Answers of Tutorial 3 of Probability and Random Processes

1. (i)Probability mass function

y=2	(4.3)/(52.51)	0	0
y=1	2(4.44)/(52.51)	2(4.4)/(52.51)	0
y=0	(44.43)/(52.51)	2(4.44)/(52.51)	(4.3)/(52.51)
	x=0	x=1	x=2

(ii) Marginal Probability distribution P(y) and P(x)

P(y=2)=(4.3)/(52.51)	P(x=2)=(4.3)/(52.51)
P(y=1)=(48.2.4)/(52.51)	P(x=1)=(48.2.4)/(52.51)
P(y=0)=(44.51+12)/(52.51)	P(x=0)=(44.51+12)/(52.51)

(iii) 0 (iv) 1 (v) 364/(2268)

2 (i)3/5 (ii) 13/20 (iii) 3/40 (iv) 23/80

(v) $f_X(x) = 3/5\left(x/2 + 1/3\right), 0 \le x \le 2, 0 \text{ elsewhere}$

(vi) $f_Y(y) = 3/5\left(2y + 2y^2\right), 0 \le y \le 1, 0 \text{ elsewhere}$ (vii) X and Y are not independent.

3 (i) Yes, X and Y are independent. (ii) $\frac{2e-1}{4e}$

4 Conditional probability distribution of Y given X=x

y=2	(2/3)	(4/9)	(6/15)
y=1	(1/3)	(3/9)	(5/15)
y=0	0	(2/9)	(4/15)
	x=0	x=1	x=2

Conditional cumulative probability distribution of Y given X=x

y≥2	1	1	1
1≤y<2	(1/3)	(5/9)	(9/15)
0≤y<1	0	(2/9)	(4/15)
y<0	0	0	0
	x=0	x=1	x=2

5. (a) Joint Probability distribution

y=6	(1/36)	(1/36)	(1/36)	(1/36)	(1/36)	(6/36)
y=5	(1/36)	(1/36)	(1/36)	(1/36)	(5/36)	0
y=4	(1/36)	(1/36)	(1/36)	(4/36)	0	0
y=3	(1/36)	(1/36)	(3/36)	0	0	0
y=2	(1/36)	(2/36)	0	0	0	0
y=1	(1/36)	0	0	0	0	0
	x=1	x=2	x=3	x=4	x=5	x=6

(b) 161/36 (c) 2555/1296

6. (i) $\frac{3}{4}x_2$ (ii) $\frac{3}{80}x_2^2$

7 (i) $\frac{25}{36}+\frac{10}{36}e^{t}+\frac{1}{36}e^{2t}$. (ii) 1/3 (iii) 5/18

8 first moment=0, second moment=2, third moment=0, variance=2

9 (i) -1/64 (ii) 3/8 (iii) -15/73

10. $e^{-5+2e^{2t}+3e^{3t}}$

11. $\frac{1}{3}+\frac{1}{3}\cos(\omega_1)+\frac{1}{3}\cos(\omega_1+\omega_2)$