

# Department of Mathematics

15B11MA211

Mathematics-II

Tutorial Sheet 4

B.Tech. Core

## Fourier Series [C106.4]

1. Expand the function  $f(x) = x \sin x$  in a Fourier series in the interval  $-\pi \leq x < \pi$ . Use the series obtained to show that  $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} = \dots = \frac{\pi - 2}{4}$ .

2. Given  $f(x) = \begin{cases} -x+1 & \text{for } -\pi < x \leq 0 \\ x+1, & \text{for } 0 \leq x \leq \pi \end{cases}$ , find a Fourier series for  $f(x)$  and hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

3. Find the Fourier series expansion of the function  $f(x) = \begin{cases} \pi x & \text{for } 0 \leq x < 1 \\ 0 & x = 1 \\ \pi(x-2) & \text{for } 1 < x < 2. \end{cases}$  in the interval  $[0, 2]$ .

4. Find the Fourier series expansion of the function  $f(x) = e^{-4x}$  in the interval  $[-2, 2]$ .

5. Find the Fourier series expansion of the function  $f(x) = x - x^2$  in the interval  $-1 < x \leq 1$ .

6. Find the half range sine series for the function  $f(x) = x^2$  for  $0 < x < \pi$ .

7. Find the half range cosine series for the function  $f(x) = 2x - 1$  for  $0 < x < 1$ .

8. Find the Fourier series expansion of the function  $f(x) = \begin{cases} 0 & \text{for } 0 \leq x < l \\ x & \text{for } l \leq x < 2l \end{cases}$  in the interval  $[0, 2l]$ .

Answers. (3)  $f(x) = 2(\sin \pi x - \frac{\sin 2\pi x}{2} + \frac{\sin 3\pi x}{3} - \dots)$ ,

(4)  $a_0 = (e^8 - e^{-8})/8$ ,  $a_n = (e^8 - e^{-8}) \cdot 8 \cdot (-1)^n / (64 + \pi^2 n^2)$ ,  $b_n = (e^8 - e^{-8}) \cdot n \cdot \pi \cdot (-1)^n / (64 + \pi^2 n^2)$

(5)  $f(x) = -\frac{1}{3} + \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{\cos(2n+1)\pi x}{(2n+1)^2} + \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi x}{(2n+1)}$ ,

(6)  $f(x) = \frac{2}{\pi} \left\{ (\pi^2 - 4) \sin x - \frac{\pi^2 \sin 2x}{2} + \frac{1}{3} (\pi^2 - \frac{4}{3^2}) \sin 3x - \dots \right\}$

(7)  $f(x) = -\frac{8}{\pi^2} \left( \cos \pi x + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \dots \right)$

(8)  $f(x) = \frac{3l}{4} + \frac{l}{\pi^2} \sum_{n=0}^{\infty} \frac{\cos(2n+1)\pi x / l}{(2n+1)^2} - \frac{l}{\pi} \left\{ \frac{3 \sin \pi x / l}{1} + \frac{\sin 2\pi x / l}{2} + \frac{3 \sin 3\pi x / l}{3} + \frac{\sin 4\pi x / l}{4} + \dots \right\}$