Department of Mathematics

15B11MA211 **Mathematics-2**

Tutorial Sheet 9 **B. Tech. Core**

Classification of PDE and Variable Separable Method

1. Classify the following partial differential equations:
    - $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$
    - $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} = 0$
    - $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial u}{\partial x} = 0$
    - $\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} - 5\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$

2. Solve the following partial differential equations by Method of Separable variable
    - $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial t} = 0$, where $u(x,0) = 4e^{-x}$.
    - $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, where $u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\left(\frac{n\pi x}{l}\right)$.
3. Solve the partial differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation, subject to the following conditions:
    I. u is not infinite for $t \to \infty$
    II. $\left(\frac{\partial u}{\partial x}\right)_{x=0,l} = 0$
    III. $u = lx - x^2$ for $t = 0$ between $x = 0, x = l$.
4. A rectangular plate with insulated surface is $10\ cm$ wide and so long compared to its width that it may be considered infinite in length without introducing an appropriate error. If the temperature of the short edge at $y = 0$ is given by
$u(x,0) = \begin{cases} 5x, & 0 < x \le 5 \\ 5(10-x), & 5 \le x < 10 \end{cases}$
And the two long edges $x = 0, x = 10$ as well as the short edge at infinity are kept at $0^o C$, prove that the steady state temperature distribution at any point (x,y) is given by
$$u(x,y) = \frac{200}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\left(\frac{(2n-1)\pi x}{10}\right) e^{\left(\frac{(2n-1)\pi y}{10}\right)}.$$

Answers:

1.

- Parabolic PDE
- Elliptic PDE
- Hyperbolic PDE
- Elliptic if $x > 0$; Hyperbolic if $x < 0$; Parabolic if $x = 0$.

2.

1. $u(x,t) = 4\,e^{\left(-x+\left(\frac{3}{2}\right)t\right)}$.
2. $u(x,y) = \dfrac{\sin\left(\frac{n\pi x}{l}\right)\sinh\left(\frac{n\pi y}{l}\right)}{\sinh\left(\frac{n\pi a}{l}\right)}$.

3. $u(x,t) = \frac{l^2}{6} - \frac{l^2}{\pi^2}\sum_{\boldsymbol{n=1}}^{\infty}\cos\left(\frac{2n\pi x}{l}\right)\ \boldsymbol{e}^{-\left(\frac{\boldsymbol{4\alpha^2 n^2\pi^2}}{\boldsymbol{l^2}}\right)\boldsymbol{t}}$.