

# Optional Tutorial: RNN Maths Simplified

Before we jump into RNN math, it is important to look at the previous layer that is involved before the RNN layer.

Let's take an example:

- "AI Planet Bootcamps are free and available to all."

## Step 1: Tokenization

During this step, we first, make each unique word as a single token:

**['AI','Planet','Bootcamps','are','free','and','available','to','all']**

Once this is done we assign each token with the index i.e., each word is now a Vocabulary. The vocab size is very important for embedding layer

```
1  {  
2      "AI": 1  
3      "Planet": 2  
4      "Bootcamps": 3  
5      "are": 4  
6      "free": 5  
7      "and": 6  
8      "available": 7  
9      "to": 8  
10     "all": 9  
11 }
```



So, our sentence becomes: **[1, 2, 3, 4, 5, 6, 7, 8, 9]**.

## Step 2: Embedding layer

In NLP, we usually convert words into dense vectors that capture their semantic meaning. This is like giving each word a unique "signature".

Assuming an embedding size of 3 (just for simplicity), the word "**AI**" might be represented as

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Got it



In the real-time example, **the Embedding size varies from 128 to 1024.**

Now the entire '**AI Planet bootcamps are free and available to all**' sentence becomes:



```
1  [[0.1, 0.2, 0.3],
2   [0.4, 0.5, 0.6],
3   [0.7, 0.8, 0.9],
4   ...
5   [0.7, 0.8, 0.9],
6   [0.4, 0.5, 0.6]]
```

## Step 3: RNN Layer – Math Simplified

Understanding the Notations involved in RNN:

- **t**: denotes the time step
- **h(t)**: denotes the hidden state to current time step
- **h(t-1)**: denotes the previous hidden state at time step **t-1**
- **x(t)**: denotes the input at time step t. This is the input that is received from embedding layer. In the below example observe carefully, the shape matches with that of embedding
- **tanh**: Is the activation function that is used to calculate the next hidden state value i.e., **h(t)** in dependent to **h(t-1)**

The RNN processes the sequences step by step, maintaining an internal state that captures information from previous time steps.

We need to define the hidden state size [hyperparameter]. Assume our RNN has a hidden state size of 2.

Let's simplify the RNN math for one time step.

Given an input at time step t, denoted as **x(t)**, and the previous hidden state at time step **t-1**, denoted as **h(t-1)**, the calculations in the RNN are as follows:

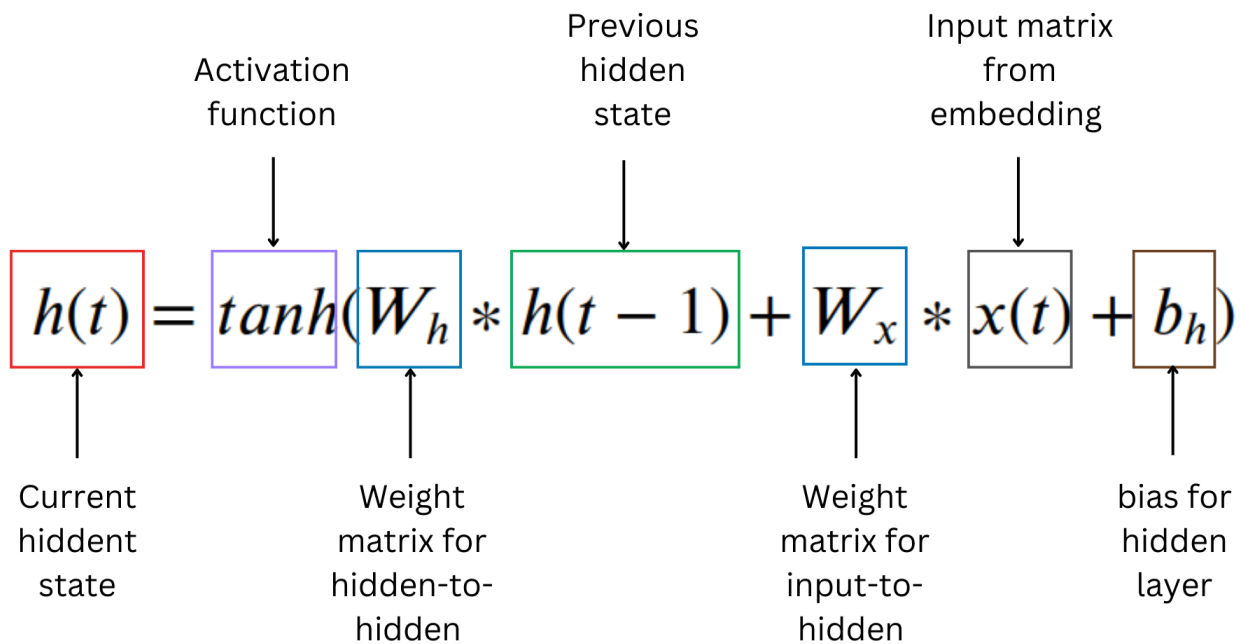
### 1. Calculate the new hidden state and output:

$$h(t) = \tanh(W_h * h(t-1) + W_x * x(t) + b_h)$$

Note: When we calculate **h(1)**, the initial hidden state value **h(0)** is filled with zeroes.

$$y_t = \text{sigmoid}(W_y \cdot h(t) + b_y)$$





## 2. Initial hidden state ( $h(0)$ ):

```
1 [0, 0]
```



### Sample Python code snippet

```
1 ht = np.zeros((self.hidden_size,1))
```



## 3. Initialize weight matrix randomly

a)  $W_h$ : weight matrix that defines the transformation applied to the previous hidden state  $h(t-1)$  at each time step.

```
1 [[0.1, 0.2],
2  [0.3, 0.4]]
```



b)  $W_x$ : Weight matrix that defines the transformation applied to the input data at each time step.

```
1 [[0.5, 0.6, 0.7],
2  [0.8, 0.9, 1.0]]
```





```
1  [[0.2, 0.3],
2  [0.4, 0.5]]
```

Note: A kind reminder, the weights are usually **initialized randomly**. Also notice the size of the weight matrix is 2x3 that is corresponding to **hidden\_size x embedding\_size**

#### Sample Python Code snippet



```
1  Wx = np.random.randn(self.hidden_size, self.input_size.shape[2])
2  Wh = np.random.randn(self.hidden_size, self.hidden_size)
3  Wy = np.random.randn(self.output_size.shape[1], self.hidden_size)
```

#### Bias

- Bias for hidden state ( $b_h$ ):



```
1  [0.1, 0.2]
```

- Bias for output ( $b_y$ ):



```
1  [0.3, 0.4]
```

## Step 3. Calculate the new hidden state $h(1)$ and the output $y(1)$ at the first time step

We have total 9 vocabulary in the Embeddings, let's just take the first word ( $x(1)$ ):



```
1  [0.1, 0.2, 0.3]
```

Now let's Calculate the weighted sum of the previous hidden state ( $h(0)$ ) and the input ( $x(1)$ ) along with the bias for the hidden state: ( $b(h)$ ):



```
1  weighted_sum = W_h * h(t-1) + W_x * x(t) + b_h
2
3  weighted_sum = [[0.1, 0.2], [0.3, 0.4]] * [0, 0] +
4                  [[0.5, 0.6, 0.7],
5                  [0.8, 0.9, 1.0]] * [0.1, 0.2, 0.3] +
```

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```

8      = [0.0, 0.0] + [0.38, 0.56] + [0.1, 0.2]
9      = [0.48, 0.76]

```

### Let's apply tanh function and find out h(1)

```

1  h(1) = tanh([0.21, 0.46])
2      ≈ [0.44624361, 0.64107696]

```



### Let's apply sigmoid function and find out y(1)

- $y(t) = \text{sigmoid}(W_y \cdot h(t) + b_y)$

Calculate:

```

1  y(1) = sigmoid([0.2, 0.3], [0.4, 0.5]) * [0.206, 0.440] + [0.3, 0.4]
2      = sigmoid([0.216, 0.355] + [0.3, 0.4])
3      = sigmoid([0.516, 0.755])
4      ≈ [0.61614087, 0.66871967]

```



## Verification via code

```

1  import numpy as np
2
3  W_hXh_t_1 = np.dot(np.array([0.1, 0.2], [0.3, 0.4]), np.array([0, 0]))

```



```

1  W_xXx_t = np.dot(np.array([0.5, 0.6, 0.7],
2      [0.8, 0.9, 1.0]), np.array([0.1, 0.2, 0.3]))

```



```

1  b_h = np.array([0.1, 0.2])

```



```

1  weighted_sum = W_hXh_t_1 + W_xXx_t + b_h
2  weighted_sum

```



```

1  array([0.48, 0.76])

```



```

1  h_1 = np.tanh(weighted_sum)

```



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```
1 array([0.44624361, 0.64107696])
```



```
1 def sigmoid(x):  
2     return 1 / (1 + np.exp(-x))
```



```
1 y_1 = sigmoid(  
2     np.dot(  
3         np.array([[0.2, 0.3], [0.4, 0.5]]),  
4         np.array([0.206, 0.440])  
5     )  
6     + np.array([0.3, 0.4])  
7 )  
8 y_1
```



```
1 array([0.61614087, 0.66871967])
```

## Step 5. Loss Function:

The loss function measures the difference between the predicted output and the actual target output. In the context of sequence tasks like language modeling, sentiment analysis, or sequence-to-sequence tasks, a common choice for the loss function is the categorical cross-entropy loss.

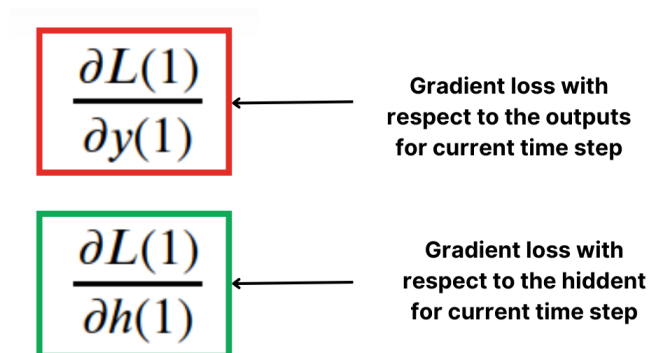
**$L(t) = - \sum(y_{\text{actual}}(t) * \log(y(t)))$**

Here let's say our use case is sentiment analysis, considering our example "AI Planet Bootcamps are free and available to all." is a positive sentiment thus:

**$y_{\text{actual}} = 1$**

## Step 6. Backwardpropagation through time (BPTT):





- First we calculate forward pass as above
- Then we calculate the loss for each time step using the predicted outputs ( $y(t)$ ) and the true target outputs ( $y(\text{actual})$ ).
- Begin the backpropagation process by computing the gradient (partial derivations) of the loss with respect to the outputs and hidden states at the last time step ( $T$ ).
- Iterate backward through time steps from  $T$  down to 1. For each time step  $t$ , calculate the gradients of the loss with respect to the hidden state and input at time step  $t$ , and update the gradients with respect to the parameters ( $W_x$ ,  $W_h$ ,  $W_y$ , biases, etc.).
- Use **learning rate** to add the step size to update the weights ( $W_x$ ,  $W_h$ ,  $W_y$ ).
- Continue iterating over mini-batches of sequences and performing forward and backward passes until convergence i.e., end of time step or a specified number of epochs.

### Sample Python code snippet

```

1  dWy = np.dot(dyt, self.hidden_states[-1].T)
2  dht = np.dot(dyt, self.Wy).T
3  dWx = np.zeros(self.Wx.shape)
4  dWh = np.zeros(self.Wh.shape)
5
6  for step in reversed(range(n)):

```



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```
9      dWh += np.dot(temp, self.hidden_states[step].T)
10     dht = np.dot(self.Wh, temp)
11
12     #gradient clipping: this is used to tackle Exploding Gradient problem
13     dWy = np.clip(dWy, -1, 1)
14     dWx = np.clip(dWx, -1, 1)
15     dWh = np.clip(dWh, -1, 1)
16     #weights updation using learning rate(step size)
17     self.Wy -= self.lr * dWy
18     self.Wx -= self.lr * dWx
19     self.Wh -= self.lr * dWh
```

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