Department of Mathematics

15B11MA211 **Mathematics-2**
B.Tech. Core

Tutorial Sheet 2 (Differential Equations with Variable Coefficients)

1. Solve the following ordinary differential equations when an integral of complementary function is known
   i. $y'' + y = \sec x$,
   ii. $x^2 y'' + xy' - y = 2x^2$,
   iii. $xy'' + (1-x)y' - y = e^x$.
2. By changing the dependent variable solve the following differential equation
   $$y'' - 2\tan x\, y' + 8y = e^x \sec x.$$
3. Solve the following differential equations by changing the independent variable
   $$xy'' - y' - 4x^3 y = 8x^3 \sin x^2.$$
4. Using the method of variation of parameters find the general solution of
   $$y'' - 6y' + 9y = \frac{e^{3x}}{x}.$$
5. To apply variation of parameters, find two linearly independent solutions of the corresponding homogeneous differential equation of
   $$(x^2+1)\, y'' - 2xy' + 2y = 6(x^2+1)^2.$$

Answers:

1.
   i. $y = x\sin x + \cos x \log\cos x + a\sin x + b\cos x$,
   ii. $y = \dfrac{2}{3}x^2 - \dfrac{a}{2x} + bx$,
   iii. $y = e^x \log x + a e^x \displaystyle\int \frac{e^{-x}}{x}\,dx + b e^x$.
2. $y = \sec x\left(C_1 \cos 3x + C_2 \sin 3x + \dfrac{e^x}{10} \right)$.
3. $y = C_1 e^{x^2} + C_2 e^{-x^2} - \sin x^2$.
4. $y = (C_1 + C_2 x)e^{3x} + (\log x - 1)\, x e^{3x}$.
5. $\phi = (x^2 - 1)$, $\psi = x$.