Department of Mathematics
Special Semester 2020-21
Probability and Random Processes

Tutorial Sheet 5 **B.Tech Core**

(Moment Generating Function, Characteristic function, Covariance and Correlation)

1. A pair of fair dice is thrown and let X be the number of 6's turned up. Find the moment generating function (MGF), mean and variance of X.

   (Ans. MGF= $\frac{25}{36}+\frac{10}{36}(1+\frac{t}{1!}+\frac{t^2}{2!}+\ldots)+\frac{1}{36}(1+\frac{2t}{1!}+\frac{4t^2}{2!}+\ldots)$; Mean=1/3, Var=5/18)

2. Find MGF of X whose probability density function is given by $f(x) = k\,\frac{e^{-|x|}}{5}$, $-\infty < x < \infty$. Find first three moments of X about the origin. What is the variance of X? (Ans. Three moments are 0, 2,0; Var(X)= 2)

3. The joint pdf of a two dimensional random variables (X, Y) is given as:

   $f(x,y) = \begin{cases}\frac{3}{2}(x^2+y^2), if\ 0 \le x, y \le 1\end{cases}$ and $f(x,y) = 0, otherwise.$

   Find C_{XY} , E (XY) and ρ_{XY}. (Ans. $C_{XY} = -\frac{1}{64}$, E (XY) $= \frac{3}{8}, \rho_{XY} = -\frac{15}{73}$)

4. The joint pdf of a two dimensional random variables (X,Y) is $P_X(k) = \frac{1}{k!}e^{-2}2^k$ and $P_Y(k) = \frac{1}{k!}e^{-3}3^k$. Compute the MGF of Z = 2X + 3Y. (Ans. $e^{(2e^{2t}+3e^{2t}-5)}$)

5. Compute the characteristic function of discrete random variables X and Y if the joint probability mass function is given as

$$P_{X,Y}(k,l) = \begin{cases} \frac{1}{3}, & k = l = 0 \\ \frac{1}{6}, & k = \pm 1, l = 0 \\ \frac{1}{6}, & k = l = \pm 1 \\ 0, & else \end{cases}$$

   (Ans. $\phi_{X,Y}(\omega_1, \omega_2) = \frac{1}{3} + \frac{1}{3}\cos\omega_1 + \frac{1}{3}\cos(\omega_1 + \omega_2)$)

6. Find the density function of the distribution for which the characteristic function is given by $\phi(t) = e^{-\frac{\sigma^2 t^2}{2}}$. (Ans. $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2}$, $-\infty < x < \infty$)

7. Find characteristic function of of random variable X whose probability density function is given by $f(x) = \lambda e^{-\lambda x}$, $x \ge 0, \lambda > 0$ and hence find first twocentral moments.

   (Ans. $\phi(\omega) = \frac{\lambda}{\lambda - i\omega}$, $\mu_1 = 0, \mu_2 = \frac{1}{\lambda^2}$.)