Bayesian Learning

Lab 3

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Question 1

```
rainfall <- read.table("../data/rainfall.dat", header=F)
```

Prior

$$\mu \sim N(\mu_0, \tau_0^2)$$
$$\sigma^2 \sim \text{Inv}\text{–}\chi^2(\nu_0, \sigma_0^2)$$

Likelihood

$$\mathbf{y}|\mu, \sigma^2 \sim N(\mu, \sigma^2)$$

Posterior

$$\mu|\sigma^2, \mathbf{x} \sim N(\mu_n, \tau_n^2)$$
$$\sigma^2|\mu, \mathbf{x} \sim \text{Inv}\text{–}\chi^2\left(\nu_n, \frac{\nu_0\sigma_0^2 + \sum_{i=1}^{n}(x_i - \mu)^2}{n + \nu_0}\right)$$

where

$$\mu_n = \frac{\frac{1}{\tau_0^2}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}\mu_0 + \frac{\frac{n}{\sigma^2}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}\bar{x}$$
$$\frac{1}{\tau_n^2} = \frac{1}{\tau_0^2} + \frac{n}{\sigma^2}$$
$$\nu_n = \nu_0 + n$$

a)

```
library(geoR)

mun <- function(x, sigmasq, hyperparams){
    n <- length(x)
    denom <- ((1 / hyperparams$tausq0) + (n / sigmasq))
```

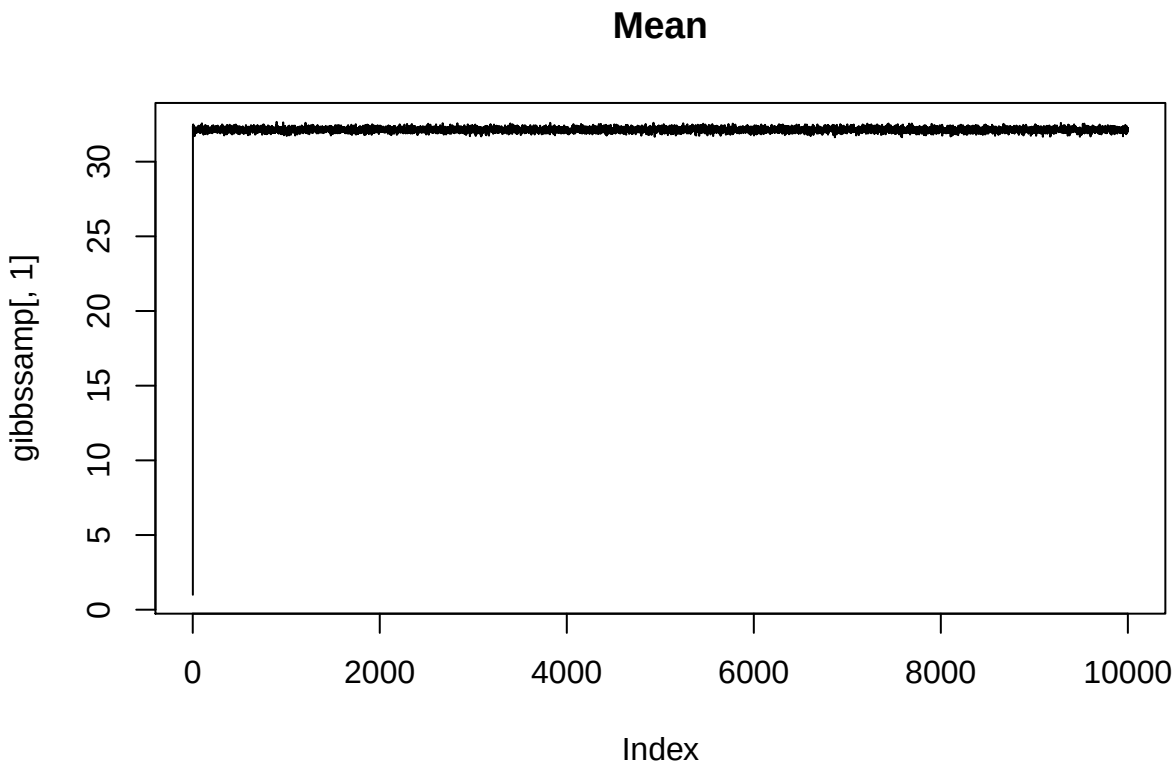
```
    pt1 <- ((1 / hyperparams$tausq0) / denom)  * hyperparams$mu0
    pt2 <- ((n / sigmasq) / denom) * mean(x)

    pt1 + pt2
}


taun <- function(x, sigmasq, hyperparams){
    hyperparams$tausq0 + (sigmasq / length(x))
    1 / hyperparams$tausq0
}

nun <- function(x, hyperparams){
    hyperparams$nu0 + length(x)
}


sigmasqn <- function(x, mu, hyperparams){
    (hyperparams$nu0 * hyperparams$sigmasq0 + sum((x - mu)^2 )) / (length(x) + hyperparams$nu0)
}


musampler <- function(x,sigmasq, hyperparams){
    mu <- mun(x, sigmasq, hyperparams)
    sigma <- sqrt(taun(x,sigmasq, hyperparams))
    rnorm(1, mu, sigma)
}

sigmasampler <- function(x, mu, hyperparams){
    scale  <- sigmasqn(x, mu, hyperparams)
    df <- nun(x, hyperparams)
    rinvchisq(1, df, scale)
}
```

Now we start start sampling

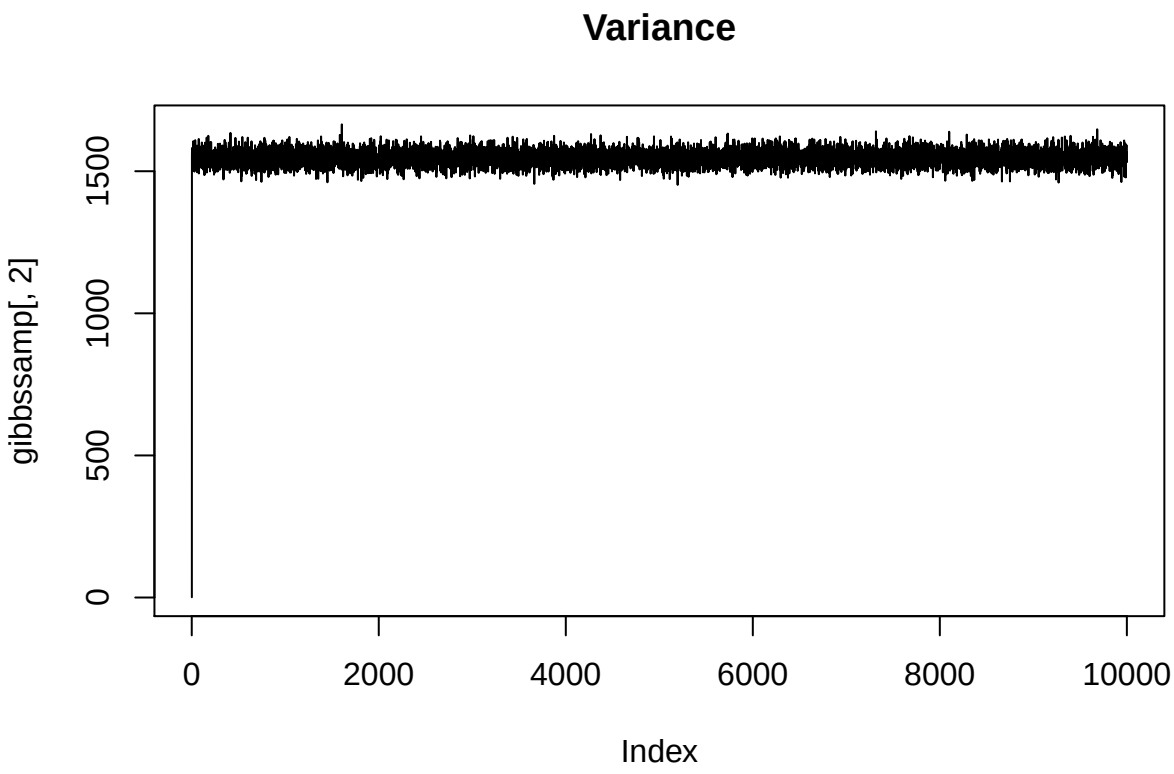
```
gibbs <- function(x, iter, init, hyperparams){
    samples <- matrix(NA, ncol = 2, nrow = iter + 1)
    samples[1,] <- init

    for (i in 2:(iter+1)){
        mu <- musampler(x, samples[i-1, 2], hyperparams)
        sigma <- sigmasampler(x, mu, hyperparams)
        samples[i,] <- c(mu, sigma)
    }

    samples
}
```

```
set.seed(123456)

hyperparams <- list(mu0=0, tausq0=50, nu0=1, sigmasq0=50)
gibbssamp <- gibbs(x=rainfall$V1, iter = 10000, init = c(1,1), hyperparams)
```

```r
plot(gibbssamp[,1], type = "l", main="Mean")
```




```r
plot(gibbssamp[,2], type = "l", main="Variance")
```



b)

```
nn <- function(I){
    n1 <- sum(I == 1)
    n2 <- sum(I == 2)
    list(n1 = n1,n2 = n2)
}

dd <- function(x, I) {
    d1 <- x[I == 1]
    d2 <- x[I == 2]
    list(d1=d1, d2=d2)
}

pisampler <- function(x, I, hyperparams){
    n <- nn(I)
    rbeta(1, shape1 = n$n1 + hyperparams$a1 , shape2 = n$n2 + hyperparams$a2)
}

sigmasq2sampler <- function(x, I, hyperparams){
    n <- nn(I)
    d <- dd(x, I)

    vnsn1 <-  hyperparams$nu0 * hyperparams$sigmasq0 +
```

```
        (n$n1 - 1) * var(d$d1) +
        (((1 / hyperparams$tausq0) * n$n1) / ((1 / hyperparams$tausq0) + n$n1)) *
        ((mean(d$d1) - hyperparams$mu0)^2)

    vnsn2 <-  hyperparams$nu0 * hyperparams$sigmasq0 +
        (n$n2 - 1) * var(d$d2) +
        (((1 / hyperparams$tausq0) * n$n2) / ((1 / hyperparams$tausq0) + n$n2)) *
        ((mean(d$d2) - hyperparams$mu0)^2)

    vn1 <-  hyperparams$nu0 + n$n1
    vn2 <-  hyperparams$nu0 + n$n2

    sn1 <- vnsn1 / vn1
    sn2 <- vnsn2 / vn2

    sigmasq1 <- rinvchisq(1, vn1, sn1)
    sigmasq2 <- rinvchisq(1, vn2, sn2)

    c(sigmasq1, sigmasq2)
}

mu2sampler <- function(x, I, sigmasq, hyperparams){
    d <- dd(x, I)

    tau1 <- sqrt(taun(d$d1, sigmasq[1], hyperparams))
    mun1 <- mun(d$d1, sigmasq[1], hyperparams)

    tau2 <- sqrt(taun(d$d2, sigmasq[2], hyperparams))
    mun2 <- mun(d$d2, sigmasq[2], hyperparams)

    mu1 <- rnorm(1, mun1, tau1)
    mu2 <- rnorm(1, mun2, tau2)

    c(mu1, mu2)
}

Isampler <- function(x, pi, mu, sigmasq){
    nom <- (1 - pi) * dnorm(x, mu[2], sqrt(sigmasq[2]))
    denom <- pi * dnorm(x, mu[1], sqrt(sigmasq[1])) + nom
    theta <- nom / denom
    rbinom(length(x), prob = theta, size = 1) + 1 # to get them into 1 and 2
}

ysampler <- function(pi, mu, sigmasq){
    pi * rnorm(1, mu[1], sqrt(sigmasq[1])) +
        (1 - pi) * rnorm(1, mu[2], sqrt(sigmasq[2]))
}
```

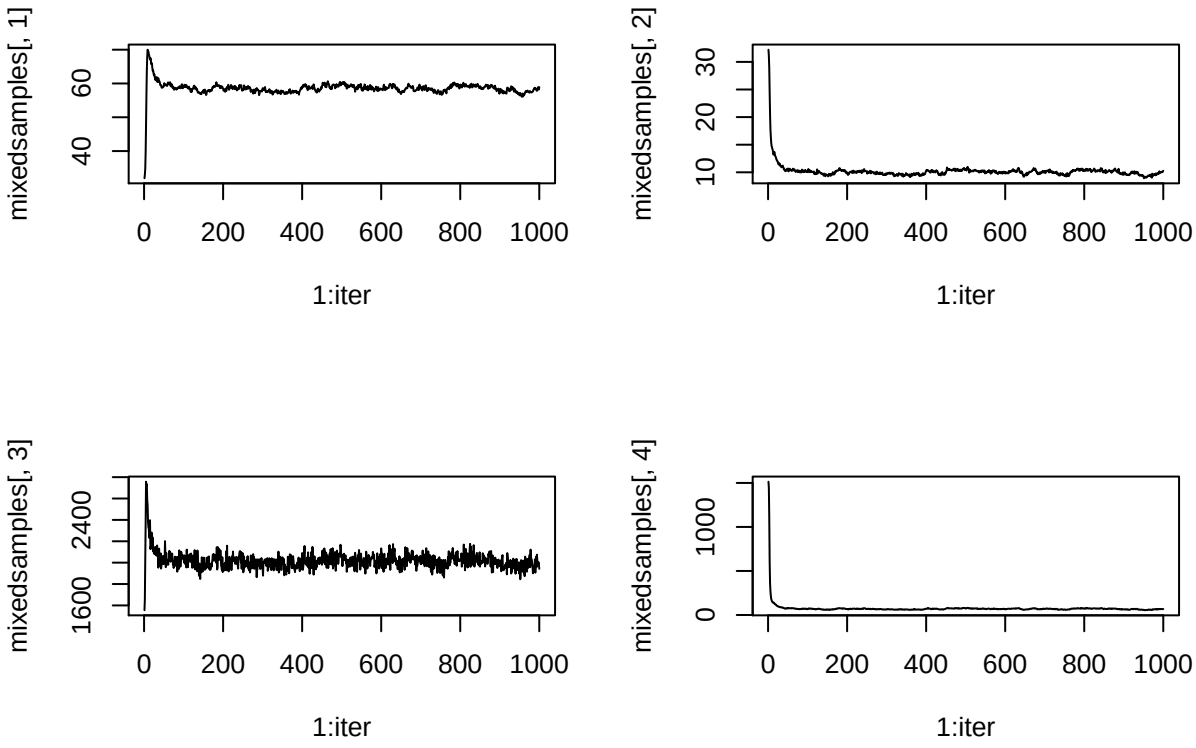
And now we sample.

```
mixedgibbs <- function(x, iter, init, hyperparams){
    samples <- matrix(NA, ncol = 4, nrow = iter)
    I <- init
```

```
    for (i in 1:iter){
        pi <- pisampler(x, I, hyperparams)
        sigmasq <- sigmasq2sampler(x, I, hyperparams)
        mu <- mu2sampler(x, I, sigmasq, hyperparams)
        I <- Isampler(x, pi, mu, sigmasq)

        samples[i,] <- c(mu, sigmasq)
    }

    samples
}
```

```
set.seed(123456)

hyperparams <- list(a1=1, a2=1, mu0=1, tausq0=50, nu0=1, sigmasq0=50)
init <- sample(c(1, 2), size=length(rainfall$V1), replace=TRUE)
iter <- 1000

mixedsamples <- mixedgibbs(rainfall$V1, iter, init, hyperparams)
```

```
par(mfrow = c(2,2))
plot(x = 1:iter, mixedsamples[,1], type = "l")
plot(x = 1:iter, mixedsamples[,2], type = "l")
plot(x = 1:iter, mixedsamples[,3], type = "l")
plot(x = 1:iter, mixedsamples[,4], type = "l")
```



```r
par(mfrow = c(1,1))
```

c)

Question 2

a)

b)

c)