Jaypee Institute of Information and Technology
Department of Mathematics

Course: Matrix Computations (16B1NMA533)

Tutorial Sheet 6 [C301-3.3]

(Topics covered: spanning set, dimension, basis, inner product space, norm, parallelogram law)

1. Show that the vectors (1,1,1), (1,1,0) and (1,0,0) span R^3.
2. Prove that $W = \{(a,b,c) : a+b+c=0\}$ is a subspace of R^3. Find a basis and the dimension of W.
3. Let W be the subspace of R^4 spanned by the vectors (1,-2,5,-3), (2,3,1,-4), (3,8,-3,-5). Find a basis and the dimension of W.
4. Prove that R^2 is an inner product space with respect to the inner product defined as $\langle u,v\rangle = 5x_1x_2 - x_1y_2 - x_2y_1 + 5y_1y_2,$ where $u = (x_1,\ y_1)$ and $v = (x_2,\ y_2)$.
5. Let $V(C)$ be the vector space of all continuous complex valued functions on the unit interval, $0 \le t \le 1$. Show that the following defined product is an inner product on $V(C)$.
$$\text{for any } f(t), g(t) \in V,\ \langle f(t), g(t)\rangle = \int_0^1 f(t)\overline{g(t)}dt$$
6. Let $\langle\ ,\ \rangle$ be the standard inner product on R^2. Let $\alpha = (1,\ 2),\ \beta = (-1,\ 1)$. If γ is a vector such that $\langle \alpha, \gamma\rangle = -1$ and $\langle \beta, \gamma\rangle = 3$, find γ.
7. Prove or disprove: there is an inner product on R^2 such that the associated norm is given by $\|(a,b)\| = \max\{|a|,|b|\},\ \ \forall\ (a,b) \in R^2$.
8. Suppose $u, v \in V$ are such that $\|u\| = 3,\ \|u+v\| = 4,\ \|u-v\| = 6$. What number does $\|v\|$ equal?