Probability and Random Processes **15B11MA301**
Tutorial Sheet 11 **C201.5**

Random Process

1. Define a random process and classify them with suitable examples.

2. In an experiment of two fair dice, the process $\{X(t)\}$ is defined as $X(t) = \sin \pi t$, if the experiment shows a prime sum and $X(t) = 2t + 1$, otherwise. Find the mean of the process. Is the process stationary? [Ans: not stationary]

3. Let $X(t) = A\cos\lambda t + B\sin\lambda t$, with random variable A taking values 1 and 3 with equal probabilities and random variable B taking values -1 and 1 with probabilities ¼ and ¾ respectively. Test the process $\{X(t)\}$ for stationarity. [Ans: not stationary]

4. Test the random processes $\{X(t)\}$ and $\{Y(t)\}$ for WSS when:

   (i) $X(t) = \cos(\lambda t + Y)$, where λ is a constant and Y is uniform in $(0, 2\pi)$
   [Ans: WSS]

   (ii) $Y(t) = X\sin(\lambda t)$, where λ is a constant and X is uniform in $(-1, 1)$.
   [Ans: not WSS]

5. Find auto correlation functions of the processes $\{X(t)\}$ and $\{Y(t)\}$ such that $X(t) = A\cos\lambda t + B\sin\lambda t$ and $Y(t) = B\cos\lambda t - A\sin\lambda t$, where A and B are uncorrelated random variables taking value -4 and 4 with equal probabilities. Prove that $\{X(t)\}$ and $\{Y(t)\}$ are jointly WSS. [Ans: $16\cos\lambda(t_1 - t_2)$]

6. If $X(t) = A\sin\omega(\omega t + \theta)$ where A and ω are constants and θ is a random variable, uniformly distributed over $(-\pi, \pi)$, find the autocorrelation of $\{Y(t)\}$ where $Y(t) = X^2(t)$
   [Ans: $R(t_1, t_2) = \frac{A^4}{8}\{2 + \cos 2\omega(t_1 - t_2)\}$]

7. If $\{X(t)\}$ is a WSS process with $E\{X(t)\} = 2$ and $R_{XX}(\tau) = 4 + e^{-|\tau|/10}$, find the variance of X(1), X(2) and X(3). Also compute the second order moment about origin of X(1) + X(2) + X(3). [Ans: $Var(X(1)) = Var(X(2)) = Var(X(3)) = 5 - 4 = 1$ and $39 + 4e^{-1/10} + 2e^{-1/5}$]

8. Define a Random walk and prove that the limiting form of a random walk is Wiener process.

Weiner process can never take negative value [illegible] value of time..