

Department of Mathematics

16B1NMA533

Tutorial Sheet 8 [C301-3.5]

Matrix Computations

B.Tech. Elective

(Eigenvalues and Eigenvectors, Power Method, Inverse Power Method, Gershgorin's theorem)

1. Find all the eigenvalues and the eigenvectors of the matrices:

$$(i) \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \quad (ii) \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad (iii) \begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

2. Use power method to estimate the largest eigenvalue and the corresponding eigenvector of following matrices. (Consider all the components of initial vector as 1.)

$$(i) \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix} \quad (iii) \begin{bmatrix} 8 & 2 \\ 3 & 3 \end{bmatrix} \quad (ii) \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

3. Use inverse power method on the matrix defined in Q.2 and stop after five iterations (wherever necessary).
4. State and prove Gershgorin's theorem.
5. Using Gershgorin's theorem, find and sketch disks and intervals, that contains the eigenvalues of

$$(i) \begin{bmatrix} 1 & 2+4i & 1-i \\ 2-4i & -5 & 3-5i \\ 1+i & 3+5i & 6 \end{bmatrix} \quad (ii) \begin{bmatrix} 1 & 1 & -2 \\ 1 & -5 & 3 \\ -2 & 3 & 4 \end{bmatrix}.$$

Also verify the results by finding the eigenvalues.