TUT 9

Q2] $\lambda = 40/\text{hour}$ $t = 25 \text{ min} = \frac{25}{60} \text{ hour}$

$$\lambda t = \cancel{40}^{2} \times \frac{25}{\cancel{60}_{3}} = \frac{100}{6} = \frac{50}{3}$$

$$P[x = 6] = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$$

$$= \frac{e^{-50/3}(50/3)^6}{6!}$$

Q3] $\lambda_m = 5/h$ $\lambda_f = 10/h$

a) $x \leq 4$ $t = 30 \text{ min} = 1/2 \text{ hour}$

$$P[x \leq 4] = \sum_{x=0}^{4} \frac{e^{-5/2}(5/2)^x}{x!}$$

b) $x \leq 4$ $t = 1/2\, h$

$$P[x \leq 4] = \sum_{x=0}^{4} \frac{e^{-5}(5)^x}{x!}$$

c) Inter-arrival time follow exp. dist

$$P[T \geq 15/60] = e^{-(5/4)}$$

Q4] $\lambda = 5/\text{min}$

i) $P[T>3] = e^{-5\times 3} = e^{-15}$

ii) $P[4<T<7] = \int_4^7 5e^{-5T}\,dT$

$= 5\left|\dfrac{e^{-5T}}{-5}\right|_4^7 = e^{-20} - e^{-35}$

iii) $P[T<6] = 1 - e^{-30}$

Q5]

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{ccc} 1 & 2 & 3 \\ \end{array}$$

$$P = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} .2 & .3 & .5 \\ .4 & .4 & .4 \\ .4 & .6 & 0 \end{bmatrix} \end{array} \qquad P^2 = \begin{bmatrix} & \otimes_{a_{12}} & \\ & & \\ & & \end{bmatrix}$$

a) $a_{13}^{(1)} = \underline{0.5}$

b) $a_{12}^{(2)} = .3\times.2 + .4\times.3 + .6\times.5$

$= 0.06 + 0.12 + 0.30$

$= \underline{0.48}$

Q 6]

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & .75 & .25 & 0 \\ 1 & .25 & .50 & .25 \\ 2 & 0 & .75 & .25 \end{array}$$

$$P_0 = \begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix}$$

i) $P[X_2 = 2]$

$$\textcircled{0} \xrightarrow{P^2} \textcircled{2} \qquad \textcircled{1} \xrightarrow{P^2} \textcircled{2} \qquad \textcircled{2} \xrightarrow{P^2} \textcircled{2}$$

$$P^2 = \begin{bmatrix} 0.625 & 0.3125 & 0.0625 \\ 0.3125 & 0.5 & 0.1875 \\ 0.1875 & 0.5625 & 0.25 \end{bmatrix}$$

$$P[X_2 = 2] = \underline{0.16666}$$

ii) $P[X_0 = 2, X_1 = 1, X_2 = 2, X_3 = 1]$

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}$$

$$\overset{1/3}{\textcircled{2}} \xrightarrow{0.75} \textcircled{1} \xrightarrow{.25} \textcircled{2} \xrightarrow{0.75} \textcircled{1} = (1/3)(0.75) \times (0.25)(0.75)$$

$$= \underline{0.046875}$$

Q7]

$$P = \begin{array}{c|c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} \end{array}$$

Let stead state vector $\pi = [\pi_1 \quad \pi_2 \quad \pi_3]$

$$\pi P = \pi$$

$$[\pi_1 \quad \pi_2 \quad \pi_3] \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} = [\pi_1 \quad \pi_2 \quad \pi_3]$$

$\pi_1 + \pi_2 + \pi_3 = 1$ — 1

$2/3\,\pi_2 + 2/3\,\pi_3 = \pi_1$ — 2

$2\pi_2 + 2\pi_3 = 3\pi_1$

$\pi_1 + 1/3\,\pi_3 = \pi_2$

$1/3\,\pi_2 = \pi_3$ — 3

$$2\pi_1 + \cancel{2\pi_2} + \cancel{2\pi_3} = 2$$

$$-\cancel{2\pi_2} - \cancel{2\pi_3} = -3\pi_1$$

$$5\pi_1 = 2$$

$$\boxed{\pi_1 = \frac{2}{5}}$$

$$\frac{2}{5} + \pi_2 + \frac{1}{3}\pi_2 = 1$$

$$\frac{2}{5} + \frac{4}{3}\pi_2 = 1$$

$$\frac{4}{3}\pi_2 = \frac{3}{5}$$

$$\boxed{\pi_2 = \frac{9}{20}}$$

$$\boxed{\pi_3 = \frac{3}{20}}$$

Q8]

$$P = \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 2/3 & 0 \end{bmatrix} \end{array} \quad n(\text{zero}) = 5$$

The Prob of $P[X_n]$ depends only on $P[X_{n-1}]$ hence it is <u>markov chain</u>.

$$p^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix} \quad n(\text{zero}) = 4$$

$$p^3 = \begin{bmatrix} 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \\ 2/9 & 4/9 & 3/9 \end{bmatrix} \quad n(\text{zero}) = 2$$

Matrix is <u>Irroducible</u> because

$a_{11} > 0 \rightarrow p^3$ $\quad$ $a_{21} > 0 \rightarrow p^2$ $\quad$ $a_{31} > 0 \rightarrow p, p^3$

$a_{12} > 0 \rightarrow p, p^3$ $\quad$ $a_{22} > 0 \rightarrow p^2, p^3$ $\quad$ $a_{32} > 0 \rightarrow p, p^2, p^3$

$a_{13} > 0 \rightarrow p^2$ $\quad$ $a_{23} > 0 \rightarrow p, p^3$ $\quad$ $a_{33} > 0 \rightarrow p^2, p^3$

$$p^4 = \begin{bmatrix} 0 & 1/3 & 2/3 \\ 2/9 & 4/9 & 3/9 \\ 1/9 & 4/9 & 4/9 \end{bmatrix} \quad n(\text{zero}) = 1$$

$$p^5 = \begin{bmatrix} 2/9 & 4/9 & 3/9 \\ 1/9 & 4/9 & 4/9 \\ 4/27 & 11/27 & 12/27 \end{bmatrix} \quad n(\text{zero}) = 0$$

Thus matrix is <u>Regular</u>

Matrix is finite & Regular, so state of $a_{11} = a_{22} = a_{33}$

State of a_{22} = gcd $(2, 3, 4, 5) = 1$

thus all states are <u>Aperiodic</u>

Matrix is finite, regular, aperiodic hence <u>Ergodic</u> in nature.

Q9]

$$P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 11/16 & 5/16 \\ 5/8 & 3/8 \end{bmatrix}$$

a) To find stationary dist.

$$\pi P = \pi$$

$$[\pi_1 \ \ \pi_2] \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = [\pi_1 \ \ \pi_2]$$

$$\pi_1 + \pi_2 = 1 \quad \text{—— (1)}$$

$$3/4\, \pi_1 + 1/2\, \pi_2 = \pi_1 \quad \text{—— (2)}$$

$$1/2\, \pi_2 = 1/4\, \pi_1$$

$$2\pi_2 = \pi_1 \longrightarrow \boxed{\begin{matrix} \pi_2 = 1/3 \\ \pi_1 = 2/3 \end{matrix}}$$

$$\pi = [2/3 \ \ 1/3]$$

b)

\# If chain is stationary, it's period is infinite

9.10]

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

a) $\pi P = \pi$

$$[\pi_1 \quad \pi_2 \quad \pi_3] \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = [\pi_1 \quad \pi_2 \quad \pi_3]$$

$\pi_1 + \pi_2 + \pi_3 = 1$ —— ①

$\pi_2 + \pi_3 = 2\pi_1$ —— ②

$4\pi_1 + 3\pi_3 = 6\pi_2$

$$\begin{aligned} \cancel{4\pi_1} + 3\pi_3 &= 6\pi_2 \\ 2\pi_2 + 2\pi_3 &= \cancel{4\pi_1} \\ \hline 5\pi_3 &= 4\pi_2 \end{aligned}$$

$1 - \pi_1 = 2\pi_1 \Rightarrow \boxed{\pi_1 = 1/3}$

$\pi_2 + \frac{4}{5}\pi_2 = \frac{2}{3} \Rightarrow \boxed{\pi_2 = \frac{10}{27}}$

$\pi_3 = \frac{4}{5} \times \frac{10}{27} \Rightarrow \boxed{\pi_3 = \frac{8}{27}}$

$$\pi = \begin{bmatrix} \frac{9}{27} & \frac{10}{27} & \frac{8}{27} \end{bmatrix}$$

This markov chain has a stationary distribution, hence all states are <u>non null persistant</u>.

$$P^2 = \begin{bmatrix} 3/6 & 1/6 & 2/6 \\ 3/12 & 7/12 & 2/12 \\ 3/12 & 4/12 & 5/12 \end{bmatrix}$$

↳ Regular

↳ Irreducible

period = ∞ (as stationary)

or Non - periodic