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Predicates: (Open propositions)

" Predicate is a proposition except for the fact that it contains variables, whose values are to be taken from some universe of discourse."
or set

these are all proposition except the variable and the value of variables is taken from universal set.

$$\left.\begin{array}{l} x+2=7 \\ x+y \leq 8 \\ x \text{ loves } y \\ x \text{ can fool } y \text{ at time } t \end{array}\right\}$$

→ they are called predicate because the subject is absent and predicate is present that's y it is called predicate.

→ open proposition because it open to become proposition by putting the value of variable.

In English
Declarative sentence
Subject: about which the assertion is made
Predicate: the property the subject has

properties

$E(x)$: $x+2=7$

$LE(x,y)$: $x+y \leq 8$

$loves(x,y)$: x loves y

$canfool(x,y,t)$: x can fool y at time t.

$E \rightarrow$ equal to
$LE \rightarrow$ less than equal to.

→ "A predicate involving one variable is called one place predicate."
(1-place)

Always ahead...

→ A predicate involving 2-variables is called 2-place predicate.

Similarly,

⇒ n-place — n-variables.

Predicates to Propositions (conversion)

1. Substitution
2. Quantification

Quantifiers ⇒

1. Universal quantifier, ($\forall$). — for all / for every } read as
2. Existential quantifier, $\exists$ — there exist / for some } read as

Q. Consider the following predicate:-
$$P(x) = x+4 \leq 7$$
$$U = \{1, 2, 3, 4\}$$

1st method: **Substitution:**

$p: P(1): 1+4 \leq 7$ [T]
$q: P(2): 2+4 \leq 7$ [T]
$s: P(3): 3+4 \leq 7$ [T]
$r: P(4): 4+4 \leq 7$ [F]

} propositions.

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2nd method :

Quantification :

→ for every x, $P(x)$ is true [F]
or $\boxed{\forall x, P(x)}$

→ for some x, $P(x)$ is true [T]
or $\boxed{\exists x, P(x)}$

converted to proposition

Ex:- $P(x): x+4 \leq 7$, $U = \{1, 2, 3\}$

Substitution

$P(1): 5 \leq 7$ [T]
$P(2): 6 \leq 7$ [T]
$P(3): 7 \leq 7$ [T]

Quantification

for every x, $P(x)$ is true [T]
or $\boxed{\forall x, P(x)}$

for some x, $P(x)$ is true [T]
or $\boxed{\exists x, P(x)}$

{ $\forall x, P(x) \Rightarrow$ $\underbrace{\text{for every } x, P(x) \text{ is true}}_{\text{meaning of } \forall x, P(x)}$ }

it is universal or general not depend on truth value of sentence.

It is true or not is decided when the predicate & universe is given.

Always ahead...

Form	Meaning
1. $\forall x\, P(x)$	all true
2. $\exists x\, P(x)$	some, true. or atleast one, true
3. $\neg \forall x, P(x)$	not all true
4. $\neg \exists x\, P(x)$	none. true
5. $\forall x, \neg P(x)$	all false
6. $\exists x\, \neg P(x)$	Some false or atle…
7. $\neg \forall x, \neg P(x)$	not all false
8. $\neg \exists x, \neg P(x)$	none false

$\underbrace{\text{All - atleastone}}_{\text{none.}}$

(Red line shows equivalnces)

Equivalences:

$$\boxed{\begin{aligned} \forall x\, P(x) &\equiv \neg \exists x, \neg P(x) \\ \exists x\, P(x) &\equiv \neg \forall x, \neg P(x) \\ \neg \forall x\, P(x) &\equiv \exists x\, \neg P(x) \end{aligned}}$$

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$$\boxed{\neg\exists x\, P(x) \equiv \forall x, \neg P(x)}$$

IDEA. Negating quantified predicates.

Practice

- $\neg\forall x\, P(x) \equiv \exists x\, \neg P(x)$
- $\exists x\, \neg P(x) \equiv \neg\forall x\, P(x)$

two-sided.

- $\neg\exists x\, P(x) \equiv \forall x\, \neg P(x)$
- $\forall x\, \neg P(x) \equiv \neg\exists x\, P(x)$

two-sided.

Problems:

Imp ①. $\neg\forall x\, [P(x) \rightarrow Q(x)]$

$\equiv \exists x\, \neg[P(x) \rightarrow Q(x)]$

$\exists x\, \neg[\neg P(x) \vee Q(x)]$

$\exists x\, [P(x) \wedge \neg Q(x)]$

$$\boxed{\neg\forall x\, [P(x) \rightarrow Q(x)] \equiv \exists x\, [P(x) \wedge \neg Q(x)]}$$

②. $\neg\exists x\, [P(x) \wedge Q(x)]$

$\equiv \forall x\, \neg[P(x) \wedge Q(x)]$

$\equiv \forall x\, [\neg P(x) \vee \neg Q(x)]$

$\equiv \forall x\, [P(x) \rightarrow \neg Q(x)]$

$$\boxed{\neg\exists x\, [P(x) \wedge Q(x)] \equiv \forall x\, [P(x) \rightarrow \neg Q(x)]}$$