

Proficiency Exam 6 - Eigenvalues and Eigenvectors

You will have 30 minutes to complete the exam. You may use a calculator, but you must show all steps done to get full credit for completing the problem. This means that if you use your calculator for anything other than arithmetic, you must indicate on your test paper what you did on the calculator.

1. Compute the determinant of the following matrix.

$$\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 5 \\ -3 & 3 & -9 \end{bmatrix}$$

2. The matrix A given below has an eigenvalue of 3. Find a basis for the eigenspace (the subspace of all eigenvectors for the eigenvalue of 3).

$$A = \begin{bmatrix} 43 & -15 & -10 \\ -120 & 48 & 30 \\ 360 & -135 & -87 \end{bmatrix}$$

3. (TRUE or FALSE) Consider the statement and decide if it is true or false. If true, provide reasoning. If false, provide a counterexample.

“ For any 2×2 matrix, $\det(A)$ is equal to the product of the eigenvalues. ”

4. Find the general solution to the following system of LODEs.

$$\begin{aligned} \frac{dx}{dt} &= 10x(t) + 3y(t) \\ \frac{dy}{dt} &= -3x(t) - 5y(t) \end{aligned}$$