Proficiency Exam 1 - Systems of Equations

You will have 30 minutes to complete the exam. You may use a calculator, but you must show all steps done to get full credit for completing the problem. This means that if you use your calculator for anything other than arithmetic, you must indicate on your test paper what you did on the calculator.

1. Consider a degree 4 polynomial $p(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 + p_4 x^4$ which passes through the points (-2,-1), (-1,-8), (0,0), (1,4), and (2,0). Write, but **do not solve**, a system of linear equations, in standard form, that can be solved to find the coefficients of $p(x)$.

   Answer: For each point, plug it in to get an equation.

$$\begin{aligned} p_0 - 2p_1 + 4p_2 - 8p_3 + 16p_4 &= -1 \\ p_0 - p_1 + p_2 - p_3 + p_4 &= -8 \\ p_0 &= 0 \\ p_0 + p_1 + p_2 + p_3 + p_4 &= 4 \\ p_0 + 2p_1 + 4p_2 + 8p_3 + 16p_4 &= 0 \end{aligned}$$

2. Solve the system of equations below. Make sure at each step you indicate what you are doing.

$$\begin{aligned} 2x + 2y + 4z &= 10 \\ 2x - y + z &= 1 \\ -3x + y + 3z &= -6 \end{aligned}$$

   Answer: Use elimination to get the RREF and read off the answer.

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 10 \\ 2 & -1 & 1 & 1 \\ -3 & 1 & 3 & -6 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 2 & -1 & 1 & 1 \\ -3 & 1 & 3 & -6 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & -3 & -3 & -9 \\ 0 & 4 & 9 & 9 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 4 & 9 & 9 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 5 & -3 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & -0.6 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2.6 \\ 0 & 1 & 0 & 3.6 \\ 0 & 0 & 1 & -0.6 \end{array}\right]$$

   Thus $x = 2.6$, $y = 3.6$, and $z = -0.6$.

3. (TRUE or FALSE) Consider the following statement and decide if it is true or false. If it is true, provide reasoning. If it is false, provide a counterexample.

   "If the reduced row-echelon form of the **coefficient matrix** has a pivot in every row, then the system has a solution."

   Answer: TRUE. Having a pivot in every row of the *coefficient* matrix means that there cannot be a pivot in the solution column of the *system* matrix. Since the system matrix cannot have a pivot in the solution column, the system is consistent for any choice of right hand side vector.

4. Identify the pivot columns of the following matrix. (Hint: reduce to row echelon form.)

$$A = \begin{bmatrix} 3 & 5 & 3 & -1 \\ -2 & 0 & -4 & 6 \\ -1 & 3 & 1 & 3 \end{bmatrix}.$$

   Answer: Find the RREF. The pivot columns of the matrix will be in the same position as those of the RREF.

$$\begin{bmatrix} 3 & 5 & 3 & -1 \\ -2 & 0 & -4 & 6 \\ -1 & 3 & 1 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

   So the pivots are in columns 1, 2, and 3.