Tugas Praktikum Analisis Algoritma

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Studi Kasus

5. Mencari Pasangan Titik Terdekat (Closest Pair of Points)

a. Buatlah program untuk menyelesaikan problem closest pair of points menggunakan algoritma divide & conquer yang diberikan. Gunakan bahasa C++

```
/*
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*/

// A divide and conquer program in C++
// to find the smallest distance from a
// given set of points.

#include <bits/stdc++.h>
using namespace std;

// A structure to represent a Point in 2D plane
class Point
{
  public:
  int x, y;
};

/* Following two functions are needed for library function qsort().
Refer: http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */

// Needed to sort array of points
// according to X coordinate
int compareX(const void* a, const void* b)
{
  Point *p1 = (Point *)a, *p2 = (Point *)b;
  return (p1->x - p2->x);
}

// Needed to sort array of points according to Y coordinate
int compareY(const void* a, const void* b)
{
  Point *p1 = (Point *)a, *p2 = (Point *)b;
  return (p1->y - p2->y);
}

// A utility function to find the
// distance between two points
float dist(Point p1, Point p2)
```
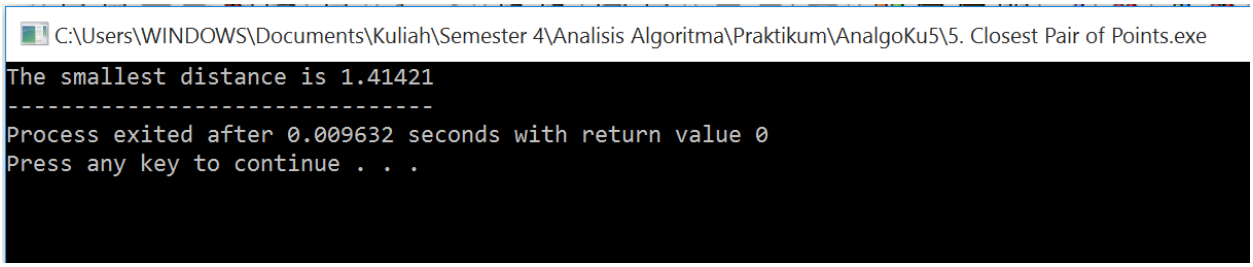

```
{
  return sqrt( (p1.x - p2.x)*(p1.x - p2.x) +
                (p1.y - p2.y)*(p1.y - p2.y)
             );
}

// A Brute Force method to return the
// smallest distance between two points
// in P[] of size n
float bruteForce(Point P[], int n)
{
  float min = FLT_MAX;
  for (int i = 0; i < n; ++i)
      for (int j = i+1; j < n; ++j)
          if (dist(P[i], P[j]) < min)
              min = dist(P[i], P[j]);
  return min;
}

// A utility function to find
// minimum of two float values
float min(float x, float y)
{
  return (x < y)? x : y;
}


// A utility function to find the
// distance beween the closest points of
// strip of given size. All points in
// strip[] are sorted accordint to
// y coordinate. They all have an upper
// bound on minimum distance as d.
// Note that this method seems to be
// a O(n^2) method, but it's a O(n)
// method as the inner loop runs at most 6 times
float stripClosest(Point strip[], int size, float d)
{
  float min = d; // Initialize the minimum distance as d

  qsort(strip, size, sizeof(Point), compareY);

  // Pick all points one by one and try the next points till the difference
  // between y coordinates is smaller than d.
  // This is a proven fact that this loop runs at most 6 times
  for (int i = 0; i < size; ++i)
      for (int j = i+1; j < size && (strip[j].y - strip[i].y) < min; ++j)
          if (dist(strip[i],strip[j]) < min)
              min = dist(strip[i], strip[j]);
```

```
    return min;
}

// A recursive function to find the
// smallest distance. The array P contains
// all points sorted according to x coordinate
float closestUtil(Point P[], int n)
{
    // If there are 2 or 3 points, then use brute force
    if (n <= 3)
        return bruteForce(P, n);

    // Find the middle point
    int mid = n/2;
    Point midPoint = P[mid];

    // Consider the vertical line passing
    // through the middle point calculate
    // the smallest distance dl on left
    // of middle point and dr on right side
    float dl = closestUtil(P, mid);
    float dr = closestUtil(P + mid, n - mid);

    // Find the smaller of two distances
    float d = min(dl, dr);

    // Build an array strip[] that contains
    // points close (closer than d)
    // to the line passing through the middle point
    Point strip[n];
    int j = 0;
    for (int i = 0; i < n; i++)
        if (abs(P[i].x - midPoint.x) < d)
            strip[j] = P[i], j++;

    // Find the closest points in strip.
    // Return the minimum of d and closest
    // distance is strip[]
    return min(d, stripClosest(strip, j, d) );
}

// The main functin that finds the smallest distance
// This method mainly uses closestUtil()
float closest(Point P[], int n)
{
    qsort(P, n, sizeof(Point), compareX);

    // Use recursive function closestUtil()
```

```
  // to find the smallest distance
  return closestUtil(P, n);
}

// Driver code
int main()
{
  Point P[] = {{2, 3}, {12, 30}, {40, 50}, {5, 1}, {12, 10}, {3, 4}};
  int n = sizeof(P) / sizeof(P[0]);
  cout << "The smallest distance is " << closest(P, n);
  return 0;
}
```

Screenshot:



b. Tentukan rekurensi dari algoritma tersebut, dan selesaikan rekurensinya menggunakan metode recursion tree untuk membuktikan bahwa algoritma tersebut memiliki Big-O (n lg n)

- Let's try divide and conquer.
  - Divide each number into two halves.
    - $x = x_H r^{n/2} + x_L$
    - $y = y_H r^{n/2} + y_L$
  - Then:

$$xy = (x_H r^{n/2} + x_L)\, y_H r^{n/2} + y_L$$
$$= x_H y_H r^n + (x_H y_L + x_L y_H) r^{n/2} + x_L y_L$$

  - Runtime?
    - T(n) = 4 T(n/2) + O(n)
    - T(n) = O(n^2)
- Instead of 4 subproblems, we only need 3 (with the help of clever insight).
- Three subproblems:
  - $a = x_H y_H$
  - $d = x_L y_L$
  - $e = (x_H + x_L)(y_H + y_L) - a - d$
- Then $xy = a r^n + e r^{n/2} + d$
- $T(n) = 3 T(n/2) + O(n)$
- $T(n) = O(n^{\log 3}) = O(n^{1.584\ldots})$

6. Algoritma Karatsuba untuk Perkalian Cepat

a. Buatlah program untuk menyelesaikan fast multiplication menggunakan algoritma divide & conquer yang diberikan (Algoritma Karatsuba). Gunakan bahasa C++

```
/*
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*/

// C++ implementation of Karatsuba algorithm for bit string multiplication.
#include<iostream>
#include<stdio.h>

using namespace std;

// FOLLOWING TWO FUNCTIONS ARE COPIED FROM http://goo.gl/q0OhZ
// Helper method: given two unequal sized bit strings, converts them to
// same length by adding leading 0s in the smaller string. Returns the
// the new length
int makeEqualLength(string &str1, string &str2)
{
  int len1 = str1.size();
  int len2 = str2.size();
  if (len1 < len2)
  {
        for (int i = 0 ; i < len2 - len1 ; i++)
              str1 = '0' + str1;
        return len2;
  }
  else if (len1 > len2)
  {
        for (int i = 0 ; i < len1 - len2 ; i++)
              str2 = '0' + str2;
  }
  return len1; // If len1 >= len2
}

// The main function that adds two bit sequences and returns the addition
string addBitStrings( string first, string second )
{
  string result; // To store the sum bits

  // make the lengths same before adding
  int length = makeEqualLength(first, second);
  int carry = 0; // Initialize carry

  // Add all bits one by one
```
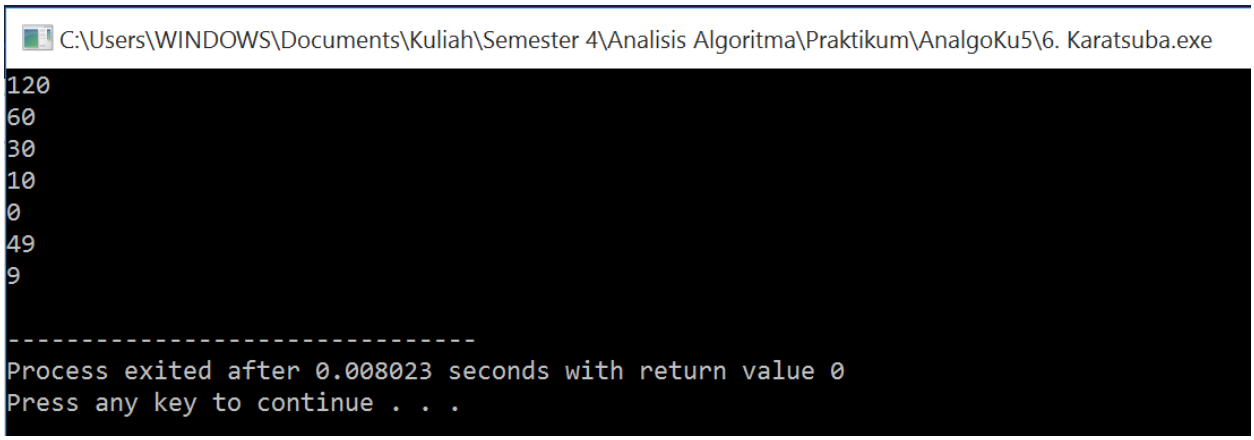

```
    for (int i = length-1 ; i >= 0 ; i--)
    {
            int firstBit = first.at(i) - '0';
            int secondBit = second.at(i) - '0';

            // boolean expression for sum of 3 bits
            int sum = (firstBit ^ secondBit ^ carry)+'0';

            result = (char)sum + result;

            // boolean expression for 3-bit addition
            carry = (firstBit&secondBit) | (secondBit&carry) | (firstBit&carry);
    }

    // if overflow, then add a leading 1
    if (carry) result = '1' + result;

    return result;
}

// A utility function to multiply single bits of strings a and b
int multiplyiSingleBit(string a, string b)
{ return (a[0] - '0')*(b[0] - '0'); }

// The main function that multiplies two bit strings X and Y and returns
// result as long integer
long int multiply(string X, string Y)
{
    // Find the maximum of lengths of x and Y and make length
    // of smaller string same as that of larger string
    int n = makeEqualLength(X, Y);

    // Base cases
    if (n == 0) return 0;
    if (n == 1) return multiplyiSingleBit(X, Y);

    int fh = n/2; // First half of string, floor(n/2)
    int sh = (n-fh); // Second half of string, ceil(n/2)

    // Find the first half and second half of first string.
    // Refer http://goo.gl/lLmgn for substr method
    string Xl = X.substr(0, fh);
    string Xr = X.substr(fh, sh);

    // Find the first half and second half of second string
    string Yl = Y.substr(0, fh);
    string Yr = Y.substr(fh, sh);

    // Recursively calculate the three products of inputs of size n/2
```

```
    long int P1 = multiply(Xl, Yl);
    long int P2 = multiply(Xr, Yr);
    long int P3 = multiply(addBitStrings(Xl, Xr), addBitStrings(Yl, Yr));

    // Combine the three products to get the final result.
    return P1*(1<<(2*sh)) + (P3 - P1 - P2)*(1<<sh) + P2;
}

// Driver program to test aboev functions
int main()
{
    printf ("%ld\n", multiply("1100", "1010"));
    printf ("%ld\n", multiply("110", "1010"));
    printf ("%ld\n", multiply("11", "1010"));
    printf ("%ld\n", multiply("1", "1010"));
    printf ("%ld\n", multiply("0", "1010"));
    printf ("%ld\n", multiply("111", "111"));
    printf ("%ld\n", multiply("11", "11"));
}
```

Screenshot:



b. Rekurensi dari algoritma tersebut adalah T (n) = 3T (n / 2) + O (n), dan selesaikan rekurensinya menggunakan metode substitusi untuk membuktikan bahwa algoritma tersebut memiliki Big-O (n lg n)

- Let's try divide and conquer.
  - Divide each number into two halves.
    - $x = x_H r^{n/2} + x_L$
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  - Then:

$$xy = (x_H r^{n/2} + x_L)\, y_H r^{n/2} + y_L$$
$$= x_H y_H r^n + (x_H y_L + x_L y_H) r^{n/2} + x_L y_L$$

  - Runtime?
    - T(n) = 4 T(n/2) + O(n)
    - T(n) = O(n^2)
- Instead of 4 subproblems, we only need 3 (with the help of clever insight).
- Three subproblems:
  - $a = x_H y_H$
  - $d = x_L y_L$
  - $e = (x_H + x_L)(y_H + y_L) - a - d$
- Then $xy = a r^n + e r^{n/2} + d$
- T(n) = 3 T(n/2) + O(n)
- $T(n) = O(n^{\log 3}) = O(n^{1.584\ldots})$

7. Permasalahan Tata Letak Keramik Lantai

a. Buatlah program untuk menyelesaikan problem tilling menggunakan algoritma divide & conquer yang diberikan. Gunakan bahasa C++

```
/*
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*/

// C++ implementation to count number of ways to
// tile a floor of size n x m using 1 x m tiles
#include <bits/stdc++.h>

using namespace std;

// function to count the total number of ways
int countWays(int n, int m)
{

    // table to store values
    // of subproblems
    int count[n + 1];
    count[0] = 0;

    // Fill the table upto value n
    for (int i = 1; i <= n; i++) {
        // recurrence relation
        if (i > m)
            count[i] = count[i - 1] + count[i - m];

        // base cases
        else if (i < m)
            count[i] = 1;

        // i = = m
        else
            count[i] = 2;
    }

    // required number of ways
    return count[n];
}

// Driver program to test above
int main()
{
    int n = 2, m = 2;
    cout << "Number of ways = "
         << countWays(n, m);
```
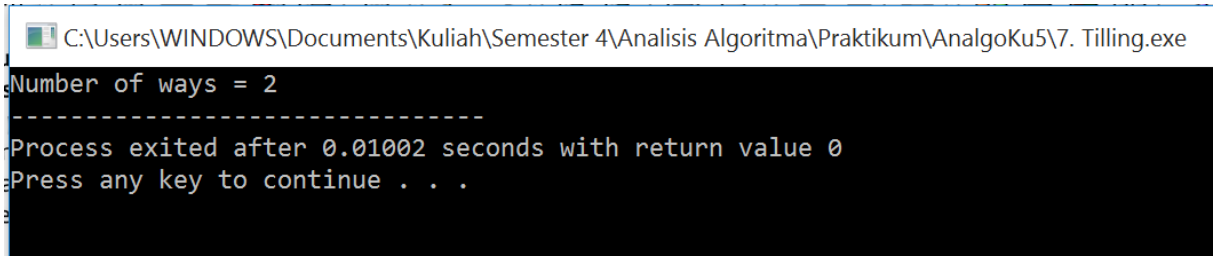

```
    return 0;
}
```

Screenshot:



b. Relasi rekurensi untuk algoritma rekursif diatas dapat ditulis seperti dibawah ini. C adalah konstanta. T(n)=4T (n/2) + C. Selesaikan rekurensi tersebut dengan Metode Master

Kompleksitas Waktu:

Relasi perulangan untuk algoritma rekursif di atas dapat ditulis seperti di bawah ini. C adalah konstanta.

T (n) = 4T (n / 2) + C

Rekursi di atas dapat diselesaikan dengan menggunakan Metode Master dan kompleksitas waktu adalah O (n2)

Bagaimana cara kerjanya?

Pengerjaan algoritma Divide and Conquer dapat dibuktikan menggunakan Mathematical Induction. Biarkan kuadrat input berukuran 2k x 2k di mana k> = 1.

Kasus Dasar: Kita tahu bahwa masalahnya dapat diselesaikan untuk k = 1. Kami memiliki 2 x 2 persegi dengan satu sel hilang.

Hipotesis Induksi: Biarkan masalah dapat diselesaikan untuk k-1.

Sekarang perlu dibuktikan untuk membuktikan bahwa masalah dapat diselesaikan untuk k jika dapat diselesaikan untuk k-1. Untuk k, ditempatkan ubin berbentuk L di tengah dan memiliki empat subsqure dengan dimensi 2k-1 x 2k-1 seperti yang ditunjukkan pada gambar 2 di atas. Jadi jika dapat menyelesaikan 4 subskuares, dapat menyelesaikan kuadrat lengkap.