PRACTICE PROBLEMS BASED ON MULTILEVEL PAGING-

Problem-01:

Consider a system using multilevel paging scheme. The page size is 1 MB. The memory is byte addressable and virtual address is 64 bits long. The page table entry size is 4 bytes.

Find-

1. How many levels of page table will be required?
2. Give the divided physical address and virtual address.

Solution-

Given-

- Virtual Address = 64 bits
- Page size = 1 MB
- Page table entry size = 4 bytes

Number of Bits in Frame Number-

We have,
Page table entry size

= 4 bytes

= 32 bits

Thus, Number of bits in frame number = 32 bits

Number of Frames in Main Memory-

We have, Number of bits in frame number = 32 bits
Thus,

Number of frames in main memory

= 2^{32} frames

Size of Main Memory-

Size of main memory

= Total number of frames x Frame size

= 2^{32} x 1 MB

= 2^{52} B

Thus, Number of bits in physical address = 52 bits

Number of Bits in Page Offset-

We have,

Page size

= 1 MB

= 2^{20} B

Thus, Number of bits in page offset = 20 bits

Alternatively,

Number of bits in page offset

= Number of bits in physical address – Number of bits in frame number

= 52 bits – 32 bits

= 20 bits

Process Size-

Number of bits in virtual address = 64 bits

Thus,

Process size

$= 2^{64}$ bytes

Number of Pages of Process-

Number of pages the process is divided

= Process size / Page size

$= 2^{64}$ B / 1 MB

$= 2^{64}$ B / 2^{20} B

$= 2^{44}$ pages

Inner Page Table Size-

Inner page table keeps track of the frames storing the pages of process.

Inner page table size
= Number of entries in inner page table x Page table entry size

= Number of pages the process is divided x Page table entry size

$= 2^{44}$ x 4 bytes

$= 2^{46}$ bytes

Now, we can observe-

- The size of inner page table is greater than the frame size (1 MB).
- Thus, inner page table can not be stored in a single frame.
- So, inner page table has to be divided into pages.

Number of Pages of Inner Page Table-

Number of pages the inner page table is divided

= Inner page table size / Page size

$= 2^{46}$ B / 1 MB
$= 2^{46}$ B / 2^{20} B

$= 2^{26}$ pages

Now, these 2^{26} pages of inner page table are stored in different frames of the main memory.

Number of Page Table Entries in One Page of Inner Page Table-

Number of page table entries in one page of inner page table

= Page size / Page table entry size

= 1 MB / 4 B

= 2^{20} B / 2^2 B

= 2^{18} entries

Number of Bits Required to Search an Entry in One Page of Inner Page Table-

One page of inner page table contains 2^{18} entries.

Thus,

Number of bits required to search a particular entry in one page of inner page table = 18 bits

Outer Page Table-1 Size-

Outer page table-1 is required to keep track of the frames storing the pages of inner page table.

Outer page table-1 size

= Number of entries in outer page table-1 x Page table entry size

= Number of pages the inner page table is divided x Page table entry size

= 2^{26} x 4 bytes

= 2^{28} bytes

= 256 MB

Now, we can observe-

- The size of outer page table-1 is greater than the frame size (1 MB).
- Thus, outer page table-1 can not be stored in a single frame.
- So, outer page table-1 has to be divided into pages.

Number of Pages of Outer Page Table-1

Number of pages the outer page table-1 is divided

= Outer page table-1 size / Page size

= 256 MB / 1 MB

= 256 pages

Now, these 256 pages of outer page table-1 are stored in different frames of the main memory.

Number of Page Table Entries in One Page of Outer Page Table-1

Number of page table entries in one page of outer page table-1

= Page size / Page table entry size

= 1 MB / 4 B

= 2^{20} B / 2^{2} B
= 2^{18} entries

Number of Bits Required to Search an Entry in One Page of Outer Page Table-1

One page of outer page table-1 contains 2^{18} entries.

Thus,

Number of bits required to search a particular entry in one page of outer page table-1 = 18 bits

Outer Page Table-2 Size-

Outer page table-2 is required to keep track of the frames storing the pages of outer page table-1.

Outer page table-2 size

= Number of entries in outer page table-2 x Page table entry size

= Number of pages the outer page table-1 is divided x Page table entry size

= 256 x 4 bytes

= 1 KB

Now, we can observe-

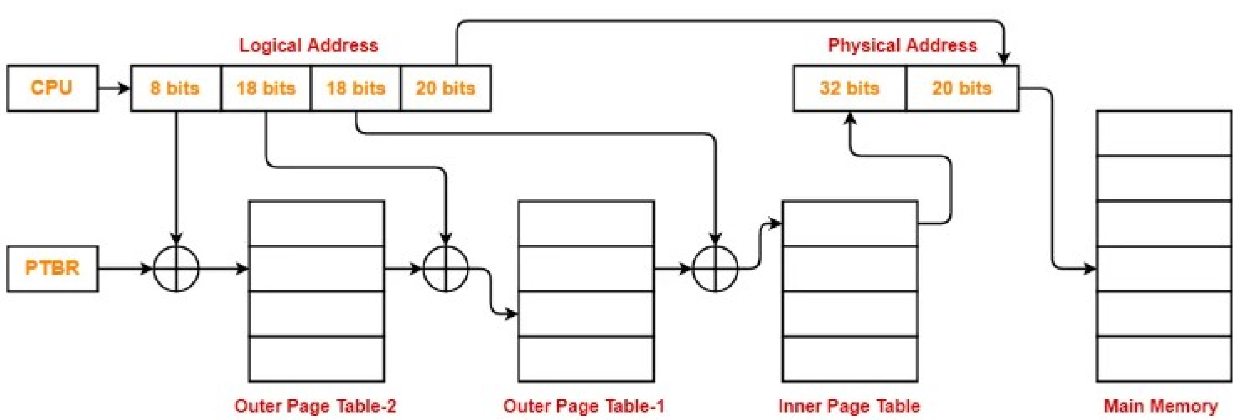
- The size of outer page table-2 is less than the frame size (16 KB).
- Thus, outer page table-2 can be stored in a single frame.
- In fact, outer page table-2 will not completely occupy one frame and some space will remain vacant.
- So, for given system, we will have three levels of page table.
- Page Table Base Register (PTBR) will store the base address of the outer page table-2.

Number of Bits Required to Search an Entry in Outer Page Table-2

Outer page table-2 contains 256 = 2^8 entries.

Thus,Number of bits required to search a particular entry in outer page table-2 = 8 bits

The paging system will look like as shown below-



Problem-02:

Consider a system using multilevel paging scheme. The page size is 1 GB. The memory is byte addressable and virtual address is 72 bits long. The page table entry size is 4 bytes.

Find-

1. How many levels of page table will be required?
2. Give the divided physical address and virtual address.

Solution-

Given-

- Virtual Address = 72 bits
- Page size = 1 GB
- Page table entry size = 4 bytes

Number of Bits in Frame Number-

We have,

Page table entry size

= 4 bytes

= 32 bits

Thus, Number of bits in frame number = 32 bits

Number of Frames in Main Memory-

We have, Number of bits in frame number = 32 bits

Thus,

Number of frames in main memory

= 2^{32} frames

Size of Main Memory-

Size of main memory

= Total number of frames x Frame size

= 2^{32} x 1 GB

= 2^{62} B

Thus, Number of bits in physical address = 62 bits

Number of Bits in Page Offset-

We have,

Page size

= 1 GB

= 2^{30} B

Thus, Number of bits in page offset = 30 bits

Alternatively,

Number of bits in page offset

= Number of bits in physical address – Number of bits in frame number

= 62 bits – 32 bits

= 30 bits

Process Size-

Number of bits in virtual address = 72 bits

Thus,

Process size

= 2^{72} bytes

Number of Pages of Process-

Number of pages the process is divided

= Process size / Page size

= 2^{72} B / 1 GB

= 2^{72} B / 2^{30} B

= 2^{42} pages

Inner Page Table Size-

Inner page table keeps track of the frames storing the pages of process.

Inner page table size

= Number of entries in inner page table x Page table entry size

= Number of pages the process is divided x Page table entry size

= 2^{42} x 4 bytes

= 2^{44} bytes

Now, we can observe-

- The size of inner page table is greater than the frame size (1 GB).
- Thus, inner page table can not be stored in a single frame.
- So, inner page table has to be divided into pages.

Number of Pages of Inner Page Table-

Number of pages the inner page table is divided

= Inner page table size / Page size

= 2^{44} B / 1 GB

= 2^{44} B / 2^{30} B

= 2^{14} pages

Now, these 2^{14} pages of inner page table are stored in different frames of the main memory.

Number of Page Table Entries in One Page of Inner Page Table-

Number of page table entries in one page of inner page table

= Page size / Page table entry size

= 1 GB / 4 B

= 2^{30} B / 2^{2} B

= 2^{28} entries

Number of Bits Required to Search an Entry in One Page of Inner Page Table-

One page of inner page table contains 2^{28} entries.

Thus,

Number of bits required to search a particular entry in one page of inner page table = 28 bits

Outer Page Table Size-

Outer page table is required to keep track of the frames storing the pages of inner page table.

Outer page table size

= Number of entries in outer page table x Page table entry size

= Number of pages the inner page table is divided x Page table entry size

= 2^{14} x 4 bytes

= 2^{16} bytes

= 64 KB

Now, we can observe-

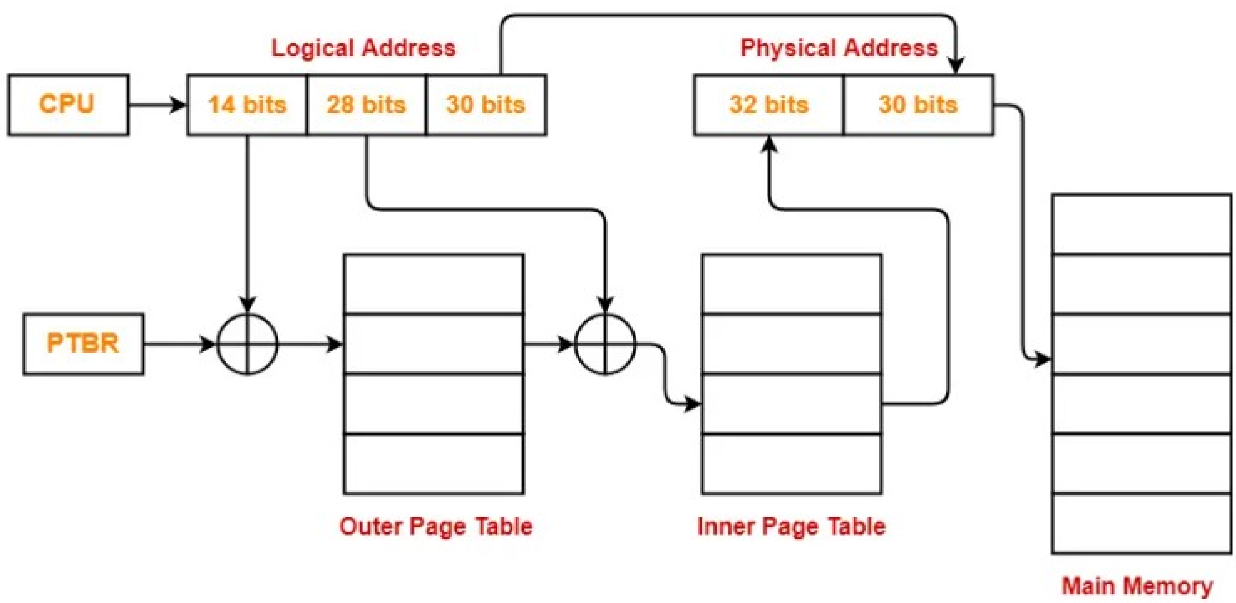
- The size of outer page table is less than the frame size (1 GB).
- Thus, outer page table can be stored in a single frame.
- In fact, outer page table will not completely occupy one frame and some space will remain vacant.
- So, for given system, we will have two levels of page table.
- Page Table Base Register (PTBR) will store the base address of the outer page table.

Number of Bits Required to Search an Entry in Outer Page Table-

Outer page table contains 2^{14} entries.

Thus, Number of bits required to search a particular entry in outer page table = 14 bits

The paging system will look like as shown below-



Problem-03:

Consider a system using multilevel paging scheme. The page size is 256 MB. The memory is byte addressable and virtual address is 72 bits long. The page table entry size is 4 bytes.

Find-

1. How many levels of page table will be required?
2. Give the divided physical address and virtual address.

Solution-

Given-

- Virtual Address = 72 bits
- Page size = 256 MB
- Page table entry size = 4 bytes

Number of Bits in Frame Number-

We have,

Page table entry size

= 4 bytes

= 32 bits

Thus, Number of bits in frame number = 32 bits

Number of Frames in Main Memory-

We have, Number of bits in frame number = 32 bits

Thus,

Number of frames in main memory

= 2^{32} frames

Size of Main Memory-

Size of main memory

= Total number of frames x Frame size

= 2^{32} x 256 MB

= 2^{60} B

Thus, Number of bits in physical address = 60 bits

Number of Bits in Page Offset-

We have,

Page size

= 256 MB

= 2^{28} B

Thus, Number of bits in page offset = 28 bits

Alternatively,

Number of bits in page offset

= Number of bits in physical address – Number of bits in frame number

= 60 bits – 32 bits

= 28 bits

Process Size-

Number of bits in virtual address = 72 bits

Thus,

Process size

= 2^{72} bytes

Number of Pages of Process-

Number of pages the process is divided

= Process size / Page size

= 2^{72} B / 256 MB

= 2^{72} B / 2^{28} B

= 2^{44} pages

Inner Page Table Size-

Inner page table keeps track of the frames storing the pages of process.

Inner page table size

= Number of entries in inner page table x Page table entry size

= Number of pages the process is divided x Page table entry size

= 2^{44} x 4 bytes

= 2^{46} bytes

Now, we can observe-

- The size of inner page table is greater than the frame size (256 MB).
- Thus, inner page table can not be stored in a single frame.
- So, inner page table has to be divided into pages.

Number of Pages of Inner Page Table-

Number of pages the inner page table is divided

= Inner page table size / Page size

= 2^{46} B / 256 MB

= 2^{46} B / 2^{28} B

= 2^{18} pages

Now, these 2^{18} pages of inner page table are stored in different frames of the main memory.

Number of Page Table Entries in One Page of Inner Page Table-

Number of page table entries in one page of inner page table

= Page size / Page table entry size

= 256 MB / 4 B

= 2^{28} B / 2^{2} B

= 2^{26} entries

Number of Bits Required to Search an Entry in One Page of Inner Page Table-

One page of inner page table contains 2^{26} entries.

Thus,

Number of bits required to search a particular entry in one page of inner page table = 26 bits

Outer Page Table Size-

Outer page table is required to keep track of the frames storing the pages of inner page table.

Outer page table size

= Number of entries in outer page table x Page table entry size

= Number of pages the inner page table is divided x Page table entry size

= 2^{18} x 4 bytes

= 2^{20} bytes

= 1 MB

Now, we can observe-

- The size of outer page table is less than the frame size (256 MB).
- Thus, outer page table can be stored in a single frame.
- In fact, outer page table will not completely occupy one frame and some space will remain vacant.
- So, for given system, we will have two levels of page table.
- Page Table Base Register (PTBR) will store the base address of the outer page table.

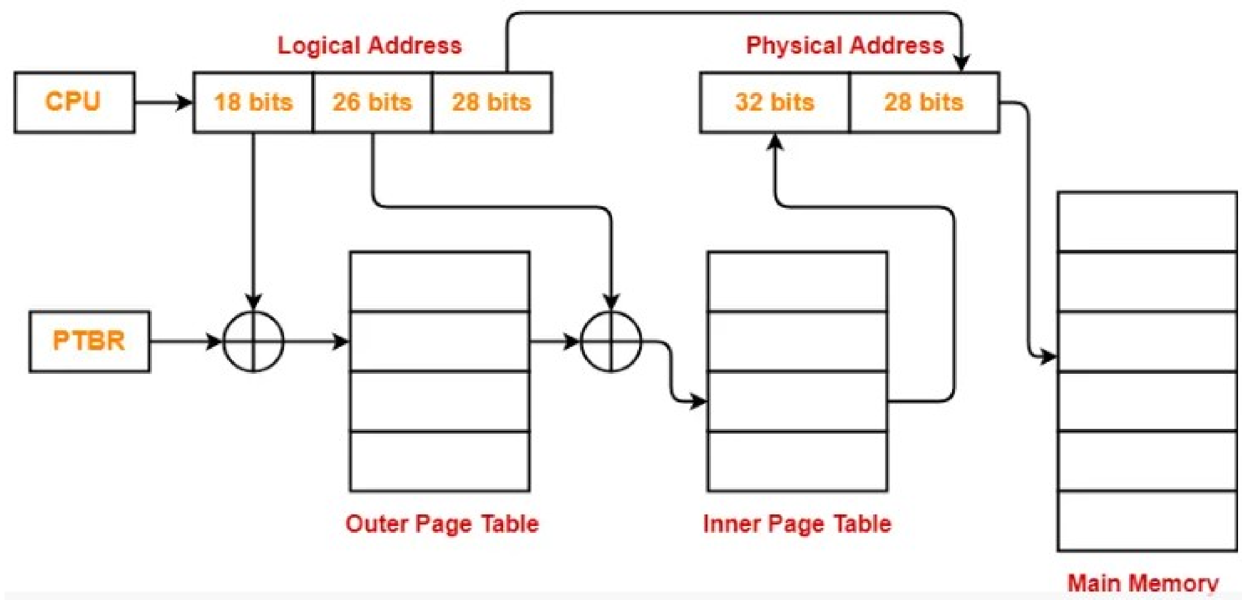
Number of Bits Required to Search an Entry in Outer Page Table-

Outer page table contains 2^{18} entries.

Thus,

Number of bits required to search a particular entry in outer page table = 18 bits

The paging system will look like as shown below-



Problem-04:

Consider a system using multilevel paging scheme. The page size is 16 MB. The memory is byte addressable and virtual address is 72 bits long. The page table entry size is 4 bytes.

Find-

1. How many levels of page table will be required?
2. Give the divided physical address and virtual address.

Solution-

Given-

- Virtual Address = 72 bits
- Page size = 16 MB
- Page table entry size = 4 bytes

Number of Bits in Frame Number-

We have,

Page table entry size

= 4 bytes

= 32 bits

Thus, Number of bits in frame number = 32 bits

Number of Frames in Main Memory-

We have, Number of bits in frame number = 32 bits

Thus,

Number of frames in main memory

= 2^{32} frames

Size of Main Memory-

Size of main memory

= Total number of frames x Frame size

= 2^{32} x 16 MB

= 2^{56} B

Thus, Number of bits in physical address = 56 bits

Number of Bits in Page Offset-

We have,

Page size

= 16 MB

= 2^{24} B

Thus, Number of bits in page offset = 24 bits

Alternatively,

Number of bits in page offset

= Number of bits in physical address – Number of bits in frame number

= 56 bits – 32 bits

= 24 bits

Process Size-

Number of bits in virtual address = 72 bits

Thus,

Process size

= 2^{72} bytes

Number of Pages of Process-

Number of pages the process is divided

= Process size / Page size

= 2^{72} B / 16 MB

= 2^{72} B / 2^{24} B

= 2^{48} pages

Inner Page Table Size-

Inner page table keeps track of the frames storing the pages of process.

Inner page table size

= Number of entries in inner page table x Page table entry size

= Number of pages the process is divided x Page table entry size

= 2^{48} x 4 bytes

= 2^{50} bytes

Now, we can observe-

- The size of inner page table is greater than the frame size (16 MB).
- Thus, inner page table can not be stored in a single frame.
- So, inner page table has to be divided into pages.

Number of Pages of Inner Page Table-

Number of pages the inner page table is divided

= Inner page table size / Page size

= 2^{50} B / 16 MB

= 2^{50} B / 2^{24} B

= 2^{26} pages

Now, these 2^{26} pages of inner page table are stored in different frames of the main memory.

Number of Page Table Entries in One Page of Inner Page Table-

Number of page table entries in one page of inner page table

= Page size / Page table entry size

= 16 MB / 4 B

= 2^{24} B / 2^{2} B

= 2^{22} entries

Number of Bits Required to Search an Entry in One Page of Inner Page Table-

One page of inner page table contains 2^{22} entries.

Thus,

Number of bits required to search a particular entry in one page of inner page table = 22 bits

Outer Page Table-1 Size-

Outer page table-1 is required to keep track of the frames storing the pages of inner page table.

Outer page table-1 size

= Number of entries in outer page table-1 x Page table entry size

= Number of pages the inner page table is divided x Page table entry size

= 2^{26} x 4 bytes

= 2^{28} bytes

= 256 MB

Now, we can observe-

- The size of outer page table-1 is greater than the frame size (16 MB).
- Thus, outer page table-1 can not be stored in a single frame.
- So, outer page table-1 has to be divided into pages.

Number of Pages of Outer Page Table-1

Number of pages the outer page table-1 is divided

= Outer page table-1 size / Page size

= 256 MB / 16 MB

= 16 pages

Now, these 16 pages of outer page table-1 are stored in different frames of the main memory.

Number of Page Table Entries in One Page of Outer Page Table-1

Number of page table entries in one page of outer page table-1

= Page size / Page table entry size

= 16 MB / 4 B

= 2^{24} B / 2^{2} B

= 2^{22} entries

Number of Bits Required to Search an Entry in One Page of Outer Page Table-1

One page of outer page table-1 contains 2^{22} entries.

Thus,

Number of bits required to search a particular entry in one page of outer page table-1 = 22 bits

Outer Page Table-2 Size-

Outer page table-2 is required to keep track of the frames storing the pages of outer page table-1.

Outer page table-2 size

= Number of entries in outer page table-2 x Page table entry size

= Number of pages the outer page table-1 is divided x Page table entry size

= 16 x 4 bytes

= 64 bytes

Now, we can observe-

- The size of outer page table-2 is less than the frame size (16 MB).
- Thus, outer page table-2 can be stored in a single frame.
- In fact, outer page table-2 will not completely occupy one frame and some space will remain vacant.
- So, for given system, we will have three levels of page table.
- Page Table Base Register (PTBR) will store the base address of the outer page table-2.

Number of Bits Required to Search an Entry in Outer Page Table-2

Outer page table-2 contains 16 = 2^4 entries.

Thus,

Number of bits required to search a particular entry in outer page table-2 = 4 bits

The paging system will look like as shown below-