732A62 Lab 2

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Assignment 1

a)

```
library(astsa)
library(kernlab)
library(TSA)
library(forecast)

set.seed(12345)
AR3 <- arima.sim(1000, model = list(order = c(3,0,0),
                                    ar = c(0.8, -0.2, 0.1)))

## The theoretical
AR3.pacf <- pacf(AR3, plot=F)
AR3.data <- ts.intersect(xt = AR3, x1 = lag(AR3, 1), x2 =lag(AR3, 2), x3= lag(AR3, 3))

AR.lm <- resid(lm(xt ~ x1 + x2, data = AR3.data))
AR.lm.lag3 <- resid(lm(x3 ~ x1 + x2 , data = AR3.data))

AR3.pacf[3]
```

```
##
## Partial autocorrelations of series 'AR3', by lag
##
##     3
## 0.117
```

```
cor(AR.lm, AR.lm.lag3)
```

```
## [1] 0.1146076
```

b)

```
set.seed(12345)
AR2 <- arima.sim(100, model = list(order = c(2,0,0),
                                   ar = c(0.8, 0.1)))

ar2.yw <- ar(AR2, order.max = 2, method = "yw", aic = FALSE)
ar2.ols <- ar(AR2, order.max = 2, method = "ols", aic = FALSE)
ar2.mle <- arima(AR2, order = c(2,0,0), method = "ML")

ar2.yw
```

```
##
## Call:
## ar(x = AR2, aic = FALSE, order.max = 2, method = "yw")
##
## Coefficients:
##      1       2
## 0.8029  0.1037
##
## Order selected 2  sigma^2 estimated as  1.267
```

```
ar2.ols
```

```
##
## Call:
## ar(x = AR2, aic = FALSE, order.max = 2, method = "ols")
##
## Coefficients:
##      1       2
## 0.8067  0.1205
##
## Intercept: -0.04401 (0.1074)
##
## Order selected 2  sigma^2 estimated as  1.129
```

```
ar2.mle
```

```
##
## Call:
## arima(x = AR2, order = c(2, 0, 0), method = "ML")
##
## Coefficients:
##          ar1     ar2  intercept
##       0.7967  0.1189     0.8290
## s.e.  0.0992  0.1000     1.1385
##
## sigma^2 estimated as 1.126:  log likelihood = -148.71,  aic = 303.41
```

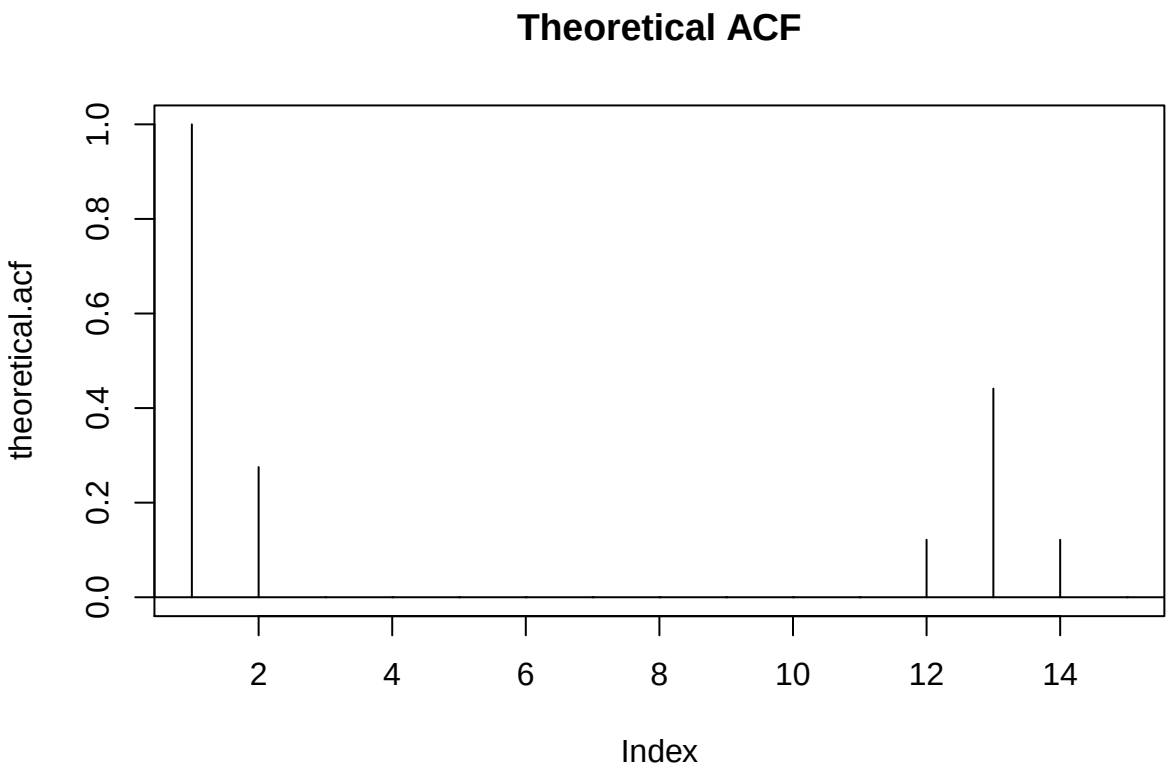
Yes, the theoretical value for

$$\phi_2$$

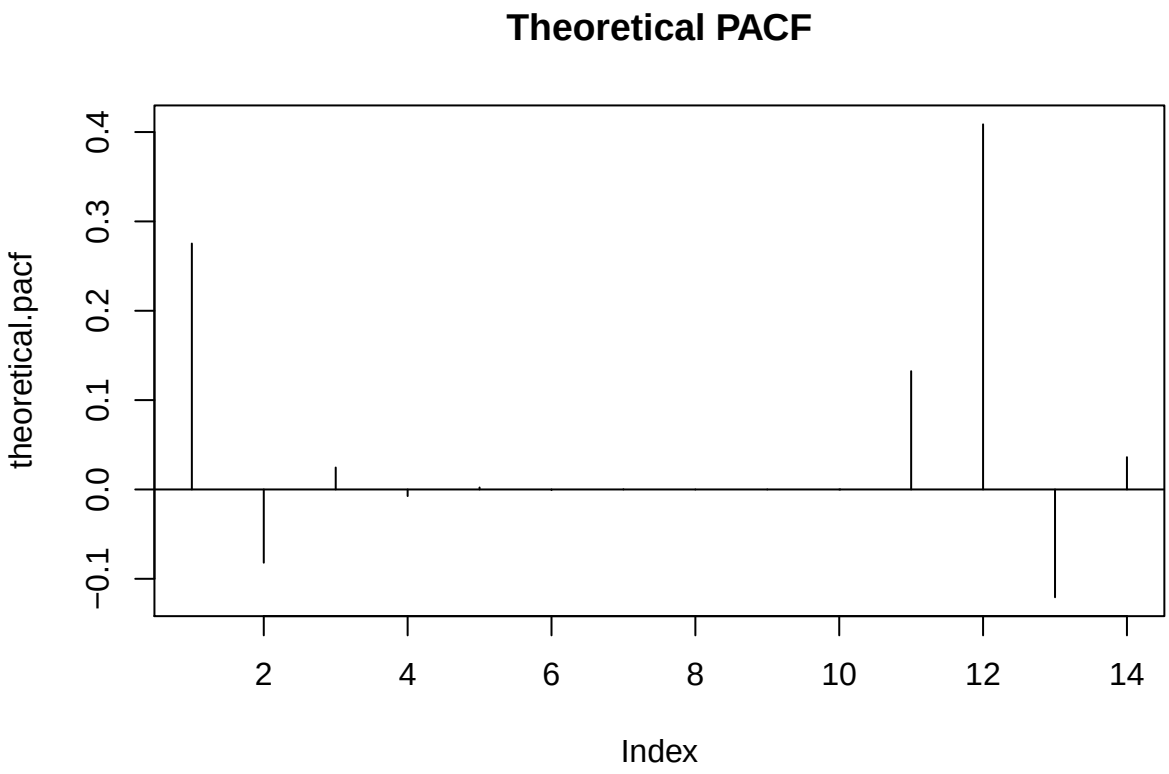
is inside the confidence-intervall for the ML estimate.

c)

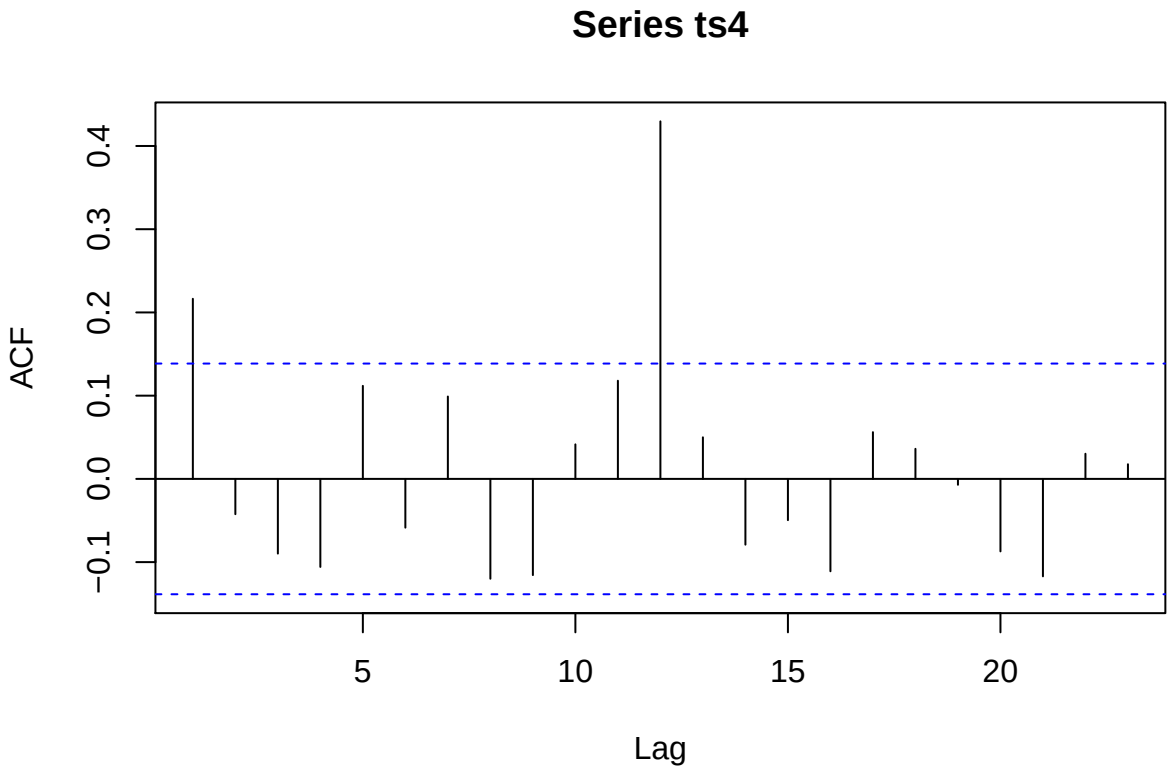
```
set.seed(12345)
ma.coef <- c(0.3, rep(0, 10), 0.6)
ts4 <- arima.sim(n=200, model=list(order=c(0, 0, 12), ma = ma.coef))

theoretical.acf <- ARMAacf(ma=c(ma.coef, 0.3 * 0.6))
theoretical.pacf <- ARMAacf(ma=c(ma.coef, 0.3 * 0.6), pacf=TRUE)

plot(theoretical.acf, type="h", main="Theoretical ACF")
abline(h=0)
```



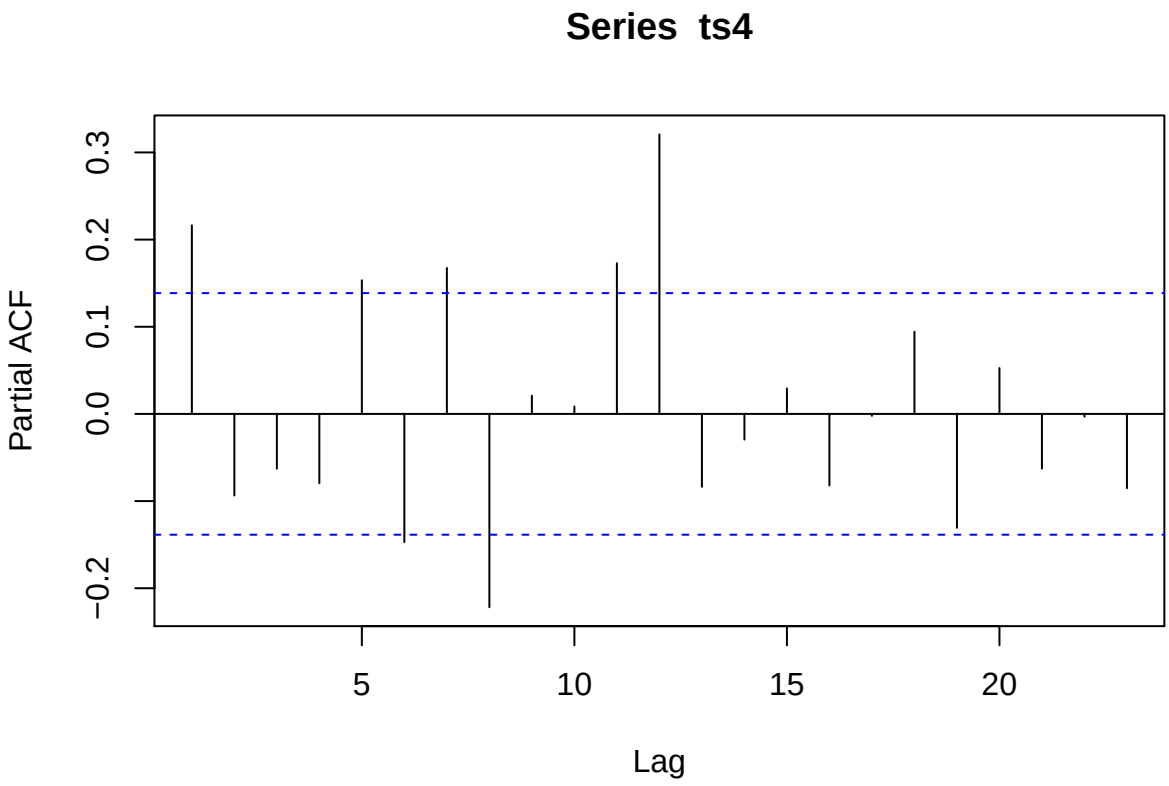

```
plot(theoretical.pacf, type="h", main="Theoretical PACF")
abline(h=0)
```




```
empirical.acf <- acf(ts4)
```

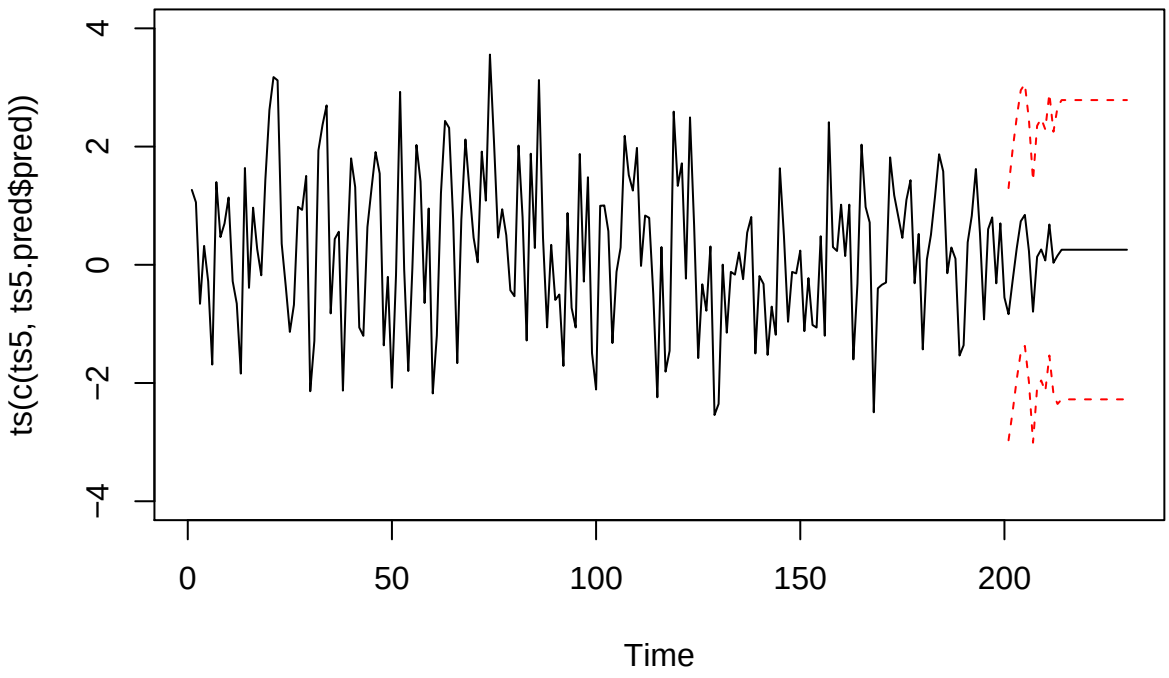



```
empirical.pacf <- pacf(ts4)
```



d)

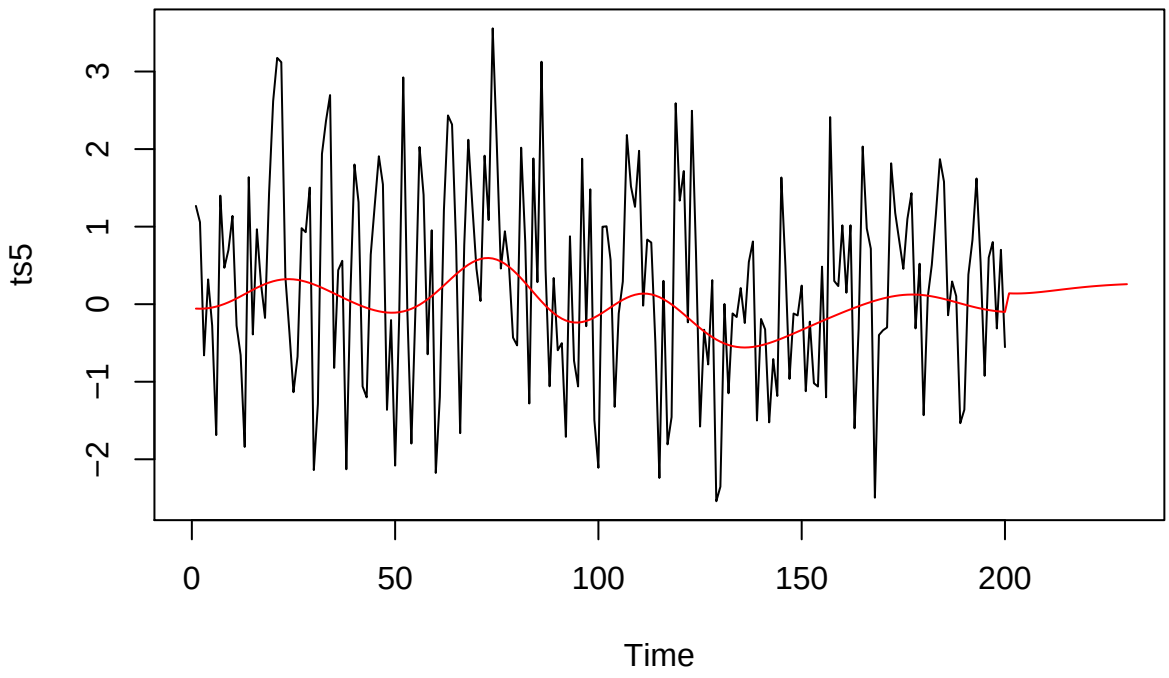
```
set.seed(12345)
ma.coef <- c(0.3, rep(0, 10), 0.6)
ts5 <- arima.sim(n=200, model=list(order=c(0, 0, 12), ma = ma.coef))

ts5.fit <- arima(ts5, order=c(0, 0, 1), seasonal=list(order=c(0, 0, 1), period=12))
ts5.pred <- predict(ts5.fit, n.ahead=30, se.fit=TRUE)

plot(ts(c(ts5, ts5.pred$pred)), ylim=c(-4, 4))
lines(200 + 1:length(ts5.pred$pred), ts5.pred$pred + 1.96 * ts5.pred$se, lty=2, col="red")
lines(200 + 1:length(ts5.pred$pred), ts5.pred$pred - 1.96 * ts5.pred$se, lty=2, col="red")
```




```
gausspr.data <- data.frame(y=ts5, x=1:200)
gausspr.fit <- gausspr(y ~ x, gausspr.data)
```

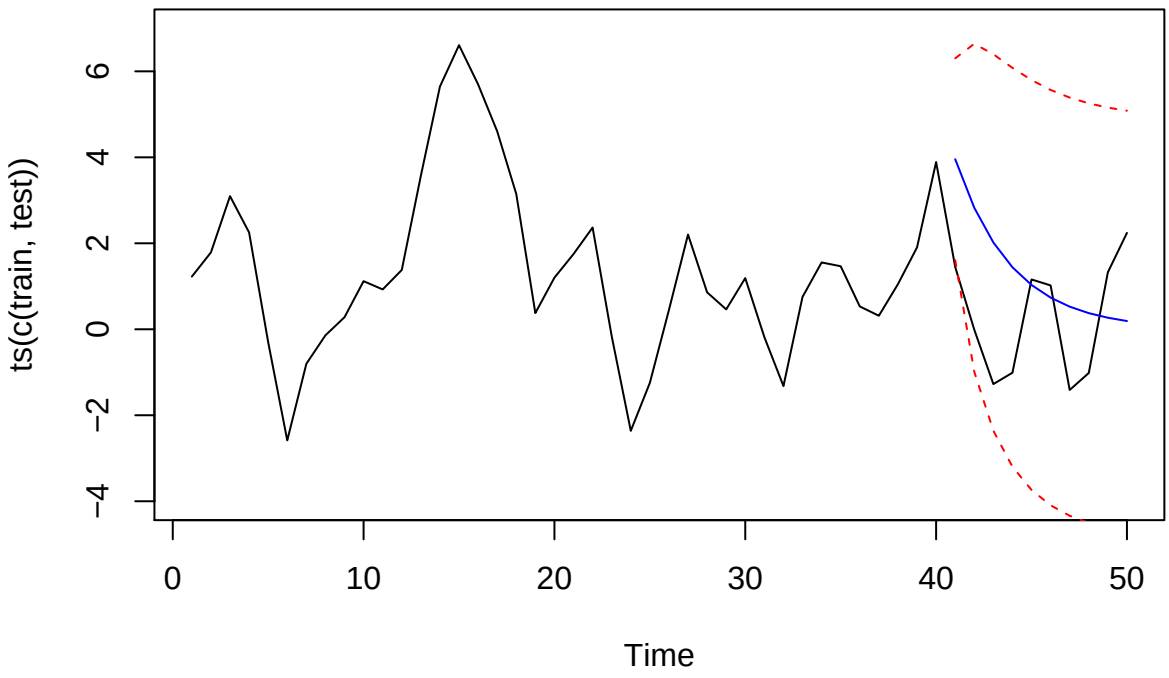
```
## Using automatic sigma estimation (sigest) for RBF or laplace kernel
```

```
gausspr.pred <- predict(gfit, data.frame(x=201:230))

plot(ts5, xlim=c(0, 230))
lines(c(fitted(gausspr.fit), gausspr.pred), , col="red")
```

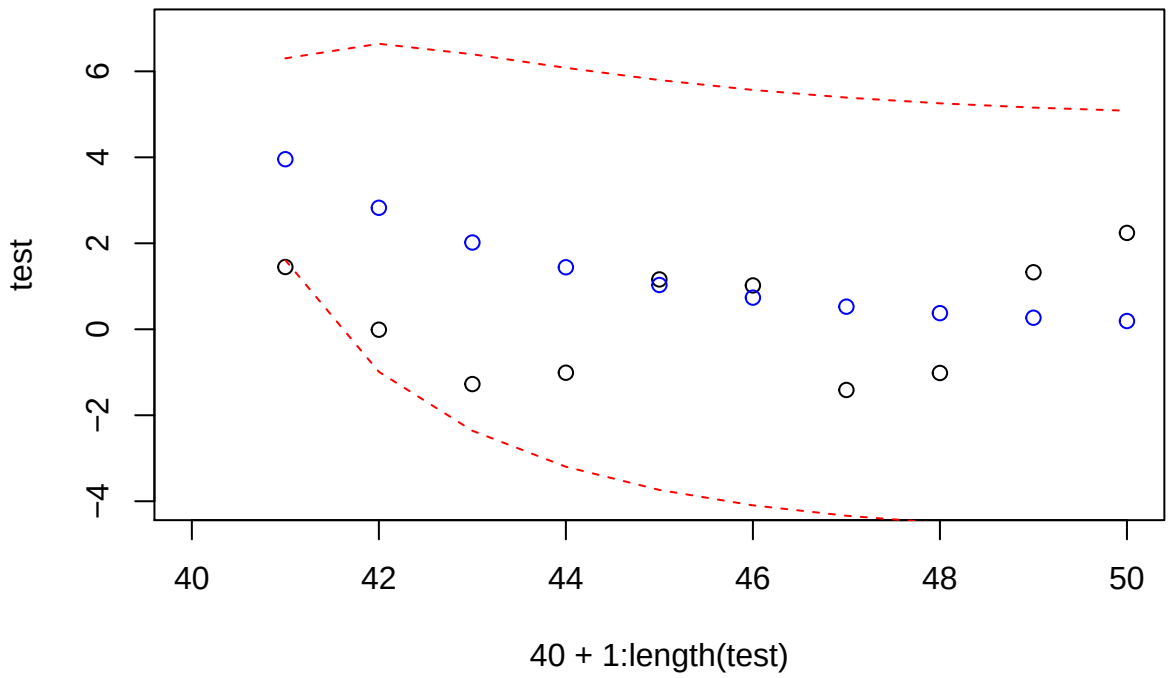


e)

```
set.seed(12345)
ts6 <- arima.sim(model=list(ma=c(0.5), ar=c(0.7)), n=50)

train <- ts(ts6[1:40])
test <- ts(ts6[41:50])

ts6.fit <- arima(train, order=c(1, 0, 1), include.mean = F)
ts6.pred <- predict(ts6.fit, n.ahead=10)

plot(ts(c(train, test)), ylim=c(-4, 7), type="l")
lines(40 + 1:length(test), ts6.pred$pred, col="blue")
lines(40 + 1:length(test), ts6.pred$pred + 1.96 * ts6.pred$se, lty=2, col="red")
lines(40 + 1:length(test), ts6.pred$pred - 1.96 * ts6.pred$se, lty=2, col="red")
```




```
plot(40 + 1:length(test), test, ylim=c(-4, 7), xlim=c(40, 50), type="p")
points(40 + 1:length(test), ts6.pred$pred, col="blue")
lines(40 + 1:length(test), ts6.pred$pred + 1.96 * ts6.pred$se, lty=2, col="red")
lines(40 + 1:length(test), ts6.pred$pred - 1.96 * ts6.pred$se, lty=2, col="red")
```

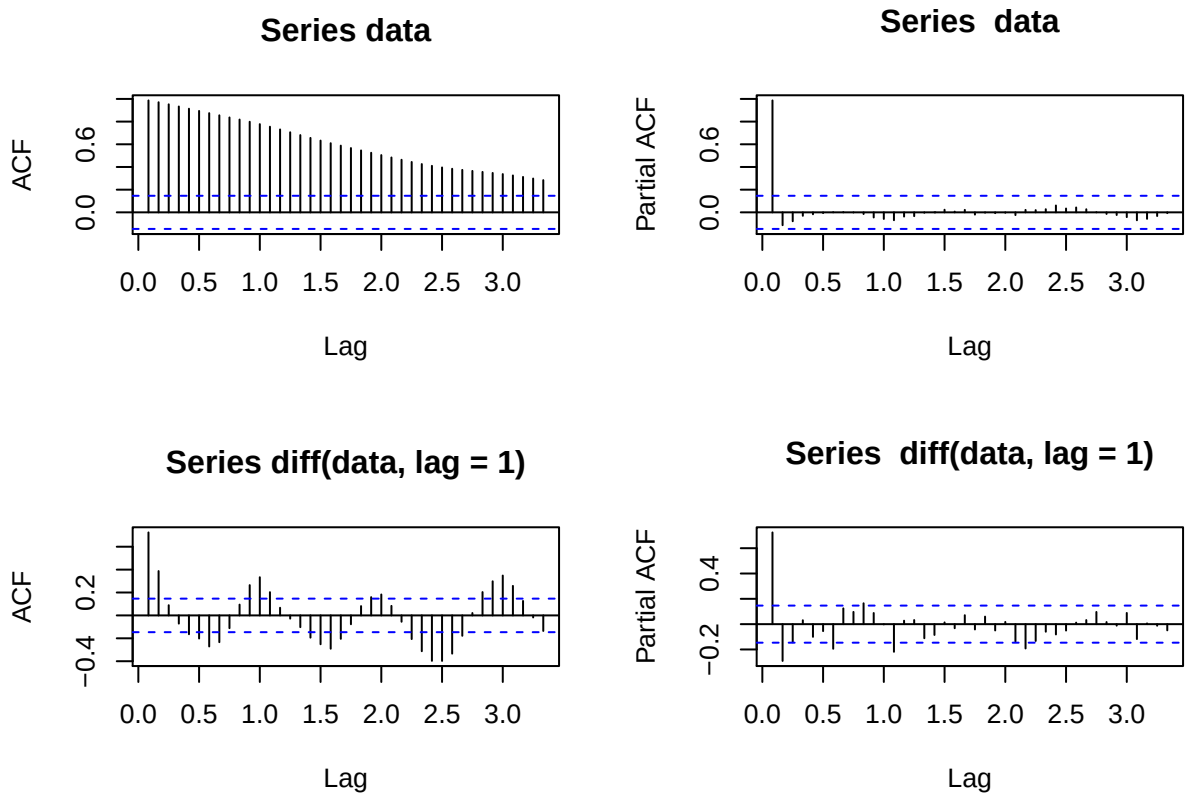

test
6
4
2
0
−2
−4
40
42
44
46
48
50
40 + 1:length(test)

Assignment 2

```
assignment2 <- function(data){
    old <- par(mfrow = c(2, 2))
    acf(data, lag.max = 40)
    pacf(data, lag.max = 40)
    acf(diff(data, lag = 1), lag.max = 40)
    pacf(diff(data, lag = 1), lag.max = 40)
    par(old)
}
```

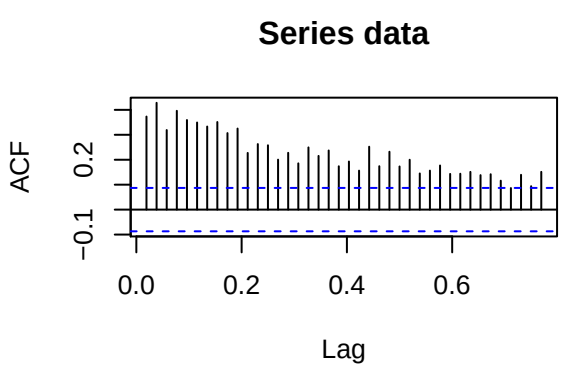
Chicken

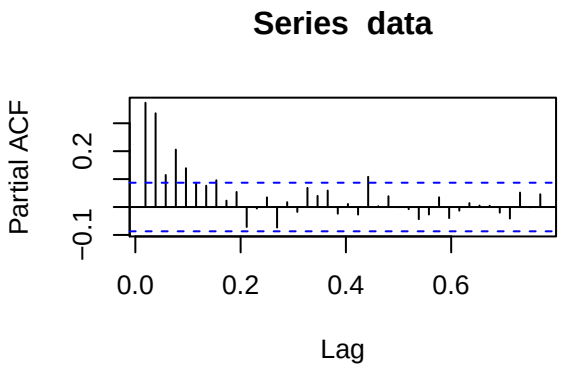
```
assignment2(chicken)
```

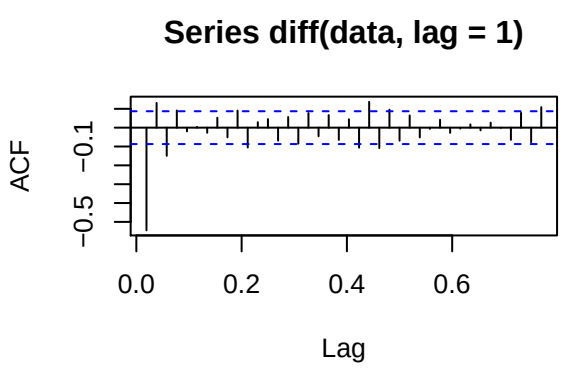


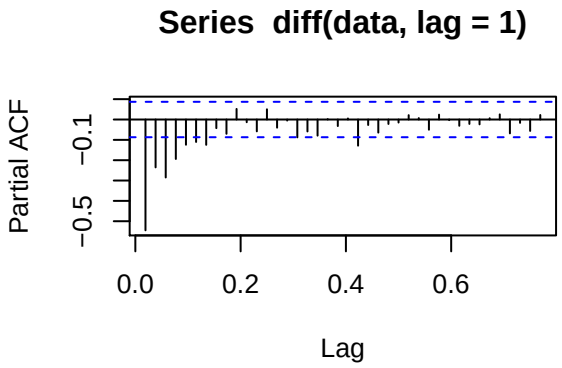
so2

```
assignment2(so2)
```



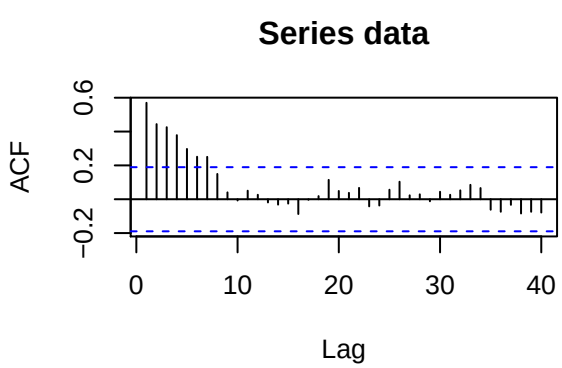


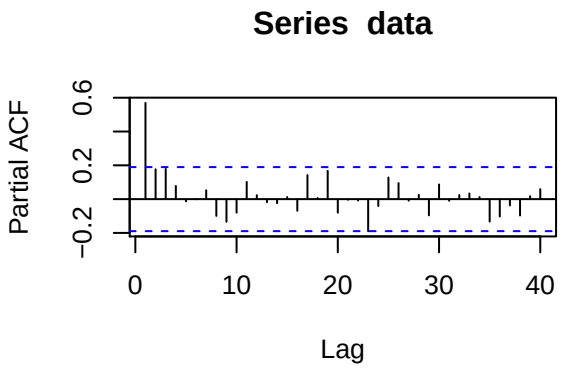


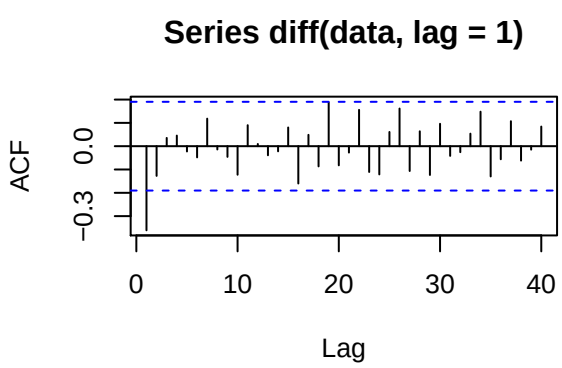


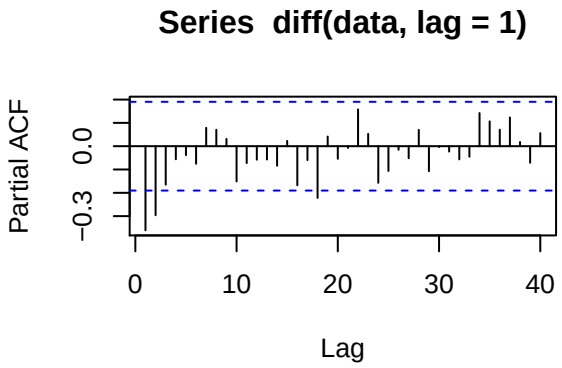
EQcount

```
assignment2(EQcount)
```



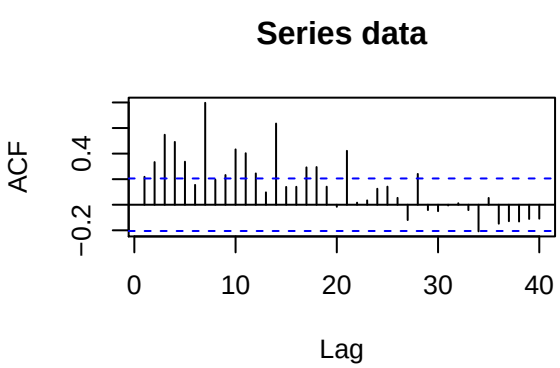


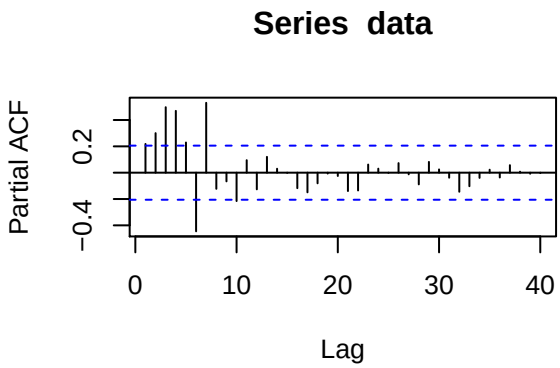


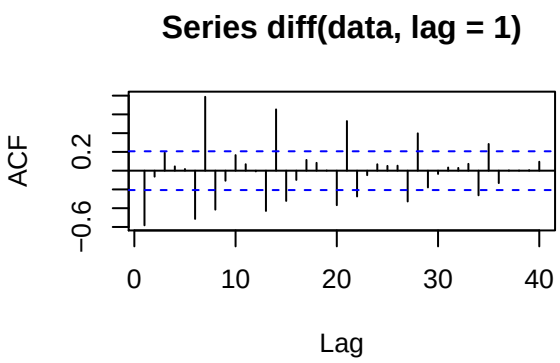


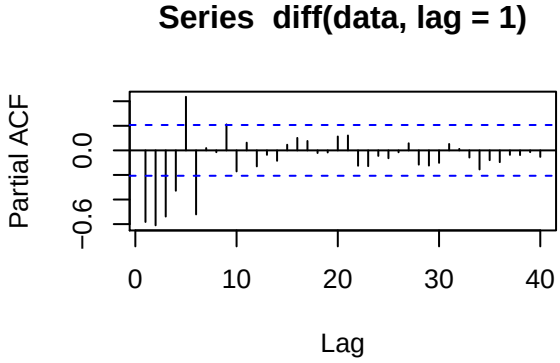
HCT

```
assignment2(HCT)
```


Series data
ACF
0.4
−0.2
0
10
20
30
40
Lag


Series data
Partial ACF
0.2
−0.4
0
10
20
30
40
Lag


Series diff(data, lag = 1)
ACF
0.2
−0.6
0
10
20
30
40
Lag

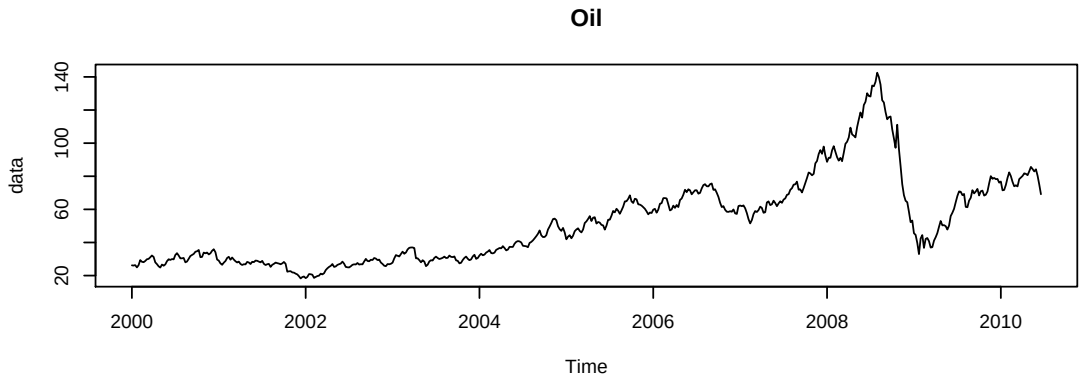

Series diff(data, lag = 1)
Partial ACF
0.0
−0.6
0
10
20
30
40
Lag

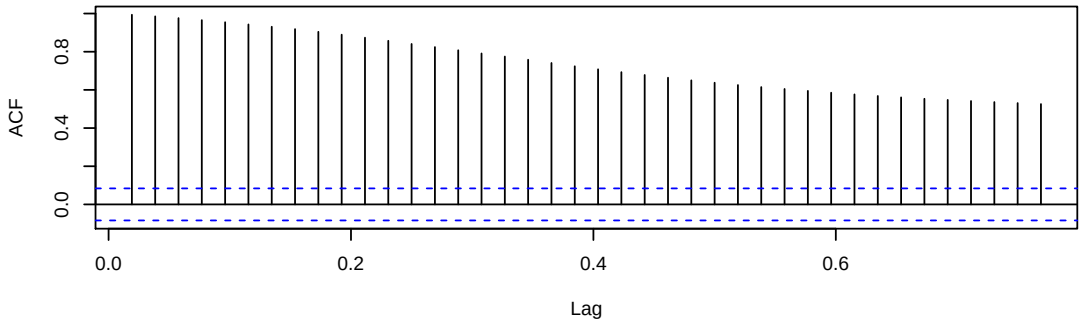
Assignment 3

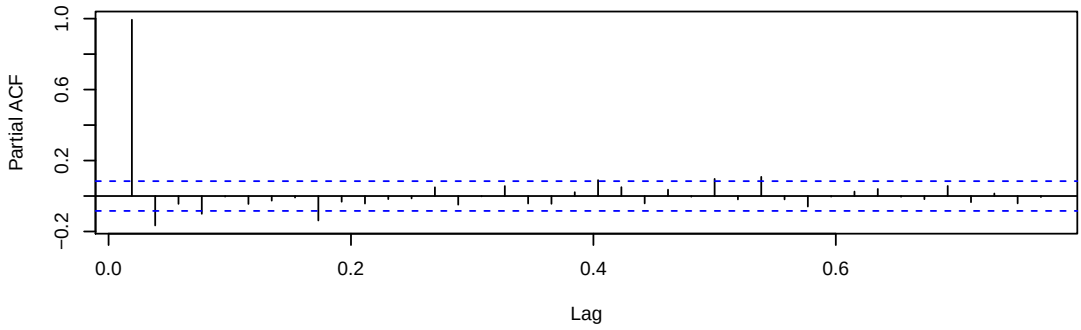
```
plot_helper <- function(data, title) {
    old <- par(mfrow=c(4, 1))
    plot(data, main=title)
    acf(data, lag.max=40, main="")
    pacf(data, lag.max=40, main="")
    qqnorm(data, main="", las=1)
    qqline(data)
    par(old)
}

test_helper <- function(data) {
    print(Box.test(data, lag = 1, type = "Ljung-Box"))
    print(suppressWarnings(adf.test(data)))
    e <- eacf(data)
}

fit_plot <- function(model) {
    pred <- predict(model, n.ahead=20, se.fit=TRUE)
    upper_band <- pred$pred + 1.96 * pred$se
    lower_band <- pred$pred - 1.96 * pred$se

    plot(c(model$x, pred$pred), type="l", xlim=c(500, length(oil) + 20), ylim=c(min(lower_band), max(up
    lines(length(oil) + 1:20, upper_band, lty=2, col="red")
    lines(length(oil) + 1:20, lower_band, lty=2, col="red")
}
```

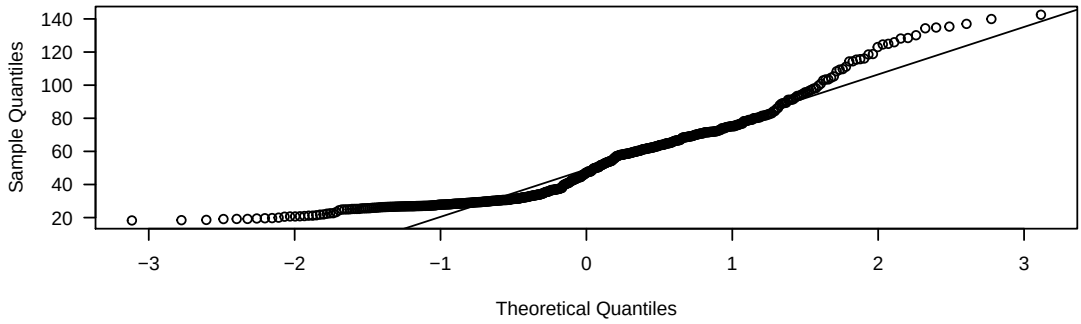
a)

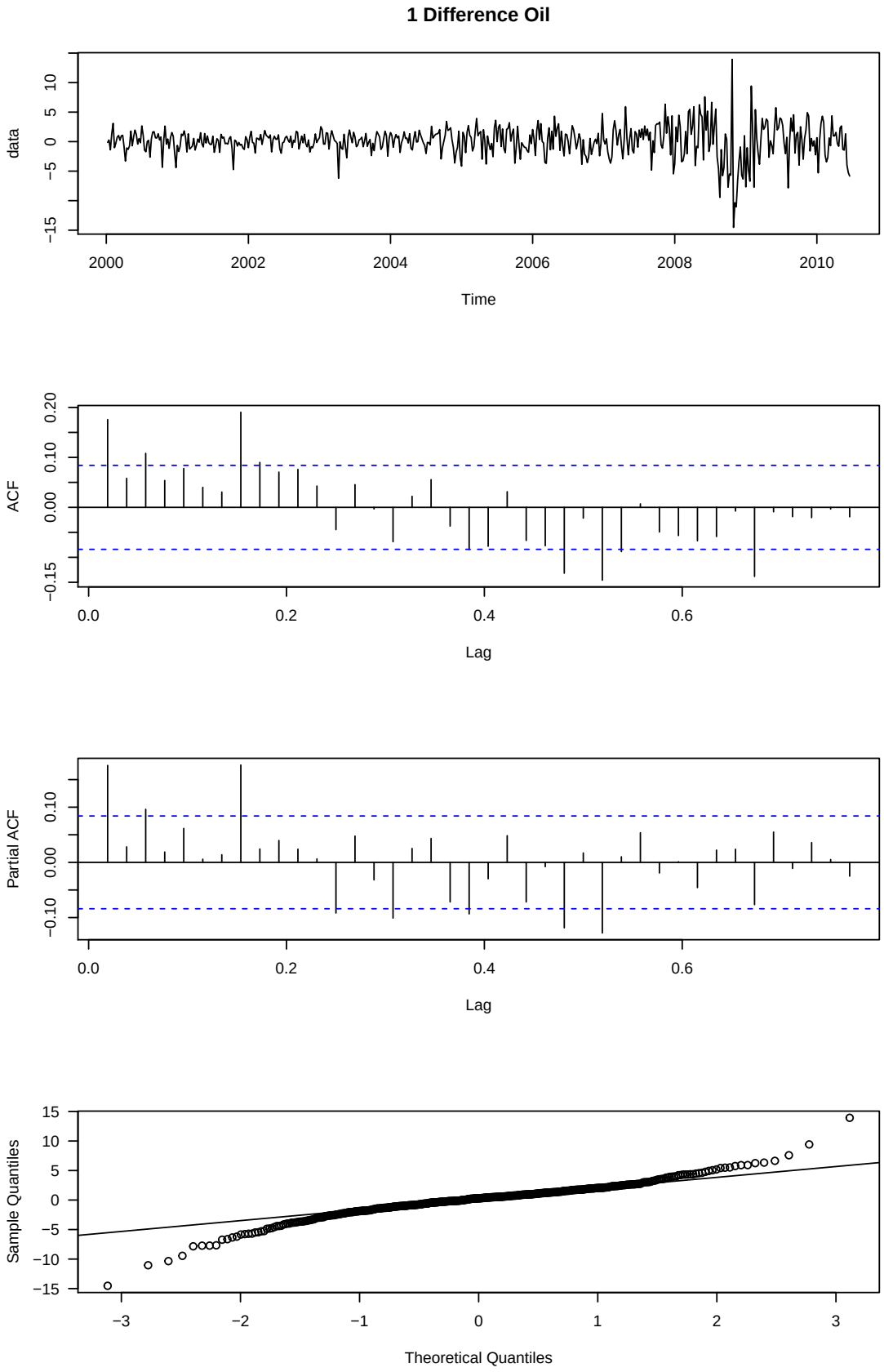
```
loil <- log(oil)
doil <- diff(oil)
ddoil <- diff(oil, 2)
dloil <- diff(loil)
ddloil <- diff(loil, 2)
```

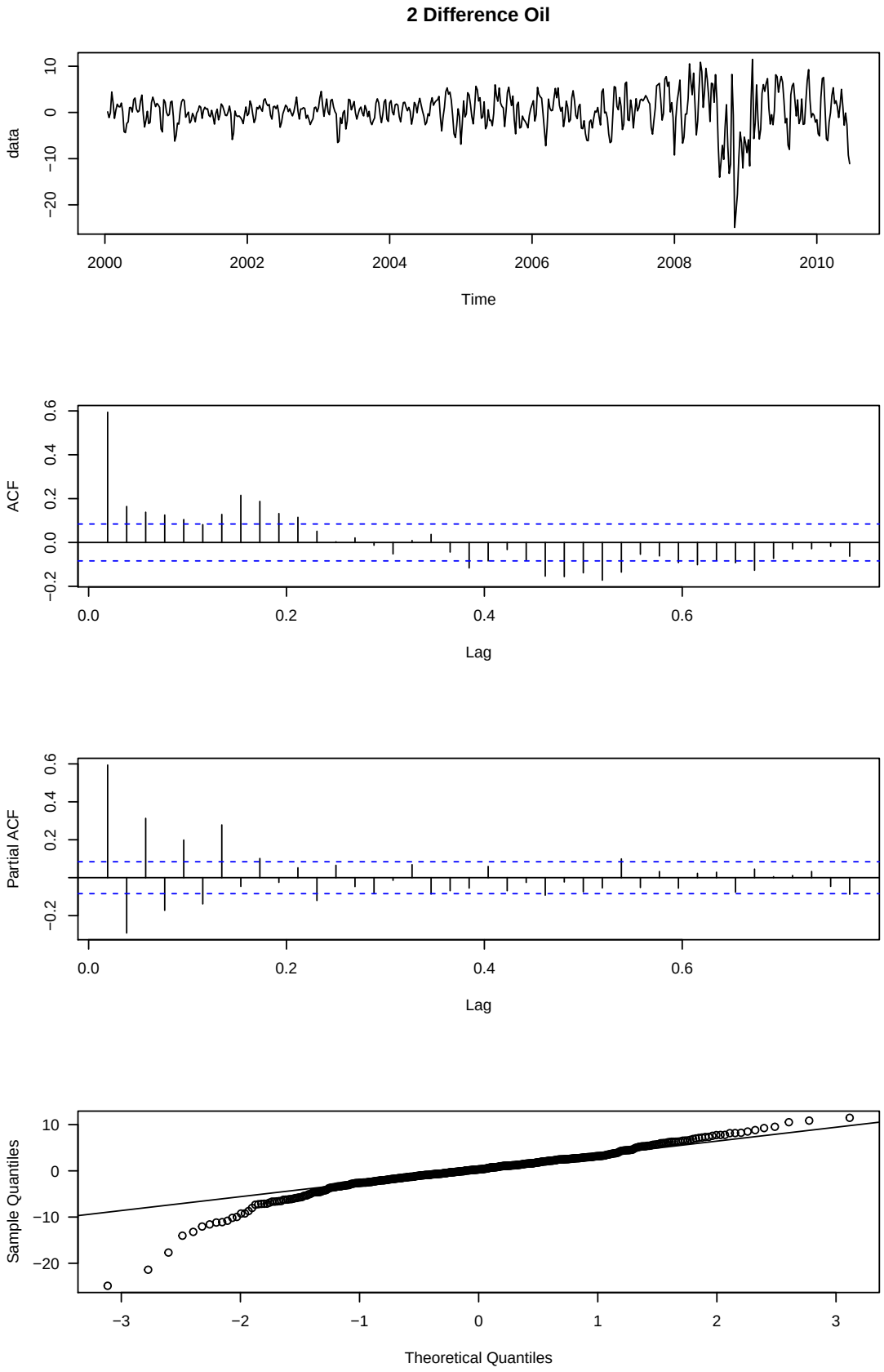

Oil
data
20
60
100
140
2000
2002
2004
2006
2008
2010
Time

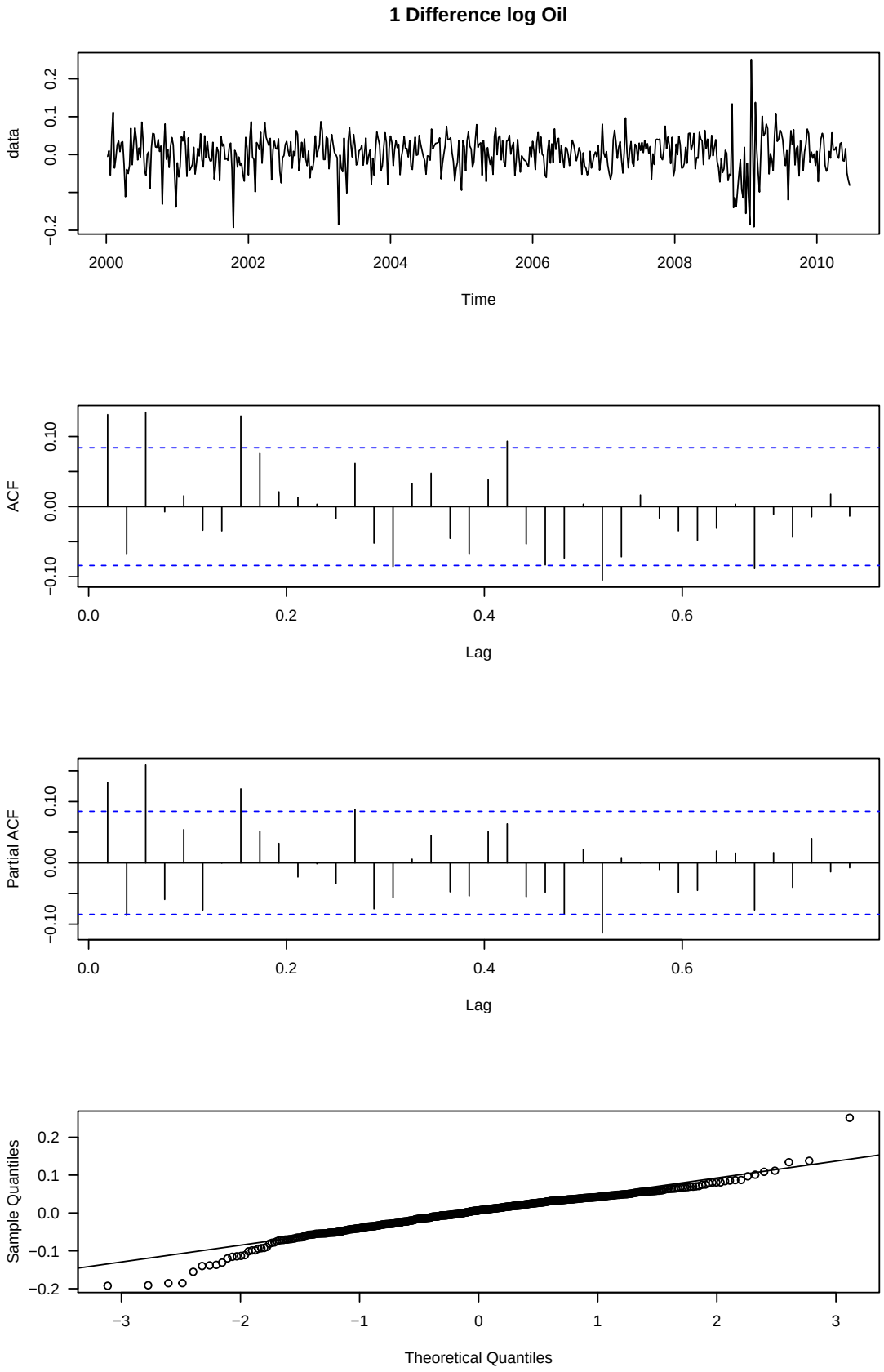

ACF
0.0
0.4
0.8
0.0
0.2
0.4
0.6
Lag

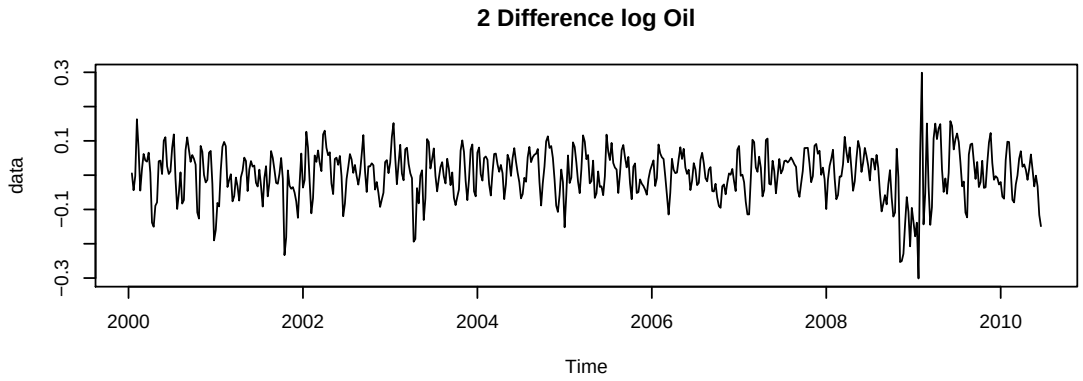

Partial ACF
−0.2
0.2
0.6
1.0
0.0
0.2
0.4
0.6
Lag

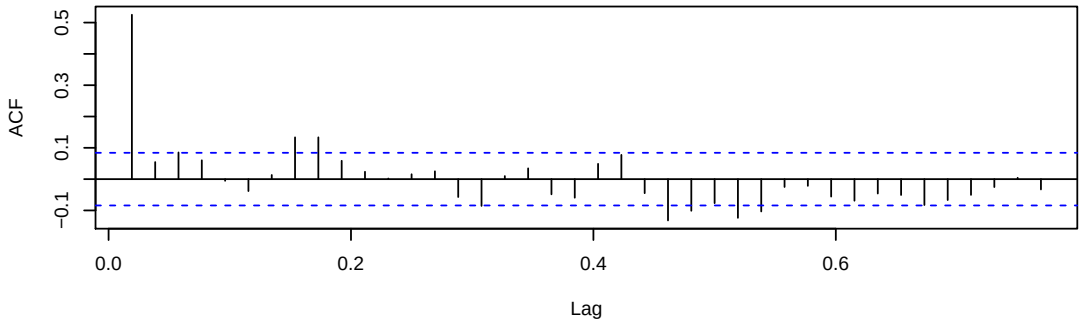

Sample Quantiles
20
40
60
80
100
120
140
−3
−2
−1
0
1
2
3
Theoretical Quantiles

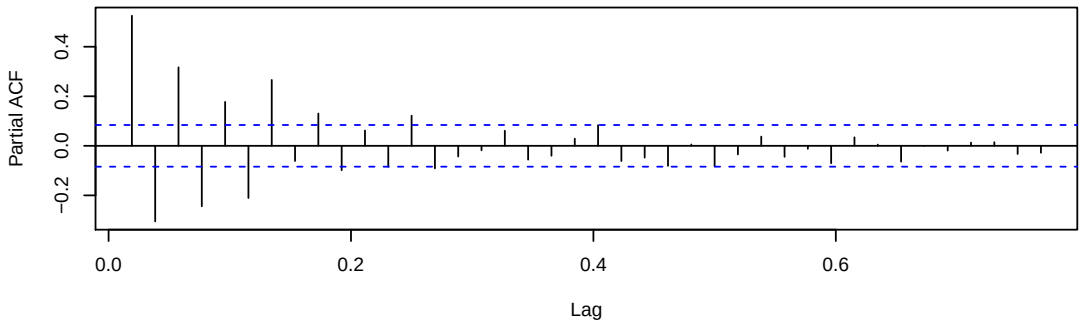

1 Difference Oil
data
Time
ACF
Lag
Partial ACF
Lag
Sample Quantiles
Theoretical Quantiles

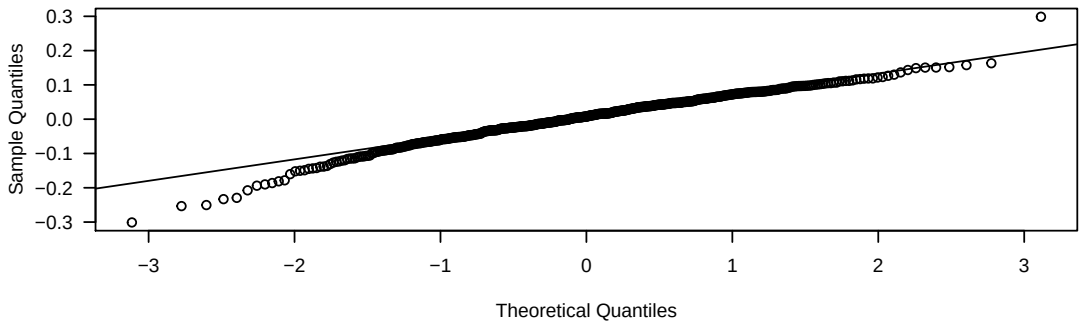

2 Difference Oil
data
10
0
−10
−20
2000
2002
2004
2006
2008
2010
Time
ACF
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
Lag
Partial ACF
0.6
0.4
0.2
−0.2
0.0
0.2
0.4
0.6
Lag
Sample Quantiles
10
0
−10
−20
−3
−2
−1
0
1
2
3
Theoretical Quantiles


1 Difference log Oil
data
Time
ACF
Lag
Partial ACF
Lag
Sample Quantiles
Theoretical Quantiles


2 Difference log Oil
data
0.3
0.1
−0.1
−0.3
2000
2002
2004
2006
2008
2010
Time


ACF
0.5
0.3
0.1
−0.1
0.0
0.2
0.4
0.6
Lag


Partial ACF
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
Lag


Sample Quantiles
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−3
−2
−1
0
1
2
3
Theoretical Quantiles

```
test_helper(doil)
```

```
##
##  Box-Ljung test
##
## data:  data
## X-squared = 16.884, df = 1, p-value = 3.974e-05
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -5.3269, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o x x o o  o  o  o
## 1 x o o o o o o x o o o  o  o  o
## 2 x x o o o o o x o o o  o  o  o
## 3 x x x o o o o x o o o  o  o  o
## 4 x x x o o o o x o o o  o  o  o
## 5 x x o o o o o x o o o  o  o  o
## 6 x o x o x o o x o o o  o  o  o
## 7 o o x o x o x x o o o  o  o  o
```

```
test_helper(ddoil)
```

```
##
##  Box-Ljung test
##
## data:  data
## X-squared = 192.72, df = 1, p-value < 2.2e-16
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -4.7773, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x o x x x x x  o  o  o
## 1 x x o o o o o x o o o  o  o  o
## 2 x x x o o o o x x o x  o  o  o
## 3 x x x o o o o x o o x  o  o  o
## 4 x x o o o o o x x o o  o  o  o
## 5 x x o x x x o x o o o  o  o  o
## 6 x x o x x x x x o o o  o  o  o
## 7 x x o x x x o x o x o  o  o  o
```

```
test_helper(dloil)
```

```
##
```

```
##  Box-Ljung test
##
## data:  data
## X-squared = 9.4307, df = 1, p-value = 0.002134
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -6.3708, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o x o o o  o  o  o
## 1 x o x o o o o x o o o  o  o  o
## 2 x x x o o o o x o o o  o  o  o
## 3 x x x o o o o x o o o  o  o  o
## 4 x o x o o o o x o o o  o  o  o
## 5 x x x o x o o x o o o  o  o  o
## 6 o x x o x x o x o o o  o  o  x
## 7 o x x x x x x x o x o  o  o  o
```

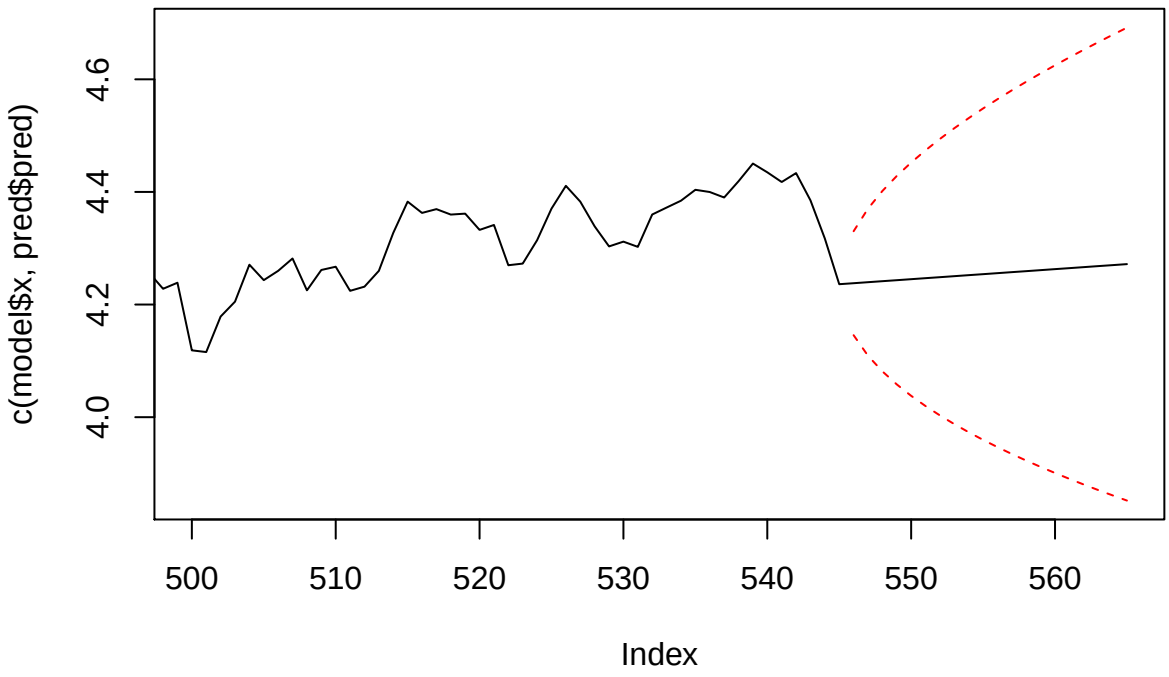
```
test_helper(ddloil)
```

```
##
##  Box-Ljung test
##
## data:  data
## X-squared = 150.51, df = 1, p-value < 2.2e-16
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -5.6251, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o x x o o  o  o  o
## 1 x x o o o o o x x o o  o  o  o
## 2 x x x o o o o x x o o  o  o  o
## 3 x x x x o o o x x o o  o  o  o
## 4 x x x x o o o x x o o  o  o  o
## 5 x o x x x o o x x o o  o  o  o
## 6 x o x x x x x x o x o  o  o  o
## 7 x x x x x x x x o x o  o  o  o
```

```
fit1 <- Arima(loil, order=c(0, 2, 1))
fit1
```

```
## Series: loil
## ARIMA(0,2,1)
##
## Coefficients:
##           ma1
##       -1.0000
## s.e.   0.0061
##
## sigma^2 estimated as 0.002213:  log likelihood=886.63
## AIC=-1769.26   AICc=-1769.24   BIC=-1760.67
```

```
fit_plot(fit1)
```

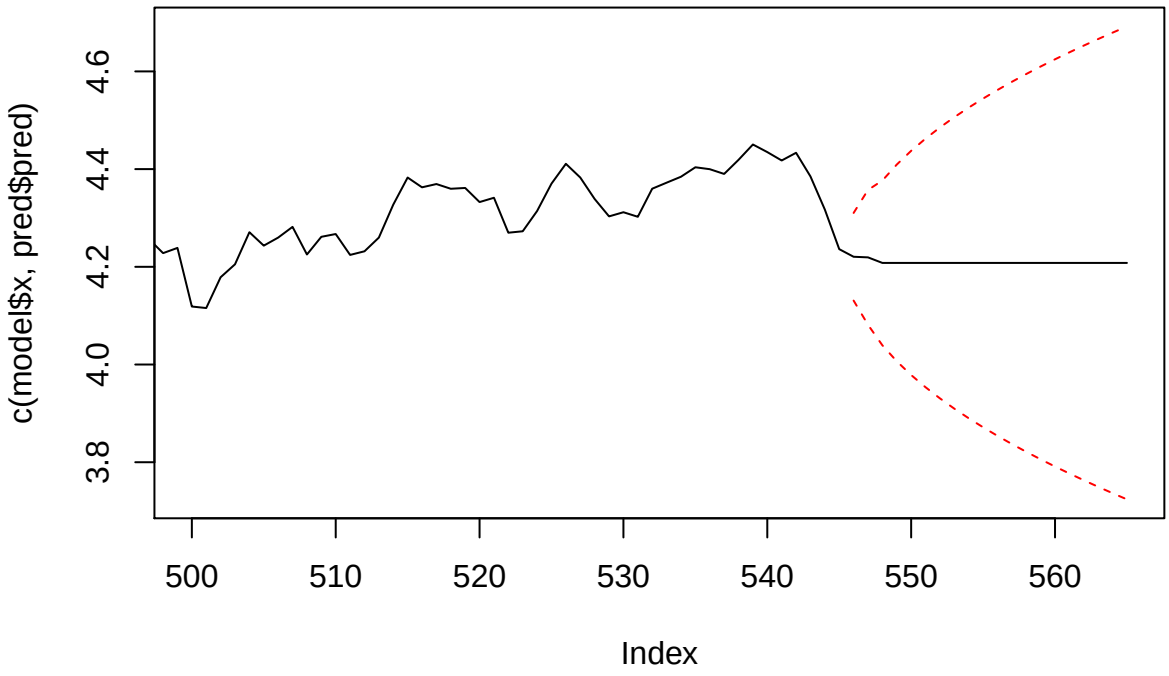


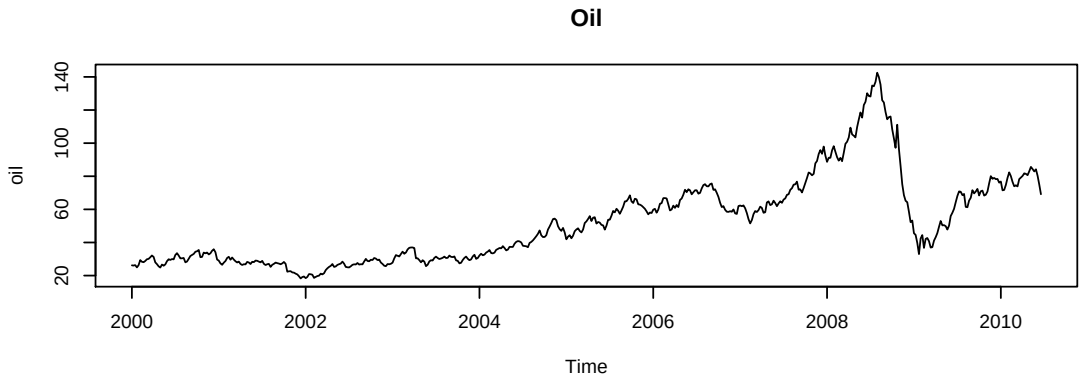

```
fit2 <- Arima(loil, order=c(0, 1, 3))
fit2
```

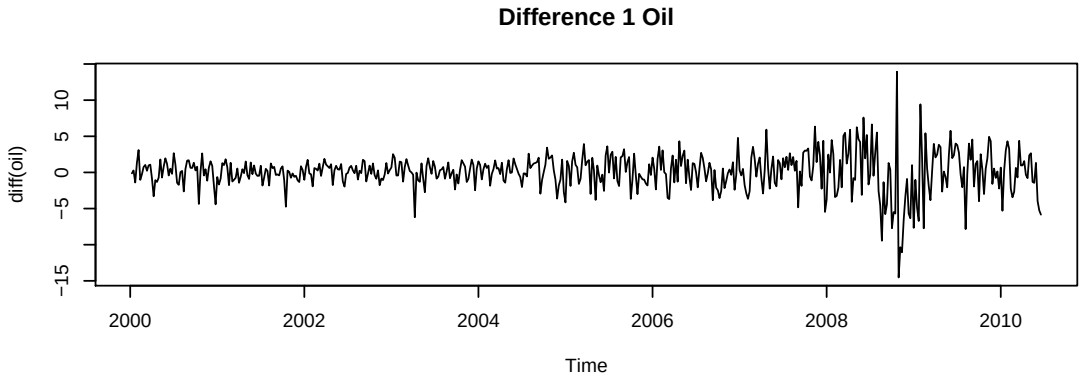
```
## Series: loil
## ARIMA(0,1,3)
##
## Coefficients:
##          ma1      ma2     ma3
##       0.1696  -0.0886  0.1458
## s.e.  0.0424   0.0424  0.0429
##
```

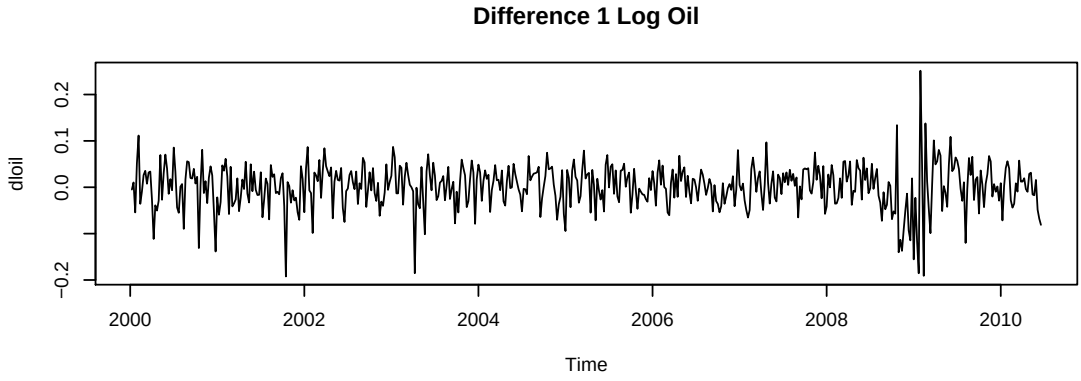
```
## sigma^2 estimated as 0.002094:  log likelihood=907.41
## AIC=-1806.83   AICc=-1806.75   BIC=-1789.63
```

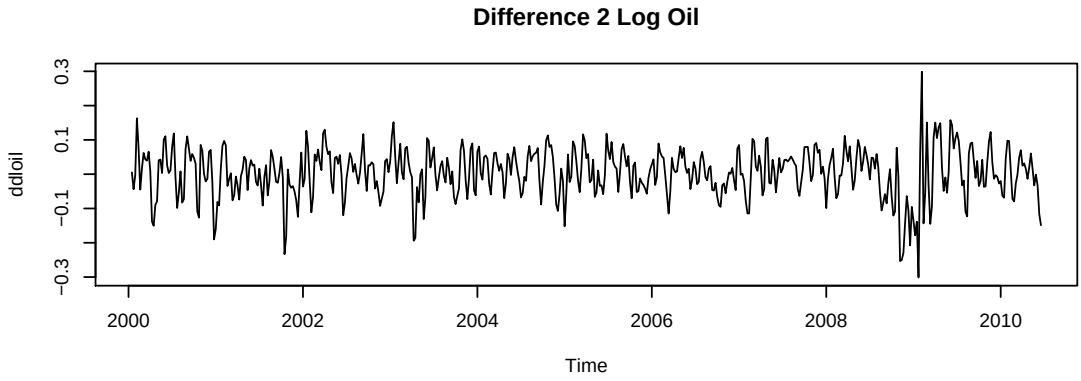
```
fit_plot(fit2)
```



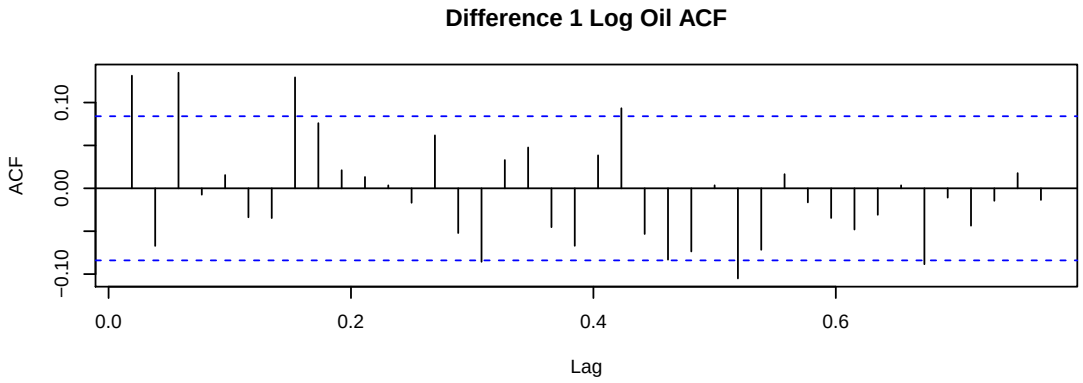

Oil
oil
20
60
100
140
2000
2002
2004
2006
2008
2010
Time

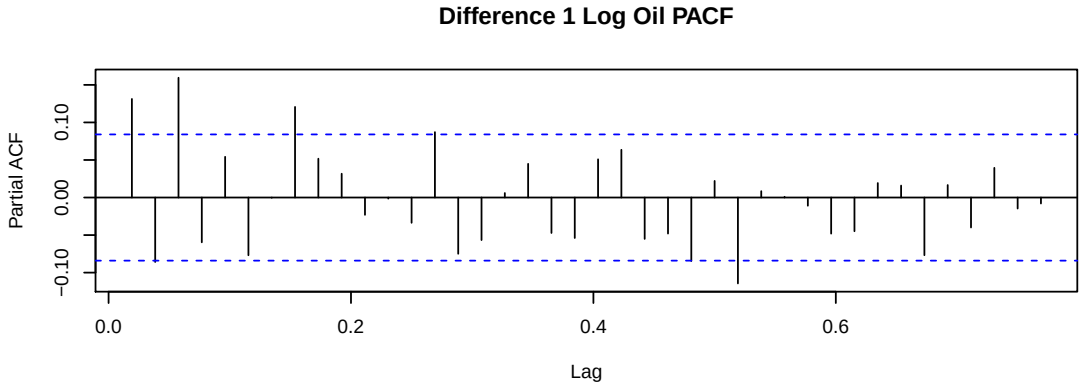

Difference 1 Oil
diff(oil)
−15
−5
0
5
10
2000
2002
2004
2006
2008
2010
Time

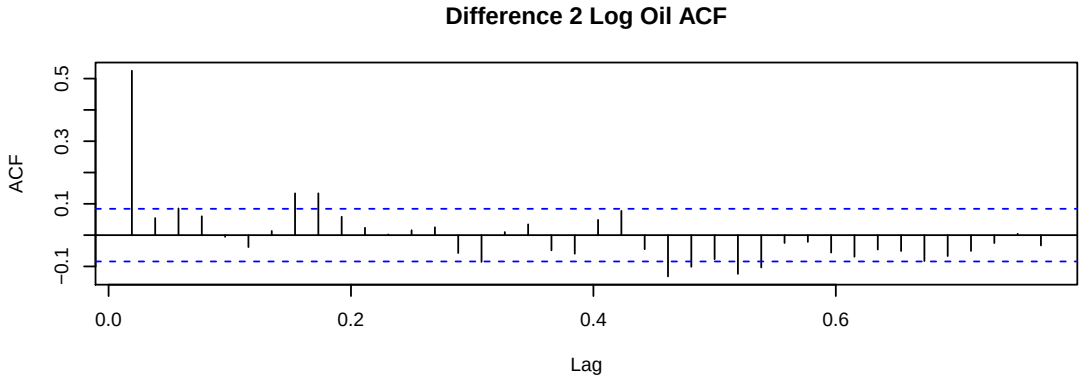

Difference 1 Log Oil
dloil
−0.2
0.0
0.1
0.2
2000
2002
2004
2006
2008
2010
Time

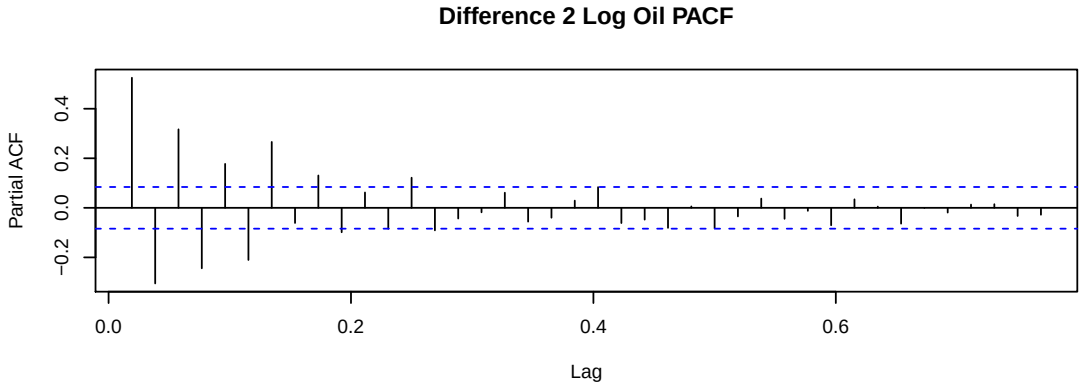

Difference 2 Log Oil
ddloil
−0.3
−0.1
0.1
0.3
2000
2002
2004
2006
2008
2010
Time

Clearly difference log is the data we should work with. bla, bla, ...


Difference 1 Log Oil ACF
ACF
0.10
0.00
−0.10
0.0
0.2
0.4
0.6
Lag


Difference 1 Log Oil PACF
Partial ACF
0.10
0.00
−0.10
0.0
0.2
0.4
0.6
Lag


Difference 2 Log Oil ACF
ACF
0.5
0.3
0.1
−0.1
0.0
0.2
0.4
0.6
Lag

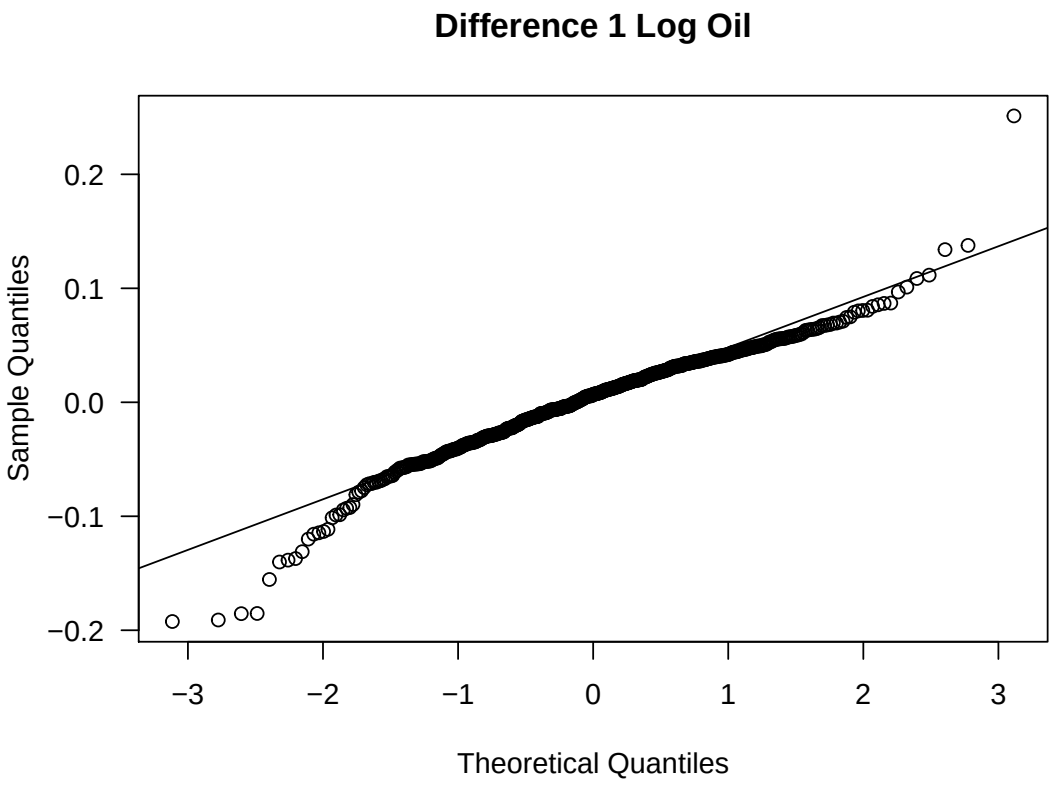

Difference 2 Log Oil PACF
Partial ACF
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
Lag

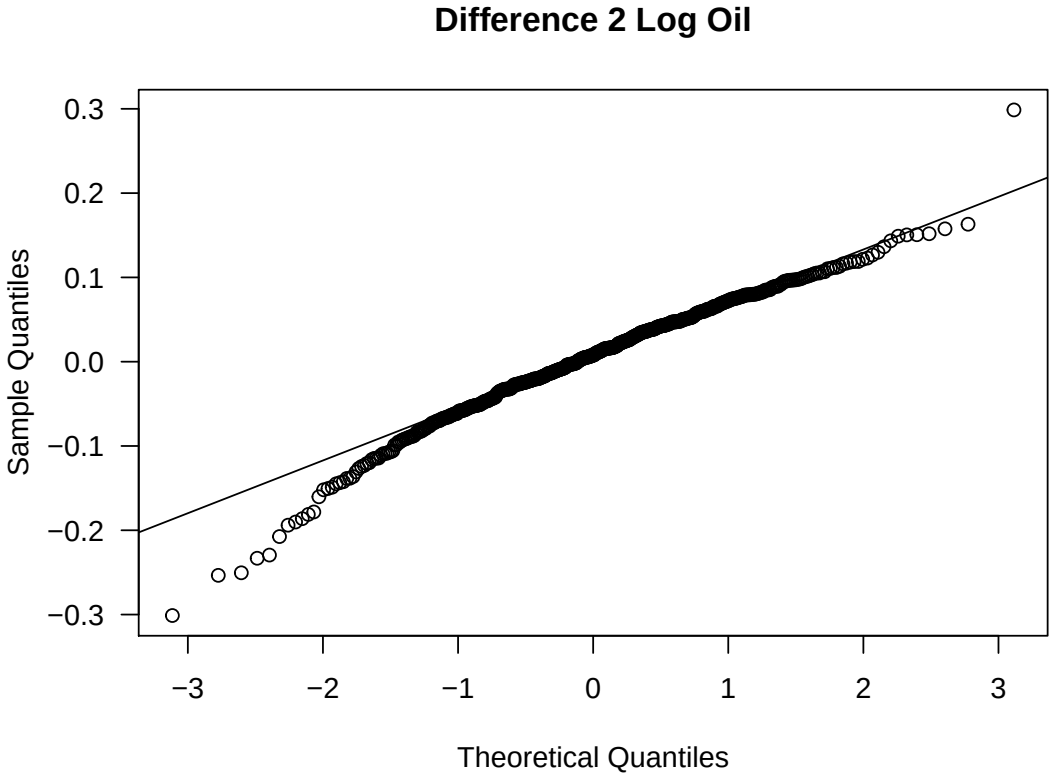
```
eacf(dloil)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o x o o o  o  o  o
## 1 x o x o o o o x o o o  o  o  o
## 2 x x x o o o o x o o o  o  o  o
## 3 x x x o o o o x o o o  o  o  o
## 4 x o x o o o o x o o o  o  o  o
## 5 x x x o x o o x o o o  o  o  o
## 6 o x x o x x o x o o o  o  o  x
## 7 o x x x x x x x o x o  o  o  o
```

```
eacf(ddloil)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o x x o o  o  o  o
## 1 x x o o o o o x x o o  o  o  o
## 2 x x x o o o o x x o o  o  o  o
## 3 x x x x o o o x x o o  o  o  o
## 4 x x x x o o o x x o o  o  o  o
## 5 x o x x x o o x x o o  o  o  o
## 6 x o x x x x x x o x o  o  o  o
## 7 x x x x x x x x o x o  o  o  o
```


Difference 1 Log Oil
Sample Quantiles
Theoretical Quantiles
0.2
0.1
0.0
−0.1
−0.2
−3
−2
−1
0
1
2
3


Difference 2 Log Oil
Sample Quantiles
Theoretical Quantiles
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−3
−2
−1
0
1
2
3

```r
fit1 <- Arima(loil, order=c(1, 1, 1))
fit1
```

```
## Series: loil
## ARIMA(1,1,1)
##
## Coefficients:
##           ar1     ma1
##       -0.5253  0.7142
## s.e.   0.0872  0.0683
##
## sigma^2 estimated as 0.002112:  log likelihood=904.58
## AIC=-1803.15   AICc=-1803.11   BIC=-1790.25
```

```r
fit2 <- Arima(loil, order=c(0, 1, 3))
fit2
```

```
## Series: loil
## ARIMA(0,1,3)
##
## Coefficients:
##          ma1      ma2     ma3
##       0.1696  -0.0886  0.1458
## s.e.  0.0424   0.0424  0.0429
##
## sigma^2 estimated as 0.002094:  log likelihood=907.41
## AIC=-1806.83   AICc=-1806.75   BIC=-1789.63
```

```r
fit3 <- Arima(loil, order=c(0, 2, 1))
fit3
```

```
## Series: loil
## ARIMA(0,2,1)
##
## Coefficients:
##           ma1
##       -1.0000
## s.e.   0.0061
##
## sigma^2 estimated as 0.002213:  log likelihood=886.63
## AIC=-1769.26   AICc=-1769.24   BIC=-1760.67
```

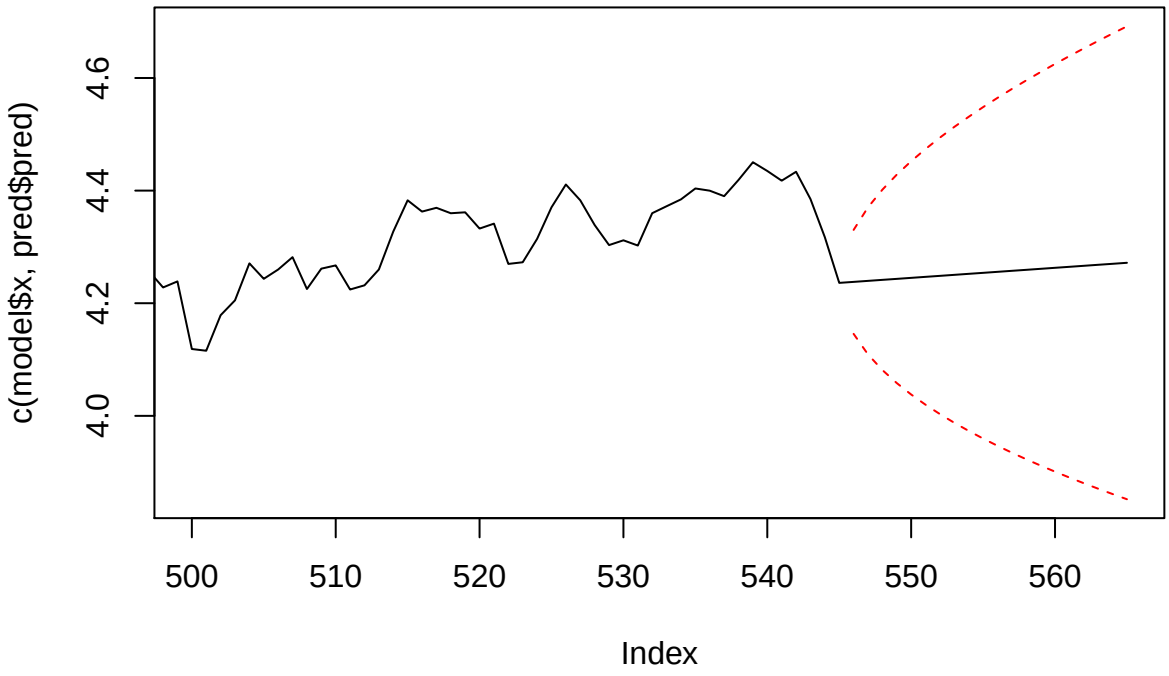
```
complex_dist <- function(x) {
    sqrt(Re(x)^2 + Im(x)^2)
}

sapply(polyroot(c(1, -2, 1)), complex_dist)
```

```
## [1] 1 1
```

```
sapply(polyroot(c(1, -1)), complex_dist)
```

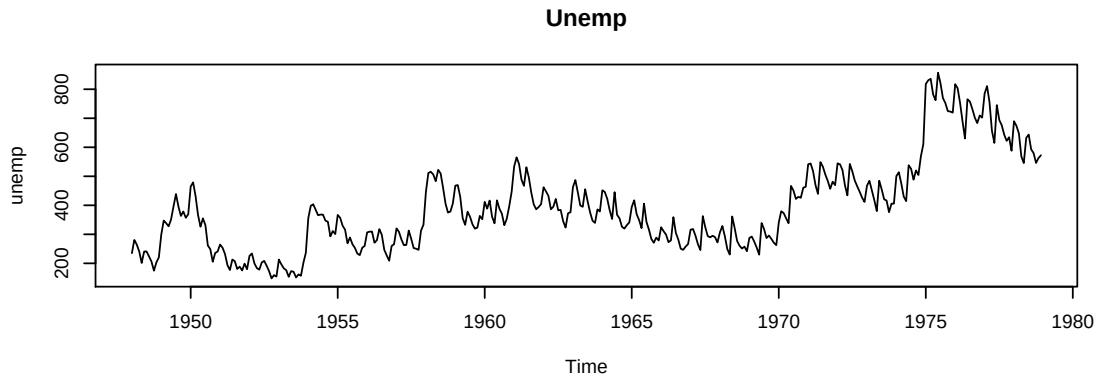
```
## [1] 1
```

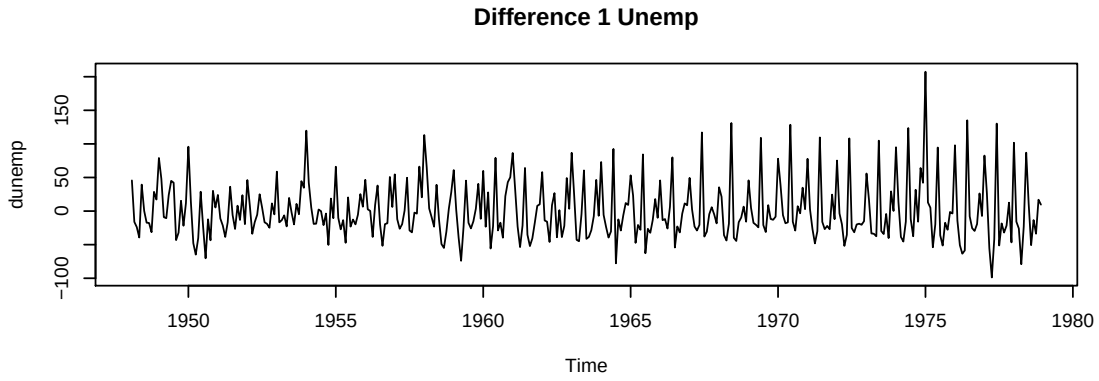
```
fit_plot(fit3)
```

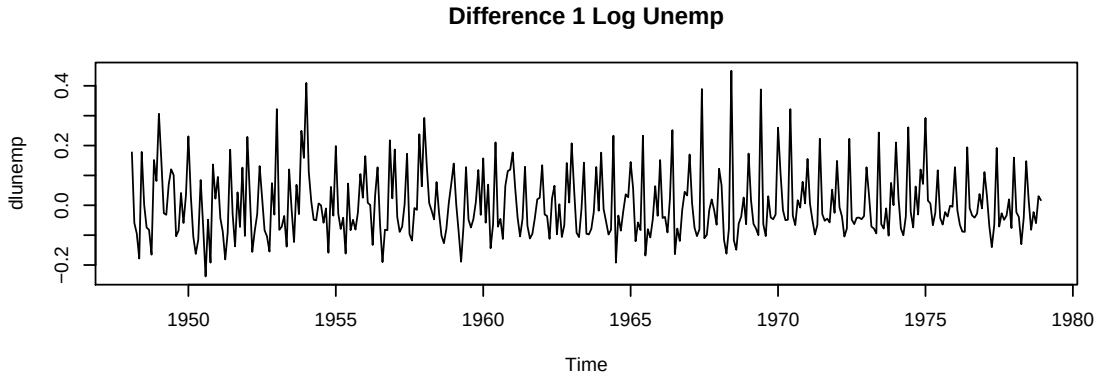


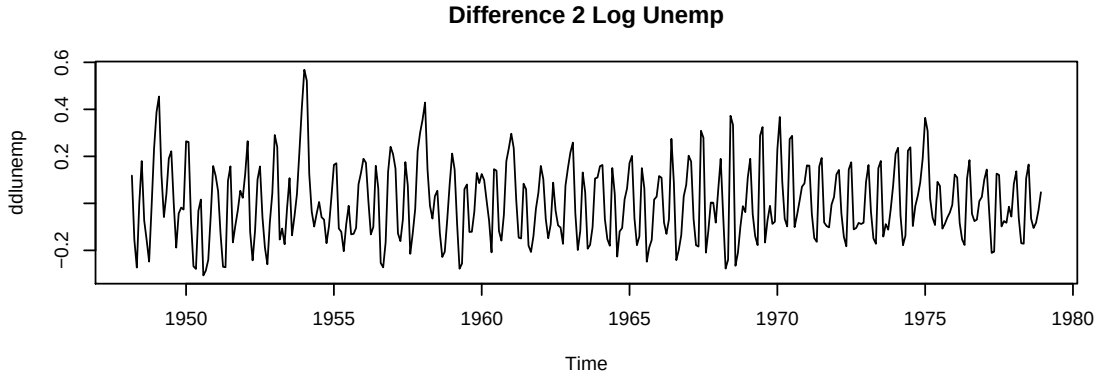
b)

```
lunemp <- log(unemp)
dunemp <- diff(unemp)
ddunemp <- diff(dunemp, 2)
dlunemp <- diff(lunemp)
ddlunemp <- diff(lunemp, 2)
```

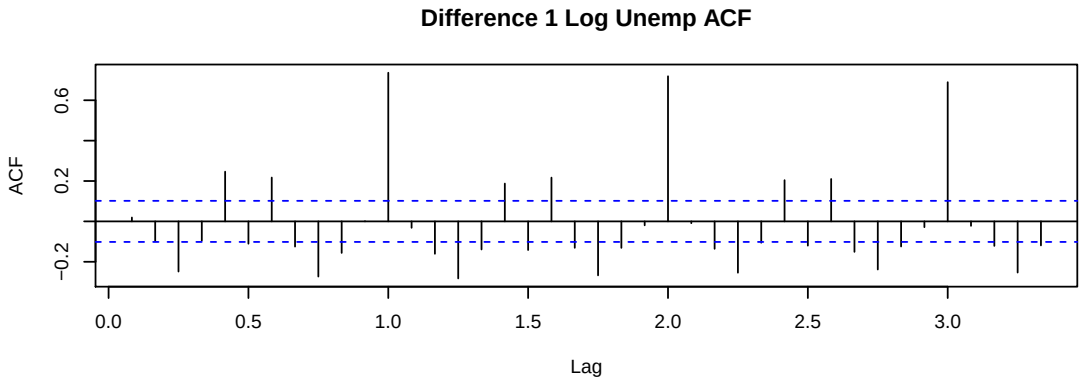

Unemp
unemp
200
400
600
800
1950
1955
1960
1965
1970
1975
1980
Time

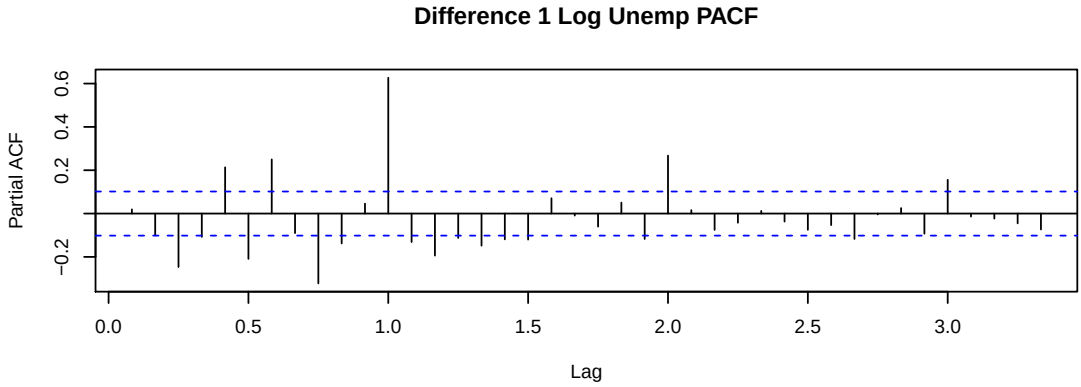

Difference 1 Unemp
dunemp
−100
0
50
150
1950
1955
1960
1965
1970
1975
1980
Time

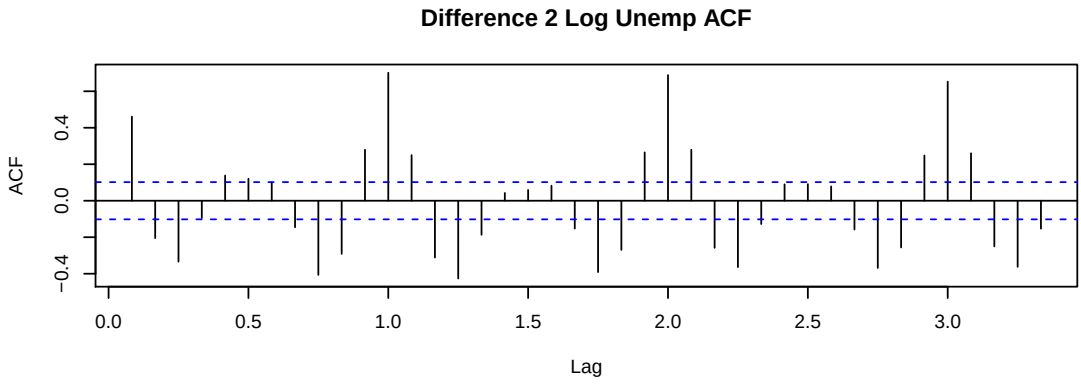

Difference 1 Log Unemp
dlunemp
−0.2
0.0
0.2
0.4
1950
1955
1960
1965
1970
1975
1980
Time

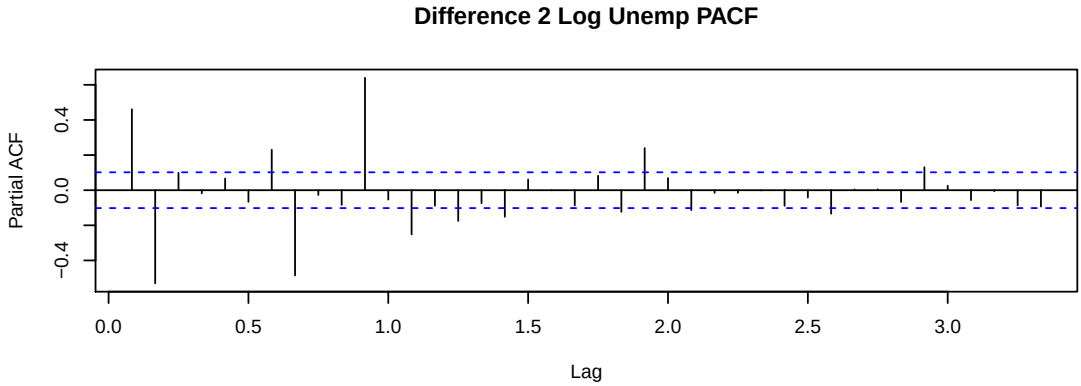

Difference 2 Log Unemp
ddlunemp
−0.2
0.2
0.4
0.6
1950
1955
1960
1965
1970
1975
1980
Time

Clearly difference log is the data we should work with. bla, bla, ...


Difference 1 Log Unemp ACF
ACF
0.6
0.2
−0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Lag


Difference 1 Log Unemp PACF
Partial ACF
0.6
0.4
0.2
−0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Lag


Difference 2 Log Unemp ACF
ACF
0.4
0.0
−0.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Lag

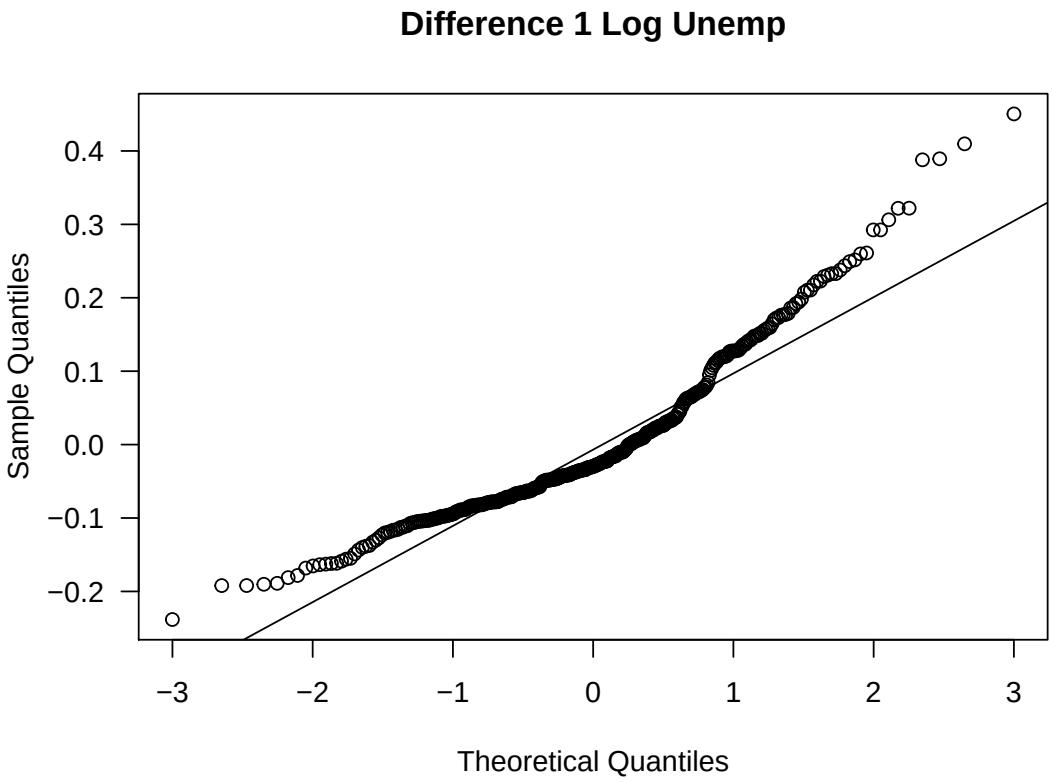

Difference 2 Log Unemp PACF
Partial ACF
0.4
0.0
−0.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Lag

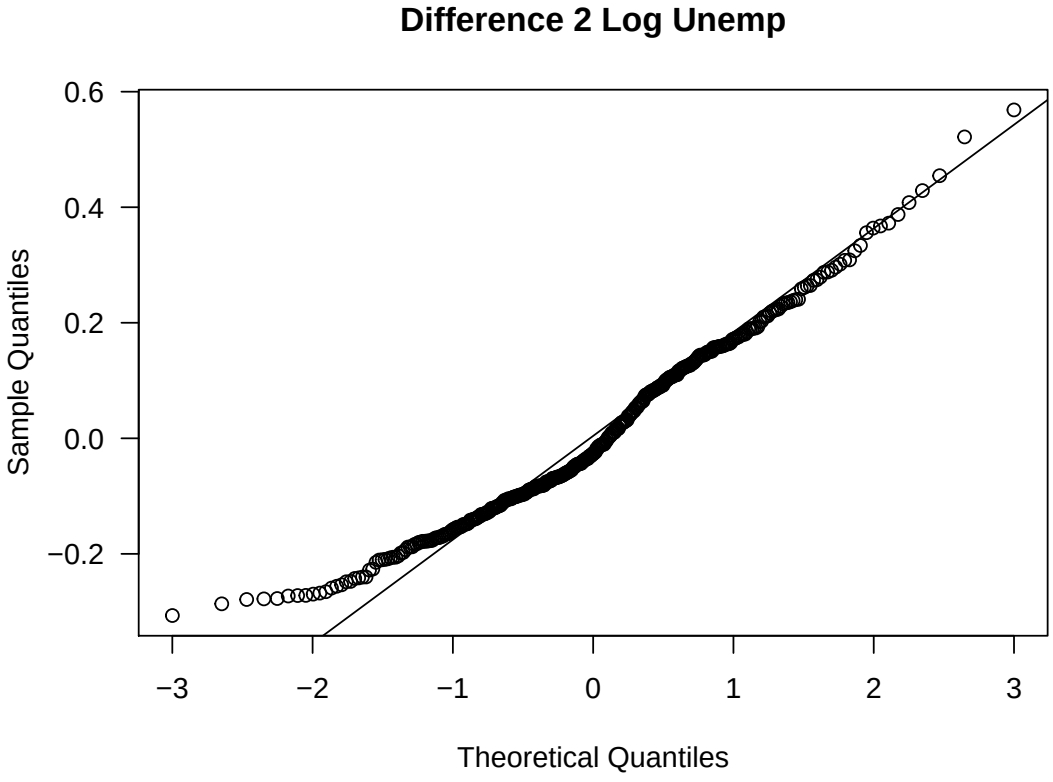
```
eacf(dlunemp)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o x o x x x x x x o  x  o  x
## 1 x o x o x o x x x x o  x  x  o
## 2 x x o x x o o x o o o  x  x  x
## 3 x x x x x o o x o o o  x  x  x
## 4 x x o x o x o o o o o  x  o  x
## 5 x x o x o x o o o o o  x  o  x
## 6 x x x o o x o o o o o  x  o  x
## 7 x x o x x o x o o o o  x  o  o
```

```
eacf(ddlunemp)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x o x x o x x x x  x  x  x
## 1 x x x x x o o o x x x  x  x  x
## 2 x o x o x o o o o o o  x  x  x
## 3 x o o o o x o o o o o  x  x  o
## 4 x x o o o x o o o o o  x  x  x
## 5 x x o o o x o x o o o  x  x  o
## 6 x x o o o x x o o o o  x  x  x
## 7 x o o o x x o x o o o  x  o  x
```


Difference 1 Log Unemp
Sample Quantiles
Theoretical Quantiles
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−3
−2
−1
0
1
2
3


Difference 2 Log Unemp
Sample Quantiles
Theoretical Quantiles
0.6
0.4
0.2
0.0
−0.2
−3
−2
−1
0
1
2
3

```r
fit1 <- Arima(lunemp, order=c(1, 1, 1))
fit1
```

```
## Series: lunemp
## ARIMA(1,1,1)
##
## Coefficients:
##           ar1     ma1
##       -0.7592  0.8157
## s.e.   0.0952  0.0796
##
## sigma^2 estimated as 0.01289:  log likelihood=281.71
## AIC=-557.42   AICc=-557.35   BIC=-545.67
```

```r
fit2 <- Arima(lunemp, order=c(0, 1, 3))
fit2
```

```
## Series: lunemp
## ARIMA(0,1,3)
##
## Coefficients:
##           ma1     ma2      ma3
##       -0.0079  0.0277  -0.3629
## s.e.   0.0470  0.0506   0.0481
##
## sigma^2 estimated as 0.01177:  log likelihood=298.85
## AIC=-589.7   AICc=-589.59   BIC=-574.03
```

```r
fit3 <- Arima(lunemp, order=c(0, 2, 1))
fit3
```

```
## Series: lunemp
## ARIMA(0,2,1)
##
## Coefficients:
##          ma1
##       -1.000
## s.e.   0.007
##
## sigma^2 estimated as 0.01298:  log likelihood=276.19
## AIC=-548.39   AICc=-548.36   BIC=-540.56
```