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Digital Signature Algorithm (DSA)

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Example 1 for Digital Signature Algorithm (DSA): DSA is a United States Federal Government standard for digital signatures and was proposed by NIST.

STEP 1: Key generation

- Choose a prime number q , which is called the prime divisor. Suppose $q = 3$
- Choose another prime number p , such that $p-1 \bmod q = 0$. p is called the prime modulus. $p = 7$ [Choose an N -bit prime q . [Choose an L -bit prime p such that $p - 1$ is a multiple of q]
- Choose an integer randomly from and Compute $g = h^{(p-1)/q} \bmod p$ as if g is not equal to 2. Say $h=2$, Therefore, $g = h^{(7-1)/2} = 2^2 = 4$. The algorithm parameters are (p, q, g) . These may be shared between different users of the system.
- Choose an integer, such that $0 < x < q$. Choose an integer randomly from . Compute $y = g^x \bmod p$. Let's pick $x=5$. So, $y = 4^5 \bmod 7 = 1024 \bmod 7 = 2$.
- Package the public key as $\{p, q, g, y\} = \{7, 3, 4, 2\}$.
- Package the private key as $\{p, q, g, x\} = \{7, 3, 4, 5\}$.

STEP 2: Signature generation using private keys $\{p, q, g, x\} = \{7, 3, 4, 5\}$.

- Generate the message digest h , using a hash algorithm like SHA1. Let's say Hash value of a message digest $H(m) = 3$
- Choose an integer randomly from . \therefore , pick $k = 2$.
- Compute $r = \left(g^k \bmod p \right) \bmod q$. In the unlikely case that $r=0$, start again with a different random . $\therefore r = (4^2 \bmod 7) \bmod 3 = 2 \bmod 3 = 2$.
- Compute $s = \left(k^{-1} \left(H(m) + x \right) \right) \bmod q$. In the unlikely case that , start again with a different random . $\therefore s = \{2^{-1}(3 + 5x2)\} \bmod 3 = 2(3+10) \bmod 3 = 26 \bmod 3 = 2$.
- Package the digital signature as $\{r, s\} = \{2, 2\}$.

STEP 3: Verification: using the public key $\{p, q, g, y\} = \{7, 3, 4, 2\}$.

- One can verify that a signature $\{r, s\} = (2, 2)$ is a valid signature for a message as follows:

- Generate the message digest h , using the same hash algorithm. Suppose we are given the same message, so it will produce the same digest, $H(m) = 3$.

$$w = a^{-1} \cdot H(m) \bmod q$$

- Compute $w = 2^{-1} \bmod 3 = 2$

$$u_1 = H(m) \cdot w \bmod q$$

$$u_2 = r \cdot w \bmod q$$

- Compute $u_1 = 3 \times 2 \bmod 3 = 0$ and Compute $u_2 = 2 \times 2 \bmod 3 = 1$.

$$v = (u_1 \cdot g + u_2 \cdot r) \bmod p$$

- Compute $v = (4^0 \times 2^1 \bmod 7) \bmod 3 = (1 \times 2 \bmod 7) \bmod 3 = 2$ and the signature is valid if and only if $v = 2$.

Pseudocode:

Example 2:

Key Gen:


- Let a Prime divisor, $q = 11$ and it should multiple of $(p-1)$, lets calculate $p = 2 \times q + 1 = 23$.
- Pick integer h randomly from 2 to 21 , Let's take $h = 2$, and Compute $g = h^{(p-1)/q} \bmod p$. h should within 2 to 21 , Let's take $h = 2$, $\therefore g = 2^{22/11} \bmod 23 = 4$.
- Take an integer x randomly from 2 to 21 , Compute $y = g^x \bmod p$. Let's pick $x = 7$.
So, $y = 4^7 \bmod 23 = 16384 \bmod 23 = 8$.
- \therefore public key as $\{p, q, g, y\} = \{23, 11, 4, 8\}$.
- \therefore private key as $\{p, q, g, x\} = \{23, 11, 4, 7\}$.

Sign:

- Let's say Hash value of a message digest $H(m) = 3$. pick $k = 5$.
 $r = \left((g^k \bmod p) \right) \bmod q$
- $\therefore r = (4^5 \bmod 23) \bmod 11 = 12 \bmod 11 = 1$.
 $s = \left((k^{-1} \cdot H(m) + x) \cdot r^{-1} \right) \bmod q$
- $\therefore s = \{5^{-1}(3 + 7 \times 1)\} \bmod 11 = 9(3+7) \bmod 11 = 90 \bmod 11 = 2$.
- Package the digital signature as $\{r, s\} = \{1, 2\}$.


Verify:

- $\therefore H(m) = 3$ and given signature $(r, s) = (1, 2)$
 $w = a^{-1} \cdot H(m) \bmod q$
- $\therefore w = 2^{-1} \bmod 11 = 6$


$$\dots$$

$$\dots$$

- $\therefore u_1 = 3 \times 6 \bmod 11 = 7$ and Compute
 $\therefore u_2 = 1 \times 6 \bmod 11 = 6.$


$$\dots$$

- $\therefore v = (4^7 \times 8^6 \bmod 23) \bmod 11 = (16384 \times 262144 \bmod 23) \bmod 11 = (8 \times 13 \bmod 23) \bmod 11 = (104 \bmod 23) \bmod 11 = 12 \bmod 11 = 1$ and the signature is valid.
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