Digital Systems
18B11EC213

Module 1: Boolean Function Minimization Techniques and Combinational Circuits-10

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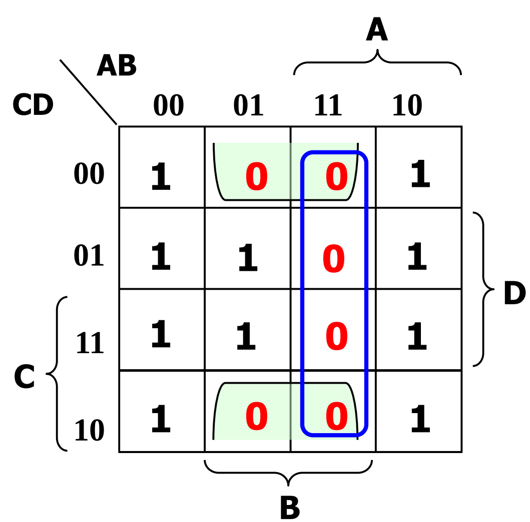
Cont.. K-Maps

Getting POS Expressions

- Simplified POS expression can be obtained by grouping the maxterms (i.e., 0s) of given function.
- Example:

Given F (A,B,C,D) =$\sum$m (0,1,2,3,5,7,8,9,10,11), we first draw the K-map, then group the maxterms together:



Getting POS Expressions

K-map of **F**

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	1	1	0	1
10	1	0	0	1

K-map of **F'**

CD \ AB	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	0	0	1	0
10	0	1	1	0

- This gives the SOP of F' to be:

  F' = B.D' + A.B

- To get POS of F, we have:

  F = (B.D' + A.B)'

  = (B.D')'.(A.B)'   DeMorgan

  = (B'+D).(A'+B')   DeMorgan

Don’t-care Conditions

- In certain problems, some outputs are not specified.
- These outputs can be either ‘1’ or ‘0’.
- They are called don’t-care conditions, denoted by X (or sometimes by d).
- Example: An odd parity generator for BCD code which has 6 unused combinations.

No.	A	B	C	D	**P**
0	0	0	0	0	**1**
1	0	0	0	1	**0**
2	0	0	1	0	**0**
3	0	0	1	1	**1**
4	0	1	0	0	**0**
5	0	1	0	1	**1**
6	0	1	1	0	**1**
7	0	1	1	1	**0**
8	1	0	0	0	**0**
9	1	0	0	1	**1**
10	1	0	1	0	**X**
11	1	0	1	1	**X**
12	1	1	0	0	**X**
13	1	1	0	1	**X**
14	1	1	1	0	**X**
15	1	1	1	1	**X**

Don’t-care Conditions

- Don’t-care conditions can be used to help simplify Boolean expression further in K-maps.
- They could be chosen to be either ‘1’ or ‘0’, depending on which gives the simpler expression.

Don’t-care Conditions

- For comparison:
  - Without Don’t-cares:

    P = A'.B'.C'.D’ + A'.B'.C.D + A'.B.C'.D + A'.B.C.D' + A.B'.C'.D

AB \ CD	00	01	11	10
00	1		1	
01		1		1
11				
10		1		

  - With Don’t-cares:

    P = A'.B'.C'.D' + B'.C.D + B.C'.D + B.C.D' + A.D

AB \ CD	00	01	11	10
00	1		1	
01		1		1
11	X	X	X	X
10		1	X	X

Examples

- Example-1:

  $$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

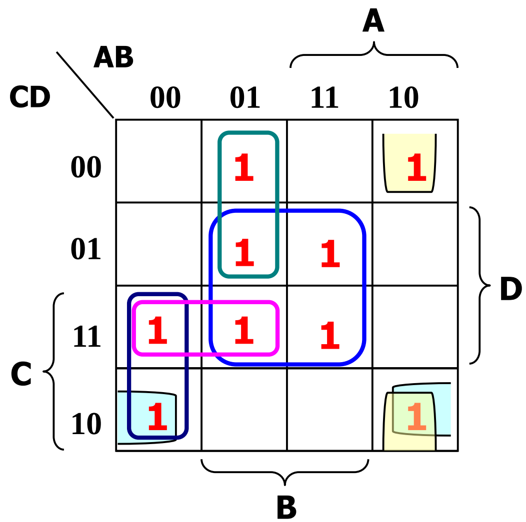
CD \ AB	00	01	11	10
00		1		1
01		1	1	
11	1	1	1	
10	1			1

Fill in the 1s

Examples

- Example-1: Cont..

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$

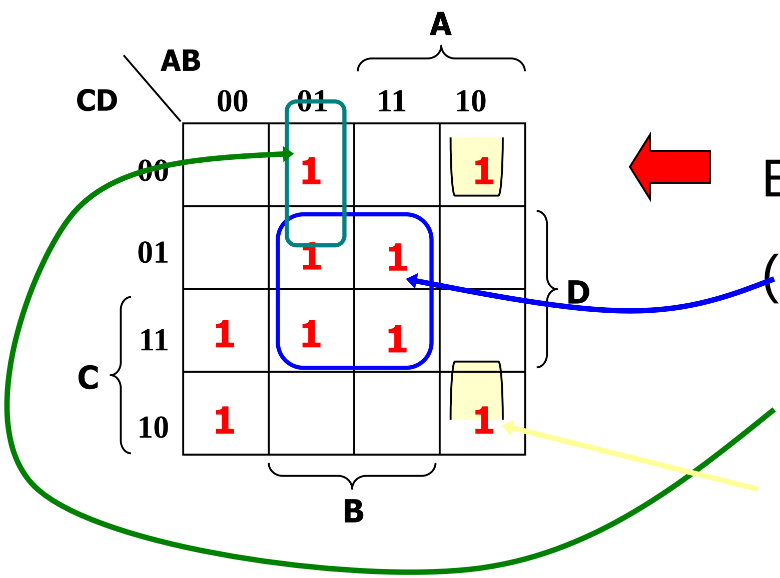


These are all the prime implicants (PIs).

Examples

- Example-1: Cont..

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$



Essential prime implicants (EPIs):

B.D

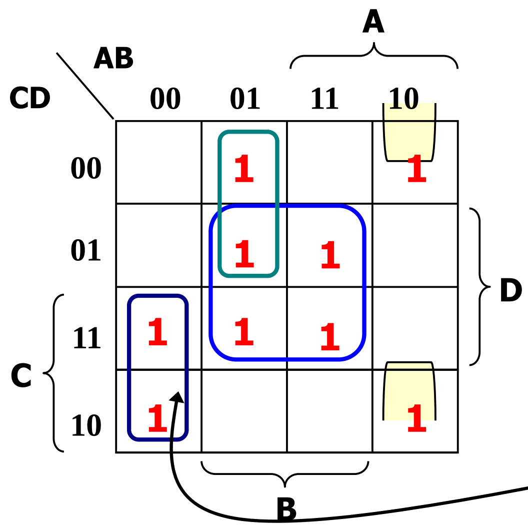
A'.B.C'

A.B'.D'

Examples

- Example-1: Cont..

$$f(A,B,C,D) = \sum m(2,3,4,5,7,8,10,13,15)$$



Minimum cover

EPIs: B.D, A'.B.C', A.B'.D'

+

A'.B'.C

$$f(A,B,C,D) = B.D + A'.B.C' + A.B'.D' + A'.B'.C$$

Examples

- Example-2:

$$f(A,B,C,D) = A.B.C + B'.C.D' + A.D + B'.C'.D'$$

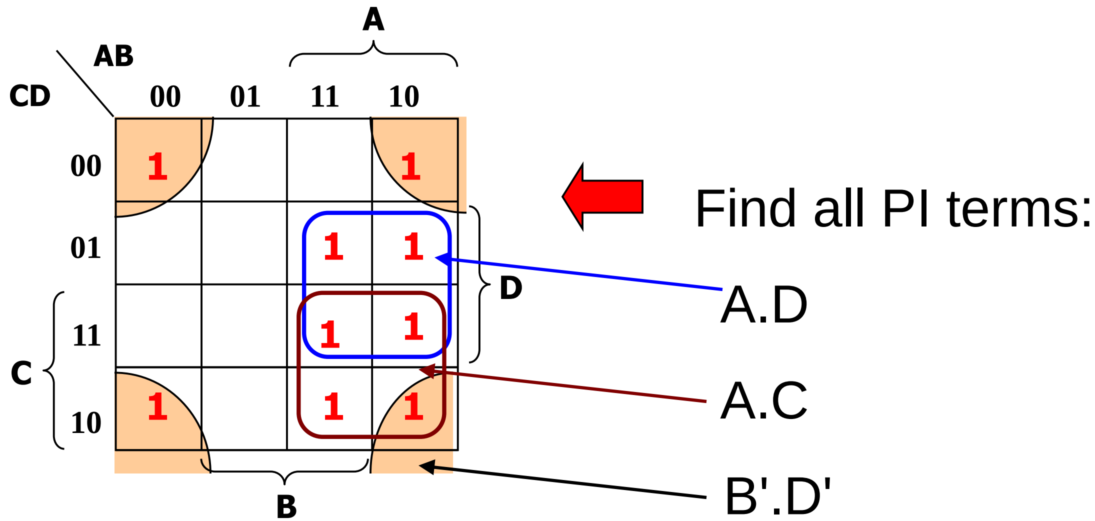
CD \ AB	00	01	11	10
00	1			1
01			1	1
11			1	1
10	1		1	1

Fill in the 1s

Examples

- Example-2: Cont..

  f(A,B,C,D) = A.B.C + B'.C.D' + A.D + B'.C'.D'



EPI terms: A.D, A.C, B'.D'

Simplified function: f(A,B,C,D) = A.D + A.C + B'.D'

Examples

- Example-3 (with don't cares):

$$f(A,B,C,D) = \sum m(2,8,10,15) + \sum d(0,1,3,7)$$

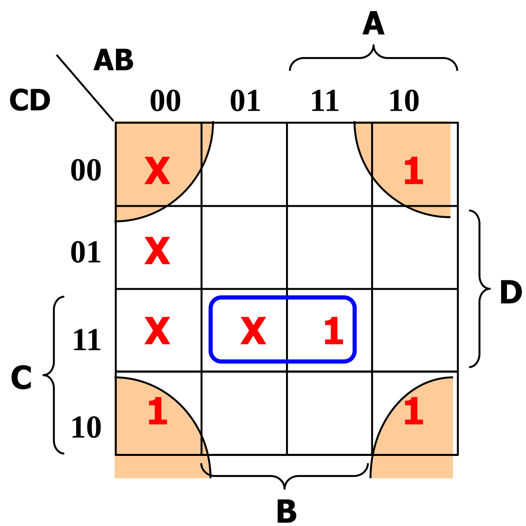
CD \ AB	00	01	11	10
00	X			1
01	X			
11	X	X	1	
10	1			1

Fill in the 1's and X's

Examples

- Example-3 (with don't cares): Cont..

  $f(A,B,C,D) = \sum m(2,8,10,15) + \sum d(0,1,3,7)$



Do we need to have an additional term A'.B' to cover the 2 remaining x's?

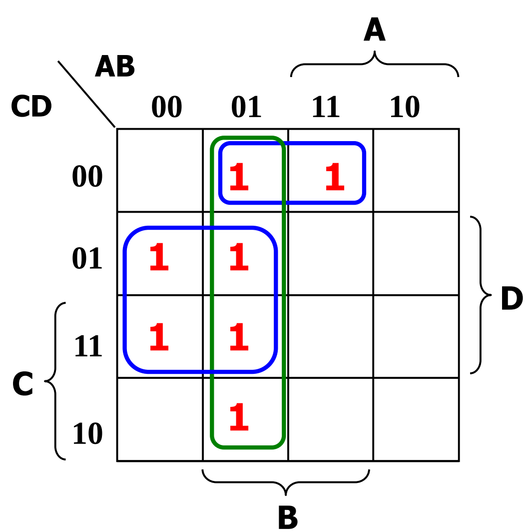
No, because all the 1's (minterms) have been covered.

f(A,B,C,D) = B'.D' + B.C.D

Examples

- To find the minimized POS expression for example-2:

  f(A,B,C,D) = A.B.C + B'.C.D' + A.D + B'.C'.D'
- Draw the K-map of the complement of f, i.e., for f '.



From K-map,

f ' = A'.B + A'.D + B.C'.D'

Using DeMorgan’s theorem,

f = (A'.B + A'.D + B.C'.D')'

= (A+B').(A+D').(B'+C+D)

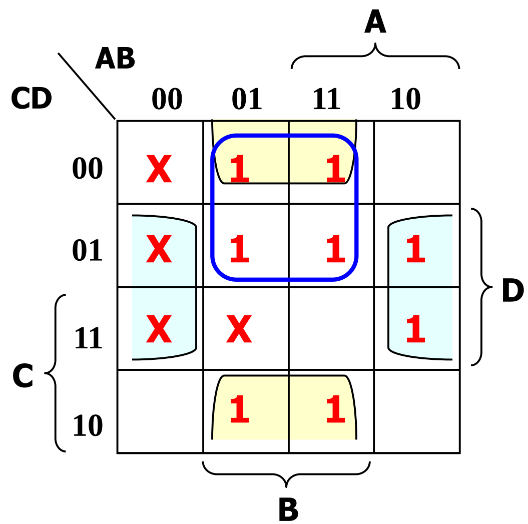
Examples

- To find simplest POS expression for example-3:

  $f(A,B,C,D) = \sum m(2,8,10,15) + \sum d(0,1,3,7)$

- Draw the K-map of the complement of f, i.e., for f '.

  $f'(A,B,C,D) = \sum m(4,5,6,9,11,12,13,14) + \sum d(0,1,3,7)$



From K-map,

$f' = B.C' + B.D' + B'.D$

Using DeMorgan's theorem,

$f = (B.C' + B.D' + B'.D)'$

$= (B'+C).(B'+D).(B+D')$

References

- M. M. Mano, *Digital Logic and Computer Design*, 5th ed., Pearson Prentice Hall, 2013.
- R. P. Jain, *Modern Digital Electronics*, 4th ed., Tata McGraw-Hill Education, 2009.