① (a) Ratio of spont. emission rate to stim. emission rate,

$$R = (e^{h\nu/kT} - 1)$$

If $R = 1$, $\Rightarrow (e^{h\nu/kT} - 1) = 1$

$$\Rightarrow e^{hc/\lambda kT} = 2$$

Given, $\lambda = 500 \times 10^{-9}\ m \Rightarrow \underline{T = 41,573\ K}$

(b) If $R = 1 \Rightarrow e^{hc/\lambda kT} = 2$

Given, $T = 300\ K \Rightarrow \underline{\lambda = 69.8 \times 10^{-6}\ m = 69.8\ \mu m}$

② $R = (e^{h\nu/kT} - 1) = (e^{hc/\lambda kT} - 1)$

Given, $T = 50\ K$ & $\lambda = 10^{-5}\ m \Rightarrow \underline{R = e^{28.8} - 1 \simeq e^{28.8}}$

③ No. of photons emitted per second

$$= \frac{5 \times 10^{-3}\ \text{watt}}{\left(\frac{hc}{632.8 \times 10^{-9}}\right)\ \text{Joule}} = \underline{1.6 \times 10^{19}}$$

④ $$\frac{N_2}{N_1} = \frac{N_0\, e^{-E_2/kT}}{N_0\, e^{-E_1/kT}} = e^{-(E_2 - E_1)/kT} = e^{-hc/\lambda kT}$$

$$\Rightarrow \frac{N_2}{N_1} = e^{-69.155} = \underline{9.25 \times 10^{-31}}$$

⑤ $A_{21} = \frac{1}{10^{-6}\ s} = 10^6\ s^{-1}$, $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} = \frac{8\pi h}{\lambda^3}$

Given $\lambda = 600 \times 10^{-9}\ m \Rightarrow \underline{B_{21} = 1.3 \times 10^{19}}$

⑥ $r = \frac{1.3}{2}\ mm = 0.65\ mm = 0.65 \times 10^{-3}\ m$

$$\therefore I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{20 \times 10^{-3}}{3.14 \times (0.65 \times 10^{-3})^2} = \underline{1.5 \times 10^3\ W/m^2}$$

⑦ $$\frac{N_2}{N_1} = 1.059 \times 10^{-30} = \frac{N_0\, e^{-E_2/kT}}{N_0\, e^{-E_1/kT}} = e^{-(E_2 - E_1)/kT}$$

$$\Rightarrow 1.059 \times 10^{-30} = e^{-hc/\lambda kT} \Rightarrow \underline{\lambda = 1.1 \times 10^{-7}\ m}$$