Department of Mathematics

15B11MA211 **Mathematics-2**

Tutorial Sheet 7 **B.Tech. Core**

Bessel's Functions

1. Express $J_{-5/2}(x)$ in terms of trigonometric functions.
2. Evaluate the following integrals in terms of the Bessel's functions
   (a) $\int J_3(x)\,dx$, (b) $\int xJ_0^2(x)\,dx.$
3. Solve the differential equation $x^2y''+xy'+(8x-1)y=0$ in terms of Bessel's functions.
4. Show that
   $$\cos(x\cos\theta)=J_0-2J_2\cos 2\theta+2J_4\cos 4\theta-...$$
   and $$\sin(x\cos\theta)=2\left[J_1\cos\theta-J_3\cos 3\theta+...\right].$$
5. Show that $\frac{1}{\pi}\int_0^{\pi}\cos(x\cos\theta)\,d\theta=J_0$.

Answers:

1. $J_{-5/2}(x)=\sqrt{\frac{2}{\pi x}}\left[\frac{1}{x^2}\left(3-x^2\right)\cos x+\frac{3}{x}\sin x\right].$
2. (a) $\int J_3(x)\,dx=c-J_2(x)-\frac{2}{x}J_1(x)$, (b) $\int xJ_0^2(x)\,dx=\frac{x^2}{2}\left(J_0^2+J_1^2\right)+c.$
3. $y=c_1J_2\left(4\sqrt{2x}\right)+c_2J_{-2}\left(4\sqrt{2x}\right)$.