Digital Systems
18B11EC213

Module 1: Boolean Function Minimization Techniques and Combinational Circuits-1

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Number Systems

- Many number systems are in use in digital technology.

- The most common number systems are:

❖ Decimal

❖ Binary

❖ Octal

❖ Hexadecimal (Hex)

Radix (Base) of Number Systems

- Decimal - Base is 10
- Binary - Base is 2
- Octal - Base is 8
- Hexadecimal (Hex) - Base is 16

Symbols in Number Systems

- Decimal (Base 10) - 10 symbols (0 to 9)
- Binary (Base 2) - 2 symbols (0 and 1)

  A digit in base 2 is also called a bit.
- Octal (Base 8) - 8 symbols (0 to 7)
- Hexadecimal (Base 16) - 16 symbols (0 to 9, A to F).

  Use letters A to F to represent values 10 to 15.

  (Alphanumeric number system)

In general, a digit in base R can range from 0 to R-1.

Positional Notation

- The value of a number is determined by multiplying each digit by a weight and then by adding. The weight of each digit is a power of the base and is determined by position.

Example:

In decimal number system, 3874 means

3 times 1000

plus 8 time 100

plus 7 times 10

plus 4 times 1

Cont..

- In decimal, there are 10 symbols and the value of each position is a power of 10.

  $10^0 = 1$ = value of the unity position

  $10^1 = 10$ = value of next position to the left etc.
- In binary, there are 2 symbols, and the value of each position is a power of 2.
- In octal, 8 symbols, and powers of 8.
- In hexadecimal, 16 symbols, and powers of 16.

Base 10, Base 2, Base 16

$953.78_{10} = 9 \times 10^2 + 5 \times 10^1 + 3 \times 10^0 + 7 \times 10^{-1} + 8 \times 10^{-2}$
$= 900 + 50 + 3 + .7 + .08 = 953.78$ (decimal)

$1011.11_2 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$
$= 8 + 0 + 2 + 1 + 0.5 + 0.25$
$= 11.75_{10}$ (binary to decimal)

$A2F_{16} = 10x16^2 + 2x16^1 + 15x16^0$
$= 10 \times 256 + 2 \times 16 + 15 \times 1$
$= 2560 + 32 + 15 = 2607_{10}$ (hex to decimal)

Conversion of Any Base to Decimal

Converting from any base to decimal is done by multiplying each digit by its weight and by adding.

Examples:

- Binary to decimal

$$(1011.11)_2 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$$
$$= 8 + 0 + 2 + 1 + 0.5 + 0.25$$
$$= (11.75)_{10}$$

- Hex to decimal

$$(A2F)_{16} = 10x16^2 + 2x16^1 + 15x16^0$$
$$= 10x256 + 2x16 + 15x1$$
$$= 2560 + 32 + 15 = (2607)_{10}$$

Least Significant Digit
Most Significant Digit

Example: For a binary number $(110101)_2$

- Least significant digit has weight of 2^0 or 1

  For base 2 (binary), also called least significant bit (LSB)

  Always right most digit

- Most significant digit has weight of 2^5 or 32

  For base 2 (binary), also called most significant bit (MSB)

  Always left most digit

Conversion of Decimal Integer To Any Base

- Divide Number N by base R until quotient is 0.
- Remainder at each step is a digit in base R, from least significant digit to most significant digit.

Cont..

Example: Convert $(53)_{10}$ to binary.

53/2 = 26,	rem = 1	Least significant digit
26/2 = 13,	rem = 0	
13/2 = 6,	rem = 1	
6/2 = 3,	rem = 0	
3/2 = 1,	rem = 1	
1/2 = 0,	rem = 1	Most significant digit

$53_{10} = 110101_2$

$= 1x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0$

$= 32 + 16 + 0 + 4 + 0 + 1 = 53_{10}$

Conversion of Decimal Fraction To Any Base

- It is accomplished by a method similar to that used for integers.
- Multiplication is used instead of division.
- Integers are accumulated instead of remainders.

Cont..

Example-1: Convert $(0.6875)_{10}$ to binary.

	Integer		Fraction	Coefficient
0.6875 * 2 =	1	+	0.3750	$a_{-1} = 1$
0.3750 * 2 =	0	+	0.7500	$a_{-2} = 0$
0.7500 * 2 =	1	+	0.5000	$a_{-3} = 1$
0.5000 * 2 =	1	+	0.0000	$a_{-4} = 1$

Answer: $(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$

Cont..

Example-2: Convert $(0.513)_{10}$ to octal.

	Integer		Fraction	Coefficient
0.513 * 8	= 4	+	0.104	$a_{-1} = 4$
0.104 * 8	= 0	+	0.832	$a_{-2} = 0$
0.832 * 8	= 6	+	0.656	$a_{-3} = 6$
0.656 * 8	= 5	+	0.248	$a_{-4} = 5$
0.248 * 8	= 1	+	0.984	$a_{-5} = 1$
0.984 * 8	= 7	+	0.872	$a_{-6} = 7$

Answer: $(0.513)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}a_{-5}a_{-6}\ldots.)_8$

$= (0.406517\ldots\ldots)_8$

Hex to Binary Conversion

- Each hexadecimal digit represents 4 bits.
- To convert a hexadecimal number to binary number, simply convert each hex digit to its four-bit binary equivalent.

Hex to binary:

$0_{16} = 0000_2$
$1_{16} = 0001_2$
$2_{16} = 0010_2$
$3_{16} = 0011_2$
$4_{16} = 0100_2$
$5_{16} = 0101_2$
$6_{16} = 0110_2$
$7_{16} = 0111_2$
$8_{16} = 1000_2$
$9_{16} = 1001_2$

Cont..

$A_{16} = 1010_2$
$B_{16} = 1011_2$
$C_{16} = 1100_2$
$D_{16} = 1101_2$
$E_{16} = 1110_2$
$F_{16} = 1111_2$

Binary and Hex Conversion

- Hexadecimal to binary conversion:

Examples: $(A2F)_{16} = (1010\ \ 0010\ \ 1111)_2$

$(345)_{16} = (0011\ \ 0100\ \ 0101)_2$

- Binary to hexadecimal conversion is just the opposite, create groups of 4 bits starting from LSB.

  If last group does not have 4 bits, then pad with zeros for unsigned numbers.

Example:

$$(1010001)_2 = (0101\ 0001)_2 = (51)_{16}$$

Padded with a zero

Binary and Octal Conversion

- Octal to binary conversion - by converting each octal digit into its 3-bit equivalent binary.

Example: $(736)_8 = (111\ 011\ 110)_2$

- Binary to octal conversion - by making groups of three bits starting from LSB.

Example: $(1001110)_2 = (001\ 001\ 110)_2$

$= (116)_8$

References

- M. M. Mano, *Digital Logic and Computer Design*, 5th ed., Pearson Prentice Hall, 2013.
- R. P. Jain, *Modern Digital Electronics*, 4th ed., Tata McGraw-Hill Education, 2009.