TFCS KANISHK BHAT - 20103233 - B8

Tut:-5

1. $1^2+3^2+5^2+\cdots+(2n+1)^2 = \frac{(n+1)(2n+1)(3n+1)}{3}$

basis step:- P(0) is true because $1^2=1$

Induction step:- Assume that P(K) is true

Then

$P(K+1) = 1^2+3^2+\cdots+(2(2K+1)+1)^2$

$= \frac{(2K+1+1)(2(2K+1)+1)(2(2K+1)+1)}{3}$

$= \frac{(2K+2)(4K+3)(6K+3)}{3}$

H.P

2. Basis Step:- P(5) is true as $32 > 25$

Inductive step:- Assume that P(K) is true, $2^K > K^2$

Then $2^{K+1} = 2^K \cdot 2 > K^2+K^2 > K^2+4K \geq K^2+2K+1$

$=(K+1)^2$ because $K>4$.

3. (A) $A_{n+1} = A_n+6$ for $n \geq 1$ ($A_1=6$)

(B) $A_{n+1} = A_n+2$ for $n \geq 1$ ($A_1=3$)

(C) $A_{n+1} = 10A_n$ for $n \geq 1$ ($A_1=10$)

(d) $A_{n+1} = A_n$ for $n \geq 1$ ($A_1=5$)

4. Characteristic equation

$r^2-r-2=0$

roots $=-1, 2$

General Sol:

$C_1 = 2^n + C_2(-1)^n$

Solve for C_1 & C_2

$C_1+C_2 = 2$

$2C_1 - C_2 = 7$

$\boxed{C_1 = 3, C_2 = -1}$. $\boxed{3.2^n - (-1)^n}$

5. $r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$

$r = 3, 3$

$x_n = C_1 \cdot 3^n + C_2 n \cdot 3^n$

$x_0 = 2$ & $x_1 = 3$ imply that $C_1 = 2$ & $C_2 = -1$

$$\boxed{x_n = (2-n)3^n, \; n \geq 0}$$

6. $t^2 - 10t + 25 = 0$

$(t-5)^2 = 0$

$t = 5, 5$

$x_n = An^2 5^n$

$10(A(-2n+1)5 - 25A(-4n+4) + 8 \cdot 5^2 = 0$

$\Rightarrow \boxed{A = 4}$

$$\boxed{x_n = 4 \cdot n^2 \cdot 5^n}$$

$x_n = 4n5^n + C_1 \cdot 5^n + C_2 \cdot n \cdot 5^n$

$C_1 = 6, \; C_2 = -8$

$\Rightarrow x_n = (4n^2 - 8n + 6)5^n$