(ii) Let X be the random variable with MGF given as $M_X(t) = e^{t+(t^2/4)}$. Find $P(Y \leq 2)$ with $Y=X^2+1$.

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$$m_{x}(s) = e^{t + \frac{t^2}{4}} \Rightarrow \cdot e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$\mu = 1$$

$$\sigma^2 = \frac{1}{2}$$

$$\sigma = \frac{1}{\sqrt{2}}$$

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$P(x^2+1 \leq 2)$

$P(x^2 \leq 1)$

$P(-1 \leq x < 1)$

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$$P(-1 \le x < 1)$$

$$P\left(\frac{-1-1}{\frac{1}{\sqrt{2}}} < \frac{x-1}{\frac{1}{\sqrt{2}}} < \frac{1-1}{\frac{1}{\sqrt{2}}}\right)$$

$$P\left(-2\sqrt{2} < \frac{x-1}{\frac{1}{\sqrt{2}}} < 0\right)$$

$$P\left(-2.82 < \frac{x-1}{\frac{1}{\sqrt{2}}} < 0\right)$$

$$\Rightarrow \quad - \frac{1}{2} - 0.0024$$

$$\underline{0.1976}$$

Q2. Trains **X** and Y arrive at a station at random between 8 A.M and 8:30 A.M. Train **X** stops for five minutes and Train Y stops for seven minutes. Assuming that the train arrival times are independent of each other and are uniformly distributed, find the probability that both the trains will meet?

(4)

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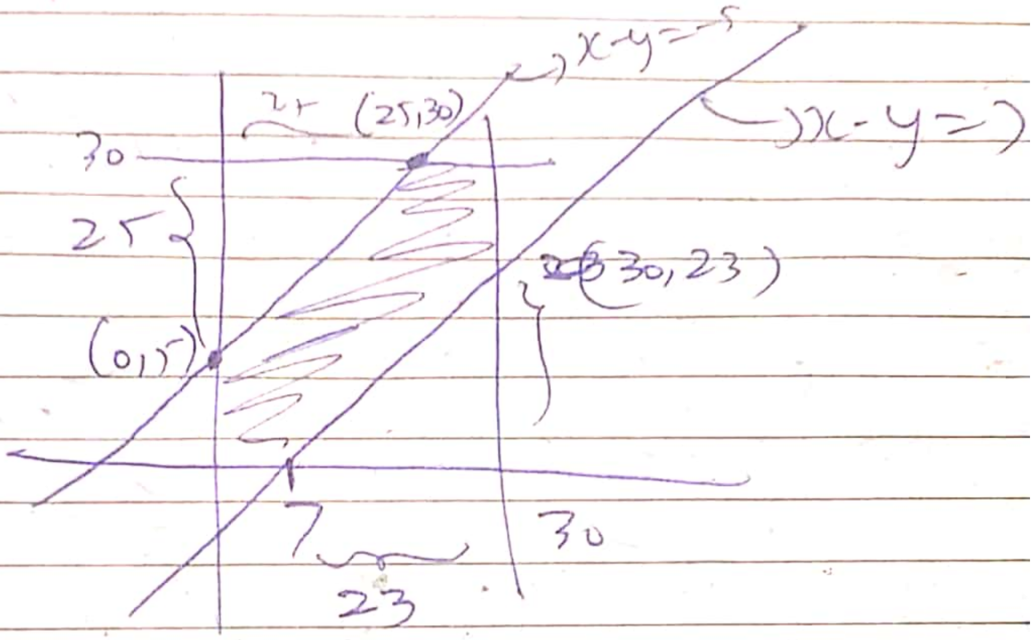
23:14

You
25 April, 21:47

$-5 \le X - Y \le 7$



$$\frac{30^2 - \left(\frac{1}{2}\times 23\times 23 + \frac{1}{2}\times 25\times 2\right.}{30}$$

0.35

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Example 7 *Night watchmen carry an industrial flashlight 8 hours per night, 7 nights per week. It is estimated that on the average the flashlight is turned on about 20 minutes per 8-hour shift. The flashlight is assumed to have a constant failure rate of 0.08/hour while it is turned on and of 0.005/hour when it is turned off but being carried.*

(a) *Estimate the MTTF of the light in working hours.*

(b) *What is the probability of the light's failing during one 8-hour shift?*

(c) *What is the probability of its failing during one month (30 days) of 8-hour shifts?*

Note: In the earlier problems, we have not taken into account the possibility that an item may fail while it is not operating. Often such failure rates are small enough to be neglected. However, for items that are operated only a small fraction of time, failure during non-operation may be quite significant and so should be taken into account. In this situation, the composite failure rate λ_c is computed as $\lambda_c = f\lambda_0 + (1-f)\lambda_N$, where f is the fraction of time the unit is operating and λ_0 and λ_N are the failure rates during operating and non-operating times]

Since the flashlight is used only a small fraction of time (20 minutes out of 8 hours), we compute the composite failure rate λ_c and use it for other calculations.

$$\lambda_c = f\lambda_0 + (1-f)\lambda_N$$

$$= \frac{1}{24} \times 0.08 + \frac{23}{24} \times 0.005 = 0.008125/\text{hour}$$

(a) $$\text{MTTF} = \frac{1}{\lambda_c} = \frac{1}{0.008125} = 123 \text{ hours}$$

(b) $$P(T \le 8) = \int_0^8 \lambda_c e^{-\lambda_c t}\, dt = 1 - e^{-8\lambda_c} = 0.0629$$

(c) p = probability of failure in one day (night) = 0.0629

$$q = 1 - p = 0.9371$$

$$n = 30$$

$\therefore$ Probability of failure in 30 days

$$= 1 - \text{Probability of no failure in 30 days}$$

$$= 1 - nC_0\, p^0\, q^n, \text{ by binomial law}$$

$$= 1 - (0.9371)^{30}$$

$$= 0.8576$$

Example 8 *A relay circuit has an MTBF of 0.8 year. Assuming random failures,*

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23:27

You

27 April, 21:47

31. The one month reliability on an indicator lamp is 0.95 with the failure rate specified as constant. What is the probability that more than two spare bulbs will be needed during the first year of operation (Ignore replacement time).

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23:29

You
28 April, 08:19

$$1 - \sum_{0}^{2} \frac{e^{-\lambda} \lambda^{r}}{r!}$$

$$= 1 - e^{-0.6} - e^{-0.6} \cdot 0.6 - e^{-0.6} \times \frac{(0.6)^2}{2!}$$

$$= 0.02311$$

Yeh rha 😭

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Example 12 *The wearout (time to failure) of a machine part is normally distributed with 90% of the failures occurring symmetrically between 200 and 270 hours of use.*

(a) Find the MTTF and standard deviation of failure times.

(b) What is the reliability if the part is to be used for 210 hours and then replaced?

(c) Determine the design life if no more than a 1% probability of failure prior to the replacement is to be tolerated.

(d) Compute the reliability for a 10-hour use if the part has been operating for 200 hours.

Let the time to failure T follow N(μ, s), where σ is the MTTF and σ is the S.D.

Solution

(a) 90% of the failures lie symmetrically between 200 and 270

$$\therefore \quad \int_{200}^{235} f(t)dt = \int_{235}^{270} f(t)dt = 0.45, \text{ where } \mu = 235 \text{ hours}$$

Putting $$z = \frac{t - 235}{\sigma}, \text{ we get } \int_{0}^{35/\sigma} \phi(z)\, dz = 0.45$$

From the normal tables, $\frac{35}{\sigma} = 1.645$

$$\therefore \quad \sigma = \frac{35}{1.645} = 21.28 \text{ hours}$$

(b) $$R(t) = \int_{t}^{\infty} f(t)dt$$

$$\therefore \quad R(210) = \int_{210}^{\infty} f(t)dt = \int_{-1.17}^{\infty} \phi(z)dz$$

$$= 0.5 + \int_{0}^{1.17} \phi(z)dz\, dz = 0.5 + 0.379 = 0.879$$

(c) If t_D is the required design life, then

$$\int_{-\infty}^{t_D} f(t)dt = 0.01 \text{ or } \int_{-\infty}^{(t_D-235)/21.28} \phi(z)dz = 0.01$$

From the normal tables, $\frac{t_D - 235}{21.28} = -2.32$

$$\therefore \quad t_D = 185.6 \text{ hours}$$

(d)

$$\frac{R(t+T_0)}{R(T_0)} = \frac{R(210)}{R(200)} = \frac{\int_{210}^{\infty} f(t)dt}{\int_{200}^{\infty} f(t)dt}$$

$$= \frac{\int_{-1.17}^{\infty} \phi(z)dz}{\int_{-1.64}^{\infty} \phi(z)dz} = \frac{0.5+0.3790}{0.5+04495} \; 0.93$$

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$$= \int_{0} \frac{1}{30}\,dx + \int_{15}^{18} \frac{1}{30}\,dx$$

$$= \frac{1}{5}$$

Example 3 *If the roots of the quadratic equation $x^2 - ax + b = 0$ are real and b is positive but otherwise unknown, what are the expected values of the roots of the equation.*

Solution Assume that b has a uniform distribution in the permissible range.

The roots of the equation $x^2 - ax + b = 0$ are given by

$$x = \frac{1}{2}\left(a \pm \sqrt{a^2 - 4b}\right)$$

Since the roots are real, $a^2 - 4b > 0$

i.e., $\quad 0 < b < \dfrac{a^2}{4} \quad (\because b > 0)$

Therefore, b is a random variable, uniformly distributed in $\left(0, \dfrac{a^2}{4}\right)$.

Therefore, its density function $f(b) = \dfrac{4}{a^2}$

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$$E\{\text{the roots}\} = E\left\{\frac{1}{2}\left(a \pm \sqrt{a^2 - 4b}\right)\right\}$$

$$= \frac{1}{2}\int_0^{a^2/4}\left(a \pm \sqrt{a^2/4 - 4b}\right)\cdot\frac{4}{a^2}\,db$$

$$= \frac{2}{a^2}\left[ab \pm \frac{(a^2-4b)^{\frac{3}{2}}}{\frac{3}{2}(-4)}\right]_0^{a^2/4}$$

$$= \frac{2}{a^2}\left[\frac{a^3}{4} \mp \frac{1}{6}\left(0 - a^3\right)\right]$$

$$= a\left(\frac{1}{2} \pm \frac{1}{3}\right)$$

$$= \frac{5a}{6} \text{ and } \frac{a}{6}$$

Example 4 *Two points are taken at random on a given straight line of length 2 units. Prove that the probability of the distance between them exceeds 1 unit*