Department of Mathematics

15B11MA211 **Mathematics-II**

Tutorial Sheet 8 **B.Tech. Core**

Fourier Series

1. Expand the function $f(x)=x\sin x$ in a Fourier series in the interval $-\pi \le x < \pi$. Use the series obtained to show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\frac{1}{7.9}=.....=\frac{\pi-2}{4}$.

2. Given $f(x)=\begin{cases}-x+1 \; for -\pi < x \le 0\\ x+1, \; for \; 0 \le x \le \pi\end{cases}$, find a Fourier series for $f(x)$ and hence show that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$$

3. Find the Fourier series expansion of the function $f(x)=\begin{cases}\pi x \; for \; 0 \le x < 1\\ 0 \quad x=1\\ \pi(x-2) \; for \; 1 < x < 2.\end{cases}$ in the interval [0,2].

4. Find the Fourier series expansion of the function $f(x)=e^{-4x}$ in the interval [-2,2].

5. Find the Fourier series expansion of the function $f(x)=x-x^2$ in the interval $-1< x \le 1$.

6. Find the half range sine series for the function $f(x)=x^2$ for $0 < x < \pi$.

7. Find the half range cosine series for the function $f(x)=2x-1$ for $0 < x < 1$.

8. Find the Fourier series expansion of the function $f(x)=\begin{cases}0 \text{ for } 0 \le x < l\\ x \text{ for } l \le x < 2l\end{cases}$ in the interval $[0,2l]$.

Answers. (3) $f(x)=2(\sin\pi x-\frac{\sin 2\pi x}{2}+\frac{\sin 3\pi x}{3}-...)$,

(4) $a_0=(e^8-e^{-8})/8$, $a_n=(e^8-e^{-8})\ 8\ (-1)^n/(64+pi^2\ n^2)$, $b_n=(e^8-e^{-8})\ n*\ pi\ (-1)^n/(64+pi^2\ n^2)$

(5) $f(x)=-\frac{1}{3}+\frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi x}{(2n+1)^2}+\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi x}{(2n+1)}$,

(6) $f(x)=\frac{2}{\pi}\left\{(\pi^2-4)\sin x-\frac{\pi^2\sin 2x}{2}+\frac{1}{3}(\pi^2-\frac{4}{3^2})\sin 3x-.....\right\}$

(7) $f(x)=-\frac{8}{\pi^2}\left(\cos\pi x+\frac{\cos 3\pi x}{3^2}+\frac{\cos 5\pi x}{5^2}+....\right)$

(8) $f(x)=\frac{3l}{4}+\frac{l}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi x/l}{(2n+1)^2}$

$$-\frac{l}{\pi}\left\{\frac{3\sin\pi x/l}{1}+\frac{\sin 2\pi x/l}{2}+\frac{3\sin 3\pi x/l}{3}+\frac{\sin 4\pi x/l}{4}+....\right\}$$