Department of Mathematics

Probability and Random Processes   **15B11MA301**

Tutorial Sheet 7   **C201.3**

Continuous Distributions (Uniform, Exponential, Erlang, Gamma and Weibull distributions)

1. A man and a Woman agree to meet at a certain place between 10 a.m. and 11 a.m. They agree that the one arriving first will have to wait 15 minutes for the other to arrive. Assuming that the arrival times are independent and uniformly distributed, find the probability that they meet. (Ans.7/16)

2. If the random variable a is uniformly distributed in the interval $(1, 7)$, what is the probability that the roots of the equation $x^2 + 2ax + (2a + 3) = 0$ are real. (Ans.2/3)

3. A straight line of length 4 units is given. Two points are taken at random on this line. Find the probability that the distance between them is greater than 3 units. (Ans.1/16)

4. The daily consumption of milk in excess of 20000 gallons is approximately exponentially distributed with $\lambda = 1/3000$. The city has a daily stock of 35000 gallons. What is the probability that of 2 days selected at random, the stock is insufficient for both days. (Ans. e^{-5})

5. The length of the shower on a tropical island during rainy season has an exponential distribution with parameter 2, time being measured in minutes. What is the probability that a shower will last more than 3 minutes? If a shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute? (Ans. (i) 0.0025, (ii)0.1353)

6. Suppose that X has an exponential distribution with parameter λ. Compute the probability that X exceeds twice its expected value. (Ans. $1/ e^2$)

7. If the service life, in hours, of a semiconductor is a RV having a Weibull distribution with the parameters $\alpha = 0.0375$ and $\beta = 0.55$, (i) How long can such a semiconductor be expected to last? (ii) What is the probability that such a semiconductor will still be in operating condition after 4000h? (Ans. 667 hr, 0.0276)

8. If the life in years of a certain type of taxi has a Weibull distribution with the parameter $\beta = 2$, find the value of the parameter α, given the probability that the life of the taxi exceeds 6 years is $e^{-0.36}$. For these value of α and β, find the mean and variance. (Ans. $\alpha = 0.01, E(X) = 5\sqrt{\pi},\ Var(X) = 100(1 - \frac{\pi}{4})$)

9. The life (in months) of a certain bacteria has an Erlang distribution with parameters $k = 3$ and $\lambda = 1/2$. What is the probability that this bacteria will survive at most one year? (Ans.0.0997)