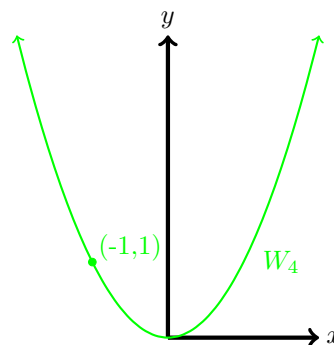
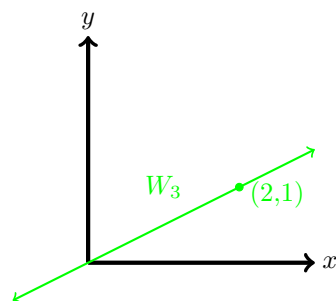
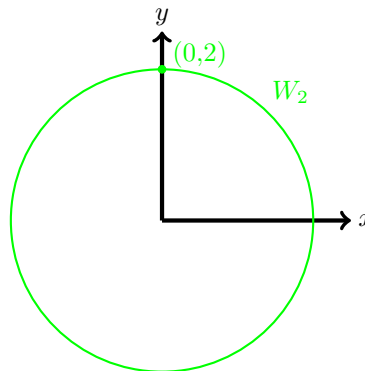
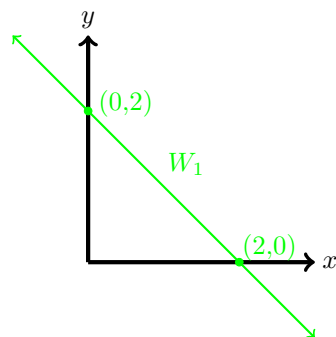


# MTH 309 - Activity 2

## Vector Spaces

A **vector space** is a set of vectors  $V$ , together with rules for how to add vectors and how to scale them. There are also many additional rules for how these two operations should behave (commutivity, associativity, distribution, etc.). A **subspace**  $W$  of the vector space  $V$  is a smaller vector space sitting inside the larger  $V$ .

1. Consider the vector space  $V = \mathbb{R}^2$  of points in the plane. Which of the following pictures represent subspaces of  $\mathbb{R}^2$ .



2. Based on what you know so far, what can you say about which sets are subspaces and which sets are not?
3. Again, consider  $V = \mathbb{R}^2$ . Which of the following sets of vectors is a subspace?
  - a.  $U_1 = \{\dots, (-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2), \dots\}$
  - b.  $U_2 = \{(3s, -4s) \mid s \in \mathbb{R}\}$
  - c.  $U_3 = \{(1, 2)t + (2, 0) \mid t \in \mathbb{R}\}$
  - d.  $U_4 = \{(x, y) \in \mathbb{R}^2 \mid 5y = 3xy\}$
  - e.  $U_5 = \{(x, y) \in \mathbb{R}^2 \mid x - 2y = 0\}$
4. Based on your findings from the first two problems, conjecture a characterization for subspaces of  $\mathbb{R}^2$ .
5. How might your characterization extend to  $\mathbb{R}^3$ .
6. Use your characterization of subspaces for  $\mathbb{R}^3$  to decide which of the following are subspaces.
  - a.  $V_1 = \{(3s - 2t, 4t, 5s + t) \mid s, t \in \mathbb{R}\}$
  - b.  $V_2 = \{(6st, s, t) \mid s, t \in \mathbb{R}\}$
  - c.  $V_3 = \{(t - 6, s - t, t + 2) \mid s, t \in \mathbb{R}\}$