Probability and Random Processes (15B11MA301)
Tutorial Sheet: 3 [C201.1]

1. The probability density function of a random variable X is $f(x) = kx(1-x)$, $0 < x < 1$. Then find (i) k and (ii) a number *'b'* such that $P(X<b)=P(X>b)$
   (Ans k=6, b=1/2)
2. A fair coin is tossed 3 times and let X be difference of the number of heads and the number of tails. Find (a) the probability mass function, (b) the cumulative distribution function of X.
   (Ans P(X=-3)=1/8, P(X=-1)=3/8, P(X=1)=3/8, P(X=3)=1/8
   CDF: 0, 1/8, 4/8, 7/8, 1)
3. A random variable X has the probability distribution defined as

X	1	2	3	4	5	6
P(X)	0.04	0.15	0.37	0.26	0.11	0.07

   Find (i) P(X Odd | X<5) (ii)P(X<5|X Odd) (iii) P(X=4 | X is not equal to 3)

Ans: (i): 41/82=1/2 (ii) 41/52 (iii) 26/63

4. Consider the function $f(x)=\begin{cases} C(x^2-2x), 0<x<5/2 \\ 0 \quad \text{elsewhere} \end{cases}$, , where C is any constant. Could $f(x)$ be a probability density function? Justify your answer.

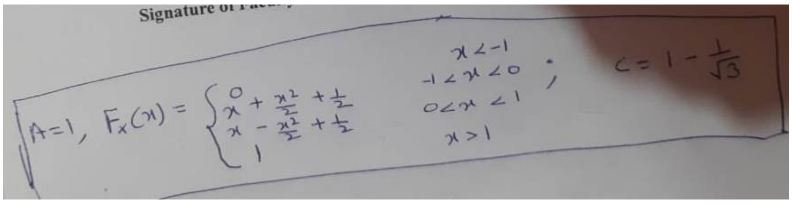
   Ans: Not possible. No value of C for which f(x) is always positive.

5. The probability density function of a random variable X is given by

$$f(x)=\begin{cases} A(1+x), -1<x\le 0 \\ A(1-x), 0<x<1 \\ 0 \quad \text{elsewhere} \end{cases}.$$

   Find (a) the value of A and plot $f(x)$, (b) the distribution function F(x), (c) the point c such that P[X > c] =P[X< c]/2

   Ans:



6. A continuous random variable X is defined as $f(x)=(ax+bx^2)$, $0<x<1$, 0 otherwise. If E[X]=0.6, then find (i) P[X<0.5], (ii) variance of X.

   Ans: a=3.6, b=-2.4, P=0.35, var=0.06

   The cumulative distribution function of a random variable X is given by $F(x)=(1-e^{-2x^2})$, $x>0$. Find (a) P(0 <X < 3) (b) P(X >1) (c) P(X=5).

   Ans: (a) $1-e^{-18}$ (b) e^{-2} (c) Discuss in class.

7. A random variable X has the following probability function:

x	0	1	2	3	4
$p(x)$	k	$3k$	$5k$	$7k$	$9k$

find (a) k (b) cdf (c) mean and variance of X and 4X+3 (d) $P(X<3)$ and $P(0<X<4)$.

Ans: k=1/25, mean (X): 70/25 Var (X): 850/625 Mean(4X+3)=280/25+3, Var(4X+3)=16*850/25 (d): 9/25, 15/25

8. Four unbiased coins are tossed and let X be the number of heads obtained. Write the probability mass function of X and find $P(X>2)$.

[Ans.: 5/16]

9. A random variable X takes the values 1, 2, 3 and 4 such that $P(X=1)=P(X=2)=2P(X=3)=3P(X=4)$.Write the probability distribution of X and find

(i) $P(X>2)$, (ii) $P(1<X<4//X>2)$, (iii)$P(X= 1 \text{or } 2)$.

[Ans.: (i) 5/17 (ii) 3/5, (iii) 12/17]

10. A random variable X is equally probable to take even or odd integral values from 1 to 6 and has the following probability mass function:

X=x	1	2	3	4	5	6
P(X=x)	*k*	2k	*2k*	3k	*?*	*k/2*

Find (i) value of *k,* (ii) $P(X=5)$ (iii) $P(X< 4/X > 2)$ (iv) F(2), (v) F(x)

[Ans.: (i) 1/11, (ii) 5/22 (iii) 4/13, (iv) 3/11]