Young's double slit experiment. The fringe width, β of interference pattern is given by

  $$\beta = \frac{\lambda D}{d}$$

  where λ is wavelength of light, d is separation of the two sources and D is distance between the source and screen.
- From this expression it is seen that the fringe pattern is well defined if the screen is placed away from the source. Phase difference between the waves remains constant over longer distance in case of coherent source. Hence the fringe pattern for coherent light is well defined than for the non coherent light.

  Also, the coherent sources are more monochromatic than non coherent source hence the fringe pattern for coherent light is well defined than for the non coherent light. Thus by comparing the fringe width two sources, light can be experimentally distinguished as coherent and non-coherent light

IMPORTANT FORMULAE

1. The coherence length, $l = c\,t = l = \frac{c}{\Delta v} = \frac{\lambda_0^2}{\Delta\lambda}$ m

   $\Delta\lambda$ is called Spectral half width or band width.
2. Energy of photon: $\varepsilon = hv, \varepsilon = hv = \frac{hc}{\lambda} J = \frac{12400}{\lambda}$ eV
3. Energy within pulse: $E = n\,hv$
4. Number of photons emitted $n = E/\varepsilon$
5. Intensity of radiation I, after it travels a distance x:

   $I = I_0\, e^{-\alpha x}$.

   α is called absorption coefficient.
6. Laser cavity length supporting m modes, $L = \frac{m\lambda}{2}$
7. Laser cavity length supporting m modes, in terms of half width, $L = \frac{mc}{2\Delta v}$

Q.38. White light has frequency range from 0.4×10^{15} Hz to 0.7×10^{15} Hz. Find the coherence-time and coherent-length. **S-95,96,W-09**

Ans. Given: 1. Range of frequency for white light:

0.4×10^{15} Hz to 0.7×10^{15} Hz.

To find: 1. Coherence time, t

2. Coherence length, l

Formulae: 1. Coherence time, $t = \frac{1}{\Delta v}$

2. Coherence length, $l = c\,t$

Soln. 1. Coherence time $t = \frac{1}{(0.7 - 0.4) \times 10^{15}}$

$= 3.33 \times 10^{-15}$ s

2. Coherence length, $l = 3 \times 10^8 \times 3.33 \times 10^{-15}$

$= 9.99 \times 10^{-7}$ m

$\therefore$ $\boxed{\begin{array}{l} \text{1. Coherence time } t = 3.33 \times 10^{-15}\text{ s} \\ \text{2. Coherence length, } l = 9.99 \times 10^{-7}\text{ m} \end{array}}$ **Ans.**

Q.39. A sodium atom radiates for 4×10^{-12} s. What is the coherence length of light from a sodium lamp? **W-97**

Ans. Given: 1. Coherence time, $t = 4 \times 10^{-12}$ s.

To find: Coherence length, l

Formula: Coherence length, $l = c \times t$

Soln. Coherence length, $l = 3 \times 10^8 \times 4 \times 10^{-12}$

$= 1.2 \times 10^{-3}$ m

$\therefore$ $\boxed{\text{Coherence length, } l = 1.2 \times 10^{-3}\text{ m}}$ **Ans.**

Q.40. A ruby laser emits light of wavelength 694.4 nm. If a laser pulse is emitted for 1.2×10^{-11} s and the energy released per pulse is 0.15 J, (i) What is the length of the pulse and (ii) How many photons are there in each pulse? **S-00**

Ans. Given: 1. Wavelength of ruby laser light,

$\lambda = 694.4$ nm $= 694.4 \times 10^{-9}$ m.

2. Coherence time, $t = 1.2 \times 10^{-11}$ s.

3. Energy per pulse, $E = 0.15$ J

To find: 1. The length of the pulse, l

2. Number of photons in each pulse, N

Formulae: 1. Length of the pulse, $l = c\,t$

2. Energy of each photon, $\varepsilon = \frac{hc}{\lambda}$

$J = \frac{12400}{\lambda}$ eV

3. Number of photons in each pulse, $N = \frac{E}{\varepsilon}$

Soln. 1. Length of the pulse, $l = 3 \times 10^8 \times 1.2 \times 10^{-11}$

$= 3.6 \times 10^{-3}$ m

Energy of each photon,

$$\varepsilon = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{694.4 \times 10^{-9}} \text{ J}$$

$$= 2.86 \times 10^{-19} \text{ J}$$

Number of photons, $n = \dfrac{0.15}{2.86 \times 10^{-19}}$

$$= 5.24 \times 10^{17}$$

$\therefore$

1. Length of the pulse, $l = 3.6 \times 10^{-3}$ m
2. Number of photons, $n = 5.24 \times 10^{17}$

Ans

Q.41. Light of certain wavelength has wave train of width 13.2×10^{-2} m. Calculate Coherence time. **S-01**

Ans. Given: 1. Coherence width, $l = 13.2 \times 10^{-2}$ m.

To find: Coherence time, t

Formula: Coherence time, $t = \dfrac{l}{c}$

Soln. Coherence time,

$$t = \frac{13.2 \times 10^{-2}}{3 \times 10^{8}} = 4.4 \times 10^{-10} \text{ s}$$

$\therefore$ $\boxed{\text{Coherence time, } t = 4.4 \times 10^{-10} \text{ s}}$ **Ans.**

Q.42. Compute the coherence length of yellow light with 5893 Å in 10^{-12} second pulse duration. Find also the Bandwidth. **S-02,08,10**

Ans. Given: 1. Coherence time $t = 10^{-12}$ s

2. Wavelength of yellow light,

$\lambda = 5893$ Å $= 5893 \times 10^{-10}$ m

To Find: 1. Coherence length, l

2. Bandwidth, $\Delta\lambda$

Formulae: 1. Coherence length, $l = c\,t$

2. Band width, $= \Delta\lambda = \dfrac{\lambda_0^2}{l}$

Soln. 1. Coherence length, $l = 3 \times 10^{8} \times 10^{-12}$

$$= 3 \times 10^{-4} \text{ m}$$

2. Bandwidth, $= \Delta\lambda = \dfrac{(5893 \times 10^{-10})^2}{3 \times 10^{-4}}$

$$= 1.157 \times 10^{-9} \text{ m}$$

$\therefore$

1. Coherence length, $l = 3 \times 10^{-4}$ m
2. Bandwidth, $\Delta\lambda = 1.157 \times 10^{-9}$ m

Ans.

Q.43. With a He-Ne laser, fringes remain clearly visible. When the path difference was increased upto 8 m. (Given $\lambda = 11.5 \times 10^{-7}$ m).

Deduce : (i) Coherence time, (ii) Spectral half width. **S-06**

Ans. Given: 1. Coherence length, $l = 8$ m

2. Wavelength of He - Ne laser light,

$\lambda = 11.5 \times 10^{-7}$ m

To Find: 1. Coherence time, t

2. Spectral width, $\Delta\lambda$

Formulae: 1. Coherence time, $t = \dfrac{l}{c}$

2. Bandwidth, $= \Delta\lambda = \dfrac{\lambda_0^2}{l}$

Soln. 1. Coherence time, $t = \dfrac{8}{3 \times 10^{8}} = 2.66 \times 10^{-8}$ s.

2. Bandwidth, $= \Delta\lambda = \dfrac{(11.5 \times 10^{-7})^2}{8}$

$$= 1.6 \times 10^{-13} \text{ m}$$

$\therefore$

1. Coherence time, $t = 2.66 \times 10^{-8}$ s
2. Bandwidth, $\Delta\lambda = 1.6 \times 10^{-13}$ m

Ans.

Q.44. A typical helium-neon laser emits radiation of $\lambda = 6328$ Å. How many photons per second would be emitted by a one milliwatt He-Ne Laser? **W-08**

Ans. Given: 1. Wavelength of He Ne laser light,

$\lambda = 6328$ Å $= 6328 \times 10^{-10}$ m

2. Power of laser $= 1$ mW $= 10^{-3}$ W

To Find: Number of photons, n

Formulae: 1. Energy of photon, $\varepsilon = \dfrac{hc}{\lambda}$

2. Energy of laser pulse, $E = P \times 1\text{s}$

3. Number of photons, $n = \dfrac{E}{\varepsilon}$

Soln. 1. Energy of photon,

$$\varepsilon = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{6328 \times 10^{-10}} = 3.1 \times 10^{-19} \text{ J}$$

2. Energy of laser pulse, $E = 10^{-3} \times 1 = 10^{-3}$ J

3. Number of photons,

$$n = \frac{10^{-3}}{3.1 \times 10^{-19}} = 3.2 \times 10^{15}$$

$\therefore$ $\boxed{\text{Number of photons, } n = 3.2 \times 10^{15}}$ **Ans.**

Q.45. Find the coherence of white light whose spectrum lies in the region of 4000 Å to 7000 Å. ***W-95***

Ans. Given: 1. Wavelength range λ_1 to λ_2

$\lambda_1 = 4000$ Å to $\lambda_2 = 7000$ Å

To Find: 1. Coherence length, l_{coh}

2. Coherence time, t_{coh}

Formula: 1. Frequency range, $\Delta v = v_2 - v_1$

$$= \frac{C}{\lambda_2} - \frac{C}{\lambda_1}$$

2. Coherence time, $t = \frac{1}{\Delta v}$

3. Coherence length, $l_{coh} = C\,t$

Soln. 1. Frequency range $\Delta v = 0.7 \times 10^{15} - 0.4 \times 10^{15}$

$= 0.3 \times 10^{15}$ Hz

2. Coherence time, t

$$t = \frac{1}{0.3 \times 10^{15}}$$

$= 3.3 \times 10^{-15}$ sec

3. Coherence length, l

$= 3 \times 10^8 \times 3.3 \times 10^{-15}$

$= 9.9 \times 10^{-7}$ m

$\therefore$

1. Coherence time $t = 3.3 \times 10^{-15}$ sec
2. Coherence length $l = 9.9 \times 10^{-7}$ m

Ans.

Q.46. Find the intensity of laser beam of 10mW power and having a diameter of 1.3 mm. Assume the intensity of the beam to be uniform across the beam.

Ans. Given: 1. Power of laser, P = 10 mW = 10^{-2} W.

2. Diameter of beam, d = 1.3mm

$= 1.3 \times 10^{-3}$ m.

To Find: intensity of the laser, I

Formula: 1. Area, $A = \frac{\pi d^2}{4}$

2. Intensity, $I = \frac{P}{A}$

Soln. Intensity, $I = \frac{4 \times 10^{-2}}{3.14 \times (1.3 \times 10^{-3})^2}$

$= 7537$ W / m^2

I = 7537 W/m^2

$\therefore$ Intensity, ~~I = 7537 W / m^2~~ Ans.

Q.47. If half width of the He Ne laser operating at a wavelength, 6328 Å is 1500 MHz, What must be the length of laser cavity to ensure that only one longitudinal mode oscillates?

Ans. Given: 1. Wavelength of laser,

$\lambda = 6328$ Å $= 6328 \times \times 10^{-10}$ m

2. Half width, $\Delta v = 1500$ MHz. $= 1500 \times 10^6$ Hz

3. Number of modes, m = 1

To Find: Length of laser cavity that supports only one mode, L.

Formula: Laser cavity length supporting m modes,

$$L = \frac{mc}{2\Delta v}$$

Soln. Laser cavity length supporting m modes,

$$L = \frac{1 \times 3 \times 10^8}{2 \times 1500 \times 10^6} = 0.1\text{m}$$

$\therefore$

Length of laser cavity that supports only one mode, L = 0.1

Ans.

COMPLETE LIST OF FORMULAE

(1) Ratio of population of the two states: normal or Boltzmann distribution: $\frac{N_2}{N_1} = e^{-\Delta E / kT}$

where, ΔE is the energy difference between two levels.

N_1 is number of atoms in lower energy level,

N_2 is number of atoms in higher energy level,

k is Boltzmann constant and

T is absolute temperature.

(2) The coherence length, $l = c\,t = l = \frac{c}{\Delta v} = \frac{\lambda_0^2}{\Delta\lambda}$ m

$\Delta\lambda$ is called Spectral half width or band width.

(3) Energy of photon: $\varepsilon = hv$, $\varepsilon = hv = \frac{hc}{\lambda} J = \frac{12400}{\lambda} eV$

(4) Energy within pulse: $E = n\,hv$

(5) Number of photons emitted $n = E/\varepsilon$

(6) Intensity of radiation I, after it travels a distance x:

$I = I_0\, e^{-\alpha x}$

α is called absorption coefficient.

(7) Laser cavity length supporting m modes, $L = \frac{m\lambda}{2}$

(8) Laser cavity length supporting m modes, in terms of half width, $L = \frac{mc}{2\Delta v}$