

MTH 309 - Activity 10
Eigenvalues and Eigenvectors

1. Go to <https://www.geogebra.org/m/mdvN0HTt>. There you will find an interactable graph containing two vectors, a unit vector (length 1) input that you can drag around, and the corresponding output vector for the linear transformation represented by the matrix.
 - (a) By dragging the input vector around the unit circle, at what point do the two vectors line up? Point in opposite directions?
 - (b) When two vectors are pointed in the same direction, what is the relationship between them? When they are pointed in opposite directions? Write your answer in the form of an equation.
 - (c) For each answer to part (a), describe the relationship between the input vector and output vector. Write an equation.
 - (d) Combine your observations to write an equation that describes the relationship between the input vector and the output vector when they are pointed in the same direction.
2. Change the matrix to each of the following and write down your observations about these special input vectors.
 - i. $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$
 - ii. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$
 - iii. $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$
3. We call these special inputs *eigenvectors*. What can you say about the eigenvectors of the following linear transformations.
 - Reflection across the x -axis.
 - Reflection across the line $y = 3x$.
 - Projection onto the y -axis.
 - Projection onto the line $y = -2x$.
 - Rotation ccw by $\frac{\pi}{2}$ radians.
 - Rotation ccw by $\frac{37\pi}{42}$ radians.