

## Department of Mathematics

### Probability and Random Processes

15B11MA301

### Tutorial Sheet 12

(Semi random and random telegraph signal processes and Ergodic Process)

1. Let  $X(t)$  is semi random telegraph signal process and  $Y(t) = \beta X(t)$ , where  $\beta$  is uniformly distributed random variable in the interval  $(-2, 2)$  and is independent of  $X(t)$ . Is  $Y(t)$  a WSS process? [Ans. Yes]
2. Find the mean and variance of a random process  $\{X(t)\}$  whose autocorrelation function is given by  $R(\tau) = 45 + \frac{4\tau^2 + 9}{\tau^2 + 2}$ . [Ans. Mean = 7, Var = 0.5]
3. For the random process  $X(t) = A \cos \omega t + B \sin \omega t$ , where  $A$  and  $B$  are random variables with  $E(A) = E(B) = 0$ ,  $E(A^2) = E(B^2) > 0$ , and  $E(AB) = 0$ . Prove that the process is mean ergodic.
4. Let  $\{X(t)\}$  be a WSS process with  $E\{X(t)\} = 2$  and  $R_{xx}(\tau) = 4 + e^{-\left(\frac{|\tau|}{10}\right)}$ , find the mean and variance of  $S = \int_0^1 X(t) dt$ . [Ans. Mean = 2; var:  $200e^{-\frac{1}{10}} - 180$ ]
5. The WSS process  $\{X(t)\}$  is given by  $X(t) = 10 \cos(100t + \theta)$ , where  $\theta$  is uniformly distributed over  $(-\pi, \pi)$ . Check whether  $\{X(t)\}$  is (i) mean ergodic random process, (ii) correlation ergodic random process. [Ans. Yes; Yes]
6. A random binary transmission process  $\{X(t)\}$  is a WSS process with zero mean and autocorrelation function  $R_{xx}(\tau) = 1 - \left(\frac{|\tau|}{T}\right)$ , where  $T$  is a constant. Find the variance of the time average of  $\{X(t)\}$  and also the mean over  $(0, T)$ . Is  $\{X(t)\}$  mean ergodic? [Ans. 2/3; 0 ; No]