Bayesian Learning

Lab 3

Emil K Svensson and Rasmus Holm

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Question 1

```
rainfall <- read.table("../data/rainfall.dat", header=F)
```

Prior

$$\mu \sim N(\mu_0, \tau_0^2)$$
$$\sigma^2 \sim \text{Inv–}\chi^2(\nu_0, \sigma_0^2)$$

Likelihood

$$\mathbf{y}|\mu, \sigma^2 \sim N(\mu, \sigma^2)$$

Posterior

$$\mu|\sigma^2, \mathbf{x} \sim N(\mu_n, \tau_n^2)$$
$$\sigma^2|\mu, \mathbf{x} \sim \text{Inv–}\chi^2\left(\nu_n, \frac{\nu_0\sigma_0^2 + \sum_{i=1}^n (x_i - \mu)^2}{n + \nu_0}\right)$$

where

$$\mu_n = \frac{\frac{1}{\tau_0^2}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}\mu_0 + \frac{\frac{n}{\sigma^2}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}\bar{x}$$
$$\frac{1}{\tau_n^2} = \frac{1}{\tau_0^2} + \frac{n}{\sigma^2}$$
$$\nu_n = \nu_0 + n$$

a)

```
library(geoR)

mun <- function(x, sigmasq, hyperparams){
    n <- length(x)
    denom <- ((1 / hyperparams$tausq0) + (n / sigmasq))
```

```r
    pt1 <- ((1 / hyperparams$tausq0) / denom)  * hyperparams$mu0
    pt2 <- ((n / sigmasq) / denom) * mean(x)

    pt1 + pt2
}


taun <- function(x, sigmasq, hyperparams){
    hyperparams$tausq0 + (sigmasq / length(x))
    1/ hyperparams$tausq0
}

nun <- function(x, hyperparams){
    hyperparams$nu0 + length(x)
}


sigmasqn <- function(x, mu, hyperparams){
    (hyperparams$nu0 * hyperparams$sigmasq0 + sum((x - mu)^2 )) / (length(x) + hyperparams$nu0)
}


musampler <- function(x,sigmasq, hyperparams){
    mu <- mun(x, sigmasq, hyperparams)
    sigma <- sqrt(taun(x,sigmasq, hyperparams))
    rnorm(1, mu, sigma)
}

sigmasampler <- function(x, mu, hyperparams){
    scale  <- sigmasqn(x, mu, hyperparams)
    df <- nun(x, hyperparams)
    rinvchisq(1, df, scale)
}
```

Now we start start sampling

```r
gibbs <- function(x, iter, init, hyperparams){
    samples <- matrix(NA, ncol = 2, nrow = iter + 1)
    samples[1,] <- init

    for (i in 2:(iter+1)){
        mu <- musampler(x, samples[i-1, 2], hyperparams)
        sigma <- sigmasampler(x, mu, hyperparams)
        samples[i,] <- c(mu, sigma)
    }

    samples
}
```
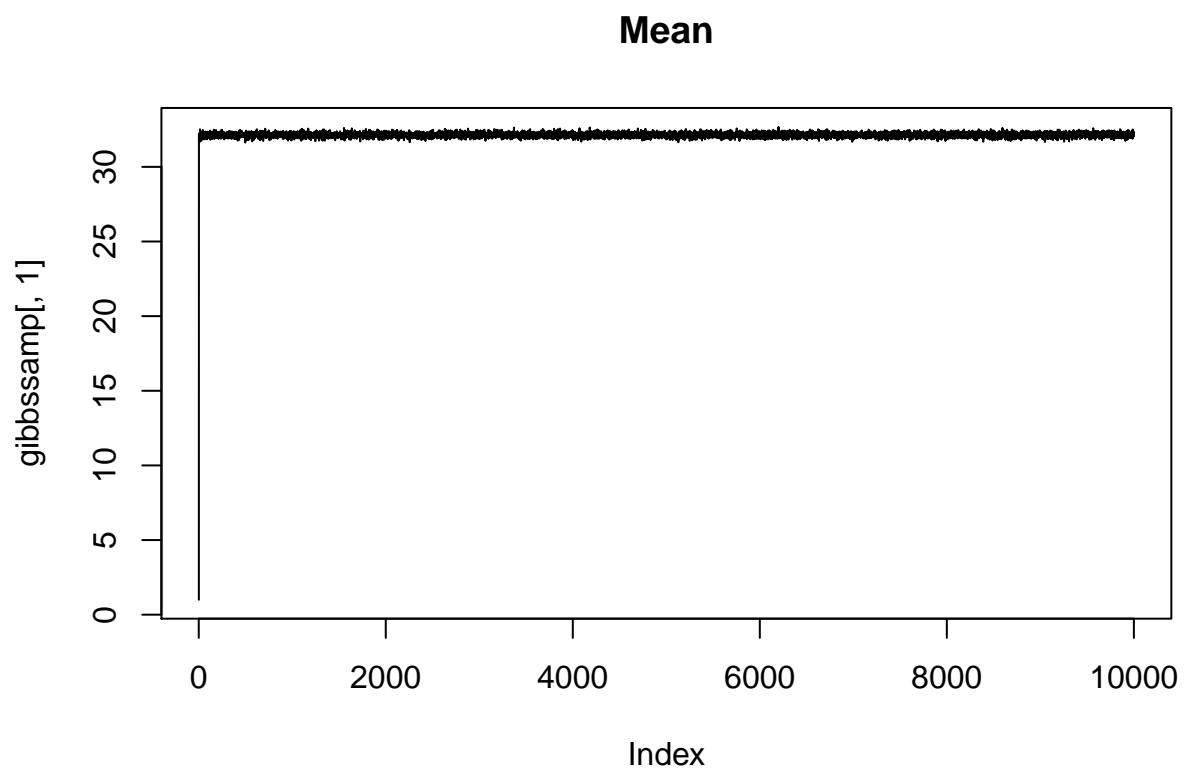
```r
hyperparams <- list(mu0=0, tausq0=50, nu0=1, sigmasq0=50)

gibbssamp <- gibbs(x=rainfall$V1, iter = 10000, init = c(1,1), hyperparams)
```
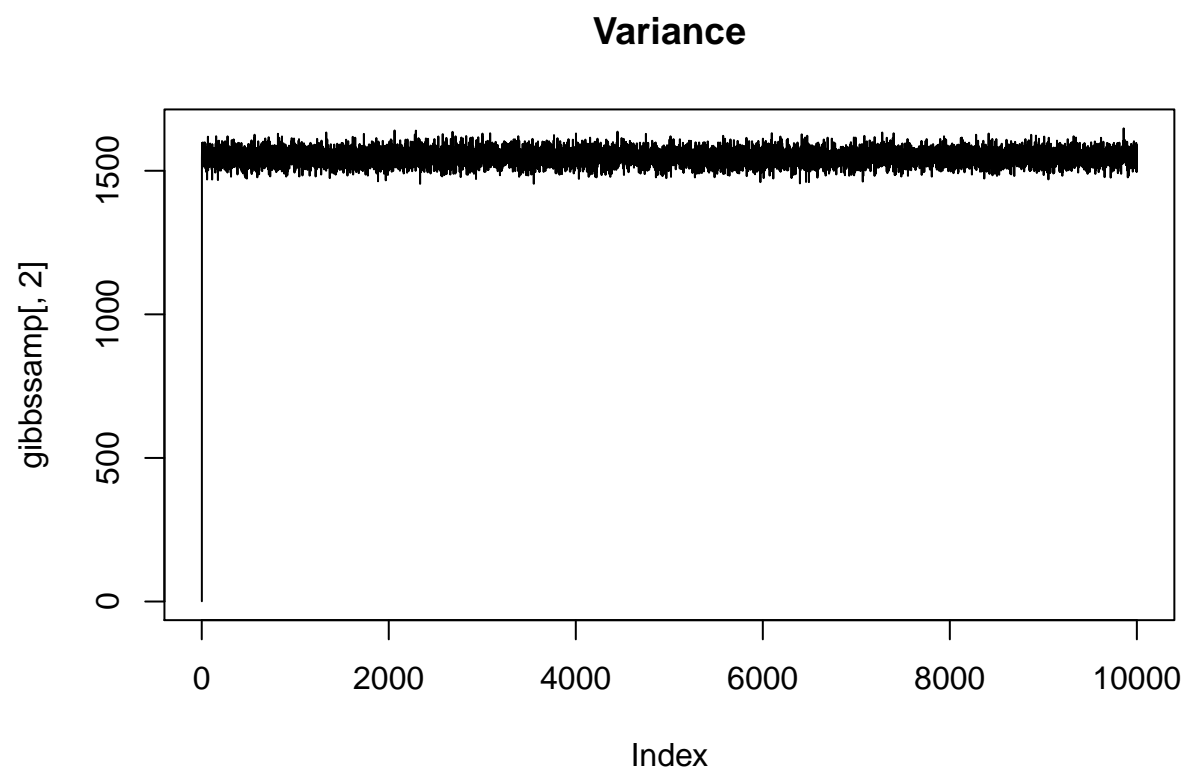
```r
plot(gibbssamp[,1], type = "l", main="Mean")
```

Mean


```r
plot(gibbssamp[,2], type = "l", main="Variance")
```

Variance

b)

c)

Question 2

a)

b)

c)