

Department of Mathematics

Odd Semester 2017

Probability and Random Processes

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Tutorial Sheet 8

15B11MA301

10B11MA411

B.Tech. Core

Random Walk, Telegraph Signal Process, Ergodic Process

1. Let $\{X_n, n = 1, 2, \dots\}$ be a sequence of independent random variables with sample space as $\{0, 1\}$, $P[X_n = 0] = 2/3$, $P[X_n = 1] = 1/3$. Let $Y_n = X_1 + X_2 + \dots + X_n$. Find first order pmf of Y_n , $E[Y_n]$, $R_{YY}(n, n+k)$, and $C_{YY}(n, n+k)$.
2. Let $X(t)$ is semi random telegraph signal process and $Y(t) = \beta X(t)$, where β is uniformly distributed random variable in the interval $(-2, 2)$. Is $Y(t)$ a WSS process?
3. A random process is defined by $X(t) = T + (1-t)$, where T is a uniform random variable in $(0, 1)$. (a) Find the cdf of $X(t)$, (b) Find $E[X(t)]$ and $C_{XX}(t_1, t_2)$.
4. If $\{X(t)\}$ is a WSS process with $E\{X(t)\} = 2$ and $R_{XX}(\tau) = 4 + e^{-|\tau|/10}$, find the mean and variance of $S = \int_0^1 X(t) dt$.
5. The WSS process $X(t)$ is given by $X(t) = 10 \cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$. Check whether $\{X(t)\}$ is (i) mean ergodic random process, (ii) correlation ergodic random process, (iii) ergodic random process?
6. A random binary transmission process $\{X(t)\}$ is a WSS process with zero mean and autocorrelation function $R(\tau) = 1 - |\tau|/T$, where T , is a constant. Find the variance of the time average of $\{X(t)\}$ and also the mean over $(0, T)$. Is $\{X(t)\}$ mean ergodic?
7. If $\{X(t)\}$ is the random telegraph signal process with $E\{X(t)\} = 0$ and $R(\tau) = \exp(-2\lambda|\tau|)$, find the mean and variance of the time average of $\{X(t)\}$ over $(-T, T)$. Is it mean ergodic?
8. Define power spectral density function (PSDF) of a stationary process. State and prove all the properties of PSDF.
9. Find the autocorrelation function of the process $\{X(t)\}$, for which the power spectral density function is given by
$$S(\omega) = \begin{cases} 1 + \omega^2, & \text{for } |\omega| \leq 1 \\ 0, & \text{for } |\omega| > 1 \end{cases}$$
.
10. Find the PSDF of a process whose autocorrelation function is given by
$$R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & \text{for } |\tau| \leq T \\ 0, & \text{for } |\tau| > T \end{cases}$$
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