Department of Mathematics

Probability and Random Processes   **15B11MA301**

   Tutorial Sheet 15   **B.Tech. Core**
Markov Chain

Q.1: The transition probability matrix (tpm) of a three state system is given as

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & .2 & .3 & .5 \\ 2 & .4 & .4 & .2 \\ 3 & .4 & .6 & 0 \end{array}$$

Find the probability of moving (a) from state 1 to state 3 in one step (b) from state 1 to state 2 in exactly two steps.

  Ans: (a) 0.5; (b) 0.48

Q.2: The transition probability matrix (tpm) of a three state 0, 1, 2 Markov chain is $P = \begin{bmatrix} .75 & .25 & 0 \\ .25 & .5 & .25 \\ 0 & .75 & .25 \end{bmatrix}$ and the initial state distribution of the chain is $P\{X_0 = i\} = \dfrac{1}{3}, i = 0,1,2.$

Find (i) $P\{X_2 = 2\}$ and (ii) $P\{X_0 = 2, X_1 = 1, X_2 = 2, X_3 = 1\}.$

Ans: (a) 0.1667; (b) 0.0469

Q.3. A businessman sells his goods in three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells either in B or C, then next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state?
Ans: [40%   40%   15%]

Q.4: Three persons P, Q and R are throwing a ball to each other. P always throws the ball to Q and Q always throws the ball to R , but R is twice as likely to throw the ball to Q as to P. Is the process Markovian? If yes, find the transition probability matrix and classify the states.
Ans. Yes, all states are ergodic.

Q.5: Consider a Markov Chain with two states and transition probability matrix $P = \begin{bmatrix} \dfrac{3}{4} & \dfrac{1}{4} \\ \dfrac{1}{2} & \dfrac{1}{2} \end{bmatrix}$,

find the stationary distribution of the chain. Is the chain periodic? If yes find its period.

Ans. aperiodic.

Q.6: Let the TPM of a Markov chain be $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$. Find the steady state

distribution of the chain. Is the chain irreducible, aperiodic and non-null persistent? Justify.

Ans. Yes.