# ELECTRICAL SCIENCE - II (15B11EC211)



## Solution

## Tutorial - 3

Q1. Solution :-

The KCL node equation at the upper node is

$$\frac{v - v_s}{R_1} + i + C\frac{dv}{dt} = 0$$

To determine equation in terms of $v$, we need a second equation in terms of current $i$. Equation for current through inductor is

$$Ri + L\frac{di}{dt} = v.$$

Using the operator $s = \frac{d}{dt}$, two equations are

$$\frac{v}{R_1} + Csv + i = \frac{v_s}{R_1}$$

and $$-v + Ri + Lsi = 0$$

Substituting parameter values and rearranging we have,

$$(10^{-3} + 10^{-3}s)\,v + i = 10^{-3}v_s$$

$$-v\,(10^{-3}s + 1)\,i = 0$$

Using cramer's rule, solve for $v$

$$v = \frac{(s+1000)v_s}{(s+1)(s+1000)+10^6} = \frac{(s+1000)v_s}{s^2+1001s+1001\times10^3}$$

Therefore, we have

$$s^2 + 1001s + 1001\times10^3]\,v = (s+1000)v_s$$

The differential equation is

Ans = $$\boxed{\frac{d^2v}{dt^2} + 1001\frac{dv}{dt} + 1001\times10^3 v = \frac{dv_s}{dt} + 1000v_s}$$

Q2 Solution:- Given

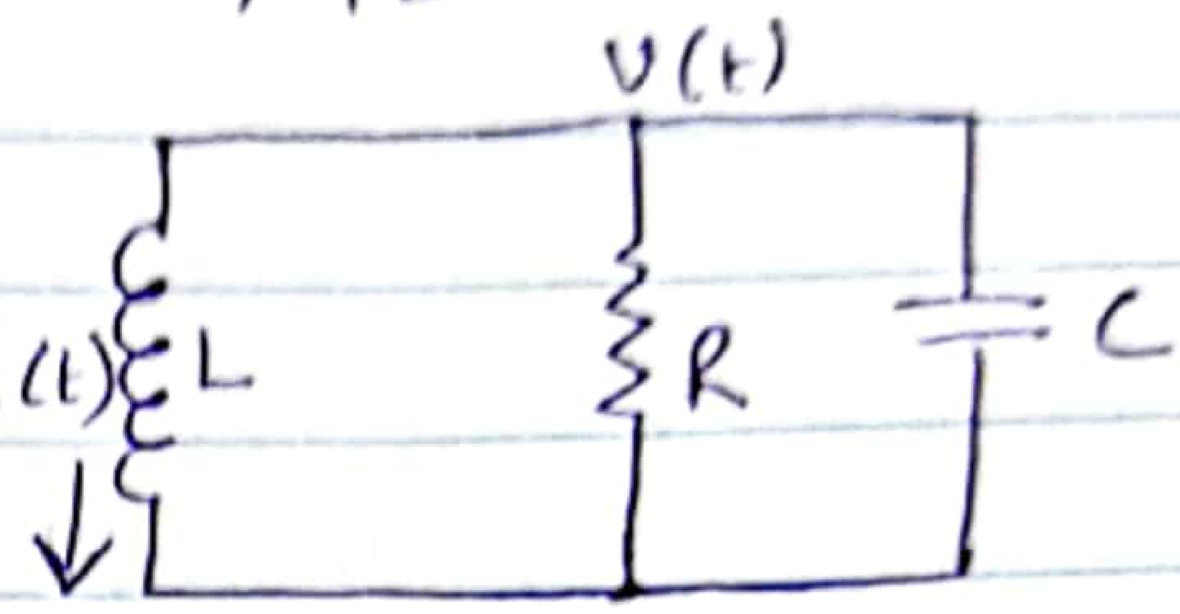
$R = 6\,\Omega$, $L = 7\,H$ & $C = 1/42\,F$

Initial cond$^n$

$V(0) = 0$

$i(0) = 10\,A$

$v_n(t) = ?$




$$\frac{dV(0)}{dt} = -\left[\frac{V(0) + R\,i(0)}{RC}\right]$$

$$= -\left[\frac{0 + 6\times 10}{6\times\frac{1}{42}}\right] = -420\ V/s$$

for source free parallel RLC ckt, two roots -

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 6\times\frac{1}{42}} = \frac{42}{12} = 3.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{6\times\frac{1}{42}}} = \sqrt{6} = 2.42$$

$\alpha > \omega_0$ overdamped case.

$$S_{1,2} = -3.5 \pm \sqrt{(3.5)^2 - 6} = -1, -6$$

$$S_1 = -1 \quad \& \quad S_2 = -6$$

for $t > 0$

$$V_n(t) = A_1 e^{-1t} + A_2 e^{-6t}$$

Differentiating $V_n(t)$

$$\frac{dv_n(t)}{dt} = -A_1 e^{-t} - 6e^{-6t}$$

Applying initial conditions

$V(0) = A_1 + A_2$ ---- (1)

$\frac{dv(0)}{dt} = -420 = -A_1 - 6A_2$ --- (2)

Solving eqn (1) & (2)

$A_1 = -84$ & $A_2 = 84$

so

$V_n(t) = -84e^{-t} + 84e^{-6t}$

Ans $\boxed{V_n(t) = -84(e^{-t} - e^{-6t})\ A.}$

Q3 Solution :-

Given $R = 10\Omega$, $C = 1mF$, $L = 0.4H$

Initial condition

$V(0) = 8V$ & $i(0) = 0$

We know that

$S_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$

$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10 \times 1 \times 10^{-3}} = 50$

$W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4 \times 1 \times 10^{-3}}} = 50$ rad/sec.

$\alpha = W_0 \Rightarrow$ Critically damped case.

$S_1 = S_2 = -\alpha = -50$

for ~~that~~ $t > 0$

$V_n(t) = (A_1 + A_2 t)e^{-\alpha t} = (A_1 + A_2 t)e^{-50t}$

$$V(0) = 8 = (A_1 + A_2 \times 0)e^0$$

$$A_1 = 8$$

$$\frac{dv(0)}{dt} = -\left[\frac{V(0) + Ri(0)}{RC}\right]$$

$$= -\left[\frac{8 + 10 \times 0}{10 \times 10^{-3}}\right]$$

$$\frac{dv(0)}{dt} = -800 \text{ v/s}$$

$$\frac{dV(t)}{dt} = (A_1 + A_2 t)(-50e^{-50t}) + A_2 e^{-50t}$$

$$\frac{dv(0)}{dt} = -800 = (8 + A_2 \times 0)(-50e^0) + A_2$$

$$-800 = -8 \times 50 + A_2$$

$$A_2 = -400$$

So Natural response

Ans $\boxed{V_n(t) = (8 - 400t)e^{-50t} \text{ Volt}}$

Q4 solution :

Given $R = 25/3\ \Omega$, $L = 0.1H$, $C = 1mF$

Initial condition

$V(0) = 10V$ & $i(0) = -0.6A$

First, we determine $\alpha^2$ & $\omega_0^2$

$$\alpha = \frac{1}{2RC} \quad \& \quad \omega_0^2 = \frac{1}{LC}$$

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$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times \frac{25}{3} \times 1 \times 10^{-3}} = 60$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{0.1 \times 1 \times 10^{-3}} = 10^4$$

$\omega_0^2 > \alpha^2$, so the Natural response is underdamped.

Damped resonant frequency $\omega_d$ is

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10^4 - 3.6 \times 10^3}$$

$$= 80 \text{ rad/s}.$$

The ~~Cara~~ characteristic roots are

$$s_1 = -\alpha + j\omega_d = -60 + j80$$
$$s_2 = -\alpha - j\omega_d = -60 - j80$$

The natural response is

$$v_n(t) = B_1 e^{-60t} \cos 80t + B_2 e^{-60t} \sin 80t$$

Because $v(0) = 10$, we have

$$B_1 = v(0) = 10$$

By using equation directly to find $B_2$

$$B_2 = \frac{\alpha}{\omega_d} B_1 - \frac{v(0)}{\omega_d RC} - \frac{i(0)}{\omega_d C}$$

$$= \frac{60 \times 10}{80} - \frac{10}{80 \times 25/3000} - \frac{-0.6}{80 \times 10^{-3}}$$

$$= 7.5 - 15.0 + 7.5 = 0$$

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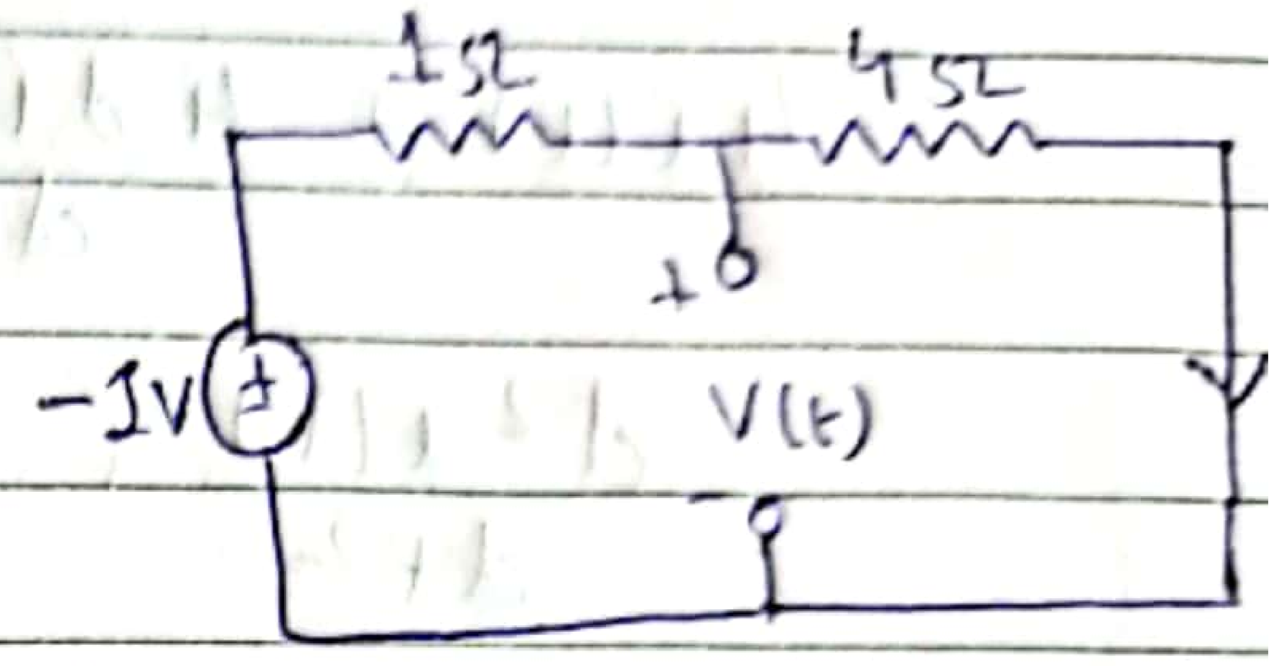
Therefore, natural response is

Ans $\boxed{v_n(t) = 10e^{-60t} \cos 80t \text{ V}}$

Q5) Solution :-

for $t < 0$, $u(t) = 0$




circuit will be in steady state at $t = 0^-$ and hence capacitor will acts as open circuit while inductor will act as short circuit.

$$i(0^-) = \frac{-1}{1+4} = -\frac{1}{5} = -0.2\,A$$

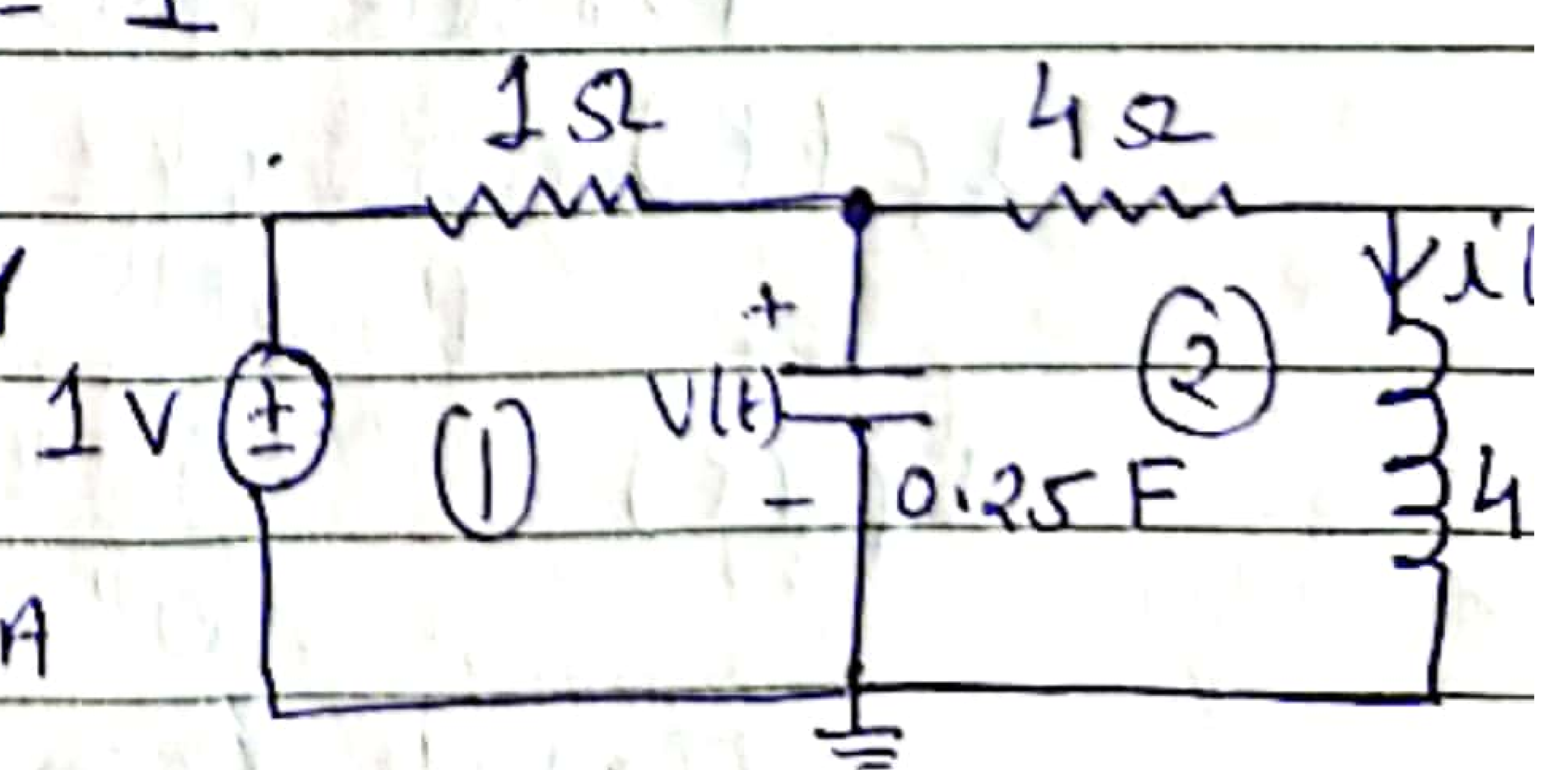
NOW

$$V(0^-) + R\,i(0^-) + 1 = 0$$

$$V(0^-) + 1 \times \left(-\frac{1}{5}\right) + 1 = 0$$

$$V(0^-) = -\frac{4}{5} \text{ volt.}$$

for $t > 0$. $u(t) = 1$




$$V(0^+) = V(0) = V(0^-) = \frac{4}{5}\cancel{V}$$

$$i(0) = i(0^-) = i(0^+) = -0.2A$$

Applying KVL in loop (2)

$$V(t) = 4i(t) + 4\frac{di(t)}{dt} \quad \text{—} \quad (1)$$

Applying KCL at top node

$$\frac{v(t)-1}{1} + 0.25\frac{dv(t)}{dt} + i(t) = 0 \quad -② $$

using eq ① & ②

$$4i(t) + 4\frac{di(t)}{dt} + 0.25\frac{d}{dt}\left[4i(t) + 4\frac{di(t)}{dt}\right] + i(t) = 0$$

$$\frac{d^2i(t)}{dt^2} + 5\frac{di(t)}{dt} + 5i(t) = 1$$

$$s = \frac{d}{dt}$$

$$(s^2 + 5s + 5)\,i(t) = 1$$

To find roots

$$s^2 + 5s + 5 = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s = -3.62, -1.38$$

Natural response

$$i_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$i_n(t) = A_1 e^{-3.62t} + A_2 e^{-1.38t} \text{ Ampere.}$$

Forced response

$$i_f(t) = \frac{1}{s^2+5s+5} \times 1e^0$$

$$i_f(t) = \frac{1}{0^2 + 0\times 5 + 5} = \frac{1}{5} = 0.2 \text{ Ampere}$$

Total Response.

$$i(t) = i_n(t) + i_f(t)$$

$$i(t) = A_1 e^{-3.62t} + A_2 e^{-1.38t} + 0.2$$

To find $A_1$ & $A_2$ initial condition is used ie, $i(0) = -0.2\,A$.

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from eqn (1)

$$V(0) = 4\,i(0) + 4\frac{d\,i(0)}{dt} \Rightarrow \frac{d\,i(0)}{dt} = \frac{V(0) - 4i(0)}{4}$$

$$\frac{d\,i(0)}{dt} = \frac{-\frac{4}{5} - 4\times(-\frac{1}{5})}{4} = \frac{0}{5\times4} = 0\ \text{A/sec.}$$

from (2)

$$i(0) = -0.2 = A_1 + A_2 + 0.2 \Rightarrow A_1 + A_2 = -0.4 \quad \text{---(3)}$$

$$\frac{d\,i(t)}{dt} = -3.62A_1 e^{-3.62t} - 1.38A_2 e^{-1.38t}$$

$$\frac{d\,i(0)}{dt} = -3.62A_1 - 1.38A_2$$

$$0 = -3.62A_1 - 1.38A_2 \Rightarrow \cancel{A_2 =}\ 3.62A_1 + 1.33A_2 = 0 \quad \text{---(4)}$$

on Solving (3) & (4)

$$A_1 = 0.246, \quad A_2 = -0.646$$

Hence

Ans $$\boxed{i(t) = 0.246\,e^{-3.62t} + (-0.646)A_2^{-1.35t} + 0.2\ \text{Amp}}$$

Q6 Solution :-

Given at $t=0^-$ the ckt is at steady state

at $t=0$, the voltage source is disconnected and the current source is connected.

$$V(0) = 10V \quad \& \quad i(0) = 0\ A.$$

Consider the circuit after time $t=0$. The first differential eqn is obtained by using KVL around the RLC mesh

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$$L\frac{di}{dt} + Ri = v$$

The second differential eq^n is obtained by using KCL at the node at top of capacitor

$$C\frac{dv}{dt} + i = i_s$$

We rewrite the two first-order differential eq^n

$$\frac{di}{dt} + \frac{R}{L}i - \frac{v}{L} = 0$$

and $$\frac{dv}{dt} + \frac{i}{C} = \frac{i_s}{C}$$

Substituting the values, we have

$$\frac{di}{dt} + 3i - v = 0 \quad \text{— (1)}$$

& $$\frac{dv}{dt} + 2i = 2i_s \quad \text{— (2)}$$

Using operator $s = d/dt$

$$(s+3)i - v = 0 \quad \text{— (4)}$$

$$2i + sv = 2i_s \quad \text{— (5)}$$

The characteristic eqn obtained from the determinant is

$$(s+3)s + 2 = 0$$

$$s^2 + 3s + 2 = 0$$

Thus roots are

$$s_1 = -2 \quad \& \quad s_2 = -1$$

To solve for $i(t)$ at $t>0$, we use cramer rule to solve eq (4) & (5)

$$i = \frac{2i_s}{s^2+3s+2}$$

Therefore, the differential eq^n is

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Therefore, differential eqn is

$$\frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i = 2i_s \quad \text{— (6)}$$

The natural response is

$$i_n = A_1 e^{-t} + A_2 e^{-2t}$$

we assume the forced response is of form

$$i_f = Be^{-3t}$$

Substituting $i_f$ in eq (6)

$$(9Be^{-3t}) + 3(-3Be^{-3t}) + 2Be^{-3t} = 2(2e^{-3t})$$

$$9B - 9B + 2B = 4 \quad \Rightarrow B = 2$$

therefore

$$i_f = 2e^{-3t}$$

The Complete respone is:

$$i = A_1 e^{-t} + A_2 e^{-2t} + 2e^{-3t}$$

As $i(0) = 0$

$$0 = A_1 + A_2 + 2 \quad \text{— (7)}$$

we need to find $\frac{di(0)}{dt}$ from eq (1)

$$\frac{di}{dt} + 3i - v = 0$$

at $t = 0$,

$$\frac{di(0)}{dt} = -3i(0) + v(0) = 10$$

The derivative of complete response at $t = 0$ is

$$\frac{di(0)}{dt} = -A_1 - 2A_2 - 6$$

as $\frac{di(0)}{dt} = 10$

$$-A_1 - 2A_2 = 16 \quad \text{— (8)}$$

using (7) & (8)

$A_1 = 12$ & $A_2 = -14$, Thus complete soln.

Ans. $\boxed{i = 12e^{-t} - 14e^{-2t} + 2e^{-3t} \text{ Amp}}$

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Q7 Solution :

The source current is applied at $t=0$

After $t=0$, the KCL eqn at the upper node is

$$i + \frac{v}{R} + C\frac{dv}{dt} = i_s \quad — (1)$$

We know

$$v = L\frac{di}{dt} \quad — (2)$$

So $$\frac{dv}{dt} = L\frac{d^2 i}{dt^2}$$

using eq (1) & (2)

$$i + \frac{L}{R}\frac{di}{dt} + CL\frac{d^2 i}{dt^2} = i_s$$

Rearranging by dividing by LC

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = \frac{i_s}{LC}$$

Substituting the values we obtain.

$$\frac{d^2 i}{dt^2} + 7\frac{di}{dt} + 6i = 48e^{-2t} \quad — (3)$$

To obtain forced response, we assume

$$i_f = Be^{-2t}$$

Where B is to be determined.

Substituting assumed value in eq (3)

$$4Be^{-2t} + 7(-2Be^{-2t}) + 6Be^{-2t} = 48e^{-2t}$$

or $$(4-14+6)Be^{-2t} = 48e^{-2t}$$

$$\Rightarrow B = -12$$

Ans. $\boxed{i_f = -12e^{-2t} \text{ Amp}}$