Solution Tute-2, Physics-2, Even 2022

① (a) $\nabla \cdot \vec{E} = \rho/\varepsilon_0$; $\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

(b) Total charge inside

(c) Nothing

(d) $\nabla \cdot \vec{E} = \rho/\varepsilon_0 \Rightarrow \nabla \cdot (-\nabla V) = \rho/\varepsilon_0 \Rightarrow \nabla^2 V = -\rho/\varepsilon_0$

(e) $\nabla \times \vec{E} = 0$ and $\vec{E} = -\nabla V$

② (a) As we move farther and further away from the plane, more and more charge comes into our 'field of view' (a cone shape extending out from our eye) and this compensates for the diminishing effect of any particular piece. Similarly for infinite line charge.

(b) Discontinuous ; σ/ε_0 ; always continuous

(c) $V(b) - V(a) = \int_a^b \vec{E} \cdot d\vec{l} = 0 \Rightarrow V(a) = V(b)$ as $\vec{E}$ within or at the surface of conductor is zero.

(d) $\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n}$; $\vec{f} = P = \frac{1}{2\varepsilon_0} \sigma^2 \hat{n} = \frac{\varepsilon_0}{2} E^2$

③ $\rho = \varepsilon_0 \nabla \cdot \vec{E} = \varepsilon_0 \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 k r^3) = 5 \varepsilon_0 k r^2$

$$Q_{enc} = \varepsilon_0 \oint_S \vec{E} \cdot d\vec{a} = \varepsilon_0 \int_0^{2\pi}\int_0^{\pi} (k R^3 \hat{r}) \cdot (R^2 \sin\theta \, d\theta \, d\phi) \hat{r} = 4\pi \varepsilon_0 k R^5$$

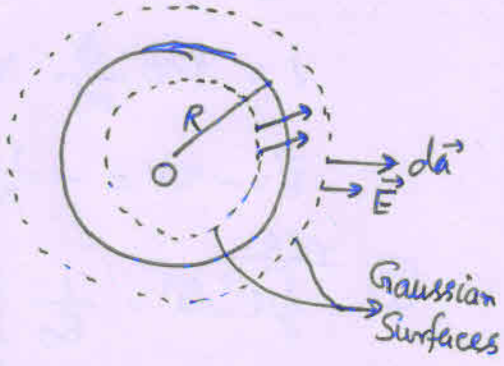
or

$$Q_{enc} = \int \rho \, dV = \int_0^R (5 \varepsilon_0 k r^2)(4\pi r^2 dr) = 4\pi \varepsilon_0 k R^5.$$

④ $\underline{r > R}$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho \, dV$$

$$\int_0^{2\pi}\int_0^{\pi} |\vec{E}| \, r^2 \sin\theta \, d\theta \, d\phi = \frac{1}{\varepsilon_0} \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$$



$$|\vec{E}|(\cancel{4\pi} r^2) = \frac{\rho}{3\varepsilon_0} (\cancel{4\pi} R^3)$$

$$\vec{E}_{r>R} = \frac{\rho}{3\varepsilon_0} \frac{R^3}{r^2} \hat{r}$$

$\underline{r = R}$

$$\vec{E}_{r=R} = \frac{\rho R}{3\varepsilon_0} \hat{r}$$

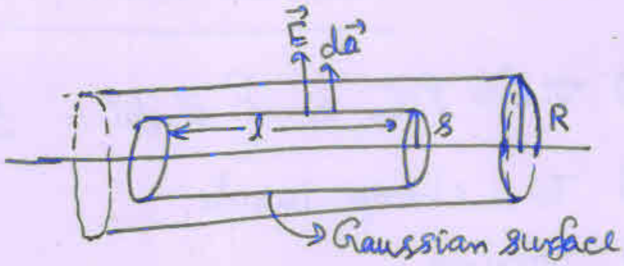
$\underline{r < R}$

$$|\vec{E}|(\cancel{4\pi} r^2) = \frac{\rho}{3\varepsilon_0} (\cancel{4\pi} r^3)$$

$$\vec{E}_{r<R} = \frac{\rho}{3\varepsilon_0} r \, \hat{r}$$

(5)

$s < R$

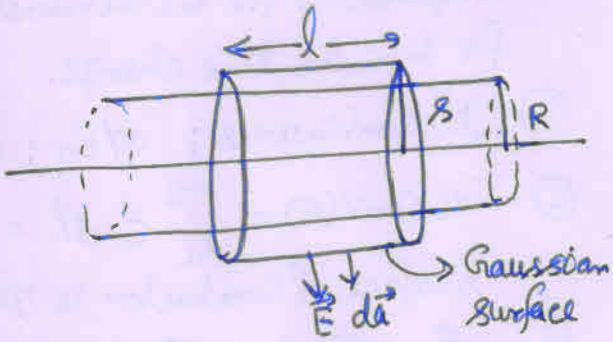


$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int_0^{l}\int_0^{2\pi}\int_0^{s} \rho \; s\, ds\, d\phi\, dz$$

$$|\vec{E}|(2\pi s l) = \frac{\rho}{\varepsilon_0}(\pi s^2 l)$$

$$\vec{E}_{s<R} = \frac{\rho}{2\varepsilon_0}\, s\, \hat{s}$$

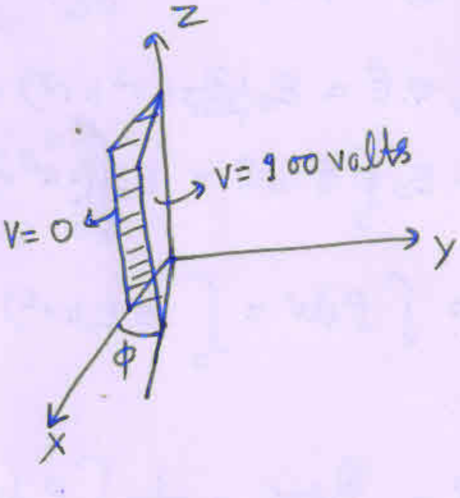
$s > R$



$$|\vec{E}|(2\pi s l) = \frac{\rho}{\varepsilon_0}(\pi R^2 l)$$

$$\vec{E}_{s>R} = \frac{\rho}{2\varepsilon_0}\,\frac{R^2}{s}\,\hat{s}$$

(6) Since the potential is constant in s and z, the Laplace's eq.



$$\frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

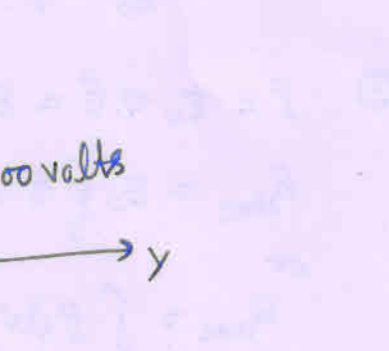
$$V = A\phi + B$$

$$B = 0, \quad A = 100/\alpha$$

$$V = 100\,\frac{\phi}{\alpha} \text{ Volt}$$

$$\vec{E} = -\nabla V = -\frac{1}{s}\frac{\partial}{\partial \phi}\left(100\,\frac{\phi}{\alpha}\right) = -\frac{100}{s\alpha}\,\hat{\phi} \quad \left(\frac{V}{m}\right)$$

(7) $r < R$



$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^{r} k r \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$

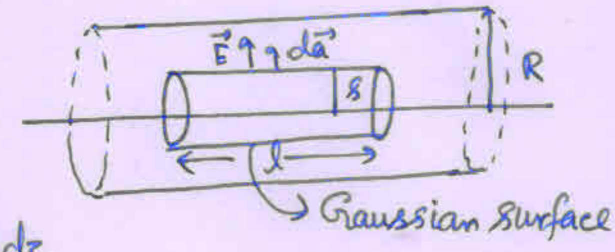
$$|\vec{E}|(4\pi r^2) = \frac{k}{\varepsilon_0}\,\frac{4\pi r^4}{4}$$

$$\vec{E}_{r<R} = \frac{1}{4\pi\varepsilon_0}\,\pi k r^2\,\hat{r}$$

$r > R$

$$|\vec{E}|(4\pi r^2) = \frac{k}{\varepsilon_0}\,\frac{4\pi R^4}{4} \Rightarrow \vec{E}_{r>R} = \frac{\pi k R^4}{4\pi\varepsilon_0}\,\frac{1}{r^2}\,\hat{r}$$

(8) $\underline{s<R}$



$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\iiint \rho\, dV$$

$$|\vec{E}|\,(2\pi s l) = \frac{k}{\varepsilon_0}\int_0^{l}\int_0^{2\pi}\int_0^{s} s\, s\, ds\, d\phi\, dz$$

$$|\vec{E}|\,(2\pi s l) = \frac{k}{\varepsilon_0}\,\frac{2\pi s^3 l}{3}$$

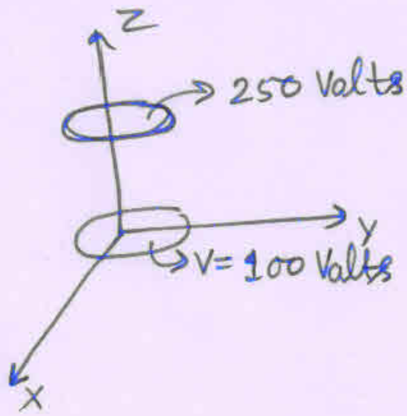
$$\vec{E}_{s<R} = \frac{k}{3\varepsilon_0}\, s^2\, \hat{s}$$

$\underline{s>R}$

$$|\vec{E}|\,(\cancel{2\pi} s \cancel{l}) = \frac{k}{\varepsilon_0}\,\frac{\cancel{2\pi} R^3 \cancel{l}}{3}$$

$$\vec{E}_{s>R} = \frac{kR^3}{3\varepsilon_0}\,\frac{1}{s}\,\hat{s}$$

(9) Since the potential is z dependent only, the Laplace's eq.



$$\frac{\partial^2 V}{\partial z^2} = 0 \Rightarrow \frac{\partial V}{\partial z} = A \quad \text{——} \ (1)$$

$$V = Az + B$$

$\underline{\text{From (1)}}$

$$\frac{\partial V}{\partial z} = A = \frac{250-100}{5\times10^{-3}} = 3\times10^{4}\ \text{V/m}$$

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial z} = -3\times10^{-4}\ \hat{z}\ \text{V/m}$$

$$\vec{D} = \varepsilon_0\varepsilon_r\vec{E} = -5.84\times10^{-7}\ \hat{z}\ \text{C/m}^2$$

Since $\vec{D}$ is constant between the plates and

$|\vec{D}| = \sigma$ at the conductor surface.

$$\sigma = \pm\ 5.84\times10^{-7}\ \text{C/m}^2$$

+ve on upper plate and −ve on lower plate.