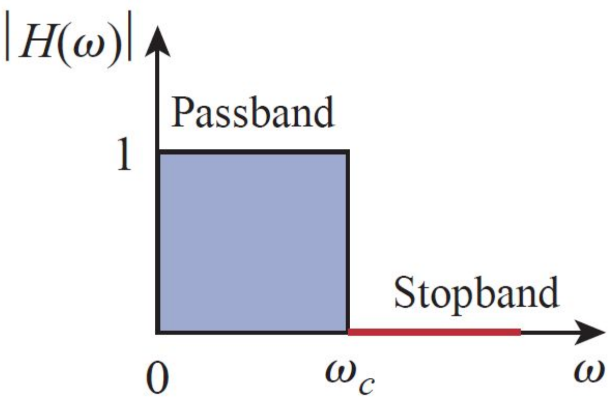
Electrical Science-2 (15B11EC211)

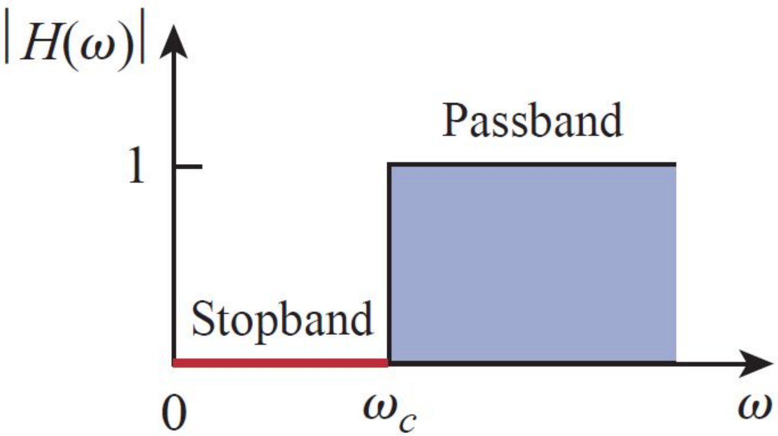
Unit-3
Operational Amplifier and Filters
Lecture-4

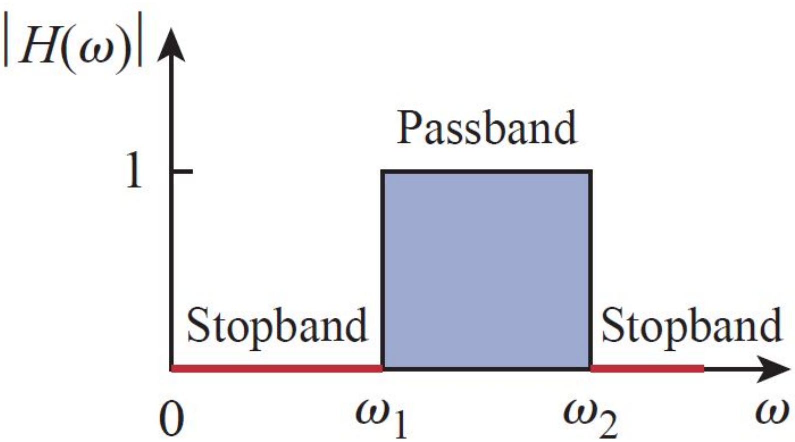
Contents

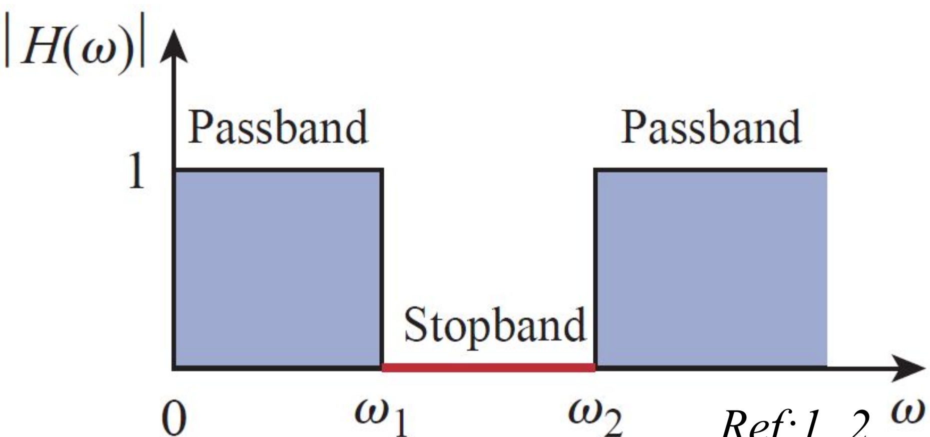
Passive Filters

- A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.
- A filter is a passive filter if it consists of only passive elements R, L, and C.
- There are four types of filters: Lowpass, Highpass, Bandpass, Bandstop









Ref:1, 2

Summary of the characteristics of ideal filters

Table 1: Summary of the characteristics of the filters.

Type of Filter	H (0)	H(∞)	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

Note: ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.

- **Be aware that the characteristics in Table are only valid for first- or second-order filter**

Ref: 1, 2

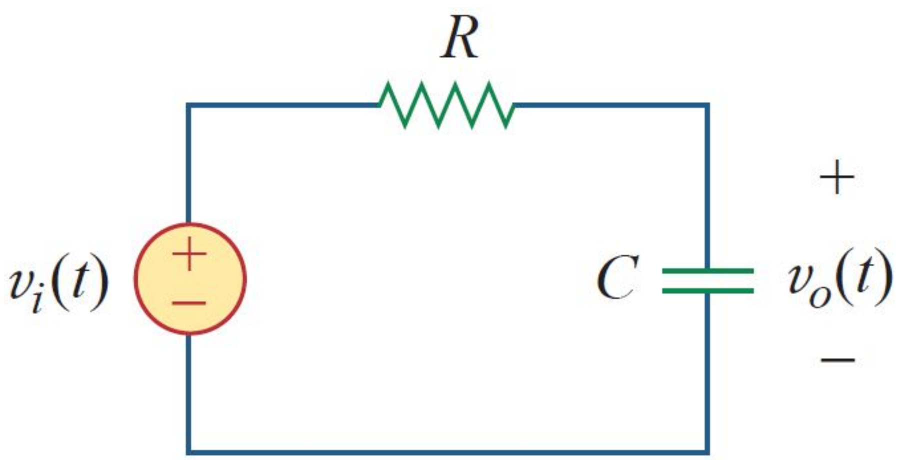
Lowpass Filter

- A lowpass filter is designed to pass only frequencies from dc up to the cutoff frequency ω_c.
- A typical lowpass filter is formed when the output of an *RC circuit is* taken off the capacitor as shown in Figure.

Transfer function H(ω)

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$



Note that H(0) = 1, H(∞) = 0

Ref: 1, 2

Lowpass Filter

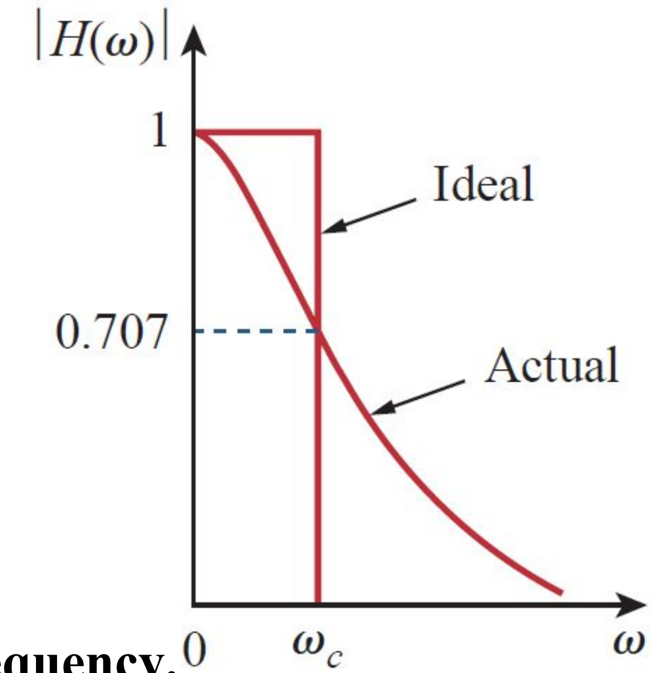
- The cutoff frequency is the frequency at which the transfer function H drops in magnitude to 70.71% of its maximum value.

Cutoff frequency ω_c is obtained by putting

$$|H(\omega)| = 1/\sqrt{2}$$

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = 1/RC$$



Note: Cutoff frequency is also called the rolloff frequency.

- It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.
- A lowpass filter can also be formed when the output of an RL circuit is taken off the resistor.

Ref:1, 2

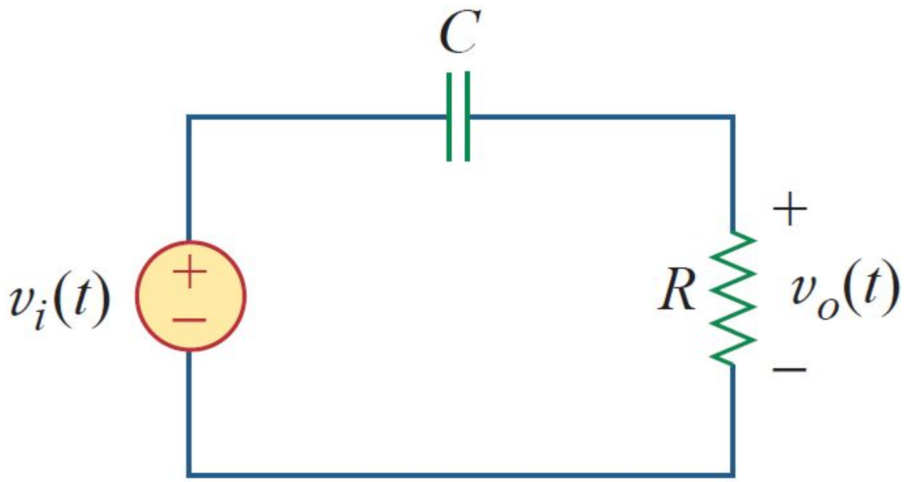
Highpass Filter

- A highpass filter is designed to pass all frequencies above its cutoff frequency ω_c.
- A highpass filter is formed when the output of an RC circuit is taken off the resistor as shown in Figure

Transfer function H(ω)

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$



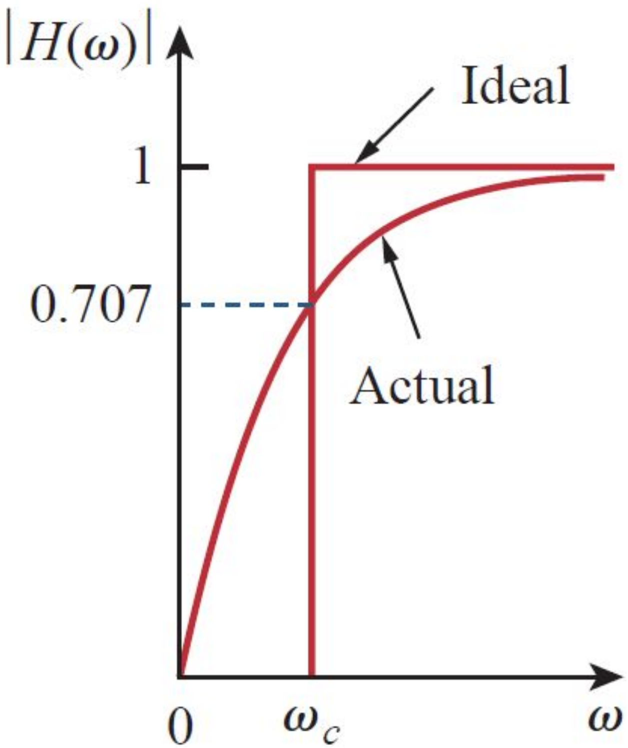
Note that H(0) = 0, H(∞) = 1

Ref:1, 2

Highpass Filter

Corner or cutoff frequency is

$$\omega_c = 1/RC$$



Note: A highpass filter can also be formed when the output of an RL circuit is taken off the inductor.

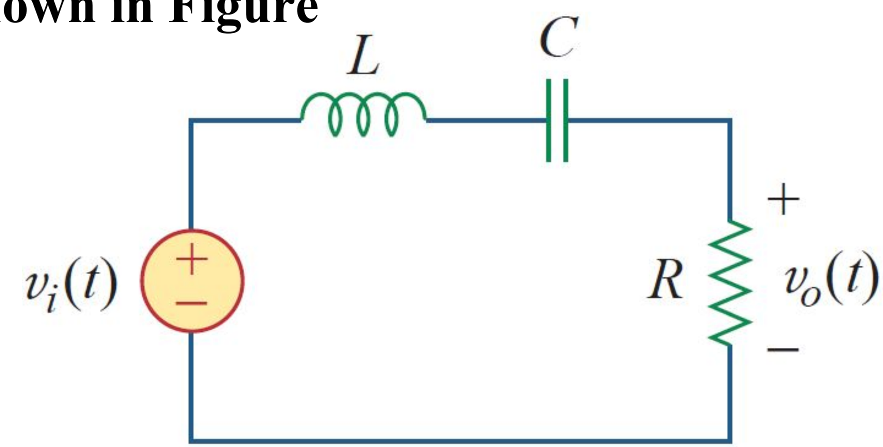
Ref:1, 2

Bandpass Filter

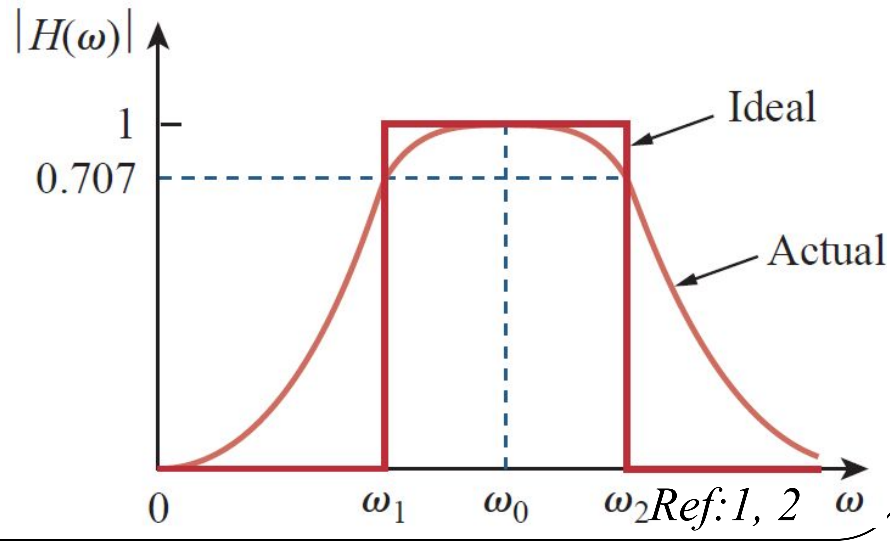
- A bandpass filter is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$
- The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Figure

Transfer function H(ω)

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$



We observed that H(0) = 0, H(∞) = 0



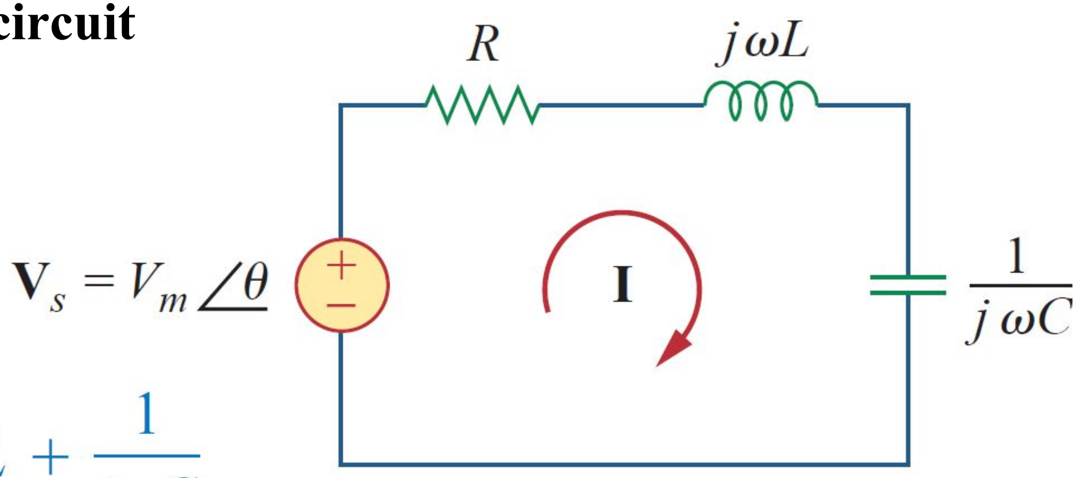
- **Center frequency ω_0**

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Ref: 1, 2

Bandpass Filter

- Since the bandpass filter is a series resonant circuit, the half-power frequencies, the bandwidth, and the quality factor can be determined by analysis of series resonant circuit



Series resonant circuit

Input impedance, Z

$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C}$$

$$\mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

- Resonance results when the imaginary part of the transfer function is zero

$$\mathrm{Im}(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0$$

Ref:1, 2

Bandpass Filter

- **The value of ω that satisfies the resonance condition is called the resonant frequency ω_0 .**

$$\omega_0 L = \frac{1}{\omega_0 C} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

- **Since $\omega_0 = 2\Pi f_0$**

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

- **The half-power frequencies (ω_1 and ω_2 are called the half-power frequencies) are obtained by setting Z = (√2)R**

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

Solving for ω , we obtain

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Ref: 1, 2

Bandpass Filter

- **Resonant frequency is the geometric mean of the half-power frequencies**

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

- **Half-power Bandwidth B**

$$\mathbf{B = \omega_2 - \omega_1}$$

- **Quality factor Q:**
  The “sharpness” of the resonance in a resonant circuit is measured quantitatively by the quality factor Q.

- **The quality factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation:**

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}$$

Ref:1, 2

Bandpass Filter

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f_0)} = \frac{2\pi f_0 L}{R}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

- **The relationship between the bandwidth *B and the quality factor Q***

$$B = \frac{R}{L} = \frac{\omega_0}{Q}$$

- **Hence, the quality factor of a resonant circuit is the ratio of its resonant frequency to its bandwidth**

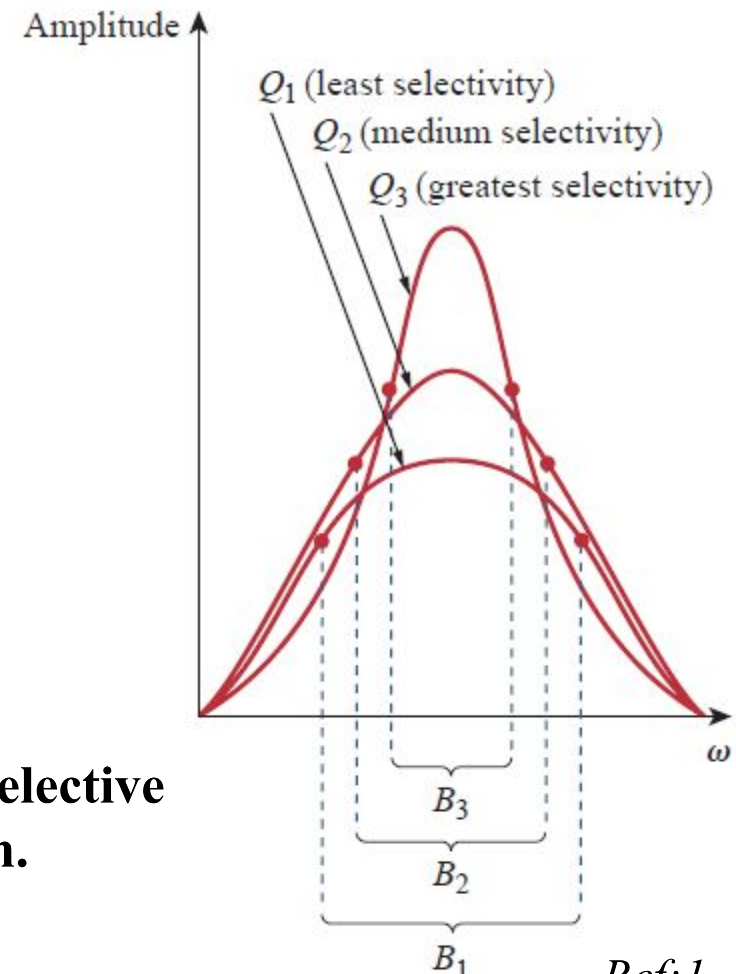


Figure higher the value of Q, the more selective the circuit is but the smaller the bandwidth.

Ref: 1, 2

Bandpass Filter

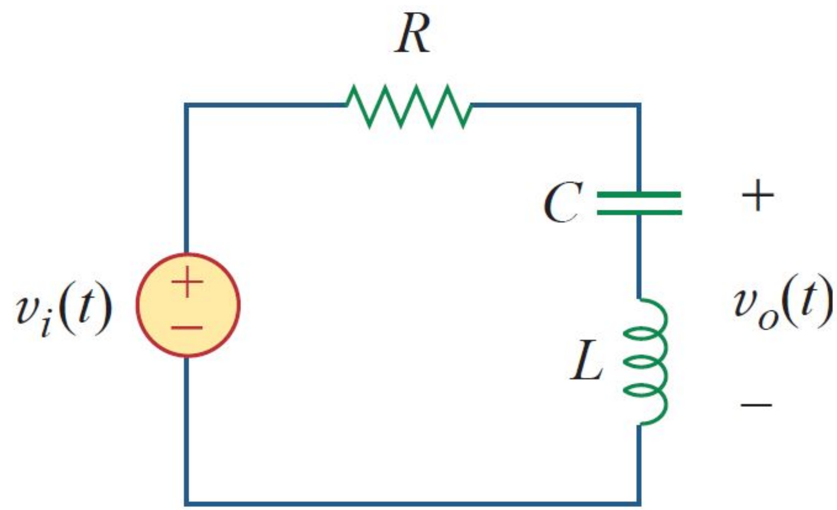
- **A resonant circuit is designed to operate at or near its resonant frequency.**
- **It is said to be a high-Q circuit when Q ≥ 10**
- **For high-Q circuits (Q ≥ 10) the half-power frequencies can be approximated as**

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

Ref: 1, 2

Bandstop Filter

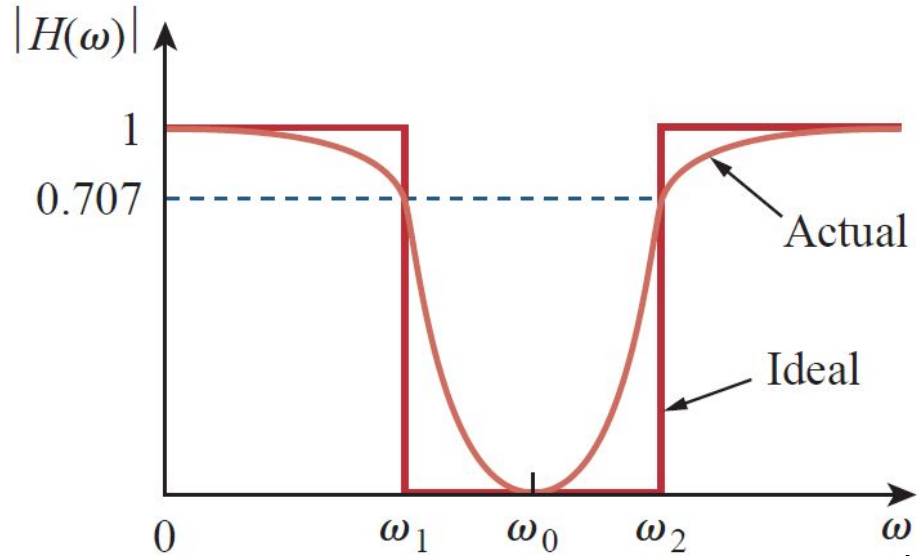
- A bandstop filter is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.
- A bandstop filter is formed when the output RLC series resonant circuit is taken off the LC series combination.



Transfer function H(ω)

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

Notice that H(0) = 1, H(∞) = 1



❖ **Center frequency ω_0**

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Bandstop Filter

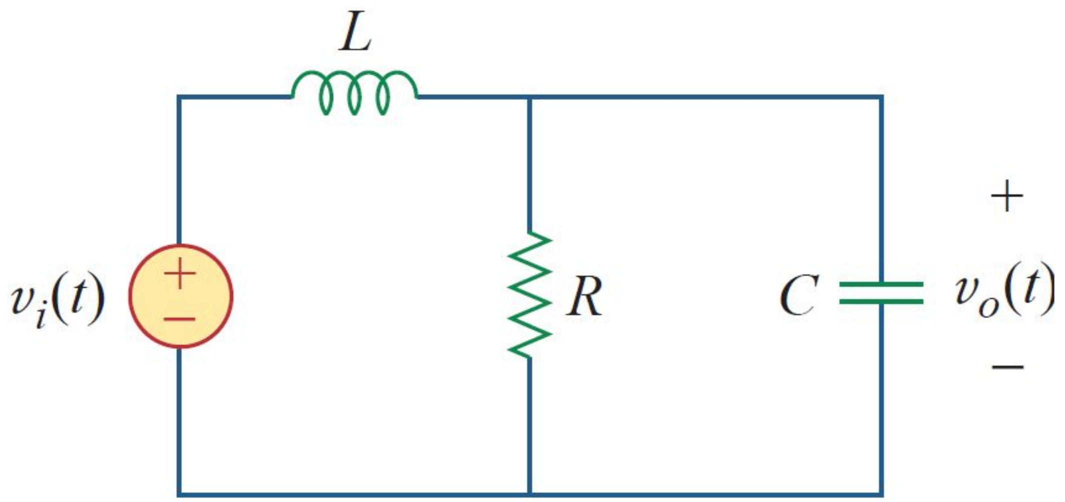
- **The half-power frequencies, the bandwidth, and the quality factor are calculated using the formulas discussed for a series resonant circuit.**

- **Here, ω_0 is called the frequency of rejection, while the corresponding bandwidth ($B = \omega_2 - \omega_1$) is known as the bandwidth of rejection.**

Ref: 1, 2

Example

Determine what type of filter is shown in Figure. Calculate the corner or cutoff frequency. Take R = 2 kΩ, L = 2 H, and C = 2 μF.



Solution:

The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \qquad s = j\omega \qquad \textbf{Eq. (1)}$$

But $$R \left\| \frac{1}{sC} = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC} \right. \qquad \textbf{Eq. (2)}$$

Ref: 1, 2

Solution Cont….

Substituting Eq. (2) into Eq. (1) gives

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \qquad s = j\omega$$

Or

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R}$$

- **Since H(0) = 1 and H(∞) = 0, we conclude from Table 1 that the circuit is a second-order lowpass filter.**

The magnitude of H is

$$H = \frac{R}{\sqrt{(R - \omega^2RLC)^2 + \omega^2L^2}} \qquad \textbf{Eq. (3)}$$

Ref:1, 2

Solution Cont….

The corner frequency is the same as the half-power frequency, i.e., where H is reduced by a factor of 1/√2

Eq. (3) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

Or

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of R, L, and C, we obtain

$$2 = (1 - \omega_c^2\, 4 \times 10^{-6})^2 + (\omega_c\, 10^{-3})^2$$

Ref: 1, 2

Solution Cont….

Assuming that ω_c is in krad/s,

$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2$$

Or

$$16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2

We get $\omega_c^2 = 0.5509 \quad \text{and} \quad -0.1134$

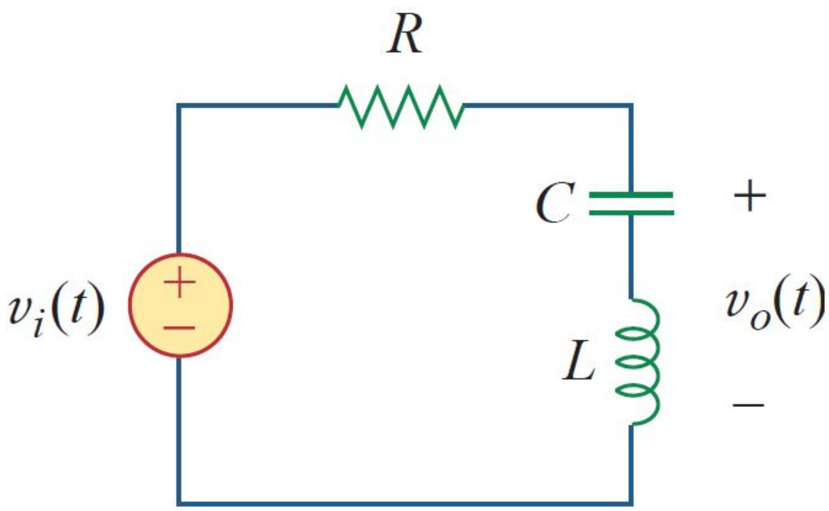
Since ω_c is real,

ω_c = 0.742 krad/s = 742 rad/s

Ref: 1, 2

Practice Example

If the bandstop filter shown in Figure is to reject a 200-Hz sinusoid while passing other frequencies, calculate the values of L and C. Take R = 150 Ω and the bandwidth as 100 Hz.



Answer:

L = 0.2387 H, C = 2.653 μF

Ref:1, 2

References

1. R.C. Dorf and James A. Svoboda, “Introduction to Electric Circuits” , Chapter 17, 9th ed, John Wiley & Sons, 2013.

2. Charles K. Alexander and Matthew N. O. Sadiku, “Fundamentals of Electric Circuits” , Chapter 19, 4th ed, Mcgraw Hill, 2009.