MTH 309 - Activity 8
Vector Geometry

1. Fix a vector v in $\mathbb{R}^2$, and consider the function

$$\text{proj}_v \colon \mathbb{R}^2 \to \mathbb{R}^2$$

where $\text{proj}_v(x)$ is the projection of x onto v.

   i. Use what we have discussed so far in class to write a formula for $\text{proj}_v(x)$ in terms of the vectors x and v.

   ii. Keeping in mind that the vector v is fixed, is proj_v a linear transformation? If yes, find its matrix representation. If no, provide a counterexample.

   iii. What happens if we apply the projection twice to the same vector? That is, what is $\text{proj}_v(\text{proj}_v(x))$?

2. Now consider the linear transformation $\text{ref}_v(x)$ that reflects its input vector x across the line spanned by the fixed vector v.

   i. Write a formula for $\text{ref}_v(x)$ in terms of the vectors x and v.

   ii. What is the matrix representation of ref_v?

   iii. What happens if we apply the reflection twice?

3. What are the eigenvalues of the projection matrix? What about the reflection matrix?

4. A matrix Q is said to be *orthogonal* if $Q^T Q = I$. Orthogonal matrices have the property that their columns form an orthonormal basis of the space. These matrices are extremely important for computation because finding its inverse boils down to taking the transpose.

   i. Are either the projection or reflection matrices orthogonal?

   ii. Are any of the matrices from previous activities orthogonal?