Tutorial Sheet – Odd Semester 2022

15B11CI212 Theoretical Foundations of Computer Science

Tutorial 1

Instructions

1. Students are advised to come prepared in tutorial by revising the lectures of that particular week so that doubts related to topics covered in that week are resolved.
2. As tutorials will be purely problem solving based, always join the tutorial with a notebook and pen with you.
3. Whatever questions are discussed in tutorial, you are supposed to submit them on your respective classrooms every week.

Introduction to Discrete Mathematics and Set Theory

1. List the members of these sets.

a) $\{x \mid x$ is a real number such that $x^2 = 1\}$
b) $\{x \mid x$ is a positive integer less than 12$\}$
c) $\{x \mid x$ is the square of an integer and $x < 100\}$
d) $\{x \mid x$ is an integer such that $x^2 = 2\}$

Solution

a) $\{-1,1\}$
b) $\{1,2,3,4,5,6,7,8,9,10,11\}$
c) $\{0,1,4, 9, 16,25, 36, 49, 64, 81\}$
d) $\emptyset$

2. Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$, $\{5, 3, 1\}$
b) $\{\{1\}\}$, $\{1, \{1\}\}$ **c)** $\emptyset$, $\{\emptyset\}$

Solution

a) Yes **b)** No **c)** No

3. Use a property to give a description of each of the following sets.

a) { a,e, i, o, u}
b) {1, 3,5, 7,9 }

Solution

a) {x|x is a vowel}
b) {n∈N | n is odd and less than 10}

4. Determine whether each of these statements is true or false.

a) $0 \in \emptyset$ **b)** $\emptyset \in \{0\}$
c) $\{0\} \subset \emptyset$ **d)** $\emptyset \subset \{0\}$
e) $\{0\} \in \{0\}$ **f)** $\{0\} \subset \{0\}$
g) $\{\emptyset\} \subseteq \{\emptyset\}$

Solution: a) False **b)** False **c)** False **d)** True **e)** False **f)** False **g)** True

5. What is the cardinality of each of these sets?

a) $\{a\}$

b) $\{\{a\}\}$

c) $\{a, \{a\}\}$

d) $\{a, \{a\}, \{a, \{a\}\}\}$

Solution:

a) 1 **b)** 1 **c)** 2 **d)** 3

6. How many elements does each of these sets have where *a* and *b* are distinct elements?

a) $P(\{a, b, \{a, b\}\})$

b) $P(\{\emptyset, a, \{a\}, \{\{a\}\}\})$

c) $P(P(\emptyset))$

Solution:

a) 8 **b)** 16 **c)** 2

7. What is the Cartesian product $A \times B \times C$, where A is the set of all airlines and B and C are both the set of all cities in the United States? Give an example of how this Cartesian product can be used.

Solution:

The set of triples (a, b, c), where a is an airline and b and c are cities. A useful subset of this set is the set of triples (a, b, c) for which a flies between b and c.

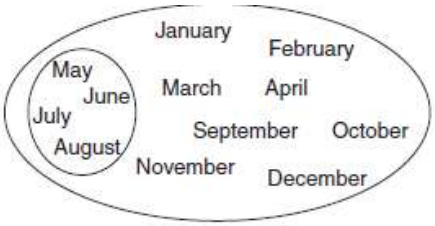
8. State whether $A \times B \times C$ and $(A \times B) \times C$ are same or not. Explain your answer

Solution:

The elements of $A \times B \times C$ consist of 3-tuples (a, b, c), where $a \in A$, $b \in B$, and $c \in C$, whereas the elements of $(A \times B) \times C$ look like $((a, b), c)$—ordered pairs, the first coordinate of which is again an ordered pair.

9. Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter *R* in the set of all months of the year.

Solution:



10. Let A, B, C be three sets as shown in the following Venn diagram. For each of the following sets, draw a Venn diagram and shade the area representing the given set

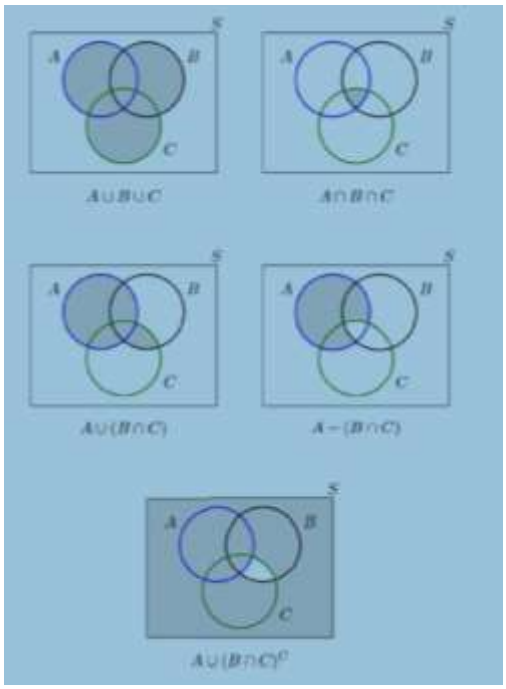
a) $A \cup B \cup C$

b) $A \cap B \cap C$

c) $A \cup (B \cap C)$

d) $A - (B \cap C)$

e) $A \cup (B \cap C)^c$



11. Using Venn Diagrams , verify the following identities

a) $A = (A \cap B) \cup (A - B)$

b) If A and B are finite sets, $|A \cup B| = |A| + |B| - |A \cap B|$