Tutorial - 7
Solution (Introduction to Semiconductor.

Q.1. In a solid, consider the energy level lying 0.01 eV below fermi level. What is the probability of this level not being occupied by an electron?

Solution. Given $E_F - E = 0.01\,eV$.

$KT = 0.026\,eV$.

$$f(E) = \frac{1}{1 + e^{-(E_F - E)/KT}}$$

$$= \frac{1}{1 + e^{-0.01eV/0.026eV}}$$

$$= 0.595.$$

$$\therefore P = 1 - f(E) = 1 - 0.595$$

$$= 0.405.$$

Q.2. Calculate the probabilities for an electronic state to be occupied at 20°c, if the energy of these states lies 0.11 eV above and 0.11 eV below the Fermi level.

Solution - Probability of occupying an energy level E

$$f(E) = \frac{1}{1 + e^{(E - E_F)/KT}}$$

probability of occupying an energy level 0.11 eV above Fermi level

$$f(E) = \frac{1}{1 + e^{0.11eV/KT}} = 0.0126.$$

below fermi level.

$$f(E) = \frac{1}{1 + e^{-0.11eV/KT}} = 0.987.$$

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Q.3. Evaluate the fermi function for energy KT above the Fermi energy.

Solution.
$$f(E) = ?$$
$$k = 1.38 \times 10^{-23} \ J/k$$
$$T = 300 K.$$
$$f(E) = \frac{1}{1 + e^{(E-E_f)/KT}}$$
Here $E - E_f = KT$. then
$$f(E) = \frac{1}{1+e^1} = \frac{1}{1+2.78} = 0.269$$

Q.4. Find the temperature at which there is 1% probability that a state with energy 0.5 eV above fermi energy.

Solution - $f(E) = 1\% = 1/100$
$$E - E_f = \cancel{0.5} \ 0.5 \ eV.$$
$$T = ?$$
$$f(E) = \frac{1}{1 + \exp(E-E_f)/k_BT}$$
$$k_B = 8.6 \times 10^{-5} \ eV/K.$$
Substituting these values.
$$\frac{1}{100} = \frac{1}{1 + \exp\left[\frac{0.5}{8.6 \times 10^{-5} T}\right]}$$
$$100 = 1 + \exp\left[\frac{5801.87}{T}\right]$$
$$\simeq \exp\left(\frac{5801.87}{T}\right)$$
Taking log both sides. $\Rightarrow$ $T = 1259.98 K$.

Q.5. Assume Si ($E_g = 1.12eV$) at room temperature (300 K) with the Fermi level located exactly in the middle of the bandgap. Answer the following :-

a) What is the probability that a state located at the bottom of the conduction band is filled?

b) What is the probability that a state located at the top of the valence band is empty?

Solution. a) $$f(E) = \frac{1}{1 + (E - E_f)/KT}.$$

Given $E = E_c$

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_f)/KT}}$$

$$\simeq e^{-(E_c - E_f)/KT}$$

(approximation taken
∵ conduction band is far above the Fermi level)

$$E_c - E_F = \frac{E_g}{2}$$

$$\therefore f(E_c) = e^{-E_g/2KT}$$

$E_g = 1.12eV$, $KT = 0.026 eV$.

$$f(E_c) = e^{-21.5}$$

$$= 4.43 \times 10^{-10}$$

b) $E = E_v$.

$E_F - E_v = E_g/2$.

$1 - f(E_v) \Rightarrow$ probability for empty state.

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$$1 - f(E_v) = 1 - \frac{1}{1 + e^{(E_v - E_F)/kT}}$$

$$= \frac{1}{1 + e^{-(E_v - E_F)/kT}}$$

$$\simeq e^{-(E_F - E_v)/kT}.$$

$$= e^{-E_g/2kT}$$

Putt the values from (a) part we get

$$1 - f(E_v) = 4.43 \times 10^{-10}.$$

Q.6 Consider a silicon crystal at room temperature (300°K) doped with arsenic atoms so that $N_D = 6 \times 10^{16}\ cm^{-3}$. Find equilibrium concentration n_o, hole concentration p_o, and Fermi level E_f with respect to the intrinsic Fermi level E_i and conduction band edge E_c.

Solution -

$$n_o \simeq N_D = 6 \times 10^{16}\ cm^{-3} \quad (\because \text{n type material})$$

Use law of mass action to find p_o

$$p_o = \frac{n_i^2}{n_o}$$

$$= \frac{2.1 \times 10^{20}}{6 \times 10^{16}} = 3.5 \times 10^{3}\ cm^{-3}$$

To find Fermi level -

$$n_o = n_i \exp\left(\frac{E_f - E_i}{KT}\right)$$

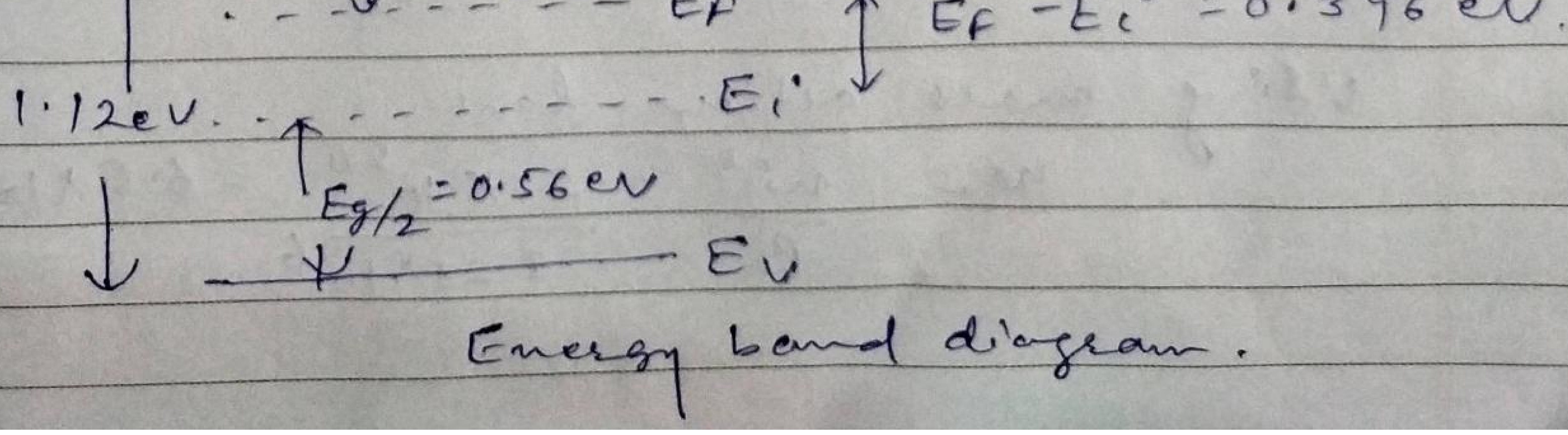
$$E_f - E_i = KT \ln\left(\frac{n_o}{n_i}\right)$$

$$KT = 8.61 \times 10^{-5} \times 300 \simeq 0.026\ eV$$

$$E_f - E_i = 0.026 \ln\left(\frac{6 \times 10^{16}}{1.45 \times 10^{10}}\right)$$

$$= 0.396\ eV$$

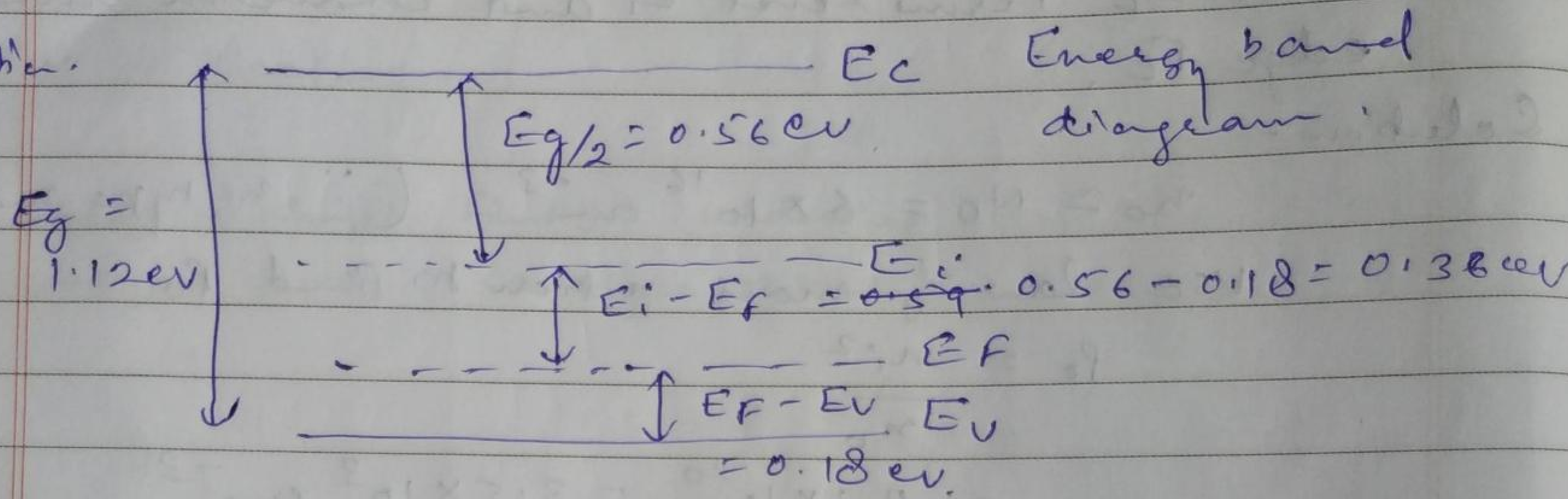
$$E_f = E_i + 0.396\ eV$$



Energy band diagram.

Q. 7. Consider a silicon crystal at 300 K with the Fermi level 0.18 eV above the valence band. What type is the material? What are the electron and hole concentration?

Solution.



$$E_i - E_V = \frac{E_g}{2} = 0.56 \text{ eV}$$

$$E_i - E_V = (E_i - E_F) + (E_F - E_V)$$

$$E_i - E_F = \frac{E_g}{2} - (E_F - E_V)$$

$$E_i - E_F = 0.56 - 0.18 = 0.38 \text{ eV}.$$

Fermi level is closer to the valence band than conduction band.

$\Rightarrow$ Therefore it is p type material.

$$p_0 = n_i \exp\left(\frac{E_i - E_F}{KT}\right)$$

$$= 1.45 \times 10^{10} \exp\left(\frac{0.38}{0.026}\right) \simeq 3.23 \times 10^{16} / cm^3$$

Using mass action law

$$n_0 = \frac{n_i^2}{p_0} = \frac{2.1 \times 10^{20}}{3.23 \times 10^{16}} = 6.5 \times 10^3 / cm^3$$

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Q.8. Consider a Silicon crystal at room temperature, doped with both donor and acceptor atoms so that $N_D = 2\times10^{15}\ cm^{-3}$, $N_A = 1\times10^{15}\ cm^{-3}$. What type of material would this yield? Find the location of the Fermi level.

Sol. Here, $N_D > N_A \Rightarrow$ n type.

$$n_o \simeq N_D - N_A$$

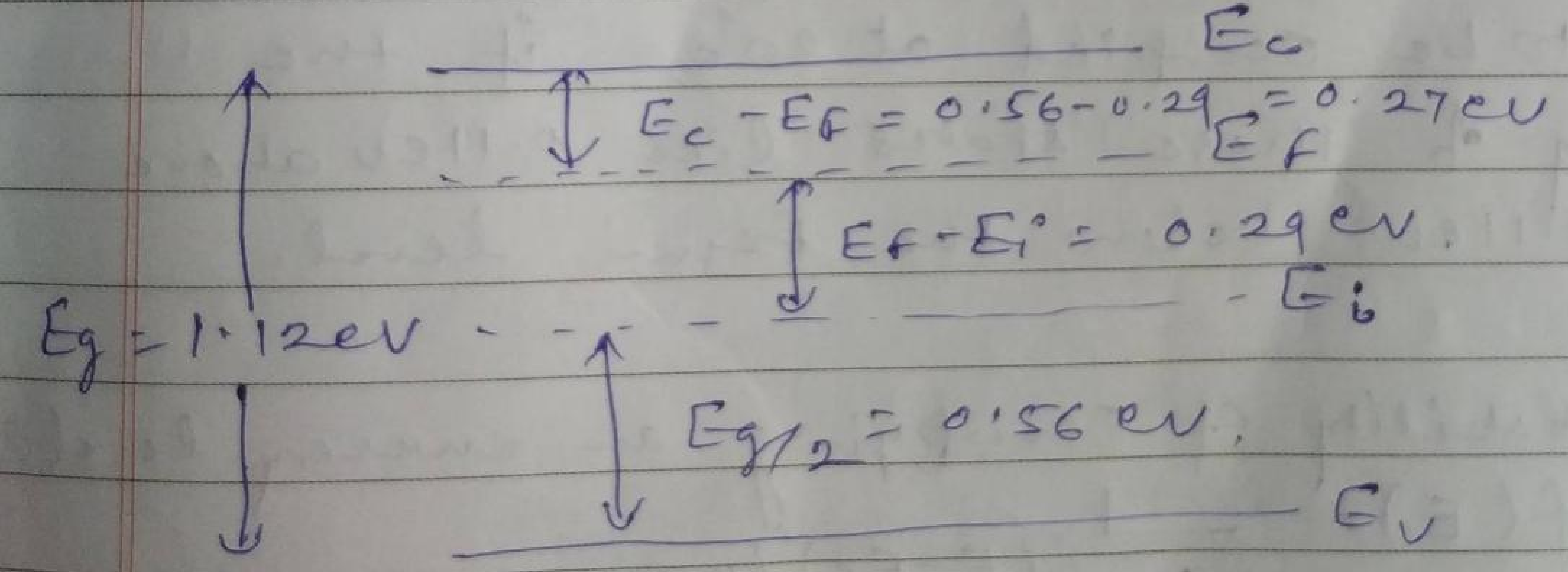
$$= 1\times10^{15}/cm^3$$

$$p_o = \frac{n_i^2}{n_o} = \frac{2.1\times10^{20}}{1\times10^{15}} = 2.1\times10^5/cm^3$$

$$E_F - E_i = kT \ln \frac{n_o}{n_i}$$

$$= 0.026 \ln\left(\frac{1\times10^{15}}{1.45\times10^{10}}\right)$$

$$= 0.29\ eV.$$



Energy band diagram.

Q.9. Consider a region of Si at ~~zero~~ room temperature. Calculate the equilibrium electron and hole concentration (n and p). Assume that the dopants are fully ionized. $n_i = 1\times 10^{10}\ cm^{-3}$.

a) For intrinsic material ($N_D = N_A = 0$)

b) $N_D = 1.00\times 10^{13}\ cm^{-3}$, $N_A = 0$.

c) $N_D = 1\times 10^{17}\ cm^{-3}$, $N_A = 3\times 10^{17}\ cm^{-3}$.

Solution a) For intrinsic material:

$$n = p = n_i = 1\times 10^{10}\ cm^{-3}.$$

b)

$$n \simeq N_D = 1\times 10^{13}\ cm^{-3}$$

$$p = \frac{n_i^2}{n}$$

$$= \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{13}} = 1\times 10^{7}\ cm^{-3}.$$

c) net p-type doping $N_A - N_D = 3\times 10^{17} - 1\times 10^{17}$
$= 2\times 10^{17}\ (\gg n_i)$

$\therefore\ p = 2\times 10^{17}$,

$n = n_i^2/p = 0.5\times 10^{3}\ cm^{-3}$

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Q. 10. For each of the cases in problem 9, Calculate the Fermi level position, with respect to the intrinsic level. $(E_F - E_i)$

Solution

$$n = n_i e^{(E_F - E_i)/KT}$$
$$p = n_i e^{(E_i - E_F)/KT}$$

$E_F - E_i$ from 1st eqn.
$$= KT \ln \frac{n}{n_i}$$

a) $n = p = n_i = 1 \times 10^{10}$ cm^{-3}
$$\frac{(E_F - E_i)}{q} = 0.026 \ln(1) = 0$$
$$E_F - E_i = 0.$$

b) $n = 1 \times 10^{13}$ cm^{-3}.
$$\frac{E_F - E_i}{q} = 0.026 \ln\left(\frac{10^{13}}{10^{10}}\right)$$
$$= \cancel{0.180}$$
$$= 0.180$$

c) $n = \cancel{1 \times 10^{17} \text{ cm}^{-3}}$. 5×10^{2} cm^{-3}
$$\frac{E_F - E_i}{q} = \cancel{0.026 \ln \frac{10^{17}}{10^{10}}}$$
$$= \cancel{0.419}.$$
$$= 0.026 \ln\left(\frac{5 \times 10^{2}}{10^{10}}\right)$$
$$= -0.437.$$

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