Department of Mathematics

Probability and Random Processes **15B11MA301**

Tutorial Sheet 12

(Semi random and random telegraph signal processes and Ergodic Process)

1. Let $X(t)$ is semi random telegraph signal process and $Y(t) = \beta X(t)$, where β is uniformly distributed random variable in the interval (-2, 2) and is independent of $X(t)$. Is $Y(t)$ a WSS process? [Ans. Yes]
2. Find the mean and variance of a random process {X(t)} whose autocorrelation function is given by $R(\tau) = 45 + \frac{4\tau^2 + 9}{\tau^2 + 2}$. [Ans. Mean = 7, Var = 0.5]
3. For the random process $X(t) = A\cos wt + B\sin wt$, where A and B are random variables with $E(A) = E(B) = 0$, $E(A^2) = E(B^2) > 0$, and $E(AB) = 0$. Prove that the process is mean ergodic.
4. Let {X(t)} be a WSS process with $E\{X(t)\} = 2$ and $R_{xx}(\tau) = 4 + e^{-\left(\frac{|\tau|}{10}\right)}$, find the mean and variance of $S = \int_0^1 X(t)dt$. [Ans. Mean = 2; var: $200e^{-\frac{1}{10}} - 180$]
5. The WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$. Check whether $\{X(t)\}$ is (i) mean ergodic random process, (ii) correlation ergodic random process. [Ans. Yes; Yes]
6. A random binary transmission process $\{X(t)\}$ is a WSS process with zero mean and autocorrelation function $R_{xx}(\tau) = 1 - \left(\frac{|\tau|}{T}\right)$, where T is a constant. Find the variance of the time average of $\{X(t)\}$ and also the mean over $(0, T)$. Is $\{X(t)\}$ mean ergodic? [Ans. 2/3; 0 ; No]

Soln $\bar{X}_T = \frac{1}{2T}\int_{-T}^{T} x(t)dt$

$R(\tau) = 1 - \frac{|\tau|}{T}$ & $E(x(t)) = E(\bar{x}_T) = 0$

$C(\tau) = R(\tau) - (E(x(t)))^2 = 1 - \frac{|\tau|}{T}$

$Var(\bar{x}_T) = \frac{1}{T}\int_{-T}^{T}\left(1 - \frac{|\tau|}{T}\right)^2 d\tau = \frac{2}{T}\int_0^T \left(1 - \frac{|\tau|}{T}\right)^2 d\tau = \left(1 + \frac{\tau^2}{T^2} - \frac{2\tau}{T}\right)$

$= \frac{2}{T}\left[\tau + \frac{\tau^3}{3T^2} - \frac{2\tau^2}{2T}\right]_0^T = \frac{2}{T}\left[T + \frac{T^3}{3T^2} - \frac{T^2}{T}\right]$

$= \frac{2}{T}\left[\frac{T}{3}\right] = \frac{2}{3} \neq 0$ $= \frac{2}{T}\left[T + \frac{T^2}{3} - T\right] = \frac{2}{3T}[3T + T^2 - 3T] = \frac{2}{3T}[T^2] = \frac{2T}{3}$

$\Rightarrow \{x(t)\}$ not mean ergodic