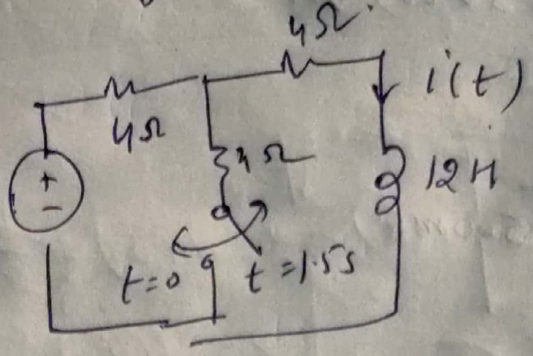
TUTE2_SOLUTION

Sequential Switching

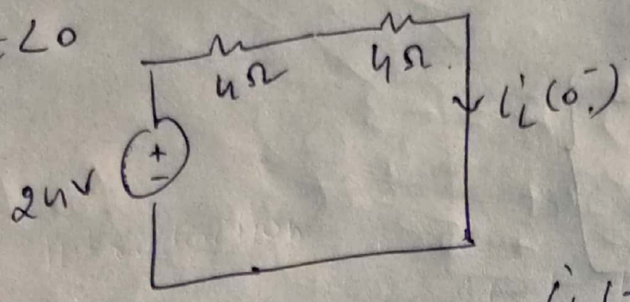
Q: Sol

Q-1

(1)



$t<0$



$i_L(0^-) = \frac{24}{8} = 3A$

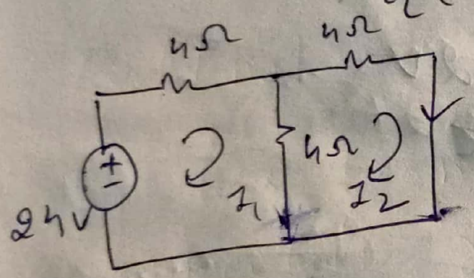
$i_L(\infty) = 3A$

$i_L(t) = i(\infty) - [i(\infty) - i(0^+)]e^{-t/\tau}$

$i_L(t) = 3A$

$0<t<1.5s$

$i_L(0^+) = 3A$



$i_L(\infty) = ?$

$24 - 4I_1 - 4(I_1 - I_2) = 0$

$-4I_2 - 4(I_2 - I_1) = 0$

$I_2 = 2A, \; I_1 = 4A$

$\boxed{i_L(\infty) = I_2 = 2A}$

$\tau = \frac{L}{R_{th}} = \frac{12}{6\Omega} = 2\,sec$

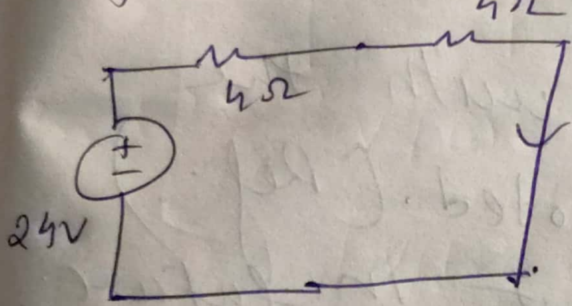
$R_{th} = \frac{4\times4}{8} + 4 = 2+4 = 6\Omega$

$i(t) = 2 - (2-3)e^{-t/2}$

$\boxed{i(t) = 2 + e^{-0.5t}\ A}$

Scanned with CamScanner

for $t > 1.5$ sec.



$$i(1.5^-) = 2 + e^{-0.5(1.5)}$$

$$i(1.5^-) = 2.47\,A = i(1.5^+)$$

$$i(\infty) = \frac{24}{8} = 3\,A$$

$$\tau = \frac{12}{8} = 1.5 \text{ sec.}$$

$$i(t) = 3 + (2.47 - 3)\,e^{-\frac{t-1.5}{1.5}}$$

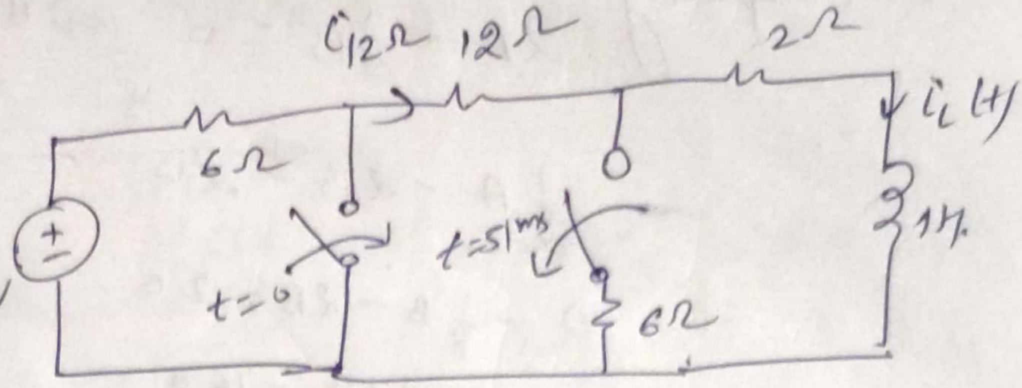
$$\boxed{i(t) = 3 - 0.53\,e^{-0.67(t-1.5)}\ A}$$

$$i(t) = \begin{cases} 3 & t < 0 \\ 2 + e^{-0.5t}\ A & 0 < t < 1.5\,s \\ 3 - 0.53\,e^{-0.67(t-1.5)}\ A & t > 1.5\,s \end{cases}$$

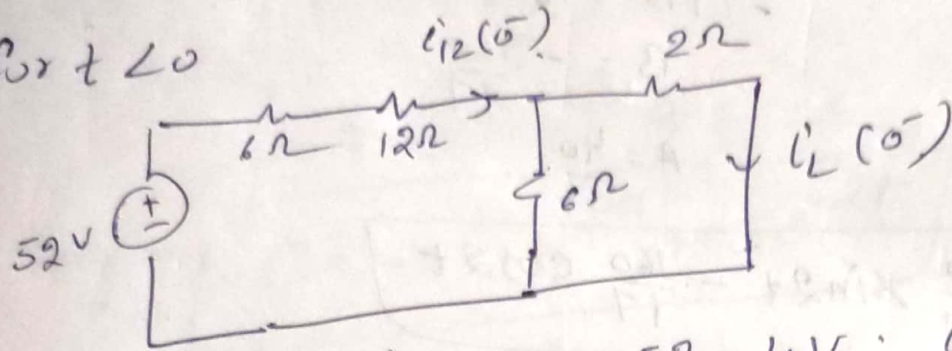
Q. 2

$i(t) = ?$

$i_{12}(t) = ?$



for $t < 0$



$$V_{2\Omega} = \frac{2\|6}{2\|6 + (6+12)} \times 52 = 4V; \quad i_L(0^-) = \frac{4}{2} = 2A$$

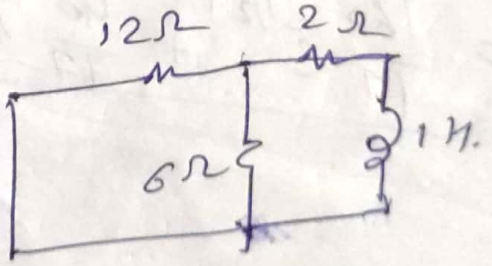
$$V_{12\Omega} = \frac{12}{2\|6 + (6+12)} \times 52 = 32V; \quad i_{12}(\Omega) = 2.67A$$

$i_L(\infty) = 2A$

$i(t) = i(\infty) - (i(\infty) - i(0^+))e^{-t/\tau}$

$\boxed{i(t) = 2A}$ for $t < 0$ & $\boxed{i_{12}(t) = 2.67A}$

for $0 < t < 0.051s$



$i_L(0^-) = i_L(0^+) = 2A$

$i_L(\infty) = 0$

$R_{th} = 6\Omega$; $\tau = \frac{L}{R} = \frac{1}{6}$

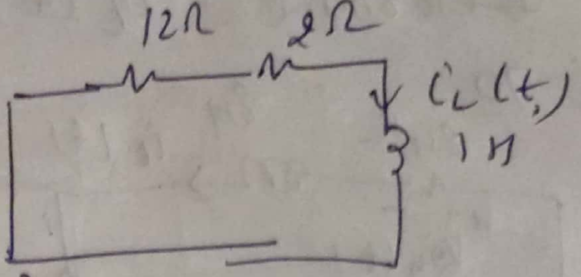
$\boxed{i_L(t) = 2e^{-6t}}$ A

$i_{12}(0 < t < 0.051s)$

$= \frac{6}{6+12} \times i_L(t)$

$\boxed{i_{12} = \frac{2}{3}e^{-6t} \text{ A}}$

for $t > 0.051$ sec.



$i_L(\infty) = 0$

$i_L(0.051^-) = 2e^{-6(0.051)} = 1.473\,A$

$R_{th} = 14\Omega$

$i_L(t) = 0-(0-1.473)e^{-14(t-0.051)}\,A$

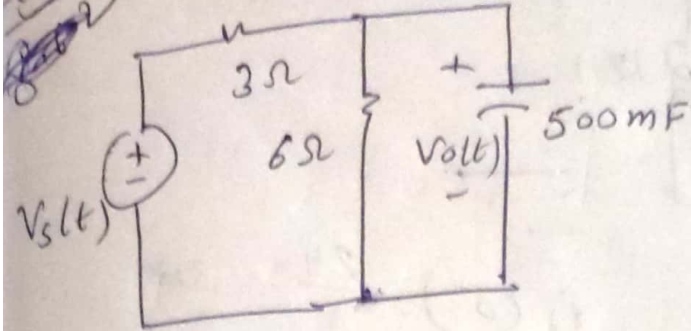
$i_L(t) = 1.473\,e^{-14(t-0.051)}\,A = i_{12}(t)$

$$i_L(t) = \begin{cases} 2 & t < 0 \\ 2e^{-6t} & 0 < t < 0.051 \\ 1.473e^{-14(t-0.051)} & t > 0.051 \end{cases}$$

$$i_{12}(t) = \begin{cases} 2.67\,A & t < 0 \\ \frac{2}{3}e^{-6t}\,A & 0 < t < 0.051 \\ 1.473\,e^{-14(t-0.051)}\,A & t > 0.051 \end{cases}$$

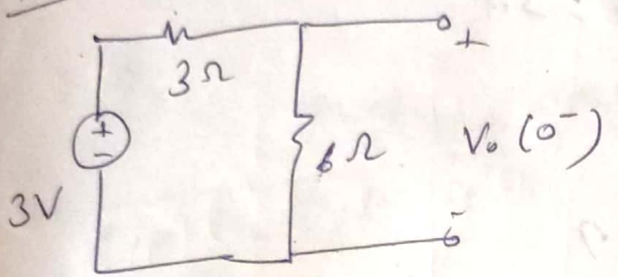
unit step

Q3) $V_o(t) = ?$, $V_s = 3 + 3u(t)$ V



Sol. $V_s = \begin{cases} 3 & t < 0 \\ 6 & t > 0 \end{cases}$

for $t < 0$,



$V_o(0^-) = \frac{6}{9} \times 3 = 2V$

$V_o(0^+) = 2V$

$V_o(\infty) = 2V$

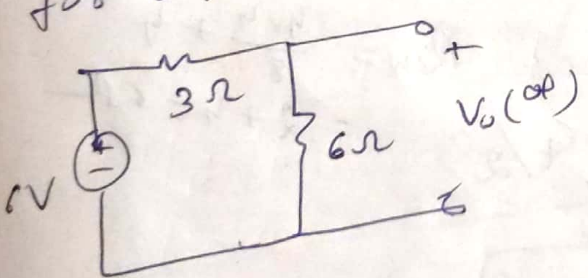
$V(t) = 2 - (2 - 2)e^{-t/\tau}$

$\boxed{V(t) = 2V}$

$V_o(\infty) = \frac{\cancel{6}^{2}}{\cancel{9}_{3}} \times \cancel{6}^{2}$

$\Rightarrow V(\infty) = 4V$

for $t > 0$



$R_{th} = \frac{3 \times 6}{9} = 2\Omega$

$C = 500mF$

$\tau = R_{th} \times C = 1$ Sec.

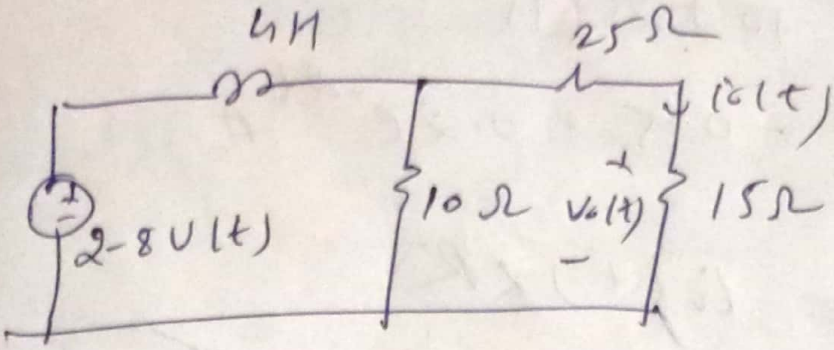
$V(t) = 4 - (4 - 2)e^{-t}$

$\boxed{V(t) = 4 - 2e^{-t}}$

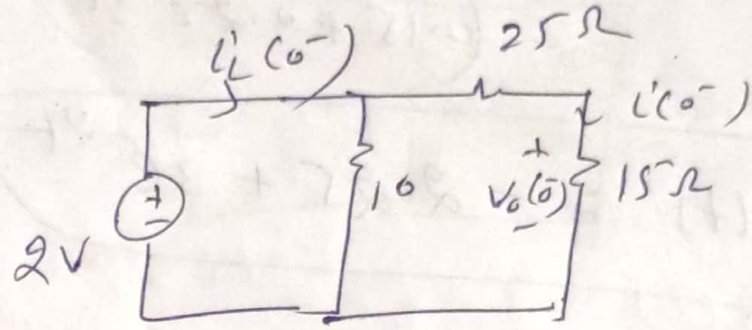
Ans:

$V(t) = \begin{cases} 2V & t < 0 \\ 4 - 2e^{-t} \ V & t > 0 \end{cases}$

Q.4. $v_o(t) = ?$

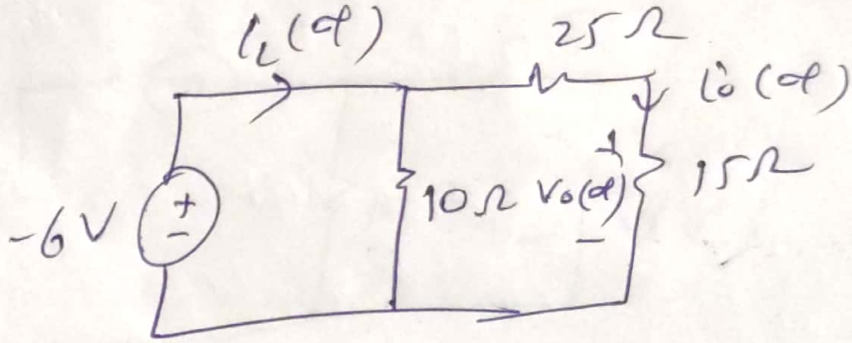


$t < 0$



$i_L(0^-) = \frac{2}{8} = \frac{1}{4}A$

$t > 0$



$i_L(\infty) = \frac{-6}{8}$

$i_L(\infty) = \frac{-3}{4}$

$R_{th} = \frac{(25+15)\times 10}{25+15+10} = 8\Omega \Rightarrow \tau = \frac{L}{R} = \frac{4}{8} = 0.5$

$i_L(t) = i_L(\infty) - (i_L(\infty) - i_L(0^+))e^{-t/\tau}$

$\boxed{i_L(t) = -0.75 + e^{-2t}}$

$$i_o(t) = \frac{10}{10+25+15} \times i_L(t)$$

$$= -0.15 + 0.2e^{-2t} \ A$$

$$V_o(t) = i_o(t) \times R$$

$$= (0.15 + 0.2e^{-2t}) \times 15$$

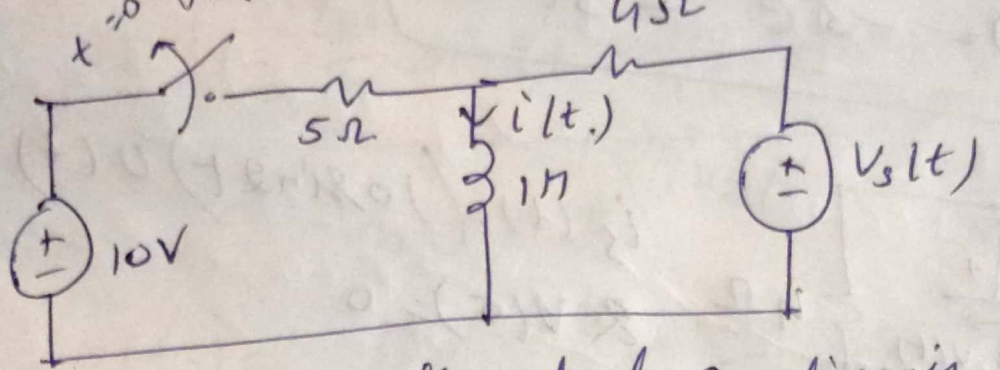
$$\boxed{V_o(t) = -2.25 + 3e^{-2t}} \ V$$

$2.25 - 0.75$

Q8 :-

for $V_s = 10e^{-2t}u(t)V$

find $i(t) = ?$



for $t \geq 0$, differential Equation is

$$\frac{di}{dt} + 4i = 10e^{-2t} \quad \text{--- (1)}$$

$i(t) = i_n(t) + i_f(t)$

<u>for $i_f(t)$</u>

$i_f = Be^{-2t}$, which satisfy Equ (1)

$$-2Be^{-2t} + 4Be^{-2t} = 10e^{-2t}$$

$$2B = 10 \Rightarrow \boxed{B = 5}$$

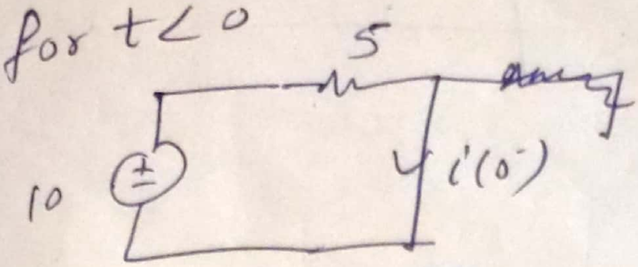
$$\boxed{i_f(t) = 5e^{-2t}}$$

$$i_n(t) = Ae^{-4t}$$

$$\boxed{i(t) = Ae^{4t} + 5e^{-2t}} \quad \text{--- (2)}$$

Now, for initial condition

for $t<0$



$i(0^-) = \frac{10}{5} = 2A$

$= i(0^+)$

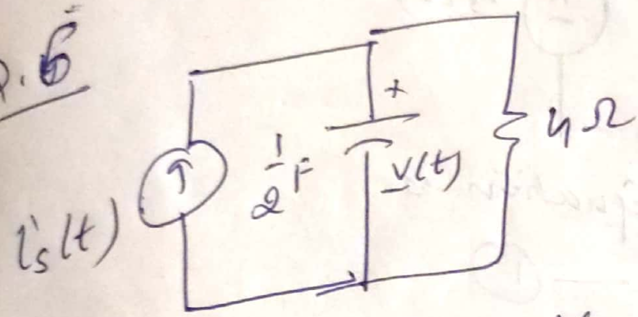
Put $t=0^+$ in Eqn (2)

$i(0^+) = A + 5$

$2 = A + 5 \Rightarrow A = -3$

therefore $\boxed{i(t) = -3e^{-4t} + 5e^{-2t}}$ A for $t>0$

Q.6



$i_s(t) = (10\sin 2t)\,u(t)$ A

& $V(0^-) = 0$

for $t>0$

$$0.5\frac{dV}{dt} + \frac{V}{4} = 10\sin 2t$$

$$\frac{dV}{dt} + \frac{V}{2} = 20\sin 2t \quad \text{--- (1)}$$

$$V(t) = V_n(t) + V_f(t)$$

$V_f(t) = A\sin 2t + B\cos 2t$, which must satisfy Eqn (1)

$$(2A\cos 2t - 2B\sin 2t) + \frac{1}{2}(A\sin 2t + B\cos 2t) = 20\sin 2t$$

$$\left(2A + \frac{1}{2}B\right)\cos 2t + \left(\frac{1}{2}A - 2B\right)\sin 2t = 20\sin 2t$$

on comparing $2A + \frac{1}{2}B = 0 \Rightarrow A = -\frac{1}{4}B$

$$\frac{1}{2}A - 2B = 20$$

$$\Rightarrow -\frac{1}{8}B - 2B = 20$$

$$-17B = 160$$

$$B = -\frac{160}{17}$$

$$A = \frac{40}{17}$$

$$\boxed{V_f(t) = \frac{40}{17}\sin 2t - \frac{160}{17}\cos 2t}$$

$$V_n(t) = \cancel{A}De^{-1/2t}$$

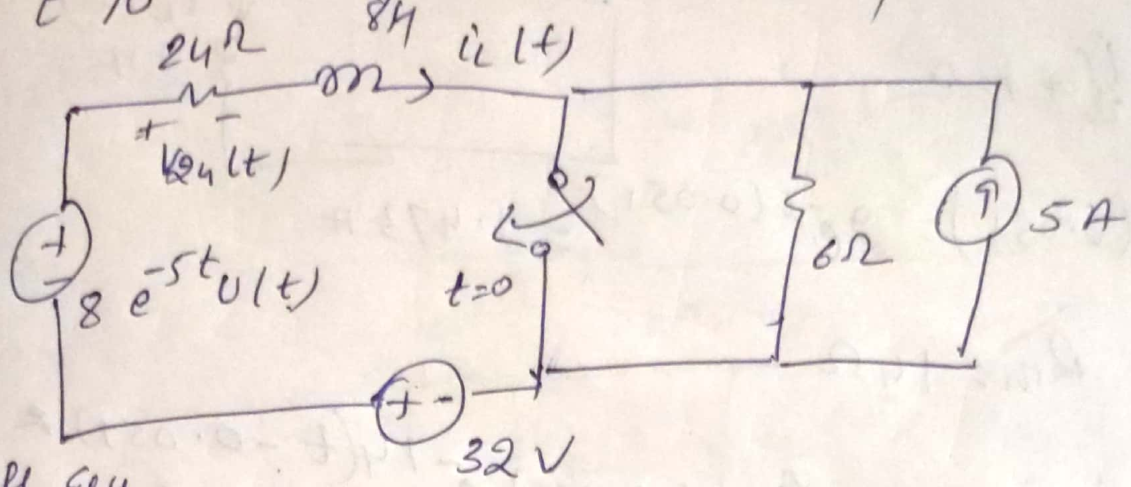
$$v(t) = \cancel{A}De^{-1/2t} + \frac{40}{17}\sin 2t - \frac{160}{17}\cos 2t$$

Put $t=0$ as $v(0^+)=0$

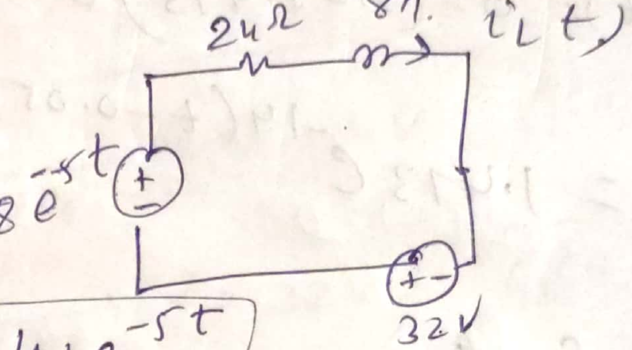
$$0 = \cancel{A}D - \frac{160}{17} \Rightarrow \cancel{A}D = \frac{160}{17}$$

$$\boxed{v(t) = \frac{160}{17}e^{-1/2t} + \frac{40}{17}\sin 2t - \frac{160}{17}\cos 2t}$$

Q7: Determine $i_L(t)$ & $V_{24}(t)$ for $t > 0$



Diff Equ
for $t > 0$



$$\boxed{\frac{di_L(t)}{dt} + 3i_L(t) = 4 + e^{-5t}} \quad — (1)$$

$$i_L(t) = i_f(t) + i_n(t)$$

$i_f(t) = B_1 e^{-5t} + B_2$, Substitute in Equ (1)

$$-5B_1 e^{-5t} + 3B_2 + 3B_1 e^{-5t} = 4 + e^{-5t}$$

$$\Rightarrow \boxed{B_2 = \frac{4}{3}} ; \boxed{B_1 = -\frac{1}{2}}$$

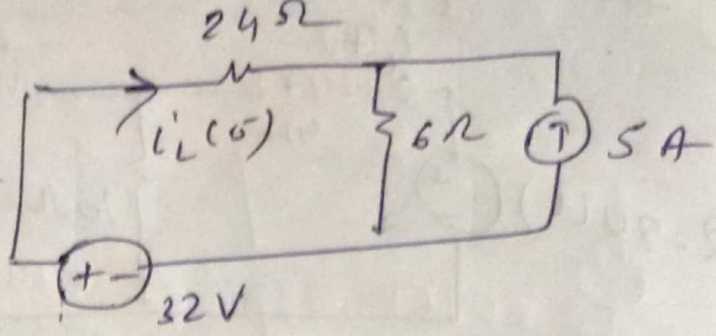
$$\boxed{i_f(t) = -\frac{1}{2}e^{-5t} + \frac{4}{3}}$$

$$i_n(t) = Ae^{-3t}$$

$$\boxed{i_L(t) = Ae^{-3t} + \frac{1}{2}e^{-5t} + \frac{4}{3}} \quad — (2)$$

for initial condition.

$t<0$



$$i_L(0^-) = \frac{32-30}{6+24} = \frac{1}{15}\,A$$

$$\boxed{i_L(0^-) = i_L(0^+) = \frac{1}{15}\,A}$$

Put $t=0^+$ & $i_L(0^+)$ in Equ (2)

$$\frac{1}{15} = A - \frac{1}{2} + \frac{4}{3} \Rightarrow \boxed{A = -\frac{23}{30}}$$

$$\boxed{i_L(t) = \frac{4}{3} - \frac{1}{2}e^{-5t} - \frac{23}{30}e^{-3t}}$$

$$\boxed{V_{24\Omega} = 24\, i_L(t) = 32 - 12e^{-5t} - \frac{92}{5}e^{-3t}\ V}$$

Since 24Ω is in series with 8H. therefore, behavior of $V_{24}(0^-)$ & $V_{24}(t)$ should be continous at $t=0$