15B11MA211   **Mathematics-II**   **B.Tech. Core**

Tutorial Sheet 3 (Convergence of Infinite Series)

1. Test the convergence of the following series:

(a) $\sum \dfrac{1}{(2n-1)^p}$,   (b) $\sum \left[1+\dfrac{1}{\sqrt{n}}\right]^{-n^{3/2}}$,

(c) $\sum \dfrac{n^n x^n}{n!}$,   (d) $\sum \dfrac{1}{n^{1+(1/n)}}$,

(e) $\sum \dfrac{x^n}{n(n+1)}$,   (f) $\sum \left[\left(n^3+1\right)^{1/3}-n\right]$,

(g) $\dfrac{x}{1}+\dfrac{1}{2}\cdot\dfrac{x^3}{3}+\dfrac{1.3}{2.4}\cdot\dfrac{x^5}{5}+\dfrac{1.3.5}{2.4.6}\cdot\dfrac{x^7}{7}+\cdots$

(h) $\dfrac{1^2}{4^2}+\dfrac{5^2}{8^2}+\dfrac{9^2}{12^2}+\dfrac{13^2}{16^2}+\cdots$

(i) $1+\dfrac{x}{2}+\dfrac{2!}{3^2}\cdot x^2+\dfrac{3!}{4^3}\cdot x^3+\dfrac{4!}{5^4}\cdot x^4+\cdots$

2. Test the convergence of the series whose n^{th} term is:

(a) $\dfrac{a^n}{x^n+a^n}$,   (b) $\dfrac{1}{\sqrt{n}+\sqrt{n+1}}$,

(c) $\dfrac{\sqrt{n+1}-1}{(n+2)^3-1}$,   (d) $\dfrac{(a+nx)^n}{n!}$,

(e) $\dfrac{1}{\sqrt{n}}\tan\dfrac{1}{n}$,   (f) $3^{-n-(-1)^n}$,

Answers:

1.(a) p >1 Conv., p ≤ 1 Div.   (b) Conv.   (c) $x<\dfrac{1}{e}$ Conv., $x\geq\dfrac{1}{e}$ Div.,
(d) Div.   (e) $x\leq 1$ Conv., $x>1$ Div.   (f) Conv.
(g) $x^2\leq 1$ $Conv.$; $x^2>1$ (h) Div.
(i) $x<e$ Conv., $x\geq e$ Div.,

2.(a) $\dfrac{x}{a}\leq 1$ Div., $\dfrac{x}{a}>1$ Conv.   (b) Div.
(c) Conv.   (d) $x<\dfrac{1}{e}$ Conv., $x\geq\dfrac{1}{e}$ Div.
(e) Conv.   (f) Conv.