

**Tutorial Sheet 4 (Alternating Series and Power Series)**

1. Test the series  $\sum (-1)^{n-1} \frac{1}{n^p}$  for (a) convergence (b) absolute convergence.
2. Show that the series  $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$  converges conditionally.
3. Discuss the convergence of the series  

$$1 - 2x + 3x^2 - 4x^3 + \dots$$
4. Show that the series  $x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$  converges if and only if  $-1 \leq x \leq 1$ .
5. Test for the uniform convergence for the series  

$$1 + a \cos x + a^2 \cos 2x + a^3 \cos 3x + \dots a^n \cos nx + \dots$$
6. Find the radius of convergence and region of convergence for the following series:  

$$\sum n(x+2)^n / 3^{n+1}$$

Ans:

1. (a) convergent for all  $p \geq 0$   
 (b) absolute convergent for  $p > 1$
- 3 convergent for  $|x| < 1$ , divergent for  $|x| \geq 1$
- 5 uniform convergent for  $|a| < 1$ .
- 6 3,  $-5 < x < 1$