Practice Sheet -2

Q1.) $fx(x) = \frac{d}{dx}\{1-(1+x)e^{\wedge}-x)$

$\Rightarrow e^{-x}-(-x-1)e^{-x}$

$\Rightarrow xe^{-x}$

To find the mean of X, we integrate x times the pdf of x from 0 to infinity

$E(X) = \int_{\infty}^{0} x(xe^{-x})\,dx \Rightarrow 2$

To find variance,

$var(X) = E(X^2) - E(X)^2$

$E(X^2) = \int_0^{\infty} x^2(xe^{-x})\,dx \Rightarrow \boxed{2}$

Q2) $fx(x) = 0.1e^{-0.1x}, x>0$

$E(X) = \int_0^{\infty} x\,fX(x)\,dx$

$10 = \int_0^{\infty} x(0.1e^{-0.1x})\,dx$

$1 = \int_0^{\infty} e^{-0.1x}\,dx$

$1 = 10$

which is not correct

$\therefore E(x) = \int_0^{\infty} x(0.1e^{-0.1x}\,dx \Rightarrow \boxed{10}$

Avg Power = 10 W

$\therefore \int 0.1e^{-0.1x}\,dx \Rightarrow e^{-1} \Rightarrow \boxed{0.368}$

Q3.) $var(X) = E[X^2] - E[X]^2 = 1$

$var(X) \neq 1$

$Cov(x, y) = E[XY]$

$Corr(X, Y) = Cov(X, Y) / (StdDev(X) * StdDev(Y))$

$\Rightarrow -0.5 * 1 * 1 = -0.5$

$VAR(X + 4Y) = 1 + 16 + 8(-0.5) = 13$

Therefore, the variance of $X + 4Y$ is 13

Q4.) (i) $C = 3/4$ Using the total probability over entire domain is equal to 1.

(ii) $P(X > 1, Y < 1/2) = \frac{9}{32}$, Integrating $f(x,y)$ over the given region.

(iii) $E(Y/X = 1) = \frac{1}{2}$; Finding the conditional PDF of Y given X=1 using Bayes' Theorem, and then finding the expected value of Y using this PDF.

(iv) The conditional PDF of X given $Y = 5$ is not defined. Using Bayes' Theorem, but the denominator is zero.

Q6.) The given moment generating function (MGF) is matched with the MGF of a Poisson distribution with parameter λ. we find that X can take values 0, 1, 2, 3, 4 with respective probabilities approximately 0.1493, 0.3188, 0.3022, 0.1699 and 0.0597.

- Therefore, the distribution of X is approx a discreet distribution.

Q5.) pmf of RV (X) is $P(X=j) = \frac{1}{2^j} \Rightarrow j = 1, 2, 3 \dots$

Charge function,

$$\phi(t) = E(e^{itx})$$

$$= \sum_{j=1}^{\infty} \left[e^{itj} \times \frac{1}{2^j} \right] = \sum_{j=1}^{\infty} \left(\frac{(e^{it})^j}{2} \right)$$

$$= \frac{e^{it}}{2} + \frac{e^{2it}}{4} + \frac{e^{3it}}{8} + \dots \quad (\infty, G, P)$$

$$\phi_x(t) = \frac{e^{it}}{2-e^{it}} \Rightarrow \phi'_x(t) = \frac{2ie^{it}}{(e^{it}-2)^2}$$

For mean, $t=0$ $\phi(0) = \frac{2 \cdot ie^0}{(1-2)^2} = 2i$

$\phi''(0) = 6$

$var(x) = \phi''(0) - (2)^2 = 6 - 4 \Rightarrow 2$

$var(x) = 2$

As $\phi(0) = 2i$

$mean = E(x) = -i\phi'(0)$

$mean = +2$