Digital Systems

18B11EC213

Module 1: Boolean Function Minimization Techniques and Combinational Circuits-7

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Standard Representations For Logical Functions

- The logical (Boolean) functions are expressed in terms of logical variables.
- Two standard forms for the representations of logical functions:

  Sum-of-Products (SOP)

  Product-of-Sums (POS)
- Literals: a variable on its own or in its complemented form. Examples: x, x', y, y'
- Product Term: a single literal or a logical product (AND) of several literals.

  Examples: x, x.y.z', A'.B, A.B

Cont..

- Sum Term: a single literal or a logical sum (OR) of several literals.

  Examples: x, x+y+z', A'+B, A+B

- Sum-of-Products (SOP) Expression: a product term or a logical sum (OR) of several product terms.

  Examples: x, x+y.z', x.y'+x’.y.z, A.B+A'.B'

- Product-of-Sums (POS) Expression: a sum term or a logical product (AND) of several sum terms.

  Examples: x, x.(y+z'), (x+y').(x'+y+z), (A+B).(A'+B')

Cont..

- Every logical (Boolean) expression can either be expressed as SOP or POS expression.

Examples:

SOP: $F1 = x'.y + x.y' + x.y.z$

POS: $F2 = (x + y').(x' + y).(x' + z')$

Minterm and Maxterm

- Consider two binary variables x, y.
- Each variable may appear as itself or in complemented form as literals in a Boolean expression, i.e., x, x' and y, y'
- For two variables, there are four possible combinations with the AND operator, namely:

  x'.y', x'.y, x.y', x.y

- These product terms are called the minterms.
- In general, *n* variables can give 2^n minterms.

Cont..

- In a similar fashion, a maxterm of *n* variables is the sum of *n* literals from the different variables.

  Examples: $x'+y'$, $x'+y$, $x+y'$, $x+y$
- In general, *n* variables can give 2^n maxterms.
- All the literals (or their complemented forms) should participate in the operations.

Cont..

- The minterms and maxterms of two variables are denoted by m0 to m3 and M0 to M3 respectively.

Minterms				Maxterms	
x	y	term	notation	term	notation
0	0	x'.y'	m0	x+y	M0
0	1	x'.y	m1	x+y'	M1
1	0	x.y'	m2	x'+y	M2
1	1	x.y	m3	x'+y'	M3

- Each minterm is the complement of the corresponding maxterm and vice versa.

Example: m2 = x.y'

m2' = (x.y')' = x' + (y')' = x'+y = M2

Canonical Forms

- If each term (i.e., minterm and maxterm) in the SOP and POS forms/expressions contains all the literals (either in normal or complemented form), then these forms/expressions are known as the canonical SOP and canonical POS forms.

  Example: For 3 variables a, b, c

  Canonical SOP form:

  $Y = abc + abc' + ab'c' + a'bc$

  Canonical POS form:

  $Y = (a+b+c).(a+b'+c).(a+b+c').(a+b'+c')$

Cont..

- Canonical Form: Sum of Minterms

Example: Given a canonical SOP form

$Y(a, b, c) = abc + abc' + ab'c' + a'bc$

We can write Y as

$$Y = m_7 + m_6 + m_4 + m_3$$
$$= m_3 + m_4 + m_6 + m_7 = \Sigma m(3, 4, 6, 7)$$

where $a'bc = m_3$

$ab'c' = m_4$

$abc' = m_6$

$abc = m_7$

Cont..

Example: Obtain the expressions of F1, F2 and F3 from the following truth table.

x	y	z	F1	F2	F3
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	1	0

Cont..

Obtain Sum-of-Minterms by summing the minterms of the function where the result is a 1.

$F1 = x.y.z' = m6$

$F2 = x'.y'.z + x.y'.z' + x.y'.z + x.y.z' + x.y.z$

$= m1 + m4 + m5 + m6 + m7$

$= \Sigma m(1, 4, 5, 6, 7)$

$F3 = x'.y'.z + x'.y.z$

$+ x.y'.z' + x.y'.z$

$= m1 + m3 + m4 + m5$

$= \Sigma m(1, 3, 4, 5)$

x	y	z	F1	F2	F3
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	1	0

Cont..

- Canonical Form: Product of Maxterms

- Maxterms are the sum terms.
- Boolean functions can be expressed as Products-of-Maxterms.
- For Boolean functions, the maxterms of a function are the terms for which the result is 0.

Cont..

Example: Given a canonical POS form:

$$Y(a, b, c) = (a+b+c)\ (a+b+c')\ (a+b'+c)\ (a'+b+c')$$

We can write Y as

$$Y = M_0 \,.\, M_1 \,.\, M_2 \,.\, M_5$$

(Since in maxterm, a is represented by 0 and a' by 1)

where $(a+b+c) = M_0$

$(a+b+c') = M_1$

$(a+b'+c) = M_2$

$(a'+b+c') = M_5$

Cont..

Example: Obtain the expressions of F2 and F3 from the following truth table.

x	y	z	F1	F2	F3
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	1	0

F2 = M0 . M2 . M3 = ΠM(0, 2, 3) = (x+y+z).(x+y'+z).(x+y'+z')

F3 = M0 . M2 . M6 . M7 = ΠM(0, 2, 6, 7)

= (x+y+z).(x+y'+z).(x'+y'+z).(x'+y'+z')

Cont..

- From the previous example (truth table), let us write the function F2 as the SOP form:

$$F2 = \Sigma m(1, 4, 5, 6, 7)$$

The complement function of F2 is:

$$F2' = m0 + m2 + m3$$

$$= \Sigma m(0, 2, 3)$$

x	y	z	F2	F2'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

- This means that the functions F2 and F2' jointly contain all possible minterms.

Cont..

From the previous slide, we obtained

$$F2' = m0 + m2 + m3$$

Therefore,

$$F2 = (m0 + m2 + m3)'$$

$= m0' \,.\, m2' \,.\, m3'$ DeMorgan's theorem

$= M0 \,.\, M2 \,.\, M3$ Since $mx' = Mx$

$= \Pi M(0, 2, 3)$

=> $\boxed{F2 = \Sigma m(1, 4, 5, 6, 7) = \Pi M(0, 2, 3)}$

- Every Boolean function can be expressed as either Sum-of-Minterms or Product-of-Maxterms.

Conversion of Canonical Forms

- Sum-of-Minterms to Product-of-Maxterms

  Example: Given F1 (A, B, C) = $\sum m(3, 4, 5, 6, 7)$ SOP

  then F1 (A, B, C) = $\prod M(0, 1, 2)$ POS

  (Replace minterm indices with indices not already used)

- Product-of-Maxterms to Sum-of-Minterms

  Example: Given F2 (A,B,C) = $\prod M(0, 3, 5, 6)$ POS

  then F2 (A,B,C) = $\sum m(1, 2, 4, 7)$ SOP

  (Replace maxterm indices with indices not already used)

Cont..

- Sum-of-Minterms of F $\Rightarrow$ Product-of-Maxterms of F'

  Example: Given F1 (A, B, C) = $\sum m(3, 4, 5, 6, 7)$

  then F1' (A, B, C) = $\prod M(3, 4, 5, 6, 7)$

  (use the same indices as in F1)

- Product-of-Maxterms of F $\Rightarrow$ Sum-of-Minterms of F'

  Example: Given F1 (A, B, C) = $\prod M(0, 1, 2)$

  then F1' (A, B, C) = $\sum m(0, 1, 2)$

  (use the same indices as in F1)

Cont..

- Sum-of-Minterms of F $\Rightarrow$ Sum-of-Minterms of F'

  Example: Given $F1(A, B, C) = \sum m(3, 4, 5, 6, 7)$

  then $F1'(A, B, C) = \sum m(0, 1, 2)$

  (list the indices not already used in F)

- Product-of-Maxterms of F $\Rightarrow$ Product-of-Maxterms of F'

  Example: Given $F1(A, B, C) = \prod M(0, 1, 2)$

  then $F1'(A, B, C) = \prod M(3, 4, 5, 6, 7)$

  (list the indices not already used in F)

References

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- R. P. Jain, *Modern Digital Electronics*, 4th ed., Tata McGraw-Hill Education, 2009.