



Digital Systems

18B11EC213

**Module 1: Boolean Function Minimization
Techniques and Combinational Circuits-11**

Dr. Saurabh Chaturvedi



Quine-McCluskey (QM) Method

- A systematic solution to K-Map when more complex function with more literals is given.
- In principle, this method can be applied to an arbitrary large number of inputs.
- One can translate Quine-McCluskey (QM) method into a computer program to perform the minimization of a Boolean expression.



Quine-McCluskey Method

- Two basic steps:
 - Finding all prime implicants (PIs) of a given Boolean function.
 - Select a minimal set of prime implicants that cover this function.



QM Method – Example-1

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

- Transform the given Boolean function into a canonical SOP function
- Convert each minterm into binary format
- Arrange each binary minterm in groups
 - All the minterms in one group contain the same number of "1"

QM Method: Grouping minterms

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	A	B	C	D	E
(0)	0	0	0	0	0
(2)	0	0	0	1	0
(4)	0	0	1	0	0
(8)	0	1	0	0	0
(16)	1	0	0	0	0
(6)	0	0	1	1	0
(10)	0	1	0	1	0
(12)	0	1	1	0	0
(18)	1	0	0	1	0
(7)	0	0	1	1	1
(11)	0	1	0	1	1
(13)	0	1	1	0	1
(14)	0	1	1	1	0
(19)	1	0	0	1	1
(29)	1	1	1	0	1
(30)	1	1	1	1	0



QM Method

- Combine terms with Hamming distance=1 from adjacent groups
- Check (✓) the terms being combined
 - The checked terms are “covered” by the combined new term
- Keep doing this till no combination is possible between adjacent groups

QM Method: Grouping minterms

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	A	B	C	D	E	
(0)	0	0	0	0	0	✓
(2)	0	0	0	1	0	✓
(4)	0	0	1	0	0	✓
(8)	0	1	0	0	0	✓
(16)	1	0	0	0	0	✓
(6)	0	0	1	1	0	✓
(10)	0	1	0	1	0	✓
(12)	0	1	1	0	0	✓
(18)	1	0	0	1	0	✓
(7)	0	0	1	1	1	✓
(11)	0	1	0	1	1	✓
(13)	0	1	1	0	1	✓
(14)	0	1	1	1	0	✓
(19)	1	0	0	1	1	✓
(29)	1	1	1	0	1	✓
(30)	1	1	1	1	0	✓

	A	B	C	D	E
(0, 2)	0	0	0	-	0
(0, 4)	0	0	-	0	0
(0, 8)	0	-	0	0	0
(0, 16)	-	0	0	0	0
(2, 6)	0	0	-	1	0
(2, 10)	0	-	0	1	0
(2, 18)	-	0	0	1	0
(4, 6)	0	0	1	-	0
(4, 12)	0	-	1	0	0
(8, 10)	0	1	0	-	0
(8, 12)	0	1	-	0	0
(16, 18)	1	0	0	-	0
(6, 7)	0	0	1	1	-
(6, 14)	0	-	1	1	0
(10, 11)	0	1	0	1	-
(10, 14)	0	1	-	1	0
(12, 13)	0	1	1	0	-
(12, 14)	0	1	1	-	0
(18, 19)	1	0	0	1	-

	A	B	C	D	E
(13, 29)	-	1	1	0	1
(14, 30)	-	1	1	1	0

QM Method: Grouping minterms

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	A	B	C	D	E	
(0, 2)	0	0	0	-	0	✓
(0, 4)	0	0	-	0	0	✓
(0, 8)	0	-	0	0	0	✓
(0, 16)	-	0	0	0	0	✓
<hr/>						
(2, 6)	0	0	-	1	0	✓
(2, 10)	0	-	0	1	0	✓
(2, 18)	-	0	0	1	0	✓
(4, 6)	0	0	1	-	0	✓
(4, 12)	0	-	1	0	0	✓
(8, 10)	0	1	0	-	0	✓
(8, 12)	0	1	-	0	0	✓
(16, 18)	1	0	0	-	0	✓
<hr/>						
(6, 7)	0	0	1	1	-	
(6, 14)	0	-	1	1	0	✓
(10, 11)	0	1	0	1	-	
(10, 14)	0	1	-	1	0	✓
(12, 13)	0	1	1	0	-	
(12, 14)	0	1	1	-	0	✓
(18, 19)	1	0	0	1	-	
<hr/>						
(13, 29)	-	1	1	0	1	
(14, 30)	-	1	1	1	0	

	A	B	C	D	E	
(0, 2, 4, 6)	0	0	-	-	0	✓
(0, 2, 8, 10)	0	-	0	-	0	✓
(0, 2, 16, 18)	-	0	0	-	0	
(0, 4, 8, 12)	0	-	-	0	0	✓
<hr/>						
(2, 6, 10, 14)	0	-	-	1	0	✓
(4, 6, 12, 14)	0	-	1	-	0	✓
(8, 10, 12, 14)	0	1	-	-	0	✓

	A	B	C	D	E
(0, 2, 4, 6, 8, 10, 12, 14)	0	-	-	-	0



Prime Implicants

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	A	B	C	D	E
(6, 7)	0	0	1	1	-
(10, 11)	0	1	0	1	-
(12, 13)	0	1	1	0	-
(18, 19)	1	0	0	1	-
(13, 29)	-	1	1	0	1
(14, 30)	-	1	1	1	0
(0, 2, 16, 18)	-	0	0	-	0
(0, 2, 4, 6, 8, 10, 12, 14)	0	-	-	-	0

- Unchecked (unticked) terms are the prime implicants.



Prime Implicants

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	A	B	C	D	E	
(6, 7)	0	0	1	1	-	$= \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}$
(10, 11)	0	1	0	1	-	$= \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}$
(12, 13)	0	1	1	0	-	$= \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}$
(18, 19)	1	0	0	1	-	$= \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}$
(13, 29)	-	1	1	0	1	$= \overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}\overline{\overline{E}}$
(14, 30)	-	1	1	1	0	$= \overline{\overline{B}}\overline{\overline{C}}\overline{\overline{D}}\overline{\overline{E}}$
(0, 2, 16, 18)	-	0	0	-	0	$= \overline{\overline{B}}\overline{\overline{C}}\overline{\overline{E}}$
(0, 2, 4, 6, 8, 10, 12, 14)	0	-	-	-	0	$= \overline{\overline{A}}\overline{\overline{E}}$

- Unchecked terms are prime implicants



QM Method

- Form a Prime Implicant Table
 - X-axis: the minterm
 - Y-axis: prime implicants
- An \times is placed at the intersection of a row and column if the corresponding prime implicant includes the corresponding product (term)

QM Method: Prime Implicant Table

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					



QM Method

- Locate the essential row from the table
 - These are essential prime implicants
 - The row consists of minterms covered by a single "x"
- Mark all minterms covered by the essential prime implicants
- Find non-essential prime implicants to cover the rest of minterms
- Form the SOP function with the prime implicants selected, which is the minimal representation



QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11)

Q-M Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18), (18,19)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18), (18,19)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18), (18,19), (13,29)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18), (18,19), (13,29)

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18), (18,19), (13,29), (14,30)
- Now all the minterms are covered by selected prime implicants !

QM Method

$$F(A, B, C, D, E) = \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30)$$

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

- Select (0,2,4,6,8,10,12,14), (6,7), (10,11), (0,2,16,18), (18,19), (13,29), (14,30)
- Now all the minterms are covered by selected prime implicants !
- Note that (12,13), a non-essential prime implicant, is not needed.

QM Method - Result

	0	2	4	6	7	8	10	11	12	13	14	16	18	19	29	30
(6,7)				X	X											
(10,11)							X	X								
(12,13)									X	X						
(18,19)													X	X		
(13,29)										X					X	
(14,30)											X					X
(0,2,16,18)	X	X										X	X			
(0,2,4,6,8,10,12,14)	X	X	X	X		X	X		X		X					

$$\begin{aligned}
 F(A, B, C, D, E) &= \sum m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30) \\
 &= (6,7) + (10,11) + (18,19) + (13,29) + (14,30) + (0,2,16,18) \\
 &\quad + (0,2,4,6,8,10,12,14) \\
 &= \overline{A}\overline{B}CD + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + BC\overline{D}E + BCDE + \overline{B}\overline{C}\overline{E} + \overline{A}E
 \end{aligned}$$

QM Method - Example-2

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

- Sometimes, simplification by K-map method could be less than optimal due to human error
- Quine-McCluskey method can guarantee an optimal answer

		CD			
		00	01	11	10
AB	00	1	1	0	0
	01	1	X	X	1
	11	1	0	0	X
	10	1	1	0	1

Grouping minterms

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

	A	B	C	D	
(0)	0	0	0	0	✓
(1)	0	0	0	1	✓
(4)	0	1	0	0	✓
(8)	1	0	0	0	✓
(5)	0	1	0	1	✓
(6)	0	1	1	0	✓
(9)	1	0	0	1	✓
(10)	1	0	1	0	✓
(12)	1	1	0	0	✓
(7)	0	1	1	1	✓
(14)	1	1	1	0	✓

	A	B	C	D
(0,1)	0	0	0	-✓
(0,4)	0	-	0	0✓
(0,8)	-	0	0	0✓
(1,5)	0	-	0	1✓
(1,9)	-	0	0	1✓
(4,5)	0	1	0	-✓
(4,6)	0	1	-	0✓
(4,12)	-	1	0	0✓
(8,9)	1	0	0	-✓
(8,10)	1	0	-	0✓
(8,12)	1	-	0	0✓
(5,7)	0	1	-	1✓
(6,7)	0	1	1	-✓
(6,14)	-	1	1	0✓
(10,14)	1	-	1	0✓
(12,14)	1	1	-	0✓

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

Prime Implicants

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

Don't Care

	0	1	4	6	8	9	10	12	5	7	14
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

Prime Implicants

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

Don't Care

	0	1	4	6	8	9	10	12	5	7	14
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

Prime Implicants

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

Don't Care

	0	1	4	6	8	9	10	12	5	7	14
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

Prime Implicants

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

	0	1	4	6	8	9	10	12	5	7	14
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

Don't Care

Prime Implicants

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

	0	1	4	6	8	9	10	12	Don't Care		
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

Prime Implicants

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

	A	B	C	D	
(0,1,4,5)	0	-	0	-	
(0,1,8,9)	-	0	0	-	Essential PI
(0,4,8,12)	-	-	0	0	
(4,5,6,7)	0	1	-	-	Non-Essential PI
(4,6,12,14)	-	1	-	0	
(8,10,12,14)	1	-	-	0	Essential PI

	0	1	4	6	8	9	10	12	Don't Care		
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

QM Method Solution

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

$$= \overline{B}\overline{C} + A\overline{D} + \overline{A}B$$

Don't Care

	0	1	4	6	8	9	10	12	5	7	14
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

Yet Another QM Method Solution

	A	B	C	D
(0,1,4,5)	0	-	0	-
(0,1,8,9)	-	0	0	-
(0,4,8,12)	-	-	0	0
(4,5,6,7)	0	1	-	-
(4,6,12,14)	-	1	-	0
(8,10,12,14)	1	-	-	0

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

$$= \overline{B}\overline{C} + A\overline{D} + B\overline{D}$$

Don't Care

	0	1	4	6	8	9	10	12	5	7	14
(0,1,4,5)	X	X	X						X		
(0,1,8,9)	X	X			X	X					
(0,4,8,12)	X		X		X			X			
(4,5,6,7)			X	X					X	X	
(4,6,12,14)			X	X				X			X
(8,10,12,14)					X		X	X			X

To Get the Same Answer with K-Map

		CD			
		00	01	11	10
AB	00	1	1	0	0
	01	1	X	X	1
	11	1	0	0	X
	10	1	1	0	1

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

$$= \overline{B}\overline{C} + A\overline{D} + \overline{A}B$$

		CD			
		00	01	11	10
AB	00	1	1	0	0
	01	1	X	X	1
	11	1	0	0	X
	10	1	1	0	1

$$F = \sum m(0,1,4,6,8,9,10,12) + d(5,7,14)$$

$$= \overline{B}\overline{C} + A\overline{D} + B\overline{D}$$



References

- M. M. Mano, *Digital Logic and Computer Design*, 5th ed., Pearson Prentice Hall, 2013.
- R. P. Jain, *Modern Digital Electronics*, 4th ed., Tata McGraw-Hill Education, 2009.