Algorithms and Problem Solving (15B11CI411)
Tutorial – 10
Week 10 (26-31 March, 2018)
(Backtracking/Dynamic Programming)

Q1. You have given an array of n integers. Elements in this array are randomly arranged. Propose a backtracking based algorithm to compute the decreasing sequence (not necessarily contiguous) of the array elements. The desired decreasing sequence should be the longest among all such sequences. E.g. for array, A = {8, 3, 12, 10, 7, 6, 14, 5, 17, 21, 23, 2, 4, 13, 3, 1}, the output should be {12, 10, 7, 6, 5, 4, 3, 1}.

Q2. You have given an arithmetical expression involving numbers (+ve integers) and two operators, + and *, e.g. Expr = 2 + 4 * 3 * 0 + 2 * 7 + 8. After evaluating the example expression, we will get the value of example expression as 24. However, we can change its value by introducing some parenthesis in between and form a new expression (we are not allowed to change the order / place of numbers and operators), e.g.

(a) Expr1 = (2 + (4 * 3)) * 0 + (2 * 7) + 8 and its value is 22

(b) Expr2 = (2 + 4) * 3 * (0 + 2) * (7 + 8) and its value is 2016.

Similarly, there can be other placements and some value might be computed for the new expression. Propose a backtracking based algorithm to place parenthesis in between the operands and operators to form new expression so that the value of new expression is highest among all possible expressions.

Q3. Apply the dynamic programming algorithm to find all the solutions to the change-making problem for the denominations 1, 3, 5 and the amount $n = 9$.

Q4. Design a dynamic programming algorithm for the following problem. Find the maximum total sale price that can be obtained by cutting a rod of n units long into integer-length pieces if the sale price of a piece i units long is pi for i = 1, 2, . . . , n. What are the time and space efficiencies of your algorithm?

Q5. Design an efficient algorithm for computing the binomial coefficient C(n, k) that uses no multiplications. What is the time efficiency of algorithm?

Q6. Professor Adam has two children who, unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately both the professor's house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school.
The professor has a map of his town. Show how to formulate the problem of determining whether both his children can go to the same school as a **maximum-flow problem**.