

Computational Statistics

Lab 4

Emil K Svensson and Rasmus Holm

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```
targetdensity <- function(x) {
  x^5 * exp(-x)
}

lognormalfuncs <- list(propsample=function(x) { rlnorm(1, meanlog=x, sdlog=1) },
  propdensity=function(x, y) { dlnorm(x, meanlog=y, sdlog=1) },
  targdensity=targetdensity)

chisquarefuncs <- list(propsample=function(x) { rchisq(1, df=floor(x + 1)) },
  propdensity=function(x, y) { dchisq(x, df=floor(y + 1)) },
  targdensity=targetdensity)

metropolis_hastings <- function(X0, iters, funcs) {
  x <- X0
  values <- rep(0, iters)

  alpha <- function(x, y) {
    numerator <- funcs$targdensity(y) * funcs$propdensity(x, y)
    denominator <- funcs$targdensity(x) * funcs$propdensity(y, x)
    numerator / denominator
  }

  for (i in 1:iters) {
    y <- funcs$propsample(x)
    u <- runif(1)

    if (u < alpha(x, y)) {
      x = y
    }

    values[i] <- x
  }

  values
}

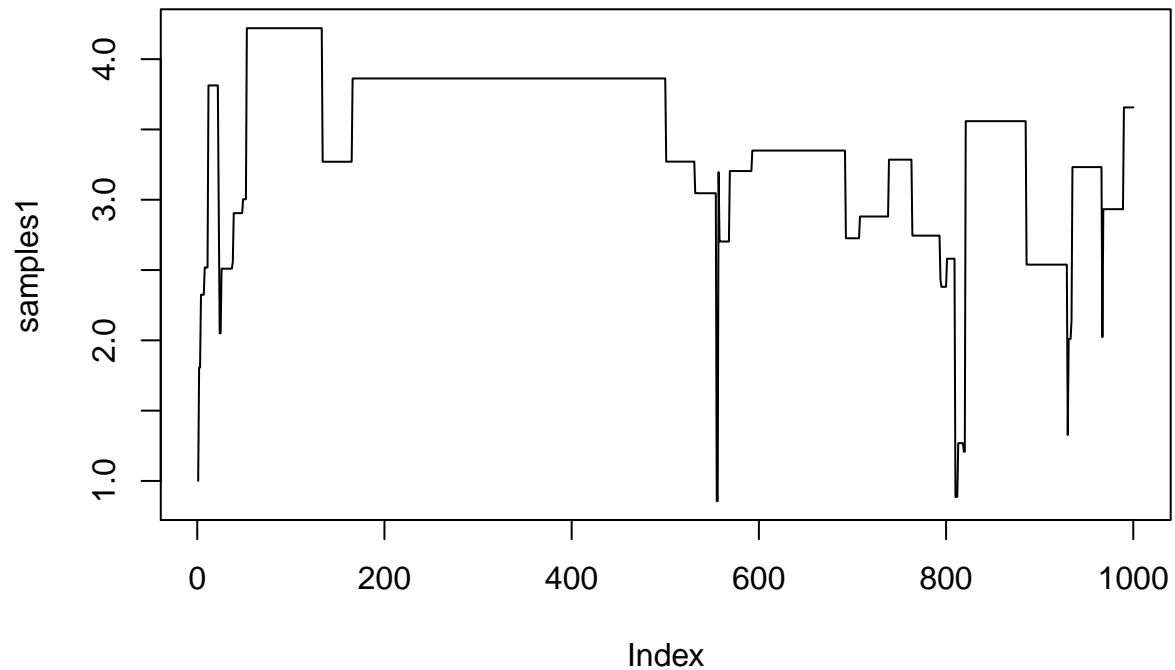
iters <- 1000
X0 <- 1

actual <- rgamma(iters, shape=6, rate=1)

set.seed(123456)
samples1 <- metropolis_hastings(X0=X0, iters=iters, funcs=lognormalfuncs)
mean(samples1)

## [1] 3.42
```

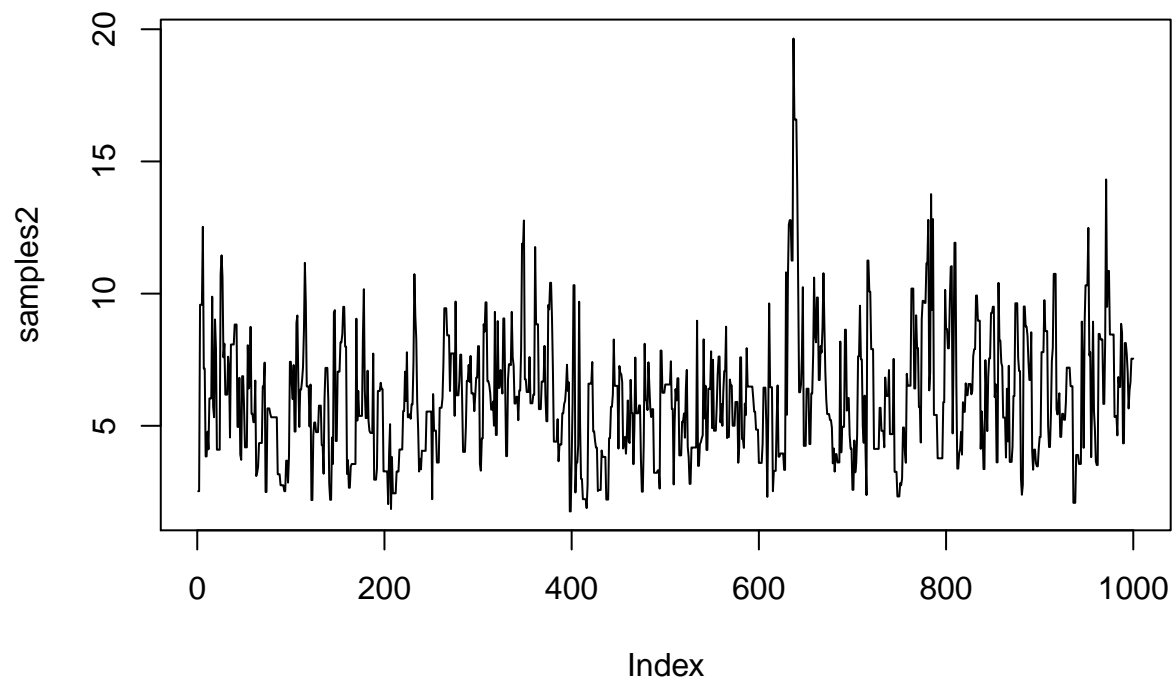
```
plot(samples1, type="l")
```



```
set.seed(123456)
samples2 <- metropolis_hastings(X0=X0, iters=iters, funcs=chisquarefuncs)
mean(samples2)
```

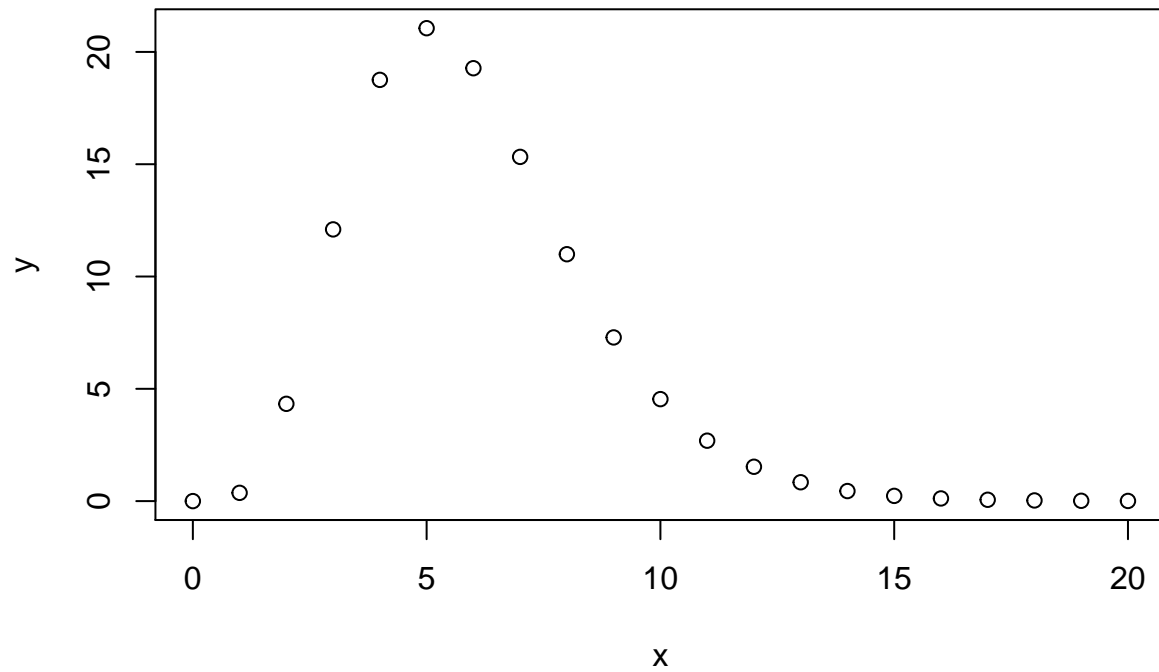
```
## [1] 6.05
```

```
plot(samples2, type="l")
```



```
x <- 0:20
y <- sapply(x, targetdensity)
```

```
plot(x, y)
```

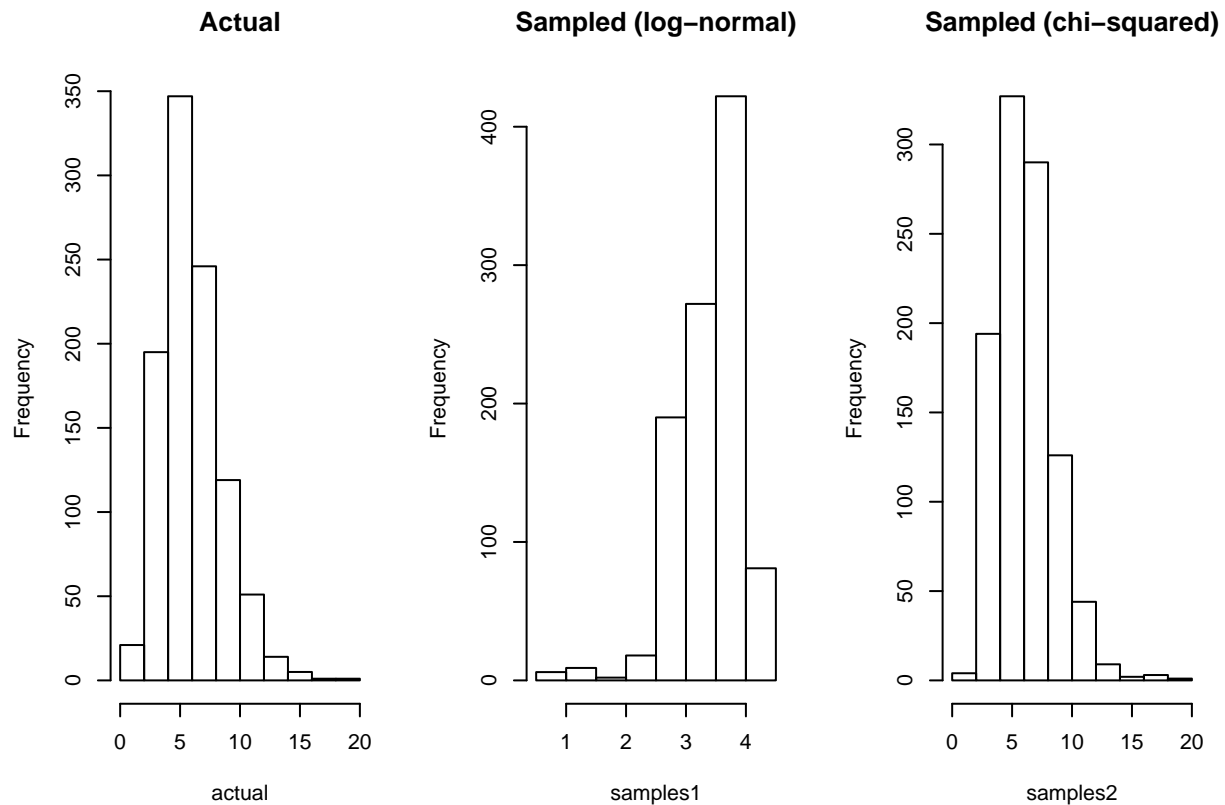


```
oldpar <- par(mfrow = c(1, 3))
```

```
hist(actual, main="Actual")
```

```
hist(samples1, main="Sampled (log-normal)")
```

```
hist(samples2, main="Sampled (chi-squared)")
```



Geldman rubin

```
Geldman<-function(x){
  k <- nrow(x)
  n <- ncol(x)

  B <- (n / (k - 1)) * sum((rowMeans(x) - mean(x))^2)

  W <- sum((x - rowMeans(x))^2) / (k * (n - 1))

  VarV <- ((n - 1) / n) * W + B / n

  sqrtR <- sqrt(VarV / W)
  sqrtR
}

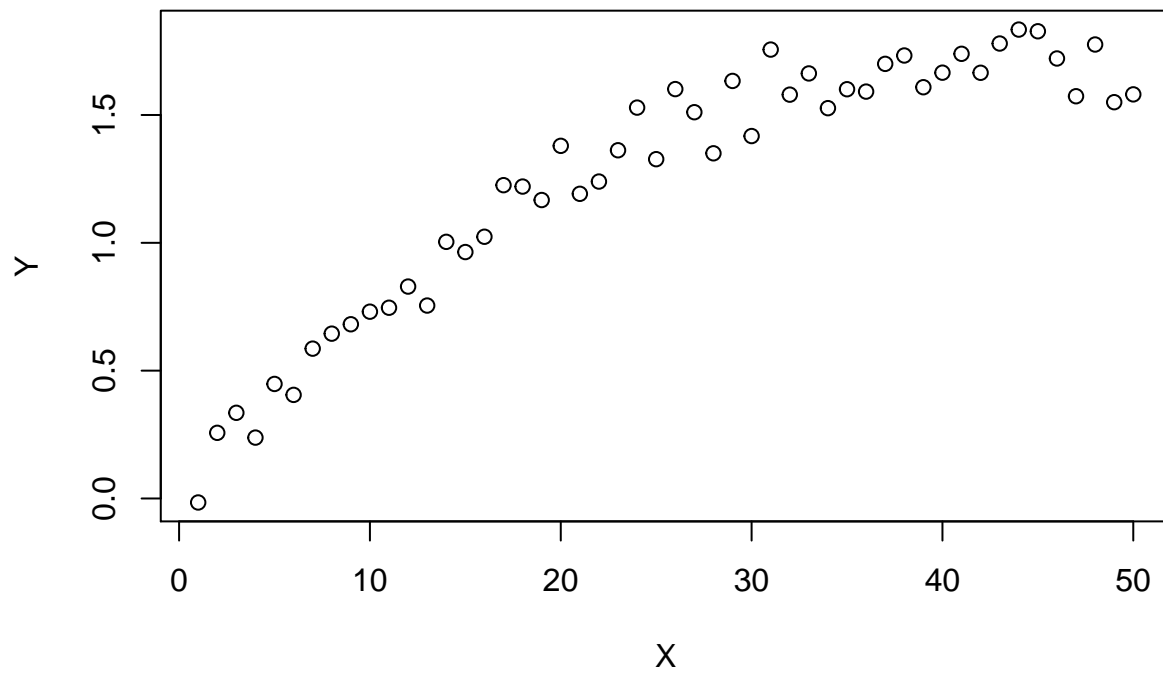
resultMatrix <- do.call(rbind, lapply(1:10, FUN = function(x)
  metropolis_hastings(X0 = x, iters = iters, funcs = chisquarefuncs)))

GeldmanRes <- Geldman(resultMatrix)
GeldmanRes

## [1] 1
```

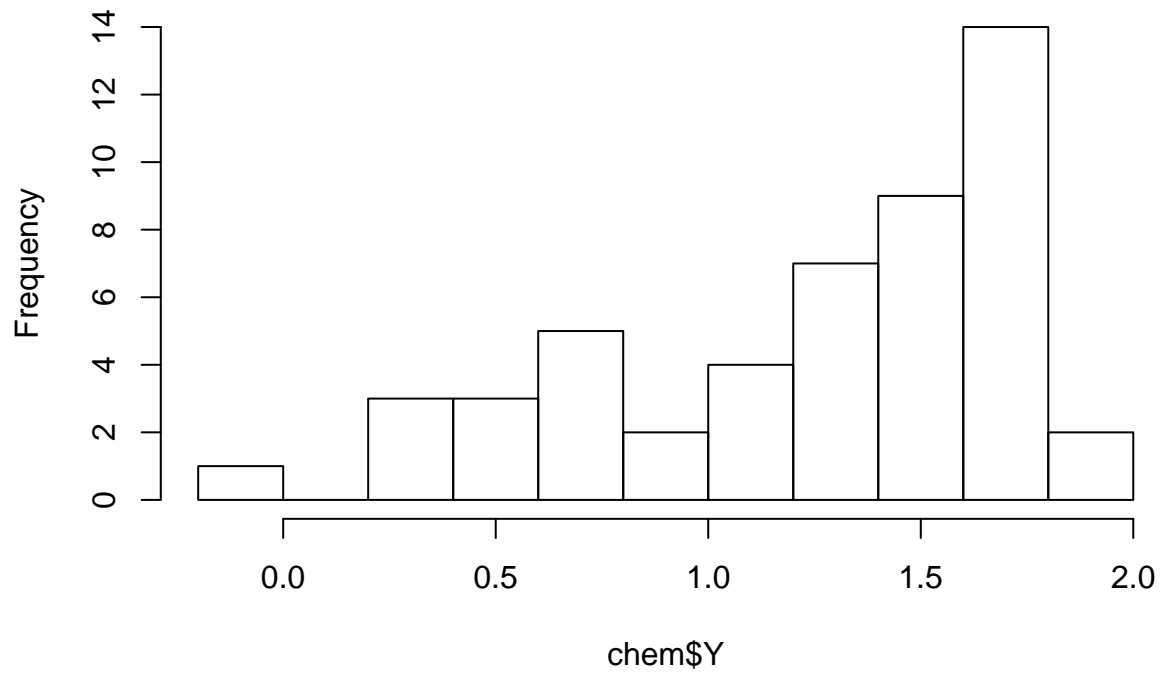
Question 2

```
load("../data/chemical.RData")  
  
chem <- data.frame(X = X , Y = Y)  
  
plot(chem)
```



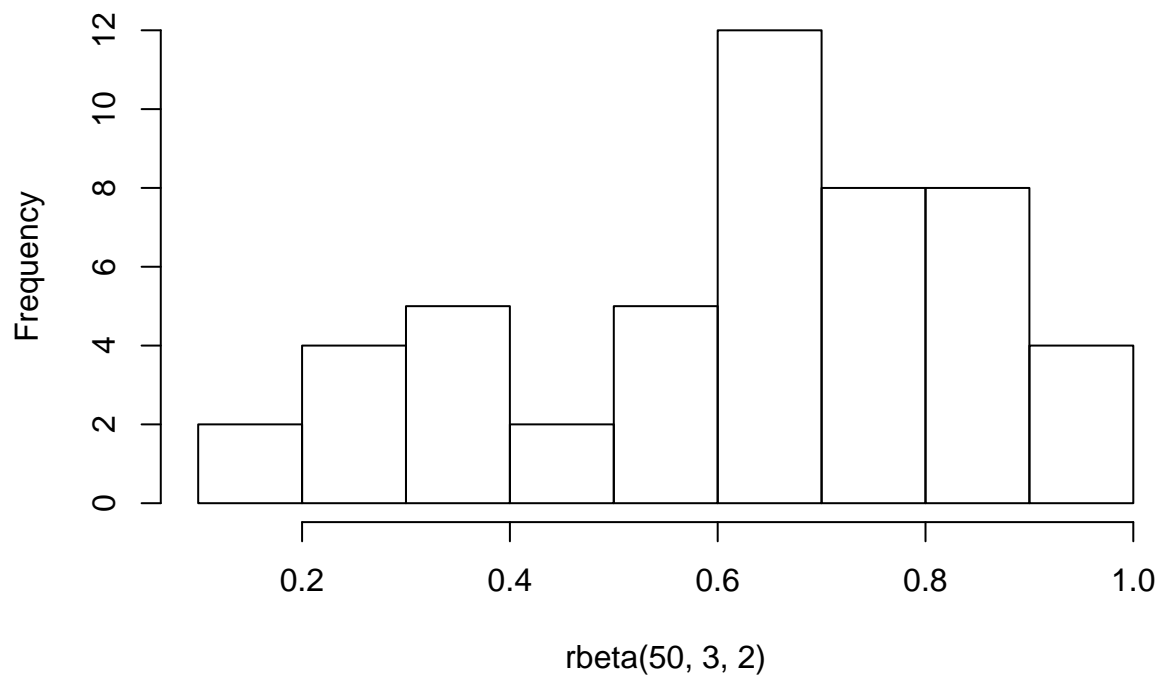
```
hist(chem$Y)
```

Histogram of chem\$Y



```
hist(rbeta(50,3,2))
```

Histogram of rbeta(50, 3, 2)



2.2

We have

$$\begin{aligned}
 p(\mu) &= p(\mu_1)p(\mu_2|\mu_1)\cdots p(\mu_n|\mu_{n-1}) \\
 &= \frac{1}{\sqrt{(2\pi\sigma^2)^{n-1}}} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=2}^n (\mu_i - \mu_{i-1})^2\right), \\
 p(y|\mu) &= p(y_1|\mu_1)p(y_2|\mu_2)\cdots p(y_n|\mu_n) \\
 &= \frac{1}{\sqrt{(2\pi\sigma^2)^n}} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^n (y_i - \mu_i)^2\right).
 \end{aligned}$$

2.3

The posterior is then

$$\begin{aligned}
 p(\mu|y) &\propto p(y|\mu)p(\mu) \\
 &\propto \exp\left(-\frac{1}{2\sigma^2} \left(\sum_{i=1}^n (y_i - \mu_i)^2 + \sum_{i=2}^n (\mu_i - \mu_{i-1})^2\right)\right) \\
 &\propto \exp\left(-\frac{2}{2\sigma^2} (\mu_1 - (y_1 + \mu_2)/2)^2 - \frac{3}{2\sigma^2} \sum_{j=2}^i (\mu_j - (y_j + \mu_{j-1} + \mu_{j+1})/3)^2 - \frac{2}{2\sigma^2} (\mu_n - (y_n + \mu_{n-1})/2)^2\right).
 \end{aligned}$$

This gives us

$$\begin{aligned}
 p(\mu_1|\mu_{-1}, y) &\propto \exp\left(-\frac{2}{2\sigma^2} (\mu_1 - (y_1 + \mu_2)/2)^2\right), \\
 p(\mu_i|\mu_{-i}, y) &\propto \exp\left(-\frac{3}{2\sigma^2} (\mu_i - (y_i + \mu_{i-1} + \mu_{i+1})/3)^2\right), \\
 p(\mu_n|\mu_{-n}, y) &\propto \exp\left(-\frac{2}{2\sigma^2} (\mu_n - (y_n + \mu_{n-1})/2)^2\right).
 \end{aligned}$$

2.4

```

posterior <- function(data, mus, index, sigmasq){
  if (index == 1){
    return(rnorm(1,
                 mean = (data[index] + mus[index + 1]) / 2,
                 sd = sqrt(sigmasq / 2)))
  }

  if (index == length(mus)){
    return(rnorm(1,
                 mean = (data[index] + mus[index - 1]) / 2,
                 sd = sqrt(sigmasq / 2)))
  }
}

```

```

    }

    return(rnorm(1,
                 mean = (data[index] + mus[index - 1] + mus[index + 1]) / 3,
                 sd = sqrt(sigmasq / 3)))
}

```

```

gibbs <- function(data, tmax){
  d <- nrow(data)
  t <- 0

  mus <- matrix(0, nrow = tmax, ncol = d)
  sigmasq <- 0.2

  for (i in 1:tmax){
    for (j in 1:d){
      mus[i, j] <- posterior(data, mus[i, ], j, sigmasq)
    }

    if (i != tmax) {
      mus[i+1,] <- mus[i,]
    }
  }

  return(mus)
}

```

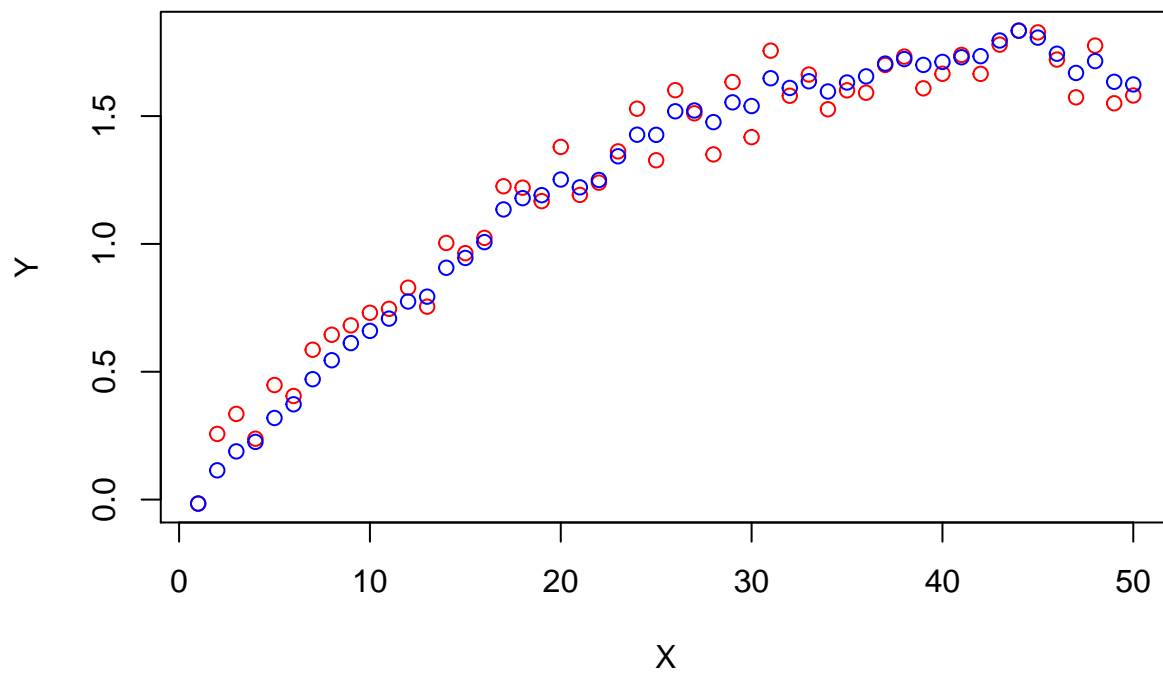
```

set.seed(123456)

d <- as.matrix(chem$Y)
mu <- gibbs(data = d, tmax = 1000)
emu <- colMeans(mu)

plot(chem, col="red", xlab="X", ylab="Y")
par(new = TRUE)
plot(chem$X, emu, col="blue", xlab="", ylab="", axes=FALSE)

```

2.5

```
library(coda)
traceplot(as.mcmc(mu[, 50]))
```

