Digital Systems
18B11EC213

Module 1: Boolean Function Minimization Techniques and Combinational Circuits-2

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Cont.. Number Systems

Example: Convert $(1001110001.100)_2$ into octal.

- Grouping of 3 bits from LSB to MSB for integer part and from binary point (dot) for fractional part:

  $\underline{001}\ \underline{001}\ \underline{110}\ \underline{001}\ .\ \underline{100}$

  Padded with zeros

- Write the octal equivalent of group of 3 bits binary number at their respective places

  1 1 6 1 . 4

=> $(\underline{001}\ \underline{001}\ \underline{110}\ \underline{001}\ .\ \underline{100})_2 = (1161.4)_8$

Signed Number Representation

- Two numbers:

❖ Unsigned numbers – Which can represent only positive numbers.

❖ Signed numbers – Which can represent both positive and negative numbers.

- The *sign* is an extra piece of information that has to be encoded in addition to the *magnitude* of a number.

Cont..

- Different schemes/methods to represent a signed number:

❖ Sign magnitude representation

❖ (r - 1)’s complement representation

❖ r’s complement representation

where r is the base (radix) of a number system.

For example, these representations are 1’s complement and 2’s complement for a binary number (r = 2).

Cont..

Sign Magnitude Representation:

- An extra bit (called as sign bit) is added to represent the sign of a number.
- The most significant bit (MSB) is used to represent the sign.
- ‘1’ is used for a ‘-’ (negative sign)

  ‘0’ is used for a ‘+’ (positive sign)

Cont..

Format of Sign Magnitude Representation:

The format of the signed number representation in 8 bits is smmmmmmm

- ‘s’ (MSB) is the sign bit.
- The other 7 bits represent the magnitude of the number.
- For a positive number, the result is the same as the unsigned binary representation.

Cont..

Sign Magnitude Representation - Examples (8 bits)

-5 = $(1\ 0000101)_2 = (85)_{16}$

+5 = $(0\ 0000101)_2 = (05)_{16}$

+127 = $(0\ 1111111)_2 = (7F)_{16}$

-127 = $(1\ 1111111)_2 = (FF)_{16}$

+ 0 = $(0\ 0000000)_2 = (00)_{16}$

- 0 = $(1\ 0000000)_2 = (80)_{16}$

- One problem/drawback is that it has two ways of representing 0 (+ 0 and - 0).

Cont..

- For N bits, the sign magnitude representation can accommodate numbers in the range

$$- \{2^{(N-1)} - 1\} \text{ to } + \{2^{(N-1)} - 1\}$$

Therefore, for 8 bits, it can represent the signed integers from -127 to +127.

Cont..

(r - 1)’s Complement Representation:

- For binary numbers, base r = 2

  => 1’s complement

- For decimal numbers, base r = 10,

  => 9’s complement

Cont..

1’s Complement Representation:

- In 1’s complement representation, a positive number is written as *straight binary form* with MSB equal to 0.
- When a number is negative, the magnitude is written by taking 1’s complement of magnitude of that number and MSB equal to 1 to represent it a negative number.
- In a binary number, if each 1 is replaced by 0 and each 0 by 1, the resulting number is known as the 1’s complement of the first number.

Cont..

Example: For 8-bit representation

Number	1's complement representation
+ 18	00010010
- 18	11101101

The straight binary representation (form) of 18 (magnitude) is 10010.

Cont..

1’s Complement - Examples (8 bits)

$+5 = (00000101)_2 = (05)_{16}$

$-5 = (11111010)_2 = (FA)_{16}$

$+127 = (01111111)_2 = (7F)_{16}$

$-127 = (10000000)_2 = (80)_{16}$

$+0 = (00000000)_2 = (00)_{16}$

$-0 = (11111111)_2 = (FF)_{16}$

- Similar to sign magnitude representation, 1’s complement representation also has two ways of representing 0 (+ 0 and - 0).
- Mathematically speaking, no such thing as two representations for zeros.

Cont..

Example: If the magnitude of a binary number is 1010, then its 1's complement will be 0101.

General method:

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   1111
 - 1010
 ______
   0101      (1's complement of 1010)
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Cont..

- Similar to the sign magnitude representation, for N bits, the 1's complement representation can accommodate numbers in the range

$$- \{2^{(N-1)} - 1\} \text{ to } + \{2^{(N-1)} - 1\}$$

Therefore, for 8 bits, it can represent the signed integers from -127 to +127.

Cont..

r’s Complement Representation:

- For binary numbers, base r = 2

  => 2’s complement

- For decimal numbers, base r = 10,

  => 10’s complement

Cont..

2’s Complement Representation:

- In 2’s complement representation, a positive number is represented as *straight binary form* with MSB equal to 0.
- When a number is negative, the magnitude is written by taking 2’s complement of magnitude of that number and MSB equal to 1 to represent it a negative number.
- 2’s complement of a binary number is obtained by adding 1 to its 1’s complement.

Cont..

Example: For 8-bit representation

Number	2's complement representation
+ 18	00010010
- 18	11101110

The straight binary representation (form) of 18 (magnitude) is 10010.

1's complement representation of -18: 11101101

Cont..

2’s Complement - Examples (8 bits)

+5 = (00000101)= $(05)_{16}$

-5 = (11111011) = $(FB)_{16}$

+127 = (01111111) = $(7F)_{16}$

-127 = (10000001) = $(81)_{16}$

+ 0 = (00000000) = $(00)_{16}$

- 0 = (00000000) = $(00)_{16}$

- 2’s complement representation contains only one type of 0. Advantage of 2’s complement representation.

References

- M. M. Mano, *Digital Logic and Computer Design*, 5th ed., Pearson Prentice Hall, 2013.
- R. P. Jain, *Modern Digital Electronics*, 4th ed., Tata McGraw-Hill Education, 2009.