

Department of Mathematics

Odd Semester 2017

Probability and Random Processes

15B11MA301

Probability and Random Processes

10B11MA411

Tutorial Sheet 7

B.Tech. Core

Random Process

1. Define a random process and classify them with suitable examples.
2. In an experiment of two fair dice, the process $\{X(t)\}$ is defined as $X(t) = \sin \pi t$, if the experiment shows a prime sum and $X(t) = 2t + 1$, otherwise. Find the mean of the process. Is the process stationary? [Ans: not stationary]
3. Let $X(t) = A \cos \lambda t + B \sin \lambda t$, with random variable A taking values 1 and 3 with equal probabilities and random variable B taking values -1 and 1 with probabilities $\frac{1}{4}$ and $\frac{3}{4}$ respectively. Test the process $\{X(t)\}$ for stationarity. [Ans: not stationary]
4. Test the random processes $\{X(t)\}$ and $\{Y(t)\}$ for WSS when:
 - (i) $X(t) = \cos(\lambda t + Y)$, where λ is a constant and Y is uniform in $(0, 2\pi)$
[Ans: WSS]
 - (ii) $Y(t) = X \sin(\lambda t)$, where λ is a constant and X is uniform in $(-1, 1)$.
[Ans: not WSS]
5. Find auto correlation functions of the processes $\{X(t)\}$ and $\{Y(t)\}$ such that $X(t) = A \cos \lambda t + B \sin \lambda t$ and $Y(t) = B \cos \lambda t - A \sin \lambda t$, where A and B are uncorrelated random variables taking value -4 and 4 with equal probabilities. Prove that $\{X(t)\}$ and $\{Y(t)\}$ are jointly WSS.
6. Given a random variable Y with characteristic function $\phi(2w) = E(e^{iwY})$ and a random process defined by $X(t) = \cos(3t + 2Y)$. Find the condition under which the process $\{X(t)\}$ is WSS. [Ans: $\phi(4) = \phi(8) = 0$]
7. If $\{X(t)\}$ is a WSS process with $E\{X(t)\} = 2$ and $R_{XX}(\tau) = 4 + e^{-|\tau|/10}$, find the variance of $X(1)$, $X(2)$ and $X(3)$. Also compute the second order moment about origin of $X(1) + X(2) + X(3)$.
8. Define a Random walk and prove that the limiting form of a random walk is Wiener process.