

Computational Statistics

Lab 4

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2017-02-21

```
targetdensity <- function(x) {
  x^5 * exp(-x)
}

lognormalfuncs <- list(propsample=function(x) { rlnorm(1, meanlog=x, sdlog=1) },
  propdensity=function(x, y) { dlnorm(x, meanlog=y, sdlog=1) },
  targdensity=targetdensity)

chisquarefuncs <- list(propsample=function(x) { rchisq(1, df=floor(x + 1)) },
  propdensity=function(x, y) { dchisq(x, df=floor(y + 1)) },
  targdensity=targetdensity)

metropolis_hastings <- function(X0, iters, funcs) {
  x <- X0
  values <- rep(0, iters)

  alpha <- function(x, y) {
    numerator <- funcs$targdensity(y) * funcs$propdensity(x, y)
    denominator <- funcs$targdensity(x) * funcs$propdensity(y, x)
    numerator / denominator
  }

  for (i in 1:iters) {
    y <- funcs$propsample(x)
    u <- runif(1)

    if (u < alpha(x, y)) {
      x = y
    }

    values[i] <- x
  }

  values
}

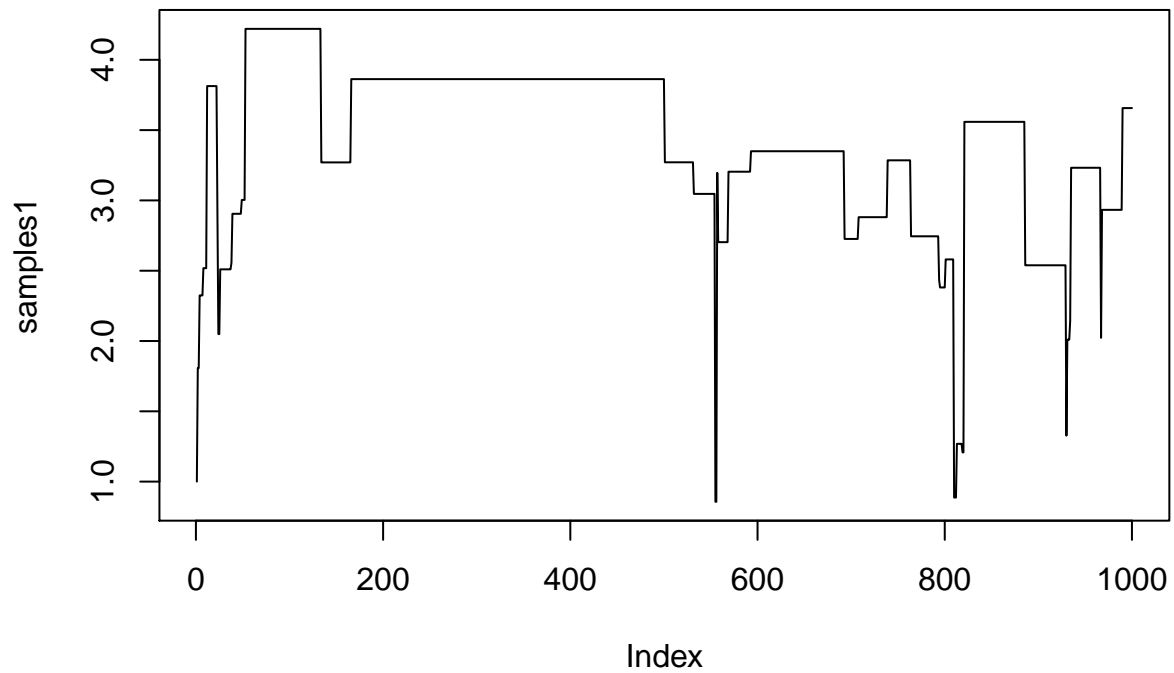
iters <- 1000
X0 <- 1

actual <- rgamma(iters, shape=6, rate=1)

set.seed(123456)
samples1 <- metropolis_hastings(X0=X0, iters=iters, funcs=lognormalfuncs)
mean(samples1)

## [1] 3.42
```

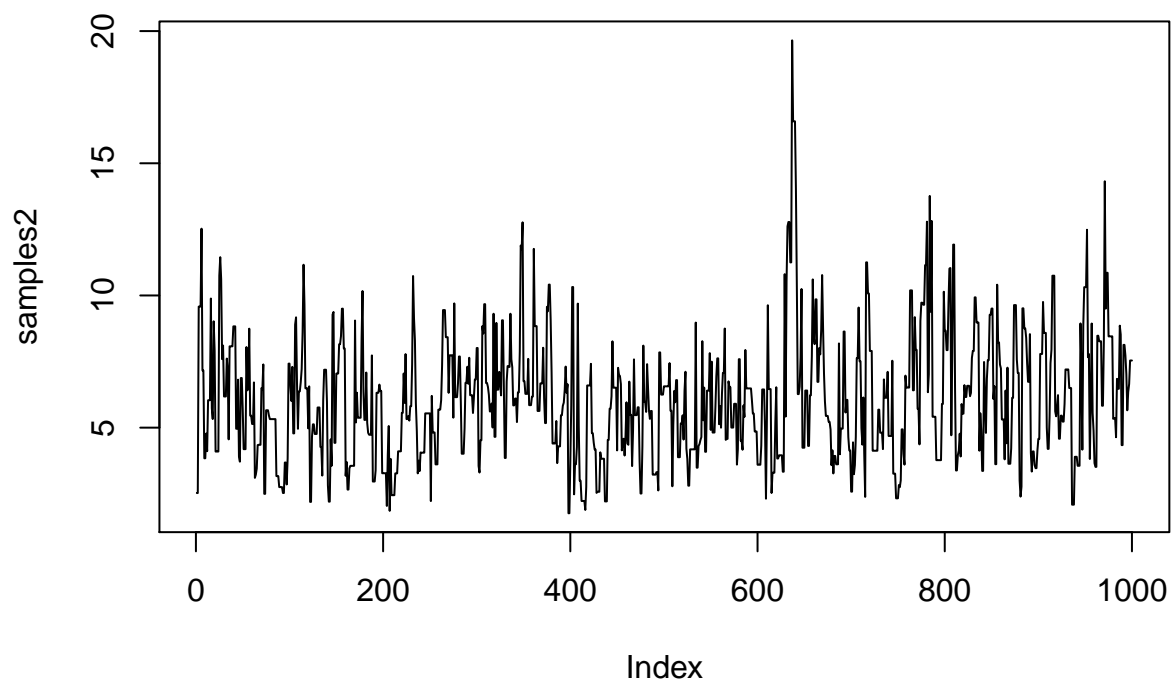
```
plot(samples1, type="l")
```



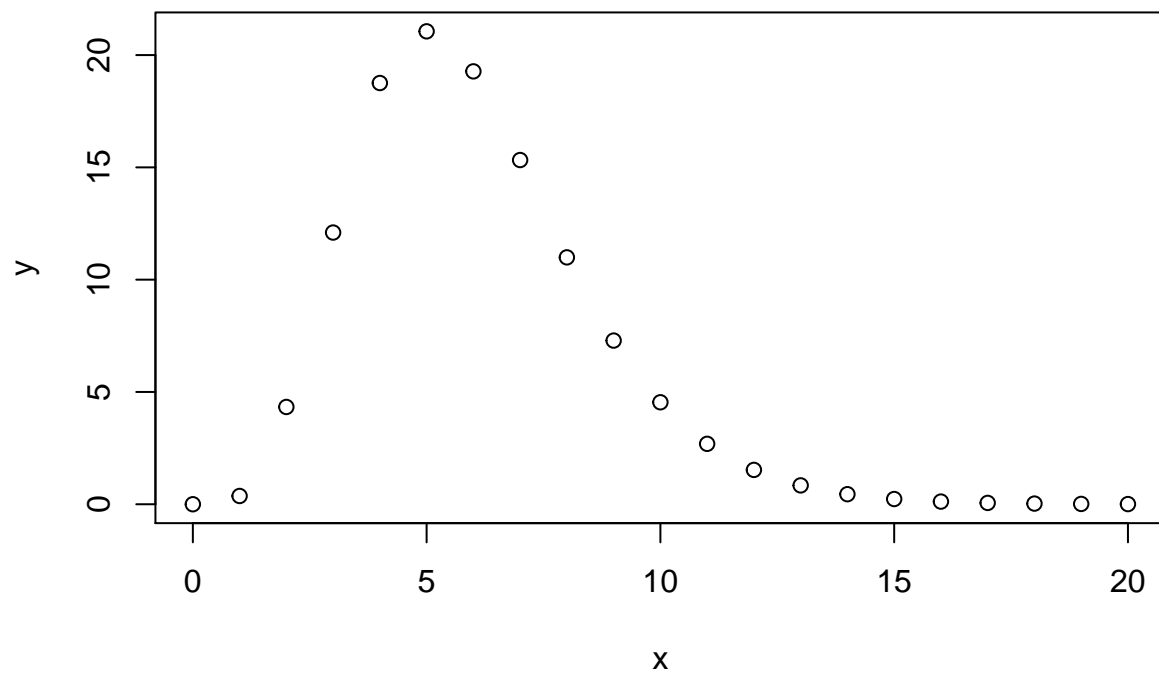
```
set.seed(123456)
samples2 <- metropolis_hastings(X0=X0, iters=iters, funcs=chisquarefuncs)
mean(samples2)
```

```
## [1] 6.05
```

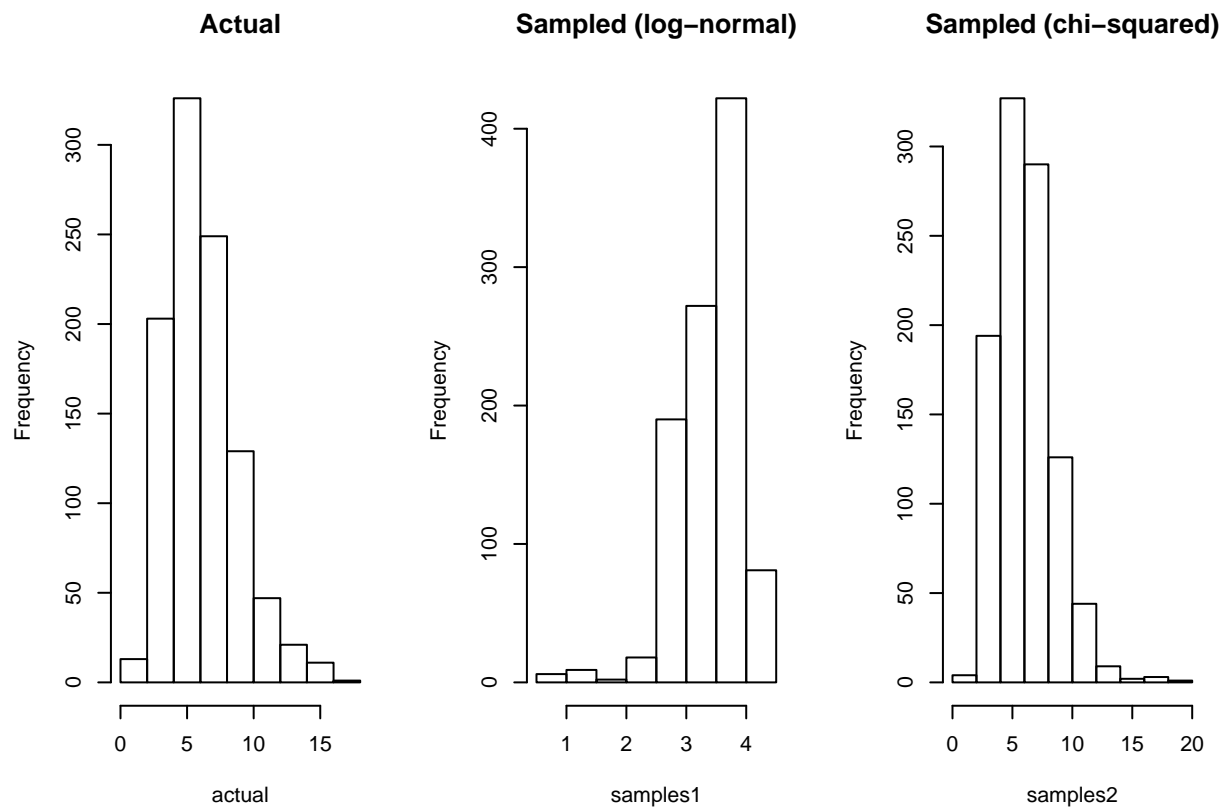
```
plot(samples2, type="l")
```



```
x <- 0:20  
y <- sapply(x, targetdensity)  
plot(x, y)
```



```
oldpar <- par(mfrow = c(1, 3))  
  
hist(actual, main="Actual")  
hist(samples1, main="Sampled (log-normal)")  
hist(samples2, main="Sampled (chi-squared)")
```



Geldman rubin

```
Geldman<-function(x){
  k <- nrow(x)
  n <- ncol(x)

  B <- (n / (k - 1)) * sum((rowMeans(x) - mean(x))^2)

  W <- sum((x - rowMeans(x))^2) / (k * (n - 1))

  VarV <- ((n - 1) / n) * W + B / n

  sqrtR <- sqrt(VarV / W)
  sqrtR
}

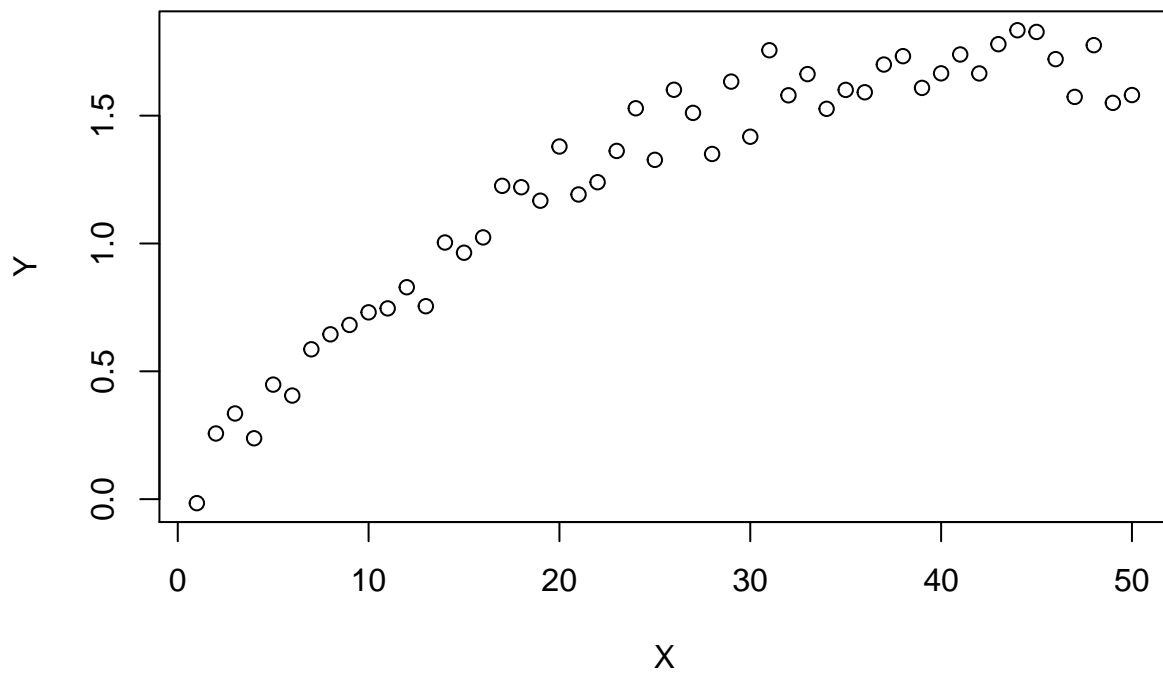
resultMatrix <- do.call(rbind, lapply(1:10, FUN = function(x)
  metropolis_hastings(X0 = x, iters = iters, funcs = chisquarefuncs)))

GeldmanRes <- Geldman(resultMatrix)
GeldmanRes

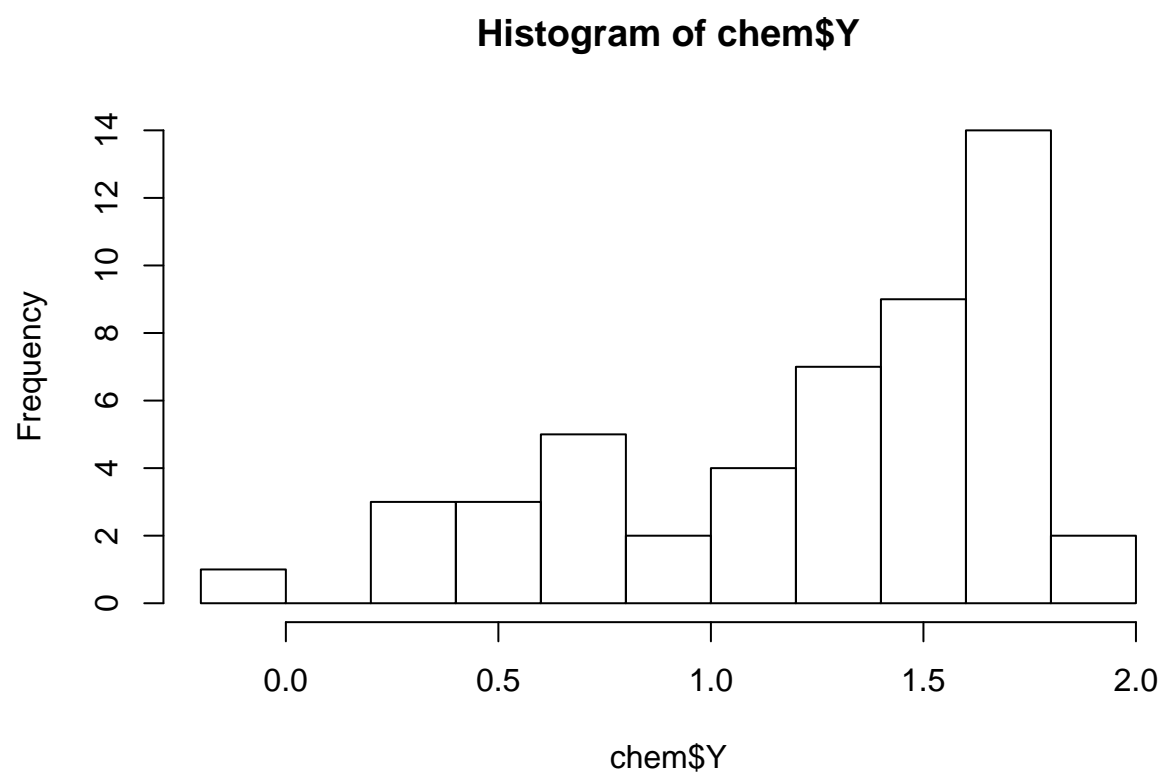
## [1] 1
```

Question 2

```
load("../data/chemical.RData")  
  
chem <- data.frame(X = X , Y = Y)  
  
plot(chem)
```

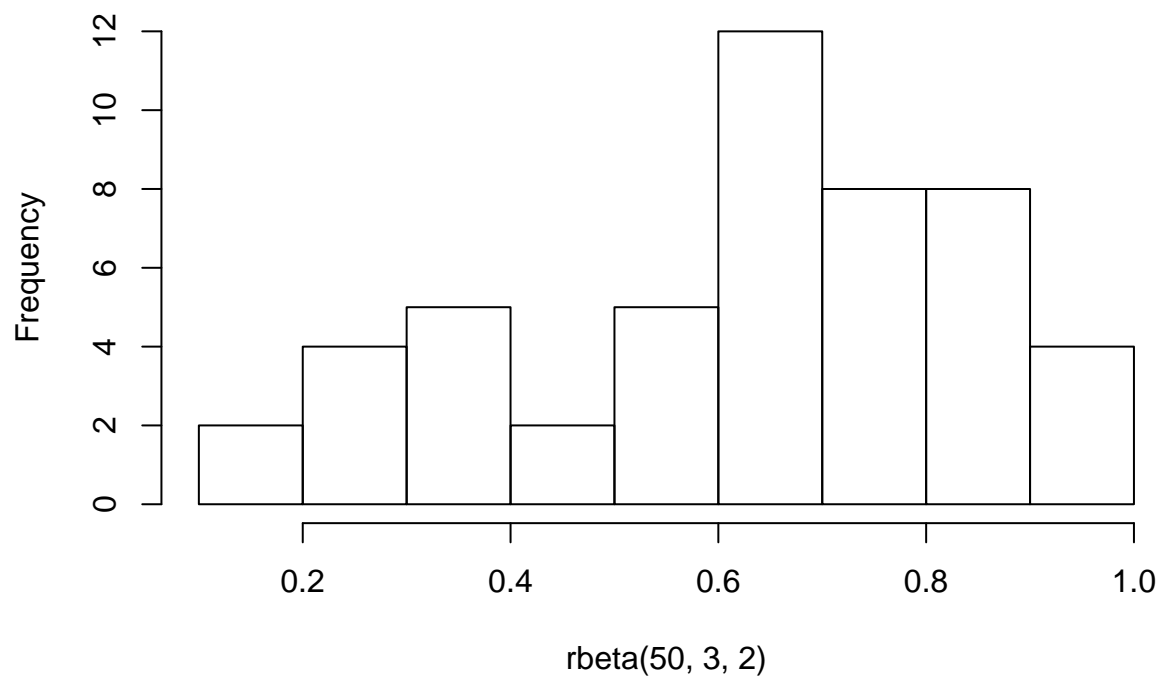


```
hist(chem$Y)
```



```
hist(rbeta(50,3,2))
```

Histogram of rbeta(50, 3, 2)



$$p(\mu) = \frac{1}{\sqrt{(0.08\pi)^n}} \exp\left(-\frac{\sum_{i=1}^{n-1} (x - \mu_i)^2}{0.08}\right)$$

$$p(y|\mu) = \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} \exp\left(-\frac{1}{2}(x - \mu)\Sigma^{-1}(x - \mu)\right)$$

where $\Sigma = 0.2I$.

$$p(\mu|y) \propto p(y|\mu)p(\mu)$$