Tutorial -5 Assignment_20103153

Reflection, Refraction Oblique incidence-s polarization

The electric field phasors for the perpendicular polarization, with reference to the system of coordinates in the figure, are given by

$$\vec{E}_i = E_{yi}\, e^{-j\beta_{ix}\cdot x - j\beta_{iz}\cdot z}\ \hat{i}_y$$

$$\vec{E}_r = E_{yr}\, e^{-j\beta_{rx}\cdot x - j\beta_{rz}\cdot z}\ \hat{i}_y$$

$$\vec{E}_t = E_{yt}\, e^{-j\beta_{tx}\cdot x - j\beta_{tz}\cdot z}\ \hat{i}_y$$

The propagation vector components in medium 1 are expressed as

$$\left|\vec{\beta}_i\right| = \sqrt{\beta_{ix}^2 + \beta_{iz}^2} = \beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$\beta_{ix} = \beta_1 \cos\theta_i \qquad \beta_{iz} = \beta_1 \sin\theta_i$$

$$\left|\vec{\beta}_r\right| = \sqrt{\beta_{rx}^2 + \beta_{rz}^2} = \beta_1$$

$$\beta_{rx} = -\beta_1 \cos\theta_r \qquad \beta_{rz} = \beta_1 \sin\theta_r$$

The propagation vector components in medium 2 are expressed as

$$\left|\vec{\beta}_t\right| = \sqrt{\beta_{tx}^2 + \beta_{tz}^2} = \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$\beta_{tx} = \beta_2 \cos\theta_t \qquad \beta_{tz} = \beta_2 \sin\theta_t$$

The magnetic field components can be obtained as

$$\vec{H}_i = \frac{\vec{\beta}_i \times \vec{E}_i}{\omega\mu_1} = \frac{E_{yi}}{\eta_1}\left(-\sin\theta_i\ \hat{i}_x + \cos\theta_i\ \hat{i}_z\right) e^{-j\beta_{ix}x - j\beta_{iz}z}$$

$$\vec{H}_r = \frac{\vec{\beta}_r \times \vec{E}_r}{\omega\mu_1} = -\frac{E_{yr}}{\eta_1}\left(\sin\theta_r\ \hat{i}_x + \cos\theta_r\ \hat{i}_z\right) e^{-j\beta_{rx}x - j\beta_{rz}z}$$

$$\vec{H}_t = \frac{\vec{\beta}_t \times \vec{E}_t}{\omega\mu_2} = \frac{E_{yt}}{\eta_2}\left(-\sin\theta_t\ \hat{i}_x + \cos\theta_t\ \hat{i}_z\right) e^{-j\beta_{tx}x - j\beta_{tz}z}$$

Assuming that the amplitude of the incident electric field is given, to completely specify the problem we need to find the amplitude of reflected and transmitted electric field.

The boundary condition at the interface (x = 0) states that the tangential electric field must be continuous. Because of the perpendicular polarization, the tangential field is also the total field

$$x = 0) \qquad E_{yi}\, e^{-j\beta_{iz} z} + E_{yr}\, e^{-j\beta_{rz} z} = E_{yt}\, e^{-j\beta_{tz} z}$$

The relation above must be valid for any choice of "z" and we must have (phase conservation law)

$$\beta_{iz} = \beta_{rz} = \beta_{tz}$$

The first equality indicates that the reflected angle is the same as the incident angle.

$$\beta_{iz} = \beta_{rz} \Rightarrow \quad \beta_1 \sin\theta_i = \beta_1 \sin\theta_r \quad \Rightarrow \quad \theta_i = \theta_r$$

The second equality provides the transmitted angle

$$\beta_{iz} = \beta_{tz} \quad \Rightarrow \quad \beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

$$\Rightarrow \quad \theta_t = \sin^{-1}\left(\sqrt{\frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}} \sin\theta_i\right)$$

Snell's Law

Since we have also

$$e^{-j\beta_{iz} z} = e^{-j\beta_{rz} z} = e^{-j\beta_{tz} z}$$

the boundary condition for the electric field becomes

$$E_{yi} + E_{yr} = E_{yt}$$

The tangential magnetic field must also be continuous at the interface. This applies in our case to the z–components

$$H_{zi} \quad + \quad H_{zr} \quad = H_{zt}$$

$$\frac{E_{yi}}{\eta_1}\cos\theta_i - \frac{E_{yr}}{\eta_1}\cos\theta_i = \frac{E_{yt}}{\eta_2}\cos\theta_t$$

$$\Rightarrow \quad E_{yi} - E_{yr} = \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i} E_{yt}$$

Solution of the system of boundary equations gives

$$\Gamma_{\perp}(E)=\frac{E_{yr}}{E_{yi}}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}$$ Reflection coefficient

$$\tau_{\perp}(E)=\frac{E_{yt}}{E_{yi}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}$$ Transmission coefficient

For the magnetic field, we can define the reflection coefficient as

$$\Gamma_{\perp}(H)=\frac{H_{zr}}{H_{zi}}=-\frac{H_r}{H_i}$$

In terms of electric field, the magnetic field components are

$$H_{zr}=\frac{-E_{yr}}{\eta_1}\cos\theta_i=-H_r\cos\theta_i$$

$$H_{zi}=\frac{E_{yi}}{\eta_1}\cos\theta_i=H_i\cos\theta_i$$

The reflection coefficient for the magnetic field is then

$$\Gamma_{\perp}(H)=\frac{-E_{yr}}{E_{yi}}=-\Gamma_{\perp}(E)=\frac{\eta_1\cos\theta_t-\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}$$

The transmission coefficient is defined as

$$\tau_{\perp}(H)=\frac{H_t}{H_i}$$

The magnetic field components are

$$H_t=\frac{E_{yt}}{\eta_2}$$

$$H_i=\frac{E_{yi}}{\eta_1}$$

The transmission coefficient for the magnetic field is then

$$\tau_{\perp}(H) = \frac{H_t}{H_i} = \frac{\eta_1}{\eta_2}\tau_{\perp}(E) = \frac{2\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$