

$$① \quad g_H = \alpha_{eff} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = 0 + \frac{1}{2 \times 0.2} \ln\left(\frac{1}{0.98}\right) = 0.05 \text{ m}^{-1}$$

$$t_c = \frac{n_0}{c g_H} = \frac{1}{3 \times 10^8 \times 0.05} = 6.67 \times 10^{-8} \text{ s} = 66.7 \text{ ns}$$

$$\text{FWHM, } \Delta \nu_p = \frac{\nu_0}{Q} = \frac{\nu_0}{t_c \cdot 2\pi \nu_0} = 2.38 \times 10^6 \text{ Hz} \approx 2.4 \text{ MHz}$$

$$\Delta \nu = \frac{c}{2 n_0 L} = \frac{3 \times 10^8}{2 \times 1 \times 0.2} = 7.5 \times 10^8 \text{ Hz} \approx 750 \text{ MHz}$$

$$② \quad g_H = \alpha_{eff} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = 0 + \frac{1}{2 \times 500 \times 10^{-6}} \ln\left(\frac{1}{0.09}\right) = 2.40 \times 10^3 \text{ m}^{-1}$$

$$t_c = \frac{n_0}{c g_H} = \frac{3.5}{3 \times 10^8 \times 2.40 \times 10^3} = 4.86 \times 10^{-12} \text{ s}$$

$$\text{FWHM, } \Delta \nu_p = \frac{\nu_0}{Q} = \frac{1}{2\pi t_c} = 3.27 \times 10^{10} \text{ Hz} \approx 3.3 \times 10^{10} \text{ Hz}$$

$$③ \quad \Delta \nu = \frac{c}{2 n_0 L}, \quad \nu = \frac{c}{\lambda} \Rightarrow |\Delta \nu| = \frac{c}{\lambda^2} |\Delta \lambda| \Rightarrow |\Delta \lambda| = \frac{\lambda^2}{2 n_0 L}$$

$$\Rightarrow 1 = \frac{\Delta \lambda_{HW}}{\Delta \lambda} \Rightarrow 1 = \frac{2 \times 10^{-12}}{[(6328 \times 10^{-10})^2 / (2 \times 1 \times L)]} \Rightarrow L = 10 \text{ cm}$$

$$④ \quad m = \frac{\Delta \lambda_{HW}}{\Delta \lambda} = \frac{\Delta \lambda_{HW}}{[\lambda^2 / (2 n_0 L)]} = \frac{2 n_0 L \Delta \lambda_{HW}}{\lambda^2} = \frac{2 \times 1 \times 0.3 \times 2 \times 10^{-15}}{(6328 \times 10^{-10})^2}$$

$$\Rightarrow m = 3$$

$$⑤ \quad (a) \quad g_1 g_2 = \left(1 - \frac{d}{R_1}\right) \left(1 - \frac{d}{R_2}\right) = 0.121 \quad (\text{stable})$$

$$(b) \quad g_1 g_2 = -0.0345 \quad (\text{unstable})$$

$$(c) \quad g_1 g_2 = -0.067 \quad (\text{unstable})$$

$$(d) \quad g_1 g_2 = 0 \quad (\text{on the edge of stability})$$

$$⑥ \quad \Delta \nu = \frac{c}{2 n_0 L} = \frac{3 \times 10^8}{2 \times 1 \times 0.1} = 1.5 \times 10^8 \text{ Hz} = 1500 \text{ MHz}, \quad \nu_0 = 6 \times 10^{14} \text{ Hz}$$

$$m = \frac{2 n_0 L \nu_0}{c} = \frac{2 \times 1 \times 0.1 \times 6 \times 10^{14}}{3 \times 10^8} = 4 \times 10^6 = 4,000,000$$

output beam will have frequencies $\nu_0 - 2\Delta\nu, \nu_0 - \Delta\nu, \nu_0, \nu_0 + \Delta\nu, \nu_0 + 2\Delta\nu$
 corresponding to $m = (39,99,998), (39,99,999), (40,00,000), (40,00,001), (40,00,002)$