Department of Mathematics

15B11MA211 **Mathematics 2**

Tutorial Sheet 5 **B.Tech. Core**

Series Solutions

1. Find the singular points of the following differential equations and classify them.
   (a) $x^2y''-5y'+3x^2y=0$ (b) $x^2y''+(\sin x)y'+(\cos x)y=0$
   (c) $(x^2+x-2)^2y''+3(x+2)y'+(x-1)y=0$ (d) $x^4y''+4x^3y'+y=0$.
2. Solve the following differential equations in series: (about $x=0$)
   (a) $(1-x^2)y''+2xy'+y=0$ (b) $xy''+y'+xy=0$ (c) $x^2y''+xy'+(x^2-n^2)y=0$
   (d) $(x-x^2)y''+(1-5x)y'-4y=0$ (e) $8x^2y''+10xy'-(1+x)y=0$
   (f) $x(1+x)y''+(x+5)y'-4y=0$.

Answers:

1(a) $x=0$, irregular singular point (b) $x=0$, regular singular point

(c) $x=1$, irregular singular point, $x=-2$ regular singular point

(d) $x=0$, irregular singular point.

2(a) $y(x)=c_0(1-\frac{1}{2}x^2+\frac{1}{8}x^4+...)+c_1(x-\frac{1}{2}x^3+\frac{1}{40}x^5+...)$

(b) $y=a(1-\frac{x^2}{2^2}+\frac{x^4}{2^2.4^2}-\frac{x^6}{2^2.4^2.6^2}+...)+b(u\log x+(\frac{x^2}{2^2}-\frac{x^2}{2^2.4^2}(1+\frac{1}{2})+...))$

(c) $y=ax^n(1-\frac{x^2}{4(n+1)}+\frac{x^4}{4.8.(n+1).(n+2)}-...)+bx^{-n}(1-\frac{x^2}{4(1-n)}+\frac{x^4}{4.8.(1-n).(2-n)}-...)$

(d) $y=a(1+2^2.x+3^2.x^2+4^2.x^3+..)+b(u\log x-2(1.2x+2.3x^2+...))$

(e) $y=ax^{\frac{1}{4}}(1+\frac{x}{14}+\frac{x^2}{14.44}+...)+bx^{\frac{-1}{2}}(1+\frac{x}{2}+\frac{x^2}{2.20}+...)$

(f) $y=a(1+\frac{4}{5}x+\frac{1}{5}x^2+...)+bx^{-4}(1+4x+5x^2+...)$