

Jaypee Institute of Information and Technology
Department of Mathematics

Course: Matrix Computations (16B1NMA533)

Tutorial Sheet 5 [C301-3.3]

(Topics covered: Vector Space and Subspace)

1. Which of the following subsets of $R^4 = \{(x_1, x_2, x_3, x_4) \mid x_1, x_2, x_3, x_4 \in R\}$ are vector spaces for coordinate-wise addition and scalar multiplication?
- a) $x_4 = 0$. b) $x_1 = 1$. c) $2x_1 + 3x_2 = 0$. d) $x_1 + \frac{2}{3}x_2 - 3x_3 + x_4 = 1$.

Sol. a) Yes b) No c) Yes d) No

2. Which of the following subsets of $P(t)$ are vector spaces? The set of all polynomials P such that
- a) degree of $P \geq 4$. b) degree of $P \leq 4$. c) $P(2) = 0$.

Sol. a) No b) Yes c) Yes

3. Which of the following sets are subspaces of R^3 ?
- a) $\{(x_1, x_2, x_3) \mid x_1 x_2 = 0\}$
b) $\{(x_1, x_2, x_3) \mid x_1 = 2x_2 \text{ and } x_3 = 3x_2\}$
c) $\{(x_1, x_2, x_3) \mid \frac{x_2}{x_1} = \sqrt{2}\}$

Sol. a) No b) Yes c) Yes

4. On R^3 define two operations: $x \oplus y = x - y$, $\alpha \cdot x = \alpha x$, $\alpha \in R$; $x, y \in R^3$. The operations on the right are usual operations. Is R^3 , with respect to \oplus and \cdot defined above, a vector space? If not, which of the axioms are not satisfied?

Sol. No, Properties 2,4,5,7

5. Determine whether $W = \{(a, b, c) : a^2 + b^2 + c^2 \leq 1\}$ is a subspace R^3 .

Sol. No