Probability and Random Processes   15B11MA301
Probability Theory and Random Processes   10B11MA411
Tutorial Sheet 3   **2-D Random Variables, MGF, CF**   **B.Tech. Core**

1. Define the following: (a) two dimensional random variable (b) marginal and conditional probability distributions (c) If X denotes the number of kings and Y denotes number of aces when two cards are drawn at random without replacement from a deck of well shuffled pack of 52 cards, find

   (i) The joint probability distribution of (X, Y). (ii) The marginal distribution
   (iii) P(X=2/Y=1) (iv) P(X<2/0<Y<2)   (v) $P(1 \leq X \leq 2/Y = 0,2)$.

2. Let the joint pdf of a random variable (X, Y) is defined as $f(x, y) = k(xy + y^2)$, $0 \leq x \leq 2$, $0 \leq y \leq 1$. Find (i) the value of k, (ii) P(X>1) (iii) P(X+Y<1) (vi) P(X<1,Y>1/2) (v) $f_X(x)$ and $f_Y(y)$. Also test whether X and Y are independent?

3. The pdf of (X,Y) be defined as $f(x, y) = (1/4)\ e^{-|x|+|y|}$, $-\infty \leq x < \infty$, $-\infty \leq y < \infty$. Are X and Y independent? Find the proability that $X \leq 1$ and $Y \leq 0$.

4. Random variable (X,Y) have a joint probability density function $f(x, y)=(2x+y)/27$, where x and y can assume only integer values 0, 1, 2. Find the conditional distribution of Y for $X = x$.

5. Two ideal dice are thrown. Let X_1 be the score on the first die and X_2 the score on the other die. Let Y denote the maximum of X_1 and X_2 i.e. max(X_1, X_2).

   (a) Write down the joint distribution of Y and X_1,   (b) Find E(Y) and Var(Y).

6. Let $f(x_1, x_2) = \begin{cases} 21x_1^2 x_2^3 & 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$ be the joint pdf of X_1 and X_2. Find the conditional mean and variance of X_1, given $X_2 = x_2$ and $0 < x_2 < 1$.

7. A pair of fair dice is thrown and let X be the number of 6's turned up. Find the moment generating function (MGF), mean and variance of X.

8. Find CF and MGF of X whose probability density function is given by $f(x) = k\ \dfrac{e^{-|x|}}{5}$ $-\infty < x < \infty$. Find first three moments of X about the origin. What is the variance of X?

9. The joint pdf of a two dimensional random variables (X, Y) is

   $$f(x, y) = \begin{cases} \dfrac{3}{2}\left(x^2 + y^2\right) & \text{if } 0 \leq x, y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find C_{XY}, $E(XY)$ and ρ_{XY}.

10. Let X and Y be two independent Poisson random variables with

    $P_X(k) = \dfrac{1}{k!} e^{-2} 2^k$, $P_X(k) = \dfrac{1}{k!} e^{-2} 2^k$. Compute the MGF of Z = 2X + 3Y.

11. Compute the characteristic function of discrete random variables X and Y if the joint pmf

    $$P_{XY}(k, l) = \begin{cases} 1/3, & k = l = 0 \\ 1/6, & k = \pm1, l = 0 \\ 1/6, & k = l = \pm1 \\ 0, & \text{else.} \end{cases}$$

12. Find the density function of the distribution for which the characteristic function is given by $\phi(t) = e^{-\sigma^2 t^2 /2}$.