

Proficiency Exam 4 - Matrix Algebra

You will have 30 minutes to complete the exam. You may use a calculator, but you must show all steps done to get full credit for completing the problem. This means that if you use your calculator for anything other than arithmetic, you must indicate on your test paper what you did on the calculator.

1. Consider the network with adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

How many paths are there of length 3 from node 3 to node 4?

Answer:

$$A^2 = \begin{bmatrix} 3 & 0 & 2 & 1 & 1 \\ 0 & 2 & 0 & 1 & 2 \\ 2 & 0 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 & 3 \end{bmatrix}.$$

$$A^3 = \begin{bmatrix} 2 & 5 & 1 & 4 & 6 \\ 5 & 0 & 4 & 2 & 1 \\ 1 & 4 & 0 & 2 & 5 \\ 5 & 2 & 2 & 2 & 4 \\ 6 & 1 & 5 & 4 & 2 \end{bmatrix}.$$

So there are 2 paths of length 3 from node 3 to node 4.

2. Find the inverse of

$$B = \begin{bmatrix} 0 & 3 & 2 \\ -1 & 4 & 1 \\ 1 & -2 & 0 \end{bmatrix}$$

Answer:

$$RREF \left(\left[\begin{array}{ccc|ccc} 0 & 3 & 2 & 1 & 0 & 0 \\ -1 & 4 & 1 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{array} \right] \right) = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 4 & 5 \\ 0 & 1 & 0 & -1 & 2 & 2 \\ 0 & 0 & 1 & 2 & -3 & -3 \end{array} \right]$$

So

$$B^{-1} = \begin{bmatrix} -2 & 4 & 5 \\ -1 & 2 & 2 \\ 2 & -3 & -3 \end{bmatrix}$$

3. (TRUE or FALSE) Consider the statement and decide if it is true or false. If true, provide reasoning. If false, provide a counterexample.

“If A is invertible, then a system of equations whose coefficient matrix is A has a unique solution.”

Answer: True. Consider the system $Ax = b$ where b can be any vector. Since A is invertible, the unique solution is $x = A^{-1}b$ since there is only one inverse matrix.

4. For what values of t is the matrix below invertible?

$$\begin{bmatrix} 5 & t+8 \\ t-3 & 2t \end{bmatrix}$$

Answer:

$$\det(A) = 10t - (t+8)(t-3) = -(t^2 - 5t - 24) = (t-8)(t+3)$$

So t can be any real number except -3 or 8.