Bayesian Learning

Lab 2

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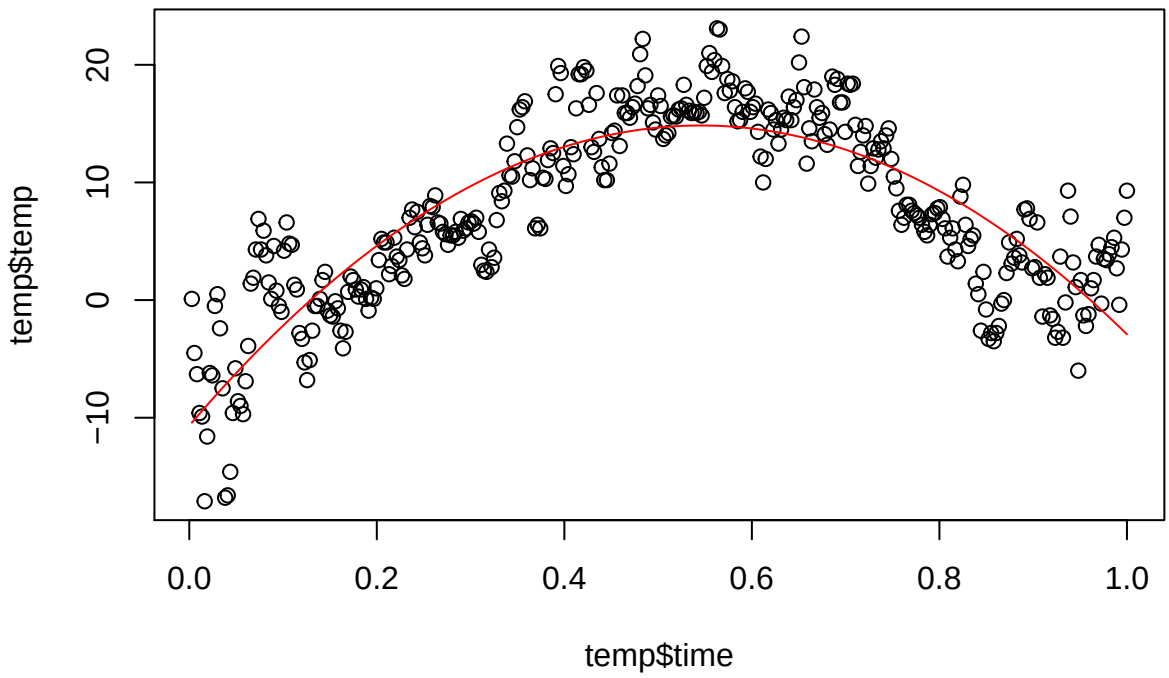
Question 1

```
temp <- read.table("../data/TempLinkoping2016.txt", header=T)

mod <- lm(temp ~ time + I(time^2), data=temp)
summary(mod)
```

```
##
## Call:
## lm(formula = temp ~ time + I(time^2), data = temp)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.0408  -2.6971  -0.1414   2.5157  12.2085
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -10.6754     0.6475  -16.49   <2e-16 ***
## time         93.5980     2.9822   31.39   <2e-16 ***
## I(time^2)   -85.8311     2.8801  -29.80   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.107 on 363 degrees of freedom
## Multiple R-squared:  0.7318, Adjusted R-squared:  0.7304
## F-statistic: 495.3 on 2 and 363 DF,  p-value: < 2.2e-16
```

```
plot(temp$time, temp$temp)
lines(sort(temp$time), fitted(mod)[order(temp$time)], col='red', type='l')
```



Prior

$$\sigma^2 \sim \text{Inv} - \chi^2(\nu_0, \sigma_0^2)$$
$$\beta|\sigma^2 \sim \text{N}(\mu_0, \sigma^2\Omega_0^{-1})$$

Likelihood

$$\mathbf{y}|\beta, \sigma^2, \mathbf{X} \sim \text{N}(\mathbf{X}\beta, \sigma^2 I_n)$$

Posterior

$$\sigma^2|\mathbf{y} \sim \text{Inv} - \chi^2(\nu_n, \sigma_n^2)$$
$$\beta|\sigma^2, \mathbf{y} \sim \text{N}(\mu_n, \sigma^2\Omega_n^{-1})$$

where

$$\mu_n = (\mathbf{X}^\intercal\mathbf{X} + \Omega_0)^{-1}(\mathbf{X}^\intercal\mathbf{X}\hat{\beta} + \Omega_0\mu_0)$$
$$\Omega_n = \mathbf{X}^\intercal\mathbf{X} + \Omega_0$$
$$\nu_n = \nu_0 + n$$
$$\nu_n\sigma_n^2 = \nu_0\sigma_0^2 + (\mathbf{y}^\intercal\mathbf{y} + \mu_0^\intercal\Omega_0\mu_0 - \mu_n^\intercal\Omega_n\mu_n)$$

a)

```
mu0 <- 0
omega0 <- 0
nu0 <- 0
sigmasq0 <- 0
```

b)

c)

d)

e)

Question 2

a)

b)

c)