# TUTORIAL 6

Q1] $\lambda(t) = 0.00034$

↳ const failure rate

↳ exponential dist.

1) $MTTF = \frac{1}{\lambda} = \frac{1}{0.00034} = \underline{\underline{2941 \text{ hr}}}$

2) Median

$$R(t) = F(t)$$

$$e^{-\lambda t} = 1 - e^{-\lambda t}$$

$$e^{-\lambda t} = 0.5$$

$$-\lambda t = \ln 0.5$$

$$-(0.00034)t = -0.6931$$

$$t = \underline{\underline{2038.66 \text{ hr}}}$$

3) $R(t) = e^{-\lambda t} = \underline{\underline{e^{-0.00034t}}}$

4) 30 days → 72 hr

$$R(72 \text{ hr}) = e^{-0.00034 \times 72} = \underline{\underline{0.78286}}$$

5) $R(t_D) = 0.95$

$$e^{-\lambda t_D} = 0.95 \Rightarrow -\lambda t = \ln(0.95)$$

$$t = \frac{\ln 0.95}{-0.00034} = \underline{\underline{150.86 \text{ hr}}}$$

Q2] $f(t) = 0.1 \quad (1+0.05t)^{-3}$

$$R(t) = \int_t^\infty 0.1 \, (1+0.05t)^{-3} \, dt$$

$$= - \frac{\cancel{0.1}^{2} \, (1+0.05t)^{-2}}{(-2)\,(\cancel{0.05})} = (1+0.05t)^{-2}$$

Now, $R(10+1/1) = R(11/1)$

$$= \frac{(1.05)^2}{(1+0.05\times 11)^2} = \underline{\underline{0.45889}}$$

$$MTTF = \int_0^\infty R(t) = \int_0^\infty (1+0.05t)^{-2}$$

$$= \left| \frac{(1+0.05t)^{-1}}{(-1)(0.05)} \right|_0^\infty = \frac{1}{\infty} - \left| \frac{1}{(-1)(0.05)} \right|$$

$= \underline{\underline{20 \text{ years}}}$

Q3]

$\lambda(t) = at + b$

| $t$ | | $R(t)$ |
|---|---|---|
| 2 days (48 hr) | $\longrightarrow$ | 10% (0.1) |
| 3 days (72 hr) | $\longrightarrow$ | 15% (0.15) |

So,

$$0.1 = 48a + b$$
$$0.15 = 72a + b$$

$$a = \frac{0.05}{24} = \underline{\underline{0.00208}}$$

& $\underline{\underline{b = 0}}$ $\Rightarrow$ $\lambda(t) = 0.00208t$

$$R(t) = e^{-\int_{-\infty}^{t} \lambda(t)dt} = e^{-\int 0.00208t \, dt}$$

$$= e^{-0.00104t^2}$$

a) $R(30) = e^{-0.00104(30)^2} = \underline{\underline{0.3916}}$

b) $R(31/30) = \dfrac{e^{-0.00104(31)^2}}{0.3916} = \dfrac{0.3674}{0.3916}$

↑
probability for working one hour more

So, Prob. for not working $= 1 - \dfrac{0.3674}{0.3916} = \underline{\underline{0.06155}}$

Q4]

$C1 \sim EXP(\lambda = 0.141)$

$C2 \sim WEIBULL(\beta = 2, \theta = ?)$

a)

$$MTTF_{C1} = MTTF_{C2}$$

$$\frac{1}{\lambda} = \theta \, \Gamma\left(\frac{1}{\beta} + 1\right)$$

$$\frac{1}{0.141} = \theta \, \Gamma\left(\frac{1}{2} + 1\right) = \theta \frac{\sqrt{\pi}}{2}$$

$$\underline{\underline{\theta = 8.0026}}$$

b) For C1

$$0.8 = e^{-0.141 t_D}$$

$$t_D = \frac{\ln 0.8}{-0.141} = 1.58257$$

For C2

$$0.8 = e^{-(t/\theta)^{\beta}}$$

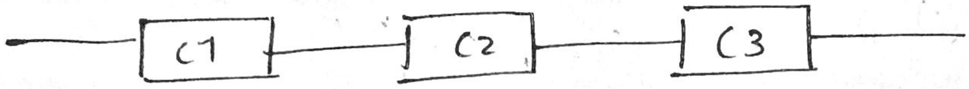
$$t_D = \theta(-\ln 0.8)^{1/\beta}$$

$$t_D = 3.78027$$

% longer $\frac{3.78027 - 1.58257}{1.58257} = 138.8693\ \%$

Q5] $t_D(sys) = 5$ year

$R(t_D\ sys) = 0.95$




$e^{-\lambda_1 t}$ $\qquad$ $e^{-\lambda_2 t}$ $\qquad$ $e^{-\lambda_3 t}$

$$\frac{\lambda_1}{2} = \frac{\lambda_2}{1} = \frac{\lambda_3}{3} \Rightarrow \lambda_2 = 0.5\lambda_1$$

$$\Rightarrow \lambda_3 = 1.5\lambda_1$$

$R_{sys} = R_{C1} \cdot R_{C2} \cdot R_{C3}$ (for series)

$R_{sys} = e^{-(\lambda_1+\lambda_2+\lambda_3)t}$

$0.95 = e^{-(\lambda_1+\lambda_2+\lambda_3)5}$

$\lambda_1 + \lambda_2 + \lambda_3 = 0.01026$

$\lambda_1 + 0.5\lambda_1 + 1.5\lambda_1 = 0.01026$

$\lambda_1 = 0.0034195$

$\lambda_2 = 0.0017097$

$\lambda_3 = 0.0051293$

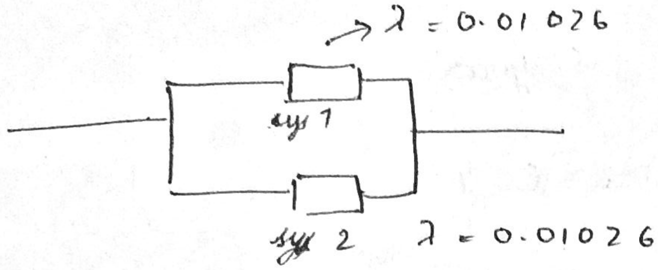
a)

$MTTF_{C1} = \frac{1}{\lambda_1} = 292.44$ yr

$MTTF_{C2} = \frac{1}{\lambda_2} = 584.89$ yr

$MTTF_{C3} = \frac{1}{\lambda_3} = 194.958$ yr b)

i)




$$R'_{sys}(t) = 1 - (1 - R(t))^{n} \quad n = 2$$

$$R'(sys)(5) = 1 - (1 - \underbrace{e^{-0.01026 \times 5}}_{0.95 \text{ (given)}})^2$$

$$= \underline{\underline{0.9975}}$$

ii) $$MTTF_{sys'} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{(\lambda_1 + \lambda_2)} \qquad \# \lambda_1 = \lambda_2$$

$$= \frac{2}{\lambda_1} - \frac{1}{2\lambda_1}$$

$$= 194.93177 - 48.73294$$

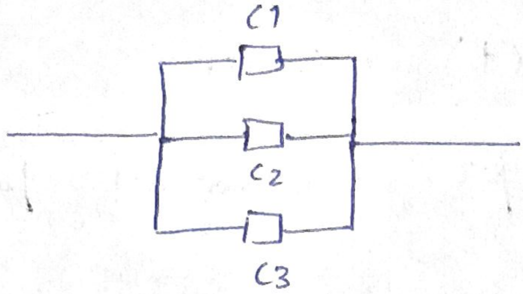
$$= \underline{\underline{146.19882}} \sim 147 \text{ years}$$

Q6]

$\lambda_1 = 5$

$\lambda_2 = 10$

$\lambda_3 = 15$




$R_1(t) = e^{-5t}$ ; $R_2(t) = e^{-10t}$ , $R_3(t) = e^{-15t}$

$$R_{sys} = 1 - (1-R_1)(1-R_2)(1-R_3)$$

$$R_{sys}(0.1) = 1 - (1-e^{-0.5})(1-e^{-1})(1-e^{-1.5})$$

$$= \underline{\underline{0.80677}}$$

$$MTTF_{sys} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} - \frac{1}{(\lambda_1+\lambda_2)} - \frac{1}{(\lambda_2+\lambda_3)} - \frac{1}{(\lambda_3+\lambda_1)}$$

$$+ \frac{1}{(\lambda_1+\lambda_2+\lambda_3)}$$

$$= \frac{1}{5} + \frac{1}{10} + \frac{1}{15} - \frac{1}{15} - \frac{1}{25} - \frac{1}{20} + \frac{1}{30}$$

$$= \frac{1}{4} - \frac{1}{25} + \frac{1}{30} = \underline{\underline{0.2433}}$$

9.7]

$R_c(t) = 0.40$

for n channel parallel system

$$R_{sys}(t) = 1 - (1 - R_c(t))^n$$

$$0.8 \le 1 - (1 - R_c)^n$$

$$0.8 \le 1 - (1 - 0.4)^n$$

$$(0.6)^n \le 0.2$$

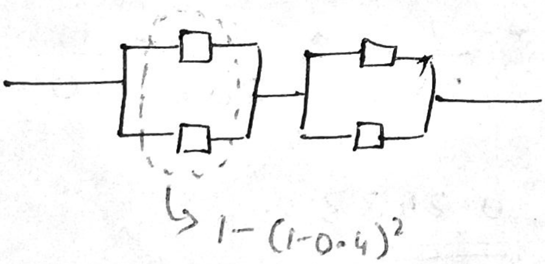
$$n \ln(0.6) \le \ln(0.2)$$

$$n \ge \frac{\ln(0.6)}{\ln(0.2)} = 3.150$$

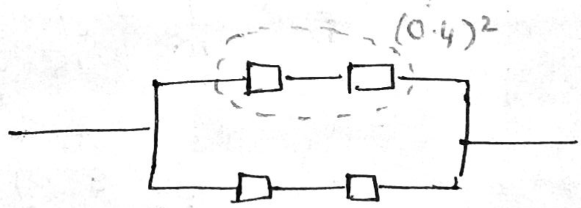
$n \ge 4$ (practically) $n \in \mathbb{Z}$

For 4 channels $\longrightarrow$ factors = $2 \times 2$

LOW LEVEL REDUNDANCY




HIGH LEVEL REDUNDANCY




$$R_{sys} = \left[1 - (1-0.4)^2\right]^2$$

$$= \underline{\underline{0.4096}}$$

$$R_{sys} = 1 - \left[1 - (0.4)^2\right]^2$$

$$= \underline{\underline{0.2944}}$$

Q8] $t = 100$ hr

i) $MTTF = \frac{1}{\lambda} = 1000$ hr

$$\lambda = \frac{1}{1000}$$

$$R_c(t) = e^{-\lambda t} = e^{-\frac{1}{1000}t}$$

$$R_c(100) = e^{-0.1}$$

$$R_{sys1} = 1 - (1-e^{-0.1})^2 = \underline{\underline{0.99094}}$$

ii) Shape $= \beta = 2$

Scale $= \theta = 10{,}000$ h

$$R_{c_1} = e^{-(t/\theta)^\beta} = e^{-(100/10{,}000)^2}$$

$$= 0.999900005$$

$$R_{c_2} = e^{-0.00005 \times 100}$$

$$= 0.995012$$

$$R_{sys2} = R_{c1} \times R_{c2} = \underline{\underline{0.994912}}$$

$\therefore$ Sys 2 has higher Realiability at end of 100 hr