Jaypee Institute of Information Technology, Noida
End Term Examination, 2017
B.Tech. IInd Semester

Course Title: Mathematics-2/Mathematics-II **Max Marks: 35**
Course Code: 15B11MA211/10B11MA201 **Time: 2 Hours**

Note: Attempt ALL questions. Marks are indicated against each question.

1. (a) Find the bilinear transformation which maps $z_1 = 1,\ z_2 = -1,\ z_3 = 0$ onto $w_1 = i,\ w_2 = 1,\ w_3 = \infty$. **(2)**

(b) Express $6P_3(x) + 4P_2(x) - 2P_1(x) - 3P_0(x)$ in powers of x. **(2)**

(c) Compute the residues of the function $f(z) = \dfrac{e^z}{(z+2)^2 (z+3)}$ at all of its singular points. **(2)**

(d) Classify the following partial differential equation: **(2)**

$$(1-x)\frac{\partial^2 u}{\partial x^2} + 2y\frac{\partial^2 u}{\partial x \partial y} + (1+x)\frac{\partial^2 u}{\partial y^2} = 0.$$

2. Solving the following differential equation by removing the first order derivative (normal form):

$$(x\log x)^2\, y'' - 2(x\log x)\, y' + \left(2 + \log x - 2(\log x)^2\right) y = x^2 (\log x)^3.$$ **(4)**

3. A rectangular plate with insulated surfaces is $\pi\ cm$ wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If its temperature along one short edge $y = 0$ is given by $u(x,0) = 100\sin x,\ 0 < x < \pi$ while other two long edges $x = 0$ and $x = \pi$ as well as the other short edge are kept at $0^0 C$, find the steady state temperature at any point of the plate. **(4)**

4. Let **(4)**

$$f(z) = \begin{cases} \dfrac{xy^2(y - ix)}{x^2 + y^4}, & z \neq 0 \\ 0, & z = 0. \end{cases}$$

Show that Cauchy-Riemann equations are satisfied at z=0. Also, show that $f'(0)$ does not exist.

5. If $f(z) = u(x,y) + i\,v(x,y)$ is analytic and $u + v = x^2 - y^2 + x - y + 2xy$, find the function $f(z)$. **(3)**

6. State and prove Cauchy's Integral Formulas. Hence evaluate the integral **(5)**

$$\oint_C \frac{1}{(z-1)^2 (2z+3)}\, dz,\quad C: |z+i| = \sqrt{3}.$$

7. Find Laurent series for $f(z) = \dfrac{z}{z^2 + z - 6}$ in the following regions: **(3)**

(i) $2 < |z| < 3$, (ii) $|z| > 3$, (iii) $|z| < 2$.

8. Evaluate the integral $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{5 + 3\cos\theta}\, d\theta$ using Residue Theorem. **(4)**
