Practice sheet CO4

1) $f(t) = 32/(t+4)^3$: $(t>0)$ is in years

(i) $R(t) = \int_t^{\infty} \frac{32}{(t+4)^3} dt = 32 \left[\frac{(t+4)^{-3+1}}{-3+1} \right]_t^{\infty} = \frac{16}{(t+4)^2}$

(ii) $\lambda(t) = \frac{f(t)}{R(t)} = \frac{32}{(t+4)^3} \times \frac{(t+4)^2}{16} = \frac{2}{(t+4)}$

(iii) $MTTF = \int_0^{\infty} R(t) dt = 16 \left[\frac{(t+4)^{-2+1}}{-2+1} \right]_0^{\infty} = 4$

Ans 2) $\lambda(t) = \frac{1}{20\sqrt{t}}$ per year, where t is in years
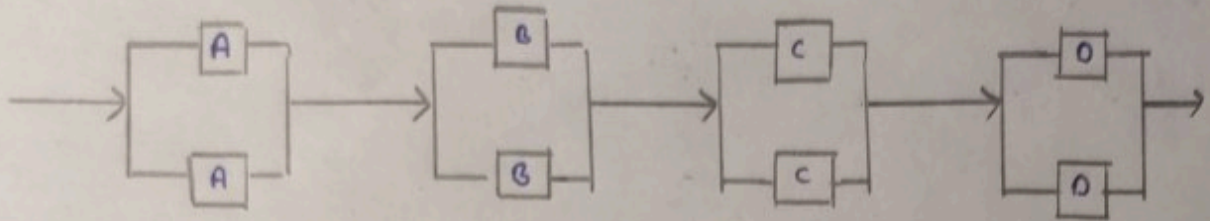
(a) $t_0 = 1$ year

$R(t_0) = \exp\left[-\int_0^1 \frac{1}{20\sqrt{t}} dt \right] = \exp\left[-\frac{1}{20} \times 2\left[\sqrt{t}\right]_0^1 \right]$

(b) $\frac{R(1)}{R(1/12)} = \exp\left[\left\{ -\int_0^1 \frac{1}{20\sqrt{t}} dt \right\} - \left\{ -\int_0^{1/12} \frac{1}{20\sqrt{t}} dt \right. \right.$

$= \exp\left[-\frac{1}{10} - \left\{ -\frac{1}{20} \times 2\sqrt{\frac{1}{12}} \right\} \right]$

$= \exp\left[\frac{1}{20\sqrt{3}} - \frac{1}{10} \right] = 0.93$

3)

$A = 0.90$ $B = 0.95$ $C = 0.96$ $D = 0.98$
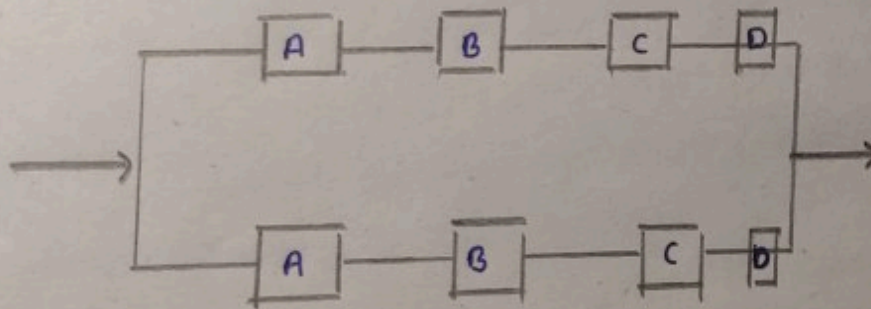
Reliability of the system =

$$\left[1- (0.10)^2\right]\left[1-(0.05)^2\right]\left[1- (0.04)^2\right]\left[1- (0.02)^2\right]$$

$$= \left[0.99\right]\left[0.9975\right]\left[0.9984\right]\left[0.9996\right]$$

$$= 0.9855$$

4)

$A = 0.95$ $B = 0.99$ $C = 0.90$ $D = 0.96$

$$\text{Reliability} = 1- \left[1- (0.95 \times 0.99 \times 0.90 \times 0.96)\right]^2$$

$$= 1- \left[1- 0.812592\right]^2$$

$$= 0.9649$$