

**Jaypee Institute of Information Technology, Noida**  
**Test – 1 Examination (Odd 2021)**  
**Course Name: Theoretical Foundation of Computer Science**  
**Course Code: 15B11CI212**  
**Maximum Marks: 20**  
**Maximum Time: 1Hour**

**Q1.[CO1, 1 Marks]** If  $n(A) = 20$  and  $n(B) = 30$  and  $n(A \cup B) = 40$  then  $n(A \cap B)$  is?

**Answer:** 10

**Explanation:** By using the formula we can calculate  $n(A \cap B)$ ,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$n(A \cap B) = 20 + 30 - 40$$

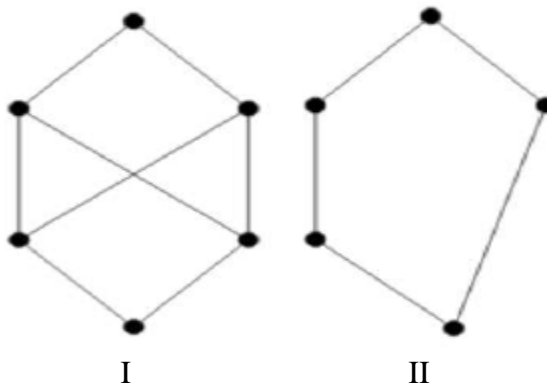
$$\text{So, } n(A \cap B) = 10$$

**Q2.[CO1, 1 Marks]** The number of transitive closure exists in the relation  $R = \{(0,1), (1,2), (2,2), (3,4), (5,3), (5,4)\}$  where  $\{1, 2, 3, 4, 5\} \in A$  is\_\_\_\_\_.

**Answer:**  $\{(0,1), (0,2), (1,2), (2,2), (3,4), (5,3), (5,4)\}$

Need to write set to get full marks

**Q3.[CO1, 1 Marks]** Which of the following represent a lattice?



Solution: II Only

**Q4.[CO1, 1 Marks]**If  $A, B, C, D$  are sets such that  $|A| = |B|$  and  $|C| = |D|$ , then following statements are true or false?

I:  $|P(A)| = |P(B)|$

II:  $|A \times C| = |B \times D|$

**Answer: Both correct**

**Q5.[CO1, 1 Marks]**Let the players who play cricket be 12, the ones who play football 10, those who play only cricket are 6, then the number of players who play only football are \_\_\_\_\_, assuming there is a total of 16 players.

**Answer: 4**

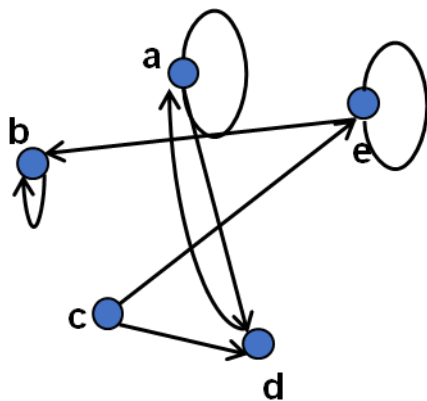
**Q6. [CO1, 1 Marks]**The cardinality of the power set of  $\{0, 1, 2, \dots, 9\}$  is \_\_\_\_\_.

**Answer: 1024**

**Q7. [CO1, 1 Marks]**Let  $f(x) = x^2 + 2$ ,  $g(x) = (f \circ f)(x)$ ,  $h(x) = (f + f)(x)$  then  $g(2)$  and  $h(2)$  are equal to

**Answer: 38, 12**

**Q8. [CO1, 1 Marks]**For the given relation  $R$  shown below, symmetric closure is



**Answer:  $\{(a, a), (a, d), (b, b), (c, d), (c, e), (d, a), (e, b), (e, e), (b, e), (e, c), (d, c)\}$**

**Need to write the edge pairs to get full marks**

**Q9. [CO1, 1 Marks]**Pick the correct statements

- I. If a relation  $R$  is both symmetric and transitive, then  $R$  is reflexive.
- II. If a relation  $R$  is asymmetric, then  $R$  is both anti-symmetric and irreflexive.

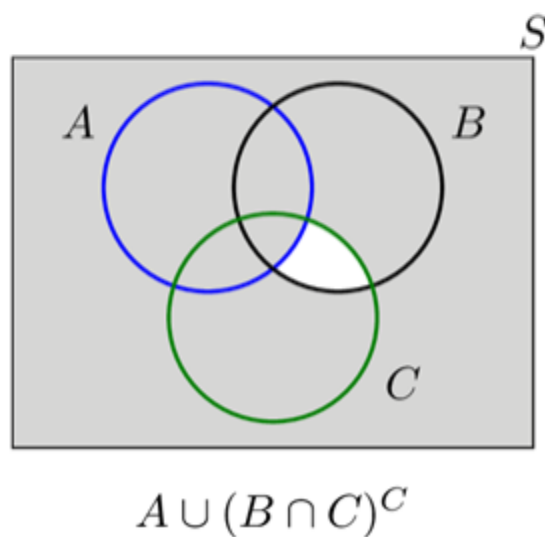
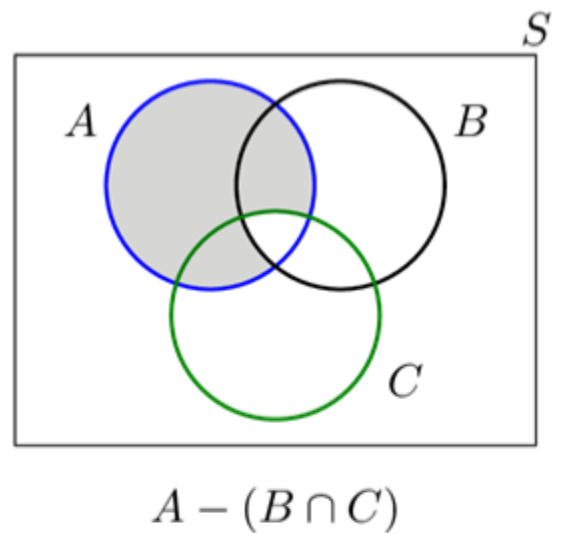
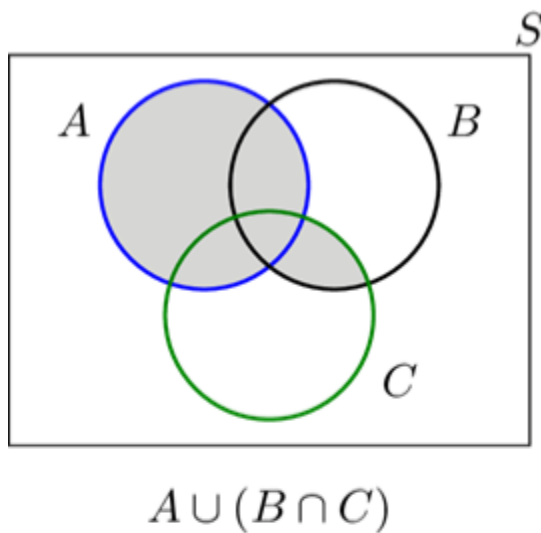
**Answer: Only II correct**

**Q10.[CO1, 1 Marks]** Consider this function  $f(x) = x^2 + 1$  from  $\mathbb{R}$  to  $\mathbb{R}$ , then identify that this function is bijective or not:

Answer: Yes function is not bijective. (need to show how it is not bijective)

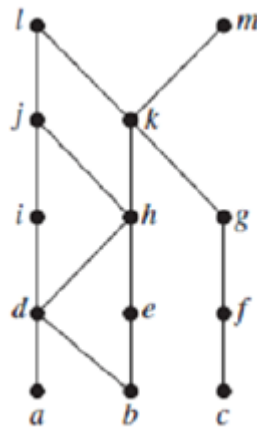
**Q11. [CO1, 3 Marks]** Let  $A, B, C$  be three sets, draw Venn diagram for each of the following sets and shade the area representing the given set.

- a.  $A \cup (B \cap C)$
- b.  $A - (B \cap C)$
- c.  $A \cup (B \cap C)'$



Q12. [CO1, 3 Marks] For the given Hasse diagram, Find the maximal, minimal, greatest, least, LB, glb, UB and lub for the following subsets

- (i) {d, k, f} (ii) {b, h, f} (iii) {a, b, c}



Solution

Set	Greatest	Least	Maximum	Minimum	UB	LB	LUB	GLB
{d, k, f}	{k}	NIL	{k}	{d, f}	{k, l, m}	NIL	{k}	NIL
{b, h, f}	NIL	NIL	{h, f}	{b, f}	{l, m}	NIL	{k}	NIL
{d}	{d}	{d}	{d}	{d}	{d, h, i, j, k, l, m}	{d, a, b}	{d}	{d}
{a, b, c}	NIL	NIL	{a, b, c}	{a, b, c}	{k, l, m}	NIL	{k}	NIL
{l, m}	NIL	NIL	{l, m}	{l, m}	NIL	{a, b, c, d, e, f, g, h, k}	NIL	{k}

Q13.[CO1, 4 Marks] Consider the following recurrence relation:

$$a_n = 9a_{n-1} - 27a_{n-2} + 27a_{n-3}$$

- Find General Solution
- Find solution with initial conditions:  $a_0 = 3, a_1 = 6, a_2 = 27$

**Solution:**

$$a_n = 9a_{n-1} - 27a_{n-2} + 27a_{n-3}$$

a) characteristic Equation

$$x^3 = 9x^2 - 27x + 27$$

$$\Rightarrow x^3 - 9x^2 + 27x - 27 = 0$$

$$\Rightarrow (x-3)^3 = 0$$

$$\text{Roots} = 3, 3, 3$$

root = 3 with multiplicity 3

General Solution

$$a_n = (A + Bn + Cn^2) 3^n$$

Ans.

b)  $a_0 = 3 \quad a_1 = 6 \quad a_2 = 27$

$n=0$

$$(A + B \times 0 + C \times 0) 3^0 = 3$$

$$\boxed{A = 3}$$

$n=1$

$$(A + B \times 1 + C \times 1) 3 = 6$$

$$A + B + C = 2$$

$$B + C = 2 - 3$$

$$B + C = -1 \quad \text{--- (i)}$$

$n=2$

$$(A + 2B + 4C) 3^2 = 27$$

$$A + 2B + 4C = 3 \quad \text{---}$$

$$2B + 4C = 0$$

$$B = -2C \quad \text{--- (ii)}$$

Substituting  $B = -2C$  in eq(i)

$$-2C + C = -1$$

$$-C = -1$$

$$\boxed{C = 1}$$

$$B = -2 \times 1 = -2$$

$\therefore$  Solution

$$\boxed{a_n = (3 - 2n + n^2) 3^n}$$