Department of Mathematics

15B11MA211 Mathematics-II **B.Tech. II Semester**

Tutorial Sheet 6 (Legendre's and Bessel's Functions)

1. Show that

(i) $\int_{-1}^{1} x^{2m} P_n \, dx = 0$ when n is odd.

(ii) $P_{2m}(x)$ has only even degree terms.

(iii) x = 0 is a root of $P_n(x)$ when n is odd.

(iv) $P_4(x) = (35x^4 - 30x^2 + 3)/8$

(v) $\sum_{r=0}^{m} \frac{(-1)^r (2n-2r)!}{r!(n-r)!(n-2r)!} = 2^n$

2. Using the ratio test show that the series of $J_v(x)$ converges for all values of x.

3. Show the following

(i) $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$,

(ii) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

(iii) $\cos x = J_0(x) - 2J_2(x) + 2J_4(x) - \ldots\ldots$

(iv) $\int_0^x t^n J_{n-1}(t)dt = x^n J_n(x)$

(v) $J_0'' = (J_2 - J_0)/2$