MTH 309 - Activity 4
Matrix Algebra

1. Consider the linear transformation $R_\theta \colon \mathbb{R}^2 \to \mathbb{R}^2$ that rotates the space by θ radians counterclockwise.
   (a) What is the matrix representation of R_θ?
   (b) Describe in words what the effect of the composition $R_\theta \circ R_\theta$ is on the space.
   (c) What is the matrix representation of the composition $R_\theta \circ R_\theta$ based on your answer to part (b)?
   (d) What is the matrix representation of the composition $R_\theta \circ R_\theta$ based on the definition?
   (e) What can you deduce by comparing these two matrix representations?
2. Redo parts (b) - (e) from problem 1 for the composition $R_\theta \circ R_\phi$.
3. Consider the inverse of R_θ.
   (a) Describe in words what the inverse of R_θ does to the space.
   (b) What is the matrix representation of R_θ^{-1} based on your description from part (a)?
   (c) Describe in words what the effect of the composition $R_\theta^{-1} \circ R_\theta$ is on the space.
   (d) What is the matrix representation of the composition $R_\theta^{-1} \circ R_\theta$ based on your description in part (c)?
   (e) What is the matrix representation of the composition $R_\theta^{-1} \circ R_\theta$ based on the definition?
   (f) Write an equation that relates the matrix representatiuons of R_θ, R_θ^{-1}, and the composition $R_\theta^{-1} \circ R_\theta$.
4. Generalize problem 3 part (f) to a generic invertible linear transformation $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ that has matrix representation A.
5. How might you compute the inverse of a matrix? Below are some examples to try with.

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix},$$
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 10 \end{bmatrix}$$