

1. Closure

Associativity

Identity

Inverse

i.e. y exists if and only if $xb = -1$. Since -1 doesn't have an inverse under $*$, $(R, *)$ is not a group.

2. (A) $a \odot b \in G \forall a, b \in G$

(b) $(a \odot b) \odot c = a \odot (b \odot c) \forall a, b \in G$

(c) $a \odot 1 = a \forall a \in G$

(d) The inverse elements of 1, 3, 7, 9 are 1, 7, 3, 9 respectively.

3. $ab = ba = (a^2 b^2)$ apply a^{-1} from left & b^{-1} from right. We obtain $ba = ab$ for all $a, b \in G$, hence G is abelian.

4. $(A, +, \cdot)$ is a ring

(A) Let 0 be the additive identity.

Then $a \cdot 0 = 0 \cdot a = 0$ for all a in A . $(a+a) \cdot a = a \cdot a + a \cdot a$

$a \cdot a = a + a = a \cdot (a+a)$ hence $a+a=0$.

(B) $(a+b) \cdot (a+b) = a+b$

$a \cdot a + a \cdot b + b \cdot a + b \cdot b = a+b$

$a + b \cdot a + a \cdot b + b = a+b$

Hence $a \cdot b + b \cdot a = 0$

Hence commutative.

5. (A) $xyz + xy\bar{z} + x\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}y\bar{z}$.

(B) $xyz + xy\bar{z} + x\bar{y}z$

(C) $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z}$

(D) $x\bar{y}z + x\bar{y}\bar{z}$

