MATH 316 — Dr. Bogacki

SAMPLE TEST 2

Spring 2008

1. Consider the set of vectors $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \right\}$ in R^3.

   Find the dimension of span S and describe span S geometrically (as a line, a plane, etc.). If possible, find an example of a vector that is outside span S.

2. Find a basis for R^3 that includes the vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

3. Consider $A = \begin{bmatrix} 1 & 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 3 & 2 \\ 0 & 0 & 2 & 2 & 0 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}$.

   (a) Find a basis for the null space of A.

   (b) Find the nullity of A.

   (c) Find a basis for the row space of A.

   (d) Find a basis for the column space of A.

   (e) Find the rank of A.

   (f) Are the rows of A linearly independent? Do they span R^5?

4. Consider the set of vectors $S = \{\overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3}\}$ where $\overrightarrow{u_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\overrightarrow{u_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\overrightarrow{u_3} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

   (a) Show that the set S is a basis for R^3.

   (b) Find $\left[\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right]_S$.

5. Consider the basis $T = \left\{ \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} \right\}$ for the vector space M_{22}.

   Find the vector $\overrightarrow{v}$ such that $[\overrightarrow{v}]_T = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}$.

6. Decide if each of the following is a linear transformation:

   (a) $F : R^2 \to R^3$ defined by $F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ x_2 - x_1 \end{bmatrix}$.

   (b) $G : M_{22} \to R$ defined by $G(A) = \det A$.

   (c) $H : R^2 \to P_1$ defined by $H\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 t$.

7. Given the linear transformation $F(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = \begin{bmatrix} x_1 + 2x_2 \\ 2x_1 + x_2 \end{bmatrix}$,

   (a) find a basis for the kernel of F and a basis for the range of F,

   (b) is F one-to-one?; is F onto R^2?

   (c) find the matrix of the transformation with respect to the basis $S = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$,

   (d) use the matrix obtained in part (c) to find $F(\begin{bmatrix} -3 \\ 4 \end{bmatrix})$,

   (e) compute $F(\begin{bmatrix} -3 \\ 4 \end{bmatrix})$ directly from the definition of F.

8. Consider the following orthogonal basis for R^5: $T = \{ \begin{bmatrix} 3 \\ 0 \\ -4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} \}$.

   (a) Transform T into an orthonormal basis S for R^5.

   (b) Given the vector $\overrightarrow{u} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, find $[\overrightarrow{u}]_S$

9. Use the Gram-Schmidt process to find an orthogonal basis for the subspace of R^4 with a basis: $\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 4 \end{bmatrix} \}$.

10. Find a basis for the orthogonal complement of span$\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -3 \end{bmatrix} \}$ in R^4.

11. (a) Exercises 33-38, p.183.

    (b) Exercises 6-14, p. 206.

    (c) Exercises 21-24, p. 238.

    (d) Exercises 11-15, p. 248.