Computational Statistics

Lab 4

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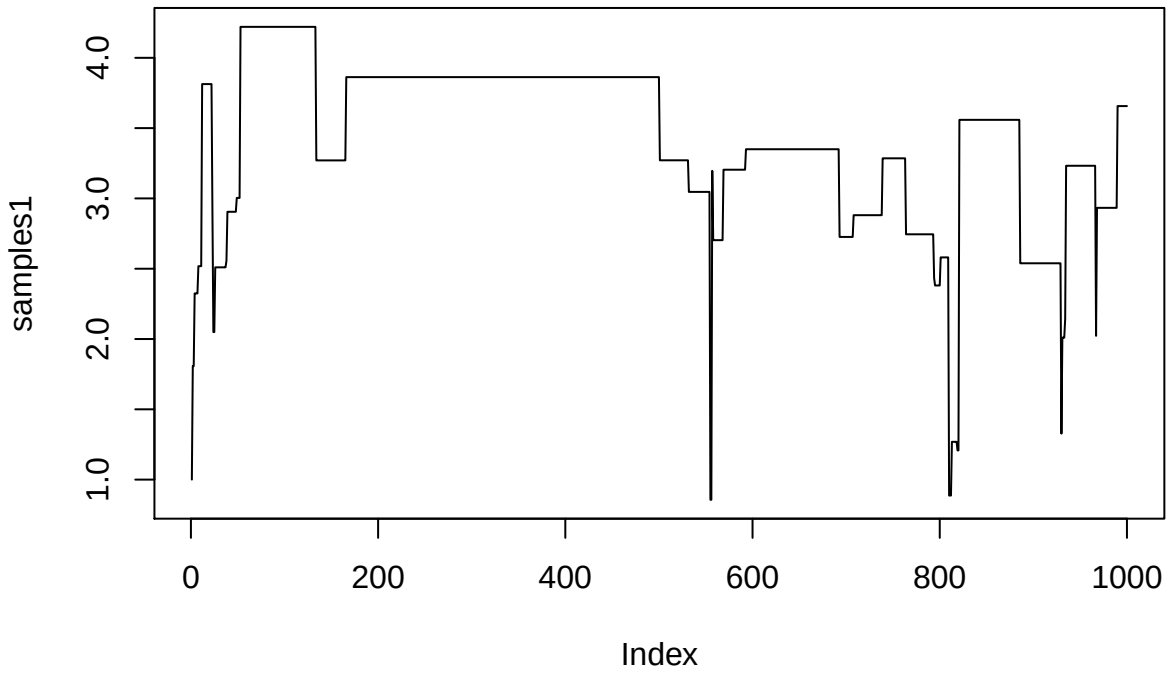
```
targetdensity <- function(x) {
    x^5 * exp(-x)
}

lognormalfuncs <- list(propsample=function(x) { rlnorm(1, meanlog=x, sdlog=1) },
                       propdensity=function(x, y) { dlnorm(x, meanlog=y, sdlog=1) },
                       targdensity=targetdensity)

chisquarefuncs <- list(propsample=function(x) { rchisq(1, df=floor(x + 1)) },
                       propdensity=function(x, y) { dchisq(x, df=floor(y + 1)) },
                       targdensity=targetdensity)

metropolis_hastings <- function(X0, iters, funcs) {
    x <- X0
    values <- rep(0, iters)

    alpha <- function(x, y) {
        numerator <- funcs$targdensity(y) * funcs$propdensity(x, y)
        denominator <- funcs$targdensity(x) * funcs$propdensity(y, x)
        numerator / denominator
    }

    for (i in 1:iters) {
        y <- funcs$propsample(x)
        u <- runif(1)

        if (u < alpha(x, y)) {
            x = y
        }

        values[i] <- x
    }

    values
}

iters <- 1000
X0 <- 1

actual <- rgamma(iters, shape=6, rate=1)
```

```
set.seed(123456)
samples1 <- metropolis_hastings(X0=X0, iters=iters, funcs=lognormalfuncs)
mean(samples1)
```

```
## [1] 3.42
```

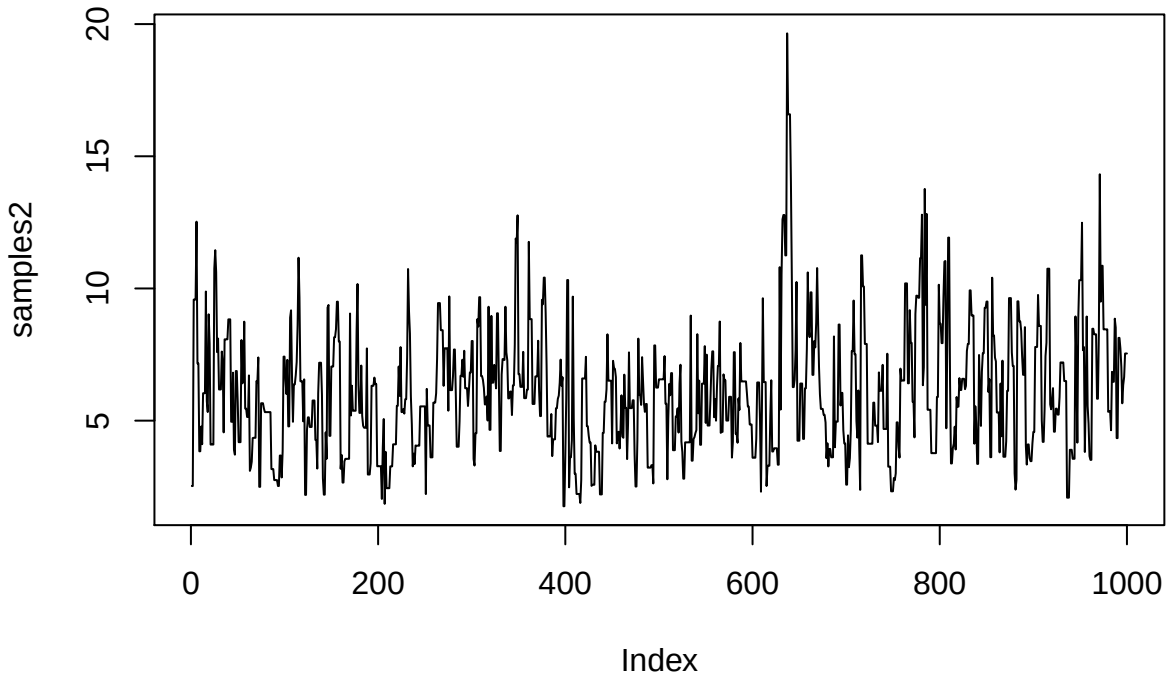
```
plot(samples1, type="l")
```



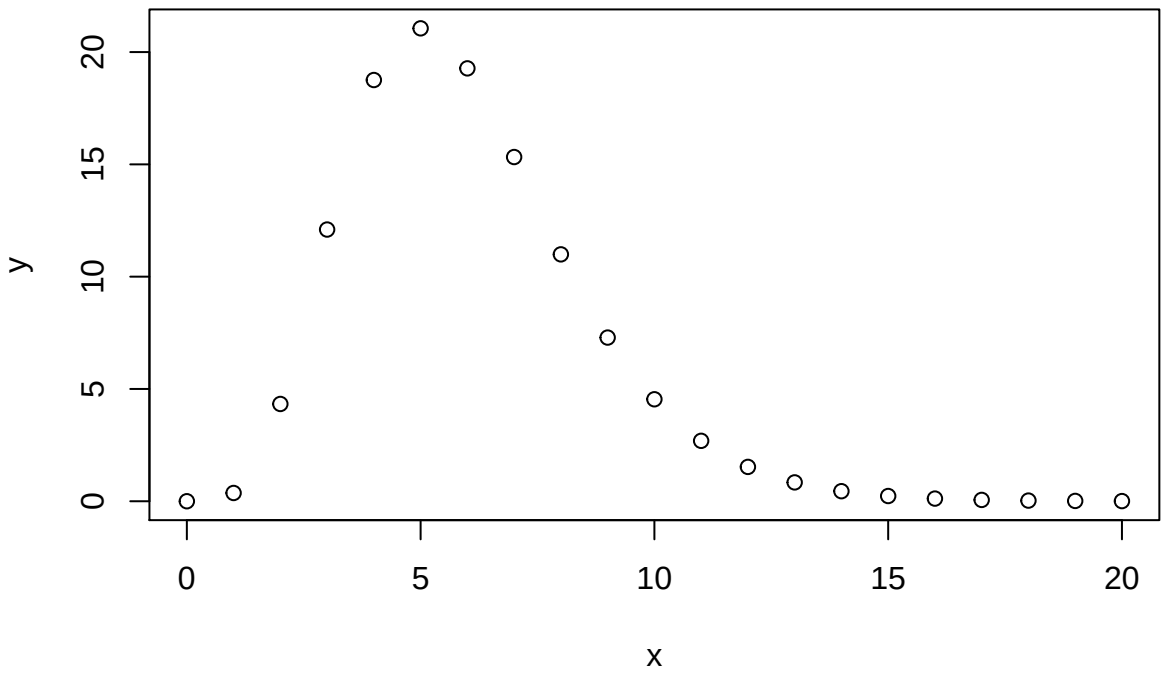

```
set.seed(123456)
samples2 <- metropolis_hastings(X0=X0, iters=iters, funcs=chisquarefuncs)
mean(samples2)
```

```
## [1] 6.05
```

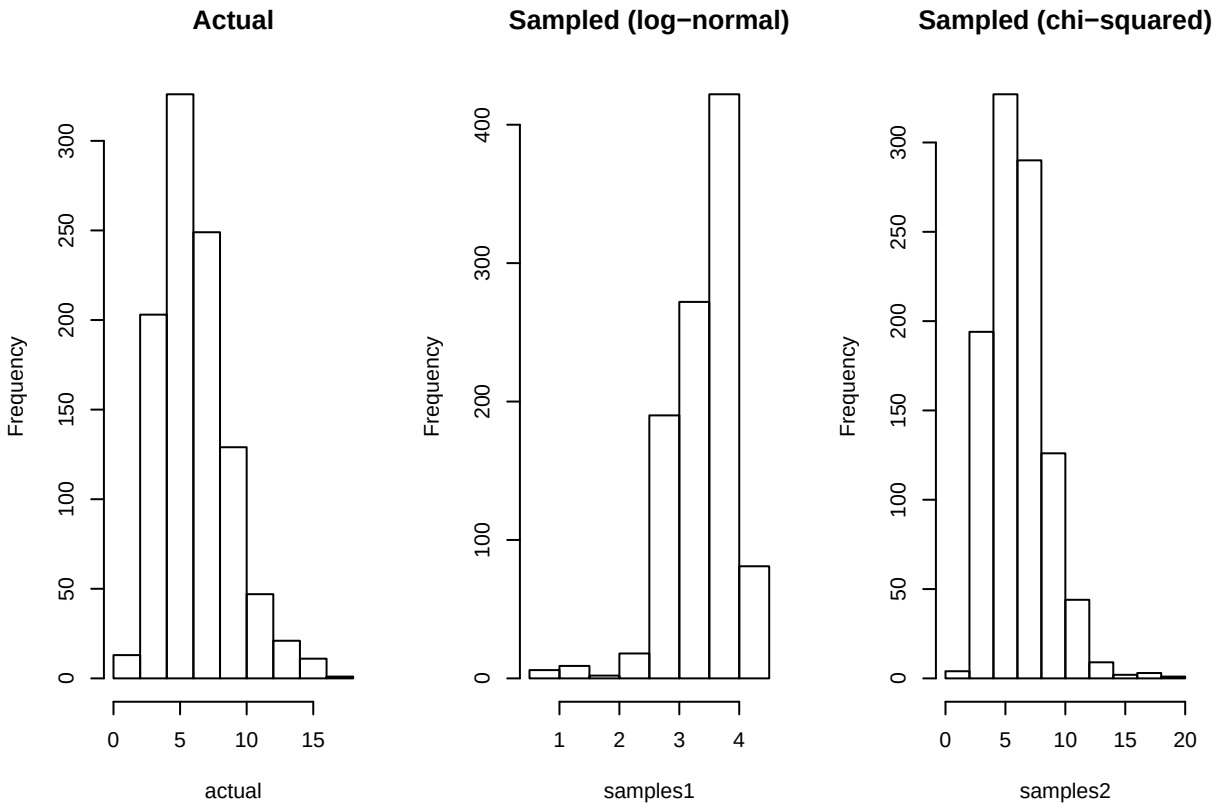
```
plot(samples2, type="l")
```




```
x <- 0:20
y <- sapply(x, targetdensity)
plot(x, y)
```




```
oldpar <- par(mfrow = c(1, 3))

hist(actual, main="Actual")
hist(samples1, main="Sampled (log-normal)")
hist(samples2, main="Sampled (chi-squared)")
```



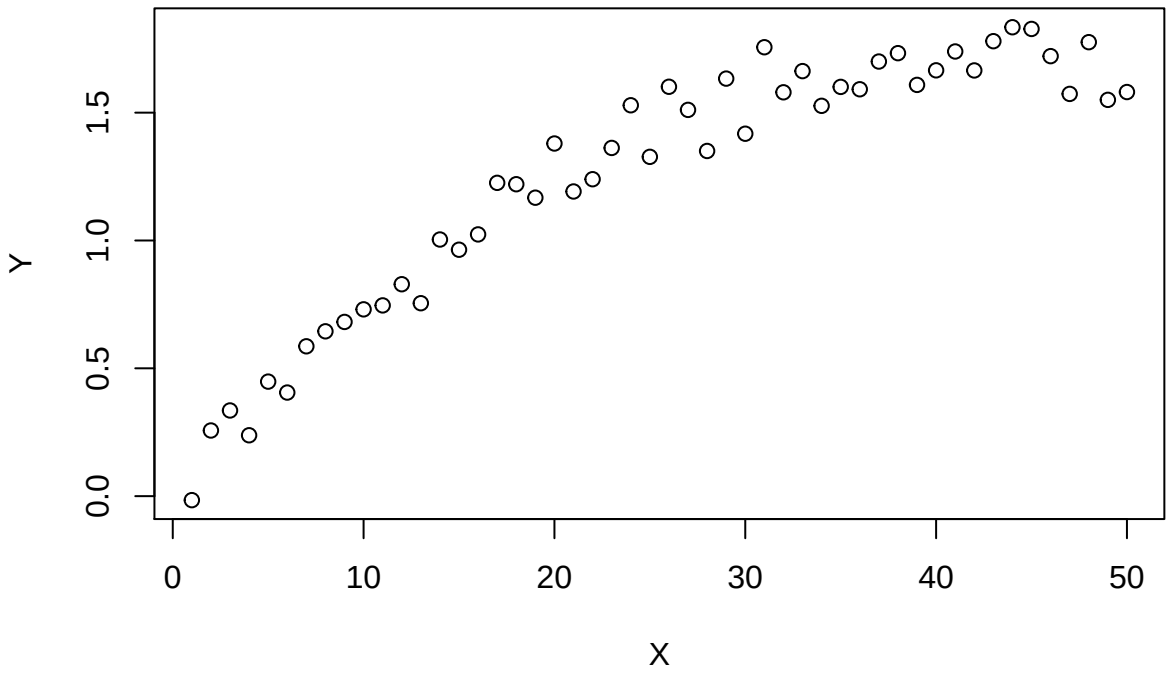
Geldman rubin

```
Geldman<-function(x){
  k <- nrow(x)
  n <- ncol(x)

  B <- (n / (k - 1)) * sum((rowMeans(x) - mean(x))^2)

  W <- sum((x - rowMeans(x))^2) / (k * (n - 1))

  VarV <- ((n - 1) / n) * W + B / n

  sqrtR <- sqrt(VarV / W)
  sqrtR
}

resultMatrix <- do.call(rbind, lapply(1:10, FUN = function(x)
    metropolis_hastings(X0 = x, iters = iters, funcs = chisquarefuncs)))

GeldmanRes <- Geldman(resultMatrix)
GeldmanRes
```

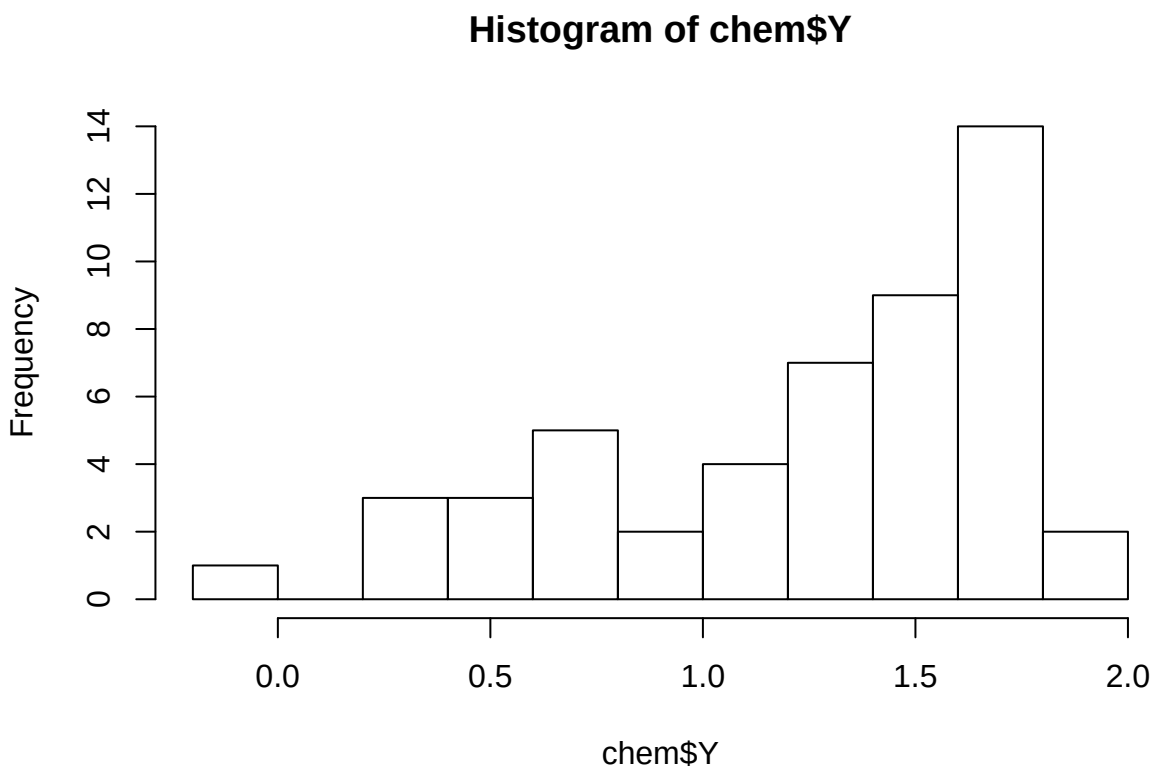
```
## [1] 1
```

Question 2

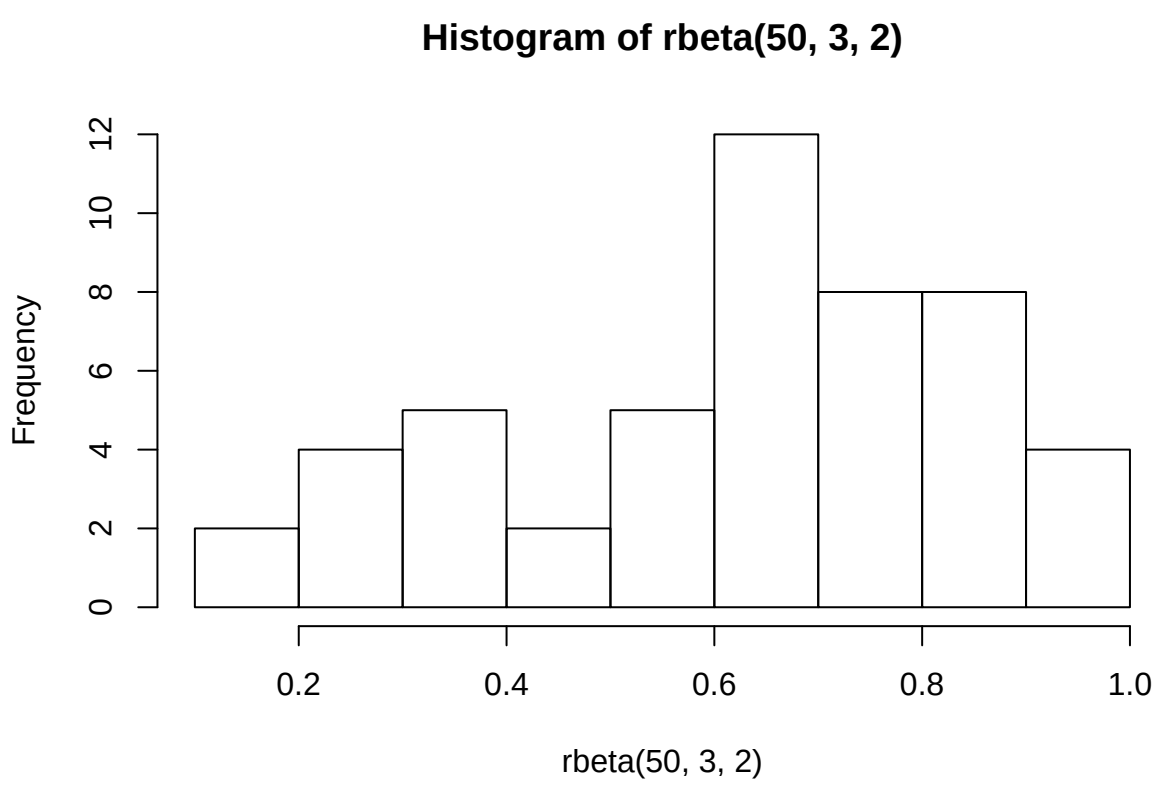
```
load("../data/chemical.RData")

chem <- data.frame(X = X , Y = Y)


plot(chem)
```




```
hist(chem$Y)
```




```
hist(rbeta(50,3,2))
```



$$
\begin{aligned}
p(\mu) &= p(\mu_1)p(\mu_2|\mu_1)\cdots p(\mu_n|\mu_{n-1}) \\
&= \frac{1}{\sqrt{(2\pi\sigma^2)^{n-1}}} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=2}^{n}(\mu_i - \mu_{i-1})^2\right) \\
p(y|\mu) &= p(y_1|\mu_1)p(y_2|\mu_2)\cdots p(y_n|\mu_n) \\
&= \frac{1}{\sqrt{(2\pi\sigma^2)^{n}}} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \mu_i)^2\right)
\end{aligned}
$$

$$
\begin{aligned}
p(\mu|y) &\propto p(y|\mu)p(\mu) \\
&\propto \exp\left(-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n}(y_i - \mu_i)^2 + \sum_{i=2}^{n}(\mu_i - \mu_{i-1})^2\right)\right)
\end{aligned}
$$

This

$$
\begin{aligned}
p(\mu_1|\mu_{-1}, y) &\propto \exp\left(-\frac{1}{\sigma^2}(\mu_1 - (y_1 + \mu_2)/2)^2\right) \\
p(\mu_i|\mu_{-i}, y) &\propto p(\mu_1|\mu_{-1}, y) \exp\left(-\frac{3}{2\sigma^2}\sum_{j=2}^{i}(\mu_j - (y_j + \mu_{j-1} + \mu_{j+1})/3)^2\right) \\
p(\mu_n|\mu_{-n}, y) &\propto p(\mu_{n-1}|\mu_{-n-1}, y) \exp\left(-\frac{1}{\sigma^2}(\mu_n - (y_n + \mu_{n-1})/2)^2\right)
\end{aligned}
$$

Alternatively this

$$
\begin{aligned}
p(\mu_1|\mu_{-1}, y) &\propto \exp\left(-\frac{1}{\sigma^2}(\mu_1 - (y_1 + \mu_2)/2)^2\right) \\
p(\mu_i|\mu_{-i}, y) &\propto \exp\left(-\frac{3}{2\sigma^2}(\mu_i - (y_i + \mu_{i-1} + \mu_{i+1})/3)^2\right) \\
p(\mu_n|\mu_{-n}, y) &\propto \exp\left(-\frac{1}{\sigma^2}(\mu_n - (y_n + \mu_{n-1})/2)^2\right)
\end{aligned}
$$