(Q1.) $q = \frac{2L \cdot n_0}{\lambda} = \frac{2 \times 0.35 \times 1.3}{400 \times 10^{-9}} = 2.275 \times 10^6$ modes.

-for $\lambda = 400nm \Rightarrow$

~~for~~ $q = \frac{2 L n_0}{\lambda} = \frac{2 \times 0.35 \times 1.3}{700 \times 10^{-9}} = 1.3 \times 10^6$ modes.

(Q2) (a) $\lambda = \frac{2 n_0 L}{q} = \frac{2 \times 1.6 \times 21.7 \times 10^{-2}}{10^6} = 6944 \mathring{A}$ (Ruby).

(b) $\lambda = \frac{2 n_0 L}{q} = \frac{2 \times 1.055 \times 1.2}{4 \times 10^6} = 6330 \mathring{A}$ (He-Ne)

(Q3) (a) For three dimensional cavity with sides a, b, d, Propagation constant in each direction (under standing wave conditions) are given by -

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}, \quad k_z = \frac{q\pi}{d}$$

For standing waves where $k = \frac{2\pi}{\lambda} = \frac{r\pi}{L}$

$\Rightarrow \boxed{2L \approx r \cdot \lambda}$

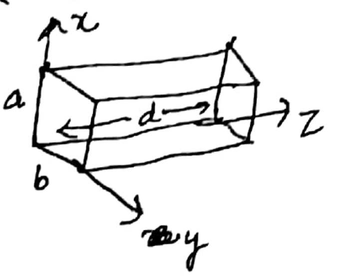
Dispersion relation $\rightarrow$

$n_0 \rightarrow$ refractive index of medium

$$\omega = v \cdot k \quad \text{or} \quad \omega^2 = v^2 \cdot k^2$$

$$\text{or } \omega^2 = \frac{c^2}{n_0^2} \cdot (k_x^2 + k_y^2 + k_z^2)$$

as k is a vector.



$$(2\pi \nu)^2 = \frac{c^2}{n_0^2} \pi^2 \left[\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{q^2}{d^2} \right]$$

$$\nu^2 = \frac{c^2}{4 \cdot n_0^2} \left[\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{q^2}{d^2} \right]$$

$$\Rightarrow \boxed{\nu_{mnq} = \frac{c}{2n_0} \left[\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{q^2}{d^2} \right]^{1/2}} \quad \text{------} \quad (X)$$

(b) If $m, n << q \Rightarrow \boxed{\nu_q = \frac{c}{2n_0} \frac{q}{d}}$ and $\Delta\nu_q = \nu_{q+1} - \nu_q$

[redacted]

for open cavity

$$\boxed{\Delta \nu_q = \frac{c}{2n_0 d}}$$

(c)

(c) If $m, n \ll q$, by using Binomial expansion in eqn (X),

$$\nu_{mnq} = \frac{c}{2n_0}\left[\frac{q}{d} + \frac{d}{2q}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\right]$$

for a pair of square mirror, $a = b$ (open cavity)

$$\nu_{mnq} = \left[\frac{c}{2n_0}\frac{q}{d} + \frac{c}{2n_0}\frac{d}{2q}\frac{(m^2+n^2)}{a^2}\right]$$

$$\Rightarrow \quad \nu_{(m+1)nq} = \left[\frac{cq}{2n_0 d} + \frac{cd}{4n_0 q}\frac{((m+1)^2+n^2)}{a^2}\right]$$

$$\nu_{mnq} = \frac{cq}{2n_0 d} + \frac{cd}{4n_0 q a^2}(m^2+n^2)\Big]$$

$$\Rightarrow \quad \Delta\nu_m = \frac{cd}{4n_0 q \cdot a^2}(\cancel{m^2}+1+2m-\cancel{m^2})$$

$$\Delta\nu_m = \frac{cd}{2n_0 q \cdot a^2}\left(m+\frac{1}{2}\right)$$

$$\Delta\nu_m = \frac{c}{2n_0 d}\left(\frac{d^2}{q}\right)\frac{1}{a^2}\left(m+\frac{1}{2}\right) \qquad \left[\text{using } \Delta\nu_q = \frac{c}{2n_0 d}\right]$$

$$\Delta\nu_m = \Delta\nu_q\left(\frac{d^2}{q \cdot a^2}\right)\left(m+\frac{1}{2}\right)$$

put $q = \frac{2d}{\lambda}$ $\Rightarrow$ $\Delta\nu_m = \Delta\nu_q \cdot \left(\frac{d^{\cancel{2}} \times \lambda}{2 \cdot \cancel{d} \times a^2}\right)\left(m+\frac{1}{2}\right)$

$\left(\text{as } k_z = \frac{2\pi}{\lambda} = \frac{q\pi}{d}\right)$ $\Rightarrow$ $\boxed{\frac{\Delta\nu_m}{\Delta\nu_q} = \frac{\lambda \cdot d}{2a^2}\left(m+\frac{1}{2}\right)}$

as $\frac{\Delta\nu_m}{\Delta\nu_q} \approx \lambda \Rightarrow \boxed{\Delta\nu_m \ll \Delta\nu_q}$ Proved.

(d) If $d = 50$ cm, $a = 2$ cm, $n_0 = 1$, $\lambda = 500$ nm

(i) $\nu_{q=5} = \frac{cq}{2d} = \frac{5c}{2d} = \frac{5 \times 3 \times 10^8}{2 \times 0.50} = 15 \times 10^8$ Hz

(ii) $\Delta\nu_q = \frac{c}{2d} = 3 \times 10^8$ Hz

(iii) $\Delta\nu_m = \frac{3 \times 10^8 \times 500 \times 10^{-9} \times 0.5}{2 \times 0.04} \times \frac{3}{2} = 1406$ Hz