Course Name: Electrical Science- II
Course Code: 15B11EC211

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Problem Solving Strategies for First Order (RC/RL) Circuits with Constant Sources

The four-step process [1,2] for solving for the capacitor voltage or inductor current is

1. Determine the capacitor's initial steady state voltage $V_C(0-)$ or inductor's initial steady state current $I_L(0^-)$ before the switch is thrown.

2. After the switch is thrown, determine the Thévenin parameters (V_{OC} or $V(\infty)$ & Rth) or Norton's parameters (Isc or $I(\infty)$ & R_N) and the inverse time constant $1/\tau$.

3. Put the voltage or current into solution form and interpret the solution:

$$V_c(t) = V_{oc} + [V_c(0^+) - V_{oc}]e^{-t/R_{th}C}$$

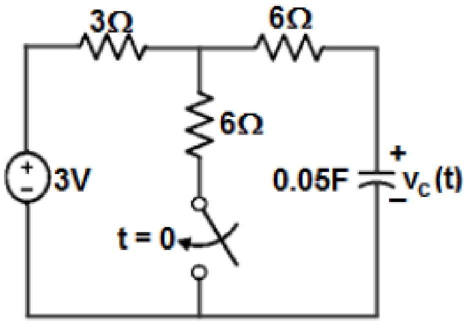
or

$$I_L(t) = I_{sc} + [I_L(0^+) - I_{sc}]e^{-tR_{th}/L}$$

4. To solve the circuit problem for the appropriate parameter, go the circuit before the RC/ RL circuit was converted into a Thévenin /Norton circuit and relate unknown parameter to $V_C(t)$ / $I_L(t)$ using KCL or KVL.

Problem-1

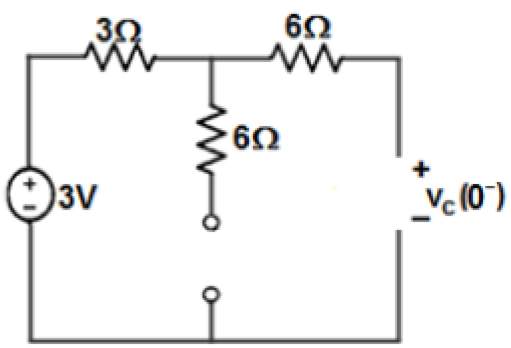
Q. 1 The circuit is in steady state before the switch closes at t=0s. Determine the capacitor voltage Vc(t) for t>0s [1]



Sol. Step 1: Determine Vc (0-)=Vc(0+),

for t<0 circuit looks like as below, from the circuit

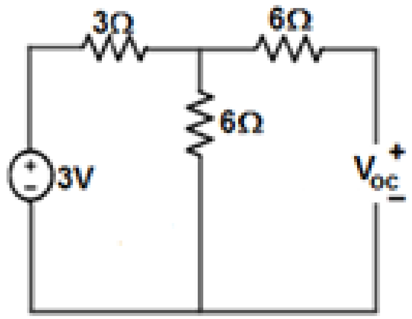
Vc(0-)=Vc(0+)=3V



Problem-1 Continues

Step 2: Determine Thevenin parameters and time constant for t>0

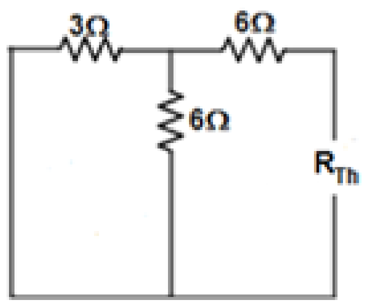
For Voc or Vc(∞), circuit is



$$V_{6\Omega} = \frac{6}{6+3}3 = 2V$$

$$V_{oc} = 2V$$

For Rth, circuit is



$$R_{th} = 6 + 6 \| 3 = 8\Omega$$

Time constant τ is

$$\tau = R_{th}C = 8(0.05) = 0.4s$$

Step 3: Solution of capacitor voltage

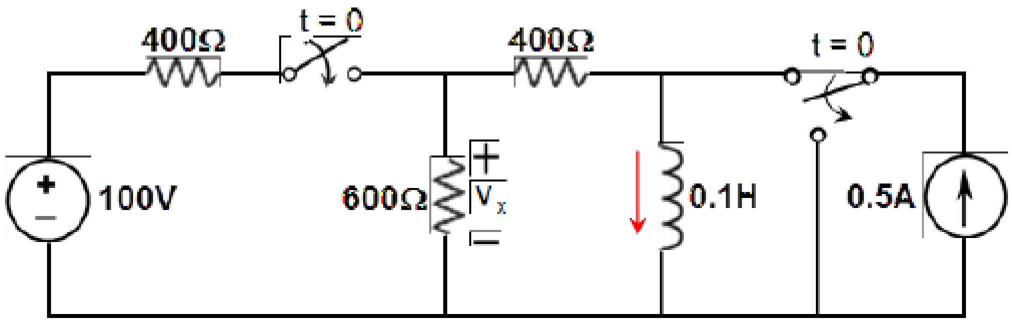
$$V_c(t) = \begin{cases} 3V & t < 0 \\ 2 + e^{-2.5t}V & t \geq 0 \end{cases}$$

$$V_c(t) = V_{oc} + [V_c(0^+) - V_{oc}]e^{-t/R_{th}C} = 2 + e^{-2.5t}V$$

Problem-2

Q. 2 The circuit is in steady state before the switch closes at t=0s. Determine the inductor current $I_L(t)$ and $V_X(t)$ for t>0s [2]

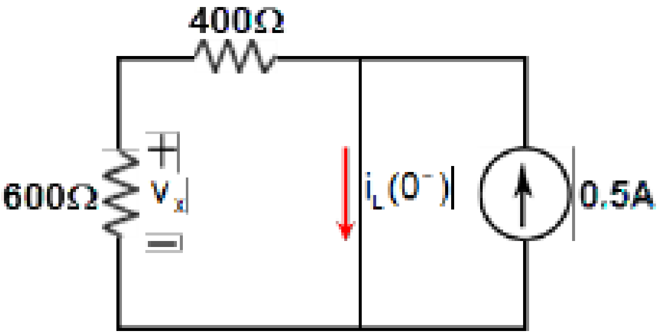
Sol.



Sol. Step 1: Determine $I_L(0^-)=I_L(0^+)$,

for t<0 circuit looks like as below, from the circuit

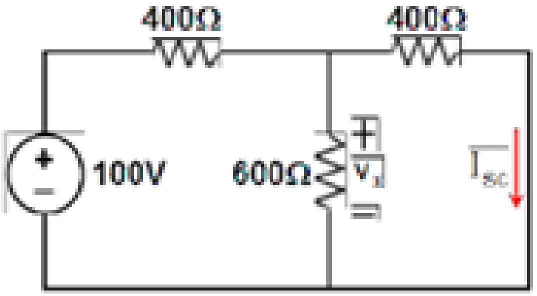
$$I_L(0^-)= I_L(0^+) = 0.5A$$



Problem-2 Continues

Step 2: Determine Norton parameters and time constant for t>0

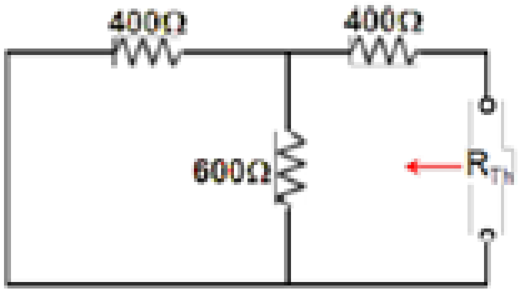
For I_{sc} or $I_L(\infty)$, circuit is



$$V_{400\Omega} = \frac{600 \,||\, 400}{600 \,||\, 400 + 400}(100V) = 37.5V$$

$$I_{sc} = I_{400\Omega} = \frac{V_{400\Omega}}{400} = 93.8mA$$

For Rth, circuit is



$$R_{th} = 400 + 400 \,||\, 600 = 640\Omega$$

Time constant τ is

$$\tau = \frac{L}{R_{th}} = \frac{0.1}{640} = 1.562 \times 10^{-4}$$

Step 3: Solution of inductor current

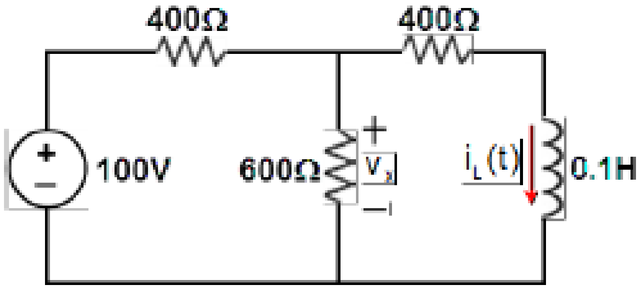
$$I_L(t) = \begin{cases} 500mA & t < 0 \\ 93.8 + 406.2e^{-6400t} mA & t \geq 0 \end{cases}$$

Problem-2 Continues

Step 4: Solving the circuit for V_X (t), apply KVL in RHS part

$$V_{600\Omega} = V_{400\Omega} + V_L(t) = 400 I_L(t) + \frac{1}{10}\frac{dI_L}{dt}$$

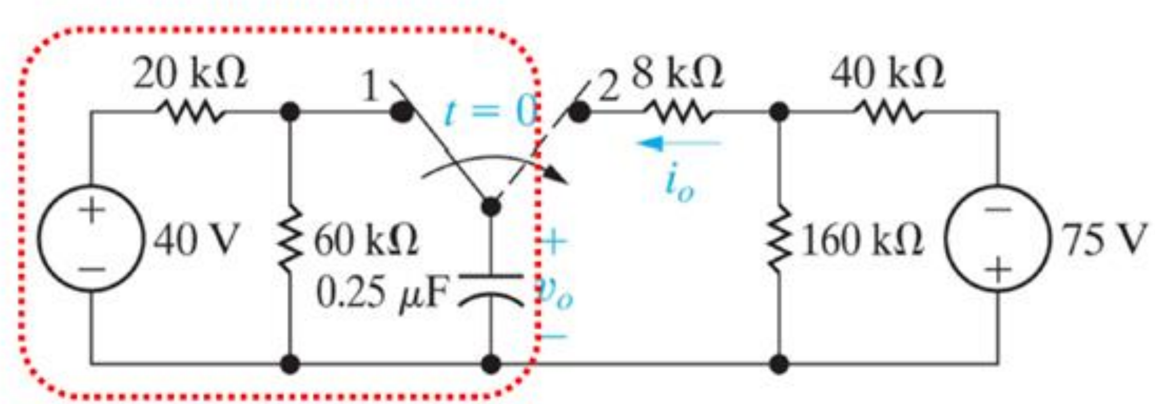
On substituting the value of I_L(t) in above equation, the voltage can be given as



$$V_{600\Omega} = V_X(t) = 37.5 - 97.5e^{-6400t} V$$

Practice Problem

Q. Find $i_o(t)$ and $v_o(t)$ in the given circuit for t>0s [1]



$$v_o(t) = -60 + 90e^{-100t} V$$

$$i_o(t) = -2.25e^{-100t} mA$$

References

[1] R.C. Dorf and J. A. Svoboda, “Introduction to Electric Circuits”, 9th ed, John Wiley & Sons, 2013.

[2] W. H. Hayt, J. E. Kemmerly and S. M. Durbin, “ Engineering Circuit Analysis,” New York: McGraw-Hill,2006