

MTH 309 - Activity 8  
Matrix Algebra

1. Consider the linear transformation  $R_\theta: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  that rotates the space by  $\theta$  radians counterclockwise.
  - (a) What is the matrix representation of  $R_\theta$ ?
  - (b) Describe in words what the effect of the composition  $R_\theta \circ R_\theta$  is on the space.
  - (c) What is the matrix representation of the composition  $R_\theta \circ R_\theta$  based on your answer to part (b)?
  - (d) What is the matrix representation of the composition  $R_\theta \circ R_\theta$  using matrix multiplication?
  - (e) What can you deduce by comparing these two matrix representations?
2. Redo parts (b) - (e) from problem 1 for the composition  $R_\theta \circ R_\phi$ .
3. Consider the inverse of  $R_\theta$ .
  - (a) Describe in words what the inverse of  $R_\theta$  does to the space.
  - (b) What is the matrix representation of  $R_\theta^{-1}$  based on your description from part (a)?
  - (c) Describe in words what the effect of the composition  $R_\theta^{-1} \circ R_\theta$  is on the space.
  - (d) What is the matrix representation of the composition  $R_\theta^{-1} \circ R_\theta$  based on your description in part (c)?
  - (e) Write an equation that relates the matrix representations of  $R_\theta$ ,  $R_\theta^{-1}$ , and the composition  $R_\theta^{-1} \circ R_\theta$ .
4. Generalize problem 3 part (e) to a generic invertible linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  that has matrix representation  $A$ .
5. How might you compute the inverse of a matrix? Below are some examples to try with.

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix},$$
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 10 \end{bmatrix}$$