Counting :-

we have two rule
Sum rule
Product rule

Sum rule ⇒ (Disjoint events)

If an event E_1 can happen in m ways and another event E_2 can happen in n ways then E_1 or E_2 can happen in $m+n$ ways.

Ex:- A - 5 f.R.
B - 4
C - 3 f.R.

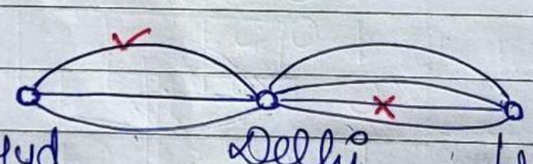
How many ways we can select a project?

Ans:- 12 ways

(Disjoint events means object must not be common)

Product rule ⇒ (Independent events)

Ex:-



No. of ways from hyd. to luck via delhi

Ans: 12 ways.

(one root from hyd to Delhi) 1 ⟶ 4 (then 4 from delhi to luck.)
3 ⟶ ?

$3 \times 4 = 12$.

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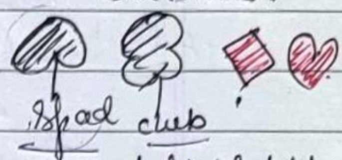
If an event E_1 can happen in <u>m</u> ways. another event E_2 can happen in <u>n</u> ways. then E_1 followed by E_2 can happen <u>m*n</u> ways.

Q ①: From a pack of 52 cards, how many ways we can select
1. Spade or diamond
2. King or Ace
3. Heart or king.

52 cards.



divided into 4 suits

each suit.

A
2
3
|
10
J
Q
K

$\begin{array}{r}13\\ \times 4\\ \hline 52\end{array}$

Sol^n: 52 cards, $\frac{\cancel{52}\ \cancel{26}\ 13}{\cancel{4}\ \cancel{2}}$.

①. $\underbrace{\text{Spade}}_{13}$ or $\underbrace{\text{diamond}}_{13}$.

$13+13=26.$

② $\underbrace{\text{king}}_{4}$ or $\underbrace{\text{Ace}}_{4}$

$4+4=8$

③ $\underbrace{\text{Heart}}_{13}$ or $\underbrace{\text{king}}_{4}$.

there is one king with heart (subtract King with heart)

so, neglect one. Result:

$(4-1) \Rightarrow 3$

$13+3=16.$

Always ahead...

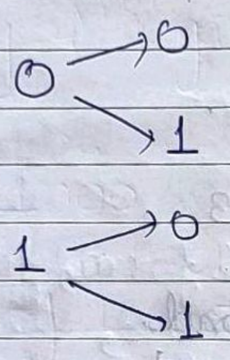
Im

Q.2. How many binary string of length 2 are there?

Sol^n: 4

$\Rightarrow$ 2 2 $\Rightarrow 2*2 = 4$ choices.

choice either 0 or 1



00
01
10
11

Q.3. bs of length 3 = 8

0 0 0
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1

* **Result:-**

No. of b.s. of length $n = \boxed{2^n}$

1 2 3 ... n

$$\underbrace{2 \quad 2 \quad 2 \cdots\cdots 2}_{n\text{-times}} \Rightarrow 2 * 2 * \cdots * 2 \Rightarrow 2^n.$$

* NO. of b.s of length 8 which start with "0".

$$\underbrace{1 \quad 2 \quad 2 \cdots\cdots 2.}_{7\text{-times}} = 1 \times 2^7.$$

Q. NO. of b.s of length 16 which start with "10".

Ans: 2^{14}

Q. NO. of b.s of length 21 which start with "110"

Ans: 2^{18}

length 24 starting with "110" $\Rightarrow 2^{21}$.

$\Rightarrow$ { IPV4 (32-bit addressing) } $\Rightarrow$ 4 byte

	net id	host id		
class A	0			

	net id		host id	
Class B	10			

	net id			host id
class C	110			

	net ids	host ids
class A	(2^7-2)	$(2^{24}-2)$
class B	(2^{14})	$(2^{16}-2)$
class C	(2^{21})	(2^8-2)

Q. How many 4 digit positive integers are there
(may or may not given)

$[1000 \quad 9999]$

$[lb \quad ub]$

No. of elements

$= ub - lb + 1$

$= 9999 - 1000 + 1$

$= 9000$

$\cancel{0}$

$\square\,\square\,\square\,\square \Rightarrow 9 * 10 * 10 * 10$

9 10 10 10 $\Rightarrow 9000.$

Q. How many four digit even numbers have no repeated digits.

a) 2240 b) 2296 c) 2620 d) 4836

sol^n:

$\square\,\square\,\square\,\square$

9 9 8 6 $\Rightarrow \cancel{9 * 8 * 10 * 10}$

due to no repetion cod^n zero already restricted

$\cancel{0}$			0
9			

or

$\cancel{0}$			

or

$\cancel{0}$			4

or

$\cancel{0}$			6

or

$\cancel{0}$			8

			0
9	8	7	1

$\cancel{0}$			2,4,6,8
8	8	7	4

(zero allowed here)

$$(9 \times 8 \times 7 \times 1) + (8 \times 8 \times 7 \times 4) = 2296.$$

Q. How many non-negative integers $< 10^5$ are there?

Sol^n:

Positive Integers
1, 2, 3, ...

Non negative integers
0, 1, 2, 3, ____

due to 0 or 0 or ... or 0 it we get all 4 digit no's

10	10	10	10	10

{max. five digit numbers}.

Ans: 10^5 (Ans)

{No. less than $< 5 \Rightarrow$ are 5}
(0, 1, 2, 3, 4)
Same Concept.

{Allowing leading zeros the five digit no. can be reduced to 4 digit, 3 digit, 2 digit and 1 digit no.'s}.

Q. How many are even? (In above question)

Sol^n:

(0, 2, 4, 6, 8)

10	10	10	10	5

$(\Rightarrow 5 \times 10^4 = 50000)$

Q. How may are odd? (In above question).

⇒ Same (5×10^4).

Indirect Counting

odd ⇒ Total − Even
$\Rightarrow 10^5 - (5 * 10^4)$
$\Rightarrow 10^4(10-5)$
$\Rightarrow 5 * 10^4$.

Q. How many non-negative integers < 10^5 contain the digit "2".

Sol^n: Not contain '2' ⇒

9	9	9	9	9

Contain = Total − Not contain
$(10^5 - 9^5)$.

Q. How many palindrome of length 5 are there using ~~Eng~~ Eng alphabets.

Soln.

26 26 26 1 1 $\Rightarrow (26)^3$.

Q. How many 5 digit palindroms are there?

~~0~~

9 10 10 1 1 $\Rightarrow$ 900.

Q. How many are even?

~~0~~, 2, 4, 6, 8.

$\Rightarrow$ 400

4 10 10 1 4

Q. How many are odd?

$\Rightarrow$ 500

(Total - Even)

5 10 10 1 1

Q. National flag need to be designed having 4 colors using any of the 6 colors with repetitions allowed but not consecutive repetitions.

sol^n:

6	5	5	5

$\Rightarrow 6 \times 5^3 \Rightarrow 750.$

Q. Two girls picked 10 roses, 14 sunflowers, 15 Daffodils. How many way they can divide flowers among them.

sol^n:

10 14 15

10 roses

G_1	G_2
10	0
9	1
8	2
7	3
6	4
⋮	
1	9
0	10

11 ways * 15 ways * 16 ways

10
14
15 - 15
~~39~~ (19)

11
15
16

$\boxed{(11 \times 15 \times 16) \text{ ways}}$

2640 ways Ans.

Q. How many positive divisors are there for 300.

2	300
2	150
5	75
5	15
3	3
	1

$2^2 \times 5^2 \times 3^1$

$3 \times 3 \times 2$

$9 \times 2 \Rightarrow 18$

* Prime factorization:-

$$\begin{array}{r|l} 2 & 300 \\ \hline 2 & 150 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \Rightarrow 2^2 \times 3^1 \times 5^2$$

$\boxed{\text{No. of positive divisors} = (2+1) \times (1+1) \times (2+1) = 18}$

$2^a \times 3^b \times 5^c \mid 300$.

$2^1 3^1 5^1 \mid 300$ (divide)

$2^0 3^0 5^0 \mid 300$ (divide)

$2^3 3^1 5^1 \mid 300$ X (Not).

$\Rightarrow$ $0 \leq a \leq 2$

$0 \leq b \leq 1$

$0 \leq c \leq 2$.

a, b, c are integer.

a	b	c
3	2	3

$\Rightarrow 18$.

Q. How many odd positive divisors are there for 300.

$\cancel{2^2} 3^1 5^2 \mid 300 \Rightarrow 3^1 * 5^2$

$(1+1) \times (2+1)$

$\Rightarrow 2 \times 3 \Rightarrow 6$.

remove the even part for odd divisor.

even×odd → even
odd×even → even
odd×odd → odd

No. of even divisors $= 18 - 6 \Rightarrow 12$.

Q. How many positive divisors are there for $2^2 \times 3^2 \times 5^2 \times 9^2$

Sol^n.

$$\cancel{2 \times 3 \times 5 \times 9}$$

$$2^2 \times 3^2 \times 5^2 \times \underbrace{(3^2)^2}_{\text{prime factorization}}$$

$$\Rightarrow 2^2 \times 3^6 \times 5^2$$

$$\Rightarrow 3 \times 7 \times 3 \Rightarrow 63.$$

* **floor of x**

$\lfloor x \rfloor$ = $\boxed{\text{Largest integer} \leq x.}$

$\lfloor 3.99 \rfloor = 3$

$\lfloor 2 \rfloor = 2$

* **ceil of x**

$\lceil x \rceil$ = $\boxed{\text{Smallest integer} \geq x.}$

$\lceil 1.00000001 \rceil = 2$

$\lceil 3 \rceil = 3$

$\lceil 3.9999 \rceil = 4$.

Q. How many positive integers ≤ 100 are

(i) Divisible by 2

(ii) Divisible by 3

(iii) Divisible by 2 and 3.

(iv) divisible by 4 & 6.

Sol^n: (i). $\frac{100}{2} + \cancel{\frac{100}{2^2}} + \cancel{\frac{100}{2^3}} \Rightarrow 50$

not taken · not taken

$\Rightarrow \left\lfloor \frac{100}{2} \right\rfloor \Rightarrow 50.$

(ii) $\left\lfloor \frac{100}{3} \right\rfloor = 33$ (iii) $\left\lfloor \frac{100}{L.C.M(2,3)} \right\rfloor = \left\lfloor \frac{100}{6} \right\rfloor = 16$

(iv) $\left\lfloor \frac{100}{L.C.M(4,6)} \right\rfloor = \left\lfloor \frac{100}{12} \right\rfloor = 8$

Q. what is the exponent of 11 in the prime factorization of 300!.

Sol^n: $300! = 300 \times 299 \times 298 \times \_\_ \times 242 \times \_\_ \times 121 \times \_\_ 99 \_\_ \times 22 \times \_\_ 11 \times \_\_ \times 1.$

$242 = 2 \times 11^2$, $121 = 11^2$

$\left\lfloor \frac{300}{11} \right\rfloor = 27 \Rightarrow 27+1+1 \Rightarrow 29.$

2nd method.

$\Rightarrow \left\lfloor \frac{300}{11} \right\rfloor + \left\lfloor \frac{300}{11^2} \right\rfloor + \left\lfloor \frac{300}{11^3} \right\rfloor \Rightarrow$ zero.

$\Rightarrow 27 + 2$
$\Rightarrow 29.$

Q. Exponent of 13 in prime factorizati of 300!.

$\left\lfloor \frac{300}{13} \right\rfloor + \left\lfloor \frac{300}{13^2} \right\rfloor \Rightarrow (23 + 1) \Rightarrow 24.$ A

Q. Exponent of 3 in p.f of 300!

$$\Rightarrow \left\lfloor \frac{300}{3} \right\rfloor + \left\lfloor \frac{300}{3^2} \right\rfloor + \left\lfloor \frac{300}{3^3} \right\rfloor + \left\lfloor \frac{300}{3^4} \right\rfloor + \ldots$$

$$\Rightarrow 100 + 33 + 11 + 3 + 1$$

$$\Rightarrow 148 \text{ Ans}$$

* { In prime factorization we only take power of prime no. ex. here 9 are not allowed }

Q. How many ways we can select a man and a woman who are not married from 10 married couples.

Sol^n:

10 couples
10 men 10 women

$M_1 \; W_1$
$M_2 \; \cancel{W_2}$
$\vdots$
$M_{10} \; W_{10}$

M	W
10	9

$\Rightarrow 90.$

GATE

Q. n couples are invited to a party with the condition that every husband should be accompanied by his wife however wife need not be accompanied by her ~~hus~~ husband. No. of different gatherings possible at the party.

(a) $^{2n}C_n(2^n)$ (b) 3^n (c) $\frac{(2n)!}{2n}$ (d) $^{2n}C_n$

Sol^n:

$M_1 \quad w_1$

$M_2 \quad w_2$

$\vdots$

$M_n \quad w_n$

M	w
$\downarrow$	$\downarrow$
$^nC_1$	n

H	w
✓	✓
×	✓
×	×
✓	×

1	2	3	— — —	n
$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$
3	3	3		3

$\rightarrow (3)^n$.