Department of Mathematics

15B11MA211 **Mathematics-II**

Tutorial Sheet 11 **B.Tech. Core**

Complex Integration

1. Integrate $\int_C (z + 2\bar{z})dz$ from $z = 0$ to $z = 1 + i$ along the following two paths

(a) line joining (0,0) and (1,1) (b) the curve $x = t, y = t^2, 0 \le t \le 1.$

2. Integrate f(z) = z in the positive sense around the squares with corners at (1,1), (2,1), (2,2) and (1,2).

3. Evaluate $\int_C |z| dz$, where C is the contour (a) straight line from z = -i to z = i; (b) the unit circle $|z - 1| = 1$.

4. Let m be an integer and C the circle $|z - z_0| = R$. Show that the integral of $(z - z_0)^m$ over C in the anticlockwise direction vanishes if $m \ne -1$ and is equal to $2\pi i$ if m = -1. Hence evaluate $\int_C [P(z)/z] dz,$ where $P(z) = 2 - z + 3z^2 + z^3$ and C is the unit circle $|z| = 1$.

5. Using Cauchy theorem or otherwise show that

(a) $\int_C \frac{dz}{z-2} = 0,$ where C is the circle $|z| = 1$

(b) $\int_C \frac{dz}{z} = 2\pi i,$ where C is a closed contour enclosing z = 0.

(c) $\int_C \frac{dz}{(z+1)^2} = 0,$ where C is the circle $|z| = 2$.

6. Using the Cauchy integral formula or otherwise show that

(a) $\int_C \frac{e^{-z}}{z+1} dz = 2\pi e i,$ where C is the circle $|z| = 2$.

(b) $\int_C \frac{e^{2z}}{(z+1)^4} dz = 8\pi i/(3e^2),$ where C is the circle $|z| = 2$.

(c) $\int_C \frac{\cos \pi z^2}{(z-1)\,(z-2)} dz = 2\pi i, 0, 4\pi i$ according as C is the circle $|z| = 3/2, 1/2$ or 3.

(d) $\int_C \frac{e^{-z}}{z^2} dz = -2\pi i,$ where C is the ellipse $2x^2 + y^2 = 2.$

Answers:

1.(a) $2 + i$ **(b)** $2 + \frac{5}{3}i$ **2.** 0 **3. (a)** i **(b)** $-\frac{8}{3} + \frac{4}{3}i$ **4.** $-4\pi i$