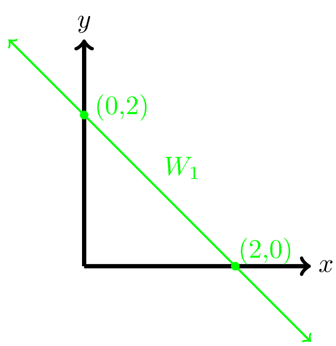
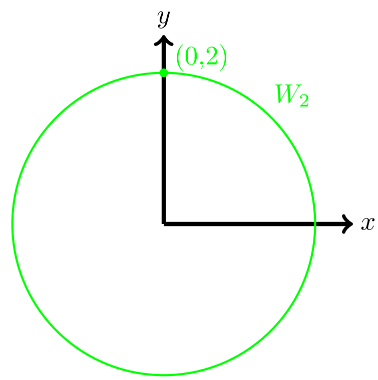
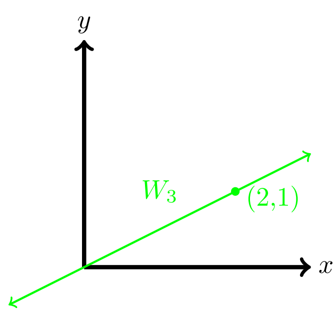
MTH 309 - Activity 2
Vector Spaces

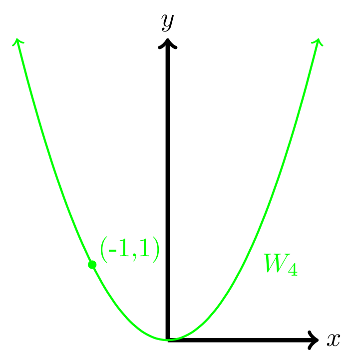
A **vector space** is a set of vectors V, together with rules for how to add vectors and how to scale them. There are also many additional rules for how these two operations should behave (commutivity, associativity, distribution, etc.). A **subspace** W of the vector space V is a smaller vector space sitting inside the larger V.

1. Consider the vector space $V = \mathbb{R}^2$ of points in the plane. Which of the following pictures represent subspaces of $\mathbb{R}^2$.









2. Based on what you know so far, what can you say about which sets are subspaces and which sets are not?

3. Again, consider $V = \mathbb{R}^2$. Which of the following sets of vectors is a subspace?

   a. $U_1 = \{\dots, (-2,-2), (-1,-1), (0,0), (1,1), (2,2), \dots\}$
   b. $U_2 = \{(3s, -4s) \mid s \in \mathbb{R}\}$
   c. $U_3 = \{(1,2)t + (2,0) \mid t \in \mathbb{R}\}$
   d. $U_4 = \{(x,y) \in \mathbb{R}^2 \mid 5y = 3xy\}$
   e. $U_5 = \{(x,y) \in \mathbb{R}^2 \mid x - 2y = 0\}$

4. Based on your findings from the first two problems, conjecture a characterization for subspaces of $\mathbb{R}^2$.

5. How might your characterization extend to $\mathbb{R}^3$.

6. Use your characterization of subspaces for $\mathbb{R}^3$ to decide which of the following are subspaces.

   a. $V_1 = \{(3s - 2t, 4t, 5s + t) \mid s, t \in \mathbb{R}\}$
   b. $V_2 = \{(6st, s, t) \mid s, t \in \mathbb{R}\}$
   c. $V_3 = \{(t - 6, s - t, t + 2) \mid s, t \in \mathbb{R}\}$