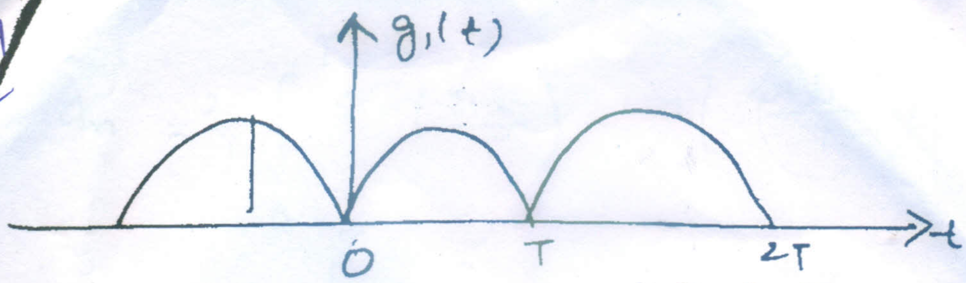
Ans 1



$$a_0 = \frac{1}{T}\int_0^T g_1(t)\,dt$$

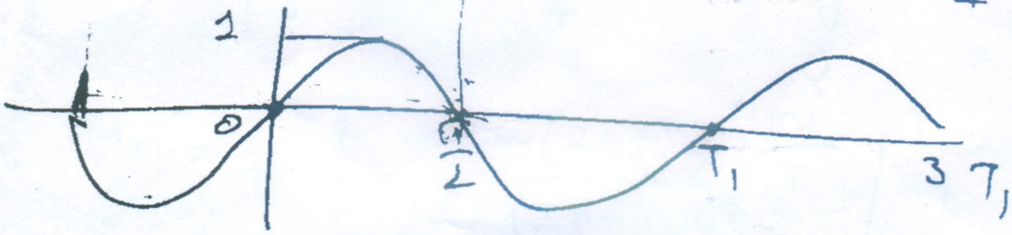
$\sin w_0 t$

$\sin \frac{2\pi}{T} t$

$$g_1(t) = \begin{cases} 1\cdot \sin\left(\frac{2\pi}{T}\cdot t\right) \end{cases}$$

$$1 \sin\left(\frac{\pi}{\pi/2}\cdot t\right)$$

$$1 \sin\left(\frac{2\pi}{T_1}\cdot t\right)$$



$g_1(t) = A\sin(w_0 t)$ where $w_0 = \frac{2\pi}{T_1}$ $\qquad \frac{T_1}{2} = T$

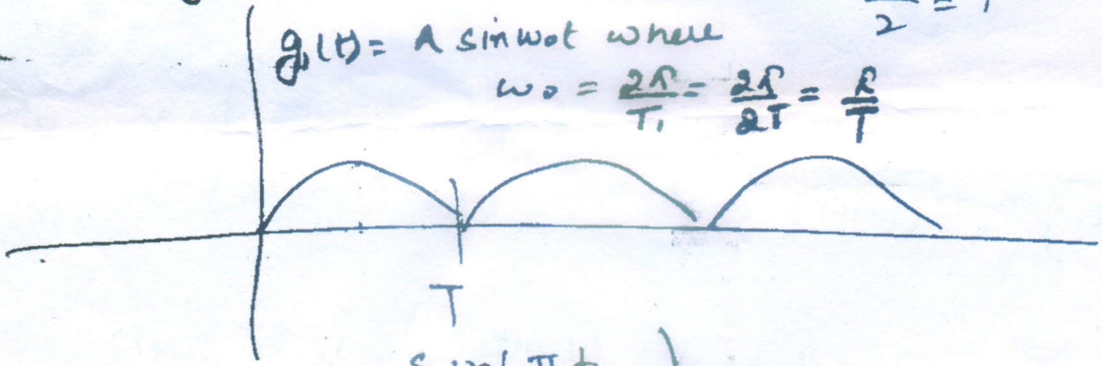
$g_1(t) = A\sin w_0 t$ where
$$w_0 = \frac{2\pi}{T_1} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

$\sin w t$

$\sin \frac{2\pi}{T}\cdot t$

$\sin \frac{2\pi}{2T}\cdot t$

$\sin\left(\frac{\pi t}{T}\right)$



$$\sin\left(\frac{\pi t}{T}\right)$$

$$a_0 = \frac{1}{T}\int_0^T 1 \sin\left(\frac{\pi t}{T}\right) dt$$

$$a_0 = \frac{1}{T} \left. \frac{-\cos\left(\frac{\pi t}{T}\right)}{\frac{\pi}{T}} \right|_0^T = -\frac{1}{T}\left[\frac{-1}{\frac{\pi}{T}} - \frac{1}{\frac{\pi}{T}}\right]$$

$$a_k = \frac{1}{T}\int_0^T \sin\left(\frac{\pi t}{t}\right) e^{-jkw_0 t}\, dt \qquad = \frac{2}{\pi}$$

$$= \frac{2}{T}\int_0^T \sin\left(\frac{\pi t}{T}\right) \cdot \cos\left(\frac{2n\pi t}{T}\right) dt$$

$$n = \frac{2}{T}\int_0^T \frac{1}{2}\left[2 \sin\left(\frac{\pi t}{T}\right) \cos\left(\frac{2\pi n t}{T}\right) dt\right.$$

$$a_n = \frac{1}{T}\int_0^T \sin\left\{(2n+1)\frac{\pi t}{T}\right\} - \sin\left\{(2n-1)\frac{\pi t}{T}\right\}\Big]$$

$$n = \frac{1}{T}\left[\int_0^T \sin\frac{(2\pi+1)\pi t}{T} - \int_0^T \sin\left\{\frac{2n-1}{T}\pi t\right\} dt\right.$$

$$a_n = \frac{1}{T}\left[-\frac{\cos(2n+1)\frac{\pi t}{T}}{\frac{(2n+1)\pi}{T}}\Bigg|_0^T + \frac{\cos(2n-1)\frac{\pi t}{T}}{\frac{(2n-1)\pi}{T}}\right.$$

$$a_n = \frac{1}{T}\left[\frac{T}{\pi(2n+1)} + \frac{T}{(2n+1)} - \frac{T}{(2n-1)\pi} - \frac{T}{(2n-1)\pi}\right.$$

$$a_n = \frac{2\times 1}{T}\left[\frac{(2n-1)-(2n+1)}{(2n+1)(2n-1)\pi}\right]\times T$$

$$a_n = \frac{2}{T}\left[\frac{2n-1-2n-1}{(4n^2-1)\pi}\right]\times T$$

$$a_n = \frac{-4}{\pi(4n^2-1)} = \frac{4}{\pi}\left(\frac{1}{1-4n^2}\right.$$

$$b_n = 0$$

$$2A + \frac{4A}{\pi}\sum \frac{1}{1-4n^2}(\cos n\omega_0 t) \quad \text{where}$$

3

$$a_0 = \frac{1}{T}\int_0^T g_2(t)\, dt$$

$$a_0 = \frac{1}{T}\left[\int_0^{T/4} 1 \cdot dt + \int_{T/4}^{3T/4} -1\, dt + \int_{\frac{3T}{4}}^{T} 1\, dt\right]$$

$$a_0 = \frac{1}{T}\left[\frac{T}{4} - \left(\frac{3T}{4} - \frac{T}{4}\right) + \left(T - \frac{3T}{4}\right)\right]$$

$$a_0 = \frac{1}{T}\left[\frac{T}{4} - \frac{3T}{4} + \frac{T}{4} + T - \frac{3T}{4}\right]$$

$$= \frac{1}{T}\left[\frac{3T}{2} - \frac{3T}{2}\right] \text{ so } a_0 = 0, \quad b_n = 0$$

$$a_n = \frac{2}{T}\int_0^T g_2(t) \cos\left(\frac{2\pi n t}{T}\right) dt$$

$$a_n = \frac{2}{T}\left[\int_0^{T/4} 1 \cdot \cos\left(\frac{2\pi n t}{T}\right) dt + \int_{T/4}^{3T/4} (-1) \cos\left(\frac{2\pi n t}{T}\right) dt\right.$$

$$= \frac{2}{T}\left[\left.\frac{-\sin\left(\frac{2\pi n t}{T}\right)}{\left(\frac{2\pi n}{T}\right)}\right|_0^{T/4} + \left.\frac{\sin\frac{2\pi n t}{T}}{\frac{2\pi n}{T}}\right|_{T/4}^{\frac{3T}{4}} - \left.\frac{\sin\left(\frac{2\pi n t}{T}\right)}{\frac{2\pi n}{T}}\right|_{\frac{3T}{4}}^{T}\right.$$

$$= \frac{2}{T}\left[-\frac{T}{2\pi n}\left[\sin\left(\frac{n\pi}{2}\right) - 0\right] + \frac{T}{2\pi n}\left[\sin\left(\frac{3\pi n}{2}\right) - \sin\left(\frac{n\pi}{2}\right)\right]\right.$$

$$- \frac{T}{2\pi n}\left[\sin(2\pi n) - \sin\left(\frac{3\pi n}{2}\right)\right.$$

$$a_n = \left[\frac{-\sin\frac{n\pi}{2}}{\pi n} + \frac{\sin\frac{3\pi n}{2}}{n\pi} - \frac{\sin\left(\frac{n\pi}{2}\right)}{\pi n} - \frac{\sin(2\pi n)}{n\pi} + \frac{\sin(3n\pi)}{n\pi}\right.$$

$$a_n = \frac{1}{\pi n}\left[-(-1)^{n/2} + (-1)^{3n} - (-1)^n - 0 + (-1)^{3n}\right] \qquad a_n = \frac{4}{\pi}(-1)^{\frac{n-1}{2}}$$

Since this waveform is having half wave symmetry so

a_2, a_4, a_6 will be zero

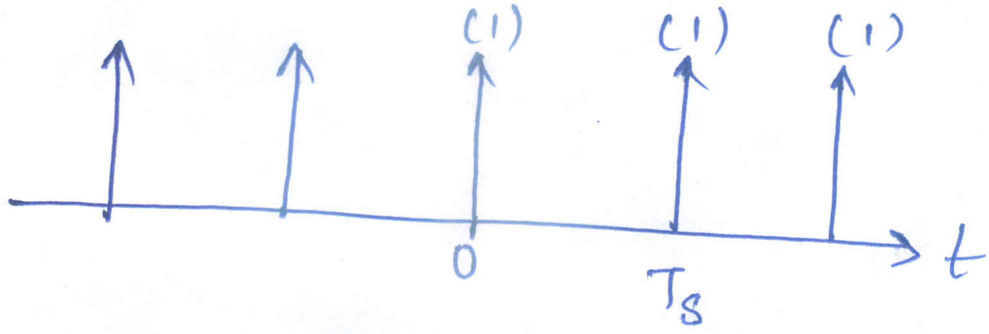
for odd value of n

$$a_n = +\frac{4}{\pi n}(-1)^{\left(\frac{n-1}{2}\right)}$$

$$g_2(t) = \frac{4}{\pi}\cos\omega t - \frac{4}{\pi}\frac{\cos 3\omega t}{3} + \frac{4}{\pi}\cdot\frac{1}{5}\cos 5\omega t$$

Ans-2 Fourier Series of unit impulse train with time period T_s

Impulse train is represented as,



$$x(t) = \sum_{k=-\alpha}^{\alpha} \delta(t - kT_s) = \sum_{k=-\alpha}^{\alpha} C_k e^{jk\omega_0 t}$$

where, $\omega_0 = \dfrac{2\pi}{T_s}$

$$C_k = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} x(t)\, e^{jk\omega_0 t}\, dt$$

$$= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t)\, e^{-jk\omega_0 T_s}\, dt.$$

$$= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t)\, dt = \frac{1}{T_s} \quad \underline{\text{Ans.}}$$

* $\int_{-T_s/2}^{T_s/2} \delta(t)\, dt$ = Area under impulse ~~func~~ functions is 1.

Ans-3(a) $\quad F\{e^{-at}u(t)\} \Longleftrightarrow \dfrac{1}{a+j\omega}$

$$F\{e^{-a(t-t_0)}u(t-t_0)\} = \frac{e^{-j\omega t_0}}{a+j\omega}.$$

$$F\{e^{-2(t-1)}u(t-1)\} = \frac{e^{-j\omega}}{2+j\omega} \quad \text{Ans}$$

3(b) $\quad g(t) = e^{-2|t-1|}$

$$F\{e^{-a|t|}\} = \frac{2a}{a^2+\omega^2}$$

$$F\{e^{-2|t|}\} = \frac{4}{4+\omega^2}$$

By applying time shifting property

$$F\{e^{-2|t-1|}\} = \frac{4e^{-j\omega}}{4+\omega^2} \quad \text{Ans.}$$

Ans - 4(a) $g_1(t) = e^{-t}\cos(\omega_c t)\, u(t)$.

$$= e^{-t}u(t)\left(\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right)$$

$$F\{e^{-t}u(t)\} = \frac{1}{1+j\omega}$$

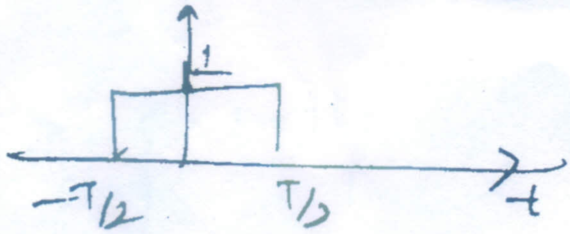
$$F\left\{\frac{1}{2}e^{-t}e^{j\omega_c t}u(t)\right\} = \frac{1}{2}\left(\frac{1}{1+j(\omega-\omega_c)}\right).$$

$$F\left\{\frac{1}{2}e^{-t}e^{-j\omega_c t}u(t)\right\} = \frac{1}{2}\left[\frac{1}{1+j(\omega+\omega_c)}\right]$$

$$F\left\{e^{-t}\cos(\omega_c t)\,u(t)\right\} = \frac{1}{2}\left[\frac{1}{1+j(\omega-\omega_c)} + \frac{1}{1+j(\omega+\omega_c)}\right]$$

Ans

1(b) $g_2(t) = \text{rect}\left(t/T\right)$ 7



$$F\left\{\text{rect}\left(\frac{t}{T}\right)\right\} = \int_{-T/2}^{T/2} 1 \cdot e^{-j2\pi f t}\, dt$$

$$= \left. \frac{e^{-j2\pi f}}{-j2\pi f} \right|_{-T/2}^{T/2}$$

$$= \frac{-e^{-j\pi T f} + e^{j\pi T f}}{j2\pi f}$$

$$= \frac{\left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j}\right]}{\pi f}$$

$$= \frac{\sin(\pi f T)}{\pi f}$$

$$= T \frac{\sin(\pi f T)}{\pi f T}$$

$$= T \cdot \text{sinc}(f T)$$

Tutorial - 5

Ans- 4(i)

$$g_3(t) = \Delta(t/T)$$

$$g_3(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| \geq T \end{cases}$$

$$= \begin{cases} \left(1 + \frac{t}{T}\right), & -T \leq t < 0 \\ \left(1 - \frac{t}{T}\right), & 0 \leq t \leq T. \\ 0, & |t| \geq T \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt.$$

$$= \int_{-T}^{0} \left(1 + \frac{t}{T}\right) e^{-j\omega t} dt + \int_{0}^{T} \left(1 - \frac{t}{T}\right) e^{-j\omega t} dt$$

$$= \int_{0}^{T} \left(1 - \frac{t}{T}\right) e^{j\omega t} dt + \int_{0}^{T} \left(1 - \frac{t}{T}\right) e^{-j\omega t} dt$$

$$= 2 \int_{0}^{T} \left(1 - \frac{t}{T}\right) \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) dt.$$

$$= 2\int_0^T \left(1-\frac{t}{T}\right)\cos\omega t\,dt$$

$$= 2\left(1-\frac{t}{T}\right)\frac{\sin\omega t}{\omega}\Bigg|_0^T + \frac{2}{\omega T}\int_0^T \sin(\omega t)\,dt$$

$$= 0 - \frac{2}{\omega^2 T}\cos(\omega t)\Bigg|_0^T$$

$$= \frac{2}{\omega^2 T}\left(1-\cos(\omega T)\right)$$

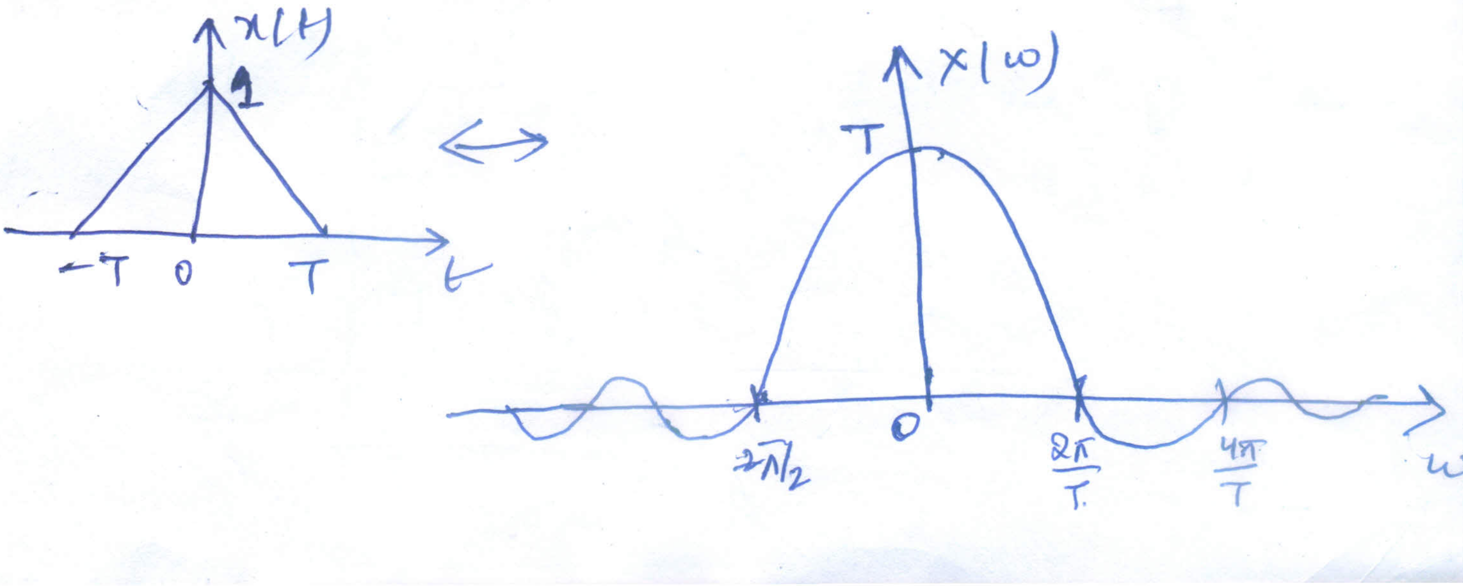
$$X(\omega) = G_3(\omega) = \frac{4}{\omega^2 T}\sin^2\left(\frac{\omega T}{2}\right)$$

$$= T\left(\frac{\sin(\omega T/2)}{(\omega T/2)}\right)^2$$

$$G_3(\omega) = T\,sa^2\left(\frac{\omega T}{2}\right) = T\,sa^2\left(\frac{\omega T}{2}\right)$$

$$= T\,\mathrm{sinc}^2(fT)$$

$$\Delta\left(\frac{t}{T}\right) \longleftrightarrow T\,\mathrm{sinc}^2\left(\frac{\omega T}{2}\right).$$



4(d) (8)

$$g_4(t) = T\ \mathrm{sinc}\left(\frac{Tt}{2}\right)$$

$$A\ \mathrm{rect}\left(\frac{t}{T}\right) \rightleftharpoons AT\ \mathrm{sinc}(fT)$$

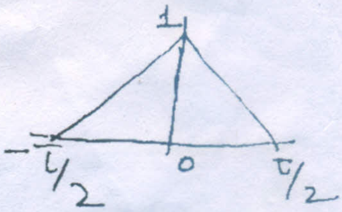
$$AT\ \mathrm{sin}(Tt \rightleftharpoons A\ \mathrm{rec}\left(\frac{f}{T}\right)$$

$$A = 2, \quad T = T/2$$

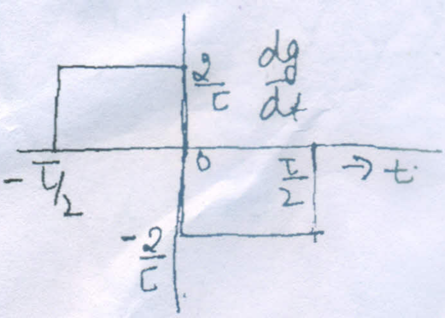
$$T\ \mathrm{sinc}\left(\frac{tT}{2}\right) \rightleftharpoons 2\ \mathrm{rect}\left(\frac{2f}{T}\right)$$

11

$$A(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ 1-2|x| & |x| < \frac{1}{2} \end{cases}$$



$$T/2\ \mathrm{sinc}^2\left(\frac{\omega\tau}{4}\right)$$



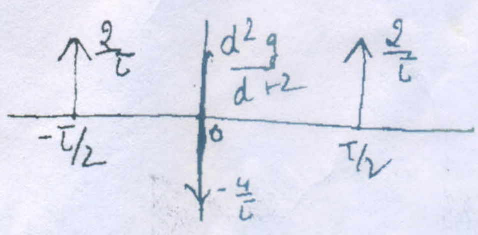
$$\frac{2}{\tau}\delta\left(t+\frac{\tau}{2}\right) + \frac{4}{\tau}\delta(t) + \frac{2}{T}\delta(t-\tau/2)$$

$g''(t) = \dots$ $\delta(t-t_0) = e^{-j\omega t_0}$

$g''(t) = \delta(t)$

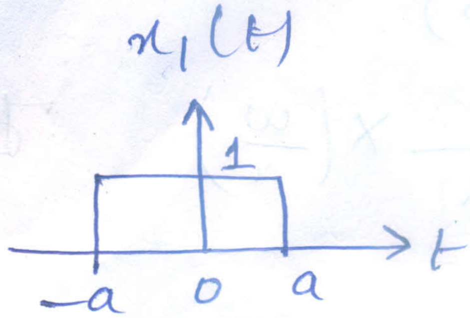
$= 1$

$j^2\omega^2 \frac{\tau}{2}$



ms-5

(i)



$$= 1 \cdot \text{rect}\left(\frac{t}{2a}\right).$$

$$= \Pi\left(\frac{t}{2a}\right) = \begin{cases} A, & |t| < a \\ 0, & |t| > a \end{cases}$$

$$X_1(\omega) = \int_{-\infty}^{\infty} x_1(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-a}^{a} e^{-j\omega t}\, dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-a}^{a}$$

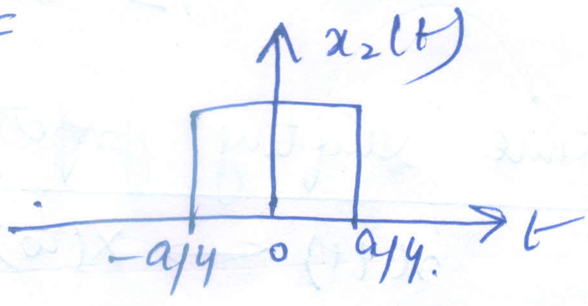
$$= \frac{-1}{j\omega}\left[e^{-j\omega a} - e^{j\omega a}\right].$$

$$= \frac{+2}{\omega}\left(\frac{e^{+j\omega a} - e^{-j\omega a}}{2j}\right).$$

$$= \frac{2}{\omega} \sin(\omega a)$$

$$\boxed{X_1(\omega) = \frac{2}{\omega} \sin(\omega a)}$$

(ii) $x_2(t) =$



Applying Time Scaling property

$$x(t) \longleftrightarrow X(\omega)$$

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right).$$

Time Scaling property

$$x_1(t) \longleftrightarrow \frac{2}{\omega} \sin(\omega a)$$

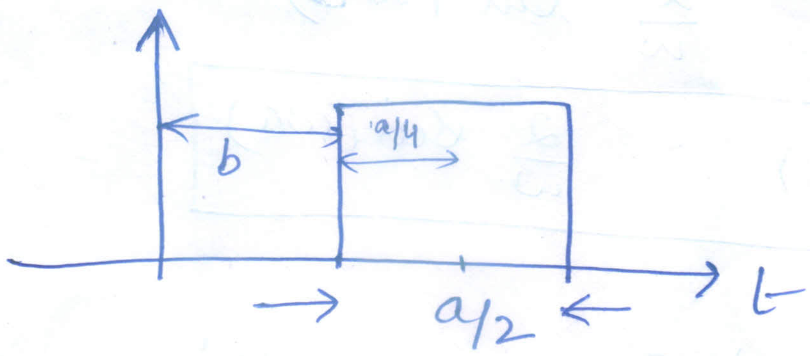
$$x_2(t) = x_1\left(\frac{t}{4}\right)$$

$$x_1(t) \longleftrightarrow \frac{2}{\omega} \sin(\omega a)$$

$$x_1\left(\frac{t}{4}\right) \longleftrightarrow \frac{2}{\omega} \cdot 4 \cdot X(4a\omega)$$

$$\boxed{x_2(t) = x_1\left(\frac{t}{4}\right) = \frac{8}{\omega} X(4a\omega)}$$

(iii) $x_3(t)$



Using time shifting property

$$x(t) \longleftrightarrow X(\omega)$$

$$x(t-t_0) \longleftrightarrow X(\omega)e^{-j\omega t_0}.$$

$$x_3(t) = x_2(t - (b + a/4))$$

$$x_2(t) \longleftrightarrow \frac{8}{\omega} X(4a\omega)$$

$$x_3(t) \longleftrightarrow x_2(t - (b + \frac{a}{4}))$$

$$x_3(t) = x_2(t - (b + \frac{a}{4})) \longleftrightarrow \frac{8}{\omega} \cdot X(4a\omega) e^{-j\omega(b + \frac{a}{4})}$$

Ans

① Solution of Tut-9

a) The DFT of the unit impulse can be calculate as —

$$X_1(k) = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

$$= \sum_{n=0}^{N-1} \delta[n] W_N^{nk}$$

$$= 1$$

b) α^n

The DFT can be calculated as

$$X_2(k) = \sum_{n=0}^{N-1} \alpha^n W_N^{nk}$$

$$= \sum_{n=0}^{N-1} (\alpha W_N^{k})^n$$

$$= \frac{1-(\alpha W_N^k)^N}{1-\alpha W_N^k}, \quad k = 0, 1, \dots N-1$$

c) $u[n] - u[n-n_0]$

DFT will be

$$X_3(k) = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

$$= \sum_{n=0}^{n_0-1} W_N^{nk} = \frac{1-W_N^{kn_0}}{1-W_N^{k}}$$

Factoring out a complex exponential $W_N^{kn_0/2}$ from the numerator and a complex exponential $W_N^{k/2}$ from the denominator, the DFT may be written as

$$X_3(k) = W_N^{k(n_0-1)/2} \frac{W_N^{-kn_0/2} - W_N^{kn_0/2}}{W_N^{-k/2} - W_N^{k/2}}$$

$$= e^{-j\frac{2\pi k}{N}\left(\frac{n_0-1}{2}\right)} \frac{\sin(n_0\pi k/N)}{\sin(\pi k/N)}, \quad k = 0, 1, \dots N-1$$

(2) 10 - point inverse DFT of

$$X(k) = \begin{cases} 3, & k = 0 \\ 1, & 1 \le k \le 9 \end{cases}$$

To find the inverse DFT, note that $X(k)$ can be represented as

$$X(k) = 1 + 2\delta(k), \quad 0 \le k \le 9$$

As we know

$$\delta[n] \xleftrightarrow{DFT} 1$$

$$1 \xleftrightarrow{DFT} N\delta(k)$$

Thus $1 + 2\delta(k)$ is the DFT of

$$\boxed{x[n] = \frac{1}{5} + \delta[n]}$$ Ans.

(3)

The DFT of this sequence may be evaluated by expanding the cosine as a sum of complex exponentials-

$$x[n] = 4 + \frac{1}{4}\left[e^{j2\pi n/N} + e^{-j2\pi n/N}\right]^2$$

$$= 4 + \frac{1}{2} + \frac{1}{4}e^{j4\pi n/N} + \frac{1}{4}e^{-j4\pi n/N}$$

of complex exponentials -

Using the periodicity

$$= \frac{9}{2} + \frac{1}{4}e^{j\frac{2\pi}{N}(2n)} + \frac{1}{4}e^{j\left(\frac{2\pi}{N}\right)(N-2)n}$$

Therefore, the DFT coefficients are

$$X(k) = \begin{cases} \frac{9}{2}N & , \quad k=0 \\ \frac{1}{4}N & , \quad k=2 \text{ and } k=N-2 \\ 0 & , \quad \text{else.} \end{cases}$$

(4)

a) The DFT of $x[n]$ can be calculated as

$$X(k) = 1 + 2W_N^{5k}$$

$$= 1 + 2e^{-j\frac{2\pi}{10}5k}$$

$$= 1 + 2(-1)^k$$

b) Multiplying $X(k)$ by a complex exponential of the form $W_N^{kn_0}$ corresponds to a circular shift of $x[n]$ by n_0. In this case, because $n_0 = -2$, $x[n]$ is circularly shifted to the left by 2, and we have

$$y[n] = x[n+2]_{10}$$

$$y[n] = 2\delta[n-3] + \delta[n-8]$$