Tutorial Sheet-ODD Semester 2022

15B11CI212 Theoretical Foundation of Computer Science

Tutorial 8 Solution

Q.1 Find the sum-of-products expansions of these Boolean functions.

a. F (x, y, z) = x + y + z
b. F (x, y, z) = (x + z)y
c. F (x, y, z) = x
d. F (x, y, z) = x y

Solution:

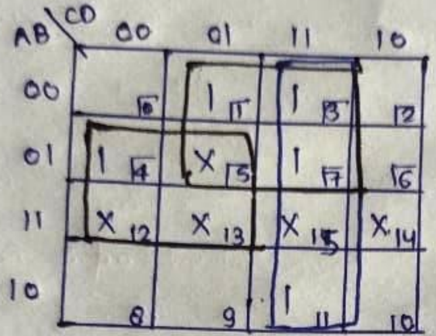
a) We want the function to have the value 1 whenever at least one of the variables has the value 1. There are seven minterms that achieve this, so the sum has seven summands: $xyz+xy\overline{z}+x\overline{y}z+\overline{x}yz+x\overline{y}\overline{z}+\overline{x}y\overline{z}+\overline{x}\overline{y}z$.
b) Here is another way to think about this problem (rather than just making a table and reading off the minterms that make the value equal to 1). If we expand the expression by the distributive law (and use the commutative law), we get $xy+yz$. Now invoking the identity laws, the law that $s+\overline{s}=1$, and the distributive and commutative laws again, we write this as $xy1+1yz=xy(z+\overline{z})+(x+\overline{x})yz=xyz+xy\overline{z}+xyz+\overline{x}yz$. Finally, we use the idempotent law to collapse the first and third term, to obtain our answer: $xyz+xy\overline{z}+\overline{x}yz$.

c) We can use either the straightforward approach or the idea used in part **(b)**. The answer is $xyz+xy\overline{z}+x\overline{y}z+x\overline{y}\overline{z}$.
d) The method discussed in part **(b)** works well here, to obtain the answer $x\overline{y}z+x\overline{y}\overline{z}$.

Q.2 Minimizing a Function with Don't Cares.

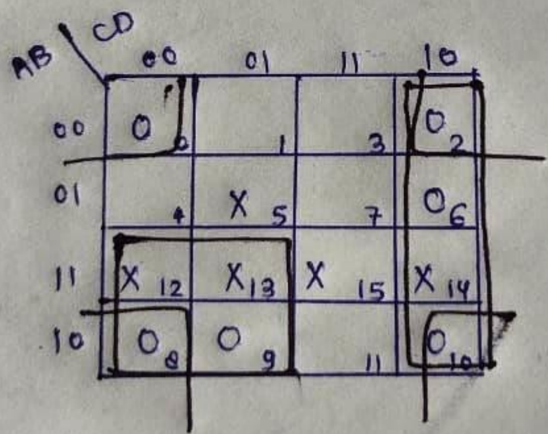
i. $f(A,B,C,D) = \sum_m(1,3,4,7,11) + d(5,12,13,14,15)$
ii. $f(A,B,C,D) = \prod_M(0,2,6,8,9,10) \cdot D(5,12,13,14,15)$

Q.2 (i) $f(A,B,C,D) = \Sigma m(1,3,4,7,11) + d(5,12,13,14,15)$



$f = A'D + BC' + CD$

(ii) $F(A,B,C,D) = \pi m(0,2,6,8,9,10) \cdot D(5,12,13,14,15)$



$f = (B+D) \cdot (C'+D) \cdot (A'+C)$

Q.3 Example 5: Simplify the Boolean expressions: (i) (X +Y) (X+ ~ Y) (~ X +Z) (ii) XYZ + X ~ Y Z + XY ~ Z
Solution:

(i) First simplify (X + Y) (X + ~ Y)

(X + Y) (X + ~ Y) = XX + X ~ Y + YX + Y ~ Y

= X + X ~ Y + YX +O, as XX =X as Y ~ Y = 0

= X + X(~ Y + Y), as ~ Y + Y =1

$= X + X . 1,$ as $X .1 = X$

$= X + X$

$= X$

Now $(X + Y) (X + \sim Y) (\sim X +Z)$

$=X(\sim X + Z)$

$=X \sim X +XZ,$ by distributive law

$= 0 + XZ$

$= XZ$

(ii) $XYZ + X \sim Y Z + XY \sim Z$

$=XZ (Y + \sim Y) + XY \sim Z$

$=XZ + XY \sim Z$, as $Y + \sim Y =1$

$=X (Z +Y \sim Z)$

$= X[(Z +Y). (Z + \sim Z)]$, (By Rule dual of distributive

$= X [(Z + Y). 1]$

$= X (Z + Y) =X (Y + Z)$, by commutative law

Q..4 Minimize the following expression by use of Boolean rules.

(a) $X = A B C + \sim A B + A B \sim C$

(b) $X = \sim A B \sim C + A \sim B \sim C + \sim A \sim B \sim C + \sim A \sim B \sim C$

Solution:

(a) $X =ABC + \sim A B + AB \sim C$

$= ABC + AB \sim C + \sim A B$

$= AB (C + \sim C) + \sim A B$

$=AB + \sim A B$ as $C + \sim C = 1$

$= (A + \sim A) B.$

$= 1. B$

$=B$

(b) $X = \sim A B \sim C + A \sim B \sim C + \sim A \sim B \sim C + \sim A \sim B \sim C$

$= \sim A B \sim C + A \sim B \sim C + \sim A \sim B \sim C$ as $\sim A + \sim A = \sim A$

$= \sim A B \sim C +(A + \sim A) \sim B \sim C$

$= \sim A\ B \sim C + 1 . B \sim C$

$= (\sim A\ B + \sim B) \sim C = [(\sim A + \sim B) . (\sim B + \sim B)] \sim C$ by the dual of distribution rules

$= (\sim A + \sim B) . 1] \sim C$

$= (\sim A + \sim B) \sim C$