Probability and Random Processes (15B11MA301)

Lecture-26

(Content Covered: Parallel-series and Series-Parallel Configuration)

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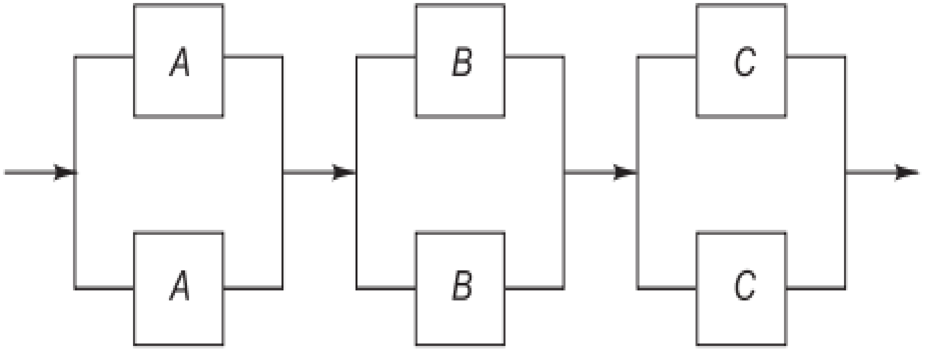
Contents

Parallel-Series Configuration

- A system in which *m* subsystems are connected in series where each subsystem has *n* components connected in parallel.
- Consider the figure below, where 3 subsystems are connected in series and each subsystem comprise of 2 components in parallel.



- The parallel series configuration is also termed as low-level redundancy.

- If R is the reliability of the individual component, the reliability of each of the subsystems (comprising of n components in parallel) is given by

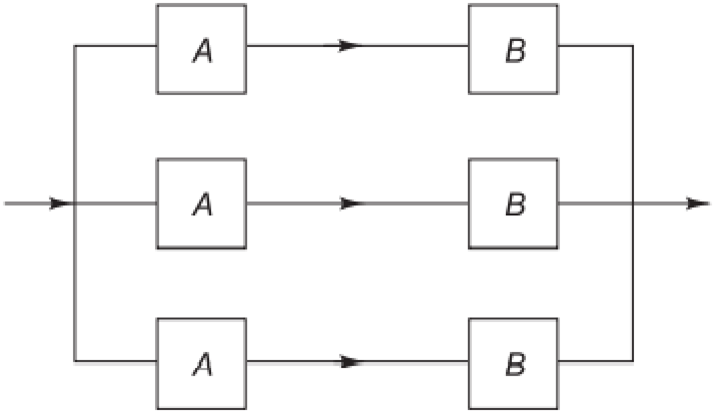
$$= (1 - (1 - R)^n) \quad \text{(In fig. } n = 2\text{)}$$

- Since m subsystems are connected in series (in fig. m=3), the system reliability for low level redundancy is given by

$$R_{LOW} = \{1 - (1 - R)^n\}^m$$

Series-Parallel Configuration

- A system in which *m* subsystems are connected in parallel where each subsystem has *n* components connected in series.
- Consider the figure below, where 3 subsystems are connected in parallel and each subsystem comprise of 2 components in series.



- The series parallel configuration is also termed as high-level redundancy.

- If R is the reliability of the individual component, the reliability of each of the subsystems (comprising of *n* components in series) is given by

$$= (R)^n \quad \text{(In fig. } n = 2\text{)}$$

- Since *m* subsystems are connected in parallel (in fig. *m*=3), the system reliability for high level redundancy is given by

$$R_{High} = 1 - (1 - R^n)^m$$

Example Six identical components with constant failure rates are connected in (a) high level redundancy with 3 components in each subsystem (b) low level redundancy with 2 components in each subsystem. Determine the component MTTF in each case, necessary to provide a system reliability of 0.90 after 100 hours of operation.

Solution: Let λ be the constant failure rate of each component. Then, $R = e^{-\lambda t}$, for each component.

For high level redundancy,

$$R_s(t) = 1 - \left[1 - \left(R(t)\right)^3\right]^2$$

$$= 1 - \left(1 - e^{-3\lambda t}\right)^2$$

$$R_s(100) = 1 - (1 - e^{-300\lambda})^2 = 0.90$$

$$(1 - e^{-300\lambda})^2 = 0.1$$

$$1 - e^{-300\lambda} = 0.31623$$

$$e^{-300\lambda} = 0.68377$$

$$300\lambda = 0.38013$$

Therefore, MTTF of each component = $\frac{1}{\lambda} = \frac{300}{0.38013}$ = 789.2 hours.

For low level redundancy

$$R_s(t) = \left[1 - \left(1 - R(t)\right)^2\right]^3$$

$$= \left[1 - \left(1 - e^{-\lambda t}\right)^2\right]^3$$

$$R_s(100) = \left[1 - \left(1 - e^{-100\lambda}\right)^2\right]^3 = 0 \cdot 90$$

Therefore, $1 - \left(1 - e^{-100\lambda}\right)^2 = 0 \cdot 96549$

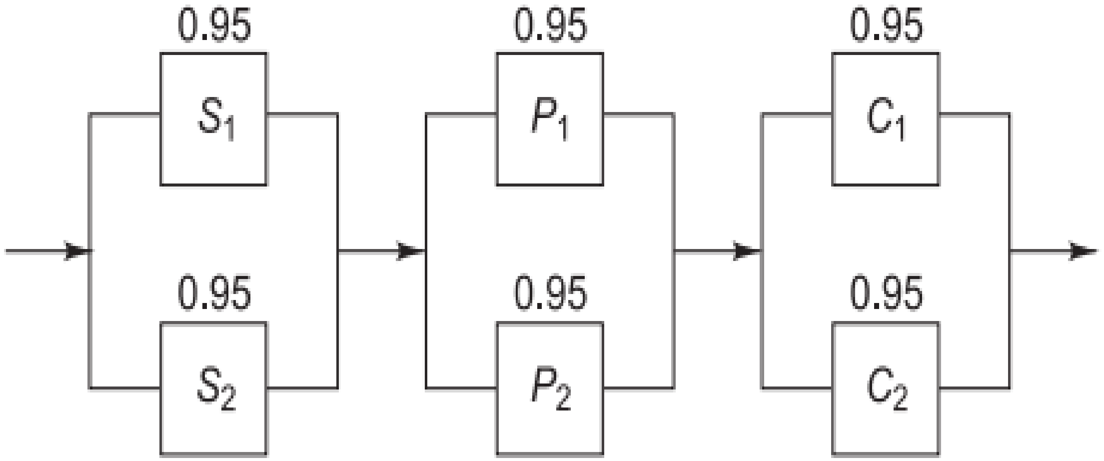
$$\left(1 - e^{-100\lambda}\right)^2 = 0.03451$$

$$e^{-100\lambda} = 0 \cdot 81423$$

$$100\lambda = 0 \cdot 20551$$

Therefore, MTTF of each component = $\frac{1}{\lambda} = \frac{100}{0.20551}$ = 486.6 hours.

Example A signal processor has a reliability of 0.90. Because of the lower reliability a redundant signal processor is. to be added. However, a signal splitter must be added before the processors and a comparator must be added after the signal processors. Each of the new components has a reliability of 0.95. Does adding a redundant signal processor increase the system reliability?



Solution: $R_1 = R(first\ subsystem) = R(S_1) \cdot R(P_1) \cdot R(C_1)$

$$= 0 \cdot 95 \times 0 \cdot 90 \times 0 \cdot 95$$

$$= 0.81225$$

$$R_2 = R(second\ subsystem) = R(S_2) \cdot R(P_2) \cdot R(C_2)$$

$$= (0 \cdot 95)^3$$

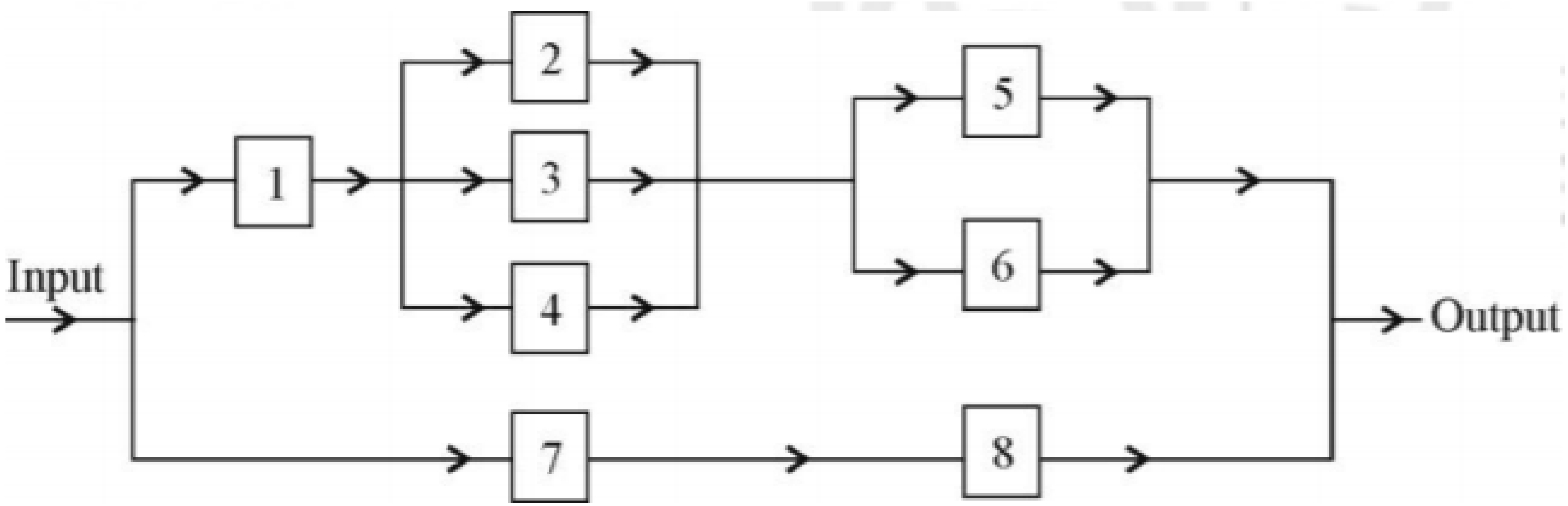
$$= 0.857375$$

$$\text{R(system)} = 1 - (1 - R_1)(1 - R_2)$$

$$= 1 - 0.0268 = 0 \cdot 9732$$

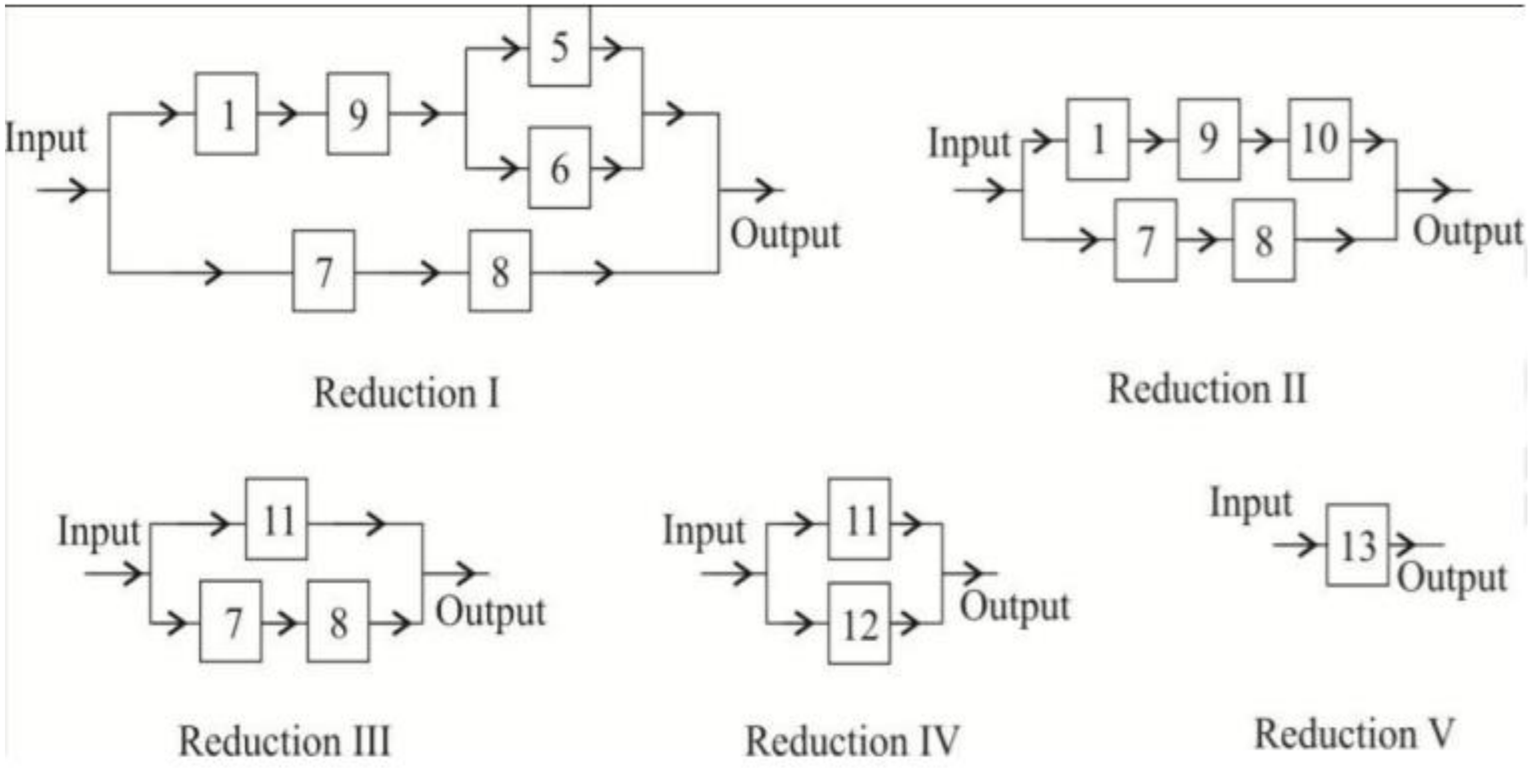
The addition of a redundant signal processor increases the reliability.

Example : Evaluate the reliability of the system composed of 8 components for which the reliability block diagram is shown in figure for a mission of 100 hours. Assume that all components are independent and the reliability of each component is given for a mission of 100 hours as follows: R1 = 0.80, R2 = 0.75, R3 = 0.50, R4 = 0.65, R5 = 0.76, R6 = 0.60, R7 = 0.95, R8 = 0.90, where Ri denotes the reliability of the component i, (i = 1, 2, 3, …,8).



Solution: The components of the given system are connected in both series and parallel configuration. So it is a mixed system. To evaluate the reliability of this mixed system, we have to break the system into subsystems such that all components of a subsystem are either in series or in parallel. This can be done as follows:

- **Reduction I:** Combine the components 2, 3, 4 in parallel configuration to form an equivalent component 9
- **Reduction II:** Combine the components 5, 6 in parallel configuration to form an equivalent component 10
- **Reduction III:** Combine the components 1, 9, 10 in series configuration to form an equivalent component 11
- **Reduction IV:** Combine the components 7, 8 in series configuration to form an equivalent component 12
- **Reduction V:** Finally, combine the components 11 and 12 in parallel configuration to form an equivalent component 13
- The component 13 represents the complete system.



The component 13 represents the complete system.

We are given that

R1 = 0.80, R2 = 0.75, R3 = 0.50, R4 = 0.65, R5 = 0.76, R6 = 0.60, R7 = 0.95, R8 = 0.90

where Ri denotes the reliability of the given component i, (i = 1, 2, 3,…, 8).

Similarly, if R9 to R13 denote the reliabilities of the equivalent components 9 to 13, then using results of reliabilities, we have :

- $R_9 = 1 - (1 - R_2)(1 - R_3)(1 - R_4)$ $\quad$ $(parallel\ configuration)$

  $= 1 - (1 - 0 \cdot 75)(1 - 0 \cdot 50)(1 - 0 \cdot 65)$ = 0.9562

- $R_{10} = 1 - (1 - R_5)(1 - R_6)$ $\quad$ $(parallel\ configuration)$

  $= 1 - (1 - 0 \cdot 76)(1 - 0 \cdot 60)$ = 0.904

- $R_{11} = R_1 \cdot R_9 \cdot R_{10}$ $\quad$ $(series\ configuration)$

  $= (0.80)(0.9562)(0.904)$ = 0.6915

- $R_{12} = R_7 \cdot R_8$ $\quad$ $(series\ configuration)$

  $= (0.95)(0.90)$ = 0.855

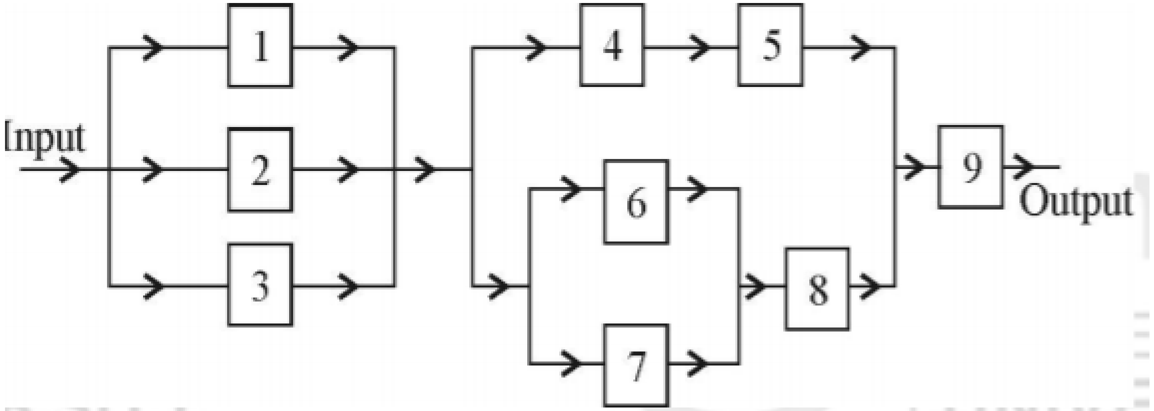
- $R_{13} = 1 - (1 - R_{11})(1 - R_{12})$ $\quad$ $(parallel\ configuration)$

  $= 1 - (1 - 0.6915)(1 - 0 \cdot 855)$ = 0.9533

Hence, the reliability of given system is 0.9533.

Practice Questions

1. Evaluate the reliability of the system for which the reliability block diagram is shown in figure, for a mission of 500 hours. Assume that all components are independent. The reliability of each component is given below for a mission of 500 hours: R1 = 0.40, R2 = 0.30, R3 = 0.60, R4 = 0.80, R5 = 0.85, R6 = 0.60, R7 = 0.70, R8 = 0.95, R9 = 0.96.



(Ans. 0.7568032)

References

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