TFCS End Examination (ODD 2021): Solutions Key: If answers are provided without proper explanation / step-by-step approach / reasoning, no marks will be provided.

Q.1

Solution:

For \$1 coin=a_{n-1} ways
For \$1 note=a_{n-1} ways
For \$5 note=a_{n-5} ways

a) $a_n = 2a_{n-1} + a_{n-5} \quad for\ n \geq 5$
b) $a_0 = 1 \qquad a_1 = 2 \qquad a_2 = 2 \times 2 = 4 \qquad a_3 = 8 \qquad a_4 = 16$
c) Solve the equation in part a

$a_{10} = 1217$

Q.2

Solution:

Solution: $U(14) = \{1, 3, 5, 9, 11, 13\}$

$3 \equiv 3(\mod 14)$, $3^2 \equiv 9(\mod 14)$, $3^3 \equiv 13(\mod 14)$, $3^4 \equiv 11(\mod 14)$, $3^5 \equiv 5(\mod 14)$, $3^6 \equiv 1(\mod 14)$

Hence $U(14) = \langle 3 \rangle$.

Now we will do similar calculation for 5. Indeed $5 \equiv 5(\mod 14), 5^2 \equiv 11(\mod 14)$, $5^3 \equiv 13(\mod 14)$, $5^4 \equiv 9(\mod 14)$, $5^5 \equiv 3(\mod 14)$, $5^6 \equiv 1(\mod 14)$.

Hence $U(14) = \langle 5 \rangle$ i.e. $U(14)$ is a cyclic group.

$\langle 11 \rangle = \{1, 11, 9\}$. So $5 \notin \langle 11 \rangle$. Hence $\langle 11 \rangle \neq U(14)$

Q.3

Solution:

(a) Sol: You can pick the 2 aces, 2 kings in C(4, 2) · C(4, 2) = 6 · 6 = 36 ways. You can pick the remaining card in any of 52 − 8 = 44 ways so the answer is 36 · 44 = 1, 584.

(b) Sol: There are 13 ways to pick the first denomination. Then are then C(4, 3)ways to pick 3 cards of that denomination. There are 12 ways to pick the second denomination and then C(4, 2) ways to pick 2 cards of that denomination.

Hence there are 13 · C(4, 3) · 12 · C(4, 2) = 13 · 4 · 12 · 6 = 3, 744

(c) Sol: There is exactly 1 way to pick a royal flush in each suit so there are 4 of them.

Q4:

(a) Sol: E→ 5; F→ 2; R→ 1; V→ 1; S→ 1; C→ 2; N→ 1. Hence there are 5 + 2 + 1 + 1 + 1 + 2 + 1 = 13 letters total and so there are

13!/2! · 5! · 2! · 1! · 1! · 1! · 1! =P(13, 5)/4 = 51, 891, 840/4 = 12, 972, 960words.

(b) Sol: There are 7 distinct letters so if repetitions are not permitted the answer is P(7, 4) = 840.

Q5:

Solution

Ans: Consider a graph where each line segment is represented as a vertex. Now two vertices of this graph are connected if the corresponding line segments intersects.
So, this graph has 9 vertices and degree of each vertex is 3.

In a graph:
* sum of degrees of all vertices = 2 * no. of edges

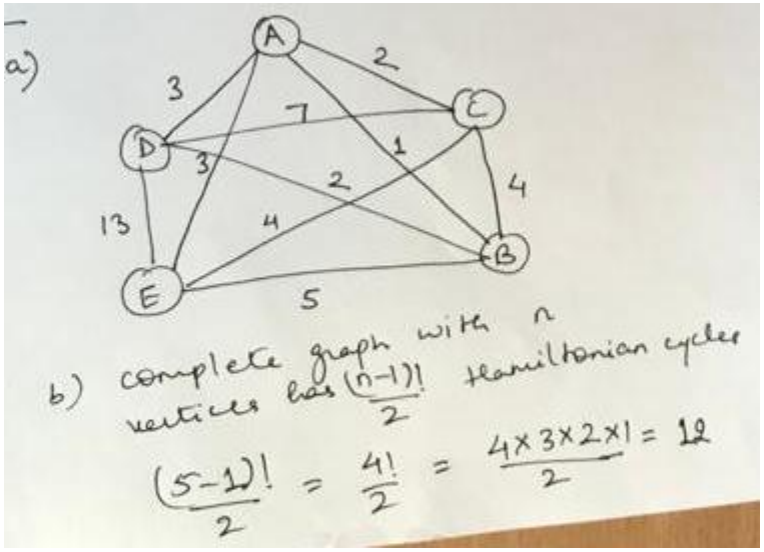
Sum of degree of all vertices = 9 × 3 = 27 = odd number

2 * no. of edges = even number

So, such an arrangement is not possible.

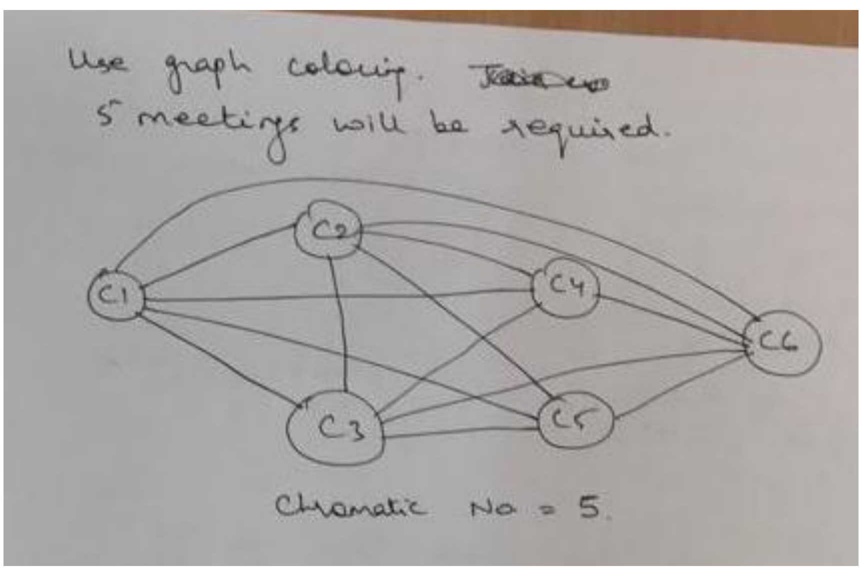
Q6:

Solution



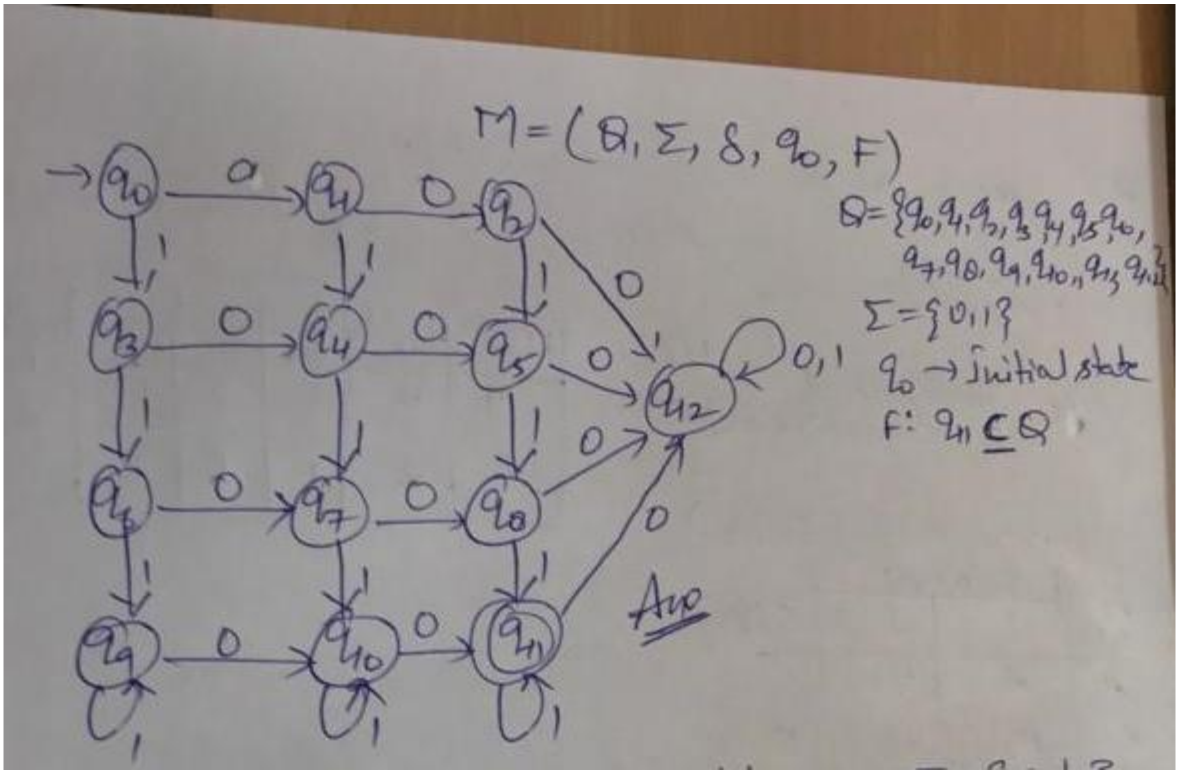
Q: 7

Solution



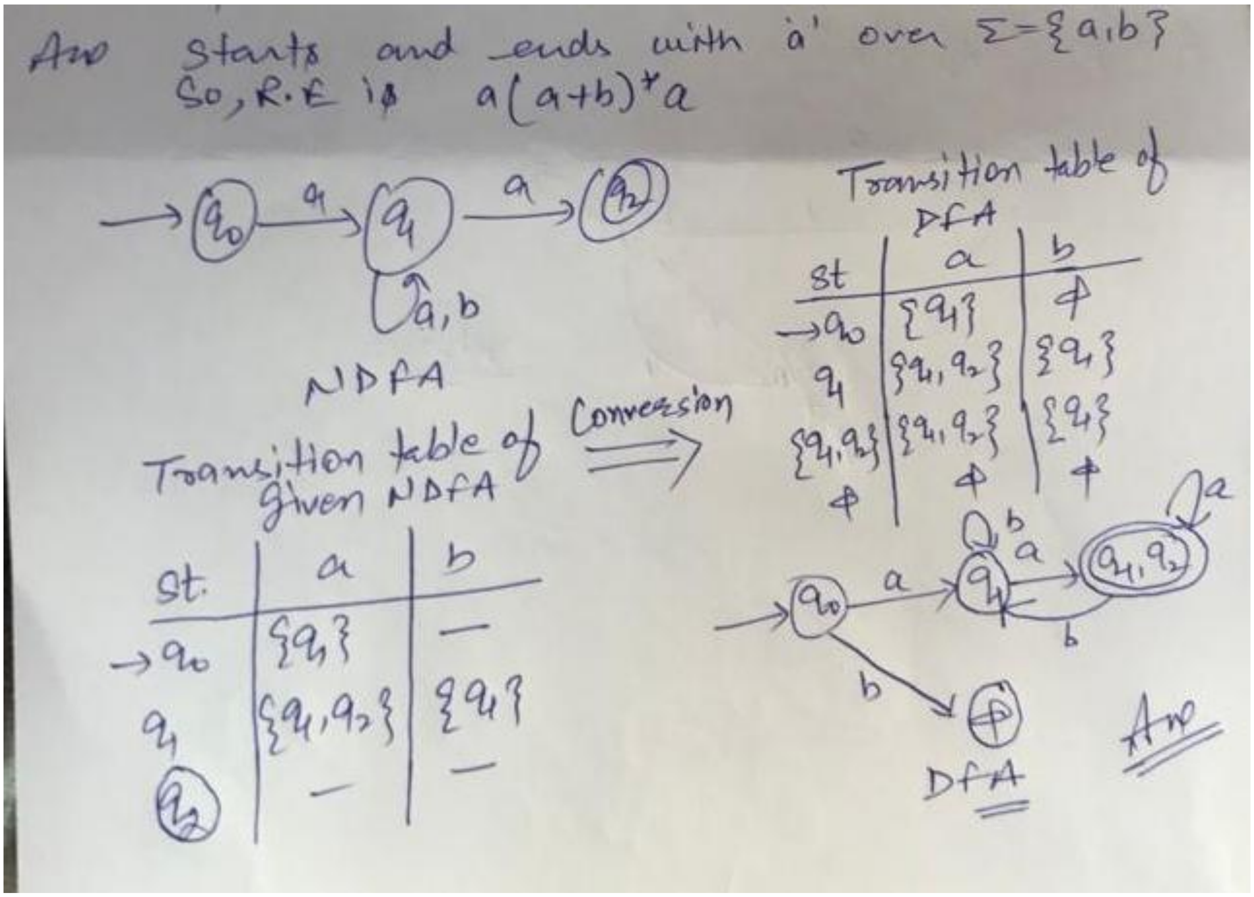
Q: 8

Solution


M = (Q, Σ, δ, q0, F)
Q = {q0, q1, q2, q3, q4, q5, q6, q7, q8, q9, q10, q11, q12}
Σ = {0, 1}
q0 → initial state
F: q11 ⊆ Q
Ans

Q: 9

Solution



Ans starts and ends with 'a' over $\Sigma = \{a, b\}$

So, R.E is $a(a+b)^*a$

Transition table of given NDFA

St.	a	b
$\rightarrow q_0$	$\{q_1\}$	—
q_1	$\{q_1, q_2\}$	$\{q_1\}$
q_2	—	—

Conversion $\Longrightarrow$

Transition table of DFA

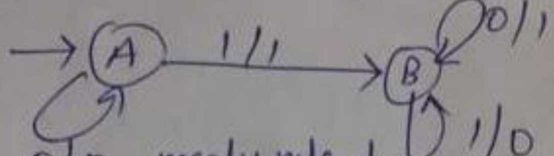
St	a	b
$\rightarrow q_0$	$\{q_1\}$	ϕ
q_1	$\{q_1, q_2\}$	$\{q_1\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1\}$
ϕ	ϕ	ϕ

Ans

Q: 10

Solution

Ans mealy m/c of 2's complement is



mealy m/c of 2's complement

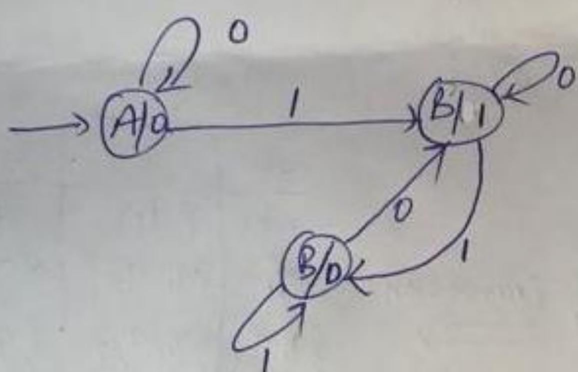
Table (mealy m/c)

Present St.	0 St.	0 O/P	1 St.	1 O/P
A	A	0	B	1
B	B	1	B	0

Let $A \to 0$
$B_0 \to 0$
$B_1 \to 1$

moore m/c (Table)

Pr. St.	Input 0	Input 1	O/P
A	A	B_1	0
B_0	B_1	B_0	0
B_1	B_1	B_0	1



Ans