MTH 309 - Activity 12
More Vector Geometry

1. Fix a vector v in $\mathbb{R}^2$, and consider the function

$$\text{proj}_v : \mathbb{R}^2 \to \mathbb{R}^2$$

where $\text{proj}_v(x)$ is the projection of x onto v.

   (a) Draw a picture depicting the vectors v, x, and $\text{proj}_v(x)$.

   (b) What is the geometric relationship between x and $\text{proj}_v(x)$? Between v and $\text{proj}_v(x)$?

   (c) Write a formula for $\text{proj}_v(x)$ in terms of v and x.

   (d) Use $\text{proj}_v(x)$ to calculate a vector n that is perpendicular to v.

   (e) What is the relationship between $\text{span}\{v, x\}$ and $\text{span}\{v, n\}$?

   (f) Given a basis for a 2-dimensional subspace $\{v_1, v_2\}$, use projection to construct an orthonormal basis $\{b_1, b_2\}$ for the subspace.

2. Now consider the linear transformation $\text{ref}_v(x)$ that reflects its input vector x across the line spanned by the fixed vector v.

   (a) Write a formula for $\text{ref}_v(x)$ in terms of the vectors x and v.

   (b) What is the matrix representation of ref_v?

   (c) Determine the eigenvalues and eigenvectors of ref_v.