Evolutionary Multi-Objective Optimization: A Parallel Computing Approach

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Outline

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Multi-Objective Problem

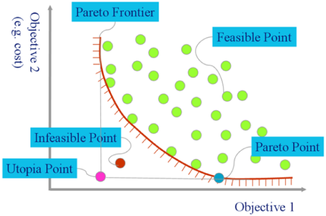

Figure: Sample Pareto Frontier

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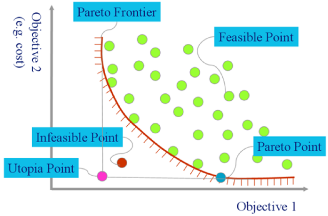

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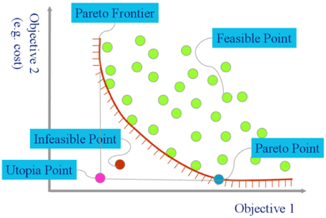

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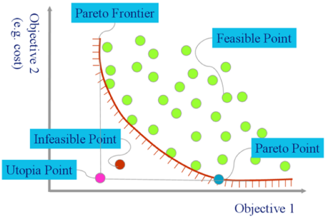

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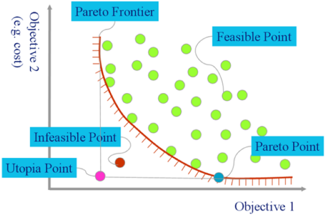

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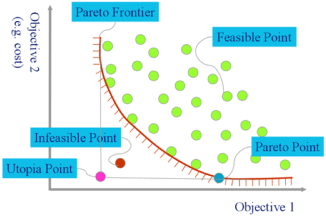
- **Utopia Point** The ideal theoretical solution we would love to reach but practically its not possible



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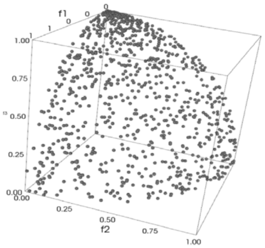
$$0 \leq x_i \leq 1 \quad \textit{where} \quad i = 1, 2, 3....30$$



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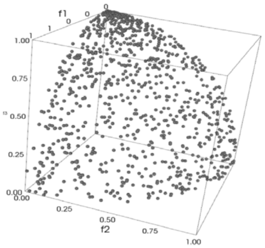

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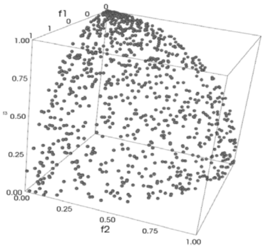
- **Optimal Solution:** Ideal Decisions are $x_i = 0.5$ where $i = 1, 2, 3...30$
Ideal objectives should satisfy the equation $\sum_{m=1}^{3} f_m^2 = 1$



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  - **Risk:** Risk involved in developing the project.

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  - **Idle:** Developers sitting idle in the project. To be Minimized.

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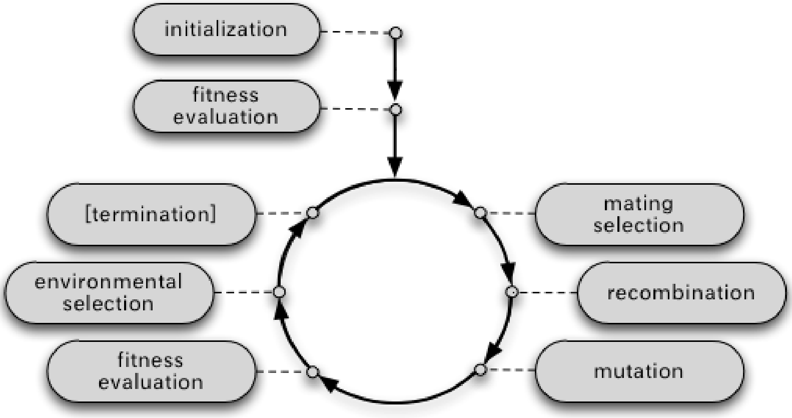
Evolutionary Algorithm



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  - Solutions are very stable.

DE - Algorithm

Pseudo-code 3.2 Differential Evolution

```
Begin
   Generate randomly an initial population of solutions.
   Calculate the fitness of the initial population.
   Repeat
      For each parent, select three solutions at random.
      Create one offspring using the DE operators.
      Do this a number of times equal to the population size.
      For each member of the next generation
         If offspring(x) is more fit than parent(x)
            Parent(x) is replaced.
   Until a stop condition is satisfied.
End.
```

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  - Concise representation of problem space.

GALE - Algorithm

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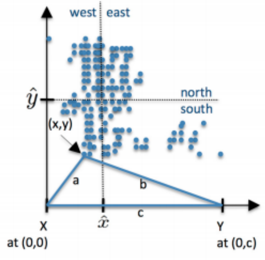

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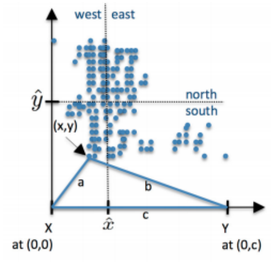

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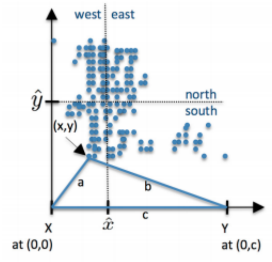

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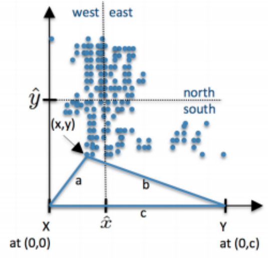

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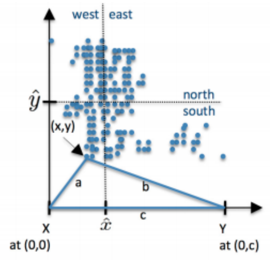

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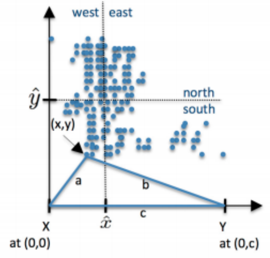

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- Repeat for **n** generations.

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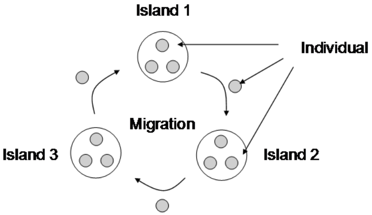

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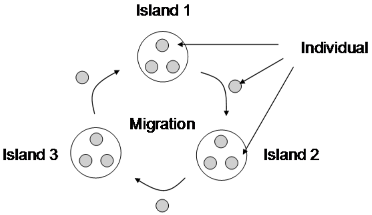

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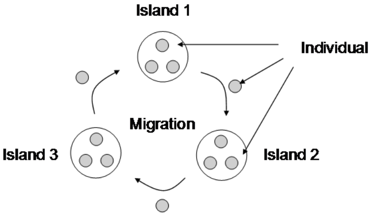

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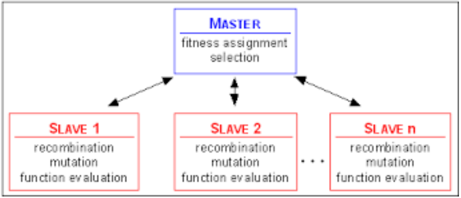

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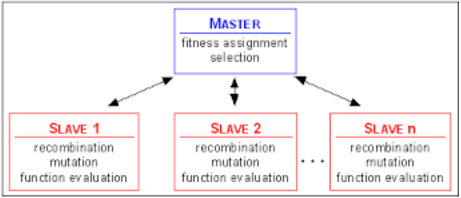

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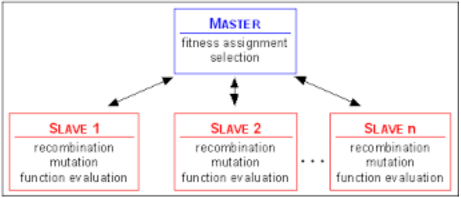

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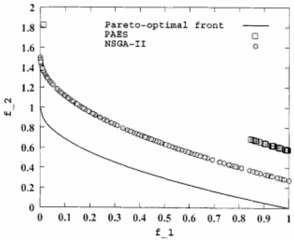

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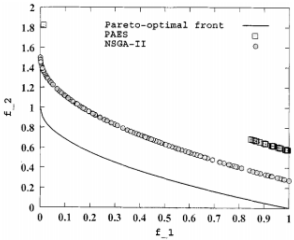

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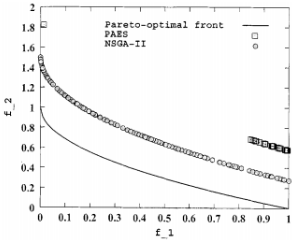

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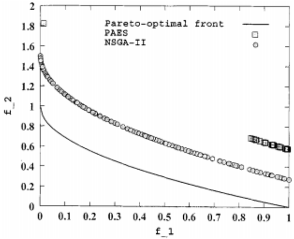

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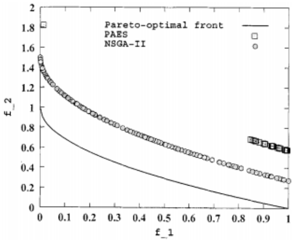

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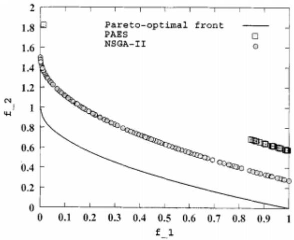

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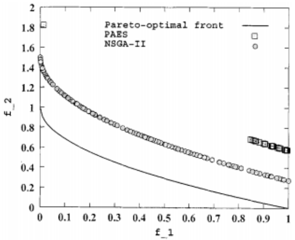
  - Ideally the solutions should be well distributed across the Pareto Frontier



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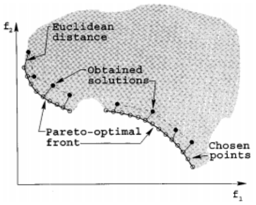
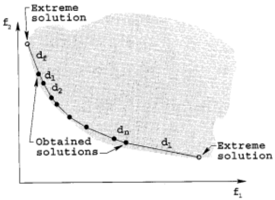
Measures

Convergence:



- Find a set of H optimal solutions.
- For each solution, compute the minimum eucledian distance from each of the solutions to a point on the Pareto Frontier.
- The average of these distances represent convergence.

Diversity:



- d_i is the distance between consecutive solutions.
- $\bar{d}$ is the mean of d_i
- d_f & d_l are distance between extreme and boundary solutions.

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}}$$

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- **HPC**
  - The henry2 shared memory linux cluster at NCSU.
  - Up to 16 shared memory processor cores and up to 128GB of memory accessible through a dedicated queue.

Outline

DTLZ-2(Island Model)

Rank	Optimizer	Median	IQR	Quartile Chart
1	DE(Parallel)	2.30×10^{-5}	2.74×10^{-6}	
1	DE(Serial)	2.36×10^{-5}	3.04×10^{-6}	
2	GALE(Serial)	5.49×10^{-4}	8.32×10^{-6}	
2	GALE(Parallel)	5.54×10^{-4}	2.21×10^{-5}	

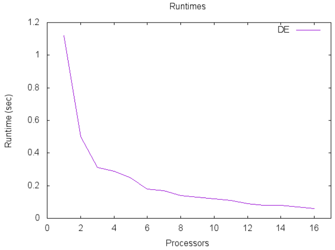
Figure: Convergence of serial & parallel DE & GALE

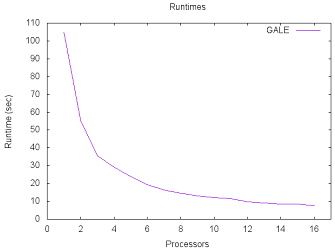
Rank	Optimizer	Median	IQR	Quartile Chart
1	GALE(Parallel)	0.416	0.070	
1	DE(Parallel)	0.417	0.049	
1	DE(Serial)	0.431	0.056	
1	GALE(Serial)	0.432	0.047	

Figure: Diversity of serial & parallel DE & GALE

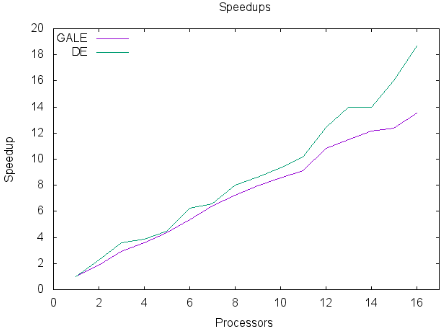
DTLZ-2(Island Model)

Runtimes:



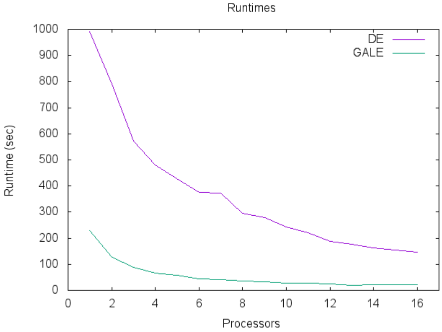


Speed Ups:



POM3(Island Model)

Runtimes:



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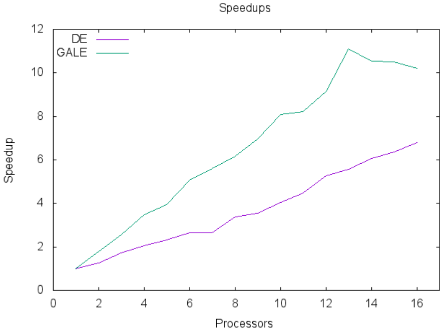
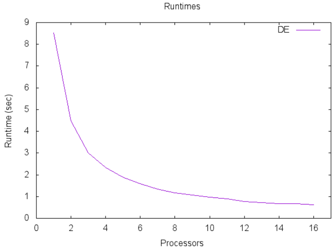
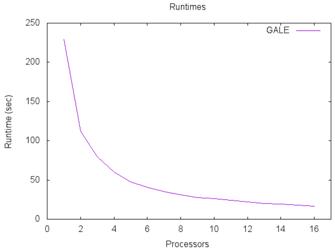


XOMO(Island Model)

Runtimes:





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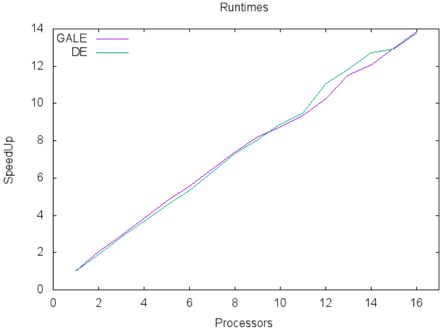
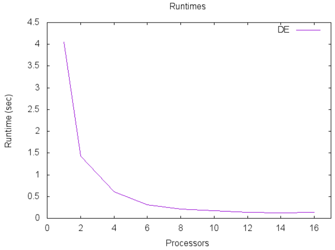
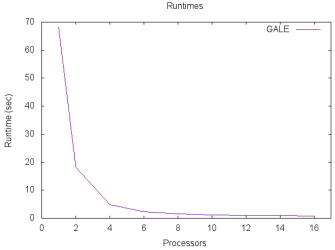


Outline

DTLZ-2(Master-Slave Model)

Runtimes:





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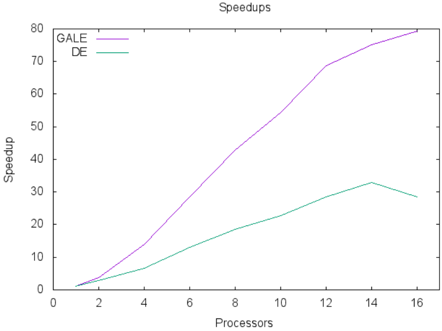

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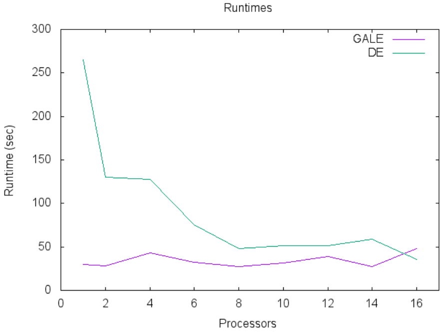

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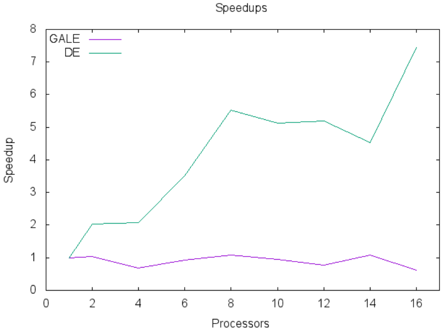

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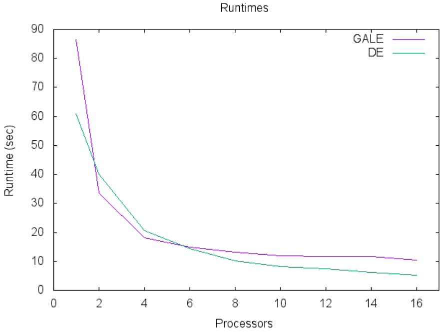

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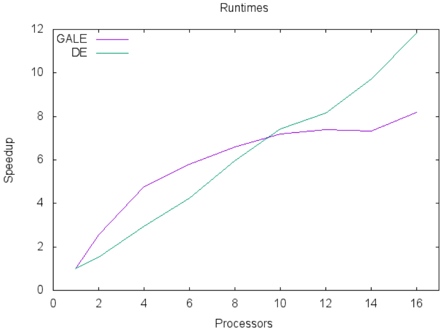

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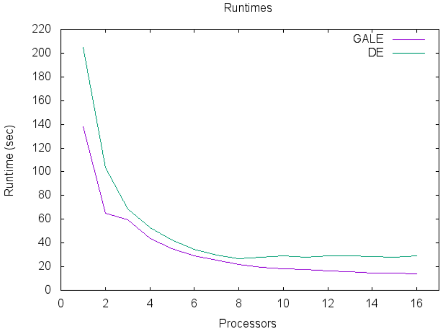
- For our experiment we use the Emergency Response(ERS) feature model, which has **35 decisions** and **3 objectives**.

Outline

Emergency Response(Island Model)

Runtimes:



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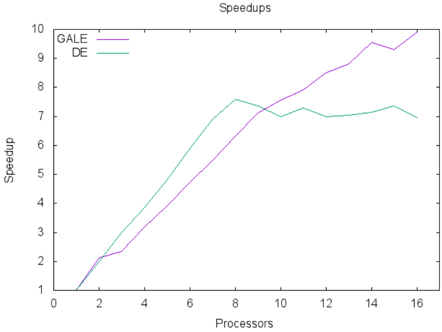

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- Strategies for efficiently dividing the feature space for more efficient parallelization.