Evolutionary Multi-Objective Optimization: A Parallel Computing Approach

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December 9, 2015

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  - → Island Model
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- Future Work
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Outline

Multi-Objective Problem

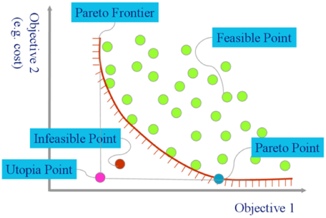

Figure: Sample Pareto Frontier

Multi-Objective Problem

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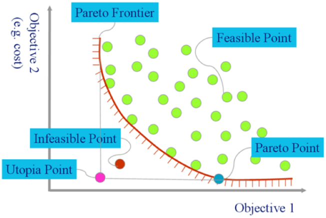

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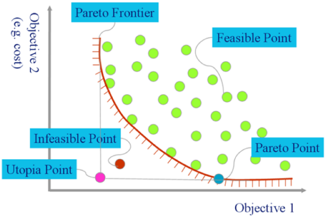

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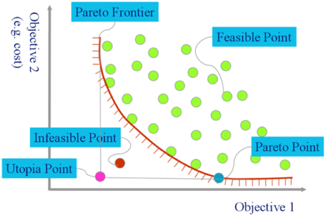

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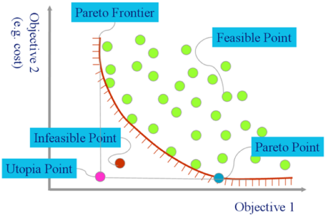

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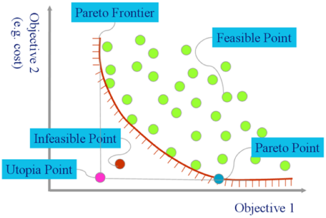
- **Utopia Point** The ideal theoretical solution we would love to reach but practically its not possible



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DTLZ-2

- **Decisions** DTLZ-2 has 30 decisions where each decision ranges between 0 and 1.

$$0 \leq x_i \leq 1 \quad \textit{where} \quad i = 1, 2, 3....30$$

- **Objectives** A point that lies on the Pareto frontier.

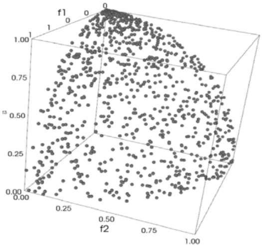
$$f_1(x) = (1 + g(x_M)) \cos(x_1\pi/2).... \cos(x_{M-1}\pi/2)$$

$$f_2(x) = (1 + g(x_M)) \cos(x_1\pi/2).... \cos(x_{M-1}\pi/2)$$

$$f_3(x) = (1 + g(x_M)) \sin(x_1\pi/2)$$

$$\textit{where} \quad g(x_M) = \sum_{x \in x_M} (x_i - 0.5)^2$$

- **Optimal Solution:** Ideal Decisions are $x_i = 0.5$ where $i = 1, 2, 3...30$
  Ideal objectives should satisfy the equation $\sum_{m=1}^{3} f_m^2 = 1$



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- 23 decisions - lines of code, storage, cyclometric complexity etc.
- 4 objectives all minimized
  - Total Developer **Effort**.
  - **Months** to complete project.
  - Total **Defects** in project.
  - **Risk** involved in project.

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POM3

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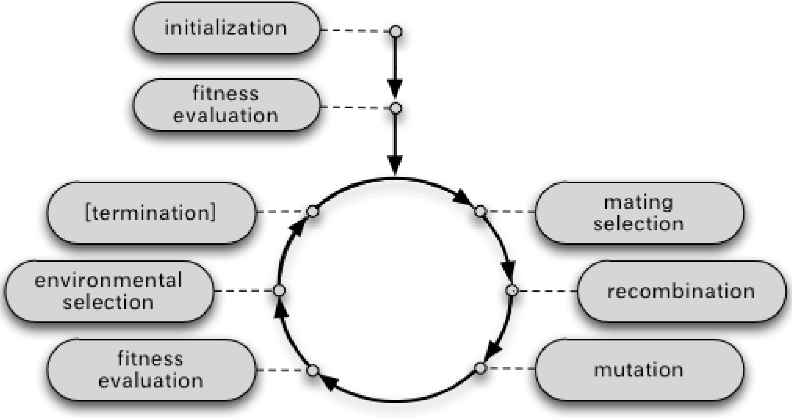
- Teams select tasks as they appear in the scrum backlog.
- 9 decisions like size of project, project plan, team size etc.
- The model contains 4 objectives
  - Minimize **Cost**.
  - Maximize **Utility**.
  - Maximize **Completion Percentage**.
  - Minimize **Idle Time** for Developers.

Outline

Evolutionary Algorithm



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- Iteratively approximates the shape of the Pareto Frontier
- Advantages:
  - Simple & Computationally Inexpensive.
  - High dimensional problems can be handled easily.
  - Solutions are very stable.

DE - Algorithm

Pseudo-code 3.2 Differential Evolution

```
Begin
   Generate randomly an initial population of solutions.
   Calculate the fitness of the initial population.
   Repeat
      For each parent, select three solutions at random.
      Create one offspring using the DE operators.
      Do this a number of times equal to the population size.
      For each member of the next generation
         If offspring(x) is more fit than parent(x)
            Parent(x) is replaced.
   Until a stop condition is satisfied.
End.
```

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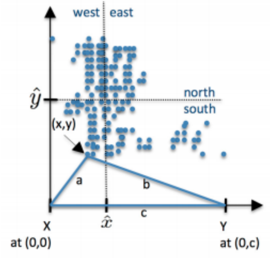
- Builds piecewise approximation to the best solutions of the pareto frontier.
- Based on WHERE which is a recursive clustering technique based on Dimensionality Reduction.
- Advantages:
  - Less Number of Computations.
  - Adept at handling problems that are **non-differentiable**, **non-liner**, **multi-dimensional** or **multi-constraint**.
  - Concise representation of problem space.

GALE - Algorithm

- Cluster data based on WHERE



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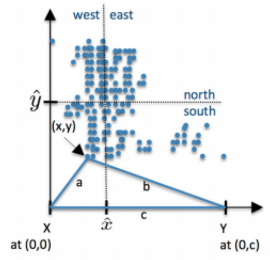

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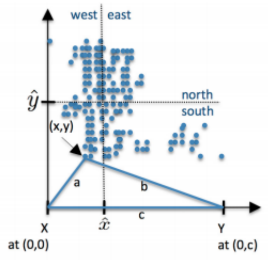

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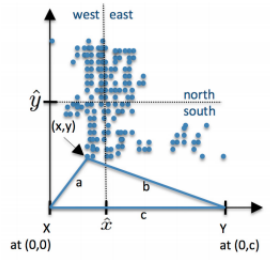

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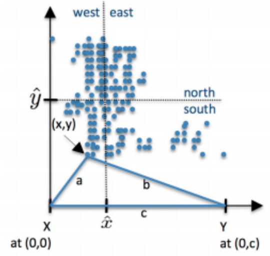

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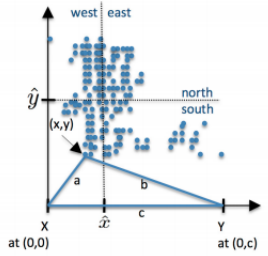

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- Repeat for **n** generations.

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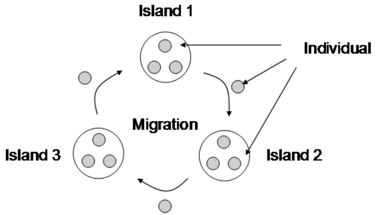
Island Model

- Divide the initial population(**N**) into sub-populations of equal size(**n**) among **k** processors.

$$n = \frac{N}{k}$$



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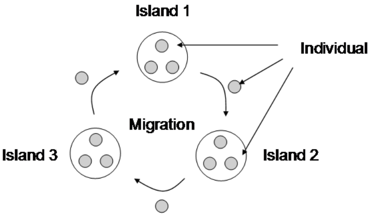

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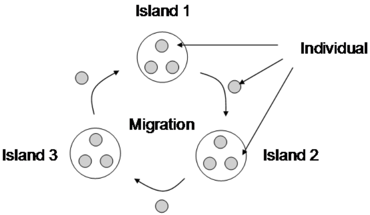

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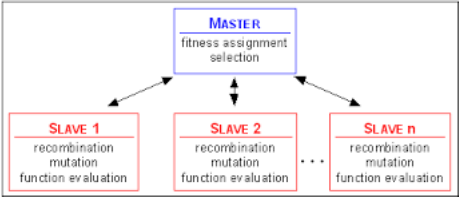
Master-Slave Model

- Master selects a population for each free slave in each generation.



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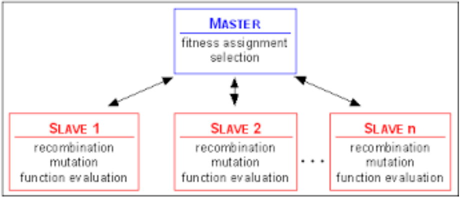

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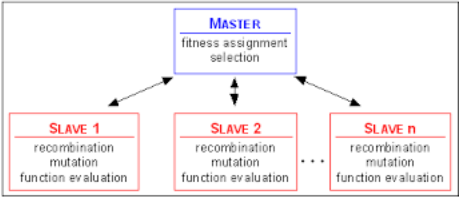
- Slave performs mutation on each population subset and sends it to master for next generation.



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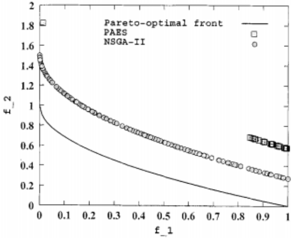

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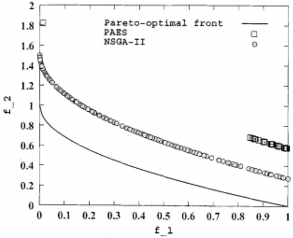

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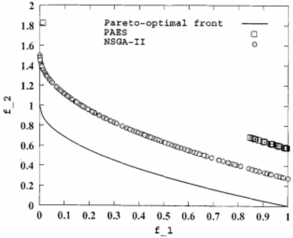
- **Diversity:**
  - Spread of the proposed solutions.
  - Ideally the solutions should be well distributed across the Pareto Frontier



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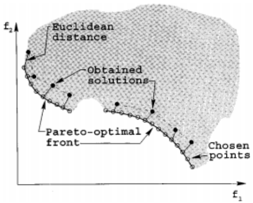
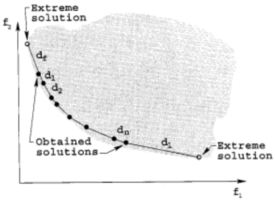
Measures

Convergence:



- Find a set of H optimal solutions.
- For each solution, compute the minimum eucledian distance from each of the solutions to a point on the Pareto Frontier.
- The average of these distances represent convergence.

Diversity:



- d_i is the distance between consecutive solutions.
- $\bar{d}$ is the mean of d_i
- d_f & d_l are distance between extreme and boundary solutions.

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}}$$

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- **Multi-Processing**
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- **HPC**
  - The henry2 shared memory linux cluster at NCSU.
  - Up to 16 shared memory processor cores and up to 128GB of memory accessible through a dedicated queue.

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DTLZ-2(Island Model)

Rank	Optimizer	Median	IQR	Quartile Chart
1	DE(Parallel)	2.30×10^{-5}	2.74×10^{-6}	
1	DE(Serial)	2.36×10^{-5}	3.04×10^{-6}	
2	GALE(Serial)	5.49×10^{-4}	8.32×10^{-6}	
2	GALE(Parallel)	5.54×10^{-4}	2.21×10^{-5}	

Figure: Convergence of serial & parallel DE & GALE

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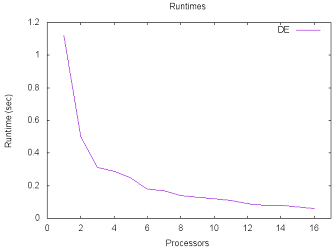
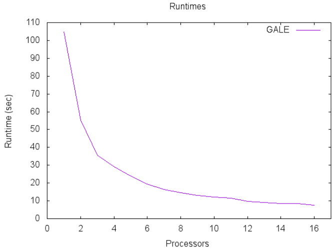
1	GALE(Parallel)	0.416	0.070	
1	DE(Parallel)	0.417	0.049	
1	DE(Serial)	0.431	0.056	
1	GALE(Serial)	0.432	0.047	

Figure: Diversity of serial & parallel DE & GALE

DTLZ-2(Island Model)

Runtimes:





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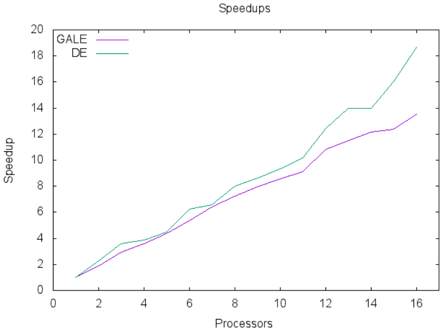
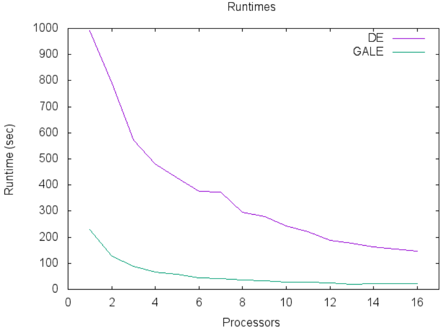


POM3(Island Model)

Runtimes:



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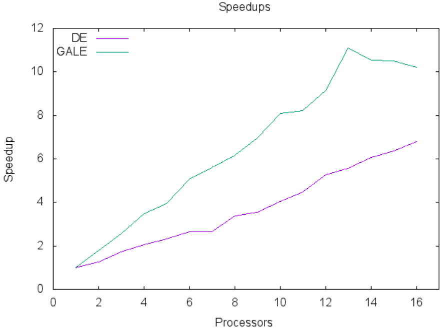
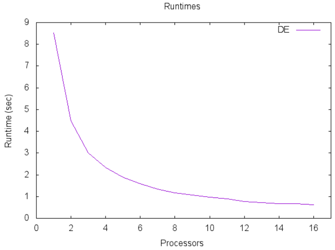
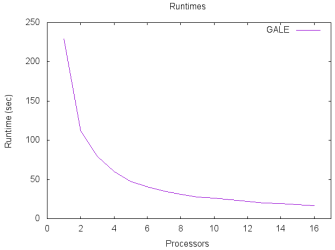


XOMO(Island Model)

Runtimes:





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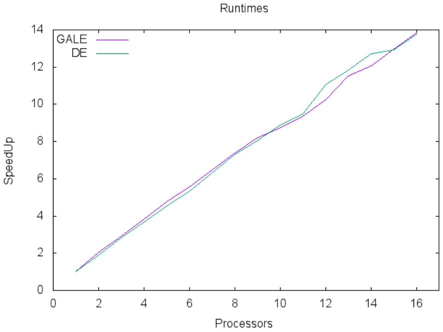

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DTLZ-2(Master-Slave Model)

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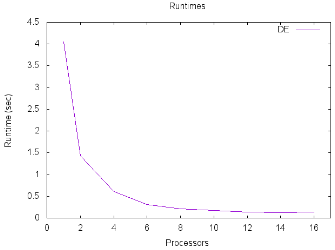

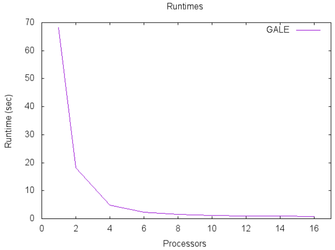

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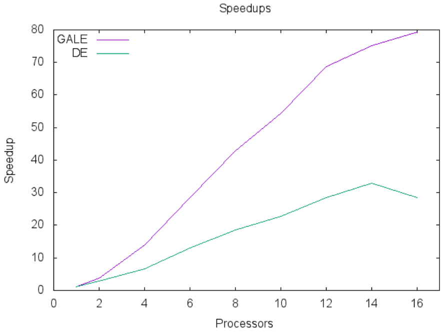

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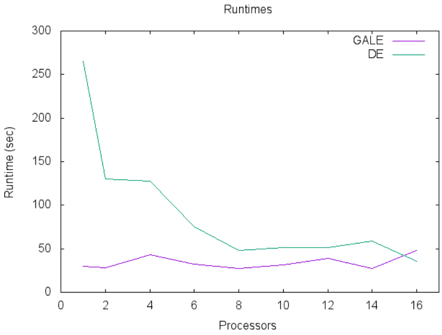

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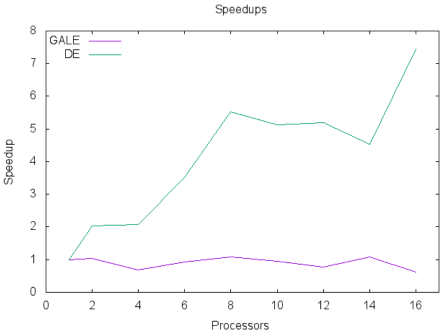
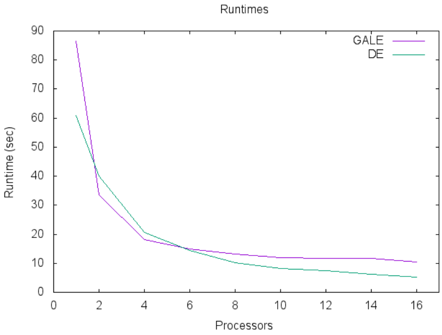
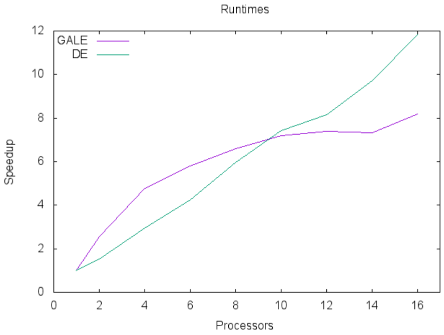


XOMO(Master-Slave Model)

Runtimes:



Speed Ups:



Outline

Feature Model

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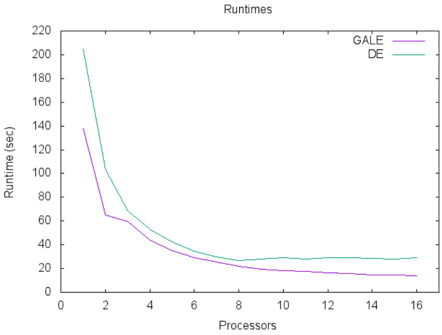
- For our experiment we use the Emergency Response(ERS) feature model, which has **35 decisions** and **3 objectives**.

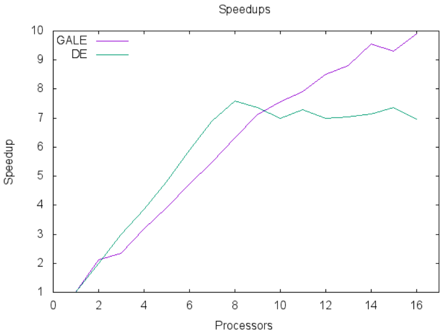
Outline

Emergency Response(Island Model)

Runtimes:



Speed Ups:



Outline

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- Extending these parallelization strategies for other Evolutionary algorithms like NSGA2, SPEA, IBEA etc
- Strategies for efficiently dividing the feature space for parallelization.

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- Parallelization is effective for Multi Objective Evolutionary Algorithms.
- Differential Evolution is preferred for Mathematical Problems.
- GALE is preferred for Real-World Problems where model evaluation is expensive.