TUTORIAL-2 (CONTD.)

Q] $y'' - 2\tan x\, y' + 8y = e^x \sec x$

* $Uv'' + (2U' + PU)v' + (U'' + PU' + QU)v = e^x \sec x$

* $P = -2\tan x \qquad Q = 8$

* $Uv'' + (2U' - 2\tan x\, U)v' + (U'' - 2\tan x\, U' + 8U)v = e^x \sec x$

~~$-2\tan x\, v''(2\sec^2 x + 8$~~

$2U' - 2\tan x\, U = 0$

$U' = 2\tan x\, U$

$\frac{1}{U}\frac{dU}{dx} = \frac{2\tan x}{2} \Rightarrow \ln U = \ln \sec x$

$\boxed{U = \sec x}$

$\sec x\, v'' + (2\sec x\tan x - 2\tan x \sec x)v'$

$+ (\sec x \tan^2 x + \sec^3 x - 2\tan^2 x \sec x + 8\sec x)v$

$= e^x \sec x$

$\Rightarrow v'' + (2\tan x - 2\tan x)v' + (\tan^2 x + \sec^2 x - 2\tan^2 x + 8)v = e^x$

* $v'' + (\sec^2 x - \tan^2 x + 8)v = e^x$

$\Rightarrow \quad v'' + 9v = e^x$

$(D^2 + 9)\, v = e^x$

$m = \pm 3i$

$CF = C_2 \sin 3x + C_1 \cos 3x$

$PI = \dfrac{e^x}{10}$

$y = uv$

$y = e^{ax}\left(C_1 \cos 3x + C_2 \sin 3x + \dfrac{e^x}{10}\right)$

Q3] $xy'' - y' - 4x^3y = 8x^2 \sin x^2$

$$y'' - \frac{1}{x}y' - 4x^2 y = 8x^2 \sin x^2$$

$P = -1/x \qquad Q = -4x^2 \qquad R = (\sin x^2)8x^2$

$$P = \frac{Z'' + PZ'}{(Z')^2} \; ; \; q = \frac{Q}{(Z')^2} \; ; \; r = \frac{R}{(Z')^2}$$

$\longrightarrow d^2y/dx^2 + P\, dy/dx + qy = r$

* we choose Z such that $P = 0$

then

* $Z'' + PZ' = 0$
* $Z'' + (-1/x)Z' = 0$

$$\Rightarrow \frac{Z''}{Z'} = \frac{1}{x}$$

$$\Rightarrow \log Z' = \log x$$

$$Z' = x$$

$$\Rightarrow Z = \frac{x^2}{2}$$

$$q = \frac{-4x^2}{(x)^2} = (-4) \; ; \; r = \frac{8x^2 \sin x^2}{x^2}$$

$$\frac{d^2y}{dx^2} + (-4)y = \frac{8x^2 \sin x^2}{x^2}$$

$$\Rightarrow (D^2 + (-4))y = 8 \sin(2z)$$

CF ⇒ $D^2 - 4 = 0$

$D^2 = 4$

$D = \pm 2$

$[C_1 e^{-2x} + C_2 e^{2x}] \rightarrow \underline{\underline{CF}}$

PI ⇒ $\dfrac{8 \sin 2x}{D^2 - 4}$ $\{D^2 = -a^2\}$ $\{D^2 = -4\}$

PI = $\dfrac{8 \sin 2x}{-8} \Rightarrow -\sin 2x$

$y = CF + PI$

$y = C_1 e^{-x^2} + C_2 e^{x^2} - \sin x^2$

Q4.] $y'' - 6y' + 9y = \frac{e^{3x}}{x}$

$m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$m = 3, 3$

$CF = (C_1 + C_2 x)e^{3x}$

$CF = C_1 e^{3x} + C_2 x e^{3x}$

$\phi = e^{3x} \qquad \psi = xe^{3x}$

$$W = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}(3x+1) \end{vmatrix}$$

$W = e^{6x}(3x+1) - 3xe^{6x}$

$e^{6x}(3x+1-3x)$

$\Rightarrow e^{6x}$

$$PI = -e^{3x}\int \frac{xe^{3x}e^{3x}}{x\,e^{6x}}\,dx + xe^{3x}\int \frac{e^{3x}e^{3x}}{x\,e^{6x}}$$

$PI = -e^{3x}(x) + xe^{3x}\ln x$

$PI = xe^{3x}(\ln x - 1)$

$$IF = e^{\int \frac{2}{x(x^2+1)}\,dx}$$

$$\frac{2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\frac{2}{x(x^2+1)} = \frac{A(x^2)+A+Bx^2+Cx}{x(x^2+1)}$$

$$A = 2$$

$$Ax^2 + Bx^2 = 0 \qquad Cx = 0$$
$$2x^2 + Bx^2 = 0 \qquad C = 0$$
$$Bx^2 = -2x^2$$
$$(B = -2)$$

$$\int \frac{2}{x} + \int \frac{(-2)x}{x^2+1}$$

$$\Rightarrow \underline{2\log x} + (-2)\int \frac{x}{x^2+1}\,dx$$

$$x^2+1 = t$$
$$2x\,dx = dt$$
$$dx = \frac{dt}{2x}$$

$$* \; 2\log x - 2\int \frac{dt}{2t}$$

$$2\log x - \log(x^2+1)$$

$$\log x^2 - \log(x^2+1)$$

$$IF = e^{\log x^2 - \log(x^2+1)} = \left(\frac{x^2}{x^2+1}\right)$$

$$z\left(\frac{x^2}{x^2+1}\right) = c$$

$$z = \frac{c(x^2+1)}{x^2}$$

$$v' = \frac{c(x^2+1)}{x^2}$$

$$v' = c + cx^{-2}$$

$$\int v' = \int c + cx^{-2} \Rightarrow v = cx - \frac{c}{x}$$

$$v = c\left(\frac{x^2-1}{x}\right) + c_2$$

$$y = uv$$

$$y = x\left[\frac{c(x^2-1) + c_2 x}{x}\right]$$

$$y = c(x^2-1) + c_2(x)$$