

**Jaypee Institute of Information Technology, Noida**  
**Test – 2 Examination (Odd 2021) Solutions Key**  
**Course Name: Theoretical Foundation of Computer Science**  
**Course Code: 15B11CI212**

**Note: Question 1- 10 are of one-line answer and 11-13 are of detailed answer type. Please write your answer in sequence of questions.**

**In PART-A one mark answers, if direct answer provided / partial correct – 0.5 Mark, explanation+steps+answer – 1 Mark is allotted.**

**In PART-B, if direct answer provided / partial correct – 1 mark, steps + partial correct/ final answer = 2 marks, correct steps + correct answer + suitable explanation = full marks are allotted.**

**Q1. [CO2, 1 Marks]** Let  $(Z, *)$  is a group with  $x * y = x + y + 1, \forall x, y \in Z$ . Then find the inverse of  $x$ .

Sol.  $-x-2$

**Q2. [CO2, 1 Marks]** Show that  $\{0, 2, 4, 6\}, + 8$  is a subgroup of the group  $[Z_8, + 8]$ .

Sol. Closure holds

+	0	2	4	6
0	0	2	4	6
2	2	4	6	0
4	4	6	0	2
6	6	0	2	4

$0 \in \{0, 2, 4\}; 0^{-1} = 0, 4^{-1} = 4$ , and 2 and 6 are inverses of each other.

**Q3. [CO2, 1 Marks]** What is the order of the group  $G$  with all  $2 \times 2$  matrices  $\begin{bmatrix} w & x \\ y & z \end{bmatrix}$  where  $wz - xy \neq 0$ , and  $w, x, y$ , and  $z$  are integers of modulo 3 relative to matrix multiplication.

Sol.

**Solution:** For the first row  $(a, b)$  of a matrix in  $G$   $a$  and  $b$  could be anything in  $Z_3$ , but we must exclude the case  $a = 0$  and  $b = 0$ . Hence  $(3 \times 3) - 1$  possibilities for the first row. The second row should be not a multiple of the first row. Hence for the second row  $(3 \times 3) - 3$  possibilities. Hence the number of elements in  $G$  is  $8 \times 6 = 48$ .

**Q4. [CO2, 1 Marks]** The values of x,y and z are given as 0,0 and 1, find the minterm and maxterm.

Sol. Minterm-  $x'y'z$  and maxterm  $x+y+z'$

**Q5. [CO4, 1 Marks]** Vipin wish to invite his 7 relatives on one function. Calculate the number of ways Vipin can send the invitation letter. if he has 4 courier boys to carry the invite?

Sol.  $16384 (4^7)$

**Q6. [CO4, 1 Marks]** Find out how many words 2 consonants and 2 vowels can be formed out of 7 consonants and 4 vowels?

Sol.  $7c2 * 4c2 = 126$

**Q7. [CO3, 1 Marks]** Write the following statements using propositions and logical connectives

- If I am late and I have money, I'll take a cab to work.

**Solution:**

- P: I am late, Q: I have money, R: I'll take a cab to work,  $(P \wedge Q) \rightarrow R$

**Q8. [CO3, 1 Marks]** Consider the following predicates:

P (x, y):  $x > y$

Q (x, y):  $x \leq y$

If the universe of discourse is the real numbers, give the truth value of the following propositions:

- (a)  $\forall x \exists y P(x, y)$
- (b)  $\exists x \forall y Q(y, x)$

**Solution:**

- (a) True
- (b) False

**Q9. [CO3, 1 Marks]** Given the following predicates on the set P of all people who ever lived

Parent (x, y): x is the parent of y

Mother (x, y): x is the mother of y

and Universe of Discourse is set of all people.

write in the following statement using propositional variable and connectives

- Every Person has two maternal grand parents

**Solution:**

(a)  $\forall p \exists x, y, z: \text{Mother}(x, p) \wedge \text{Parent}(y, x) \wedge \text{Parent}(z, x) \wedge y \neq z$

**Q10. [CO3, 1 Marks]** Find inverse, converse and contrapositive of following statement:

$|AB|^2 + |BC|^2 = |AC|^2$  whenever ABC is right-angled triangle

**Solution:**

P: ABC is right-angled triangle

Q:  $|AB|^2 + |BC|^2 = |AC|^2$

Inverse: If ABC is not a right-angled triangle then  $|AB|^2 + |BC|^2 \neq |AC|^2$

Converse: If  $|AB|^2 + |BC|^2 = |AC|^2$  then ABC is right-angled triangle

Contrapositive: If  $|AB|^2 + |BC|^2 \neq |AC|^2$  then ABC is not a right-angled triangle

**Q11. [CO2, 3 Marks]** Assume that @ is an associative binary operation on  $\{1, a, b, c, d\}$ .

Complete the following table to define a group with identity 1

@	1	a	b	c	d
1	1				
a			c		1
b		c	d		
c		d		a	
d				b	

**Sol.**

@	1	a	b	c	d
1	1	a	b	c	d
a	a	b	c	d	1
b	b	c	d	1	a
c	c	d	1	a	b
d	d	1	a	b	c

**Q12. [CO2, 3 Marks]** Find the minimized expression of following equation using K-Map.

$$\sum m(0, 1, 3, 8, 9, 13) + \sum d(2, 5, 7, 10, 15)$$

**Sol.**

$$\bar{a}\bar{b} + \bar{c}d + \bar{b}\bar{d}$$

**Q13. [CO3, 4 Marks]** Check the validity of the following argument.

- Every living thing on Earth is a plant or an animal. John has a cat which is not a plant. All animals have heart. Therefore, John's cat has a heart

**Solution:**

Universe of discourse is every living thing on Earth.

P(x): x is a plant

A(x): x is animal

H(x): x has heart

Has (x, y): x has y

Premises:  $\forall x (P(x) \vee A(x))$ .....1

Has (John, cat)  $\wedge \sim P(\text{cat})$ .....2

$\forall x A(x) \rightarrow H(x)$  .....3

Conclusion: H(cat)

P(cat)  $\vee$  A(cat)                      apply universal Instantiation on 1.....5

$\sim P(\text{cat})$                               apply simplification on 2.....6

A(cat)                                      apply Disjunctive Syllogism on 5 & 6.....7

A(cat)  $\rightarrow$  H(cat)                      apply universal Instantiation on 3.....8

H(cat)                                      apply Modus ponens on 7 & 8.

Hence, the argument is valid