Tutorial Sheet – Odd Semester 2022

15B11CI212 Theoretical Foundations of Computer Science

Tutorial 2

Introduction to Discrete Mathematics and Set Theory

Ques 1: How many numbers below 100 are divisible by 2,3, or 5?

- Let A be the set of positive integers less than 100 that are divisible by 2.
- Let B be the set of positive integers less than 100 that are divisible by 3.
- Let C be the set of positive integers less than 100 that are divisible by 5.

We want to find the size of $A \cup B \cup C$. The size of each is easy to find on its own, but the size of the union is kind of tricky to calculate directly, so we will use the technique of inclusion-exclusion.

- $|A| = \lfloor \frac{99}{2} \rfloor = 49$
- $|B| = \lfloor \frac{99}{3} \rfloor = 33$
- $|C| = \lfloor \frac{99}{5} \rfloor = 19$
- The set $A \cap B$ is the set of positive integers less than 100 that are divisible by both 2 and 3, which is equivalent to being divisible by 6. So $|A \cap B| = \lfloor \frac{99}{6} \rfloor = 16$
- $|A \cap C| = \lfloor \frac{99}{10} \rfloor = 9$
- $|B \cap C| = \lfloor \frac{99}{15} \rfloor = 6$
- $|A \cap B \cap C| = \lfloor \frac{99}{30} \rfloor = 3$

The principle of inclusion-exclusion now tells us that $|A \cup B \cup C| = \boxed{49 + 33 + 19 - 16 - 9 - 6 + 3 = 73}$.

Ques 2. Suppose, there are 1000 students who are taking admission in the university in the academic year 2022-2023. All these students are enrolled in a Math or an English course. 400 students are taking Maths and English, 600 students are taking English.

a) How many are taking maths course?

M=set of students taking maths course

E= set of students taking English course

|MUE|=|M|+|E|-|M∩E|

1000=|M|+600-400

|M|=800

b) How many are taking a math course but not and an English course?

|M-E|= |M|-|M∩E|

= 800-400

=400

Ques 3. Out of 30 students in class,15 take an TFCS course, 8 take a AI course and 6 take DBMS course. It is known that 3 students take all the three courses. Show that 7 or more students take none of the courses.

$$|A| = 15, |B| = 8, |C| = 6, |A \cap B \cap C| = 3$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

$$= 15 + 8 + 6 - |A \cap B| - |B \cap C| - |C \cap A| + 3$$

$$= 32 - |A \cap B| - |B \cap C| - |C \cap A| \quad \ldots (1)$$

But $\quad |A \cap B| \geq |A \cap B \cap C|, |B \cap C| \geq |A \cap B \cap C|, |C \cap A| \geq |A \cap B \cap C|$

Therefore, $\quad |A \cap B| + |B \cap C| + |C \cap A| \geq 3\,|A \cap B \cap C|$

From (1), we have

$$|A \cup B \cup C| \geq 32 - 3\,|A \cap B \cap C| = 32 - 3 \times 3$$

Hence $\quad |A \cup B \cup C| \geq 23$

The number of students taking atleast one course ≥ 23. The students taking none of the courses $\geq 30 - 23 = 7$.

Hence, seven or more students take none of the courses.

Ques 4: Let A = {1, 2, 3, 4, 5} and B = {0, 3, 6}. Find

a) A ∪ B.

b) A ∩ B.

c) A − B.

d) B − A.

Solution:

a) {0,1,2,3,4,5,6}

b) {3}

c) {1, 2, 4,5}

d) {0, 6}

Ques 5: Let A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}. Find

a) A ∩ B ∩ C.

b) A ∪ B ∪ C.

c) (A ∪ B) ∩ C.

d) (A ∩ B) ∪ C.

Solution:

a) {4,6}

b) {0,1,2,3,4,5,6,7,8,9,10}

c) {4, 5, 6, 8, 10}

d) {0,2,4, 5,6,7,8,9,10}

Ques 6: Prove that $A \cap (B - C) \subset A - (B \cap C)$

Let $x \in A \cap (B - C)$, then

$$x \in A \cap (B - C) \Rightarrow x \in A \text{ and } x \in (B - C)$$
$$\Rightarrow x \in A \text{ and } (x \in B, \text{ and } x \notin C)$$
$$\Rightarrow x \in A \text{ and } x \notin (B \cap C)$$
$$\Rightarrow x \in A - (B \cap C)$$

Hence $A \cap (B - C) \subset A - (B \cap C)$

Ques 7: Show that if A, B, and C are sets, then

$$(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$$

17. a) $x \in \overline{A \cap B \cap C} \equiv x \notin A \cap B \cap C \equiv x \notin A \vee x \notin B \vee x \notin C \equiv x \in \overline{A} \vee x \in \overline{B} \vee x \in \overline{C} \equiv x \in \overline{A} \cup \overline{B} \cup \overline{C}$

Ques 8: Show that if A and B are sets, then

a) $A - B = A \cap B^c$.

b) $(A \cap B) \cup (A \cap B^c) = A$.

Solution:

a) Both sides equal $\{x | x \in A \wedge x \notin B\}$

b) $A = A \cap U = A \cap (B \cup B^c) = (A \cap B) \cup (A \cap B^c)$

Ques 9: List all partitions of the following sets:

(a) {a}
(b) {a, b}

(c) {a, b, c}

(1) The only partition of {a} is the partition into one single set ({a}).

(2) This set has two partitions. The first is ({a}, {b}). The second is ({a, b}).

(3) This set has five partitions.

They are • ({a}, {b}, {c})

• ({a}, {b, c})

• ({a, b}, {c})

• ({a, c}, {b})

• ({a, b, c})

Ques 10: The bit strings for the sets {1,2,3,4,5} and {1,3,5,7,9} are 11 1110 0000 and 10 1010 1010, respectively. Use bit strings to find the union and intersection of these sets.

Solution:

Union: 11 1110 0000 $\vee$ 10 1010 1010 = 11 1110 1010, {1,2,3,4,5,7,9}

Intersection: 11 1110 0000 $\wedge$ 10 1010 1010 = 10 1010 0000, {1,3,5}