Department of Mathematics

Tutorial 12

(Taylor's & Laurent's Series Expansions)

Course Name: **Mathematics-II** Course Code: **15B11MA211**

Core: **B. Tech.**

Q1. Use the formula for the coefficients in terms of derivatives to give the Taylor's series of $f(z) = e^z$ around $z = 0$.

Q2. Expand $f(z) = z^8 e^{3z}$ in a Taylor series around $z = 0$.

Q3. Find the Taylor series of $\sin z$ around $z = 0$ (Sometimes the Taylor series around 0 is called the Maclaurin's series.).

Q4. Expand the rational function $f(z) = \frac{1+2z^2}{z^3+z^5}$ around $z = 0$.

Q5. Find the Taylor's series expansion for $f(z) = \frac{e^z}{1-z}$ around $z = 0$, also give the radius of convergence.

Q6. Find the Laurent's series for the function $f(z) = \frac{z}{z^2+1}$ around $z_0 = i$. Give the region where your answer is valid. Identify the singular (principal) part.

Q7. Compute the Laurent's series for $f(z) = \frac{z+1}{z^3(z^2+1)}$ on the region $A : 0 < |z| < 1$ centered at $z = 0$.

Answers:

1. $f(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$ **2.** $f(z) = \sum_{n=0}^{\infty} \frac{3^n}{n!} z^{n+8}$ **3.** $\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$

4. $f(z) = \left(\frac{1}{z^3} + \frac{1}{z}\right) - \sum_{n=0}^{\infty} (-1)^n z^{2n+1}$

5. $f(z) = 1 + (1+1)z + \left(1 + 1 + \frac{1}{2!}\right) z^2 + \left(1 + 1 + \frac{1}{2!} + \frac{1}{3!}\right) z^3 + \cdots$ and $|z| < 1$.

6. $f(z) = \frac{1}{2} \cdot \frac{1}{z-i} + \frac{1}{4i} \cdot \sum_{n=0}^{\infty} \left(\frac{-(z-i)}{2i}\right)^n$ The region of convergence is $0 < |z - i| < 2$

7. $f(z) = \frac{1}{z^3} + \frac{1}{z^2} - \frac{1}{z} - 1 + z + z^2 - z^3 - \cdots$