Department of Mathematics

15B11MA301 **Probability and Random Processes**
Tutorial Sheet 2 **B.Tech. Core**

Discrete and Continuous Random Variables (1D)

1. Distinguish with suitable examples between
   a) Discrete random variable and continuous random variable.
   b) Probability density function and cumulative distribution function.

2. The probability density function of a random variable X is $f(x) = kx(1-x),\ 0 < x < 1.$ Then find (i) k and (ii) a number 'b' such that $P(X<b)=P(X>b)$.

3. A fair coin is tossed 3 times and let X be difference of the number of heads and the number of tails. Find (a) the probability mass function, (b) the cumulative distribution function of X.

4. A random variable X has the probability distribution defined as

X:	1	2	3	4	5	6
P(X):	0.04	0.15	0.37	0.26	0.11	0.07

   Find (i) P(X Odd/X<5) (ii)P(X<5/X Odd) (iii) P(X=4/X is not equal to 3)

5. Consider the function $f(x) = \begin{cases} C(x^2-2x), & 0 < x < 5/2 \\ 0 & \textit{elsewhere.} \end{cases}$, where C is any constant. Could $f(x)$ be a probability density function? Justify your answer.

6. The probability density function of a random variable X is given by
$$f(x) = \begin{cases} A(1+x), -1 < x \le 0 \\ A(1-x), 0 < x < 1 \\ 0, \text{ elsewhere.} \end{cases}$$
   Find (a) the value of A and plot $f(x)$, (b) the distribution function F(x), (c) the point c such that P[X > c] =P[X < c]/2.

7. A continuous random variable X is defined as $f(x) = \begin{cases} ax+bx^2 & 0 < x < 1 \\ 0 & \textit{Otherwise} \end{cases}$ where a and b are constants. If E[X]= 0.6, find (i) $P\left\{X < \frac{1}{2}\right\}$ (ii) Var (X).

8. The cumulative distribution function of a random variable X is given by $F(x) = 1-e^{-2x^2},\ x \ge 0.$ Find (a) P(0 < X < 3) (b) P(X >1) (c) P(X =5).

9. A player tosses 3 fair coins. He wins Rs.1500 if three heads appear Rs.1000 if two heads appear and Rs.500 if one head appear. On the other hand he loses Rs.2000 if three tails appear. Find the value of the game to the player. Is the game favorable to the player?

10. A box contains 10 items of which 4 are defective. A person draws 3 items form the box. Determine the expected number of defective items he has drawn.

11. A system can function for a random amount of time X. If the density of X is given (in units of months) by $f(x) = \begin{cases} Cxe^{-x/2} & x > 0 \\ 0 & x \le 0 \end{cases}$, where C is any constant. What is the probability that the system functions for at least 5 months?