

Tutorial Sheet 1 (Differential Equations with Constant Coefficients)

1. Find the complementary function of the following equations:

- i. $(D^2 - 2D + 2)y = 0$
- ii. $(D^4 - 81)y = 0$
- iii. $(D^3 - 1)^2 y = 0$

2. Solve the following differential equations.

- i. $(D^2 - 4D + 4)y = e^{2x} + x^3 + \cos 2x$
- ii. $(D^2 - 6D + 13)y = 16e^{3x} \sin 4x + 3^x$
- iii. $(D^2 + 1)y = \operatorname{cosec} x$

3. Find the solution of the following differential equations:

- i. $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$
- ii. $x^2 y'' + 4xy' + 2y = 0$
- iii. $x^2 y'' - 5xy' + 9y = 0$
- iv. $x^3 y''' + 3x^2 y'' + xy' + y = \sin(\log x) + x$

Ans:

1.

- i. $y(x) = e^x (A \cos x + B \sin x)$
- ii. $y(x) = Ae^{3x} + Be^{-3x} + C \cos 3x + D \sin 3x$
- iii. $y(x) = (A + Bx)e^x + e^{-x/2} \left((C + Dx) \cos\left(\frac{\sqrt{3}}{2}x\right) + (E + Fx) \sin\left(\frac{\sqrt{3}}{2}x\right) \right)$

2.

- i. $y(x) = (A + Bx)e^{2x} + \frac{x^2}{2}e^x + \frac{x^3}{4} + \frac{3x^2}{4} + \frac{9x}{8} + \frac{3}{4} - \frac{\sin 2x}{8}$
- ii. $y(x) = e^{3x} (A \cos 2x + B \sin 2x) - \frac{4}{3}e^{3x} \sin 4x + \frac{3^x}{(\log 3)^2 - 6 \log 3 + 13}$
- iii. $y(x) = (A - x) \cos x + (B + \log |\sin x|) \sin x$

3.

- i. $y(x) = A \cos(\log x) + B \sin(\log x)$
- ii. $y(x) = \frac{A}{x} + \frac{B}{x^2}$
- iii. $y(x) = (A + B \log x)x^3$
- iv. $y(x) = \frac{A}{x} + x^{1/2} \left(B \cos\left(\frac{\sqrt{3}}{2} \log x\right) + C \sin\left(\frac{\sqrt{3}}{2} \log x\right) \right) + \frac{\sin(\log x) + \cos(\log x) + x}{2}$