Bayesian Learning

Lab 3

Emil K Svensson and Rasmus Holm

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Question 1

```
rainfall <- read.table("../data/rainfall.dat", header=F)
```

Prior

$$\mu \sim N(\mu_0, \tau_0^2)$$
$$\sigma^2 \sim \text{Inv}\text{–}\chi^2(\nu_0, \sigma_0^2)$$

Likelihood

$$\mathbf{y}|\mu, \sigma^2 \sim N(\mu, \sigma^2)$$

Posterior

$$\mu|\sigma^2, \mathbf{x} \sim N(\mu_n, \tau_n^2)$$
$$\sigma^2|\mu, \mathbf{x} \sim \text{Inv}\text{–}\chi^2\left(\nu_n, \frac{\nu_0\sigma_0^2 + \sum_{i=1}^{n}(x_i - \mu)^2}{n + \nu_0}\right)$$

where (?)

$$\mu_n = \frac{\frac{1}{\tau_0^2}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}\mu_0 + \frac{\frac{n}{\sigma^2}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}\bar{y}$$
$$\frac{1}{\tau_n^2} = \frac{1}{\tau_0^2} + \frac{n}{\sigma^2}$$
$$\nu_n = \nu_0 + \frac{n}{\sigma^2}$$

a)

```
library(geoR)

mun <- function(y,sigmasq,mu0,tau0sq){
  n <- length(y)
  denom <- ((1 / tau0sq) + (n/sigmasq) )
```

```
  pt1 <- ((1 / tau0sq) / denom)  * mu0
  pt2 <- ( (n/sigmasq) / denom) * mean(y)

  pt1 + pt2
}


taun <- function(y,sigmasq,tau0sq){
  tau0sq +  (sigmasq / length(y))

  1/ tau0sq
}

nun <- function(y,nu0){

  nu0 + length(y)
}


sigmasqn <- function(y,sigma0sq,mu0,nu0){

  (nu0*sigma0sq + sum( (y - mu0)^2 )) / (length(y) + nu0)

}


musampler <- function(y,sigmasq,tau0sq,mu0){

  Mean  <- mun(y,sigmasq,tau0sq,mu0)
  SigmaSq <- taun(y,sigmasq,tau0sq)

  rnorm(1,Mean,SigmaSq)

}

sigmasampler <- function(y,sigmasq,nu0,mu){
 ssn  <- sigmasqn(y,sigmasq,mu,nu0)
 nuna <- nun(y,nu0)

 rinvchisq(1, nuna, ssn)
 }
```

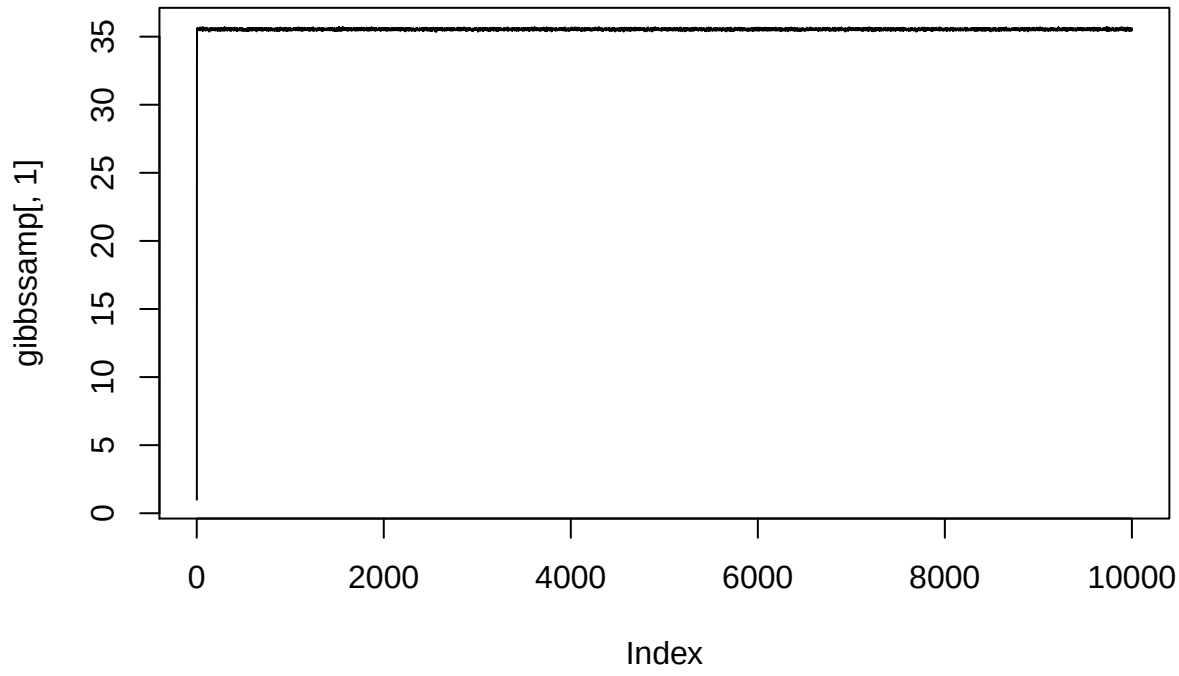
Now we start start sampling

```
gibbs <- function(y, iter, mu0, tau0sq, nu0, sigma0sq, init){

  samples <- matrix(NA,ncol = 2, nrow = iter + 1)

  samples[1,] <- init

  for (i in 2:(iter+1)){

   mu <- musampler(y,sigmasq = samples[i-1,2],tau0sq,mu0)
   sigma <- sigmasampler(y,sigmasq = sigma0sq,nu0,mu)
```

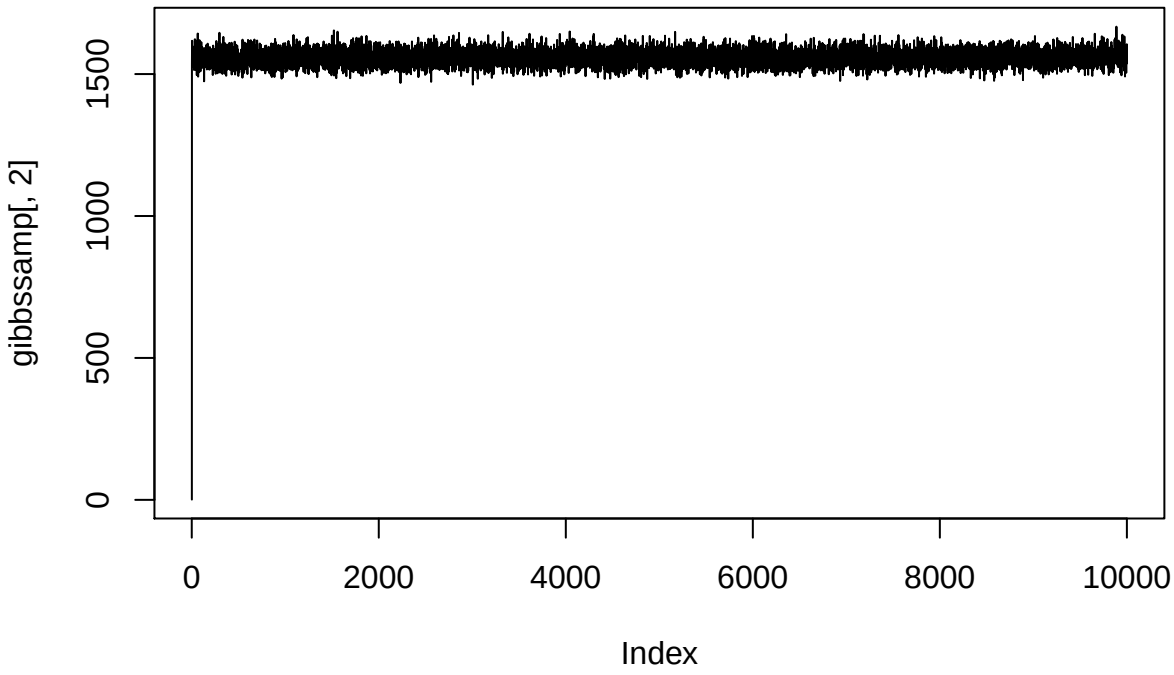
```
   samples[i,] <- c(mu,sigma)

  }

samples
}
```

b)

```
gibbssamp <- gibbs(y = rainfall$V1, iter = 10000, mu0 = 1,tau0sq = 50,nu0 = 1,sigma0sq = 20, init = c(1

plot(gibbssamp[,1], type = "l")
```




```
plot(gibbssamp[,2], type = "l")
```



c)

Question 2

a)

b)

c)