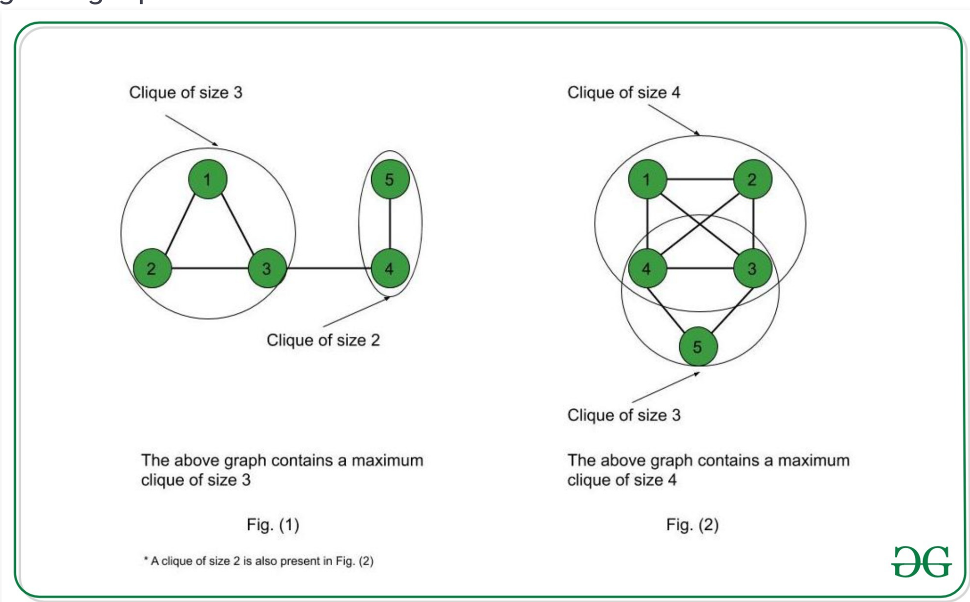
Proof that Clique Decision problem is NP-Complete

Difficulty Level : Expert • Last Updated : 13 Jun, 2020

Prerequisite: NP-Completeness

A clique is a subgraph of a graph such that all the vertices in this subgraph are connected with each other that is the subgraph is a complete graph. The Maximal Clique Problem is to find the maximum sized clique of a given graph G, that is a complete graph which is a subgraph of G and contains the maximum number of vertices. This is an optimization problem. Correspondingly, the Clique Decision Problem is to find if a clique of size k exists in the given graph or not.



To prove that a problem is NP-Complete, we have to show that it belongs to both NP and NP-Hard Classes. (Since NP-Complete problems are NP-Hard problems which also belong to NP) to verify in polynomial time if it is a solution to the problem.

Proof:

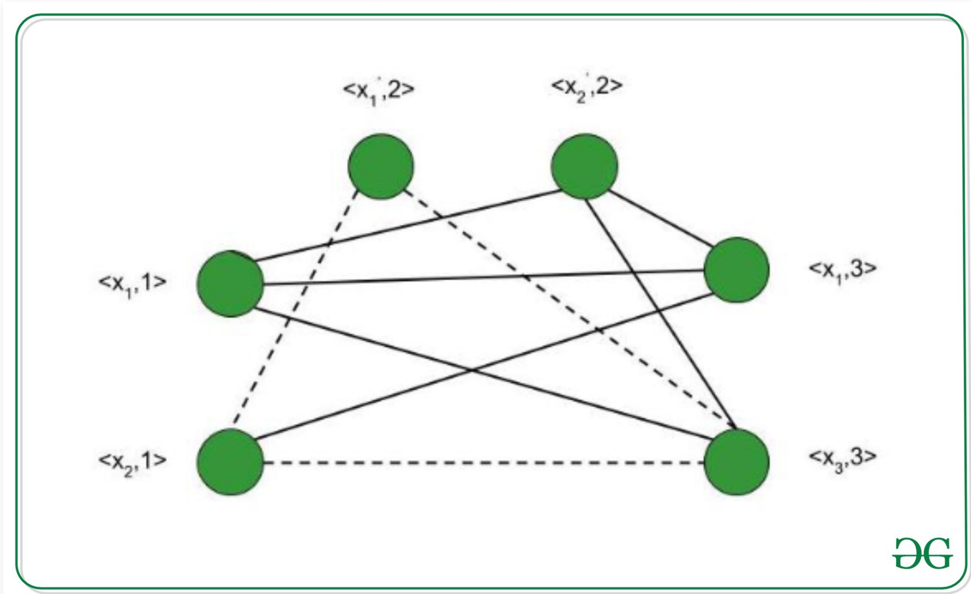
1. Certificate – Let the certificate be a set S consisting of nodes in the clique and S is a subgraph of G.
2. Verification – We have to check if there exists a clique of size k in the graph. Hence, verifying if number of nodes in S equals k, takes O(1) time. Verifying whether each vertex has an out-degree of (k-1) takes $O(k^2)$ time. (Since in a complete graph, each vertex is connected to every other vertex through an edge. Hence the total number of edges in a complete graph = ${}^kC_2 = k*(k-1)/2$). Therefore, to check if the graph formed by the k nodes in S is complete or not, it takes $O(k^2) = O(n^2)$ time (since k<=n, where n is number of vertices in G).

Therefore, the Clique Decision Problem has polynomial time verifiability and hence belongs to the NP Class.

The Clique Decision Problem belongs to NP-Hard – A problem L belongs to NP-Hard if every NP problem is reducible to L in polynomial time. Now, let the Clique Decision Problem by C. To prove that C is NP-Hard, we take an already known NP-Hard problem, say S, and reduce it to C for a particular instance. If this reduction can be done in polynomial time, then C is also an NP-Hard problem. The Boolean Satisfiability Problem (S) is an NP-Complete problem as proved by the Cook's theorem. Therefore, every problem in NP can be reduced to S in polynomial time. Thus, if S is reducible to C in polynomial time, every NP problem can be reduced to C in polynomial time, thereby proving C to be NP-Hard.

Proof that the Boolean Satisfiability problem reduces to the Clique Decision Problem
Let the boolean expression be – $F = (x_1 \vee x_2) \wedge (x_1' \vee x_2') \wedge (x_1 \vee x_3)$ where x_1, x_2, x_3 are the variables, '^' denotes logical 'and', 'v' denotes logical 'or' and x' denotes the complement of x. Let the expression within each parentheses be a clause. Hence we have three clauses – C_1, C_2 and C_3. Consider the vertices as – $<x_1, 1>$; $<x_2, 1>$; $<x_1', 2>$; $<x_2', 2>$; $<x_1, 3>$; $<x_3, 3>$ where the second term in each vertex denotes the clause number they belong to. We

2. No variable is connected to its complement.



Thus, the graph G (V, E) is constructed such that – V = { <a, i> | a belongs to C_i } and E = { (<a, i>, <b, j>) | i is not equal to j ; b is not equal to a' } Consider the subgraph of G with the vertices $<x_2, 1>$; $<x_1', 2>$; $<x_3, 3>$. It forms a clique of size 3 (Depicted by dotted line in above figure) . Corresponding to this, for the assignment – $<x_1, x_2, x_3> = <0, 1, 1>$ F evaluates to true. Therefore, if we have k clauses in our satisfiability expression, we get a max clique of size k and for the corresponding assignment of values, the satisfiability expression evaluates to true. Hence, for a particular instance, the satisfiability problem is reduced to the clique decision problem.

Therefore, the Clique Decision Problem is NP-Hard.

Conclusion
The Clique Decision Problem is NP and NP-Hard. Therefore, the Clique decision problem is NP-Complete.

Attention reader! Don't stop learning now. Get hold of all the important DSA concepts with the **DSA Self Paced Course** at a student-friendly price and become industry ready. To complete your preparation from learning a language to DS Algo and many more, please refer **Complete Interview Preparation Course**.

In case you wish to attend live classes with industry experts, please refer **Geeks Classes Live** and **Geeks Classes Live USA**

Like 0

Previous

Next

RECOMMENDED ARTICLES

Page : 1 2 3

01 **Proof that Clique Decision problem is NP-Complete | Set 2**
30, Jun 20

02 **Two Clique Problem (Check if Graph can be divided in two Cliques)**
19, Dec 16

03 **Maximal Clique Problem | Recursive Solution**
07, Feb 20

04 **Proof that traveling salesman problem is NP Hard**
03, Jun 20

05 **Proof that Subgraph Isomorphism problem is NP-Complete**
12, Jun 20

06 **Proof that Collinearity Problem is NP Complete**
14, Oct 20

07 **Proof that Hamiltonian Path is NP-Complete**
04, Jun 18

08 **Proof that vertex cover is NP complete**
03, Aug 18

Article Contributed By :

Vote for difficulty

Current difficulty : Expert

Easy Normal Medium Hard Expert

Article Tags : Algorithms-NP Complete, NP Complete, NPHard, Algorithms, Graph

Practice Tags : Graph, Algorithms

Improve Article Report Issue

Writing code in comment? Please use ide.geeksforgeeks.org, generate link and share the link here.

Load Comments

GeeksforGeeks

5th Floor, A-118,
Sector-136, Noida, Uttar Pradesh - 201305

feedback@geeksforgeeks.org