POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

Student Name ____________________ **Enrollment No.**____________

Jaypee Institute of Information Technology, Noida
End Term Examination, ODD Semester-2016
B.Tech. 3rd Semester

Course Title: Probability and Random Processes/ Probability Theory and Random Processes
Course Code: 15B11MA301/10B11MA411

Max Marks: 35
Max Time: 2 Hours

Note: Attempt all Questions.

Q1. One way to design a spam filter is to look at the words in an email. In particular, some words are more frequent in spam emails. Suppose that we have the following information: 50% of emails are spam, 1% of spam emails contain the word "refinance" and 0.001% of non-spam emails contain the word "refinance". Suppose that an email is checked and found out to contain the word refinance. What is the probability that the email is spam? (3)

Q2. The joint probability density function of two dimensional random variables (X,Y) is given by

$$f(x,y)=\begin{cases}\frac{8}{9}xy & , \quad 1<x<y<2\\ 0 & , \quad \text{otherwise}\end{cases}$$ (4)

Find the marginal density functions of X and Y. Also find the conditional mean of Y for given $X=x$.

Q3. Find the characteristic function of geometric distribution and hence find the first four moments about origin. (4)

Q4. Eight identical components with constant failure rates are connected in low level redundancy with four components in each subsystem. Determine the component MTTF, necessary to provide a system reliability of 0.90 after 100 hours of operation. (4)

Q5 (a) Prove that the inter-arrival time of a Poisson process with parameter λ follows an exponential distribution with mean $1/\lambda$. (3)

(b) Patients arrive randomly and independently at doctor's consulting room from 8 a.m at an average rate of one in 5 minute. The waiting room can hold 12 persons. What is the probability that the room will be full when the doctor arrives at 9 a.m? (2)

Q6 The three-state Markov chain is given by the transition probability matrix

$$P=\begin{pmatrix}0 & 2/3 & 1/3\\ 1/2 & 0 & 1/2\\ 1/2 & 1/2 & 0\end{pmatrix}.$$ Determine whether the chain is irreducible, non-null persistent, aperiodic or not. Justify your answer. (4)

Q7. An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follows a highly distorted signal, with no recognizable signal between, where as 20 out of 23 recognizable signals follow recognizable signals with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted (long run). (4)

Q8. Let $\{X(t)\}$ be a stationary random process with spectral density function $S_{XX}(\omega)$ and $\{Y(t)\}$ be another independent random process where $Y(t)=A\cos(\omega_0 t+\theta)$ and θ is a uniformly distributed random variable over $(-\pi,\pi)$. Find the spectral density function of $\{Z(t)\}$, where $Z(t)=X(t)Y(t)$. (4)

Q9. Prove that a random process defined by $X(t)=\cos(bt+\theta)$ is mean ergodic process, where b is a constant and θ is a uniform random variable over $(0,2\pi)$. Justify your answer. (3)

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