Digital Systems
18B11EC213

Module 1: Boolean Function Minimization Techniques and Combinational Circuits-4

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Binary Arithmetic

- Binary Addition

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$1 + 1 = (10)_2$

- Binary ‘carry overs’ are performed in a similar way as in decimal addition, i.e., it is added to the next higher binary position.

Cont..

- Examples - Binary Addition

1011 + 1100

```
  1 0 1 1
+ 1 1 0 0
---------
1 0 1 1 1
```

(11 + 12 = 23)

0101 + 1111

Carry digit 1 1 1

```
  0 1 0 1
+ 1 1 1 1
---------
1 0 1 0 0
```

(5 + 15 = 20)

Cont..

- Binary Subtraction

Minuend	Subtrahend	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

- The subtraction of numbers in binary is carried out by subtracting column by column and borrowing from the adjacent left column whenever required.

Cont..

- Examples - Binary Subtraction

1011 - 0110

```
  1 0 1 1
- 0 1 1 0
---------
  0 1 0 1
```

(11 - 6 = 5)

1010 - 0110

```
  1010
- 0110
------
  0100
```

(10 - 6 = 4)

Cont..

- Binary Multiplication

A	B	Product
0	0	0
0	1	0
1	0	0
1	1	1

Cont..

- Example - Binary Multiplication

Multiply 1001 by 1101

```
      1 0 0 1
    * 1 1 0 1
  ------------
      1 0 0 1
    0 0 0 0 *
  1 0 0 1 * *
1 0 0 1 * * *
-------------------
1 1 1 0 1 0 1
```

Subtraction Using r's Complement

If M and N are two positive numbers with same base r, the subtraction (M - N) can be done by using the following steps:

(M - N) = M + (-N)

Step 1: Represent -N in r's complement (let N*)

Step 2: Add M and N*

Step 3: Inspect

a) If an end carry occurs, discard it.

b) If an end carry does not occur, take the r's complement of the number obtained in step 2 and place a negative sign in front to get the answer.

Cont..

Example-1: Using 10’s complement, calculate

72532 - 3250

M = 72532 (5 digits)

N = 03250 (5 digits)

M - N = M + (-N)

10’s complement of N = 96750 = N*

M + N* = 72532 + 96750 = 169282 (6 digits)

Discard the end carry 1.

Therefore, the answer is 69282

Verification: Using decimal subtraction:

72532 - 3250 = 69282

Cont..

Example-2: Using 10's complement, calculate

3250 - 72532

M = 03250 (5 digits)

N = 72532 (5 digits)

M - N = M + (-N)

10's complement of N = 27468 = N*

M + N* = 03250 + 27468 = 30718 (5 digits)

Since no end carry, take the 10's complement of 30718 and place a negative sign to obtain the final answer.

Therefore, the answer is - 69282

Verification: Using decimal subtraction:

3250 - 72532 = - 69282

Cont..

Example-3: Using 2's complement, calculate 7 - 5

Steps: M - N = M + (-N) = 7 + (-5)

Straight binary representation of M = 7 with the sign bit is

0111 (minimum 4 bits can be used)

Straight binary representation of -N = -5 with the sign bit is

1101 (minimum 4 bits can be used)

2's complement representation of -N is

(1's complement of 1101) + 1 = 1010 + 1 = 1011 = N*

M + N* = 0111 + 1011 = 10010 (5 bits)

Discard the end carry 1, then we get 0010

The sign bit is 0, the answer is a positive number 010

Verification: Using decimal subtraction: 7 - 5 = 2

Cont..

Example-4: Using 2's complement, calculate 5 - 7

Steps: M - N = M + (-N) = 5 + (-7)
Straight binary representation of M = 5 with the sign bit is
0101 (minimum 4 bits can be used)
Straight binary representation of -N = -7 with the sign bit is
1111 (minimum 4 bits can be used)
2's complement representation of -N is
(1's complement of 1111) + 1 = 1000 + 1 = 1001 = N*
M + N* = 0101 + 1001 = 1110
Since no end carry, take the 2's complement of 1110
2's complement of 1110 is 1010. Since the sign bit is 1, the answer is a negative number 010
Verification: Using decimal subtraction: 5 - 7 = - 2

Subtraction Using (r-1)’s Complement

If M and N are two positive numbers with same base r, the subtraction (M - N) can be done by using the following steps:

(M - N) = M + (-N)

Step 1: Represent -N in (r-1)’s complement (let N*)

Step 2: Add M and N*

Step 3: Inspect

a) If an end carry occurs, add the carry to the least significant digit (end around carry).

b) If an end carry does not occur, take the (r-1)’s complement of the number obtained in step 2 and place a negative sign in front to get the answer.

Cont..

Example-1: Using 9's complement, calculate

72532 - 3250

M = 72532 (5 digits)

N = 03250 (5 digits)

M - N = M + (-N)

9's complement of N = 96749 = N*

M + N* = 72532 + 96749 = 169281 (6 digits)

Add the end carry 1 to the least significant digit.

Therefore, the answer is 69282

Verification: Using decimal subtraction:

72532 - 3250 = 69282

Cont..

Example-2: Using 9's complement, calculate

3250 - 72532

M = 03250 (5 digits)

N = 72532 (5 digits)

M - N = M + (-N)

9's complement of N = 27467 = N*

M + N* = 03250 + 27467 = 30717 (5 digits)

Since no end carry, take the 9's complement of 30717 and place a negative sign to obtain the final answer.

Therefore, the answer is - 69282

Verification: Using decimal subtraction:

3250 - 72532 = - 69282

Cont..

Example-3: Using 1's complement, calculate 7 - 5

Steps: M - N = M + (-N) = 7 + (-5)

Straight binary representation of M = 7 with the sign bit is 0111 (minimum 4 bits can be used)

Straight binary representation of -N = -5 with the sign bit is 1101 (minimum 4 bits can be used)

1's complement representation of -N is 1010 = N*

M + N* = 0111 + 1010 = 10001 (5 bits)

Add the end carry 1 to the least significant bit, then we get 0010

The sign bit is 0, the answer is a positive number 010

Verification: Using decimal subtraction: 7 - 5 = 2

Cont..

Example-4: Using 1’s complement, calculate 5 - 7

Steps: M - N = M + (-N) = 5 + (-7)
Straight binary representation of M = 5 with the sign bit is 0101 (minimum 4 bits can be used)
Straight binary representation of -N = -7 with the sign bit is 1111 (minimum 4 bits can be used)
1’s complement representation of -N is 1000 = N*
M + N* = 0101 + 1000 = 1101
Since no end carry, take the 1’s complement of 1101
1’s complement of 1101 is 1010. Since the sign bit is 1, the answer is a negative number 010

Verification: Using decimal subtraction: 5 - 7 = - 2

References

- M. M. Mano, *Digital Logic and Computer Design*, 5th ed., Pearson Prentice Hall, 2013.
- R. P. Jain, *Modern Digital Electronics*, 4th ed., Tata McGraw-Hill Education, 2009.