Digital Systems
18B11EC213

Module 1: Boolean Function Minimization Techniques and Combinational Circuits-3

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Cont.. Number Systems

❑ **Review - Previous Lecture**

➢ Sign Magnitude Representation:

- For N bits, the sign magnitude representation can accommodate numbers in the range

$$-\{2^{(N-1)} - 1\} \text{ to } +\{2^{(N-1)} - 1\}$$

- For N bits, total 2^N numbers or values are possible.
- For 8 bits (N = 8), total 2^8 (= 256) numbers or values are possible.
- For 8 bits (N = 8), the sign magnitude representation can represent the signed numbers (integers) from -127 to +127.

{127 + 2 + 127 = 256 numbers (integers)}

Sign magnitude representation has two ways of representing 0 (+ 0 and - 0).

Cont..

➢ 1's Complement Representation:

- Similar to the sign magnitude representation, for N bits, the 1's complement representation can accommodate numbers in the range

  $$-\{2^{(N-1)} - 1\} \text{ to } +\{2^{(N-1)} - 1\}$$

- For N bits, total 2^N numbers or values are possible.
- For 8 bits (N = 8), total 2^8 (= 256) numbers or values are possible.
- For 8 bits (N = 8), the 1's complement representation can represent the signed numbers (integers) from -127 to +127.

  {127 + 2 + 127 = 256 numbers (integers)}

  1's complement representation has two ways of representing 0 (+ 0 and - 0).

Cont..

➢ 2's Complement Representation:

- For N bits, the 2's complement representation can accommodate numbers in the range

  $$- 2^{(N-1)} \text{ to } + \{2^{(N-1)} - 1\}$$

- For N bits, total 2^N numbers or values are possible.
- For 8 bits (N = 8), total 2^8 (= 256) numbers or values are possible.
- For 8 bits (N = 8), the 2's complement representation can represent the signed numbers (integers) from -128 to +127.

  {128 + 1 + 127 = 256 numbers (integers)}

  2's complement representation contains only one type of 0.

(r-1)'s Complement

- Given a positive number N in base r with n number of digits in integer portion and m number of digits in fractional portion, the (r-1)'s complement of N is equal to

$$r^n - r^{-m} - N$$

- For decimal numbers (base r = 10), 9's complement

For binary numbers (base r = 2), 1's complement

Cont..

- 9's Complement

Examples:

9's complement of $(36360)_{10}$

$= (10^5 - 10^0 - 36360) = (10^5 - 1 - 36360)$

$= 99999 - 36360 = (63639)_{10}$

In this case, base r = 10, n = 5, m = 0

9's complement of $(25.3636)_{10}$

$= (10^2 - 10^{-4} - 25.3636) = 99.9999 - 25.3636$

$= (74.6363)_{10}$

In this case, base r = 10, n = 2, m = 4

Cont..

Direct Approach:

- Examples - 9’s Complement

❖ Example-1: Find the 9’s complement of $(36360)_{10}$.

Answer:

$$
\begin{array}{r}
99999 \\
-\ 36360 \\
\hline
= \quad (63639)_{10}
\end{array}
$$

Cont..

- Example-2: Find the 9's complement of $(25.3636)_{10}$.

Answer:

$$
\begin{array}{rr}
 & 99.9999 \\
- & 25.3636 \\
\hline
= & (74.6363)_{10}
\end{array}
$$

Cont..

- 1's Complement

Example: 1's complement of $(101100)_2$ is

$$= 2^6 - 2^0 - (101100)_2$$

$$= (2^6 - 2^0)_{10} - (101100)_2$$

$$= (64 - 1)_{10} - (101100)_2$$

$$= (111111)_2 – (101100)_2$$

$$= (010011)_2$$

In this case, base r = 2, n = 6, m = 0

Cont..

Direct Approach:

As discussed in the previous lecture class, the 1's complement of a binary number can be obtained by simply changing each bit 1 to 0 and 0 to 1.

Example: 1's complement of $(101100)_2$ is equal to $(010011)_2$

r’s Complement

- Given a positive number N in base r with n number of digits in integer portion and m number of digits in fractional portion, the r’s complement of N is defined as

$$r^n - N \quad \text{for} \quad N \neq 0$$

$$0 \quad \text{for} \quad N = 0$$

- For decimal numbers (base r = 10), 10’s complement

For binary numbers (base r = 2), 2’s complement

Cont..

- 10’s Complement

Examples:

10’s complement of $(36360)_{10}$

$= 10^5 - 36360 = (63640)_{10}$

In this case, base r = 10, n = 5

10’s complement of $(0.3534)_{10} = 10^0 - 0.3534$

$= (0.6466)_{10}$

In this case, base r = 10, n= 0

10’s complement of $(25.353)_{10} = 10^2 - 25.353$

$= 100 - 25.353 = (74.647)_{10}$

In this case, base r = 10, n = 2

Cont..

Simpler Method: As discussed in the previous lecture class, a simpler method for obtaining the r's complement of a number is defined as

r's complement of a positive number

= (r - 1)'s complement of the number + 1

Cont..

- Examples - 10's Complement

❖ Example-1: Find the 10's complement of $(36360)_{10}$.

Answer: 10's complement of a number

= 9's complement of that number + 1

9's complement of $(36360)_{10}$ is $(63639)_{10}$

Therefore, 10's complement of $(36360)_{10}$ is

$63639 + 1 = (63640)_{10}$

Cont..

- Example-2: Find the 10's complement of $(0.3534)_{10}$.

Answer: 9's complement of $(0.3534)_{10}$

$= 0.9999 - 0.3534 = (0.6465)_{10}$

10's complement of $(0.3534)_{10}$ is

$$
\begin{array}{rr}
 & 0.6465 \\
+ & 1 \\
\hline
 & (0.6466)_{10}
\end{array}
$$

- The result is same as obtained using the formula in the previous slide.

Cont..

- Example-3: Find the 10's complement of $(25.353)_{10}$.

Answer: 9's complement of $(25.353)_{10}$

$$= 99.999 - 25.353 = (74.646)_{10}$$

10's complement of $(25.353)_{10}$ is

$$\begin{array}{r} 74.646 \\ +\quad\quad 1 \\ \hline (74.647)_{10} \end{array}$$

- The result is same as obtained using the formula in the previous slide.

Cont..

- 2's Complement

Using formula, the 2's complement of $(101100)_2$

$$= r^n - N$$

$$= (2^6) - (101100)_2$$

$$= (64)_{10} - (101100)_2$$

$$= (1000000)_2 - (101100)_2 = (010100)_2$$

In this case, base r = 2, n = 6

Cont..

Using direct method / simpler method:

1's complement of $(101100)_2$ is $(010011)_2$

Therefore, 2's complement of $(101100)_2$ is

(1's complement + 1) =

$$
\begin{array}{r}
010011 \\
+ \qquad 1 \\
\hline
(010100)_2
\end{array}
$$

- The answer is same as obtained using the formula in the previous slide.

References

- M. M. Mano, *Digital Logic and Computer Design*, 5th ed., Pearson Prentice Hall, 2013.
- R. P. Jain, *Modern Digital Electronics*, 4th ed., Tata McGraw-Hill Education, 2009.