Jaypee Institute of Information and Technology
Department of Mathematics

Course: Matrix Computations (16B1NMA533)

Tutorial Sheet 7 [C301-3.4]

(Topics covered: Orthogonality, Gram-Schmidt process, QR-decomposition)

1. Let V be an inner product space and let u and v be vectors in V. Suppose that $\|u\| = \sqrt{3},\ \|v\| = 4$ and the angle between u and v is $\pi/6$. Compute $\langle u, v \rangle$ and $\langle u+v, 2u-v \rangle$.

2. Suppose we define the inner product between two continuous functions by $\langle u(x), v(x) \rangle = \int_0^{\pi/2} u(x)v(x)dx.$ If $u(x) = \sin x$ and $v(x) = x,$ find the angle between them.

3. Find vectors $u, v \in R^2$ such that u is a scalar multiple of (1, 3), v is orthogonal to (1, 3), and (1, 2) = u + v.

4. Determine angle between x_1 and x_2, projection of x_1 onto x_2 and its orthogonal component for $(i)\ x_1 = (1, 1, 0),\ x_2 = (2, 2, 1),\quad (ii)\ x_1 = (0, 1, 1, 1),\ x_2 = (1, 1, 1, 0).$

5. Use Gram-Schmidt orthonormalization process to construct an orthonormal set from the given set of linearly independent vectors of real inner product space w. r. t standard inner product. $(i)\ \{(0,1,1,1),(1,1,1,0)\},\quad (ii)\ \{(1,1,0,0),(0,1,-1,0),(0,0,-1,1),(0,1,-1,0)\}.$

6. Determine **QR-** decomposition for the following matrices:

$$(i)\begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}\quad (ii)\begin{bmatrix}1 & 2 & 2\\ 1 & 0 & 2\\ 0 & 1 & 1\end{bmatrix}\quad (iii)\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & -1 & -1\\ 0 & 0 & 0\end{bmatrix}$$