MATH 316U

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SAMPLE TEST 1

Spring 2008

1. Perform each matrix multiplication if possible:

(a) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 3 \\ 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$

2. Find the matrix of the linear transformation from R^2 to R^2 that doubles each vertical vector and reverses the orientation of each horizontal vector (keeping its length unchanged).

3. Consider the linear transformation T from R^3 to R^2 where

$$T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = x_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} - 3x_3 \begin{bmatrix} 0 \\ 4 \end{bmatrix}.$$

Find the matrix of the linear transformation A.

4. Solve each linear system, using either Gaussian elimination with backsubstitution or Gauss-Jordan reduction:

(a) $\begin{array}{rcrcrcrcr} x & + & 2y & & & + & w & = & 2 \\ -2x & - & 4y & + & z & - & 3w & = & -3 \\ 3x & + & 6y & - & z & + & 4w & = & 5 \end{array}$

(b) $\begin{array}{rcrcrcr} x & & & - & z & = & 1 \\ & & y & + & 2z & = & 2 \\ 2x & + & y & & & = & 5 \end{array}$

(c) $\begin{array}{rcrcr} x & + & 3y & = & 1 \\ 2x & + & 7y & = & 5 \\ x & + & 4y & = & 4 \end{array}$

5. Find the elementary matrices that perform the following row operations on a 3×4 matrix:

(a) $r_3 - 5r_2 \to r_3$

(b) $r_1 \leftrightarrow r_3$.

6. If possible, find the inverse of each matrix:

(a) $\begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}$, **b.** $\begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

7. Let A be a 3×3 matrix such that $A^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -3 & 0 \\ 1 & 4 & 1 \end{bmatrix}$ and let $B = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Calculate $(B^{-1}A)^{-1}$

(b) Find a 3×1 solution matrix $\overrightarrow{x}$ to the linear system $A\overrightarrow{x} = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$.

(c) Are the matrices A and B row equivalent? Justify your answer.

8. Let $V = R^2$ with the operations defined by $\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y+y' \\ x+x' \end{bmatrix}$ and $c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cy \\ cx \end{bmatrix}$.
Determine whether the following properties hold:

(a) $c(d\overrightarrow{u}) = (cd)\overrightarrow{u}$

(b) $c(\overrightarrow{u} + \overrightarrow{v}) = (c\overrightarrow{u}) + (c\overrightarrow{v})$

9. Which of the following sets W are subspaces of R^3?

(a) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x = 2y = 3z \right\}$,

(b) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x \geq y \right\}$,

(c) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : z = 4 \right\}$.

10. (a) Is the set of all polynomials $a_2t^2 + a_1t + a_0$ such that $a_0 + 3a_2 = 0$ a subspace of P_2?

(b) Is the set of all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that are **symmetric** a subspace of M_{22}?

11. Consider the set of vectors $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \right\}$ in R^3.

Find the dimension of span S and describe span S geometrically (as a line, a plane, etc.). If possible, find an example of a vector that is outside span S.

12. For each set of vectors below, determine if the set is linearly dependent. If it is, express one vector as a linear combination of the rest.

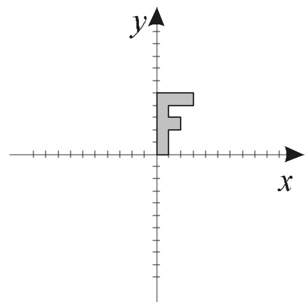
(a) Set $S = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \\ 1 \end{bmatrix} \right\}$ of vectors in R^4.

(b) Set $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$ of vectors in M_{22}.

(c) Set $S = \{t^3 + t + 2, t^3 + t^2 + t + 3, t^3 - 2t^2 + t\}$ of vectors in P_3.

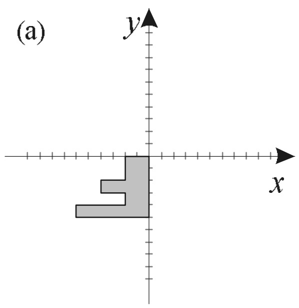
13. Find a basis for R^3 that includes the vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

14. If the position vector $\overrightarrow{v}$ of each of the corner points of the letter "F" pictured here

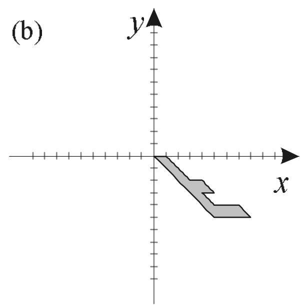


undergoes the linear transformation $F(\overrightarrow{v}) = A\overrightarrow{v}$, and the corresponding points are connected in the same way, write the matrix A that results in each transformed letter using $-2, -1, 0, 1$, or 2 as entries.

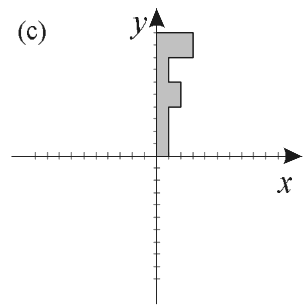
(a)



(b)



(c)



15. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a matrix whose determinant is 5. Calculate the determinant of

a. $\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$, b. $\begin{bmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{bmatrix}$, c. $(A^{-1})^2$, d. $(-3)A$.

16. Evaluate $\det \begin{bmatrix} 2 & -3 & -1 & 1 \\ 3 & 0 & 1 & 1 \\ 1 & 0 & 2 & -2 \\ -1 & 2 & 1 & 1 \end{bmatrix}$.

17. (a) Exercises 31-40, p. 13-14.
    (b) Exercises 15-26, p. 22.
    (c) Exercises 13-22, p. 28-29.
    (d) Exercises 27-37, p. 72.
    (e) Exercises 13-15, p. 84.
    (f) Exercises 23-30, p. 95.
    (g) Exercises 14-21, p. 126.
    (h) Exercises 13-17, p. 136.
    (i) Exercises 31-34, p. 158.
    (j) Exercises 21-24, p. 170.
    (k) Exercises 33-38, p. 183.