Department of Mathematics

Special Semester 2020-21

Probability and Random Processes

Tutorial Sheet 4 **B.Tech. Core**

2-D Random Variables

1. Define the following: (a) two dimensional random variable (b) marginal and conditional probability distributions.
2. If X denotes the number of kings and Y denotes number of aces when two cards are drawn at random without replacement from a deck of well shuffled pack of 52 cards, find (i) The joint probability distribution of (X, Y). (ii) The marginal distribution

   $(iii)\ P(X = 2/Y = 1)$ $(iv)\ P(X < 2/0 < Y < 2)$ $(v)\ P(1 \le X \le \frac{2}{Y} = 0,2)$

3. Let the joint pdf of a random variable (X,Y) is defined as

   $$f(x,y) = k(xy + y^2), \qquad 0 \le x \le 2, 0 \le y \le 1.$$

   Find $(i)\ the\ value\ of\ k$ $(ii)\ P(X > 1)$ $(iii)\ P(X + Y < 1)$ $(iv)\ P(X < 1, Y > 1/2)$ $(v)\ f_X(x), f_Y(y)$. Also test whether X and Y are independent?

4. The pdf of (X,Y) be defined as

   $$f(x,y) = \left(\frac{1}{4}\right) e^{-|x|+|y|}, \qquad -\infty \le x \le \infty, -\infty \le y \le \infty$$

   Are X and Y independent? Find the probability that $X \le 1\ and\ Y \le 0$.

5. Random variable (X ,Y) have a joint probability density function $f(x,y) = (2x + y)/27$, where x and y can assume only integer values 0, 1, 2. Find the conditional distribution of $Y\ for\ X = x$.

6. Two ideal dice are thrown. Let X1 be the score on the first die and X2 the score on the other die. Let Y denote the maximum of $X_1\ and\ X_2\ i.e. max(X_1, X_2)$. (a) Write down the joint distribution of Y and X_1, (b) Find E(Y) and Var(Y).

7. Let $f(x,y) = \begin{cases} 21x^2y^2, & 0 \le x < y \le 1. \\ 0 & elsewhere \end{cases}$ be the joint pdf X, Y. Find the conditional mean and variance of X given Y=y and 0<y<1.