Probability and Random Processes (15B11MA301)

Lecture-23

(Content Covered: MTTF, Conditional Reliability)

Department of Mathematics

Jaypee Institute of Information Technology, Noida

Contents

Mean time to failure (MTTF)

- The expected value of the time the failure T, denoted by $E(T)$ and variance of T, denoted by σ_T^2 are two important parameters frequently used to characterize reliability.
- $E(T)$ is called $mean\ time\ to\ failure\ and\ denoted\ by\ MTTF$.

$$MTTF = E(T) = \int_0^\infty tf(t)dt$$

$$= -\int_0^\infty tR'(t)dt$$

$$= -[tR(t)]_0^\infty + \int_0^\infty R(t)dt \qquad (1)$$

On integrating by parts

Now $\quad [tR(t)]_{t=\infty} \left[te^{-\int_0^t \lambda(t)dt} \right]_{t=\infty} = \left[\frac{t}{e^{\int_0^t \lambda(t)dt}} \right]_{t=\infty} = 0$

and $\quad [tR(t)]_{t=0} = 0 \times R(0) = 0 \times 1 = 0$

Using (1) and above results .we have MTTF and variance as:

$$\text{MTTF} = \int_0^\infty R(t)dt$$

$$\text{Var}(T)\ \sigma_T^2 = E\{T - E(T)\}^2 \text{ or } E(T^2) - \{E(t)\}^2$$

$$= \int_0^\infty t^2 f(t)dt - (MTTF)^2$$

Conditional Reliability

- Conditional reliability is used to describe the reliability of a component or system following a wear-in period (burn-in period) or after a warranty period.
- It is defined as

$$R(t/T_0) = P\{T > T_0 + t/T > T_0\}$$

$$= \frac{P\{T > T_0 + t\}}{P\{T > T_0\}} = \frac{R(T_0 + t)}{R(T_0)}$$

$$= \frac{e^{-\int_0^{T_0+t} \lambda(t)dt}}{e^{-\int_0^{T_0} \lambda(t)dt}} = e^{-\left[\int_0^{T_0+t} \lambda(t)dt - \int_0^{T_0} \lambda(t)dt\right]}$$

$$= e^{-\int_{T_0}^{T_0+t} \lambda(t)dt}$$

Design Life

- Design life is the time to failure that corresponds to a specified reliability.
- It is usually denoted by t_D.

Example 1: The density function of the time to failure in years of the gizmos (for use on widgets) manufactured by a certain company is given by

$$f(t) = \frac{200}{(t+10)^3}, \mathrm{t} \geq 0$$

(a) Derive the reliability function and determine the reliability for the first year of operation.

(b) Compute the MTTF.

(c) What is the design life for a reliability 0.95?

(d) Will a one-year burn-in period improve the reliability in part (a)? If so, what is the new reliability?

Solution:

(a)
$$f(t) = \frac{200}{(t+10)^3}, \ t \geq 0$$

$$R(t) = \int_t^{\infty} f(t)dt = \left[\frac{-100}{(t+10)^2}\right]_t^{\infty} = \frac{100}{(t+10)^2}$$

$$\therefore \quad R(1) = \frac{100}{(1+10)^2} = 0.8264.$$

(b)
$$\text{MTTF} = \int_0^{\infty} R(t)dt = \int_0^{\infty} \frac{100}{(t+10)^2} dt$$

$$= \left(\frac{-100}{t+10}\right)_0^{\infty} = 10 \text{ years.}$$

(c) Here, R=0.95

$$\therefore \quad \frac{100}{(t+10)^2} = 0.95$$

i.e., $(t_D + 10)^2 = 100.2632$

$\therefore \quad t_D = 0.2598$ year or 95 days

(d) $$R(t/1) = \frac{R(t+1)}{R(1)} = \frac{100}{(t+11)^2} \div \frac{100}{11^2} = \frac{121}{(t+11)^2}$$

Now $$R(t/1) > R(t), \text{ if } \frac{121}{(t+11)^2} > \frac{100}{(t+10)^2}$$

i.e., if $$\frac{(t+10)^2}{(t+11)^2} > \frac{100}{121}$$

i.e., if $$\frac{t+10}{t+11} > \frac{10}{11}$$

i.e., $11t > 10t$, which is true, as $t \geq 0$

$\therefore$ One year burn-in period will improve the relaibility.

Now $$R(1/1) = \frac{121}{(1+11)^2} = 0.8403 > 0.8264.$$

Example 2: The time to failure in operating hours of a critical solid-state power unit has the hazard rate function

$$\lambda l(t) = 0.003 \left(\frac{t}{500} \right)^{0.5}, \textit{for } t \geq 0.$$

(a) Determine the design life if a reliability of 0.90 is desired.

(b) Compute the MTTF.

(c) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours of operation?

Solution: (a)

$$R(t_D) = 0.90$$

$$\exp\left[-\int_0^{t_D} 0.003\left(\frac{t}{500}\right)^{0.5} dt\right] = 0.90$$

$$-\int_0^{t_D} \frac{0.003}{\sqrt{500}} t^{1/2} dt = -0.10536$$

i.e.,

$$\frac{0.003}{\sqrt{500}} \times \frac{2}{3} t_D^{3/2} = 0.10536$$

$\therefore$

$$t_D = \left\{\frac{3 \times \sqrt{500} \times 0.10536}{2 \times 0.003}\right\}^{2/3} = 111.54 \text{ hours.}$$

(b) $$\text{MTTF} = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{-\left(\frac{0.003}{\sqrt{500}}\times\frac{2}{3}\times t^{3/2}\right)}dt$$

$$= \int_0^\infty e^{-at^{3/2}}dt, \text{ where } a = \frac{0.003\times 2}{3\times\sqrt{500}}$$

$$= \int_0^\infty e^{-x}.\frac{2}{3a^{2/3}}\,x^{-1/3}\,dx, \text{ on putting } x = at^{3/2}$$

$$= \frac{2}{3a^{2/3}}\Gamma(2/3) = \frac{2}{3a^{2/3}}\frac{3}{2}\Gamma(5/3)$$

$$= \frac{0.9033}{a^{2/3}}, \text{ from the table of values of Gamma function.}$$

$$= 45.65 \text{ hours.}$$

(c) $$P(T \geq 100/T \geq 50) = \frac{P(T \geq 100)}{P(T \geq 50)} = \frac{R(100)}{R(50)}$$

$$= \exp\left[\geq \int_{50}^{100} \lambda(t)dt\right]$$

$$= \exp\left[\left\{-\frac{0.002}{\sqrt{500}} \times 100^{3/2}\right\} - \left\{\frac{-0.002}{\sqrt{500}} \times 50^{3/2}\right\}\right]$$

$$= \exp[\{-0.08944\} - \{-0.03162\}]$$

$$= 0.9438$$

Practice Question

Question: A logic circuit is known to have a decreasing failure rate of the form $\lambda(t) = \frac{1}{2\sqrt{t}}$ per year , where t is in years.

(a) If the design life is one year, what is the reliability?

(b) If the component undergoes wear in for one month before being put into operation, what will the reliability be for a one-year design life?

Ans: a) 0.905 b) 0.928

References

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3. Papoulis, A. and Pillai, S.U., Probability, Random Variables and Stochastic Processes, Tata McGraw-Hill, 2002.

4. Miller, S., Childers, D., Probability and Random Processes, Academic Press, 2012.