Bayesian Learning

Lab 4

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Question 1

```r
bid <- read.table("../data/eBayNumberOfBidderData.dat", header = TRUE)
```

a)

```r
glm_res <- glm(nBids ~ . - 1, data = bid, family = poisson(link = "log"))
summary(glm_res)
```

```
##
## Call:
## glm(formula = nBids ~ . - 1, family = poisson(link = "log"),
##     data = bid)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -3.580  -0.722  -0.044   0.527   2.461
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## Const         1.0724     0.0308   34.85  < 2e-16 ***
## PowerSeller  -0.0205     0.0368   -0.56    0.577
## VerifyID     -0.3945     0.0924   -4.27  2.0e-05 ***
## Sealed        0.4438     0.0506    8.78  < 2e-16 ***
## Minblem      -0.0522     0.0602   -0.87    0.386
## MajBlem      -0.2209     0.0914   -2.42    0.016 *
## LargNeg       0.0707     0.0563    1.25    0.210
## LogBook      -0.1207     0.0290   -4.17  3.1e-05 ***
## MinBidShare  -1.8941     0.0712  -26.59  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 6264.01  on 1000  degrees of freedom
## Residual deviance:  867.47  on  991  degrees of freedom
## AIC: 3610
##
## Number of Fisher Scoring iterations: 5
```

b)

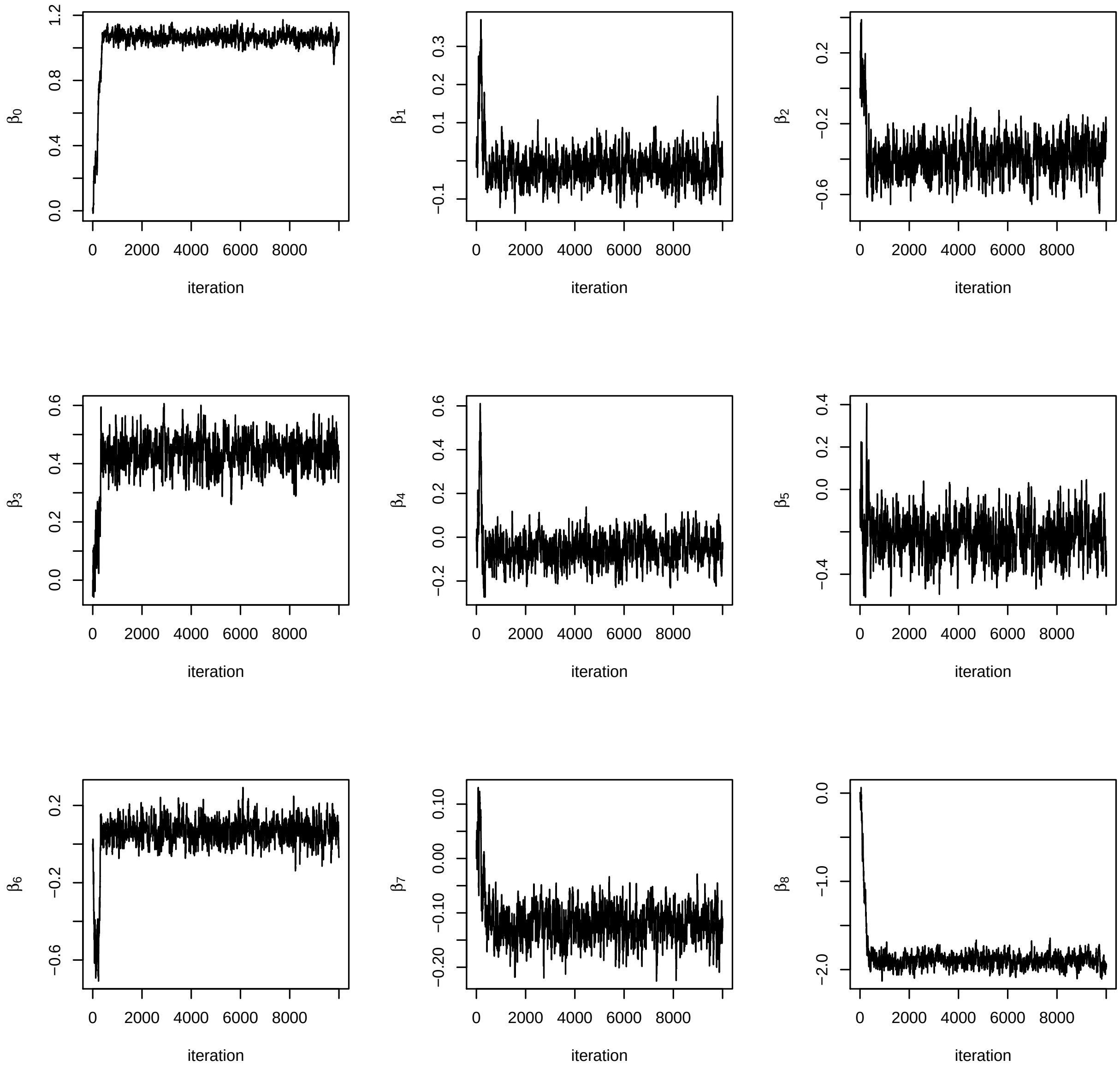
```r
library(mvtnorm)

logprior <- function(beta, mu, sigma){
    dmvnorm(beta, mean = mu, sigma = sigma, log = TRUE)
}

loglikelihood <- function(beta, X, Y){
    linear_prediction <- t(X) %*% beta
    probabilities <- Y * linear_prediction - exp(linear_prediction)
    loglike <- sum(probabilities)

    ## if (abs(loglike) == Inf)
    ##     loglike = -20000

    loglike
}

## loglikelihood <- function(beta, X, Y) {
##   linear_prediction <- t(X) %*% beta
##   probs <- dpois(Y , lambda = exp(linear_prediction), log = TRUE)
##   sum(probs)
## }

logposterior <- function(beta, X, Y, prior_mu, prior_sigma){
    loglikelihood(beta, X, Y) + logprior(beta, prior_mu, prior_sigma)
}

X <- as.matrix(bid[,-1])
Y <- as.matrix(bid[,1])

mu <- rep(0, ncol(X))
sigma <- 100 * solve(t(X) %*% X)

normal_res <- optim(par = matrix(rep(0, ncol(X)), ncol = 1),
                 fn = logposterior, method = "BFGS", hessian = TRUE,
                 X = t(X), Y = Y,
                 prior_mu= mu, prior_sigma = sigma,
                 control=list(fnscale=-1))
hessian <- normal_res$hessian
```

c)

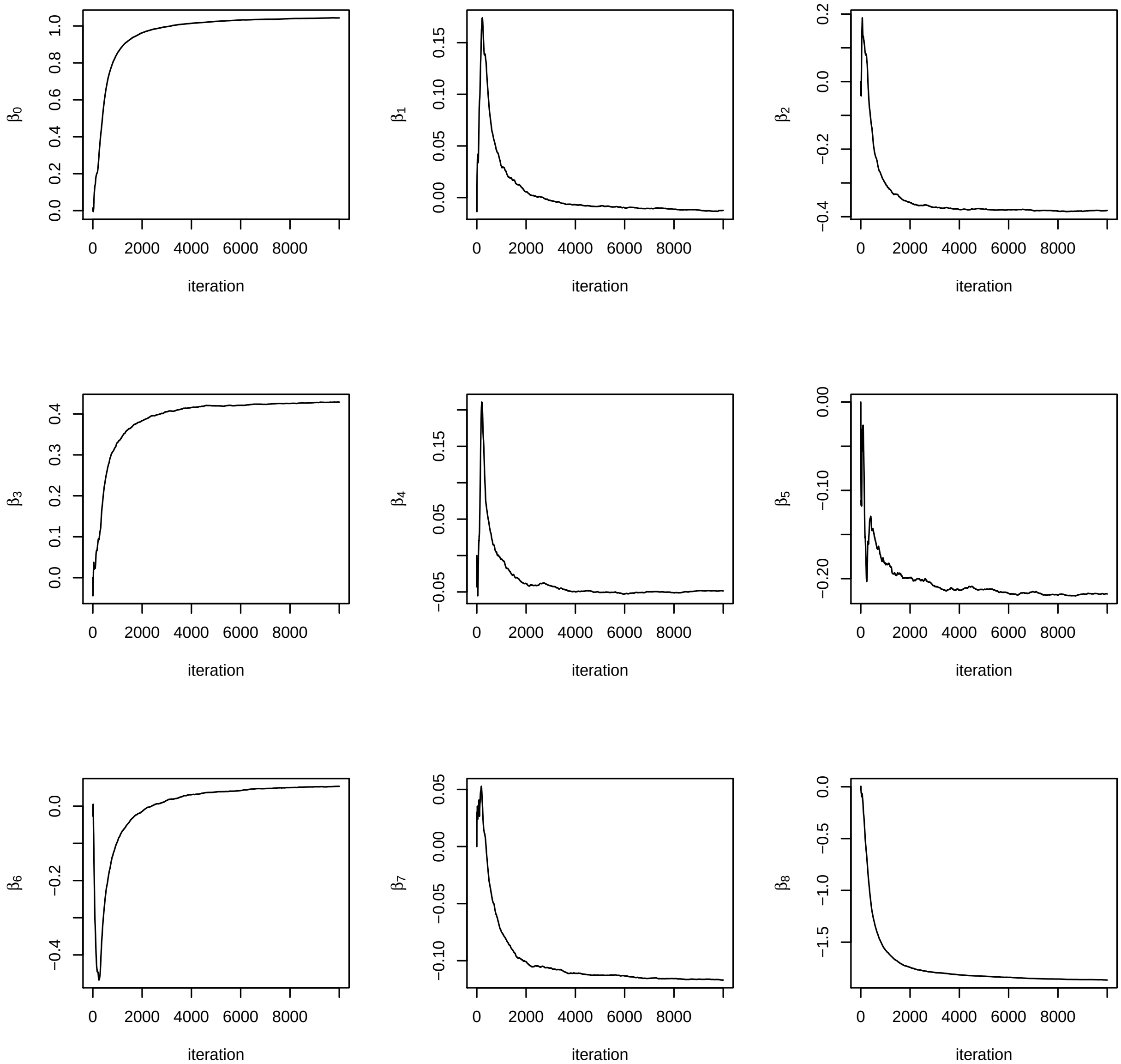
```r
targetdensity <- function(theta, prior_mu, prior_sigma, X, Y, ...) {
    likelihood <- dpois(Y, lambda = exp(t(X) %*% t(theta)), log = TRUE)
    prior <- dmvnorm(theta, mean = prior_mu, sigma = prior_sigma, log = TRUE)
    sum(likelihood) + prior
}

proposaldensity <- function(theta, mu, prop_sigma, ...){
    dmvnorm(theta, mean = mu, sigma = prop_sigma, log = TRUE)
```

```r
}

proposalsampler <- function(mu, prop_sigma, ...){
    matrix(rmvnorm(1, mean = mu, sigma = prop_sigma), nrow = 1)
}

metropolis_hastings <- function(log_targ_post_func, log_prop_func, prop_sampler,
                                X0, iters, ...){
    x <- X0
    values <- matrix(0, ncol = length(X0), nrow = iters + 1)
    values[1,] <- X0

    alpha <- function(x, y, ...) {
        numerator <- log_targ_post_func(y, ...) + log_prop_func(x, y, ...)
        denominator <- log_targ_post_func(x, ...) + log_prop_func(y, x, ...)
        exp(numerator - denominator)
    }

    for (i in 1:iters) {
        y <- prop_sampler(x, ...)
        u <- runif(1)

        if (u < alpha(x, y, ...)) {
            x <- y
        }

        values[i+1,] <- x
    }

    values
}

iters <- 10000
X0 <- rep(0, times = ncol(X))

params <- list(
    log_targ_post_func = targetdensity,
    log_prop_func = proposaldensity,
    prop_sampler = proposalsampler,
    X0 = matrix(rep(0, times = ncol(X)), nrow = 1),
    iters = iters,
    X = t(X),
    Y = Y,
    prior_mu = rep(0, times = ncol(X)),
    prior_sigma = 100 * solve(t(X) %*% X),
    prop_sigma = 0.6 * -solve(hessian)
)

metro_res <- do.call(metropolis_hastings, params)
```

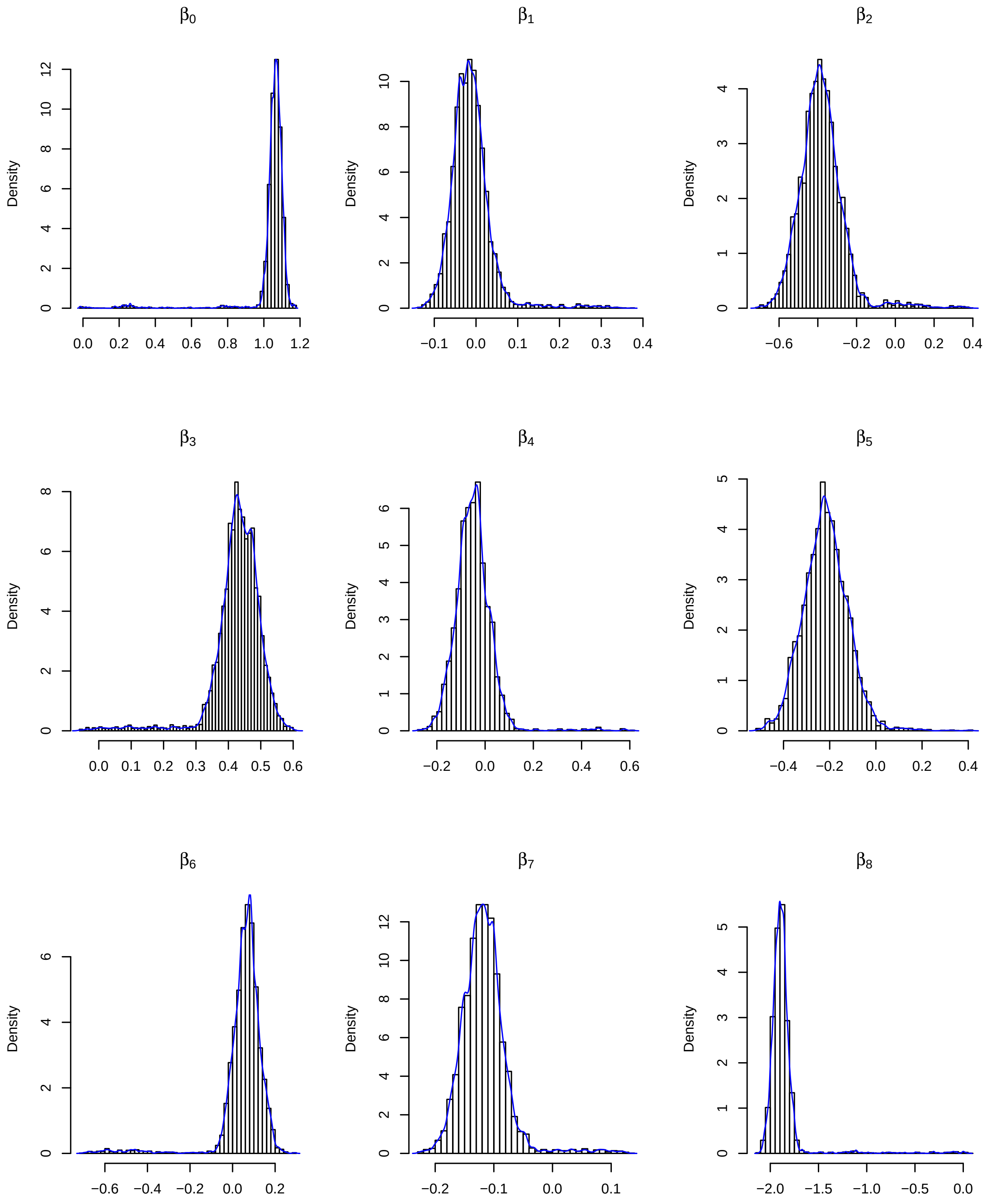
	Beta0	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6	Beta7	Beta8
glm	1.07	-0.021	-0.395	0.444	-0.052	-0.221	0.071	-0.121	-1.89
normal	1.07	-0.021	-0.393	0.444	-0.052	-0.221	0.071	-0.120	-1.89
metropolis	1.04	-0.012	-0.381	0.429	-0.049	-0.217	0.053	-0.117	-1.86

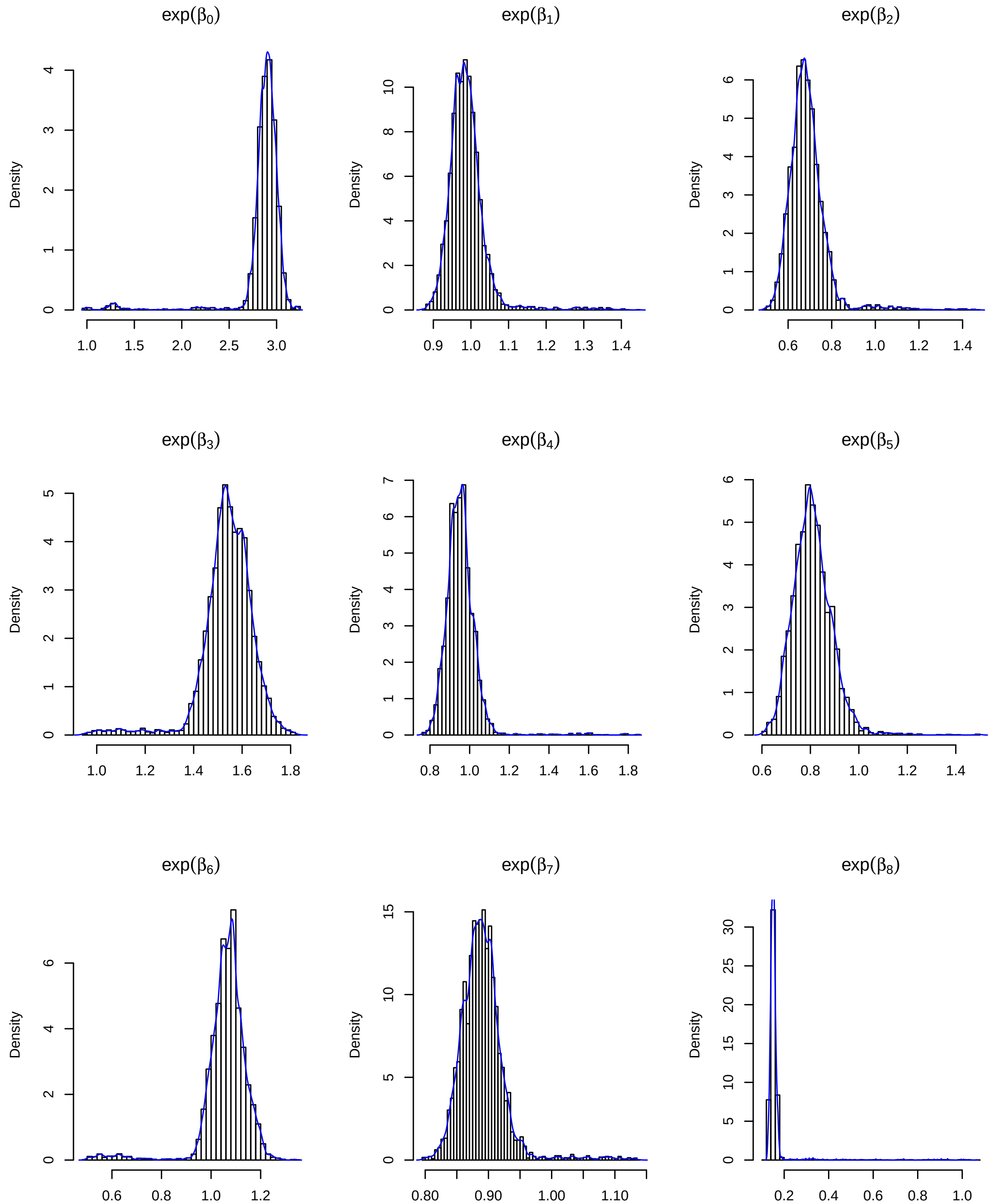


The parameters seem to converge but some of them might have some small autocorrelation left in them. Plots with cumulative means will be assessed.



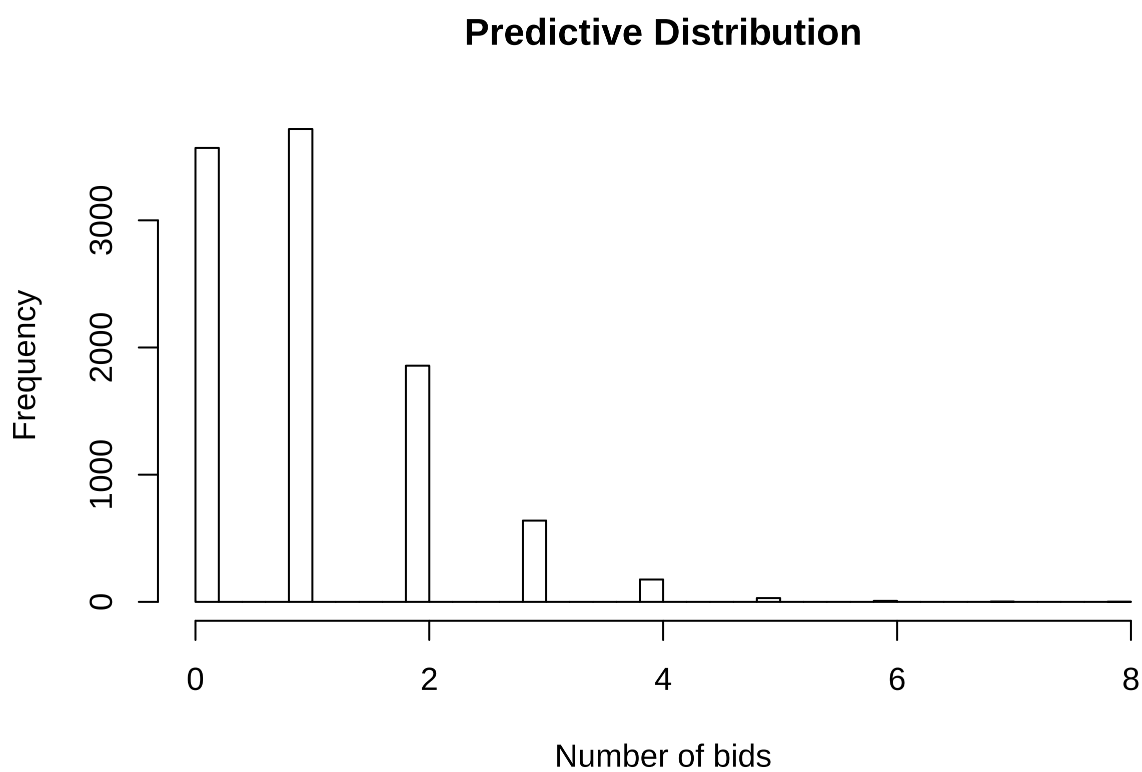
The cumulative means seem to be quite stable although not completely converged. More iterations are needed to get a more stable result.


β0
Density
0.0 0.2 0.4 0.6 0.8 1.0 1.2
0 2 4 6 8 10 12
β1
Density
−0.1 0.0 0.1 0.2 0.3 0.4
0 2 4 6 8 10
β2
Density
−0.6 −0.2 0.0 0.2 0.4
0 1 2 3 4
β3
Density
0.0 0.1 0.2 0.3 0.4 0.5 0.6
0 2 4 6 8
β4
Density
−0.2 0.0 0.2 0.4 0.6
0 1 2 3 4 5 6
β5
Density
−0.4 −0.2 0.0 0.2 0.4
0 1 2 3 4 5
β6
Density
−0.6 −0.4 −0.2 0.0 0.2
0 2 4 6
β7
Density
−0.2 −0.1 0.0 0.1
0 2 4 6 8 10 12
β8
Density
−2.0 −1.5 −1.0 −0.5 0.0
0 1 2 3 4 5


exp(β0)
exp(β1)
exp(β2)
exp(β3)
exp(β4)
exp(β5)
exp(β6)
exp(β7)
exp(β8)
Density

d

```r
Xpred <- matrix(c(1, 1, 1, 1, 0, 0, 0, 1, 0.5), nrow = 1)
predsamples <- rpois(10000, lambda = exp(Xpred %*% t(metro_res)))

hist(predsamples, breaks = 50, main="Predictive Distribution", xlab="Number of bids")
```




```r
paste("The probability that there are no bidders", mean(predsamples == 0))
```

```
## [1] "The probability that there are no bidders 0.3569"
```