Solution
Signals & Systems

Q1 a) $2+3j$

Using $z = x+iy = re^{j\theta}$

$|2+3j| = \sqrt{4+9} = \sqrt{13}$ $\arg(2+3j) = \tan^{-1}\left(\frac{3}{2}\right)$

$2+3j = \sqrt{13}\, e^{j\tan^{-1}(3/2)}$

b) $(1+j)e^{j\pi/3}$

$= \sqrt{1^2+1^2}\, e^{j\tan^{-1}(1)} e^{j\pi/3} = \sqrt{2}\, e^{j\pi/4} e^{j\pi/3} = \sqrt{2}\, e^{j7\frac{\pi}{12}}$.

c) $(\sqrt{5}+j)^2 e^{-j\pi/3}$

$= (\sqrt{5+1})^2 \left(e^{j\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)^2 e^{-j\pi/3}$

$= (\sqrt{6})^2\, e^{j2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)} \cdot e^{-j\pi/3}$

$= 6\, e^{j(\cancel{\quad}24.095 - \pi/3)} = 6e^{-j\cancel{\quad}35.91} = 6e^{-j35.91}$

d) $\frac{2-j}{1+j3} = \frac{2-j}{1+j3} \times \frac{1-j3}{1-j3} = \frac{2-j-j6-3}{1+9} = \frac{-1-j7}{10}$

$\left|\frac{-1-j7}{10}\right| = \frac{1}{\sqrt{2}}$ $\arg\left(\frac{-1-7j}{10}\right) = \pi + \tan^{-1}7$

$\frac{2-j}{1+j3} = \frac{1}{\sqrt{2}}\, e^{j(261)}$

Ques-2

(a) $x(t) = e^{j(2\pi t - \pi)}$

To be periodic, $x(t) = x(t+T) \; \forall \; t$

$x(t+T) = e^{j2\pi(t+T)} \cdot e^{-j\pi}$

$= e^{j(2\pi t - j\pi)} \cdot e^{j2\pi T}$

For periodic, $e^{j2\pi T} = 1$

$\Rightarrow e^{j2\pi T} = e^{j2\pi m}$, $m \in I$

$\Rightarrow 2\pi T = 2\pi m$

$\Rightarrow T = m$ & m is +ve integer

fundamental period = 1 s

fundamental frequency = 1 Hz.

$\because$ T satisfies all the conditions of period,

$x(t)$ is periodic.

OR

Complex exponentials are always periodic

with $T = \frac{2\pi}{|\omega_0|}$

$T = \frac{2\pi}{2\pi} = 1s$. $f = 1 Hz$.

(b) $x(t) = 3[\cos(2t)]^2$

$= 3\left[\frac{1 + \cos 4t}{2}\right] = \frac{3}{2} + \frac{3}{2}\cos 4t$

Since, sinusoids are periodic & $\omega_0 = 4 \Rightarrow T = \frac{2\pi}{4} = \frac{\pi}{2}$

Adding dc value will not effect periodicity.

$f_0 = 1/\pi/2 = 2/\pi$ Hz

(c) $x(t) = \cos 4t \;\; \sin(8t)$ ③

$$= \left(\frac{e^{j4t} + e^{-j4t}}{2}\right)\left(\frac{e^{j8t} - e^{-j8t}}{2j}\right)$$ ⑦

$$= \frac{1}{4j}\left(e^{j4t}e^{j8t} + e^{-j4t}e^{j8t} - e^{j4t}e^{-j8t} - e^{-j4t}e^{-j8t}\right)$$

$$= \frac{1}{4j}\left(e^{j12t} + e^{j4t} - e^{-j4t} - e^{-j12t}\right)$$

$$= \frac{1}{4j}\left(\sin 4t \cdot 2j + \sin 12t \cdot 2j\right)$$

$\omega t = 2\pi f t$, $f = \frac{4}{2\pi}$ or $T = \frac{2\pi}{12}$

$$= \frac{1}{2}\left(\sin 4t + \sin 12t\right)$$

$$T_1 = \frac{2\pi}{4} = \frac{\pi}{2}\,s \qquad T_2 = \frac{2\pi}{12} = \frac{\pi}{6}\,s$$

LCM of $T_1, T_2 = \frac{\pi}{2}\,s$ & $\frac{T_1}{T_2} = \frac{\pi/2}{\pi/6} = 3$ (integer)

Over all fundamental period, $T_0 = \frac{\pi}{2}\,s$.

OR

Using $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$$\cos 4t \sin 8t = \frac{1}{2}\left(\sin(12t) + \sin(+4t)\right)$$

$$T_1 = \frac{\pi}{6}\,s \qquad T_2 = \pi/2\,s$$

fundamental period $= LCM(T_1, T_2) = \pi/2\,s$

fundamental freq $= HCF\left(\frac{1}{T_1}, \frac{1}{T_2}\right) = \frac{2}{\pi}\,Hz$ or $\frac{1}{\pi/2}\,Hz$

(d) $x(t) = 4u(t) + 2\sin 3t$

$$= \begin{cases} 2\sin 3t, & t<0 \\ 4 + 2\sin 3t, & t>0 \end{cases}$$

It is independently periodic for −ve & +ve time.
In general, it is an aperiodic signal.

(4)

(e) $x(t) = 2\cos(2\pi t/3) - 2\sin(\pi t/8) + 2\cos(2\pi t + \pi/6)$

$T_1 = \frac{2\pi}{2\pi/3}$ $\quad T_2 = \frac{2\pi}{\pi/8}$ $\quad T_3 = \frac{2\pi}{2\pi/6} = 1$

$T_3 = 7$

LCM of T_1, T_2, T_3 = LCM$(3, 16, \cancel{12}\ 7)$

$= 48$

$\frac{T_1}{T_2} = \frac{3}{16}$, $\frac{T_2}{T_3} = \frac{16}{12}$, $\frac{T_3}{T_1} = \frac{12}{3}$

All ratios are ~~rational~~ rational numbers, hence sum of sinusoids is ~~always~~ also periodic with period = ~~48~~ 336

fundamental frequency = $\cancel{\frac{1}{48}}$ ~~4/3~~ $\frac{1}{336}$ Hz

Ques-3

(a) $x(t) = e^{-j2t}u(t)$

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_{-\infty}^{\infty} |e^{-j2t}|^2 dt$

$= \infty$

$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} |x(t)|^2 dt$

$= \frac{1}{\pi}\int_0^{\pi/2} dt = \frac{1}{\pi}\cdot\frac{\pi}{2} = \frac{1}{2}W$

$\because 0 < P < \infty$, the signal is power signal.

5

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

$$E = \int_0^1 t^2 dt + \int_1^2 (2-t)^2 dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 + \left[\frac{t^3}{3} + 4t - \frac{4t^2}{2}\right]_1^2$$

$$= \frac{1}{3} + \frac{37}{3}$$

$$= \frac{1}{3} + \frac{(37)}{3} = \frac{38}{3} \text{ J}$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$\because T = \infty$$

$$P = 0.$$

(c) $x(t) = \cos(4t + \pi/3)$

$$E = \int_{-\infty}^{\infty} |\cos(4t + \pi/3)|^2 dt$$

$$= \int_{-\infty}^{\infty} \frac{1}{2} + \frac{\cos(8t + 2\pi/3)}{2} dt$$

$$= \infty.$$

$$P = \frac{1}{\pi/2}\int_{-\pi/4}^{\pi/4} |\cos(4t + \pi/3)|^2 dt$$

$$= \frac{1}{\pi/2}\left(\int_{-\pi/4}^{\pi/4} \frac{1}{2} dt + \int_{-\pi/4}^{\pi/4} \frac{\cos(8t+2\pi/3)}{2} dt\right)$$

$$= \frac{1}{\pi/2}\left[\frac{2\pi}{4}\right]\cdot\frac{1}{2} + 0$$

$$= \frac{1}{2} \text{ W}$$

(d) $x(t) = (\cos(4t + \pi/3))^2$

$$= \frac{1 + \cos(8t + 2\pi/3)}{2}$$

$$E = \int_{-\infty}^{\infty} \left|\frac{1 + \cos(8t + 2\pi/3)}{2}\right|^2 dt$$

$$= \left(\int_{-\infty}^{\infty} \left|\frac{1}{2} + \frac{\cos(8t+2\pi/3)}{2}\right| \left|\frac{1}{2} + \frac{\cos(8t+2\pi/3)}{2}\right|\right) dt$$

$$= \left(\int_{-\infty}^{\infty} \frac{1}{4} + \frac{\cos^2(8t+2\pi/3)}{2^2} + \frac{\cos(8t+2\pi/3)}{2}\right) dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{8}\cos(16t + 4\pi/3) + \frac{\cos(8t+2\pi/3)}{2}\right) dt = \infty$$

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(d) $x(t) = (\cos(4t + \pi/3))^2$

$$= \frac{1}{2} + \frac{\cos(8t + 2\pi/3)}{2}$$

$$= \frac{1}{2} + \frac{e^{j(8t+2\pi/3)} + e^{-j(8t+2\pi/3)}}{2\cdot 2}$$

$$= \frac{1}{2} + \frac{1}{2\cdot 2}e^{j(8t+2\pi/3)} + \frac{1}{2^2}e^{-j(8t+2\pi/3)}$$

$$E = \int_{-\infty}^{\infty} \left| \frac{1}{2} + \frac{1}{2^2}e^{j(8t+2\pi/3)} + \frac{1}{2^2}e^{-j(8t+2\pi/3)} \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left[\frac{1}{2} + \frac{1}{2^2}e^{+j(8t+2\pi/3)} + \frac{1}{2^2}e^{-j(8t+2\pi/3)}\right)$$

$$\left(\frac{1}{2} + \frac{1}{2^2}e^{-j(8t+\frac{2\pi}{3})} + \frac{1}{2^2}e^{j(8t+\frac{2\pi}{3})}\right) dt$$

$$= \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(\frac{1}{4} + \frac{1}{8}e^{j(8t+2\pi/3)} + \frac{1}{8}e^{-j(8t+2\pi/3)}\right.$$

$$+ \frac{1}{8}e^{-j(8t+2\pi/3)} + \frac{1}{4^2} + \frac{1}{4^2}e^{-j2(8t+\frac{2\pi}{3})} + \frac{1}{8}e^{j(8t+2\pi/3)}$$

$$\left. + \frac{1}{4^2}e^{j2(8t+2\pi/3)} + \frac{1}{4^2}\right) dt$$

$$= \frac{1}{\pi/4}\int_{-\pi/8}^{\pi/8} \left(\frac{3}{8} + \frac{2}{8}e^{j(8t+2\pi/3)} + \frac{1}{4^2}e^{j2(8t+2\pi/3)}\right.$$

$$\left. + \frac{1}{2^2}e^{-j(8t+2\pi/3)} + \frac{1}{4^2}e^{-j2(8t+2\pi/3)}\right) dt$$

$$= \frac{1}{\pi/4}\int_{-\pi/8}^{\pi/8} \left(\frac{3}{8} + \frac{1}{2}\cos(8t+2\pi/3) + \frac{1}{2^3}\cos((8t+2\pi/3)2)\right) dt$$

$$= \frac{1}{\pi/4}\int_{-\pi/8}^{\pi/8} \frac{3}{8}\,dt = \frac{3}{8}$$

Ans $P = 3/8$ W

Que-1 (a) $g(t) = 2t^2 - 3t + 6$

$$g_e(t) = \frac{2t^2 - 3t + 6 + 2(-t)^2 - 3(-t) + 6}{2}$$

$$= \frac{4t^2 + 12}{2} = 2t^2 + 6$$

$$g_o(t) = \frac{2t^2 - 3t + 6 - (2(-t)^2 - 3(-t) + 6)}{2}$$

$$= \frac{-6t}{2} = -3t$$

(b) $g(t) = 20\cos(40\pi t - \frac{\pi}{4})$

$$g_e(t) = \frac{20\cos(40\pi t - \pi/4) + 20\cos(-40\pi t - \pi/4)}{2}$$

$$= 20[\cos 40\pi t \cos\pi/4 + \sin 40\pi t \sin\pi/4]/2$$

$$+ 20[\cos 40\pi t \cos\pi/4 - \sin(40\pi t)\sin\pi/4]/2$$

$$g_e(t) = 20\cos 40\pi t \cos\frac{\pi}{4} = \frac{20}{\sqrt{2}}\cos 40\pi t$$

$$g_o(t) = \frac{20\cos(40\pi t - \pi/4) - 20\cos(-40\pi t - \pi/4)}{2}$$

$$= 20[\cos 40\pi t \cos\frac{\pi}{4} + \sin 40\pi t \sin\pi/4]/2$$

$$- 20[\cos 40\pi t \cos\frac{\pi}{4} - \sin 40\pi t \sin\frac{\pi}{4}]/2$$

$$= 20\sin 40\pi t \sin\frac{\pi}{4} = \frac{20}{\sqrt{2}}\sin 40\pi t.$$

(c) $g(t) = \frac{2t^2 - 3t + 6}{1+t}$

$g_e(t) = \frac{\frac{2t^2-3t+6}{1+t} + \frac{2t^2+3t+6}{1-t}}{2} = \frac{6+5t^2}{1-t^2}$

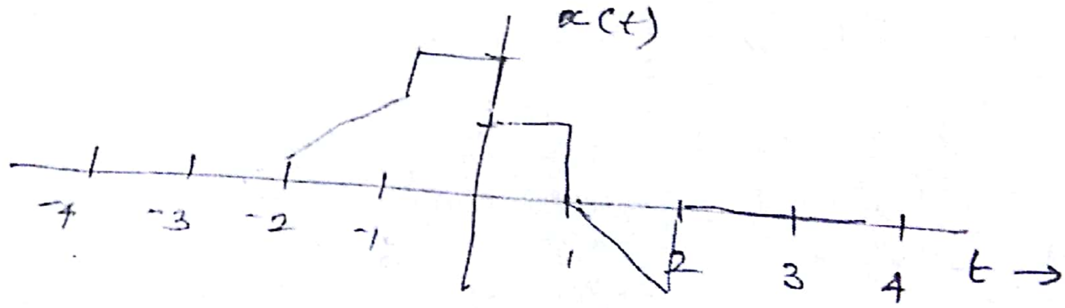
$g_o(t) = \frac{\frac{2t^2-3t+6}{1+t} - \frac{2t^2+3t+6}{1-t}}{2} = -t\,\frac{2t^2+9}{1-t^2}$

(d) $g(t) = \text{sinc}(t)$

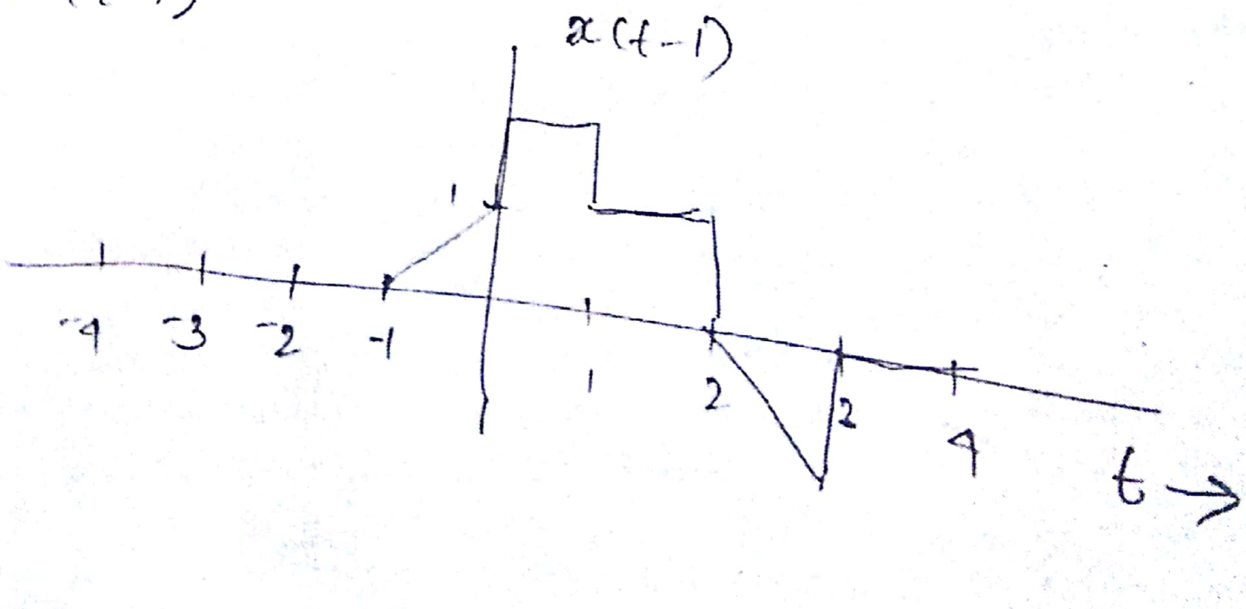
$g_e(t) = \frac{\frac{\sin \pi t}{\pi t} + \frac{\sin(-\pi t)}{-\pi t}}{2} = \frac{\sin \pi t}{\pi t}$

$g_o(t) = 0$

Q5

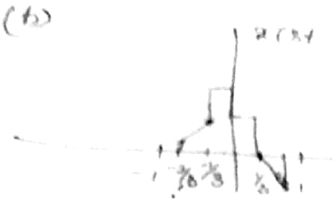


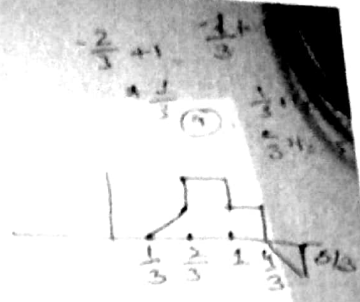
a. $x(t-1)$



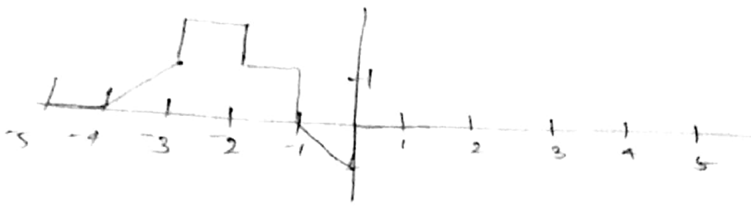
$x(a(t-b/a))$

(b)

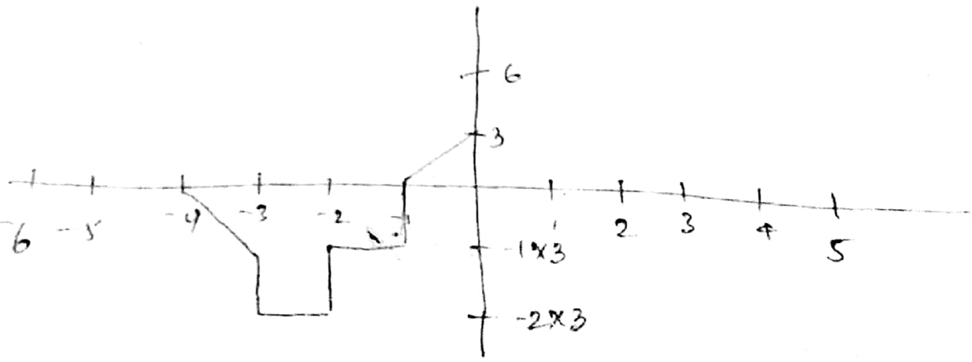




$x(t+2)$

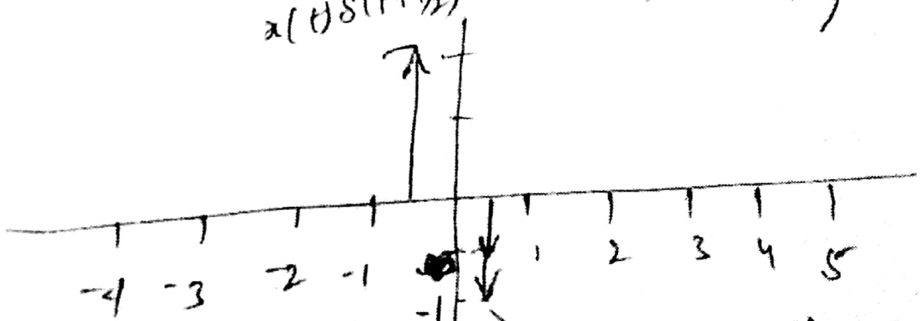


(c) $-3x(t+2)$



(d) $x(t)\,\delta(t+1/2) - \delta(t-1/2)$

$x(t)\,\delta(t+1/2) = x(-1/2)\,\delta(t+1/2)$

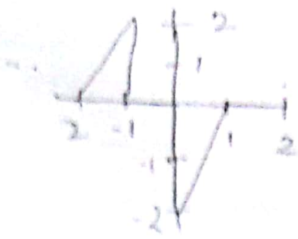


$x(t)\,\delta(t-1/2)$

$= x(1/2)\,\delta(t-1/2)$

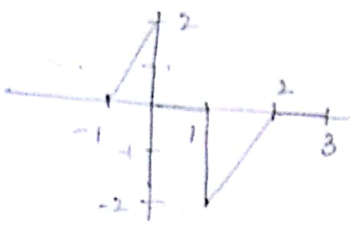
$x(t)$

$T=4$



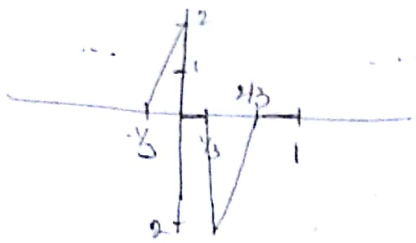
$x(t-1)$

$T=4$



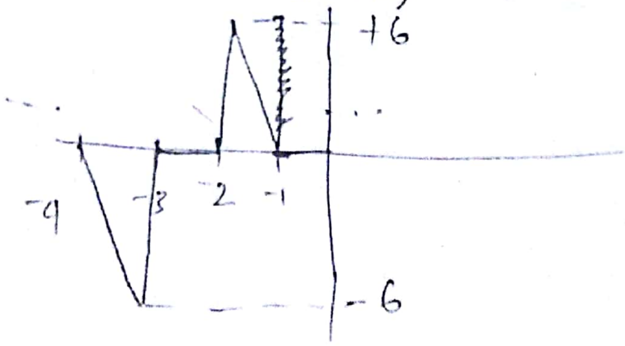
$x(3t-1)$

$T=4/3$



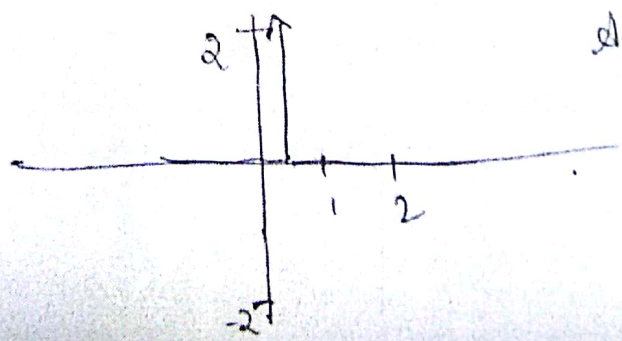
$-3x(t+2)$

$T=4/3$



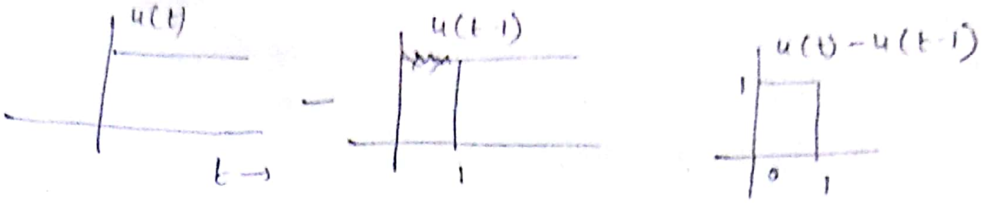
$x(t)\,\delta(t+\tfrac{1}{2}) - x(t)\,\delta(t-\tfrac{1}{2}) = -x(\tfrac{1}{2})\,\delta(t-\tfrac{1}{2})$

Aperiodic



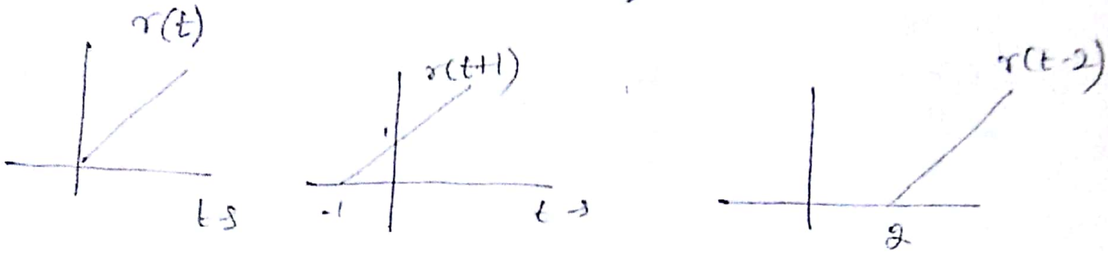
Ques. 6

(a) $x(t) = u(t) - u(t-1)$

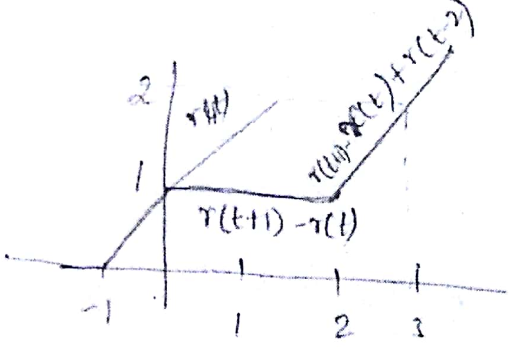


$= \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases} \qquad = \begin{cases} 1 & t>1 \\ 0, & t<1 \end{cases} \qquad = \begin{cases} 1, & 0<t<1 \\ 0, & \text{elsewhere} \end{cases}$

(b) $x(t) = r(t+1) - r(t) + r(t-2)$

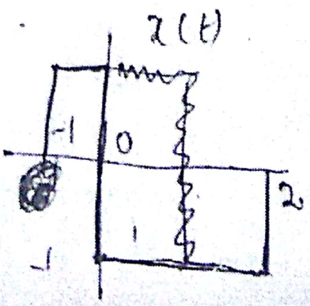


$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t<0 \end{cases} \qquad = \begin{cases} t+1, & t+1 \geq 0 \\ 0, & t+1<0 \end{cases} \qquad = \begin{cases} t-2, & t-2 \geq 0 \\ 0, & t-2<0 \end{cases}$



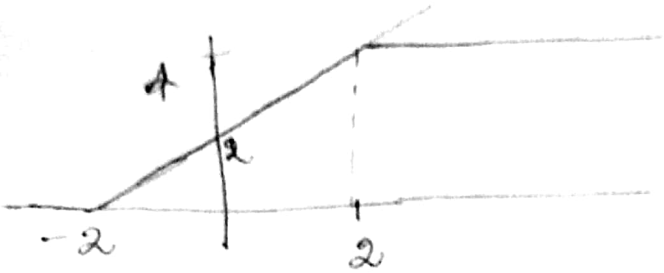
$= \begin{cases} t+1, & -1 \leq t \leq 0 \\ 1, & 0 \leq t \leq 2 \\ t-1, & t \geq 2 \\ 0, & \text{elsewhere} \end{cases}$

(c) $x(t) = u(t+1) - 2u(t) + u(t-2)$



$= \begin{cases} 0 & t<-1 \\ 1 & -1< t<0 \\ -1 & 0<t<2 \\ 0 & t>2 \end{cases}$

(d) $x(t) = r(t+2) - r(t-2)$



$$= \begin{cases} 0 & t < -2 \\ t+2 & |t| < 2 \\ t+2-t+2 = 4 & t \geq 2 \end{cases}$$

Ques-7

Sol. (a) $y(t) = x(t+2) - x(t-2)$

Memory: At $t = -1$

$y(-1) = x(1) - x(-3)$ depends on past & future

At $t = 1$,

$y(1) = x(3) - x(-1)$ depends on past & future

It has memory.

Causality: It depends on future values

Hence, Non-Causal

Time Invariance:

Delaying t by $t-t_0$, $y(t-t_0) = x(t-t_0+2) - x(t-t_0-2)$

Delaying input, $x(t) \rightarrow x(t-t_0)$

$y(t) = x(t+2-t_0) - x(t-2-t_0)$

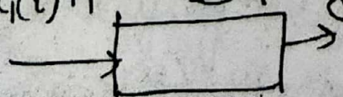
$y(t) = y(t-t_0)$ Hence, Time Invariant

Linearity: To prove:

$\alpha x_1(t) + \beta x_2(t) \longrightarrow y(t) = \alpha\{x_1(t+2) - x_1(t-2)\}$
$+ \beta\{x_2(t+2) - x_2(t-2)\} = \alpha y_1(t) + \beta y_2(t)$

$= \cancel{\alpha x_1(t+2) + \beta x_2(t+2) - \alpha x_1(t-2) - \beta x_2(t-2)}$

$\alpha x_1(t) + \beta x_2(t)$



$y(t) = \alpha x_1(t+2) + \beta x_2(t+2) + \alpha x_1(t-2) + \beta x_2(t-2)$

$= \alpha(x_1(t+2) + x_1(t-2)) + \beta(x_2(t+2) + x_2(t-2))$

$= \alpha y_1(t) + \beta y_2(t)$ Hence, linear

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Stability : Transformation is sum of two shifting operations.

To prove : $|y| \le M_y$ if $|x(t)| \le M_x$

$$|y(t)| = |x(t+2) - x(t-2)|$$
$$\le |x(t+2)| - |x(t-2)|$$
$$\le 2M_x$$

for bounded input, output is bounded

Hence, Stable.

(b) $y(t) = \sin[x(t)]$

① Memory: depends on only present values

$y(-1) = \sin[x(-1)]$
$y(0) = \sin[x(0)]$
$y(1) = \sin[x(1)]$

Hence, Memoryless

② Causality: All memoryless systems are causal system

③ Time Invariance:

Delaying t by $t-t_0$, $y(t-t_0) = \sin[x(t-t_0)]$

Delaying i/p $x(t) \to x(t-t_0)$ $y(t) = \sin[x(t-t_0)]$

$\therefore y(t) = y(t-t_0)$. Hence, time-invariant

4) Linearity :

$x_1(t) \rightarrow \boxed{} \rightarrow y_1(t) = \sin[x_1(t)]$

$x_2(t) \rightarrow \boxed{} \rightarrow y_2(t) = \sin[x_2(t)]$

$\alpha x_1(t) + \beta x_2(t) \rightarrow \boxed{} \rightarrow y(t) = \sin[\alpha x_1(t) + \beta x_2(t)]$

$$y(t) = \sin[\alpha x_1(t)]\cos[\beta x_2(t)] + \cos[\alpha x_1(t)]\sin[\beta x_2(t)]$$

$$\neq \alpha y_1(t) + \beta y_2(t).$$

Hence, Non - linear.

5) Stability: If $|x(t)| \leq M_x$
then $|y(t)| \leq M_y$ where M_x & M_y are +ve integers

$$|y(t)| = |\sin[x(t)]|$$

$$|y(t)| \leq 1$$

Hence, BIBO stable

(c) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

Memory : $y(0) = \int_{-\infty}^{t=0} x(\tau)\, d\tau$

$$y(1) = \int_{-\infty}^{1} x(\tau)\, d\tau$$

It depend on values from $-\infty$ to . Hence, System output depends on past and present values. Hence, memory

② Causality : It depends ~~for~~ value at any instant (t) on values of input from $-\infty$ to t.
Only past values not future values of input.
Hence, Causal.

③ Time Invariance:
Delaying t by t_0, $y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$

Put $t-t_0 = m$; $y(m) = \int_{-\infty}^{m} x(\tau)\, d\tau$

Delaying input, $x(t)$

$$y(t) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau$$

Put $\tau - t_0 = m$ $\quad y(t) = \int_{-\infty}^{t} x(m)\, dm = y(t-t_0)$

Hence, Time Invariant

④ Linearity:

$x_1(t) \to \boxed{} \to y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$

$x_2(t) \to \boxed{} \to y_2(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau$

$\alpha x_1(t) + \beta x_2(t) \to \boxed{} \to y(t) = \int_{-\infty}^{t} \alpha x_1(\tau) + \beta x_2(\tau)\, d\tau$

$$= \alpha \int_{-\infty}^{t} x_1(\tau)\, d\tau + \beta \int_{-\infty}^{t} x_2(\tau)\, d\tau = \alpha y_1(t) + \beta y_2(t)$$

Hence, linear.

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5 Stability:

$$|y(t)| \le \int_{-\infty}^{t} |x(\tau)|\, d\tau$$

For e.g. $x(t) = k$, some constant

$$|y(t)| \le \int_{-\infty}^{t} k\, d\tau = \infty$$ It is not bounded.

Hence, Unstable.

(d) $y(t) = x(2t/5)$

1. Memory:

for +ve time, $y(2) = x(4/5)$ past

−ve time, $y(-2) = x(-4/5)$ future

0 time, $y(0) = x(0)$ present

Output values depend on past, present and future values of input.

2. Causality: Since, output values depend on future values of input.

Hence, Non-Causal.

3. Time Invariance:

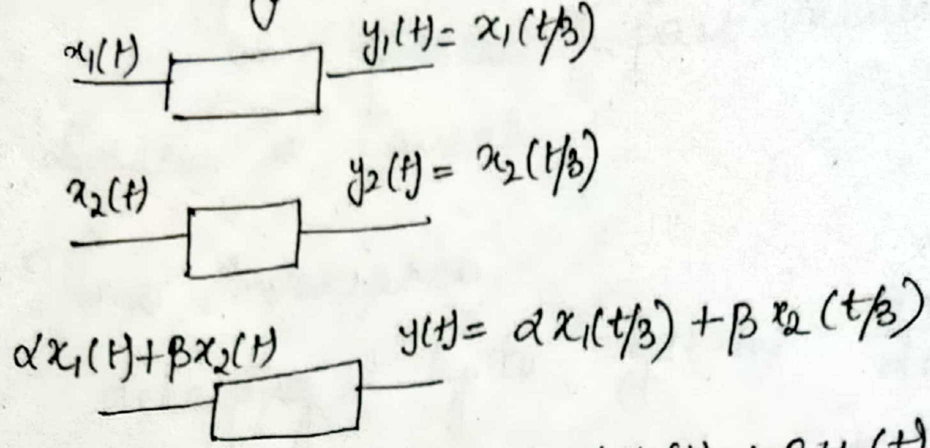
Delaying t by $t-t_0$, $y(t-t_0) = x(2(t-t_0)/5)$

Delaying input, $y(t) = x(2t/5 - t_0)$

$\therefore y(t) \neq y(t-t_0)$

Hence, Time Variant

④ Linearity:



$$\Rightarrow y(t) = \alpha y_1(t) + \beta y_2(t)$$

Hence, linear.

⑤ Stability:

$$|y(t)| = |x(t/3)|$$

If $|x(t)| \le M_x$

then $|y(t)| \le M_x$.

Since, transformation on independent variable time, does not effect bounds of amplitude.

(e) $y(t) = \dfrac{dx(t)}{dt}$

① Memory:

$$\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t) - x(t - \Delta t)}{\Delta t}$$

∵ output depends on present & past values of input. System is with memory.

(2) Causality : $\because$ Output of system depends only on present & past values of input.

System is Causal.

(3) Time Invariance:

delaying t by t_0 $\quad y(t-t_0) = \dfrac{d\,x(t-t_0)}{d(t-t_0)}$

Put $t - t_0 = \tau$ so that $d\tau = dt$

$$y(\tau) = \frac{dx(\tau)}{d\tau}$$

Delaying input by t_0, $\quad y(t) = \dfrac{dx(t-t_0)}{dt}$

$$\Rightarrow y(\tau + t_0) = \frac{d\,x(\tau)}{d\tau} = y(t-t_0)$$

Hence, Stable.

(4) Linearity:

$x_1(t) \rightarrow \boxed{} \rightarrow y_1(t) = \dfrac{dx_1(t)}{dt}$

$x_2(t) \rightarrow \boxed{} \rightarrow y_2(t) = \dfrac{dx_2(t)}{dt}$

$\alpha x_1(t) + \beta x_2(t) \rightarrow \boxed{} \rightarrow y(t) = \dfrac{d(\alpha x_1(t) + \beta x_2(t))}{dt}$

$$y(t) = \alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt} = \alpha y_1(t) + \beta y_2(t)$$

Hence, Linear.

(5) Stability:

Let $x(t) = u(t) \Rightarrow y(t) = \dfrac{dx(t)}{dt} = \delta(t)$ & $\delta(t)$ is unbounded. Since, condition must be true for all bounded inputs, the differentiation is unstable.

45

(f) $y(t) = \begin{cases} 0 & , x(t) < 0 \\ x(t) - x(t-2) & , x(t) \geq 0 \end{cases}$

(1) Memory:

for +ve time, $y(2) = x(2) - x(0)$ past & present

for -ve time $y(-2) = x(-2) - x(-4)$ past & present

$y(0) = x(0) - x(-2)$ past & present

Since, output depends on present & past values of input. System has memory.

(2) Causality: System is Causal.

(3) Time Invariance:

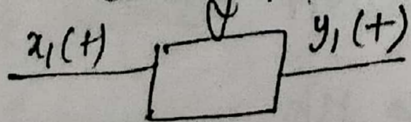
Delaying t by t_0, $y(t-t_0) = \begin{cases} 0 & , x(t) < 0 \\ x(t-t_0) - x(t-t_0-2) & , x(t-t_0) \geq 0 \end{cases}$

Delaying input,

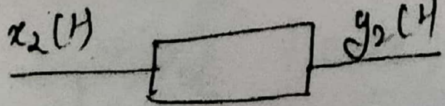
$y(t) = \begin{cases} 0 , & x(t-t_0) < 0 \\ x(t-t_0) - x(t-2-t_0) , & x(t-t_0) \geq 0 \end{cases}$

$\therefore y(t) = y(t-t_0)$ System is time invariant

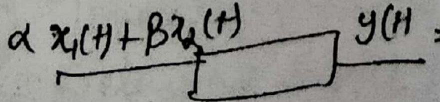
(4) Linearity:



$y_1(t) = \begin{cases} 0 & , x_1(t) < 0 \\ x_1(t) - x_1(t-2) , & x_1(t) \geq 0 \end{cases}$



$y_2(t) = \begin{cases} 0 & , x_2(t) < 0 \\ x_2(t) - x_2(t-2) , & x_2(t) \geq 0 \end{cases}$



$y(t) = \begin{cases} 0 , & \alpha x_1(t) + \beta x_2(t) < 0 \\ \alpha x_1(t) + \beta x_2(t) - (\alpha x_1(t-2) + \beta x_2(t-2)) , & \alpha x_1(t) + \beta x_2(t) \geq 0 \end{cases}$

$\neq \alpha y_1(t) + \beta y_2(t)$. Hence, Non-Linear

5) Stability :

∵ equation is subtraction of signal & its delay version. If $x(t)$ is bounded, $y(t)$ will always be bounded.

$$|y(t)| = \left| \begin{matrix} 0 & , x(t)<0 \\ x(t)-x(t-2) & , x(t)\geq 0 \end{matrix} \right| \text{ bounded}$$

Let $|x(t)| = M_x$

$$|y(t)| \leq |M_x| - |M_x| \quad , x(t) \geq 0$$

$$\leq 0.$$

Hence, stable.

(9) $y(t) = t\,x(t)$

(1) Memory :

for +ve time, $y(2) = 2x(2)$

-ve time, $y(-2) = -2x(-2)$

, $y(0) = 0$

For all-time, output depends on present values of input. Hence, Memoryless System.

(2) Causality : All memoryless systems are causal systems. Hence, Causal.

(3) Time Invariance : Delaying t by t_0 $y(t-t_0) = (t-t_0)x(t-t_0)$

Delaying input, $y(t) = t\,x(t-t_0) \neq y(t-t_0)$.

Hence, Time Variant.

④ Linearity:

$x_1(t) \to \boxed{} \to y_1(t) = t x_1(t)$ $\qquad$ $x_2(t) \to \boxed{} \to y_2(t) = t x_2(t)$

$\alpha x_1(t) + \beta x_2(t) \to \boxed{} \to y(t) = t(\alpha x_1(t) + \beta x_2(t))$

$$y(t) = \alpha t x_1(t) + \beta t x_2(t)$$

$$= \alpha y_1(t) + \beta y_2(t)$$ Hence, Linear.

⑤ Stability:

$$|y(t)| \leq |t||x(t)|$$

for $t \to \infty$, $|y(t)| \to \infty$ independent of values of input

Hence, System is Unstable.

(h) $y(t) = \sqrt{x(t)}$

① Memory: for +ve time, $y(2) = \sqrt{x(2)}$
-ve time, $y(-2) = \sqrt{x(-2)}$
0 time, $y(0) = \sqrt{x(0)}$

∵ Output depends only on present values of input. System is memoryless.

② Causality: All memoryless systems are causal.
Hence, Causal.

③ Time Invariance:
$t \to t-t_0$, $y(t-t_0) = \sqrt{x(t-t_0)}$
$x(t) \to x(t-t_0)$, $y(t) = \sqrt{x(t-t_0)} = y(t-t_0)$
Hence, time variant.

(14) Linearity:

$x_1(t) \rightarrow \boxed{} \rightarrow y_1(t) = \sqrt{x_1(t)}$ $\qquad$ $x_2(t) \rightarrow \boxed{} \rightarrow y_2(t) = \sqrt{x_2(t)}$

$$\alpha x_1(t) + \beta x_2(t) \rightarrow \boxed{} \rightarrow y(t) = \sqrt{\alpha x_1(t) + \beta x_2(t)}$$

$$\neq \sqrt{\alpha x_1(t)} + \sqrt{\beta x_2(t)}$$

Hence, Non - Linear.

(15) Stability:

Operation is square - root of signal

If $|x(t)| \leq M_x$

then $|y(t)| \leq \sqrt{M_x} < \infty$

Hence, Stable.

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