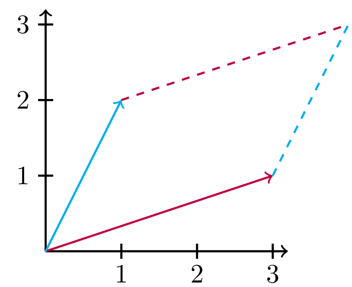
MTH 309 - Activity 9
Determinants

1. For each of the following pairs of vectors, graph the parallelogram they span and determine its area.

   i. $(4, 0), (0, 5)$
   ii. $(4, 0), (2, 5)$
   iii. $(4, 1), (2, 5)$
   iv. $(3, 4), (12, 8)$
   v. $(a, b), (c, d)$

   Example: Parallelogram spanned by (3,1) and (1,2).



2. Write a general rule for computing the area of the parallelogram spanned by two vectors in $(R)^2$.

3. Now consider the linear transformation $T\colon (R)^2 \to (R)^2$ defined by

$$T(\mathbf{x}) = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \mathbf{x}.$$

   (a) What happens to the area of each parallelogram from problem 1 when the vectors are transformed by T?
   (b) What about for the transformation given by $S(\mathbf{x}) = \begin{bmatrix} 5 & 6 \\ -2 & -3 \end{bmatrix} \mathbf{x}$?
   (c) What about for the transformation given by $R(\mathbf{x}) = \begin{bmatrix} 4 & 6 \\ -2 & -3 \end{bmatrix} \mathbf{x}$?
   (d) And for the arbitrary transformation $Q(\mathbf{x}) = A\mathbf{x}$?

4. Write a general rule that relates the area of the parallelogram after transformation to the area of the parallelogram before transformation.

5. Use your general rule, extended to $\mathbb{R}^3$ to find the areas of the following parallelopiped (3D parallelogram).

   i. Spanned by $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} \right\}$.
   ii. The unit cube (spanned by $\{e_1, e_2, e_3\}$) after being transformed by $P(\mathbf{x}) = \begin{bmatrix} 1 & 2 & 3 \\ 5 & -1 & 0 \\ 0 & -2 & -12 \end{bmatrix}$.
   iii. The parallelopiped from part i. after being transformed by the transformation from part ii.