Electrical Science-2 (15B11EC211)

Unit-3
Operational Amplifier and Filters
Lecture-5

Contents

Limitation of Passive Filters

There are three major limitations to the passive filters:

1. **They cannot generate gain greater than 1; passive elements cannot add energy to the network.**

2. **They may require bulky and expensive inductors.**

3. **They perform poorly at frequencies below the audio frequency range (300 Hz $< f <$ *3,000 Hz*).**

Nevertheless, passive filters are useful at high frequencies.

Ref: 1, 2

Advantage of Active Filters

Active filters consist of combinations of resistors, capacitors, and op amps. They offer some advantages over passive RLC filters.

They are often smaller and less expensive, because they do not require inductors. This makes feasible the integrated circuit realizations of filters.

They can provide amplifier gain in addition to providing the same frequency response as RLC filters.

Active filters can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects. This isolation allows designing the stages independently and then cascading them to realize the desired transfer function

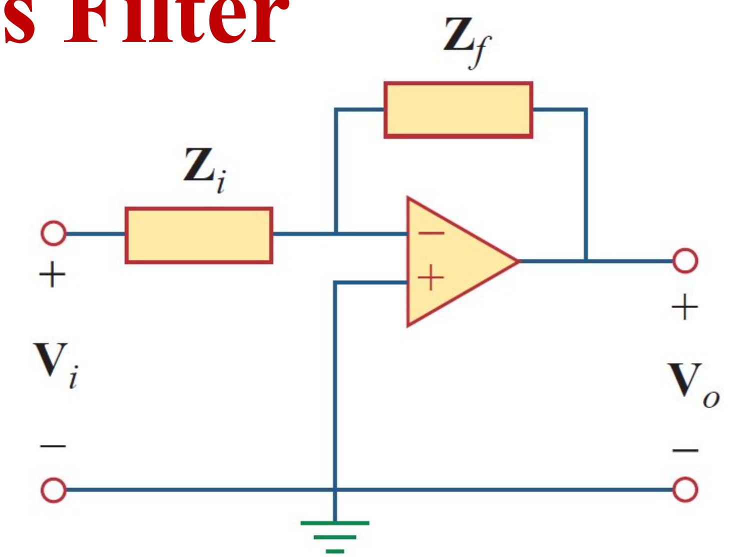
However, active filters are less reliable and less stable.

The practical limit of most active filters is about 100 kHz—most active filters operate well below that frequency

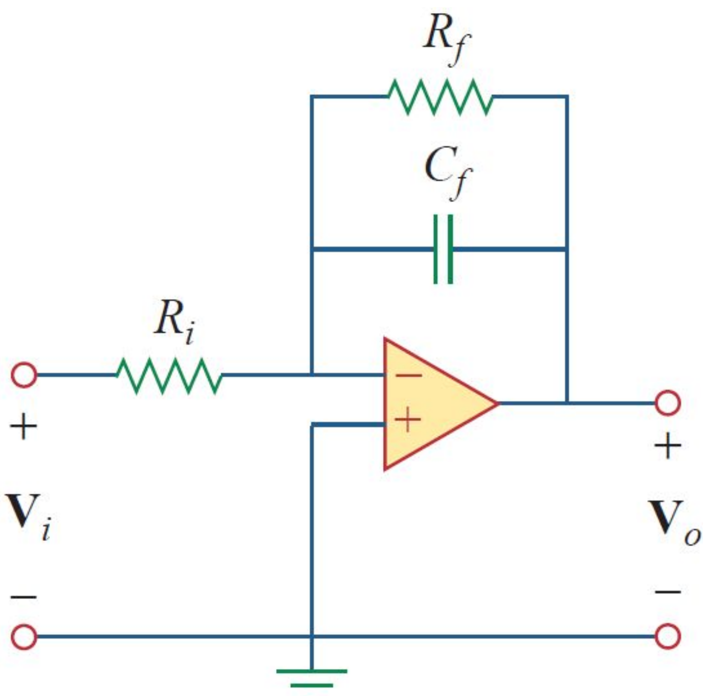
Ref:1, 2

Active Lowpass Filter

Figure shows a general first-order active filter



The components selected for Z_i and Z_f determine whether the filter is lowpass or highpass, but one of the components must be reactive.



active lowpass filter.

For this filter the transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

Where Zi = Ri , and

$$\mathbf{Z}_f = R_f \parallel \frac{1}{j\omega C_f} = \frac{R_f/j\omega C_f}{R_f + 1/j\omega C_f} = \frac{R_f}{1 + j\omega C_f R_f}$$

Ref: 1, 2

Active Lowpass Filter

Therefore, $$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f}$$ **Eq. (1)**

Low frequency gain/ dc gain ($\omega \rightarrow 0$) = $-R_f / R_i$

Corner frequency (ω_c) $$\omega_c = \frac{1}{R_f C_f}$$

Ref: 1, 2

Active Highpass Filter

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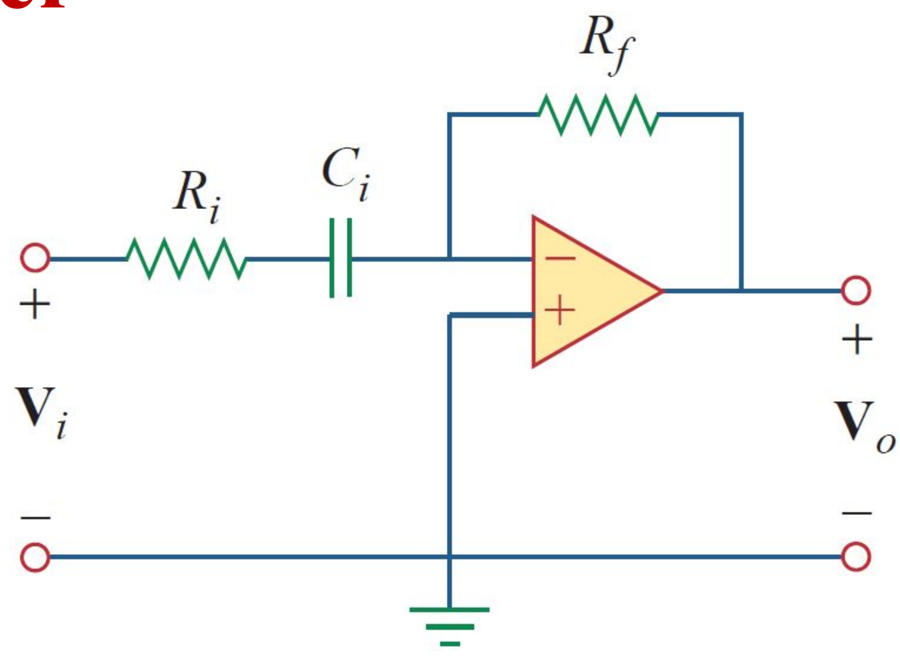


where $\mathbf{Z}_i = R_i + 1/j\omega C_i$ and $\mathbf{Z}_f = R_f$ so that

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i}$$ **Eq. (2)**

Ref:1, 2

Active Highpass Filter

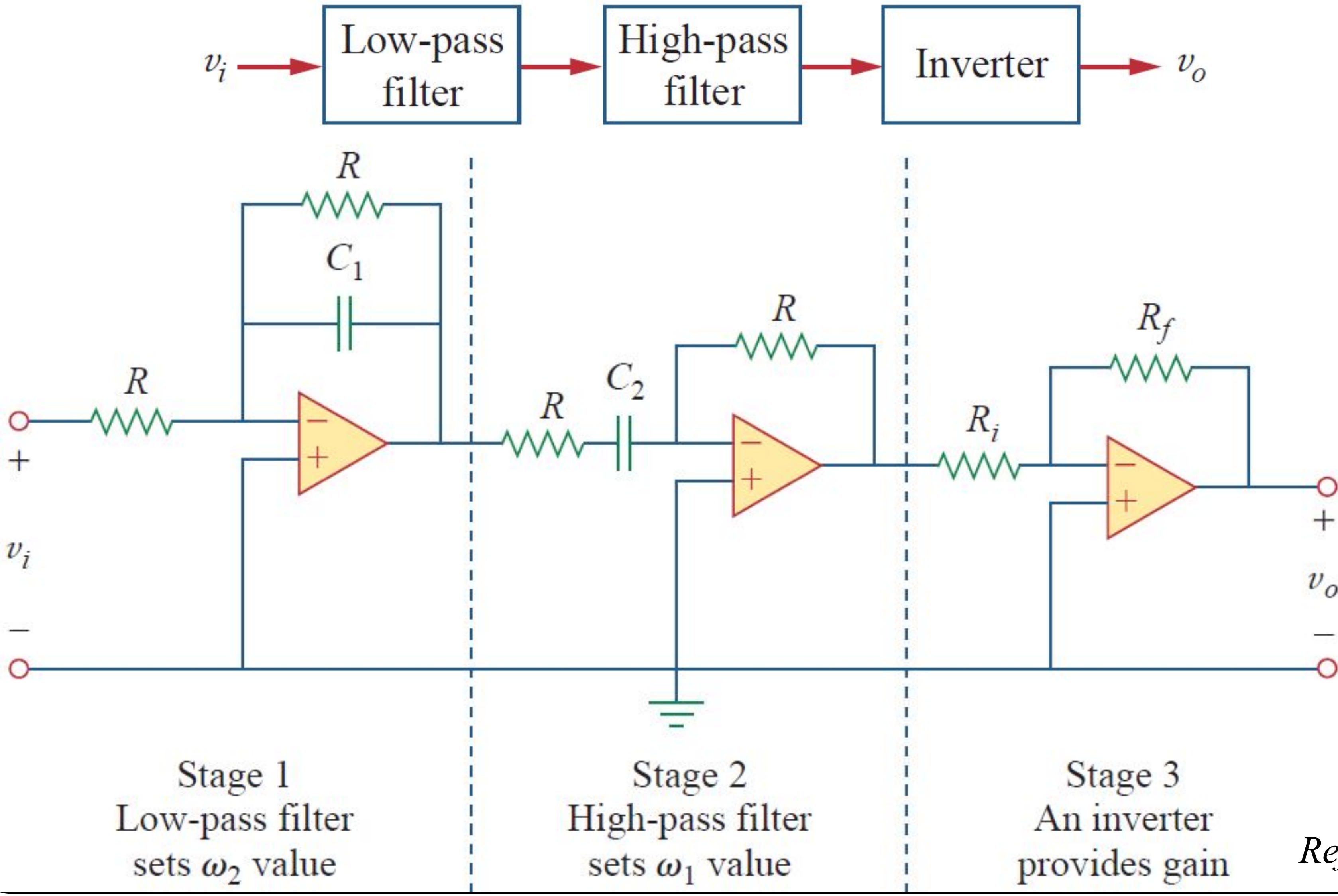
High frequency ($\omega \rightarrow \infty$) gain = $-R_f / R_i$

Corner frequency (ω_c) $$\omega_c = \frac{1}{R_i C_i}$$

Ref:1, 2

Active Bandpass Filter

Bandpass filter can be construct by cascading a unity-gain lowpass filter, a unity-gain highpass filter, and an inverter with gain $-R_f/R_i$



Ref: 1, 2

Active Bandpass Filter

Transfer function is obtained by multiplying Eqs. (1) and (2) with the gain of the inverter

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right)$$

$$= -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R} \qquad \textbf{Eq. (3)}$$

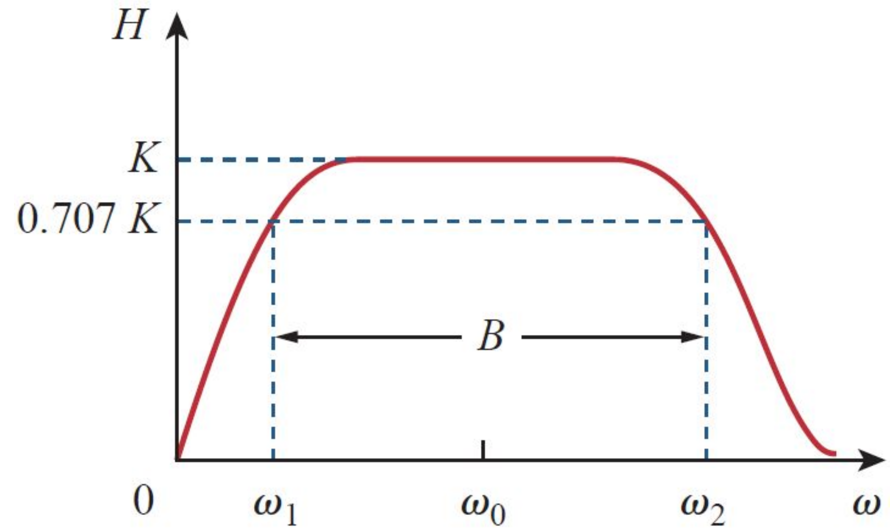
Lowpass section sets the upper corner frequency as $\omega_2 = \dfrac{1}{RC_1}$ **Eq. (4)**

Highpass section sets the lower corner frequency as $\omega_1 = \dfrac{1}{RC_2}$ **Eq. (5)**

Ref:1, 2

Active Bandpass Filter

BPF have a gain K over the required range of frequencies



FREQUENCY RESPONSE

With the help of Eqs. (4) and (5), the center frequency (ω_0), bandwidth (B), and quality factor (Q) can be calculated as follows:

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$ **Eq. (6)**

$$Q = \frac{\omega_0}{B}$$ **Eq. (8)**

$$B = \omega_2 - \omega_1$$ **Eq. (7)**

Ref:1, 2

Active Bandpass Filter

To find the passband gain K, we write Eq. (3) in the standard form as

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{j\omega/\omega_1}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)}$$

$$= -\frac{R_f}{R_i}\frac{j\omega\omega_2}{(\omega_1+j\omega)(\omega_2+j\omega)}$$

At the center frequency ω_0, the magnitude of the transfer function is

$$|\mathbf{H}(\omega_0)| = \left|\frac{R_f}{R_i}\frac{j\omega_0\omega_2}{(\omega_1+j\omega_0)(\omega_2+j\omega_0)}\right| = \left|\frac{R_f}{R_i}\frac{\omega_2}{\omega_1+\omega_2}\right.$$

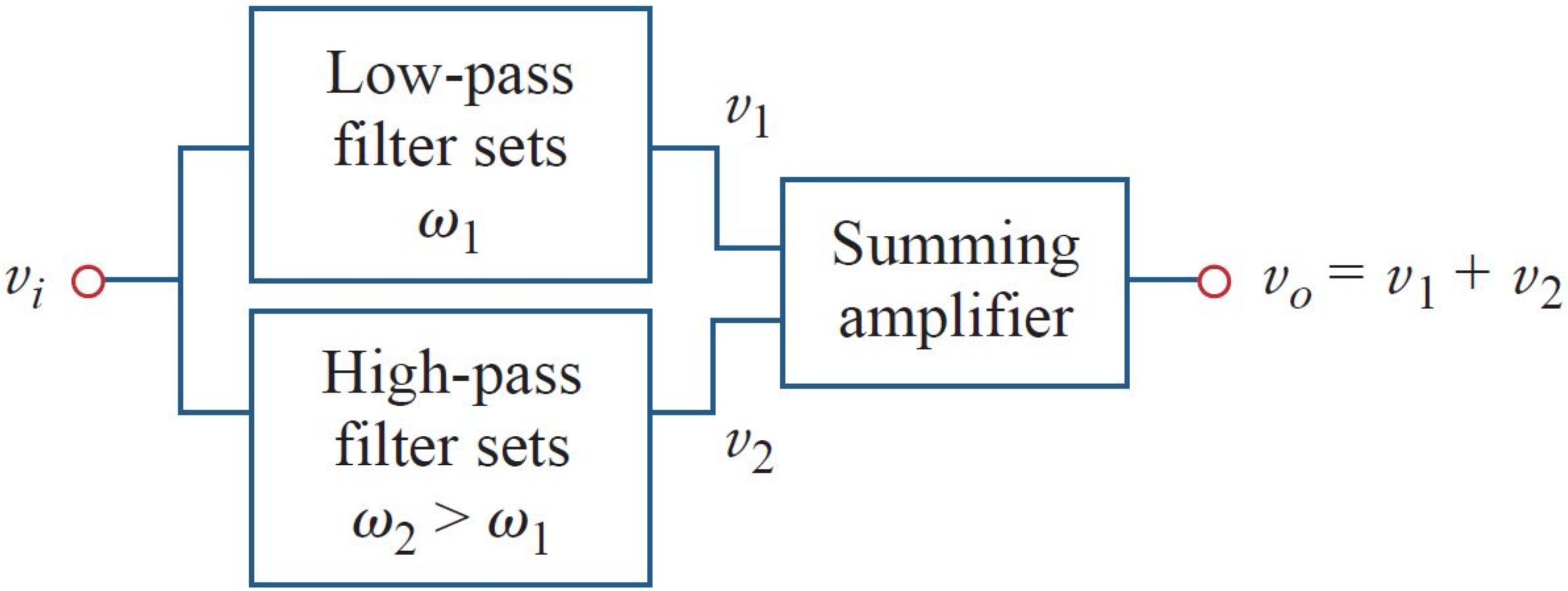
Thus, the passband gain k is

$$K = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1+\omega_2}$$

Ref: 1, 2

Active Bandstop (Notch) Filter

A bandstop filter □ constructed by parallel combination of a lowpass filter and a highpass filter and a summing amplifier



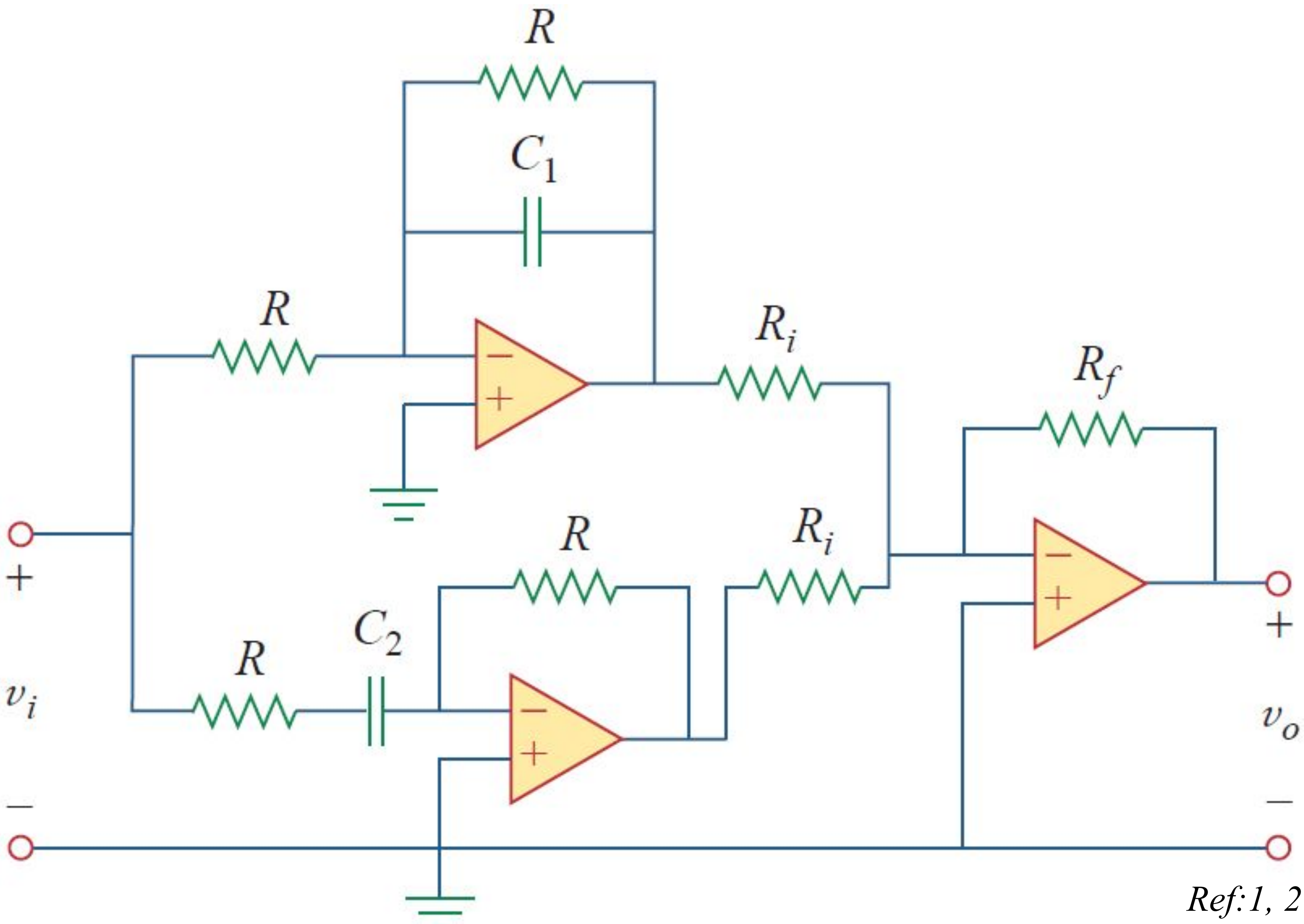
Bandstop filter circuit is designed such that □

Lower cutoff frequency ω_1 □ set by the lowpass filter

Upper cutoff frequency ω_2 □ set by the highpass filter.

Ref: 1, 2

Active Bandstop (Notch) Filter Circuit



Ref:1, 2

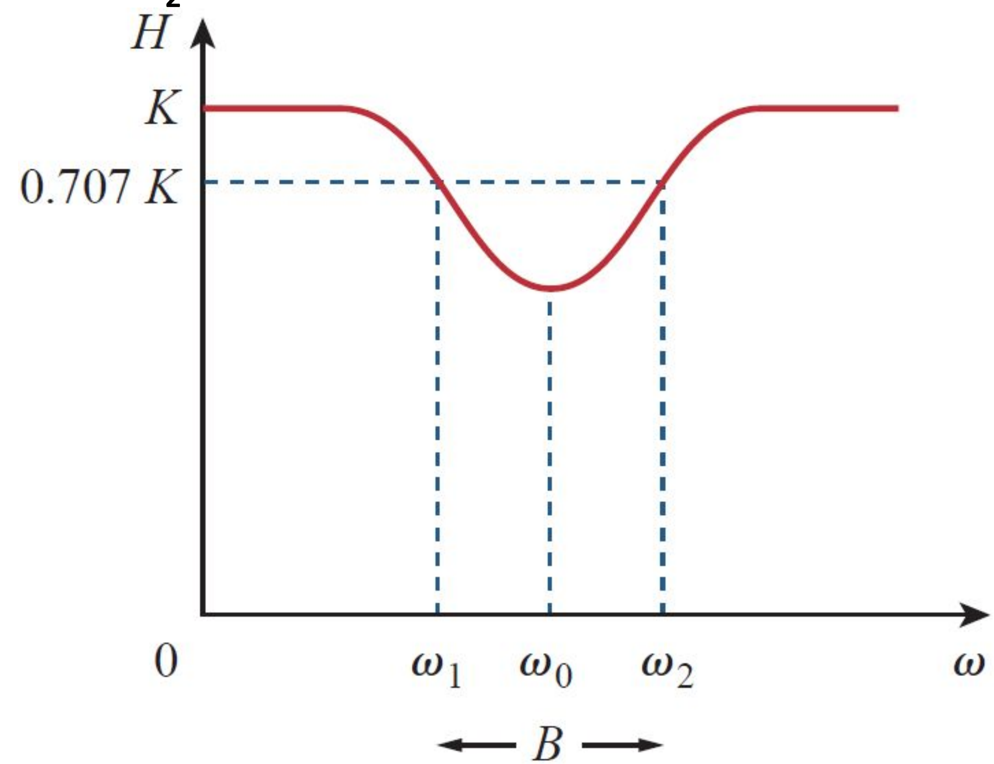
Active Bandstop (Notch) Filter

Transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{R_f}{R_i}\left(-\frac{1}{1 + j\omega C_1 R} - \frac{j\omega C_2 R}{1 + j\omega C_2 R}\right) \qquad \textbf{Eq. (9)}$$

□ **Filter passes frequencies below ω_1 and above ω_2 .**

The formulas for calculating □ the values of corner frequencies ω_1, ω_2, the center frequency, bandwidth, and quality factor are the same as in Eqs. (4) to (8).



FREQUENCY RESPONSE

Ref:1, 2

Active Bandstop (Notch) Filter

- To determine the passband gain K of the filter, we can write Eq. (9) in terms of the upper and lower corner frequencies as

$$\mathbf{H}(\omega) = \frac{R_f}{R_i}\left(\frac{1}{1 + j\omega/\omega_2} + \frac{j\omega/\omega_1}{1 + j\omega/\omega_1}\right)$$

$$= \frac{R_f}{R_i}\frac{(1 + j2\omega/\omega_1 + (j\omega)^2/\omega_1\omega_1)}{(1 + j\omega/\omega_2)(1 + j\omega/\omega_1)}$$

Eq. (10)

Comparing the Eq. (10) with the standard form,

- **Gain in the passbands**

$$K = \frac{R_f}{R_i}$$

- **Gain at the center frequency**

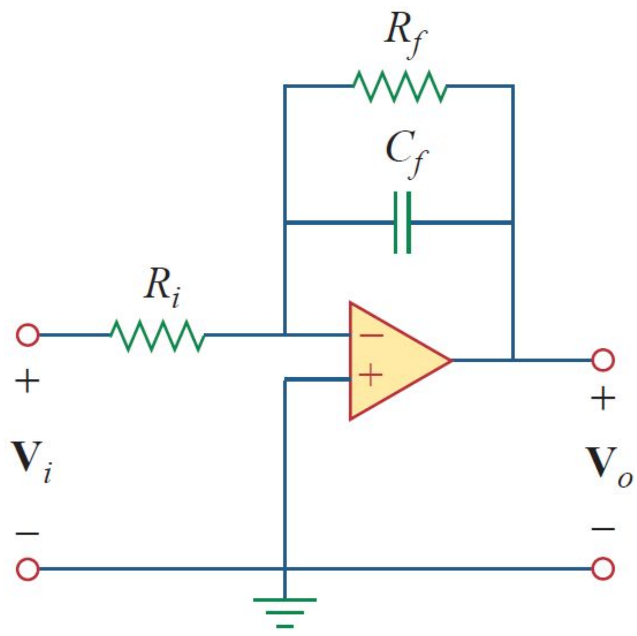
$$H(\omega_0) = \left|\frac{R_f}{R_i}\frac{(1 + j2\omega_0/\omega_1 + (j\omega_0)^2/\omega_1\omega_1)}{(1 + j\omega_0/\omega_2)(1 + j\omega_0/\omega_1)}\right|$$

$$= \frac{R_f}{R_i}\frac{2\omega_1}{\omega_1 + \omega_2}$$

Ref: 1, 2

Example

Design a lowpass active filter with a dc gain of 4 and a corner frequency of 500 Hz.



Solution:

$$\omega_c = 2\pi f_c = 2\pi(500) = \frac{1}{R_f C_f}$$

The dc gain is

$$H(0) = -\frac{R_f}{R_i} = -4$$

We have two equations and three unknowns.

If we select C_f = 0.2 μF

Then $$R_f = \frac{1}{2\pi(500)0.2 \times 10^{-6}} = 1.59\ \text{k}\Omega$$

$$R_i = \frac{R_f}{4} = 397.5\ \Omega$$

Ref: 1, 2

Practice Problem

Design a highpass filter with a high-frequency gain of 5 and a corner frequency of 2 kHz. Use a 0.1 μF capacitor in your design.

Answer

$$R_i = 800\ \Omega \quad \text{and} \quad R_f = 4\ \text{k}\Omega$$

Ref:1, 2

References

1. R.C. Dorf and James A. Svoboda, “Introduction to Electric Circuits”, 9th ed, John Wiley & Sons, 2013.

2. Charles K. Alexander and Matthew N. O. Sadiku, “Fundamentals of Electric Circuits”, Chapter 19, 4th ed, Mcgraw Hill, 2009.