

732A62 Lab 2

Emil K Svensson & Rasmus Holm

2017-10-03

Assignment 1

a)

```
library(astsa)
library(kernlab)
library(TSA)
library(forecast)

set.seed(12345)
AR3 <- arima.sim(1000, model = list(order = c(3,0,0),
                                     ar = c(0.8, -0.2, 0.1)))

## The theoretical
AR3.pacf <- pacf(AR3, plot=F)
AR3.data <- ts.intersect(xt = AR3, x1 = lag(AR3, 1), x2 = lag(AR3, 2), x3 = lag(AR3, 3))

AR.lm <- resid(lm(xt ~ x1 + x2, data = AR3.data))
AR.lm.lag3 <- resid(lm(x3 ~ x1 + x2, data = AR3.data))

AR3.pacf[3]

##
## Partial autocorrelations of series 'AR3', by lag
##
##      3
## 0.117
cat(paste("The theoretical value:", round(cor(AR.lm, AR.lm.lag3), digits = 3)))
```

The theoretical value: 0.115

As seen above the theoretical and the output from the pacf-function are very similar.

b)

```
set.seed(12345)
AR2 <- arima.sim(100, model = list(order = c(2,0,0),
                                     ar = c(0.8, 0.1)))

ar2.yw <- ar(AR2, order.max = 2, method = "yw", aic = FALSE)
ar2.ols <- ar(AR2, order.max = 2, method = "ols", aic = FALSE)
ar2.mle <- arima(AR2, order = c(2,0,0), method = "ML")
```

```
ar2.yw$ar

## [1] 0.8029146 0.1037053

ar2.ols$ar
```

```
## , , 1
##
##          [,1]
## [1,] 0.8066782
## [2,] 0.1205352
```

```
ar2.mle$coef

##          ar1          ar2 intercept
## 0.7966919 0.1188537 0.8289656
```

The Yule Walker estimate seems to have the parameters closes to the true parameters given in the assignment.

```
ar2.mle

##
## Call:
## arima(x = AR2, order = c(2, 0, 0), method = "ML")
##
## Coefficients:
##          ar1          ar2  intercept
##      0.7967  0.1189      0.8290
## s.e.  0.0992  0.1000      1.1385
##
## sigma^2 estimated as 1.126:  log likelihood = -148.71,  aic = 303.41
```

Yes, the theoretical value for

$$\phi_2$$

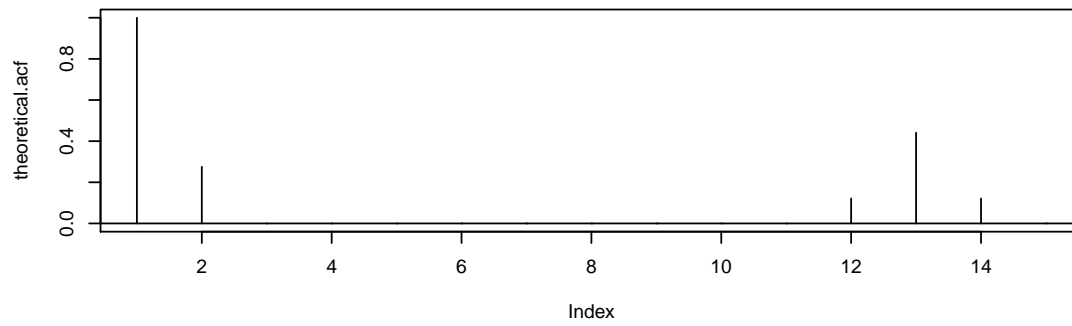
is inside the confidence-intervall for the ML estimate which can seen from the s.e times 1.96 which obviously will cover the true coefficients.

c)

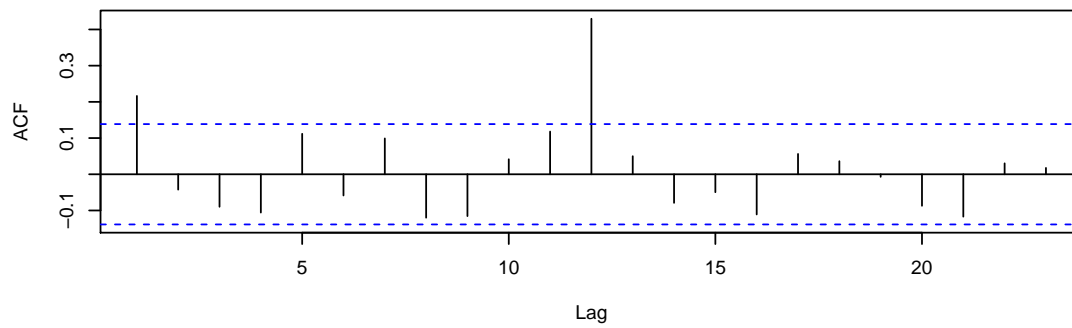
```
set.seed(12345)
ma.coef <- c(0.3, rep(0, 10), 0.6)
ts4 <- arima.sim(n=200, model=list(order=c(0, 0, 12), ma = ma.coef))

theoretical.acf <- ARMAacf(ma=c(ma.coef, 0.3 * 0.6))
theoretical.pacf <- ARMAacf(ma=c(ma.coef, 0.3 * 0.6), pacf=TRUE)
```

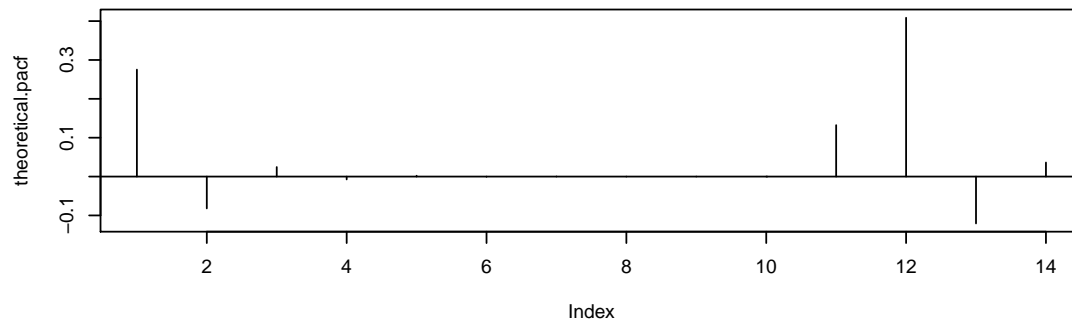
Theoretical ACF



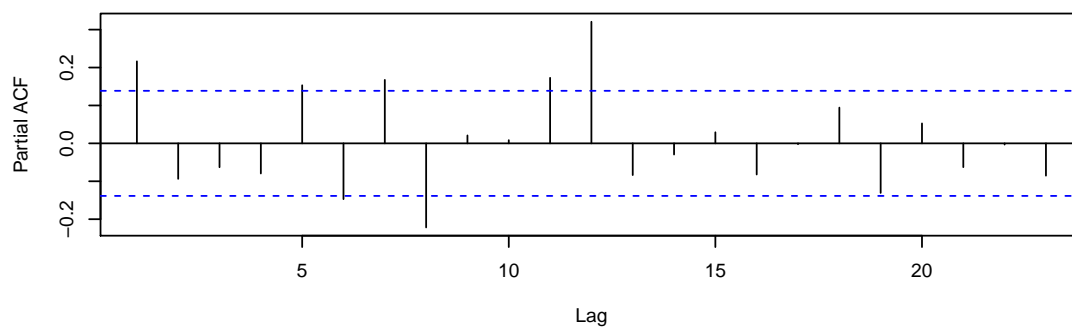
Empirical ACF



Theoretical PACF



Empirical PACF



The patterns seem to be somewhat similar. In the theoretical ACF we can see a large spike at lags 1 and 13 and in the sample ACF we have large spikes at lags 1 and 12 instead. The difference between the two being that the sample ACF has some correlation along the lags although under the confidence interval.

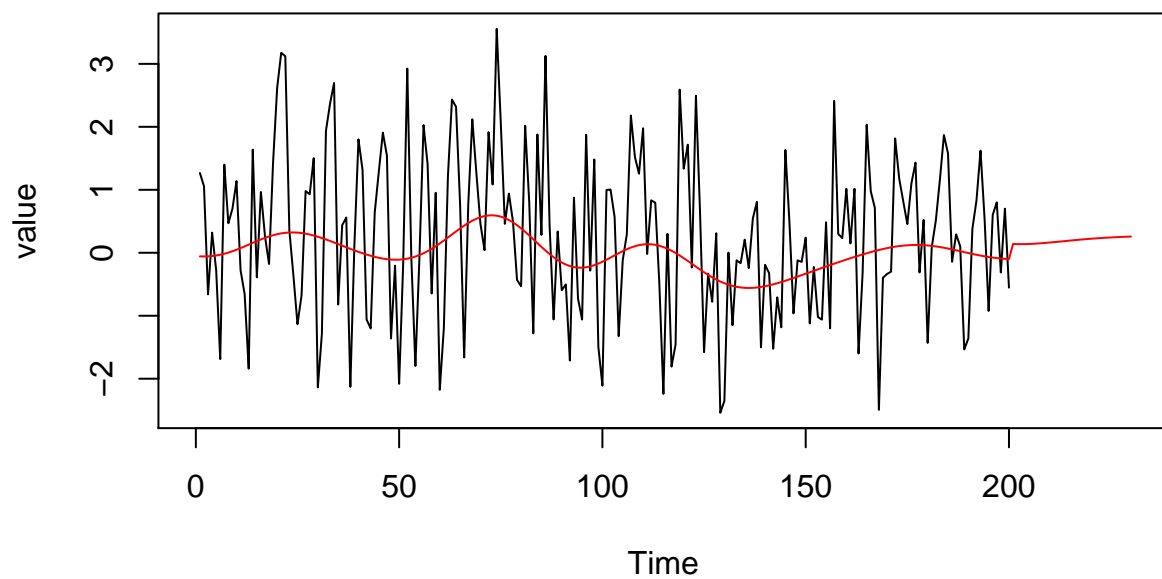
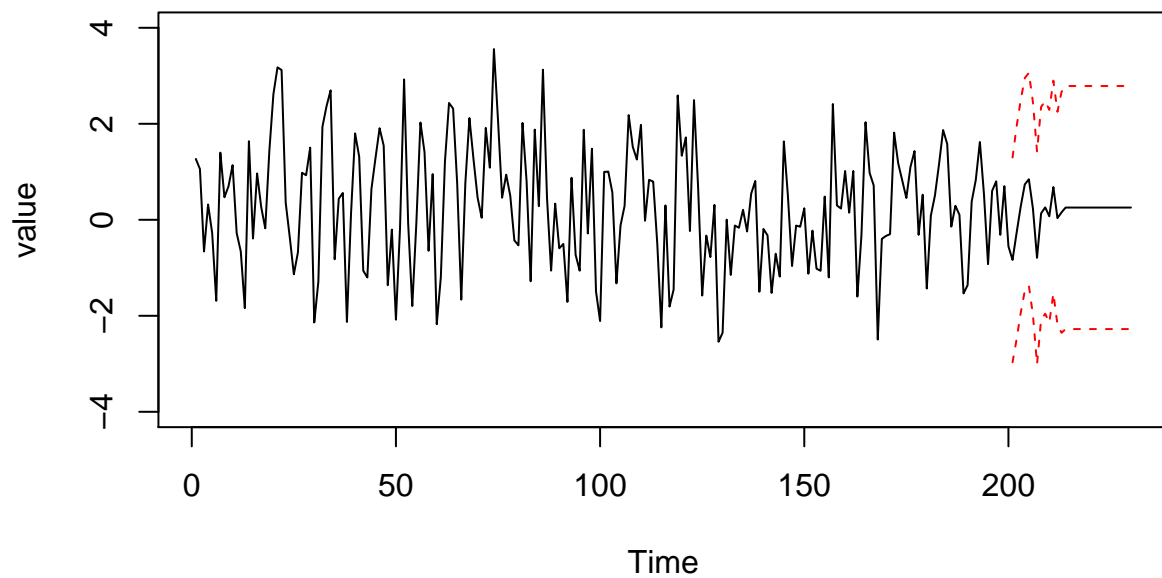
For the PACF we can see a similar observation that there are spikes at lags 1 and 12 for both the theoretical and sample PACF.

d)

```
set.seed(12345)
ma.coef <- c(0.3, rep(0, 10), 0.6)
ts5 <- arima.sim(n=200, model=list(order=c(0, 0, 12), ma = ma.coef))

ts5.fit <- arima(ts5, order=c(0, 0, 1), seasonal=list(order=c(0, 0, 1), period=12))
ts5.pred <- predict(ts5.fit, n.ahead=30, se.fit=TRUE)

gausspr.data <- data.frame(y=ts5, x=1:200)
gausspr.fit <- gausspr(y ~ x, gausspr.data)
gausspr.pred <- predict(gausspr.fit, data.frame(x=201:230))
```



In the first plot we can see the MA models predictions seem reasonable but after a while the predictions just tails of in to a mean function at $y = 0$.

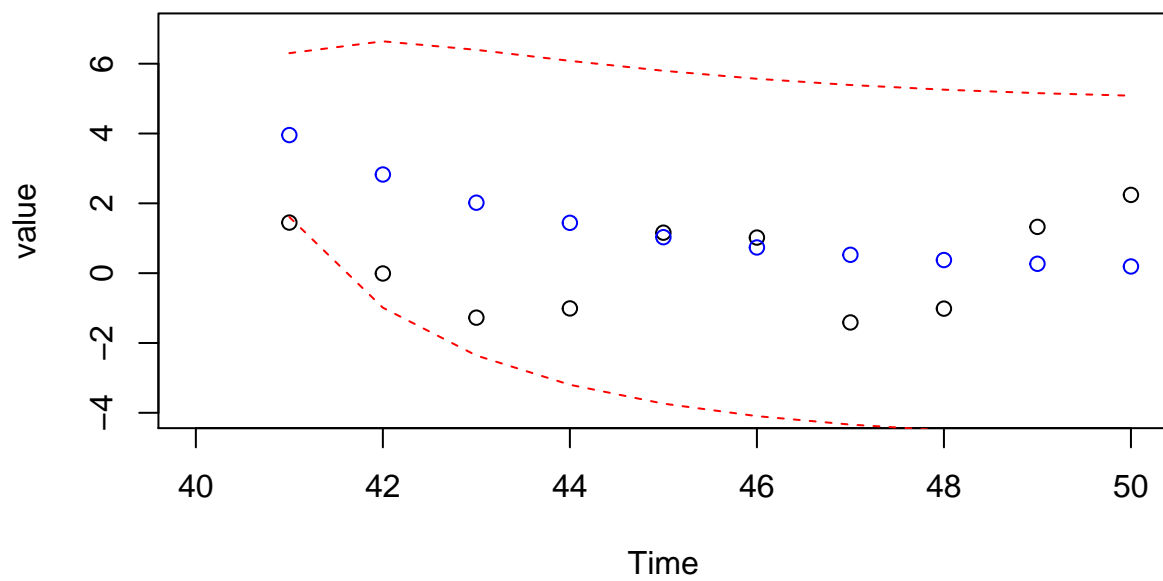
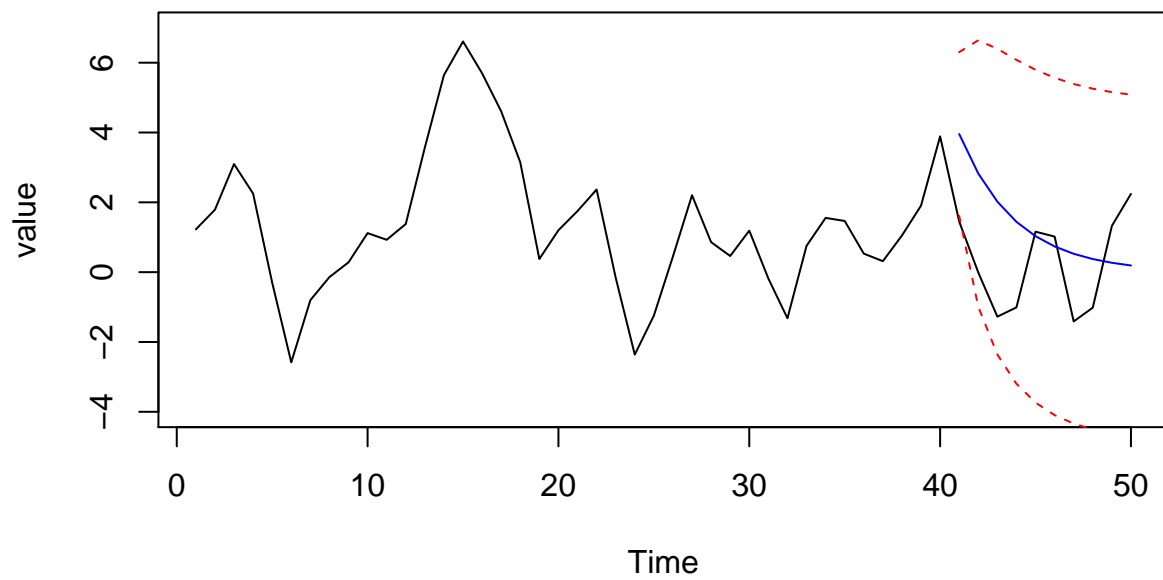
For the gaussian process there is an initial jump in the predictions and afterwards continues with a smoothed pattern similar to the fitted line in the observed data.

e)

```
set.seed(12345)
ts6 <- arima.sim(model=list(ma=c(0.5), ar=c(0.7)), n=50)

train <- ts(ts6[1:40])
test <- ts(ts6[41:50])

ts6.fit <- arima(train, order=c(1, 0, 1), include.mean = F)
ts6.pred <- predict(ts6.fit, n.ahead=10)
```



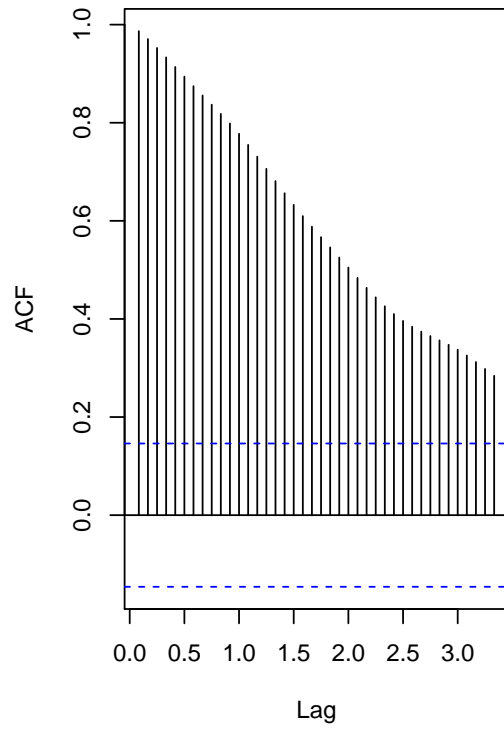
The first of the 10 observations withheld from the fitted model is the only observation that is not in the prediction interval. Since it is a 95 % prediction interval we say that we expect that 5 of 100 observations will be outside the interval so it is not unreasonable that 1 of 10 is outside the interval.

Assignment 2

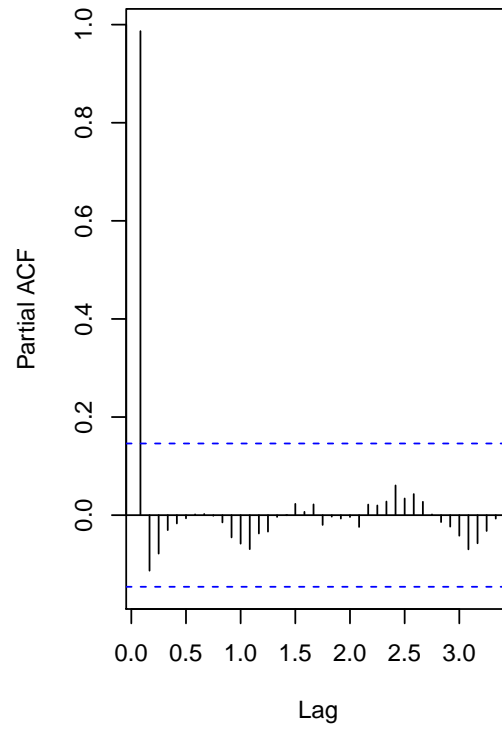
```
assignment2 <- function(data){  
  old <- par(mfrow = c(2, 2))  
  acf(data, lag.max = 40, main="Data ACF")  
  pacf(data, lag.max = 40, main="Data PACF")  
  acf(diff(data, lag = 1), lag.max = 40, main="Difference 1 Data ACF")  
  pacf(diff(data, lag = 1), lag.max = 40, main="Difference 1 Data PACF")  
  par(old)  
}
```


Chicken

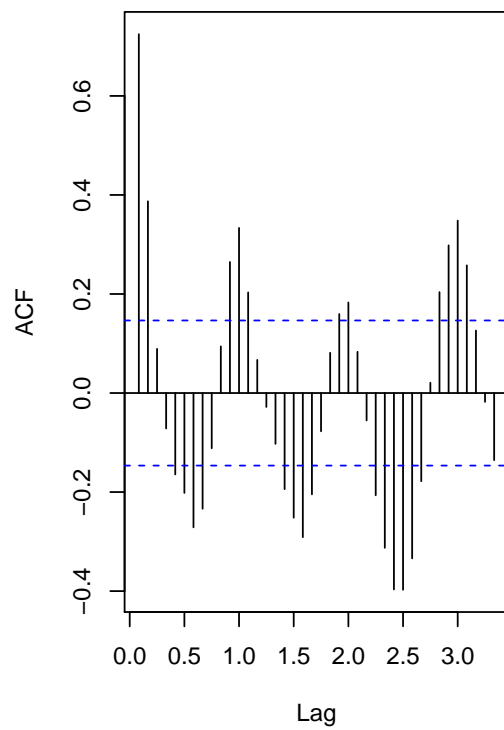
Data ACF



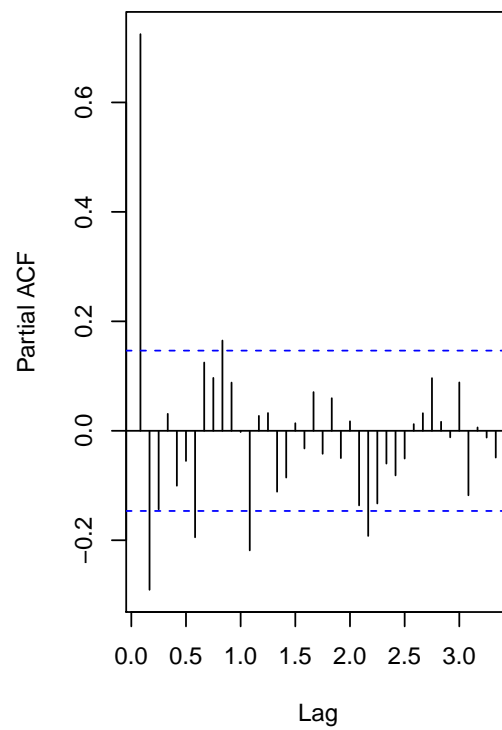
Data PACF



Difference 1 Data ACF



Difference 1 Data PACF



Data ACF

The ACF on the original data suggests an AR or ARMA model since the ACF tails off.

Data PACF

The PACF on the original data cuts off after lag 1 suggesting an AR(1) model.

Difference 1 Data ACF

After having performed difference of order 1 we can clearly see that there is a seasonal trend in the data. The ACF suggests a seasonality of 10 but it does not seem to tail off.

Difference 1 Data PACF

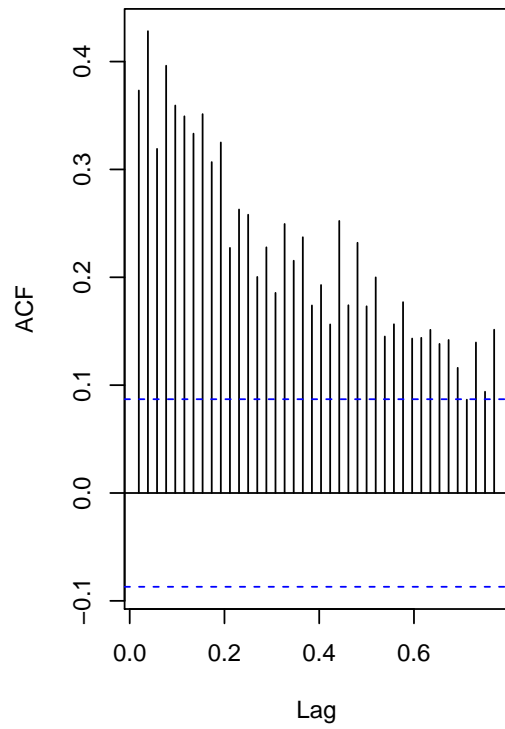
The PACF indicates that ...

Final Verdict

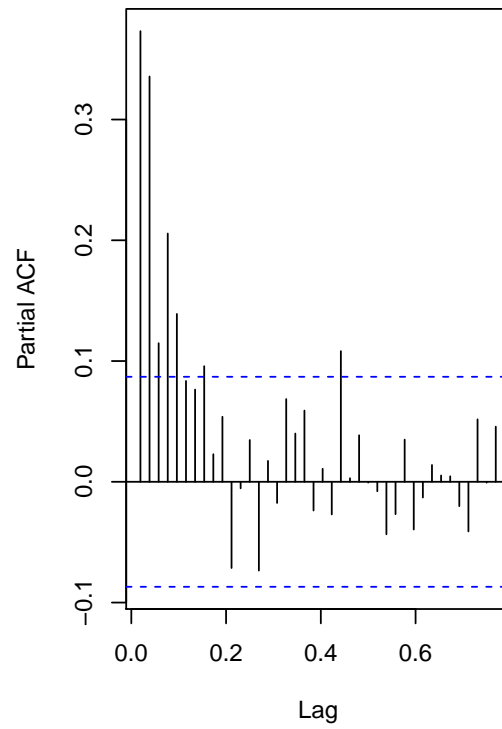
Needs further investigation $\text{ARIMA}(1, 0, 0) \times (_, 1, _)_{10}$

so2

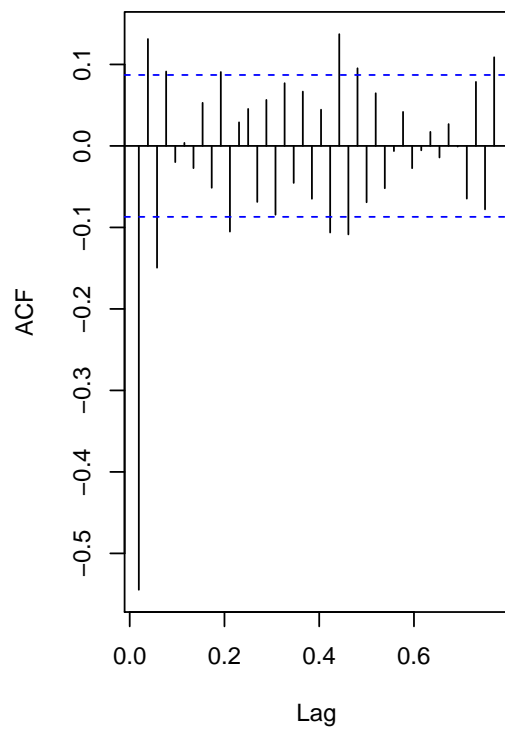
Data ACF



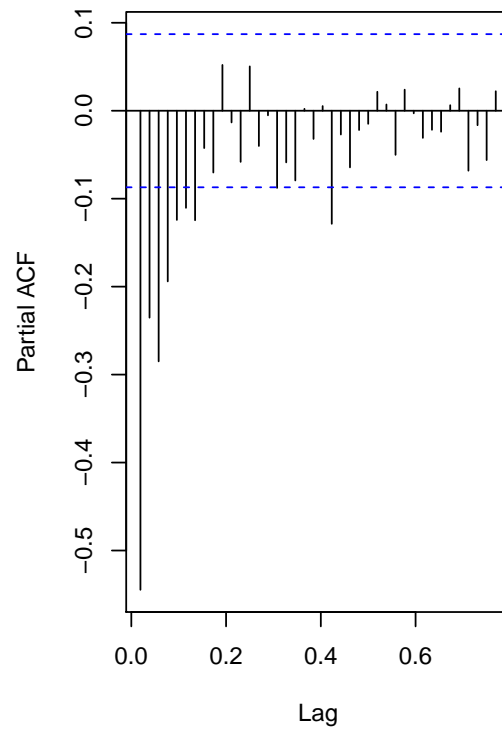
Data PACF



Difference 1 Data ACF



Difference 1 Data PACF



Data ACF

The ACF tails off suggesting either an AR or ARMA model.

Data PACF

The PACF tails off as well suggesting an ARMA model.

Difference 1 Data ACF

The ACF after difference cuts off after lag 1 suggesting a MA(1) model.

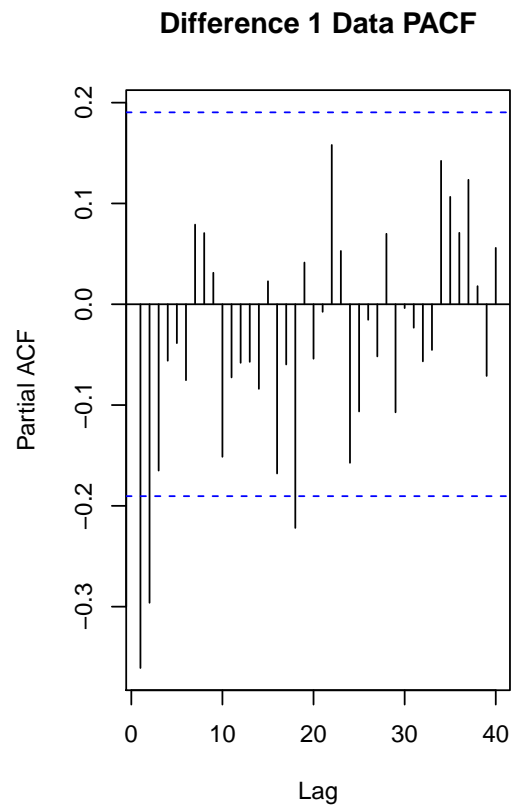
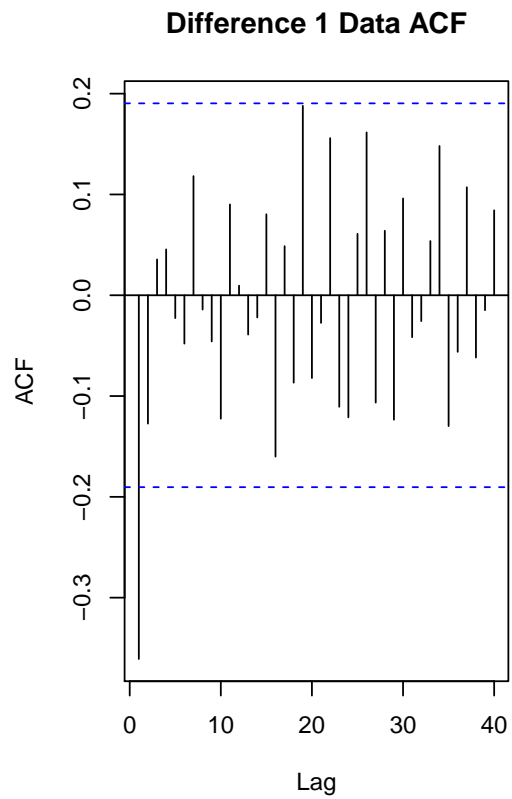
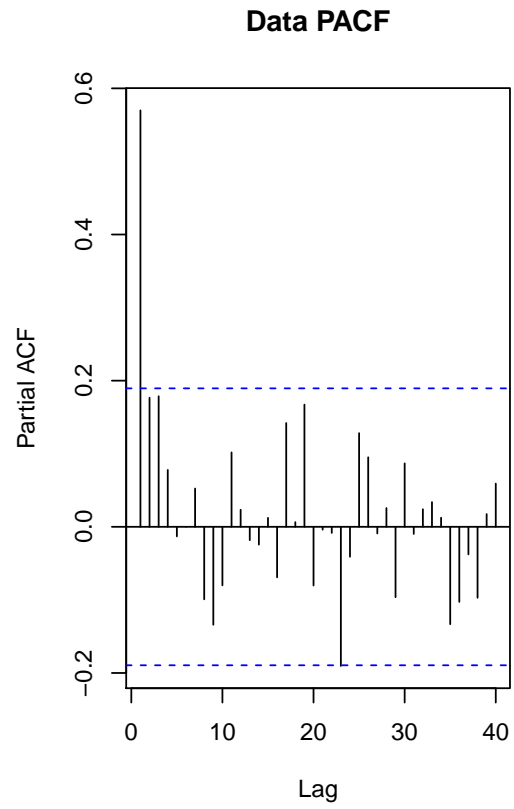
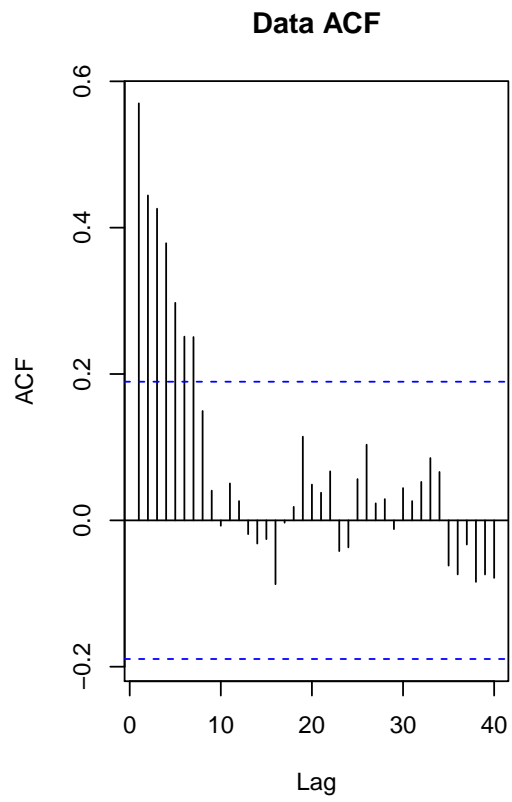
Difference 1 Data PACF

The PACF after difference tails off further suggesting a MA(1) model.

Final Verdict

ARIMA(0, 1, 1)

EQcount



Data ACF

The ACF tails off suggesting an AR or ARMA model.

Data PACF

The PACF cuts off after lag 1 suggesting AR(1) model.

Difference 1 Data ACF

The ACF after difference cuts off after lag 1 suggesting a MA(1) model.

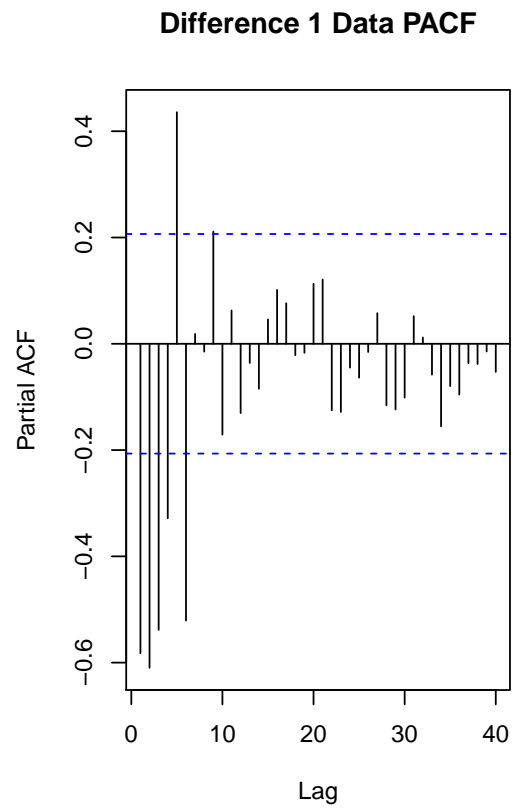
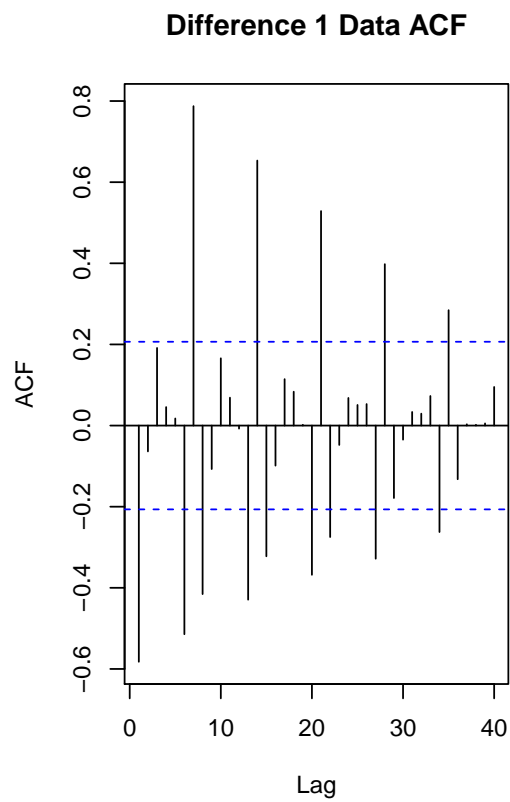
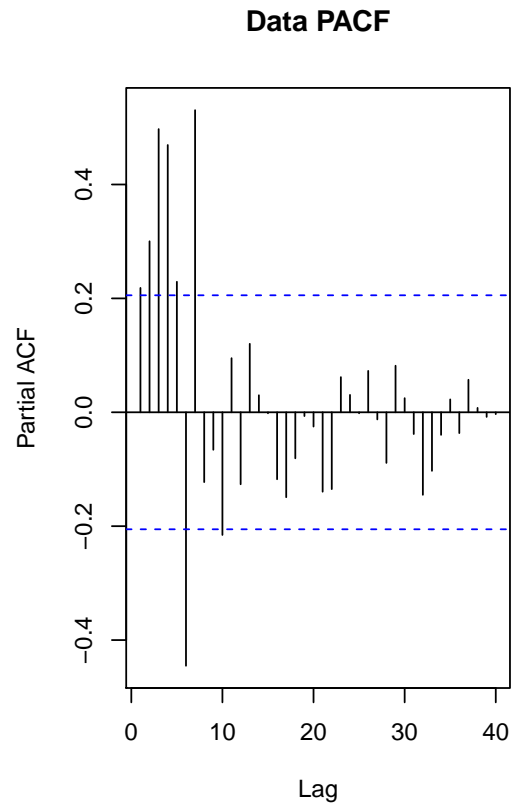
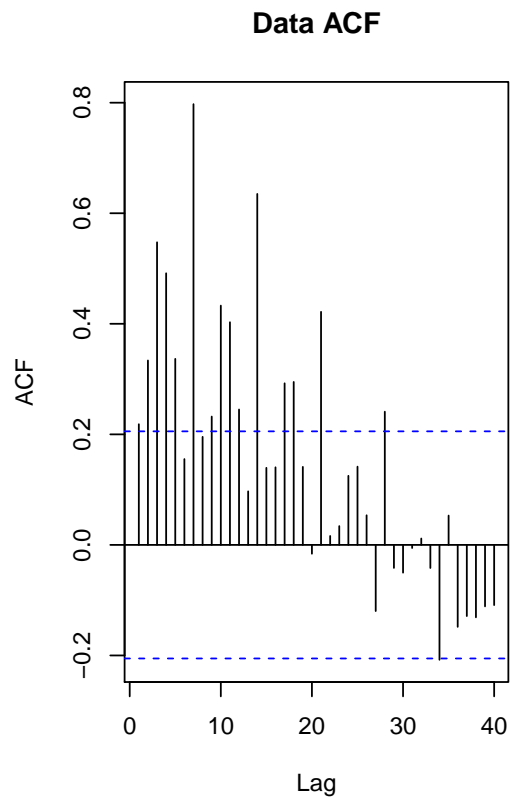
Difference 1 Data PACF

The PACF after difference tails off further suggesting a MA model.

Final Verdict

Either a ARMA(1, 0, 0) or ARMA(0, 1, 1)

HCT



Data ACF

The ACF tails off suggesting either an AR or ARMA model.

Data PACF

The PACF cuts off after lag 7 suggesting an AR(7) model.

Difference 1 Data ACF

The ACF suggests seasonality that tails off after lag 7 suggesting an seasonality of 7.

Difference 1 Data PACF

The PACF cuts off after 6 lags suggesting an AR(6) seasonality model.

Final Verdict

ARIMA(7, 0, 0) x (1, 1, 0)₇

Assignment 3

```
plot_helper <- function(data, title) {
  old <- par(mfrow=c(4, 1))
  plot(data, main=title)
  acf(data, lag.max=40, main="")
  pacf(data, lag.max=40, main="")
  qqnorm(data, main="", las=1)
  qqline(data)
  par(old)
}

test_helper <- function(data) {
  print(Box.test(data, lag = 1, type = "Ljung-Box"))
  print(suppressWarnings(adf.test(data)))
  e <- eacf(data)
}

fit_plot <- function(model) {
  pred <- predict(model, n.ahead=20, se.fit=TRUE)
  upper_band <- pred$pred + 1.96 * pred$se
  lower_band <- pred$pred - 1.96 * pred$se

  plot(c(model$x, pred$pred), type="l",
       xlim=c(500, length(oil) + 20),
       ylim=c(min(lower_band), max(upper_band)))
  lines(length(oil) + 1:20, upper_band, lty=2, col="red")
  lines(length(oil) + 1:20, lower_band, lty=2, col="red")
}
```

a)

```
loil <- log(oil)
doil <- diff(oil)
ddoil <- diff(oil, 2)
dloil <- diff(loil)
ddloil <- diff(loil, 2)
```

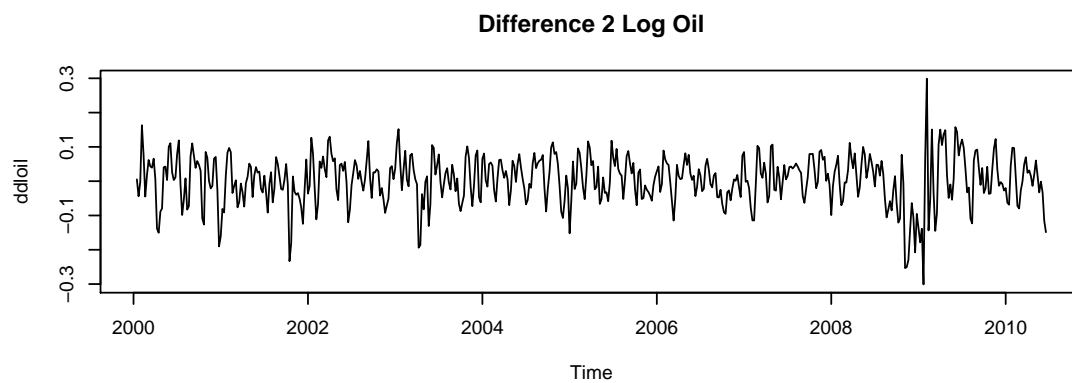
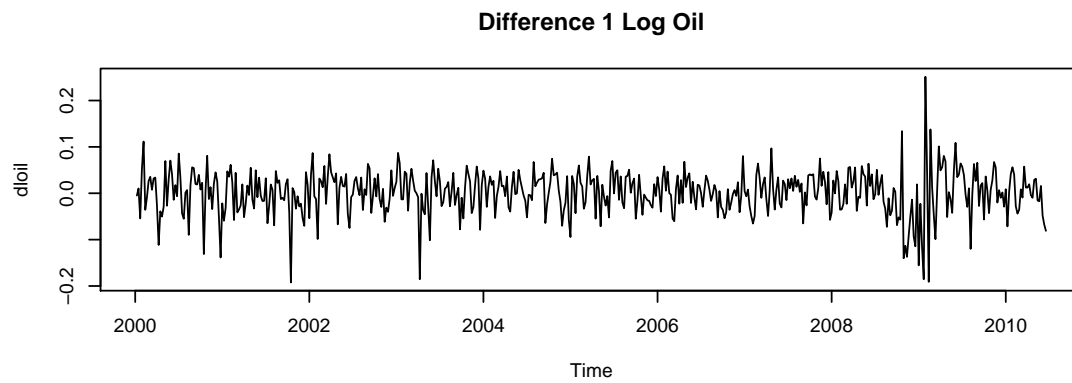
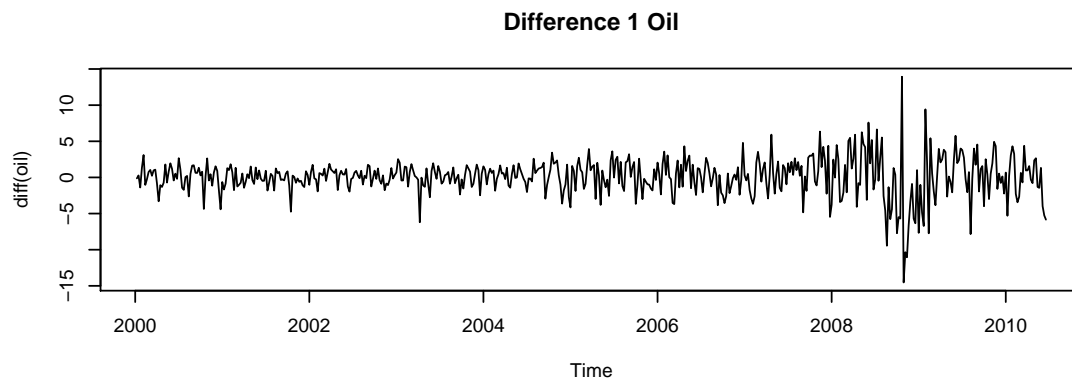
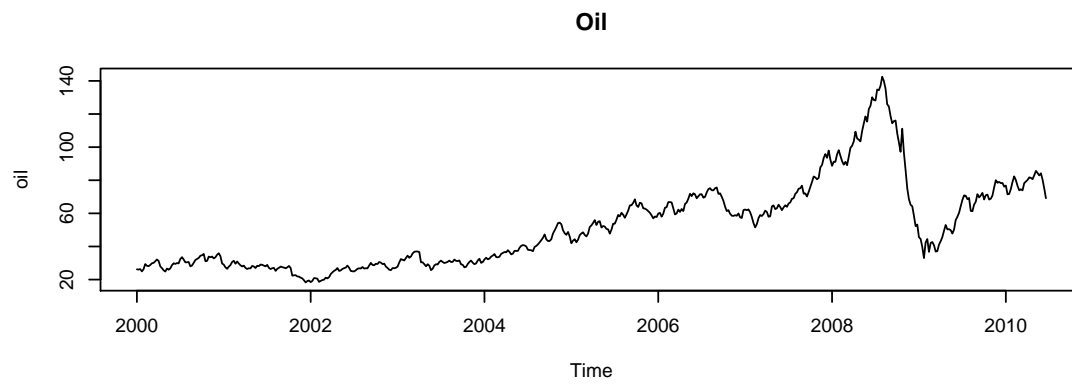
```
test_helper(doil)
test_helper(ddoil)
test_helper(dloil)
test_helper(ddloil)
```

```
fit1 <- Arima(loil, order=c(0, 2, 1))
fit1
```

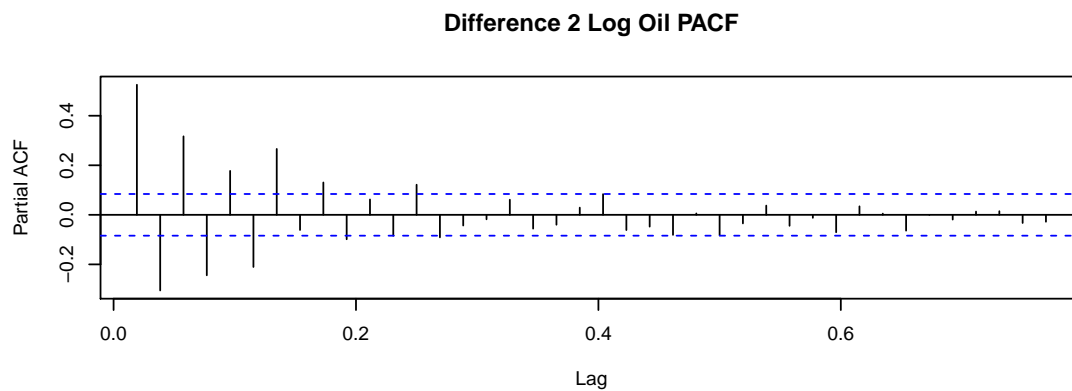
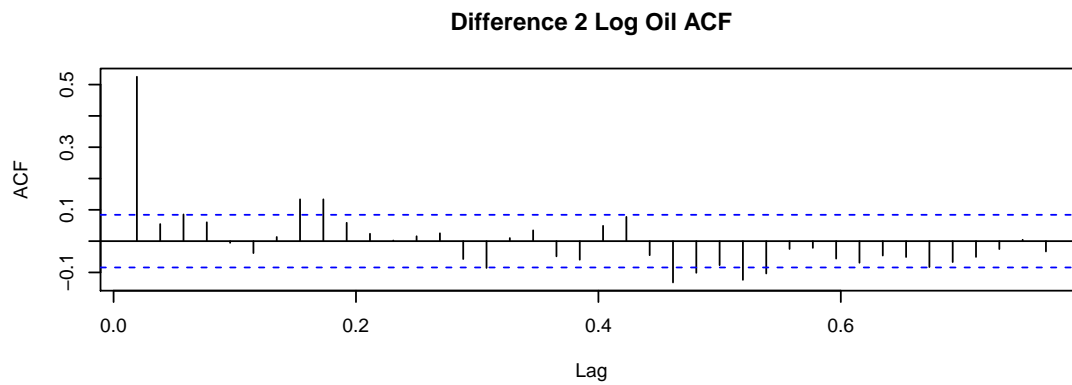
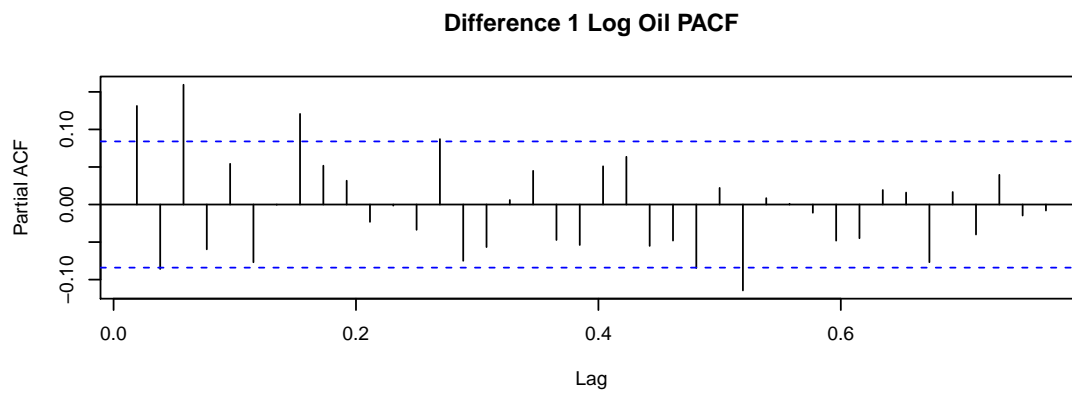
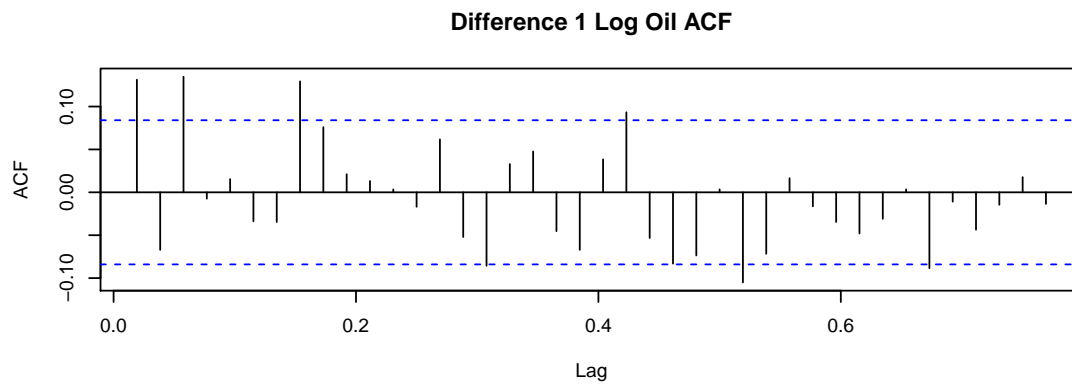
```
fit_plot(fit1)
```

```
fit2 <- Arima(loil, order=c(0, 1, 3))
fit2
```

```
fit_plot(fit2)
```



Clearly difference log is the data we should work with, a second difference does not do any visual change to the data that can already be considered stationary at diff 1 with a log transformation but we keep both and keep both as tentative models.



Here the ACF and PACF for the first order differencing doesn't seem to have any clear defined patterns compared to the second order differencing where we can see that the ACF cuts off at lag 1 and trails off in the PACF suggesting a $ARIMA(0,2,1)$.

```
eacf(dloil)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o x o o o o o o
## 1 x o x o o o o x o o o o o o
## 2 x x x o o o o x o o o o o o
## 3 x x x o o o o x o o o o o o
## 4 x o x o o o o x o o o o o o
## 5 x x x o x o o x o o o o o o
## 6 o x x o x x o x o o o o o x
## 7 o x x x x x x x o x o o o o
```

```
eacf(ddloil)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o x x o o o o o
## 1 x x o o o o o x x o o o o o
## 2 x x x o o o o x x o o o o o
## 3 x x x x o o o x x o o o o o
## 4 x x x x o o o x x o o o o o
## 5 x o x x x o o x x o o o o o
## 6 x o x x x x x x o x o o o o
## 7 x x x x x x x x o x o o o o
```

The EACF doesn't seem to help alleviate the ambiguous ACF/PACF for the first order diff series and suggest a wide range of models making the general pattern more like a box than a triangular shape.

For the second order diff we can see the triangular shape with a point at a $AR(1)$ which would confirm previous statements in the ACF.

```
fit1 <- Arima(loil, order=c(1, 1, 1))
fit1
```

```
## Series: loil
## ARIMA(1,1,1)
##
## Coefficients:
##          ar1      ma1
##      -0.5253  0.7142
## s.e.   0.0872  0.0683
##
## sigma^2 estimated as 0.002112:  log likelihood=904.58
## AIC=-1803.15  AICc=-1803.11  BIC=-1790.25
```

```
fit2 <- Arima(loil, order=c(1, 2, 2))
fit2
```

```
## Series: loil
## ARIMA(1,2,2)
##
## Coefficients:
##          ar1      ma1      ma2
##      -0.5247 -0.2861 -0.7139
## s.e.   0.0873  0.0686  0.0685
##
## sigma^2 estimated as 0.002118:  log likelihood=899.7
## AIC=-1791.4  AICc=-1791.32  BIC=-1774.21
```

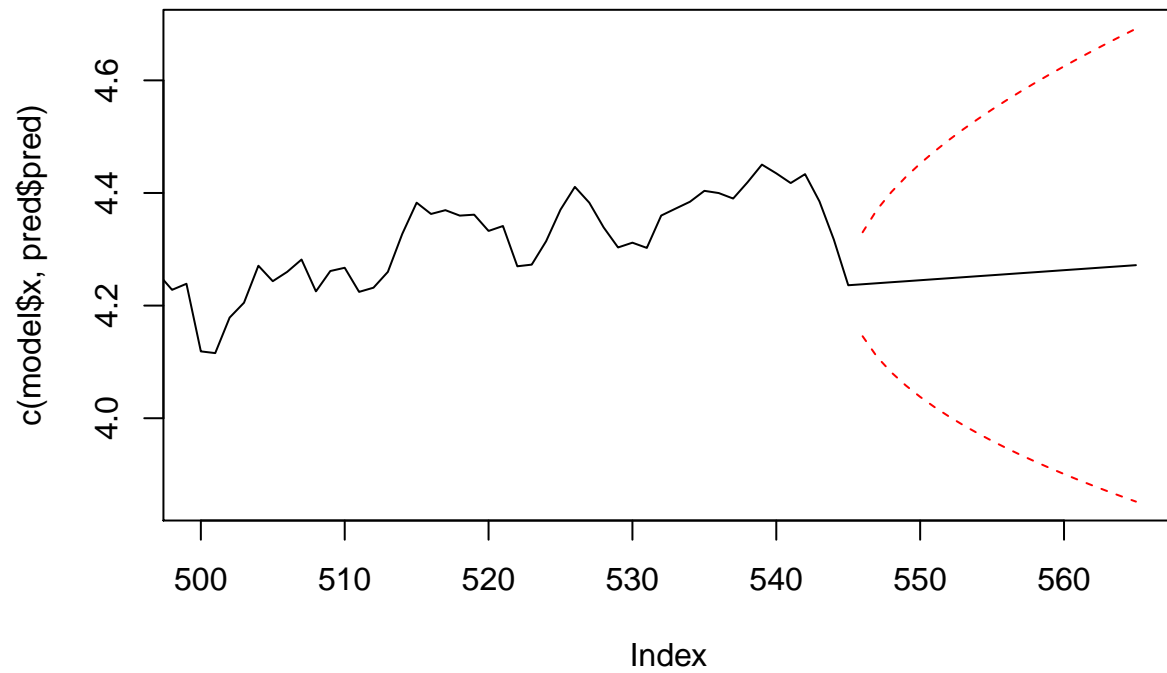
```
fit3 <- Arima(loil, order=c(0, 2, 1))
fit3
```

```
## Series: loil
## ARIMA(0,2,1)
##
## Coefficients:
##          ma1
##      -1.0000
## s.e.   0.0061
##
## sigma^2 estimated as 0.002213:  log likelihood=886.63
## AIC=-1769.26  AICc=-1769.24  BIC=-1760.67
```

The best model if we evaluate the BIC criterion we choose the ARIMA(0,2,1) which looks like following

$$\nabla^2 x_t = (1 + \theta B)w_t$$

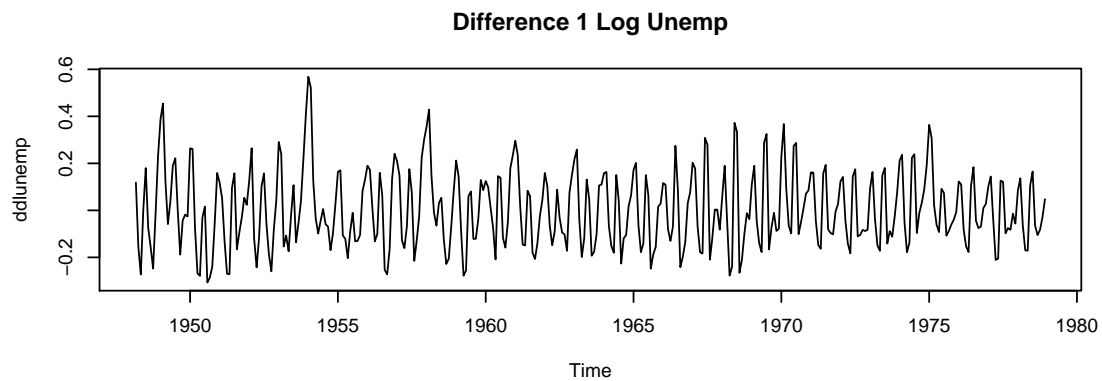
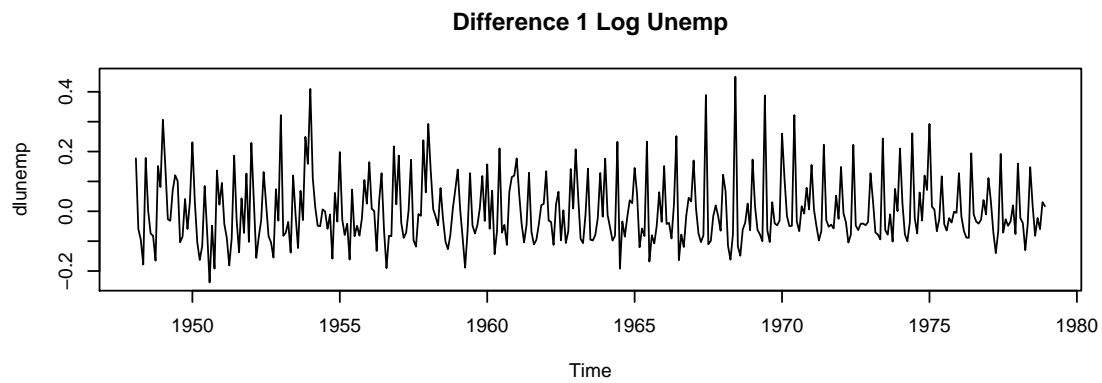
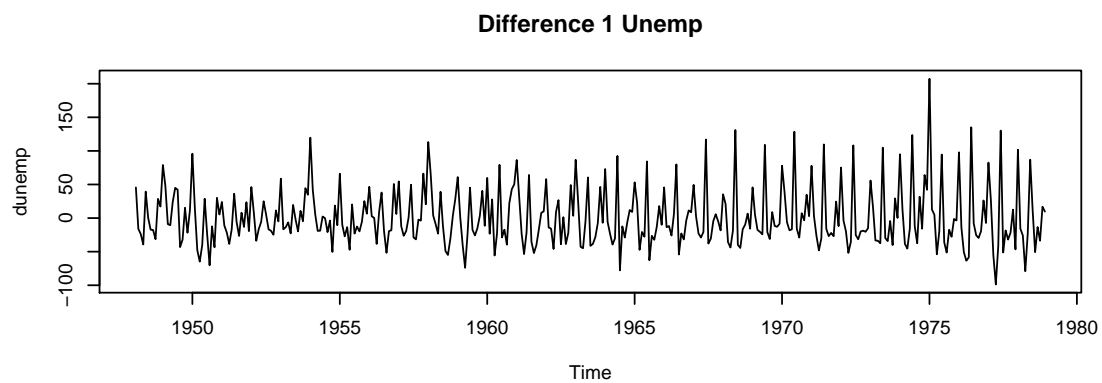
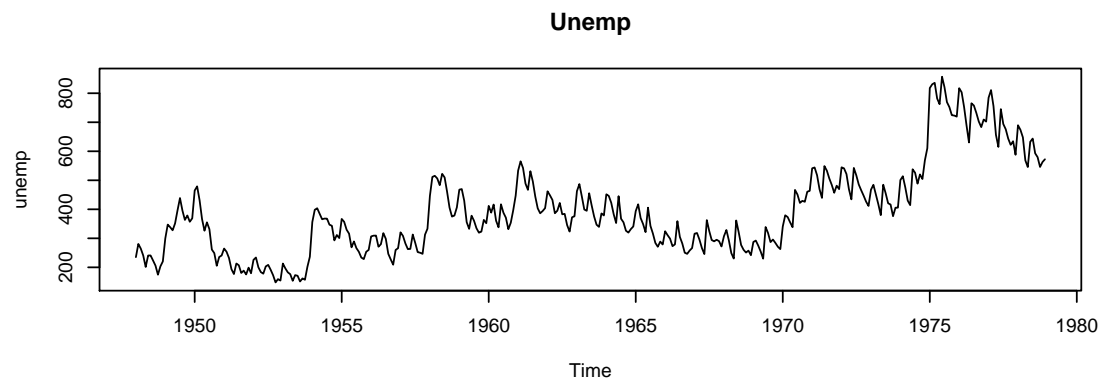
```
fit_plot(fit3)
```



The figure above displays 20 steps ahead prediction. The intervals looks resonable and the prediction aswell.

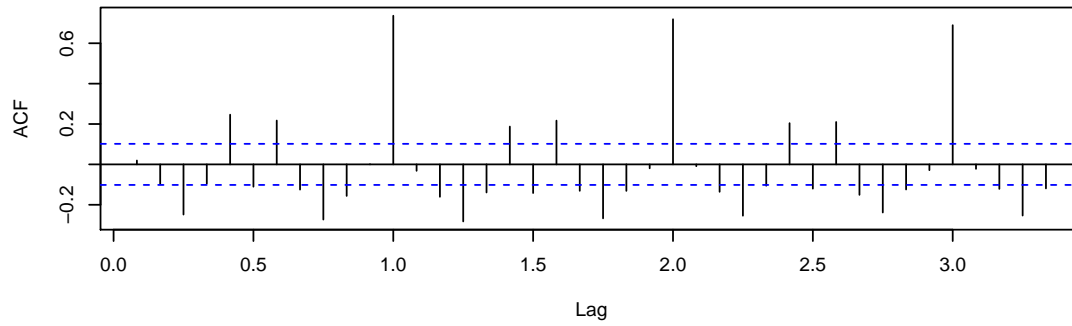
b)

```
lunemp <- log(unemp)
dunemp <- diff(unemp)
dlunemp <- diff(lunemp, 1)
ddlunemp <- diff(lunemp, 2)
```

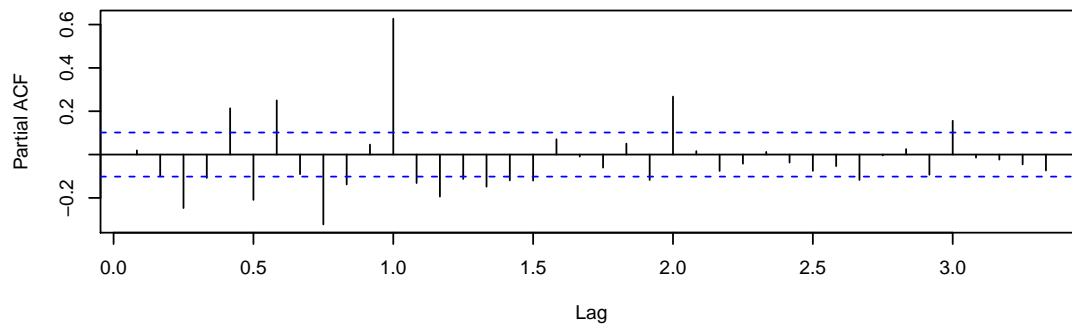



The data is clearly not stationary and to try to make it stationary we do differencing of order 1. The variance seem to increase by time which we try to negative by transforming the data on log-scale. We also used differencing of order 2 which gives a smoother result than differencing of order 1.

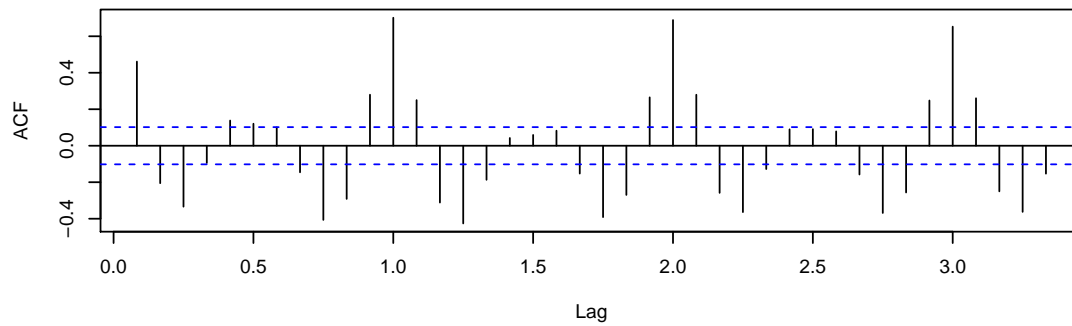
Difference 1 Log Unemp ACF



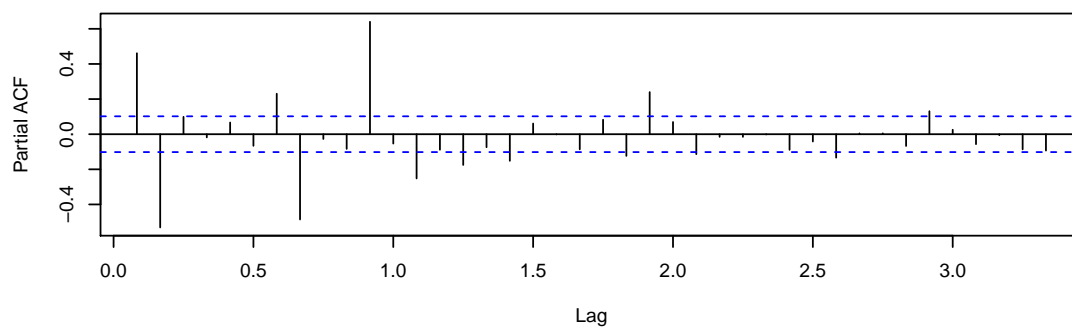
Difference 1 Log Unemp PACF



Difference 2 Log Unemp ACF



Difference 2 Log Unemp PACF



Difference 1

Seasonality behavior: The ACF plot suggests a seasonality trend of 12 lags that tails off both in the ACF and the PACF. This suggests an $ARMA_12$ seasonality component. The PACF spikes at 3 multiples which is indicative of AR(3).

There is a seasonality trend that occurs at lags 3, 5, 7, 9, 12, 15, 17, 19, 21, 24.

Non-seasonality behavior:

Difference 2

Seasonality behavior:

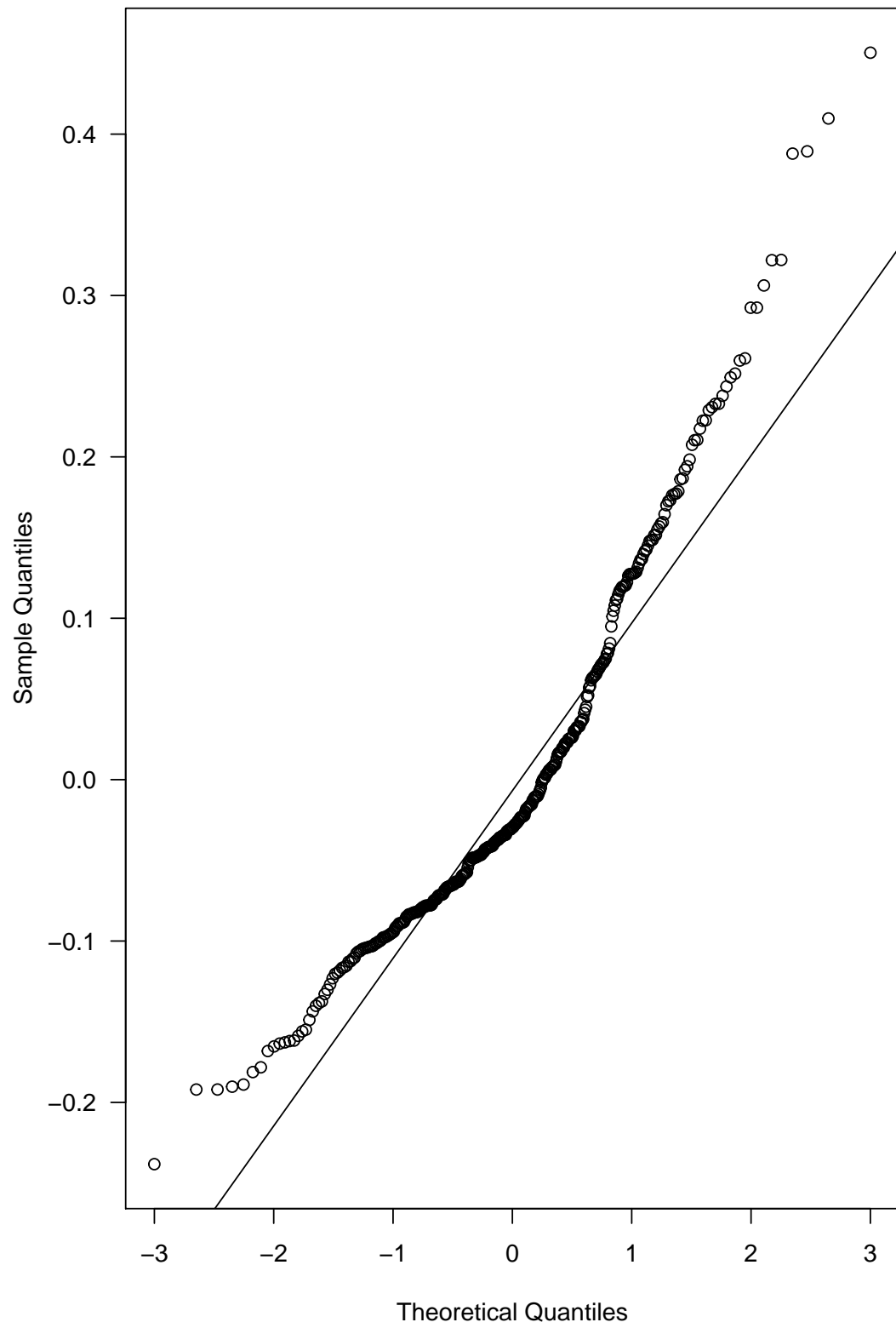
Non-seasonality behavior: Apart from the seasonal behavior the ACF tails off and the PACF cuts off after lag 2 indicating an AR(2) model.

```
eacf(dlunemp)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o x o x x x x x o x o x
## 1 x o x o x o x x x x o x x o
## 2 x x o x x o o x o o o x x x
## 3 x x x x x o o x o o o x x x
## 4 x x o x o x o o o o o x o x
## 5 x x o x o x o o o o o x o x
## 6 x x x o o x o o o o o x o x
## 7 x x o x x o x o o o o x o o
```

The EACF further suggest that there is no normal ARMA model that is well suited for the data.

Difference 1 Log Unemp



```

fit1 <- Arima(lunemp, order=c(1, 1, 1))
fit1

## Series: lunemp
## ARIMA(1,1,1)
##
## Coefficients:
##          ar1      ma1
##      -0.7592  0.8157
## s.e.   0.0952  0.0796
##
## sigma^2 estimated as 0.01289: log likelihood=281.71
## AIC=-557.42  AICc=-557.35  BIC=-545.67

fit2 <- Arima(lunemp, order=c(0, 1, 3))
fit2

## Series: lunemp
## ARIMA(0,1,3)
##
## Coefficients:
##          ma1      ma2      ma3
##      -0.0079  0.0277 -0.3629
## s.e.   0.0470  0.0506   0.0481
##
## sigma^2 estimated as 0.01177: log likelihood=298.85
## AIC=-589.7   AICc=-589.59  BIC=-574.03

fit3 <- Arima(lunemp, order=c(0, 2, 1))
fit3

## Series: lunemp
## ARIMA(0,2,1)
##
## Coefficients:
##          ma1
##      -1.000
## s.e.   0.007
##
## sigma^2 estimated as 0.01298: log likelihood=276.19
## AIC=-548.39  AICc=-548.36  BIC=-540.56

```