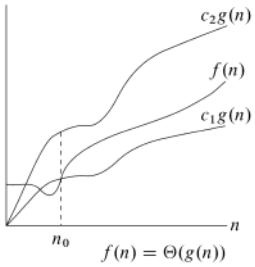
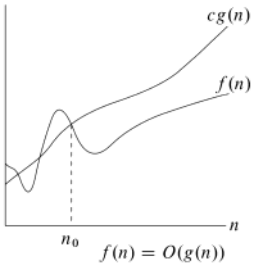
Csi503-s21-lecture03

Tuesday, February 9, 2021

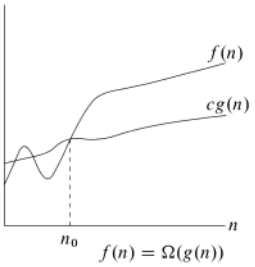
Asymptotic Notation



(a)



(b)



(c)

Θ-notation : asymptotic upper bound

$g(n)$ is asymptotic upper bound for $f(n)$:

$f(n) = O(g(n)) \longleftrightarrow O(g(n)) = \{ f(n) :$ there exists c & n_0 :
$0 \leq f(n) \leq c\,g(n)$ for all $n \geq n_0 \}$

$O(n^2) = \{ n^2, 2n, 2n^2 + 3n, n_2 \ldots \}$

$f(n) = n \qquad f(n) = O(n^2)$

$g(n)$ is asymptotic low bound for $f(n)$,

$f(n) = \Omega(g(n)) \longleftrightarrow \Omega(g(n)) = \{ f(n) : \quad c, n_0$
$0 \leq c\,g(n) \leq f(n)$ for all $n \geq n_0 \}$

quiz: $\Omega(n^2) = \{ n^2, n^2 + 5n, n, n \lg n, n^3 + n^2 \}$

$n^2(n+1)$

$f(n) = 2n$
$f(n) = O(n^2)$

$0 \leq 2n \leq cn^2, \quad n \geq n_0$

$c = 1, \; n_0 = 2 \quad n = 2 \longrightarrow 2 \times 2 \leq 1 \times 2^2$
$n = 3 \longrightarrow 2 \times 3 \leq 3^2 = 9$

Using O/Ω in equations

$2n^2 + 3n + 1 = 2n^2 + \Theta(n)$

$2n^2 + \Theta(n) = \Theta(n^2)$

$2n^2 + \Theta(n) = \Theta(n^2)$

o-notation

$o(g(n)) = \{f(n)$: for any c, there exists n_0 : $0 \leq f(n) < c\,g(n)$ for all $n \geq n_0\}$

$o(n^2) = \{ \cancel{n^2}, n^{1.99}, \ldots \}$

alternative definition: $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$n^{1.99} < c n^2 \iff \frac{1}{n^{.01}} < c$

$\lim_{n \to \infty} \frac{n^{1.99}}{n^2} = \frac{1}{n^{0.01}} = 0$ $\qquad \lim_{n \to \infty} \frac{n^2}{n^2} = 1$

ω-notation defined similarly.

$f(n) = \omega(g(n)) \iff \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$

- Check out properties of $O, \Omega, \Theta, o, \omega$ in the book.
  transitivity, reflexivity, ...