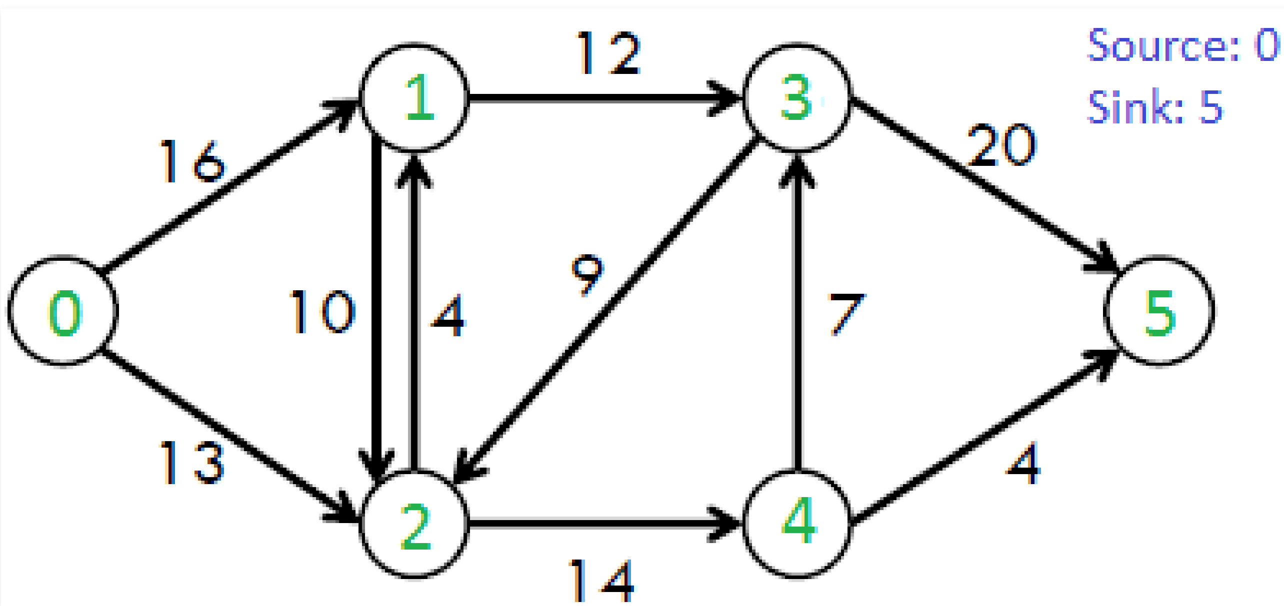
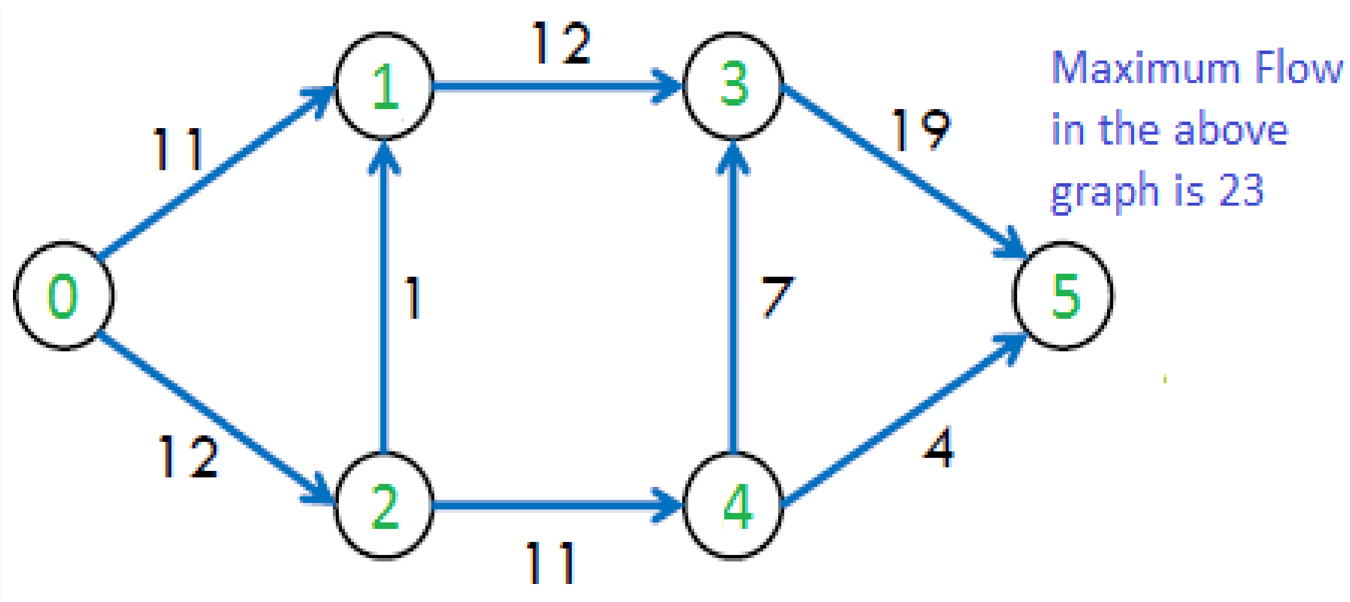
# Ford-Fulkerson Algorithm for Maximum Flow Problem

Difficulty Level : Hard • Last Updated : 04 Mar, 2021

Given a graph which represents a flow network where every edge has a capacity. Also given two vertices *source* 's' and *sink* 't' in the graph, find the maximum possible flow from s to t with following constraints:
**a)** Flow on an edge doesn't exceed the given capacity of the edge.
**b)** Incoming flow is equal to outgoing flow for every vertex except s and t.
For example, consider the following graph from CLRS book.




The maximum possible flow in the above graph is 23.




[Recommended: Please solve it on "***PRACTICE***" first, before moving on to the solution.]

Prerequisite : **[Max Flow Problem Introduction]**

```
Ford-Fulkerson Algorithm
The following is simple idea of Ford-Fulkerson algorithm:
1) Start with initial flow as 0.
2) While there is a augmenting path from source to sink.
           Add this path-flow to flow.
3) Return flow.
```

**Time Complexity:** Time complexity of the above algorithm is O(max_flow * E). We run a loop while there is an augmenting path. In worst case, we may add 1 unit flow in every iteration. Therefore the time complexity becomes O(max_flow * E).

**How to implement the above simple algorithm?**

Let us first define the concept of Residual Graph which is needed for understanding the implementation.

***Residual Graph*** of a flow network is a graph which indicates additional possible flow. If capacity of the edge minus current flow. Residual capacity is basically the current capacity of the edge.

Let us now talk about implementation details. Residual capacity is 0 if there is no edge between two vertices of residual graph. We can initialize the residual graph as original graph as there is no initial flow and initially residual capacity is equal to original capacity. To find an augmenting path, we can either do a BFS or DFS of the residual graph. We have used BFS in below implementation. Using BFS, we can find out if there is a path from source to sink. BFS also builds parent[] array. Using the parent[] array, we traverse through the found path and find possible flow through this path by finding minimum residual capacity along the path. We later add the found path flow to overall flow.

The important thing is, we need to update residual capacities in the residual graph. We subtract path flow from all edges along the path and we add path flow along the reverse edges We need to add path flow along reverse edges because may later need to send flow in reverse direction (See following link for example).

https://www.geeksforgeeks.org/max-flow-problem-introduction/

Below is the implementation of Ford-Fulkerson algorithm. To keep things simple, graph is represented as a 2D matrix.

---

**C++**

```
// C++ program for implementation of Ford Fulkerson
// algorithm
#include <iostream>
#include <limits.h>
#include <queue>
#include <string.h>
using namespace std;

// Number of vertices in given graph
#define V 6

/* Returns true if there is a path from source 's' to sink
  't' in residual graph. Also fills parent[] to store the
  path */
bool bfs(int rGraph[V][V], int s, int t, int parent[])
{
    // Create a visited array and mark all vertices as not
```

```
    // Create a queue, enqueue source vertex and mark source
    // vertex as visited
    queue<int> q;
    q.push(s);
    visited[s] = true;
    parent[s] = -1;

    // Standard BFS Loop
    while (!q.empty()) {
        int u = q.front();
        q.pop();

        for (int v = 0; v < V; v++) {
            if (visited[v] == false && rGraph[u][v] > 0) {
                // If we find a connection to the sink node,
                // then there is no point in BFS anymore We
                // just have to set its parent and can return
                // true
                if (v == t) {
                    parent[v] = u;
                    return true;
                }
                q.push(v);
                parent[v] = u;
                visited[v] = true;
            }
        }
    }

    // We didn't reach sink in BFS starting from source, so
    // return false
    return false;
}

// Returns the maximum flow from s to t in the given graph
int fordFulkerson(int graph[V][V], int s, int t)
{
    int u, v;

    // Create a residual graph and fill the residual graph
    // with given capacities in the original graph as
    // residual capacities in residual graph
    int rGraph[V]
              [V]; // Residual graph where rGraph[i][j]
                   // indicates residual capacity of edge
                   // from i to j (if there is an edge. If
```

```
    int parent[V]; // This array is filled by BFS and to
                   // store path

    int max_flow = 0; // There is no flow initially

    // Augment the flow while tere is path from source to
    // sink
    while (bfs(rGraph, s, t, parent)) {
        // Find minimum residual capacity of the edges along
        // the path filled by BFS. Or we can say find the
        // maximum flow through the path found.
        int path_flow = INT_MAX;
        for (v = t; v != s; v = parent[v]) {
            u = parent[v];
            path_flow = min(path_flow, rGraph[u][v]);
        }

        // update residual capacities of the edges and
        // reverse edges along the path
        for (v = t; v != s; v = parent[v]) {
            u = parent[v];
            rGraph[u][v] -= path_flow;
            rGraph[v][u] += path_flow;
        }

        // Add path flow to overall flow
        max_flow += path_flow;
    }

    // Return the overall flow
    return max_flow;
}

// Driver program to test above functions
int main()
{
    // Let us create a graph shown in the above example
    int graph[V][V]
        = { { 0, 16, 13, 0, 0, 0 }, { 0, 0, 10, 12, 0, 0 },
            { 0, 4, 0, 0, 14, 0 },  { 0, 0, 9, 0, 0, 20 },
            { 0, 0, 0, 7, 0, 4 },   { 0, 0, 0, 0, 0, 0 } };

    cout << "The maximum possible flow is "
         << fordFulkerson(graph, 0, 5);

    return 0;
```

## Java

```
// Java program for implementation of Ford Fulkerson
// algorithm
import java.io.*;
import java.lang.*;
import java.util.*;
import java.util.LinkedList;

class MaxFlow {
    static final int V = 6; // Number of vertices in graph

    /* Returns true if there is a path from source 's' to
      sink 't' in residual graph. Also fills parent[] to
      store the path */
    boolean bfs(int rGraph[][], int s, int t, int parent[])
    {
        // Create a visited array and mark all vertices as
        // not visited
        boolean visited[] = new boolean[V];
        for (int i = 0; i < V; ++i)
            visited[i] = false;

        // Create a queue, enqueue source vertex and mark
        // source vertex as visited
        LinkedList<Integer> queue
            = new LinkedList<Integer>();
        queue.add(s);
        visited[s] = true;
        parent[s] = -1;

        // Standard BFS Loop
        while (queue.size() != 0) {
            int u = queue.poll();

            for (int v = 0; v < V; v++) {
                if (visited[v] == false
                    && rGraph[u][v] > 0) {
                    // If we find a connection to the sink
                    // node, then there is no point in BFS
                    // anymore We just have to set its parent
                    // and can return true
                    if (v == t) {
                        parent[v] = u;
                        return true;
                    }
```

```
                }
            }
        }

        // We didn't reach sink in BFS starting from source,
        // so return false
        return false;
    }

    // Returns tne maximum flow from s to t in the given
    // graph
    int fordFulkerson(int graph[][], int s, int t)
    {
        int u, v;

        // Create a residual graph and fill the residual
        // graph with given capacities in the original graph
        // as residual capacities in residual graph

        // Residual graph where rGraph[i][j] indicates
        // residual capacity of edge from i to j (if there
        // is an edge. If rGraph[i][j] is 0, then there is
        // not)
        int rGraph[][] = new int[V][V];

        for (u = 0; u < V; u++)
            for (v = 0; v < V; v++)
                rGraph[u][v] = graph[u][v];

        // This array is filled by BFS and to store path
        int parent[] = new int[V];

        int max_flow = 0; // There is no flow initially

        // Augment the flow while tere is path from source
        // to sink
        while (bfs(rGraph, s, t, parent)) {
            // Find minimum residual capacity of the edhes
            // along the path filled by BFS. Or we can say
            // find the maximum flow through the path found.
            int path_flow = Integer.MAX_VALUE;
            for (v = t; v != s; v = parent[v]) {
                u = parent[v];
                path_flow
                    = Math.min(path_flow, rGraph[u][v]);
            }
```

```java
                rGraph[u][v] -= path_flow;
                rGraph[v][u] += path_flow;
            }

            // Add path flow to overall flow
            max_flow += path_flow;
        }

        // Return the overall flow
        return max_flow;
    }

    // Driver program to test above functions
    public static void main(String[] args)
        throws java.lang.Exception
    {
        // Let us create a graph shown in the above example
        int graph[][] = new int[][] {
            { 0, 16, 13, 0, 0, 0 }, { 0, 0, 10, 12, 0, 0 },
            { 0, 4, 0, 0, 14, 0 },  { 0, 0, 9, 0, 0, 20 },
            { 0, 0, 0, 7, 0, 4 },   { 0, 0, 0, 0, 0, 0 }
        };
        MaxFlow m = new MaxFlow();

        System.out.println("The maximum possible flow is "
                           + m.fordFulkerson(graph, 0, 5));
    }
}
```

## Python

```python
# Python program for implementation
# of Ford Fulkerson algorithm
from collections import defaultdict

# This class represents a directed graph
# using adjacency matrix representation
class Graph:

    def __init__(self, graph):
        self.graph = graph  # residual graph
        self. ROW = len(graph)
        # self.COL = len(gr[0])

    '''Returns true if there is a path from source 's' to sink 't' in
```

```
        # Mark all the vertices as not visited
        visited = [False]*(self.ROW)

        # Create a queue for BFS
        queue = []

        # Mark the source node as visited and enqueue it
        queue.append(s)
        visited[s] = True

         # Standard BFS Loop
        while queue:

            # Dequeue a vertex from queue and print it
            u = queue.pop(0)

            # Get all adjacent vertices of the dequeued vertex u
            # If a adjacent has not been visited, then mark it
            # visited and enqueue it
            for ind, val in enumerate(self.graph[u]):
                if visited[ind] == False and val > 0:
                      # If we find a connection to the sink node,
                    # then there is no point in BFS anymore
                    # We just have to set its parent and can return true
                      if ind == t:
                        visited[ind] = True
                        return True
                    queue.append(ind)
                    visited[ind] = True
                    parent[ind] = u

        # We didn't reach sink in BFS starting
        # from source, so return false
        return False


    # Returns tne maximum flow from s to t in the given graph
    def FordFulkerson(self, source, sink):

        # This array is filled by BFS and to store path
        parent = [-1]*(self.ROW)

        max_flow = 0 # There is no flow initially

        # Augment the flow while there is path from source to sink
        while self.BFS(source, sink, parent) :
```

```
            s = sink
            while(s !=  source):
                path_flow = min (path_flow, self.graph[parent[s]][s])
                s = parent[s]

            # Add path flow to overall flow
            max_flow +=  path_flow

            # update residual capacities of the edges and reverse edges
            # along the path
            v = sink
            while(v !=  source):
                u = parent[v]
                self.graph[u][v] -= path_flow
                self.graph[v][u] += path_flow
                v = parent[v]

        return max_flow


# Create a graph given in the above diagram

graph = [[0, 16, 13, 0, 0, 0],
        [0, 0, 10, 12, 0, 0],
        [0, 4, 0, 0, 14, 0],
        [0, 0, 9, 0, 0, 20],
        [0, 0, 0, 7, 0, 4],
        [0, 0, 0, 0, 0, 0]]

g = Graph(graph)

source = 0; sink = 5

print ("The maximum possible flow is %d " % g.FordFulkerson(source, sink))

# This code is contributed by Neelam Yadav
```

## C#

```
// C# program for implementation
// of Ford Fulkerson algorithm
using System;
using System.Collections.Generic;

public class MaxFlow {
```

```
from source 's' to sink 't' in residual
graph. Also fills parent[] to store the
path */
bool bfs(int[, ] rGraph, int s, int t, int[] parent)
{
    // Create a visited array and mark
    // all vertices as not visited
    bool[] visited = new bool[V];
    for (int i = 0; i < V; ++i)
        visited[i] = false;

    // Create a queue, enqueue source vertex and mark
    // source vertex as visited
    List<int> queue = new List<int>();
    queue.Add(s);
    visited[s] = true;
    parent[s] = -1;

    // Standard BFS Loop
    while (queue.Count != 0) {
        int u = queue[0];
        queue.RemoveAt(0);

        for (int v = 0; v < V; v++) {
            if (visited[v] == false
                && rGraph[u, v] > 0) {
                // If we find a connection to the sink
                // node, then there is no point in BFS
                // anymore We just have to set its parent
                // and can return true
                if (v == t) {
                    parent[v] = u;
                    return true;
                }
                queue.Add(v);
                parent[v] = u;
                visited[v] = true;
            }
        }
    }

    // We didn't reach sink in BFS starting from source,
    // so return false
    return false;
}

// Returns tne maximum flow
```

```
        // Create a residual graph and fill
        // the residual graph with given
        // capacities in the original graph as
        // residual capacities in residual graph

        // Residual graph where rGraph[i,j]
        // indicates residual capacity of
        // edge from i to j (if there is an
        // edge. If rGraph[i,j] is 0, then
        // there is not)
        int[, ] rGraph = new int[V, V];

        for (u = 0; u < V; u++)
            for (v = 0; v < V; v++)
                rGraph[u, v] = graph[u, v];

        // This array is filled by BFS and to store path
        int[] parent = new int[V];

        int max_flow = 0; // There is no flow initially

        // Augment the flow while tere is path from source
        // to sink
        while (bfs(rGraph, s, t, parent)) {
            // Find minimum residual capacity of the edhes
            // along the path filled by BFS. Or we can say
            // find the maximum flow through the path found.
            int path_flow = int.MaxValue;
            for (v = t; v != s; v = parent[v]) {
                u = parent[v];
                path_flow
                    = Math.Min(path_flow, rGraph[u, v]);
            }

            // update residual capacities of the edges and
            // reverse edges along the path
            for (v = t; v != s; v = parent[v]) {
                u = parent[v];
                rGraph[u, v] -= path_flow;
                rGraph[v, u] += path_flow;
            }

            // Add path flow to overall flow
            max_flow += path_flow;
        }
```

```
    // Driver code
    public static void Main()
    {
        // Let us create a graph shown in the above example
        int[, ] graph = new int[, ] {
            { 0, 16, 13, 0, 0, 0 }, { 0, 0, 10, 12, 0, 0 },
            { 0, 4, 0, 0, 14, 0 },  { 0, 0, 9, 0, 0, 20 },
            { 0, 0, 0, 7, 0, 4 },   { 0, 0, 0, 0, 0, 0 }
        };
        MaxFlow m = new MaxFlow();

        Console.WriteLine("The maximum possible flow is "
                          + m.fordFulkerson(graph, 0, 5));
    }
}

/* This code contributed by PrinciRaj1992 */
```

**Output:**

```
The maximum possible flow is 23
```

The above implementation of Ford Fulkerson Algorithm is called **Edmonds-Karp Algorithm**. The idea of Edmonds-Karp is to use BFS in Ford Fulkerson implementation as BFS always picks a path with minimum number of edges. When BFS is used, the worst case time complexity can be reduced to $O(VE^2)$. The above implementation uses adjacency matrix representation though where BFS takes $O(V^2)$ time, the time complexity of the above implementation is $O(EV^3)$ (Refer CLRS book for proof of time complexity)
This is an important problem as it arises in many practical situations. Examples include, maximizing the transportation with given traffic limits, maximizing packet flow in computer networks.
Dinc's Algorithm for Max-Flow.
**Exercise:**
Modify the above implementation so that it that runs in $O(VE^2)$ time.
**References:**
http://www.stanford.edu/class/cs97si/08-network-flow-problems.pdf
Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E.

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19, Feb 17

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