Raman

END Term, Odd 2021
B.Tech. Vth Semester

Course Code: 16B1NMA533
Course Name: Matrix Computations

Maximum Marks: 35
Maximum Time: 02 Hrs.

Course Outcomes: After the successful completion of this course, the student will be able to:	
CO1	Explain the basics of matrix algebra and inverse of a matrix by partitioning
CO2	Solve the system of equations using direct and iterative methods
CO3	Explain the vector spaces and their dimension, inner product space, norm of a vector and matrix
CO4	Apply the Gram-Schmidt process to construct orthonormal basis and Q-R decomposition of a matrix
CO5	Construct Greshgorin's circles and solve eigenvalue problem using Jacobi, Givens, Householder, power and inverse power methods
CO6	Analyze the systems of differential and difference equations arising in dynamical systems using matrix calculus.

Note: 1. All questions are compulsory.
2. Use of basic scientific calculator (non-programmable) is allowed.

Q.1 Find the determinant and trace of the following block diagonal matrix **(CO1, 2M)**

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Q.2 Solve the following system of equations using Gauss-Seidel method by performing two iterations only. **(CO2, 3M)**

$x + 5y + 3z = 28;\ 3x + 7y + 13z = 76;\ 12x + 3y - 5z = 1$. Take initial guess as $[1, 0, 1]^T$.

Q.3 Verify parallelogram law for $f(x) = x^2 - 1$ and $g(x) = x$ in $C[-1,1]$, where $C[a, b]$ denotes the space of all complex valued continuous functions defined on $[a, b]$. For $f, g \in C[a, b]$, inner product is defined as $\langle f, g\rangle = \int_a^b f(x)\overline{g(x)}\, dx$. **(CO3, 3M)**

Q.4 Use the Gram-Schmidt orthonormalization process to construct an orthonormal set of vectors from the linearly independent set of vectors $\{x_1, x_2, x_3\}$ where $x_1 = [1,2,1]^T$, $x_2 = [0,1,1]^T$ and $x_3 = [1,0,2]^T$. **(CO4, 3M)**

Q.5 Using Greshgorin's theorem, find the bounds of eigenvalues λ of the matrix $A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 5 & -2 \\ 4 & -2 & 1 \end{bmatrix}$. Verify the obtained bounds of eigenvalues by determining exact eigenvalues of A using JACOBI METHOD. **(CO5, 6M)**

Q.6 Reduce the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ to a similar tri-diagonal matrix using Householder's transformation. **(CO5, 3M)**

Q.7 Use Power method to find the largest eigenvalue and corresponding eigenvector of matrix $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$. Take initial approximation vector as $X^{(0)} = [1,1,1]^T$. Perform the iterations till $|\lambda_i - \lambda_{i+1}| < 0.2$, where $i = 1,2, \ldots$. **(CO5, 3M)**

Q.8 Solve the given differential equation using matrix method $\ddot{x}(t) = 2x(t) - \dot{x}(t) + 3$ with $x(0) = 0$ and $\dot{x}(0) = 1$. **(CO6, 6M)**

Q.9 Solve the following discrete dynamical system using matrix method **(CO6, 6M)**

$$x_1(t+1) = 3x_1(t) - 7x_2(t) + 5$$
$$x_2(t+1) = x_1(t) - 5x_2(t) - 3$$