## **Department of Mathematics**

## Probability and Random Processes Tutorial Sheet 13

15B11MA301 C201.5

## **Random Process**

1. If the autocorrelation function of a WSS process is  $R(\tau) = \rho e^{-\rho|\tau|}$ , Show that its spectral density is

$$S(\omega) = \frac{2}{1 + \left(\frac{\omega}{\rho}\right)^2}$$

2. Find the power density spectral of a stationary process  $\{X(t)\}$  with

$$R_{XX}(\tau) = \sigma^2 e^{-\alpha|\tau|}$$
.

Ans:

$$=\frac{2\sigma^2\alpha}{\alpha^2+\omega^2}$$

3. A WSS noise process N(t) has an ACF  $R_{NN}(\tau) = \rho e^{-3|\tau|}$ , where  $\tau$  is a constant. Find its power spectrum.

Ans:

$$=\frac{6\rho}{9+\omega^2}$$

4. Find autocorrelation function  $R(\tau)$  of a random process if the power spectral density of random process  $\{X(t)\}$  is given by

$$S(\omega) = \begin{cases} A, & |\omega| \le \beta \\ 0, & |\omega| > \beta \end{cases}$$

Ans:

$$=\frac{A}{\pi}\bigg(\frac{\sin\beta\tau}{\tau}\bigg)$$

5. Find the spectral density of random process  $\{X(t)\}$  whose autocorrelation is

$$R(\tau) = \begin{cases} -1, & -3 < \tau < 3 \\ 0, & otherwise \end{cases}$$

$$=\frac{-2\sin 3\omega}{\omega}$$

Ans

- 6. If  $\{X(t)\}$  is a constant random process with  $R_{XX}(\tau) = m^2$ ,  $\forall \tau$  where m is a constant, show that the spectral density of the process is  $S(\omega) = 2\pi m^2 \delta(\omega)$ .
- 7. Find the spectral density of random process  $\{X(t)\}$  whose autocorrelation is  $R_{XX}(\tau) = e^{-\alpha|\tau|}cos\beta\tau$ .

Ans: 
$$= \frac{\alpha}{\alpha^2 + (\omega - \beta)^2} + \frac{\alpha}{\alpha^2 + (\omega + \beta)^2}$$