732A62 Lab 2

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Assignment 1

a)

```
library(astsa)
library(kernlab)
library(TSA)
library(forecast)

set.seed(12345)
AR3 <- arima.sim(1000, model = list(order = c(3,0,0),
                                    ar = c(0.8, -0.2, 0.1)))

## The theoretical
AR3.pacf <- pacf(AR3, plot=F)
AR3.data <- ts.intersect(xt = AR3, x1 = lag(AR3, 1), x2 =lag(AR3, 2), x3= lag(AR3, 3))

AR.lm <- resid(lm(xt ~ x1 + x2, data = AR3.data))
AR.lm.lag3 <- resid(lm(x3 ~ x1 + x2 , data = AR3.data))

AR3.pacf[3]
```

```
##
## Partial autocorrelations of series 'AR3', by lag
##
##     3
## 0.117
```

```
cor(AR.lm, AR.lm.lag3)
```

```
## [1] 0.1146076
```

b)

```
set.seed(12345)
AR2 <- arima.sim(100, model = list(order = c(2,0,0),
                                   ar = c(0.8, 0.1)))

ar2.yw <- ar(AR2, order.max = 2, method = "yw", aic = FALSE)
ar2.ols <- ar(AR2, order.max = 2, method = "ols", aic = FALSE)
ar2.mle <- arima(AR2, order = c(2,0,0), method = "ML")

ar2.yw
```

```
##
## Call:
## ar(x = AR2, aic = FALSE, order.max = 2, method = "yw")
##
## Coefficients:
##      1       2
## 0.8029  0.1037
##
## Order selected 2  sigma^2 estimated as  1.267
```

```r
ar2.ols
```

```
##
## Call:
## ar(x = AR2, aic = FALSE, order.max = 2, method = "ols")
##
## Coefficients:
##      1       2
## 0.8067  0.1205
##
## Intercept: -0.04401 (0.1074)
##
## Order selected 2  sigma^2 estimated as  1.129
```

```r
ar2.mle
```

```
##
## Call:
## arima(x = AR2, order = c(2, 0, 0), method = "ML")
##
## Coefficients:
##          ar1     ar2  intercept
##       0.7967  0.1189     0.8290
## s.e.  0.0992  0.1000     1.1385
##
## sigma^2 estimated as 1.126:  log likelihood = -148.71,  aic = 303.41
```

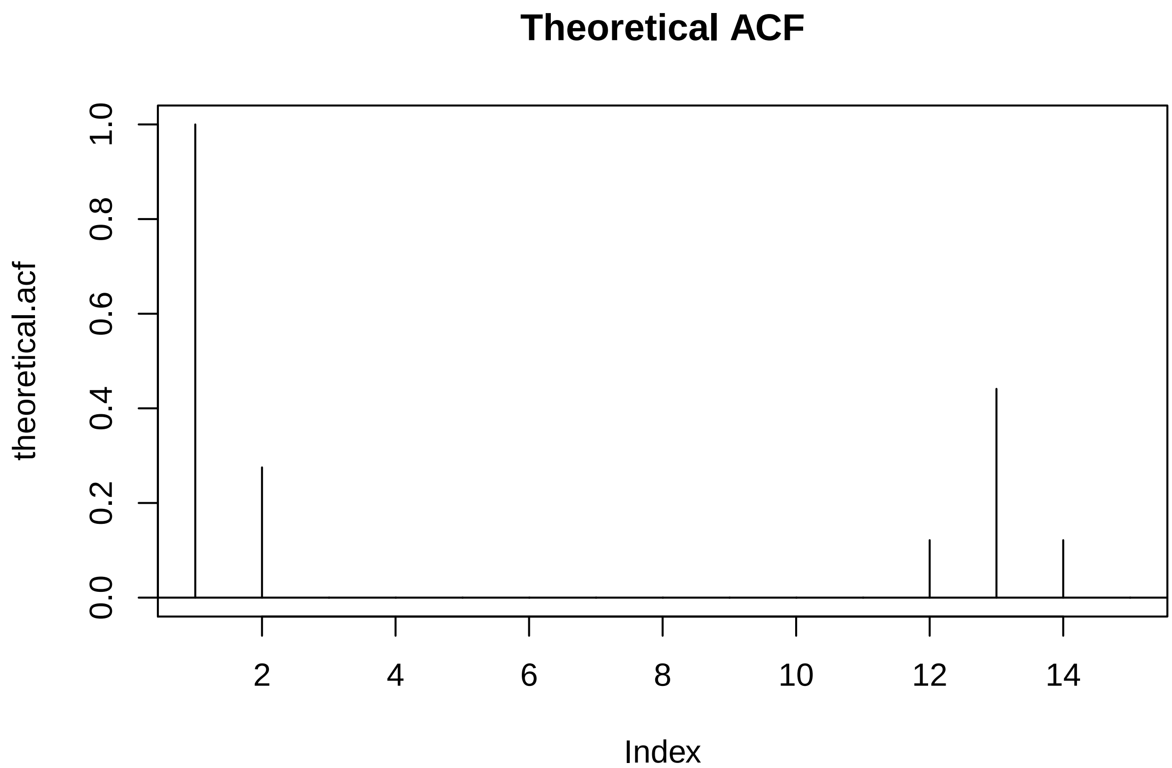
Yes, the theoretical value for

$$\phi_2$$

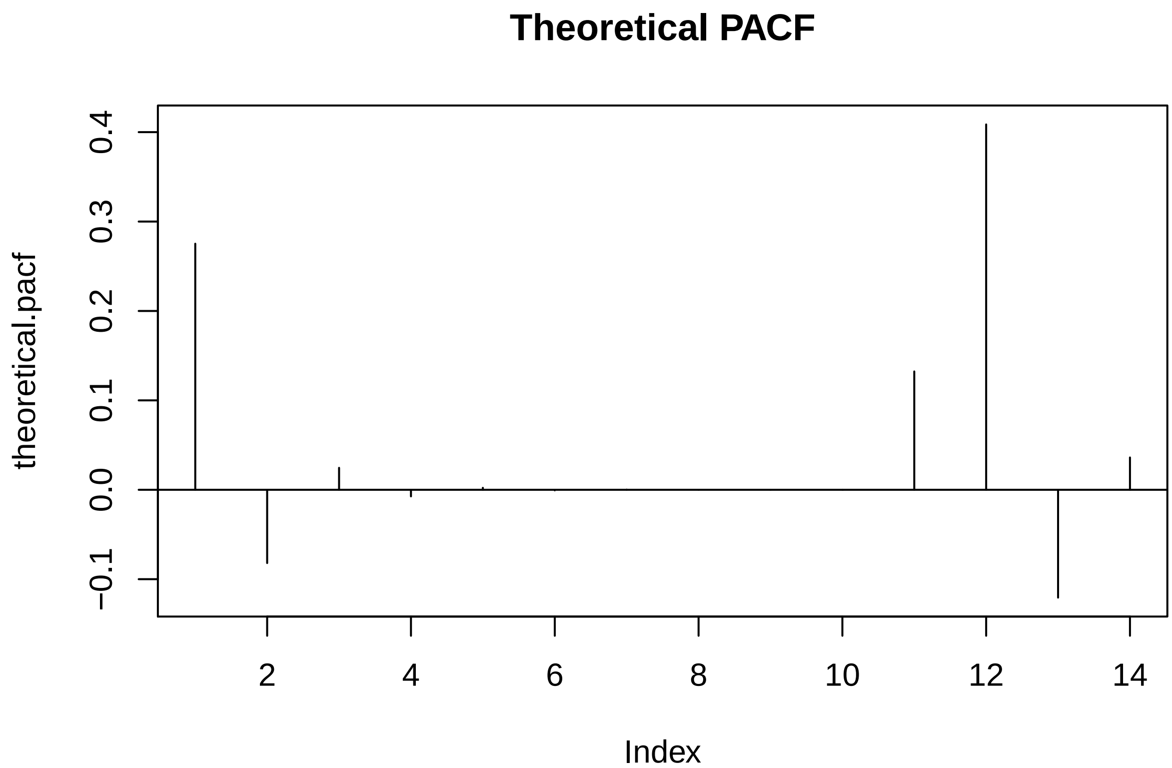
is inside the confidence-intervall for the ML estimate.

c)

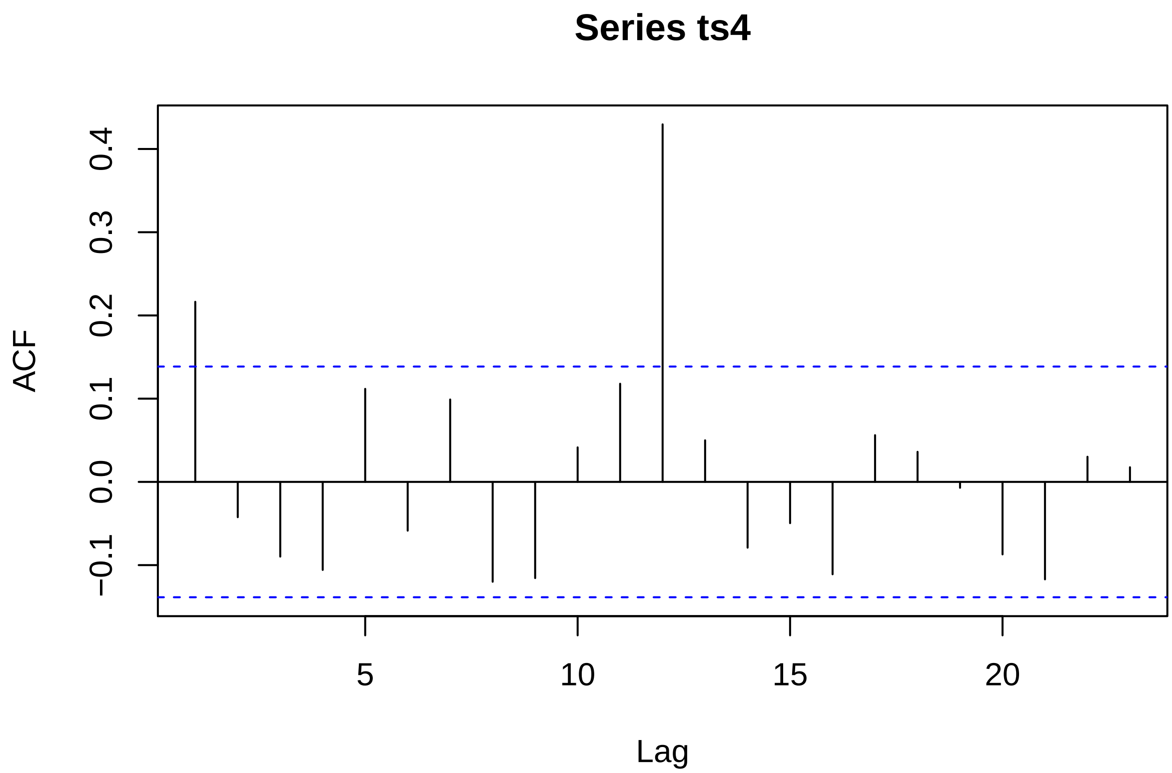
```r
set.seed(12345)
ma.coef <- c(0.3, rep(0, 10), 0.6)
ts4 <- arima.sim(n=200, model=list(order=c(0, 0, 12), ma = ma.coef))

theoretical.acf <- ARMAacf(ma=c(ma.coef, 0.3 * 0.6))
theoretical.pacf <- ARMAacf(ma=c(ma.coef, 0.3 * 0.6), pacf=TRUE)

plot(theoretical.acf, type="h", main="Theoretical ACF")
abline(h=0)
```



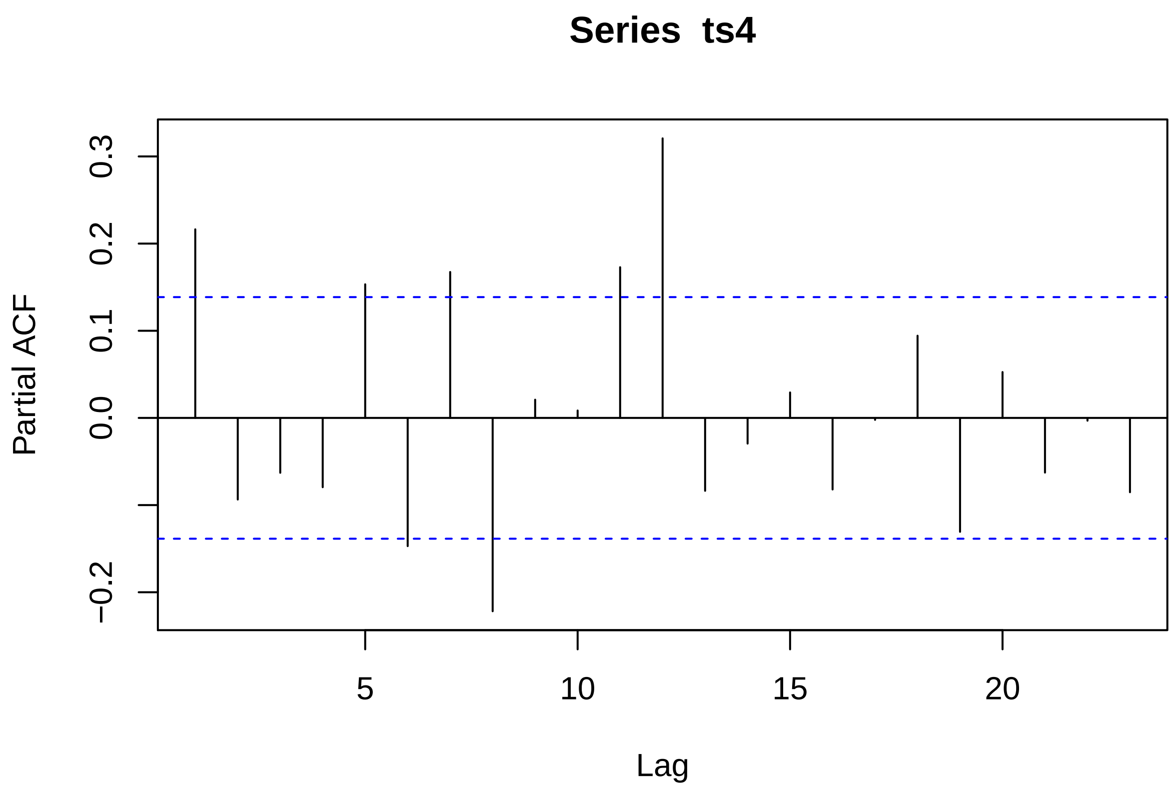

```
plot(theoretical.pacf, type="h", main="Theoretical PACF")
abline(h=0)
```




```r
empirical.acf <- acf(ts4)
```

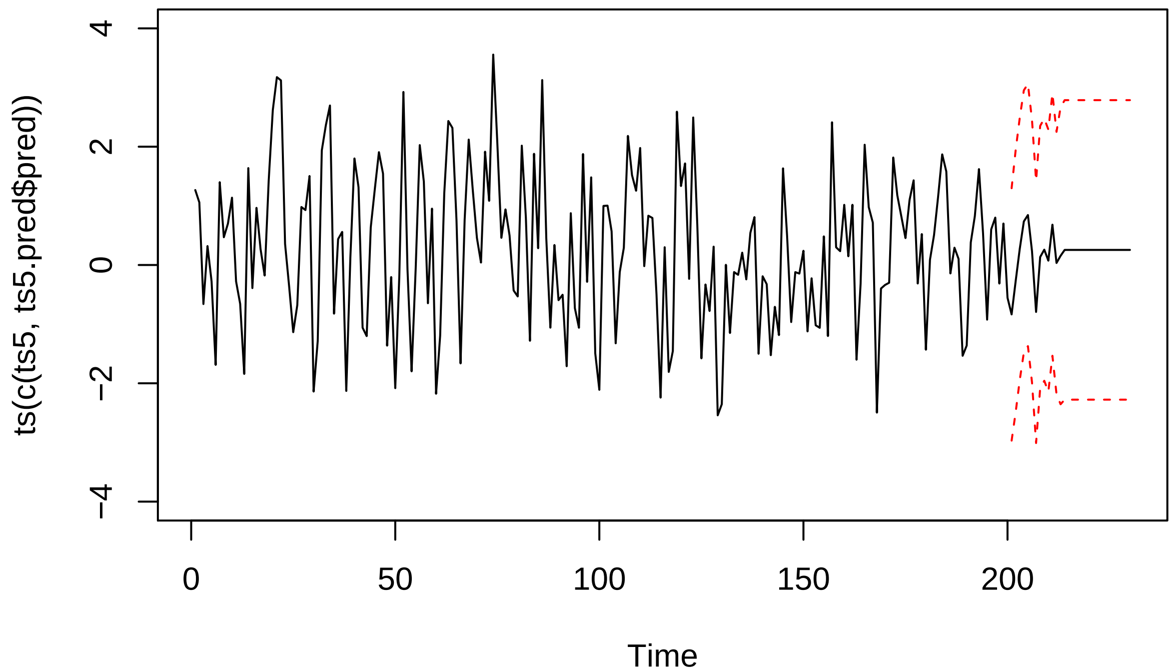



```
empirical.pacf <- pacf(ts4)
```



d)

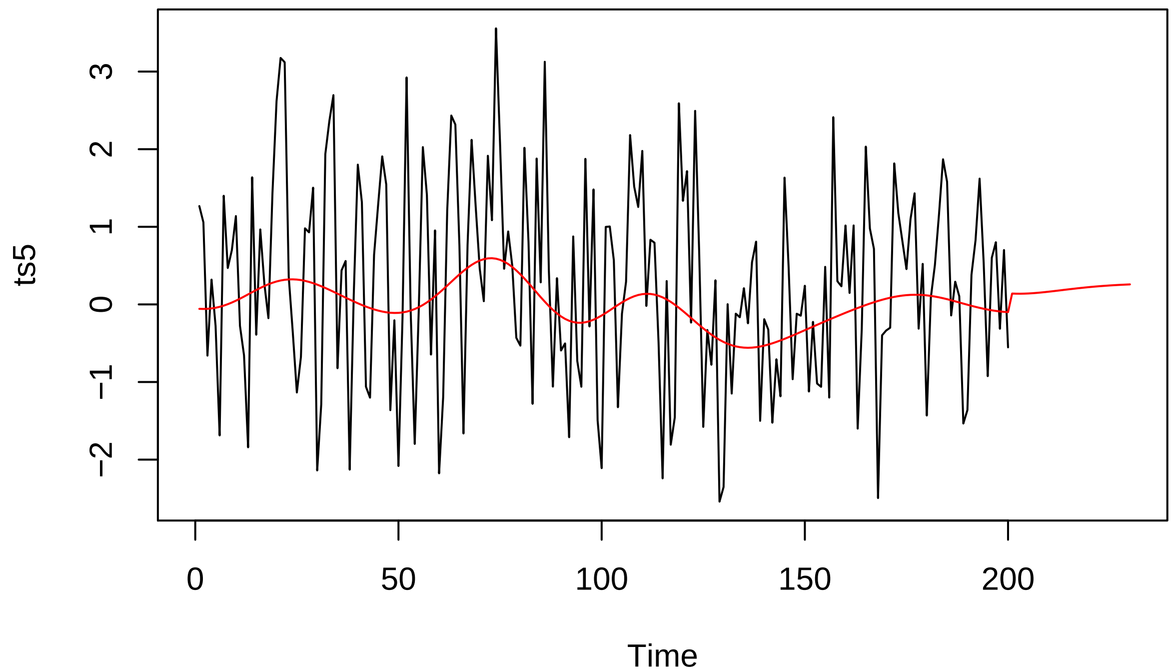
```
set.seed(12345)
ma.coef <- c(0.3, rep(0, 10), 0.6)
ts5 <- arima.sim(n=200, model=list(order=c(0, 0, 12), ma = ma.coef))

ts5.fit <- arima(ts5, order=c(0, 0, 1), seasonal=list(order=c(0, 0, 1), period=12))
ts5.pred <- predict(ts5.fit, n.ahead=30, se.fit=TRUE)

plot(ts(c(ts5, ts5.pred$pred)), ylim=c(-4, 4))
lines(200 + 1:length(ts5.pred$pred), ts5.pred$pred + 1.96 * ts5.pred$se, lty=2, col="red")
lines(200 + 1:length(ts5.pred$pred), ts5.pred$pred - 1.96 * ts5.pred$se, lty=2, col="red")
```




```
gausspr.data <- data.frame(y=ts5, x=1:200)
gausspr.fit <- gausspr(y ~ x, gausspr.data)
```

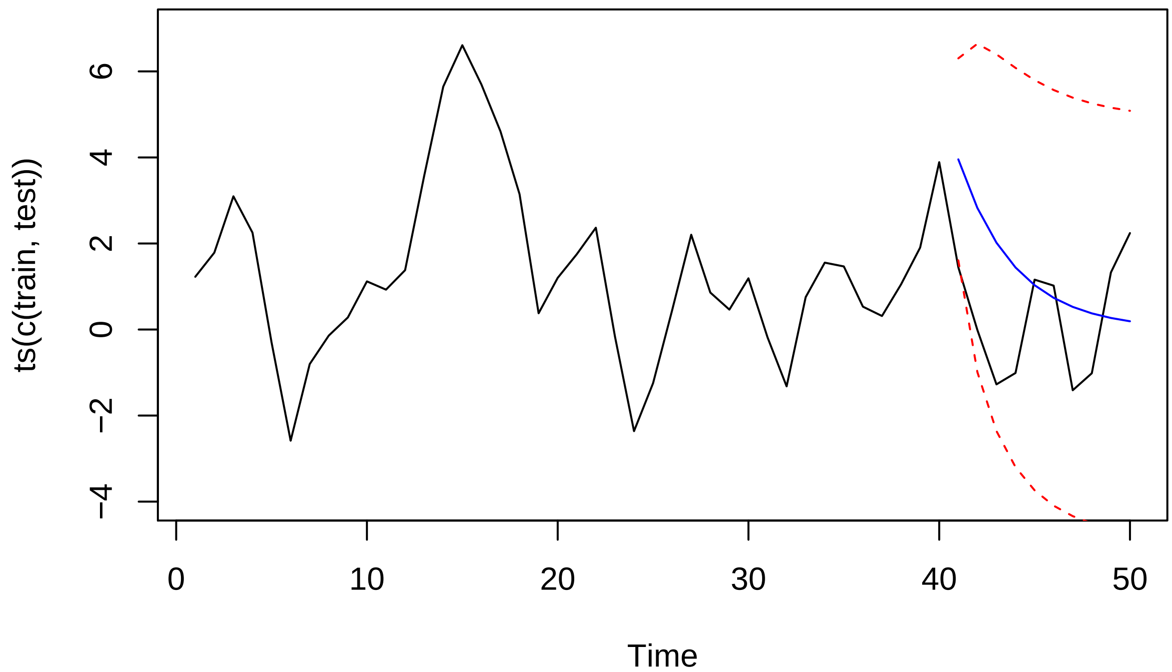
```
## Using automatic sigma estimation (sigest) for RBF or laplace kernel
```

```
gausspr.pred <- predict(gfit, data.frame(x=201:230))

plot(ts5, xlim=c(0, 230))
lines(c(fitted(gausspr.fit), gausspr.pred), , col="red")
```

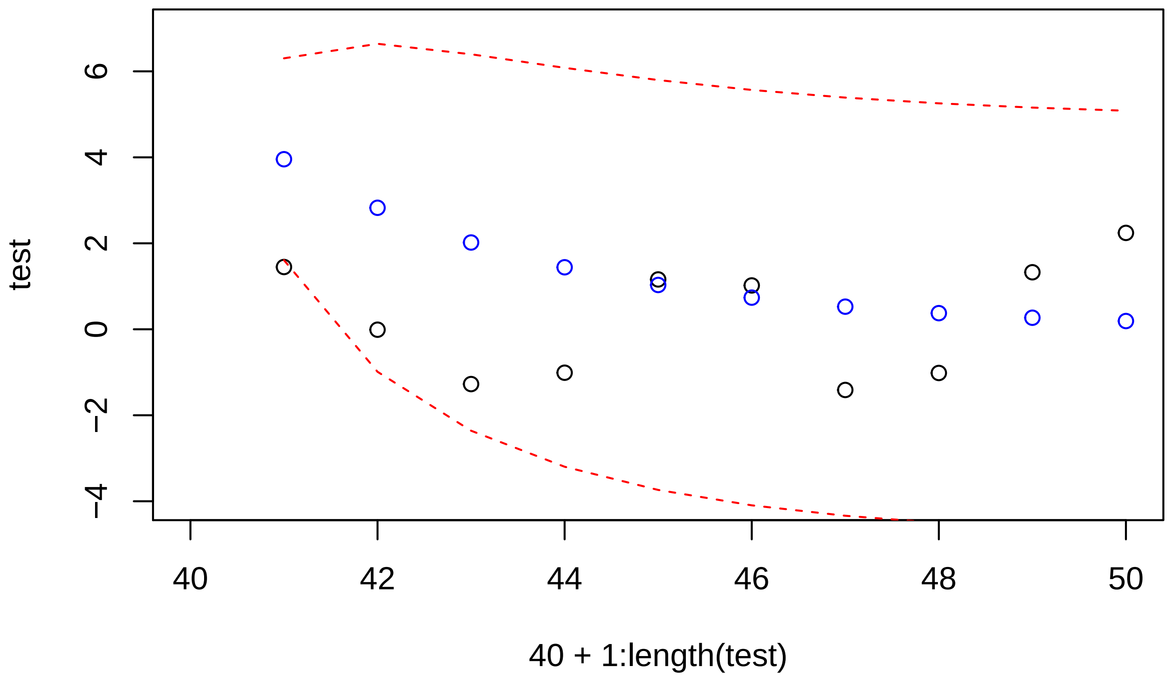


e)

```
set.seed(12345)
ts6 <- arima.sim(model=list(ma=c(0.5), ar=c(0.7)), n=50)

train <- ts(ts6[1:40])
test <- ts(ts6[41:50])

ts6.fit <- arima(train, order=c(1, 0, 1), include.mean = F)
ts6.pred <- predict(ts6.fit, n.ahead=10)

plot(ts(c(train, test)), ylim=c(-4, 7), type="l")
lines(40 + 1:length(test), ts6.pred$pred, col="blue")
lines(40 + 1:length(test), ts6.pred$pred + 1.96 * ts6.pred$se, lty=2, col="red")
lines(40 + 1:length(test), ts6.pred$pred - 1.96 * ts6.pred$se, lty=2, col="red")
```



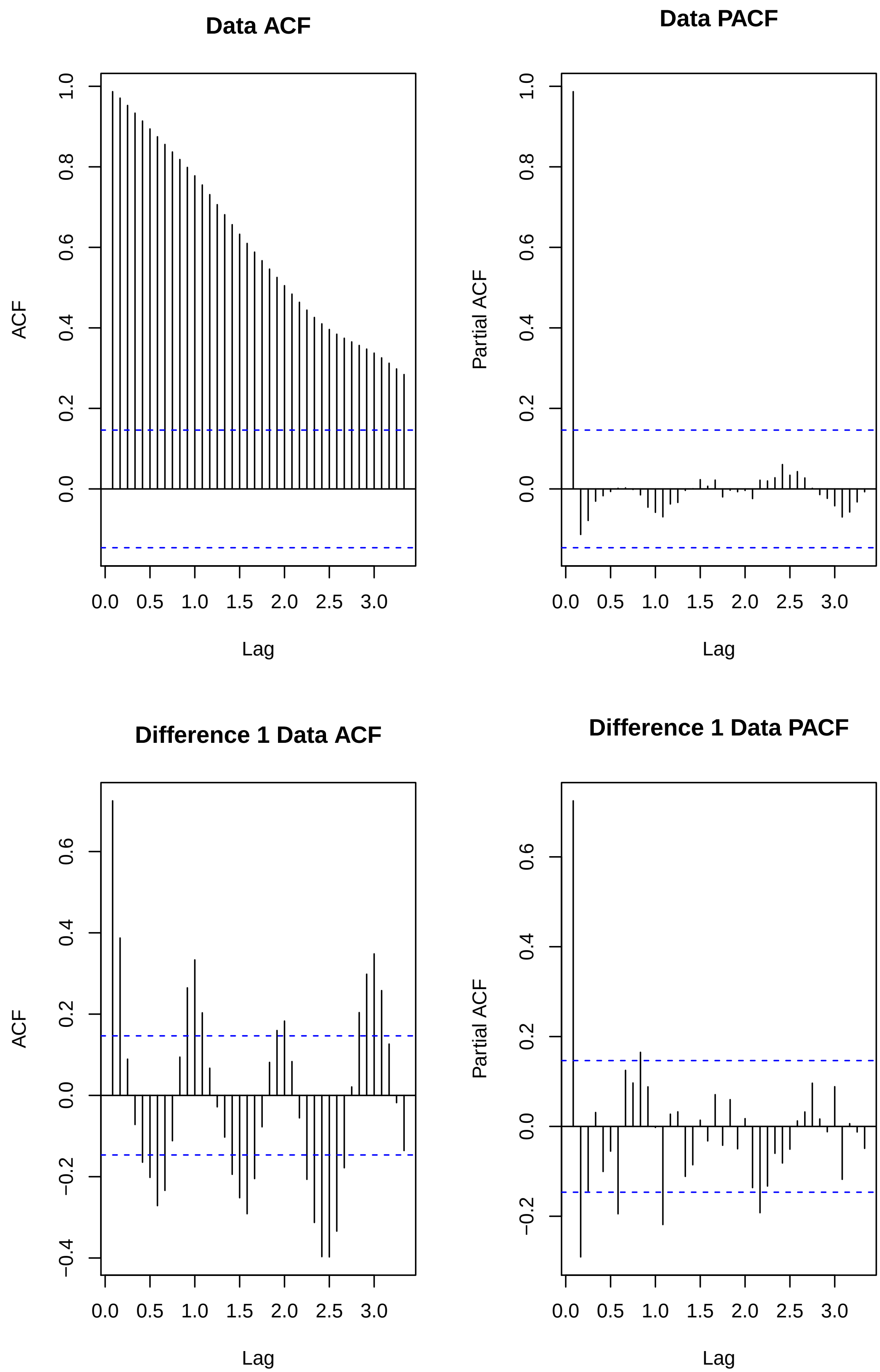

```
plot(40 + 1:length(test), test, ylim=c(-4, 7), xlim=c(40, 50), type="p")
points(40 + 1:length(test), ts6.pred$pred, col="blue")
lines(40 + 1:length(test), ts6.pred$pred + 1.96 * ts6.pred$se, lty=2, col="red")
lines(40 + 1:length(test), ts6.pred$pred - 1.96 * ts6.pred$se, lty=2, col="red")
```


test
6
4
2
0
−2
−4
40
42
44
46
48
50
40 + 1:length(test)

Assignment 2

```r
assignment2 <- function(data){
    old <- par(mfrow = c(2, 2))
    acf(data, lag.max = 40, main="Data ACF")
    pacf(data, lag.max = 40, main="Data PACF")
    acf(diff(data, lag = 1), lag.max = 40, main="Difference 1 Data ACF")
    pacf(diff(data, lag = 1), lag.max = 40, main="Difference 1 Data PACF")
    par(old)
}
```

Chicken



Data ACF
ACF
Lag
Data PACF
Partial ACF
Lag
Difference 1 Data ACF
ACF
Lag
Difference 1 Data PACF
Partial ACF
Lag

Data ACF

The ACF on the original data suggests an AR or ARMA model since the ACF tails off.

Data PACF

The PACF on the original data cuts off after lag 1 suggesting an AR(1) model.

Difference 1 Data ACF

After having performed difference of order 1 we can clearly see that there is a seasonal trend in the data. The ACF suggests a seasonality of 10 but it does not seem to tail off.

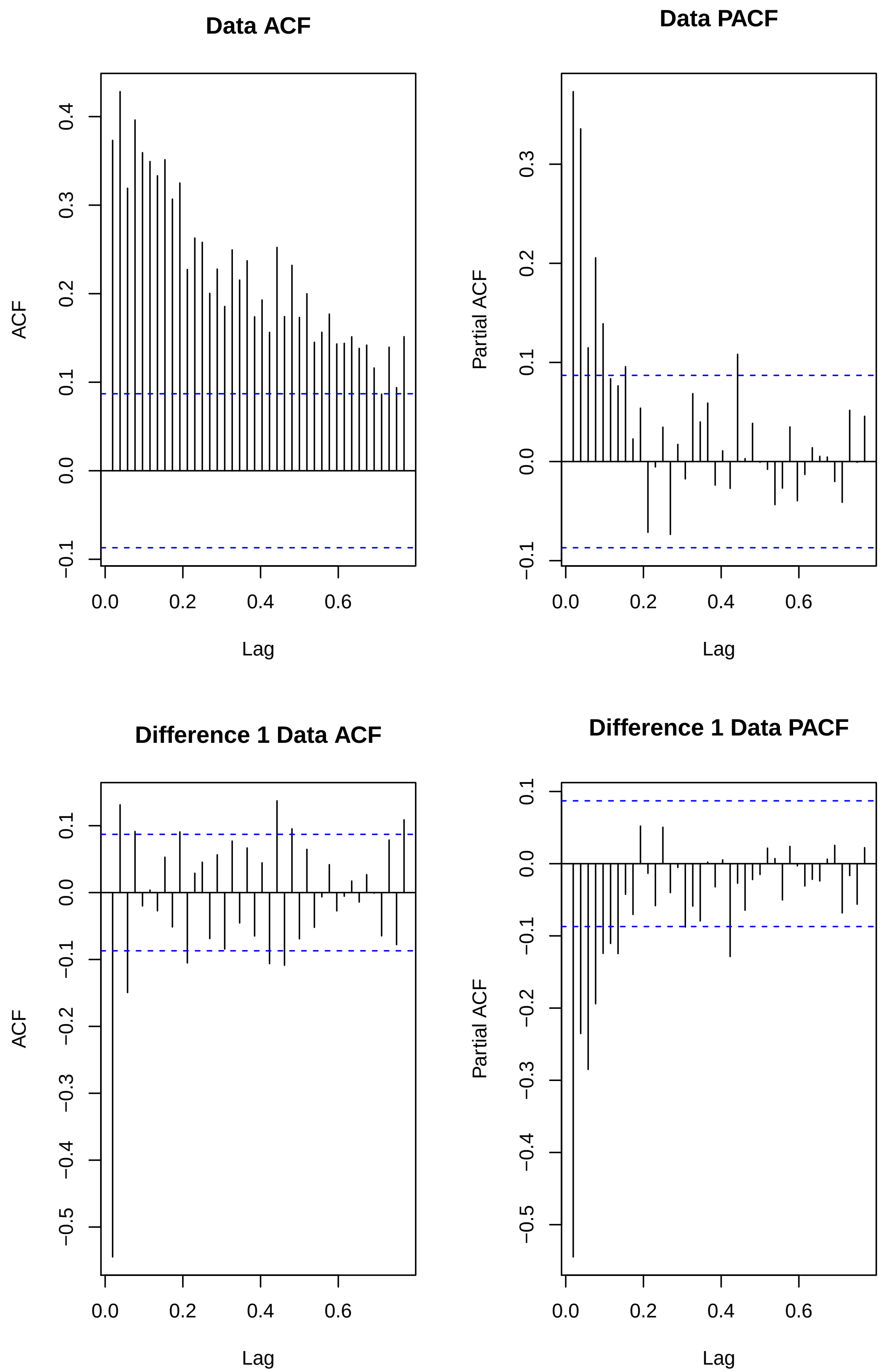
Difference 1 Data PACF

The PACF indicates that ...

Final Verdict

ARIMA(1, 0, 0) x $(\_, 1, \_)_1 0$

so2


Data ACF
ACF
Lag
Data PACF
Partial ACF
Lag
Difference 1 Data ACF
ACF
Lag
Difference 1 Data PACF
Partial ACF
Lag

Data ACF

The ACF tails off suggesting either an AR or ARMA model.

Data PACF

The PACF tails off as well suggesting an ARMA model.

Difference 1 Data ACF

The ACF after difference cuts off after lag 1 suggesting a MA(1) model.

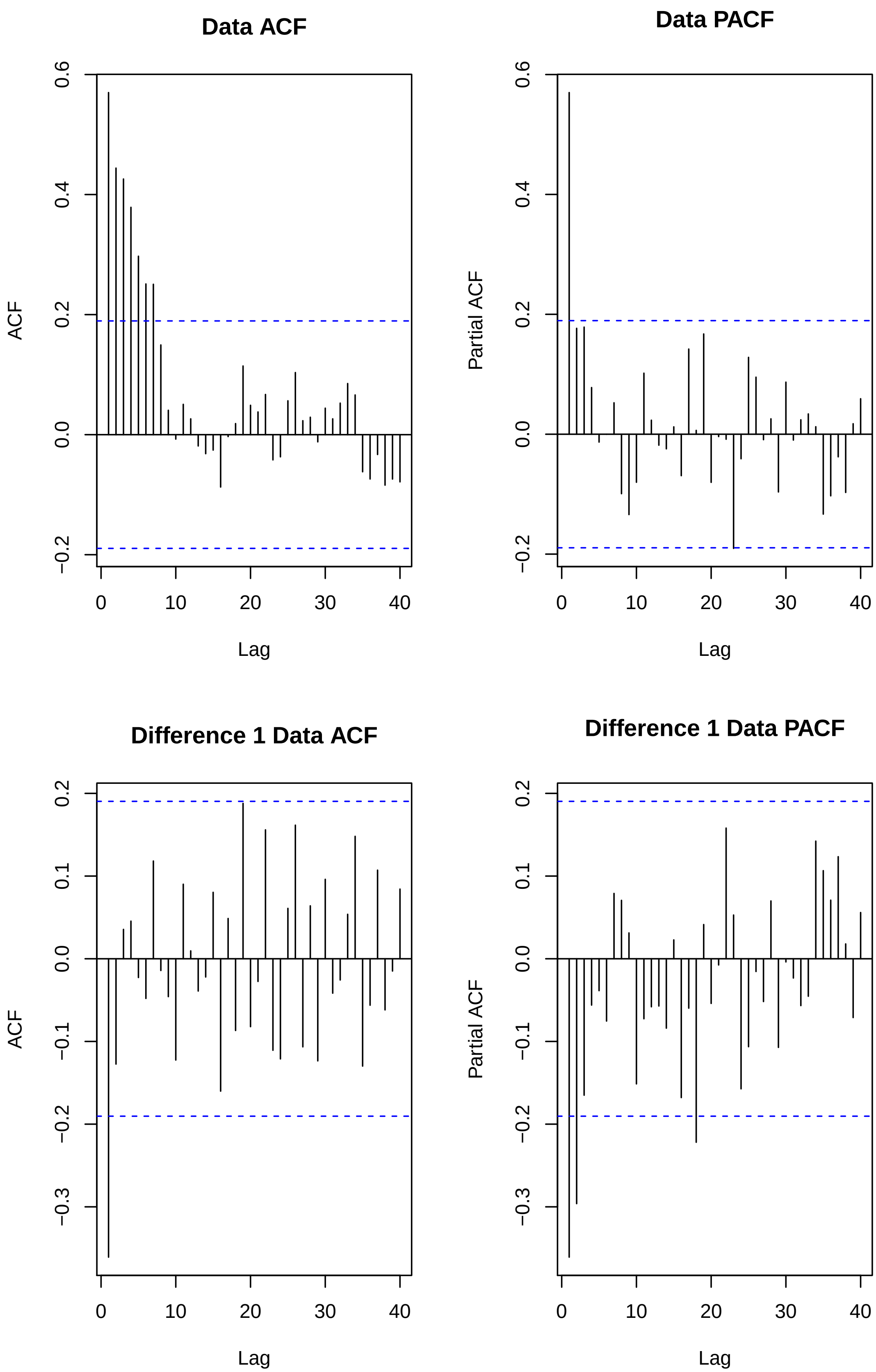
Difference 1 Data PACF

The PACF after difference tails off further suggesting a MA(1) model.

Final Verdict

ARIMA(0, 1, 1)

EQcount


Data ACF
Data PACF
Difference 1 Data ACF
Difference 1 Data PACF
ACF
Partial ACF
Lag
0
10
20
30
40
0.6
0.4
0.2
0.0
−0.2
0.1
−0.1
−0.3

Data ACF

The ACF tails off suggesting an AR or ARMA model.

Data PACF

The PACF cuts off after lag 1 suggesting AR(1) model.

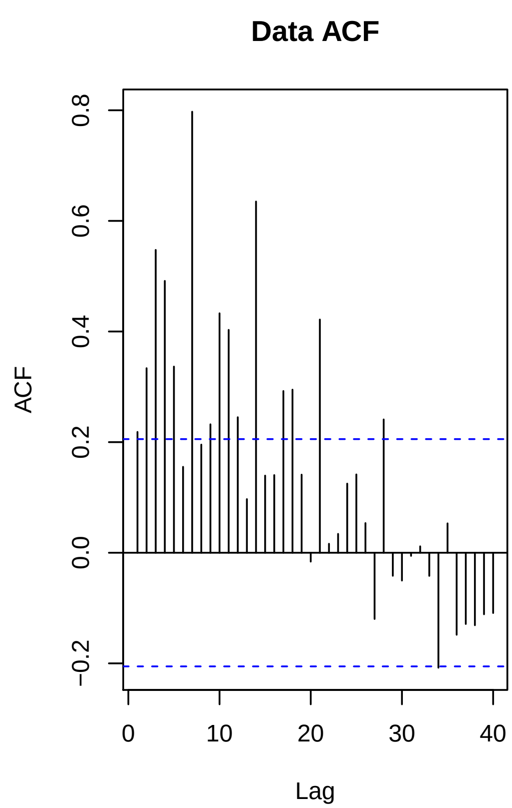
Difference 1 Data ACF

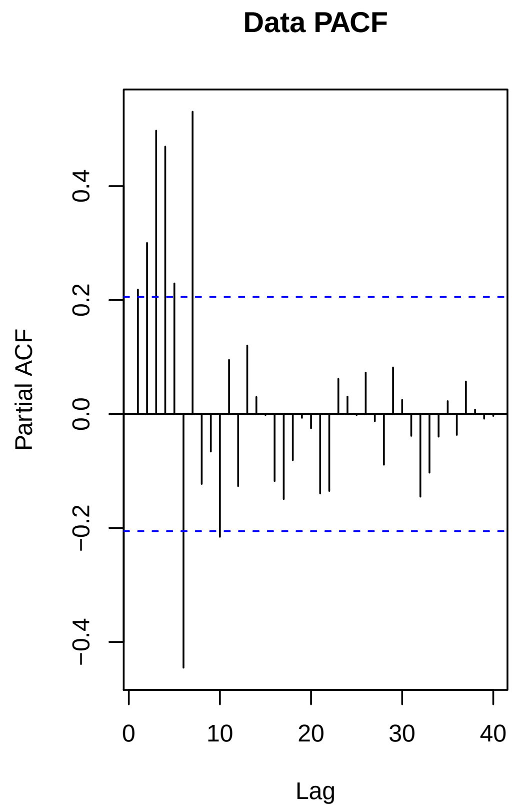
The ACF after difference cuts off after lag 1 suggesting a MA(1) model.

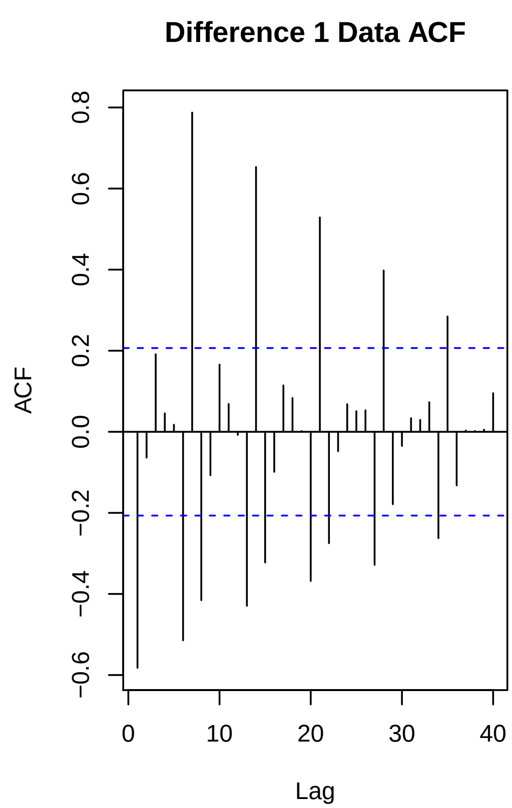
Difference 1 Data PACF

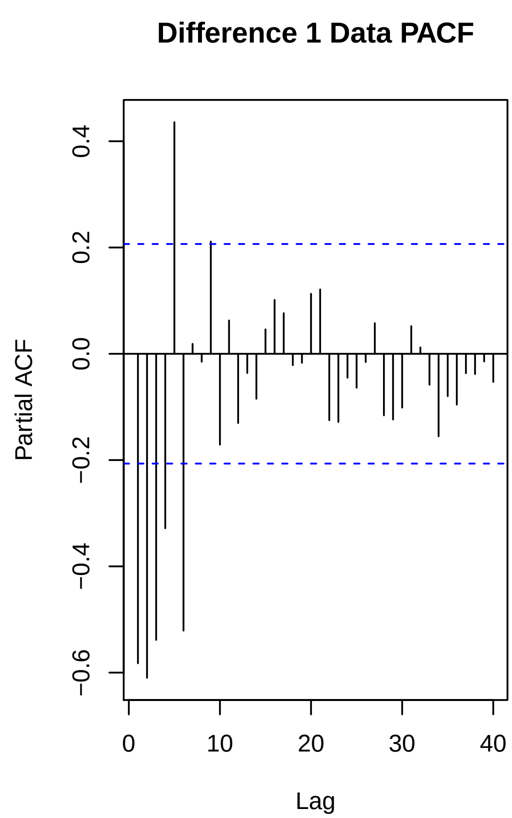
Final Verdict

HCT


Data ACF
ACF
Lag
0 10 20 30 40
−0.2 0.0 0.2 0.4 0.6 0.8


Data PACF
Partial ACF
Lag
0 10 20 30 40
−0.4 −0.2 0.0 0.2 0.4


Difference 1 Data ACF
ACF
Lag
0 10 20 30 40
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6 0.8


Difference 1 Data PACF
Partial ACF
Lag
0 10 20 30 40
−0.6 −0.4 −0.2 0.0 0.2 0.4

Data ACF

The ACF tails off suggesting either an AR or ARMA model.

Data PACF

The PACF cuts off after lag 7 suggesting an AR(7) model.

Difference 1 Data ACF

The ACF suggests seasonality that tails off after lag 7 suggesting an seasonality of 7.

Difference 1 Data PACF

The PACF cuts off after 6 lags suggesting an AR(6) seasonality model.

Final Verdict

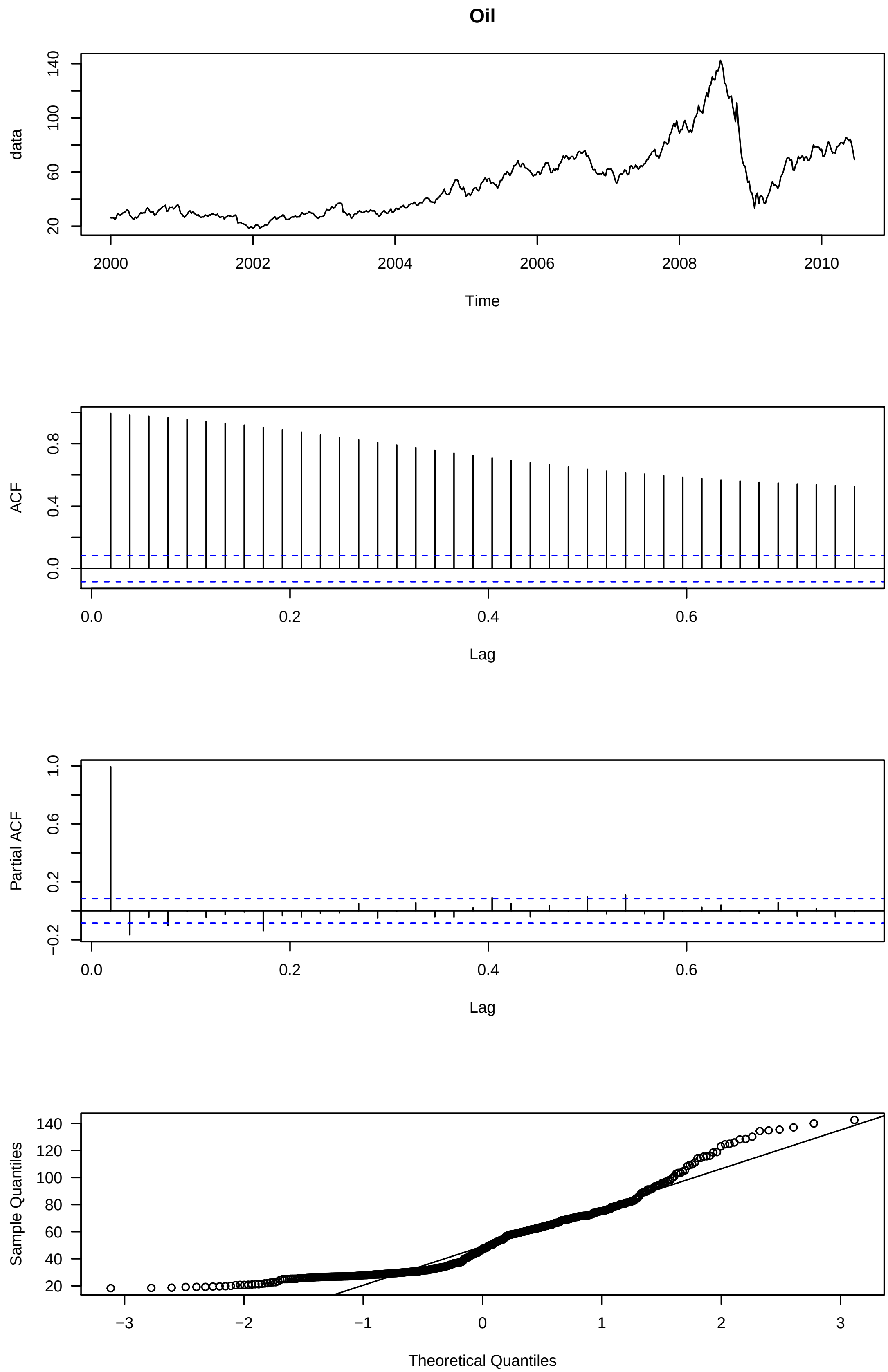
ARIMA(7, 0, 0) x $(6, 1, 0)_7$

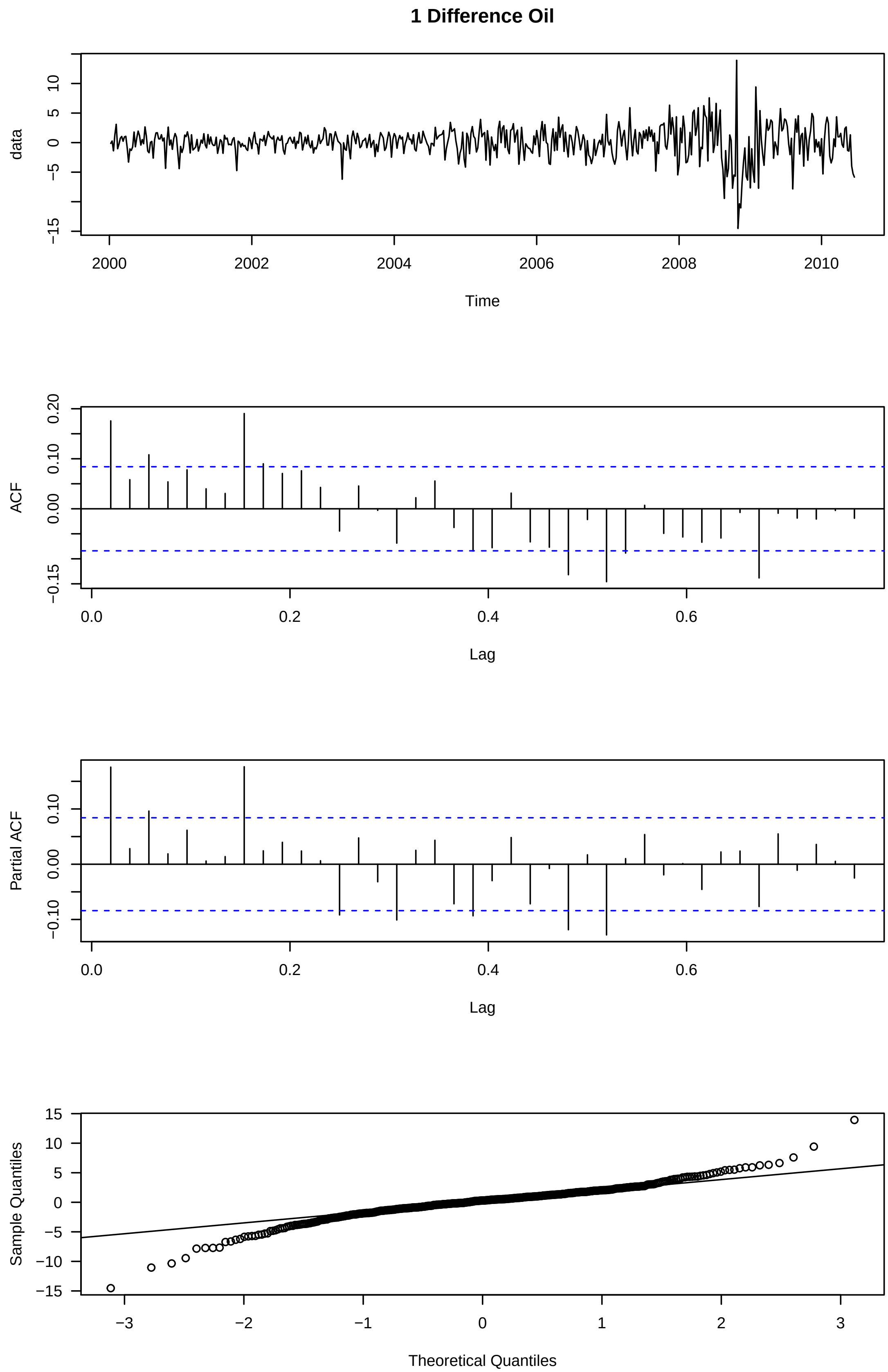
Assignment 3

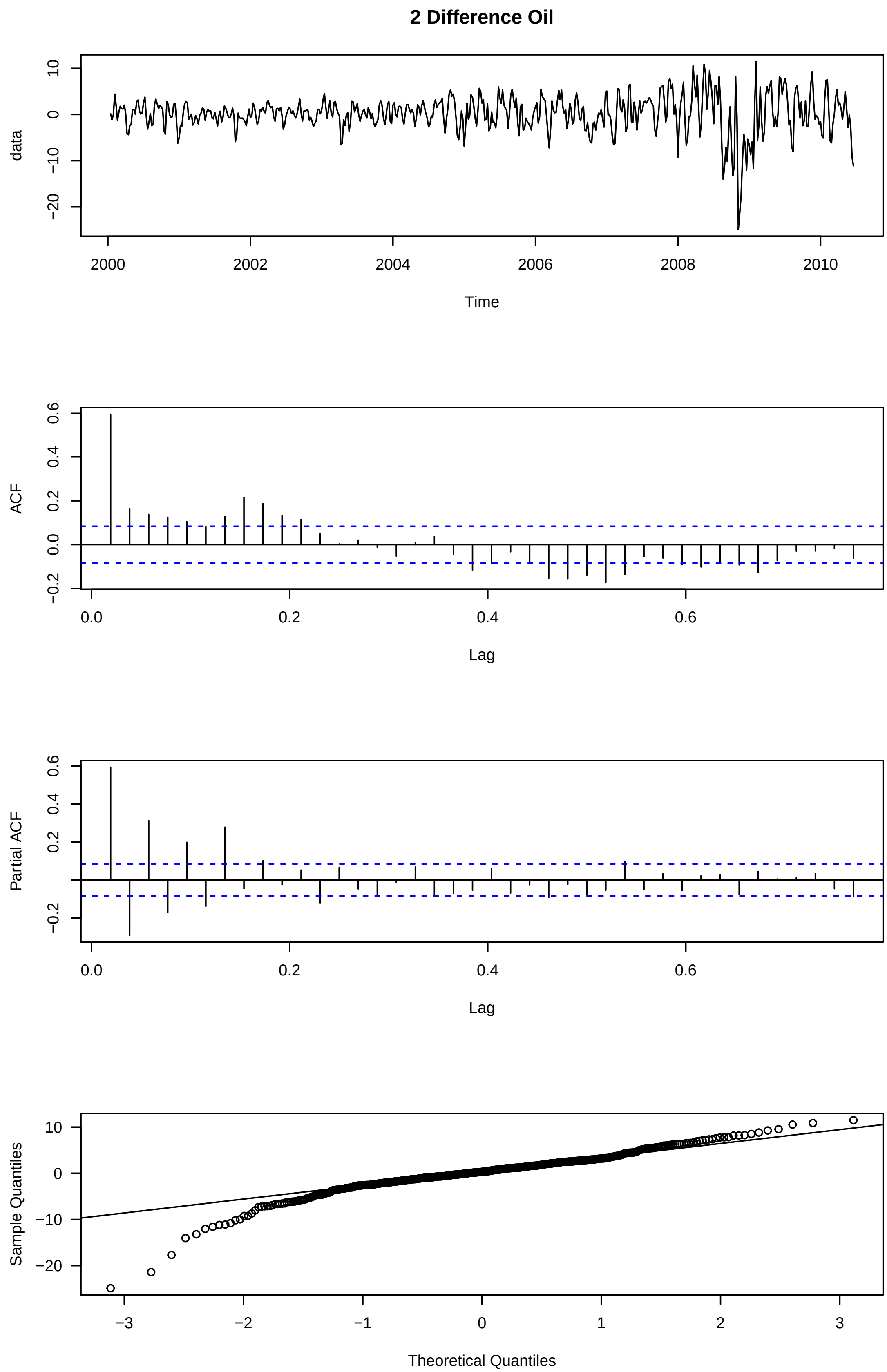
```
plot_helper <- function(data, title) {
    old <- par(mfrow=c(4, 1))
    plot(data, main=title)
    acf(data, lag.max=40, main="")
    pacf(data, lag.max=40, main="")
    qqnorm(data, main="", las=1)
    qqline(data)
    par(old)
}

test_helper <- function(data) {
    print(Box.test(data, lag = 1, type = "Ljung-Box"))
    print(suppressWarnings(adf.test(data)))
    e <- eacf(data)
}

fit_plot <- function(model) {
    pred <- predict(model, n.ahead=20, se.fit=TRUE)
    upper_band <- pred$pred + 1.96 * pred$se
    lower_band <- pred$pred - 1.96 * pred$se

    plot(c(model$x, pred$pred), type="l", xlim=c(500, length(oil) + 20), ylim=c(min(lower_band), max(up
    lines(length(oil) + 1:20, upper_band, lty=2, col="red")
    lines(length(oil) + 1:20, lower_band, lty=2, col="red")
}
```

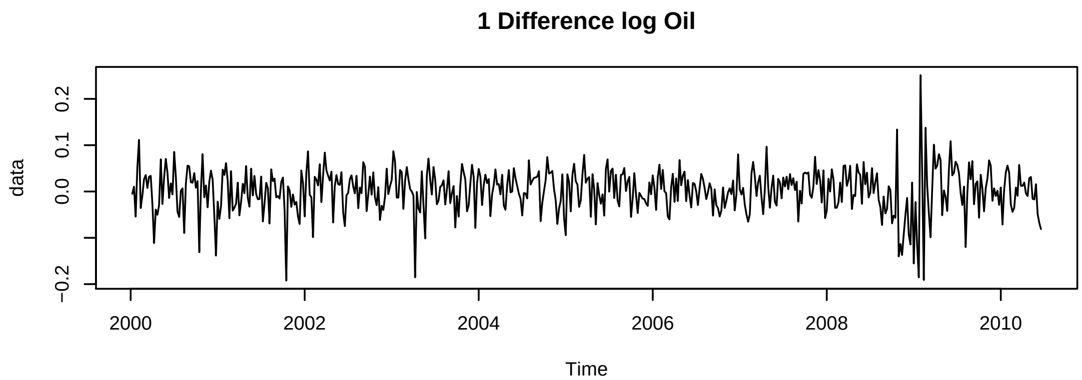
a)

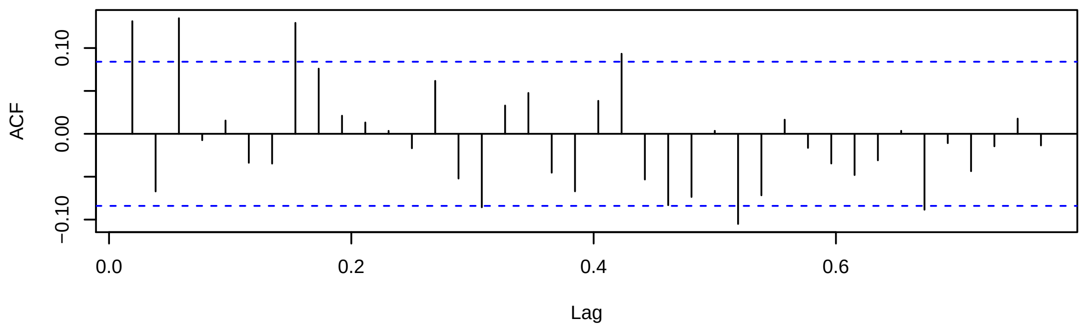
```
loil <- log(oil)
doil <- diff(oil)
ddoil <- diff(oil, 2)
dloil <- diff(loil)
ddloil <- diff(loil, 2)
```

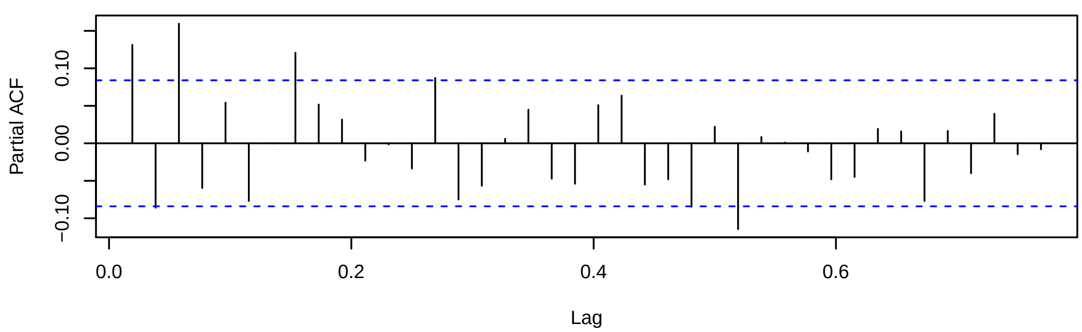

Oil
data
20
60
100
140
2000
2002
2004
2006
2008
2010
Time
ACF
0.0
0.4
0.8
0.0
0.2
0.4
0.6
Lag
Partial ACF
−0.2
0.2
0.6
1.0
0.0
0.2
0.4
0.6
Lag
Sample Quantiles
20
40
60
80
100
120
140
−3
−2
−1
0
1
2
3
Theoretical Quantiles

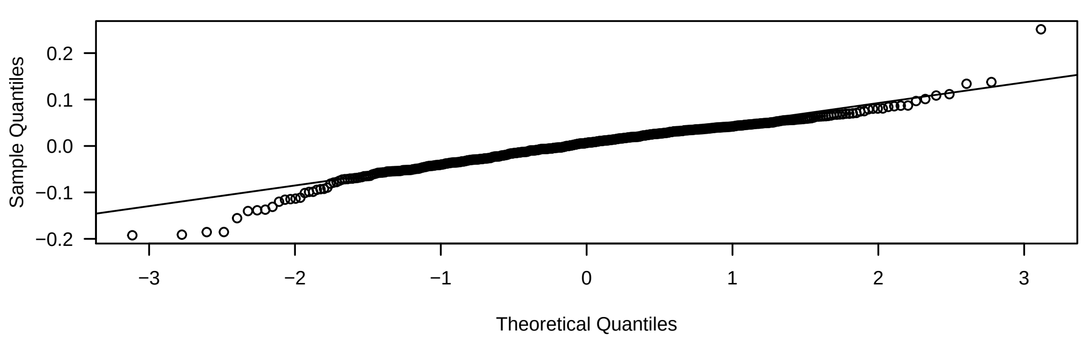

1 Difference Oil
data
−15
−5
0
5
10
2000
2002
2004
2006
2008
2010
Time
ACF
−0.15
0.00
0.10
0.20
0.0
0.2
0.4
0.6
Lag
Partial ACF
−0.10
0.00
0.10
0.0
0.2
0.4
0.6
Lag
Sample Quantiles
−15
−10
−5
0
5
10
15
−3
−2
−1
0
1
2
3
Theoretical Quantiles


2 Difference Oil
data
Time
ACF
Lag
Partial ACF
Lag
Sample Quantiles
Theoretical Quantiles

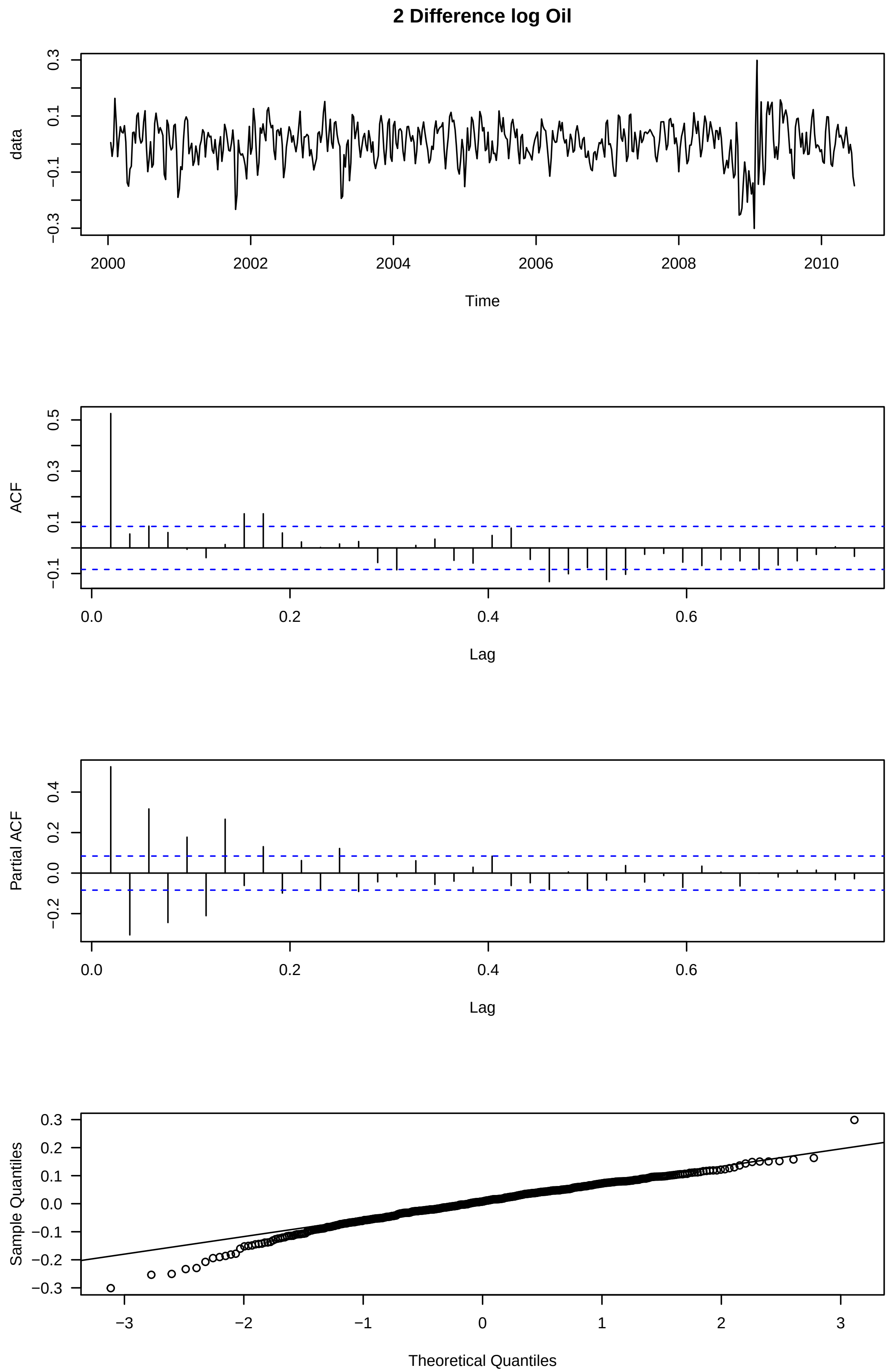

1 Difference log Oil
data
−0.2
0.0
0.1
0.2
2000
2002
2004
2006
2008
2010
Time


ACF
−0.10
0.00
0.10
0.0
0.2
0.4
0.6
Lag


Partial ACF
−0.10
0.00
0.10
0.0
0.2
0.4
0.6
Lag


Sample Quantiles
−0.2
−0.1
0.0
0.1
0.2
−3
−2
−1
0
1
2
3
Theoretical Quantiles

2 Difference log Oil



```r
test_helper(doil)
```

```
##
##  Box-Ljung test
##
## data:  data
## X-squared = 16.884, df = 1, p-value = 3.974e-05
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -5.3269, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o x x o o  o  o  o
## 1 x o o o o o o x o o o  o  o  o
## 2 x x o o o o o x o o o  o  o  o
## 3 x x x o o o o x o o o  o  o  o
## 4 x x x o o o o x o o o  o  o  o
## 5 x x o o o o o x o o o  o  o  o
## 6 x o x o x o o x o o o  o  o  o
## 7 o o x o x o x x o o o  o  o  o
```

```r
test_helper(ddoil)
```

```
##
##  Box-Ljung test
##
## data:  data
## X-squared = 192.72, df = 1, p-value < 2.2e-16
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -4.7773, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x o x x x x x  o  o  o
## 1 x x o o o o o x o o o  o  o  o
## 2 x x x o o o o x x o x  o  o  o
## 3 x x x o o o o x o o x  o  o  o
## 4 x x o o o o o x x o o  o  o  o
## 5 x x o x x x o x o o o  o  o  o
## 6 x x o x x x x x o o o  o  o  o
## 7 x x o x x x o x o x o  o  o  o
```

```r
test_helper(dloil)
```

```
##
```

```
##  Box-Ljung test
##
## data:  data
## X-squared = 9.4307, df = 1, p-value = 0.002134
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -6.3708, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o x o o o  o  o  o
## 1 x o x o o o o x o o o  o  o  o
## 2 x x x o o o o x o o o  o  o  o
## 3 x x x o o o o x o o o  o  o  o
## 4 x o x o o o o x o o o  o  o  o
## 5 x x x o x o o x o o o  o  o  o
## 6 o x x o x x o x o o o  o  o  x
## 7 o x x x x x x x o x o  o  o  o
```

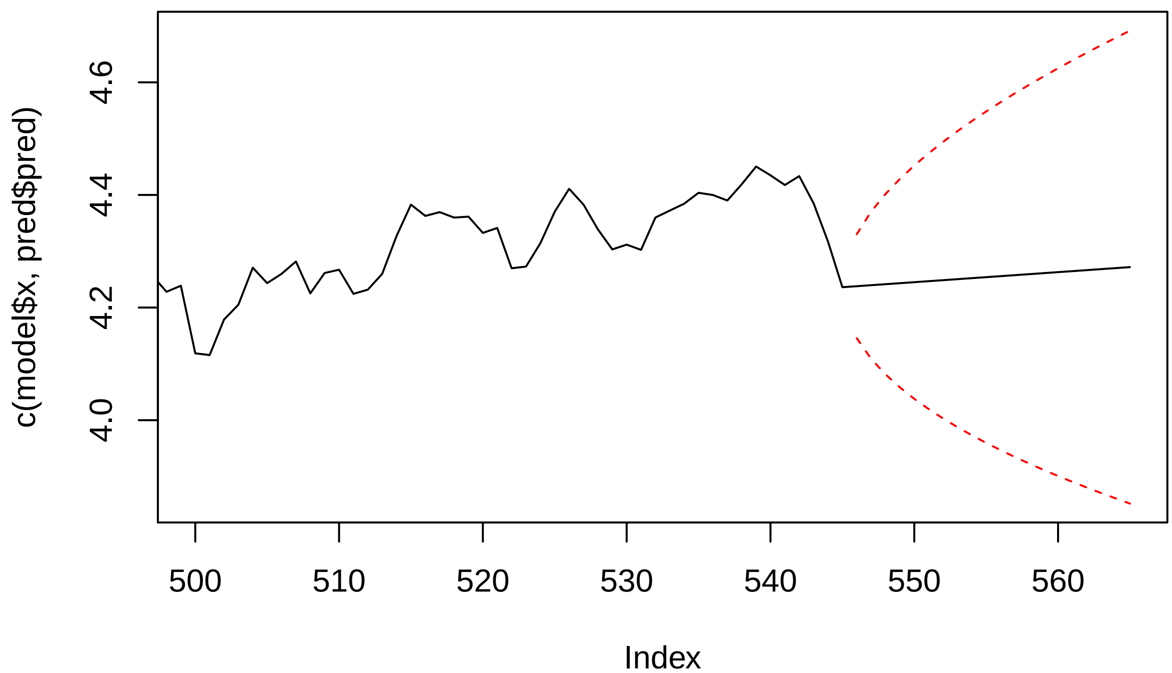
```r
test_helper(ddloil)
```

```
##
##  Box-Ljung test
##
## data:  data
## X-squared = 150.51, df = 1, p-value < 2.2e-16
##
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -5.6251, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
##
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o x x o o  o  o  o
## 1 x x o o o o o x x o o  o  o  o
## 2 x x x o o o o x x o o  o  o  o
## 3 x x x x o o o x x o o  o  o  o
## 4 x x x x o o o x x o o  o  o  o
## 5 x o x x x o o x x o o  o  o  o
## 6 x o x x x x x x o x o  o  o  o
## 7 x x x x x x x x o x o  o  o  o
```

```r
fit1 <- Arima(loil, order=c(0, 2, 1))
fit1
```

```
## Series: loil
## ARIMA(0,2,1)
##
## Coefficients:
##           ma1
##       -1.0000
## s.e.   0.0061
##
## sigma^2 estimated as 0.002213:  log likelihood=886.63
## AIC=-1769.26   AICc=-1769.24   BIC=-1760.67
```

```r
fit_plot(fit1)
```

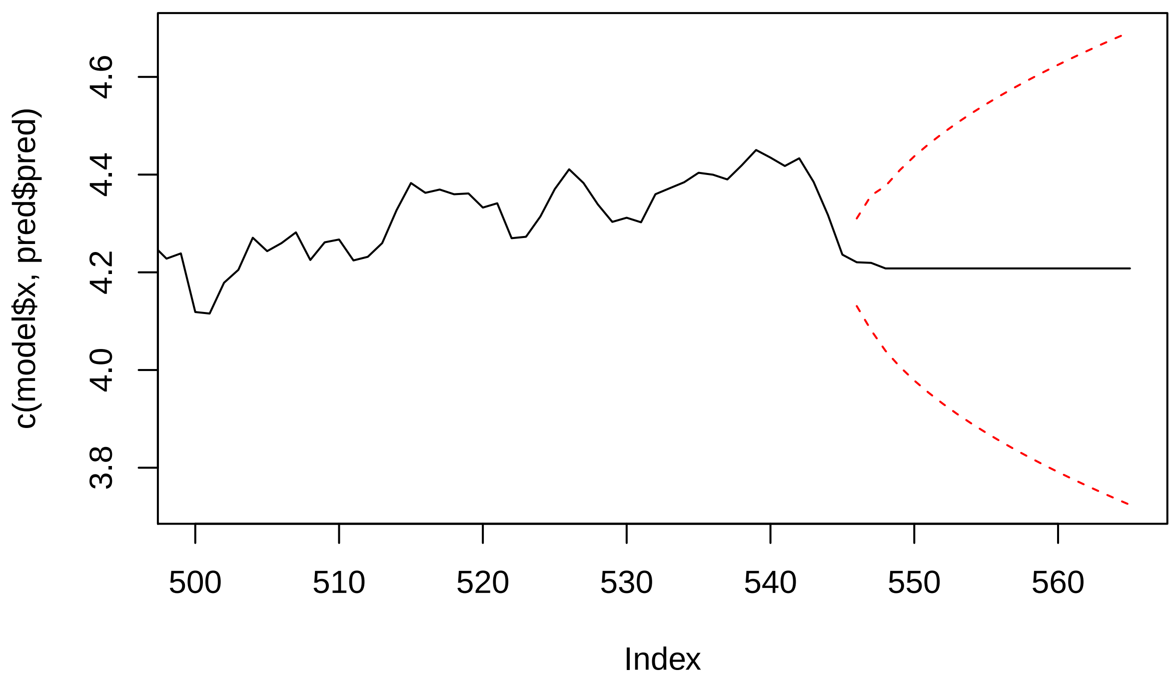


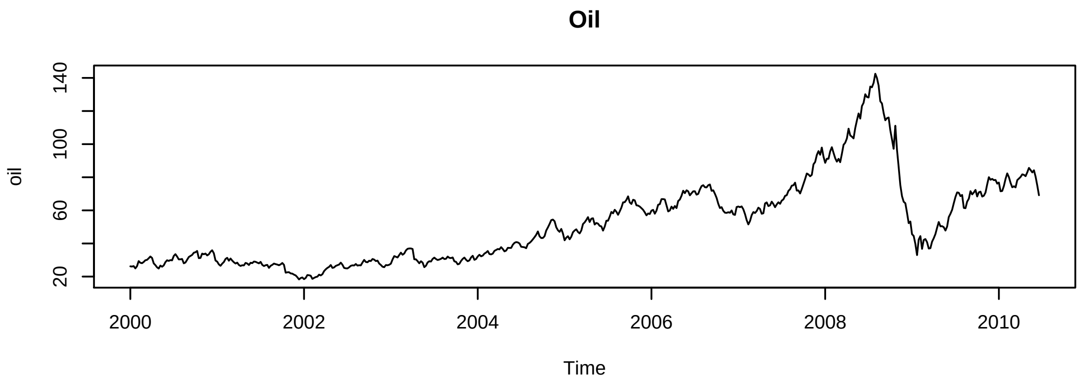

```r
fit2 <- Arima(loil, order=c(0, 1, 3))
fit2
```

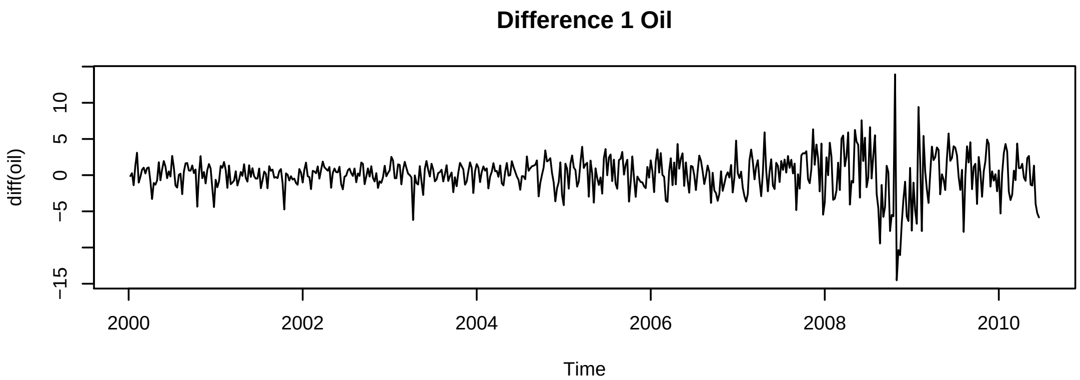
```
## Series: loil
## ARIMA(0,1,3)
##
## Coefficients:
##          ma1      ma2     ma3
##       0.1696  -0.0886  0.1458
## s.e.  0.0424   0.0424  0.0429
##
```

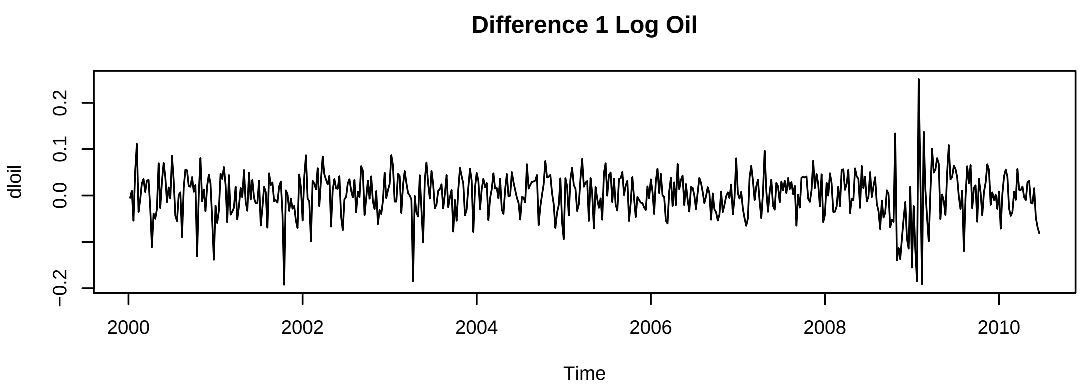
```
## sigma^2 estimated as 0.002094:  log likelihood=907.41
## AIC=-1806.83   AICc=-1806.75   BIC=-1789.63
```

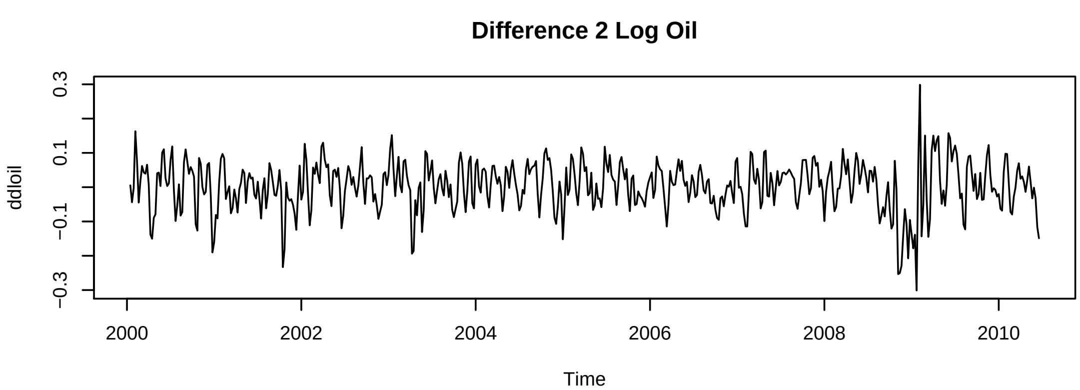
```r
fit_plot(fit2)
```




Oil
oil
20
60
100
140
2000
2002
2004
2006
2008
2010
Time

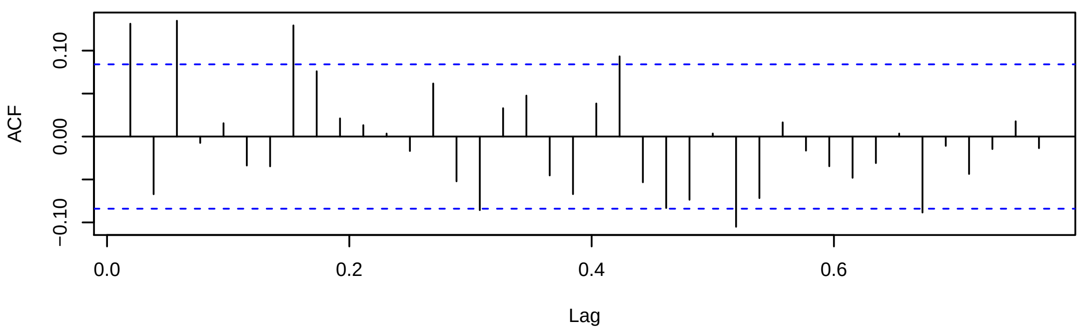

Difference 1 Oil
diff(oil)
−15
−5
0
5
10
2000
2002
2004
2006
2008
2010
Time


Difference 1 Log Oil
dloil
−0.2
0.0
0.1
0.2
2000
2002
2004
2006
2008
2010
Time

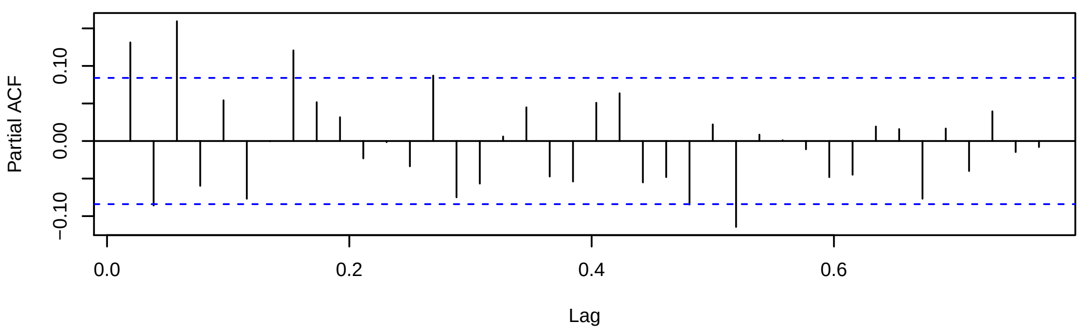

Difference 2 Log Oil
ddloil
−0.3
−0.1
0.1
0.3
2000
2002
2004
2006
2008
2010
Time

Clearly difference log is the data we should work with. bla, bla, ...

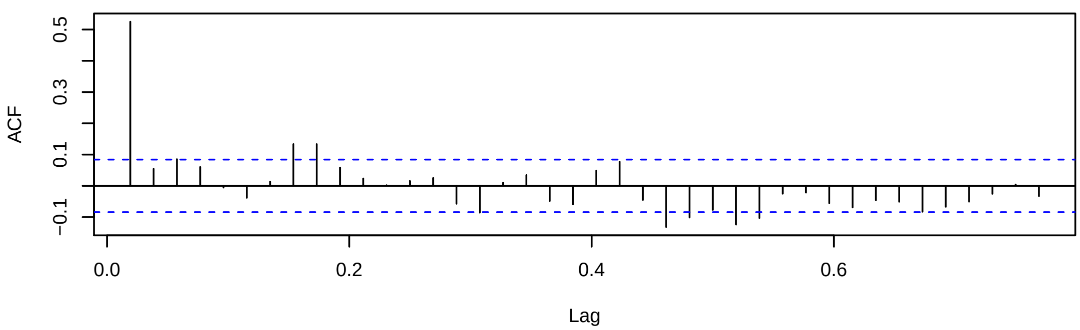
Difference 1 Log Oil ACF



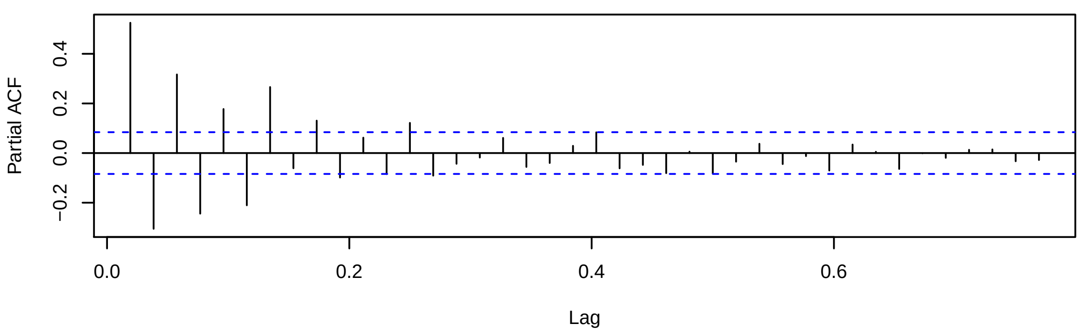
Difference 1 Log Oil PACF



Difference 2 Log Oil ACF



Difference 2 Log Oil PACF



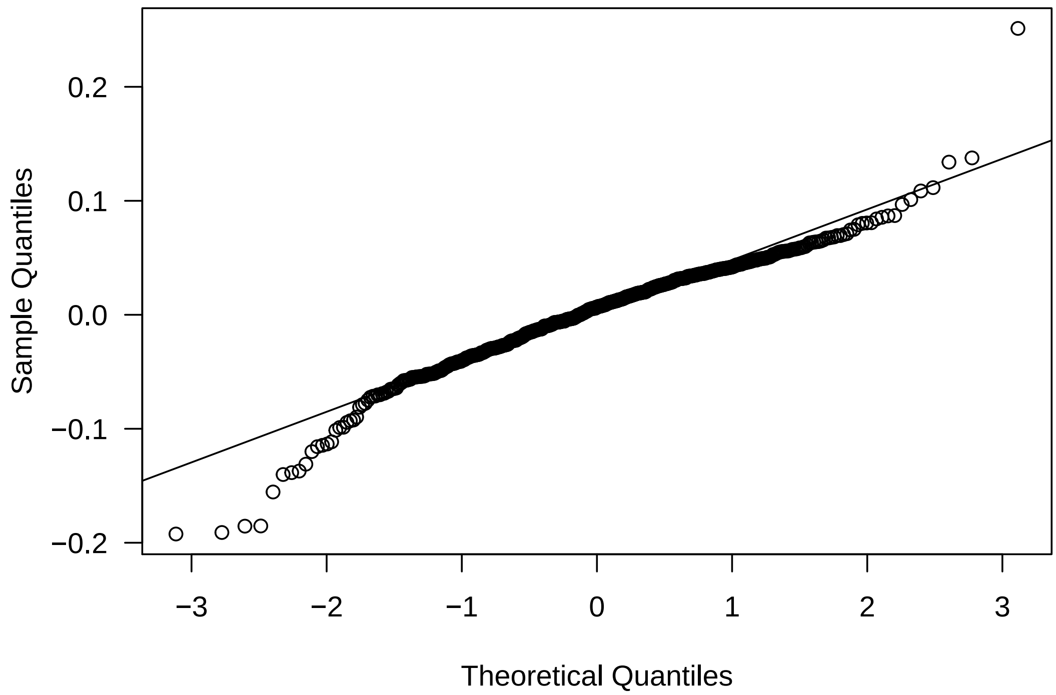
```r
eacf(dloil)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o x o o o  o  o  o
## 1 x o x o o o o x o o o  o  o  o
## 2 x x x o o o o x o o o  o  o  o
## 3 x x x o o o o x o o o  o  o  o
## 4 x o x o o o o x o o o  o  o  o
## 5 x x x o x o o x o o o  o  o  o
## 6 o x x o x x o x o o o  o  o  x
## 7 o x x x x x x x o x o  o  o  o
```

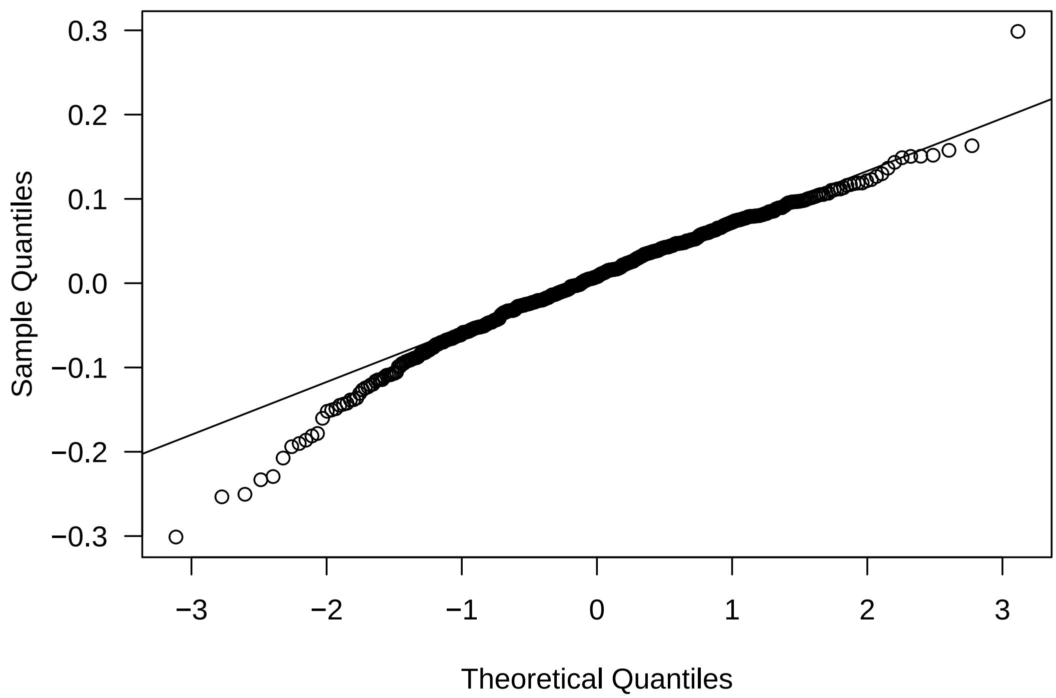
```r
eacf(ddloil)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o x x o o  o  o  o
## 1 x x o o o o o x x o o  o  o  o
## 2 x x x o o o o x x o o  o  o  o
## 3 x x x x o o o x x o o  o  o  o
## 4 x x x x o o o x x o o  o  o  o
## 5 x o x x x o o x x o o  o  o  o
## 6 x o x x x x x x o x o  o  o  o
## 7 x x x x x x x x o x o  o  o  o
```

Difference 1 Log Oil



Difference 2 Log Oil



```r
fit1 <- Arima(loil, order=c(1, 1, 1))
fit1
```

```
## Series: loil
## ARIMA(1,1,1)
##
## Coefficients:
##           ar1     ma1
##       -0.5253  0.7142
## s.e.   0.0872  0.0683
##
## sigma^2 estimated as 0.002112:  log likelihood=904.58
## AIC=-1803.15   AICc=-1803.11   BIC=-1790.25
```

```r
fit2 <- Arima(loil, order=c(0, 1, 3))
fit2
```

```
## Series: loil
## ARIMA(0,1,3)
##
## Coefficients:
##          ma1      ma2     ma3
##       0.1696  -0.0886  0.1458
## s.e.  0.0424   0.0424  0.0429
##
## sigma^2 estimated as 0.002094:  log likelihood=907.41
## AIC=-1806.83   AICc=-1806.75   BIC=-1789.63
```

```r
fit3 <- Arima(loil, order=c(0, 2, 1))
fit3
```

```
## Series: loil
## ARIMA(0,2,1)
##
## Coefficients:
##           ma1
##       -1.0000
## s.e.   0.0061
##
## sigma^2 estimated as 0.002213:  log likelihood=886.63
## AIC=-1769.26   AICc=-1769.24   BIC=-1760.67
```

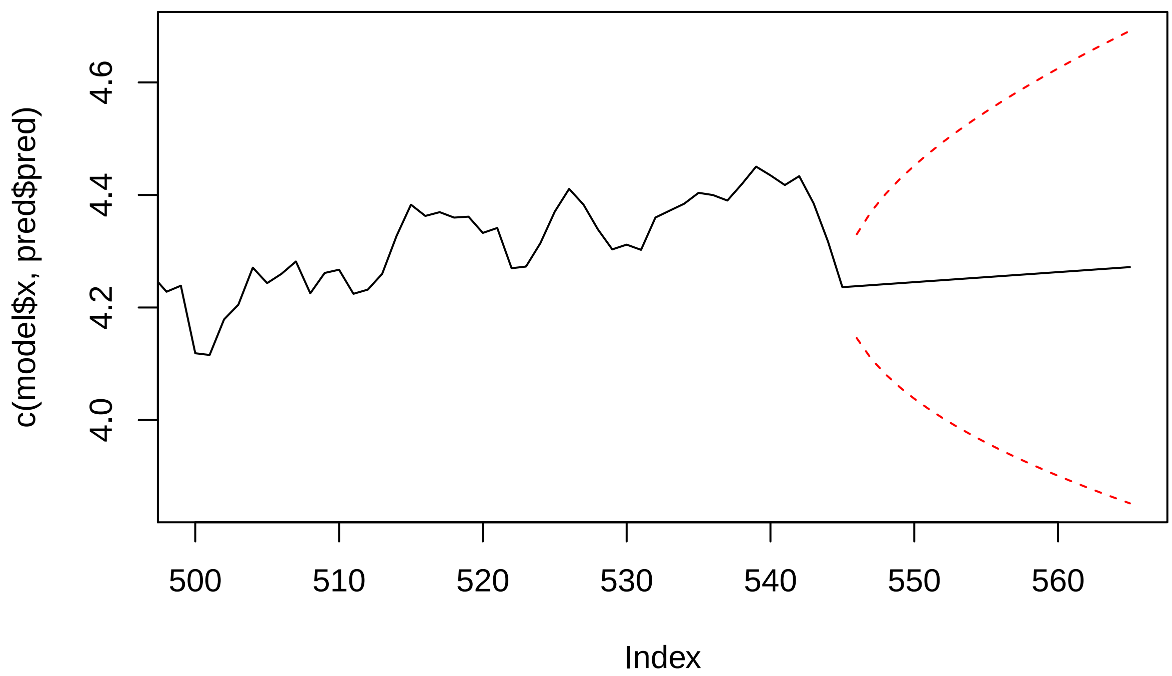
```r
complex_dist <- function(x) {
    sqrt(Re(x)^2 + Im(x)^2)
}

sapply(polyroot(c(1, -2, 1)), complex_dist)
```

```
## [1] 1 1
```

```r
sapply(polyroot(c(1, -1)), complex_dist)
```

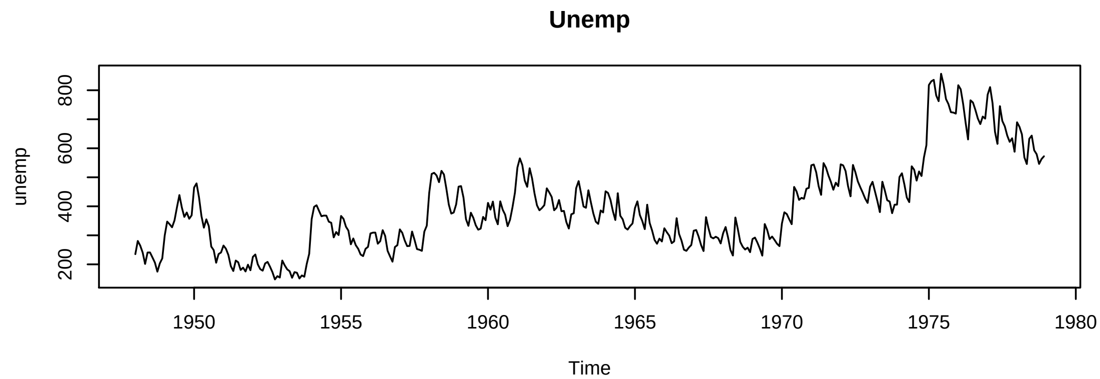
```
## [1] 1
```

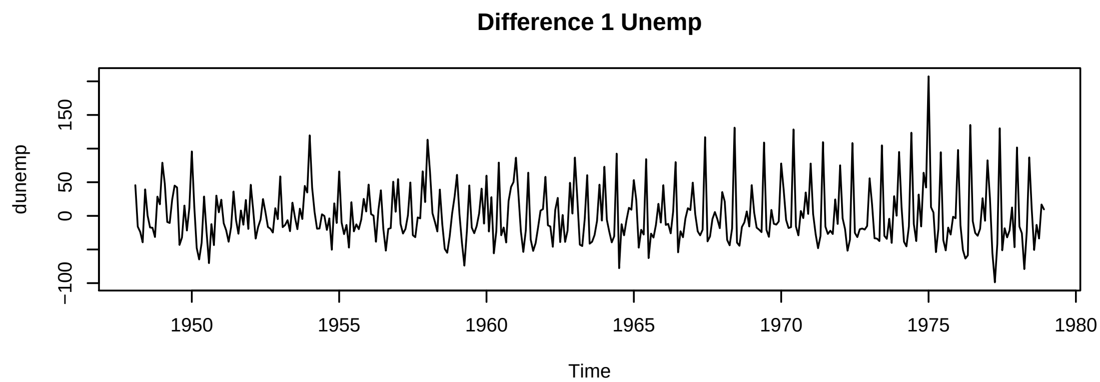
```r
fit_plot(fit3)
```

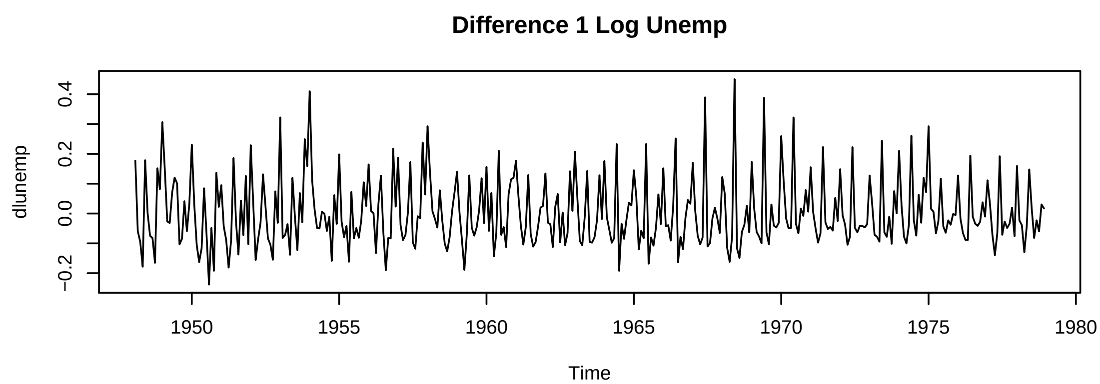


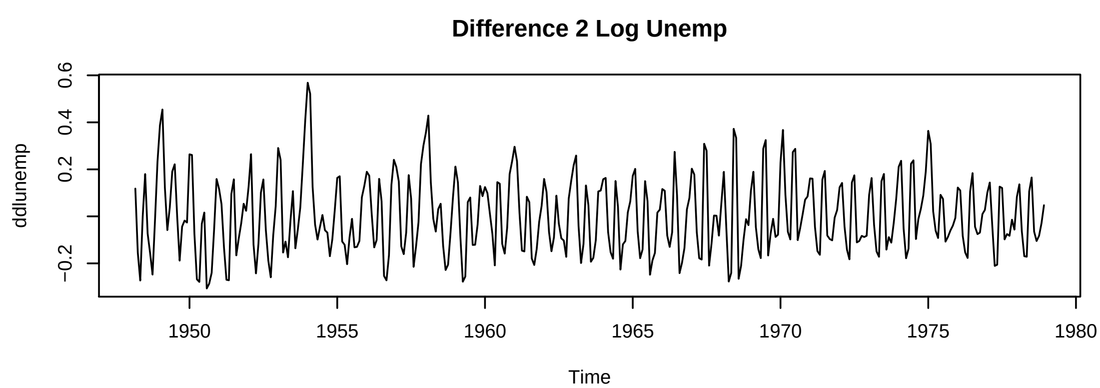
b)

```r
lunemp <- log(unemp)
dunemp <- diff(unemp)
ddunemp <- diff(dunemp, 2)
dlunemp <- diff(lunemp)
ddlunemp <- diff(lunemp, 2)
```


Unemp
unemp
200
400
600
800
1950
1955
1960
1965
1970
1975
1980
Time

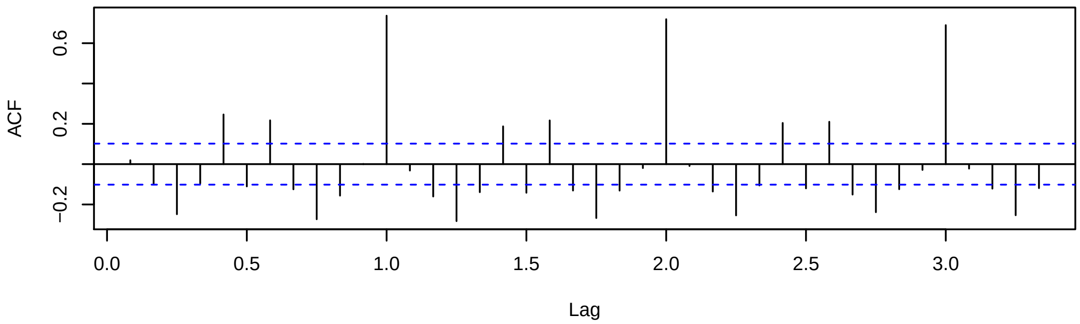

Difference 1 Unemp
dunemp
−100
0
50
150
1950
1955
1960
1965
1970
1975
1980
Time


Difference 1 Log Unemp
dlunemp
−0.2
0.0
0.2
0.4
1950
1955
1960
1965
1970
1975
1980
Time

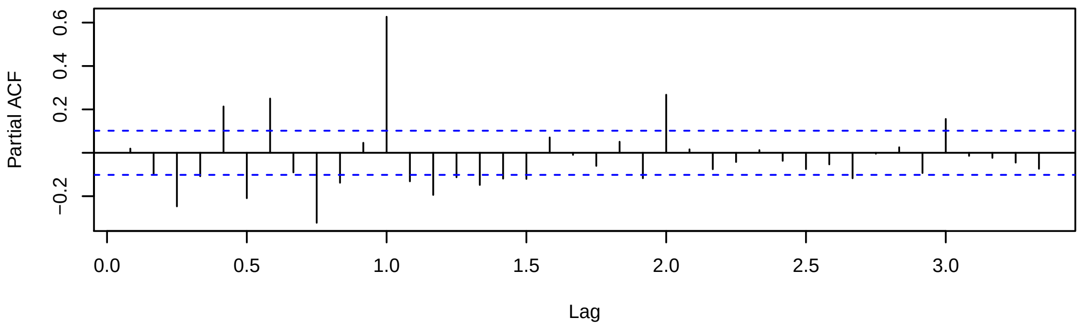

Difference 2 Log Unemp
ddlunemp
−0.2
0.2
0.4
0.6
1950
1955
1960
1965
1970
1975
1980
Time

Clearly difference log is the data we should work with. bla, bla, . . .

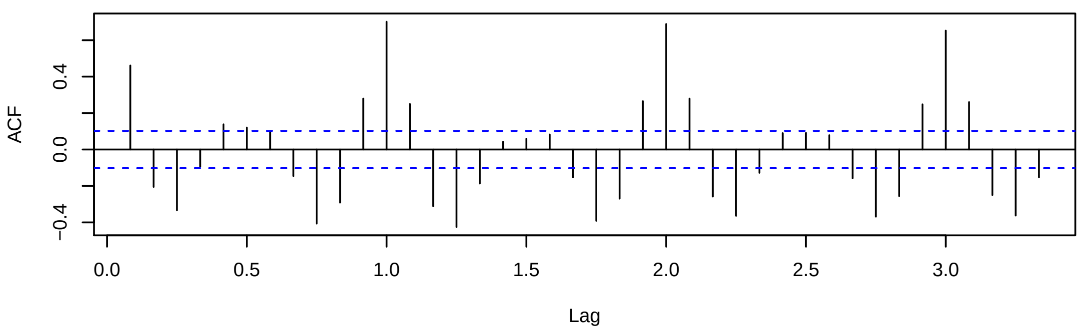
Difference 1 Log Unemp ACF



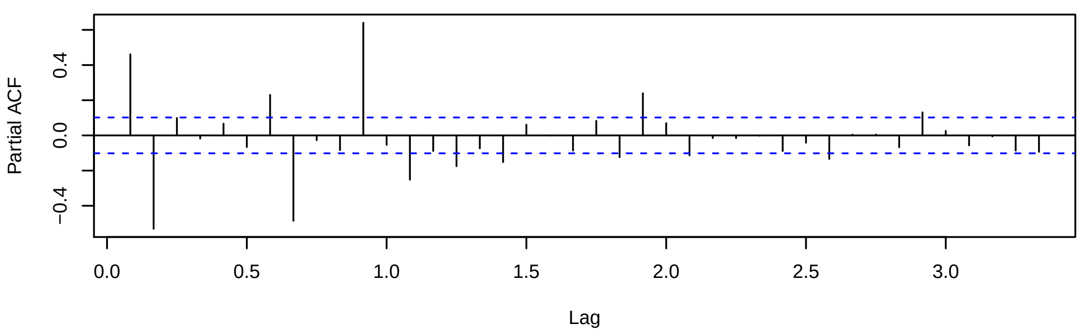
Difference 1 Log Unemp PACF



Difference 2 Log Unemp ACF



Difference 2 Log Unemp PACF

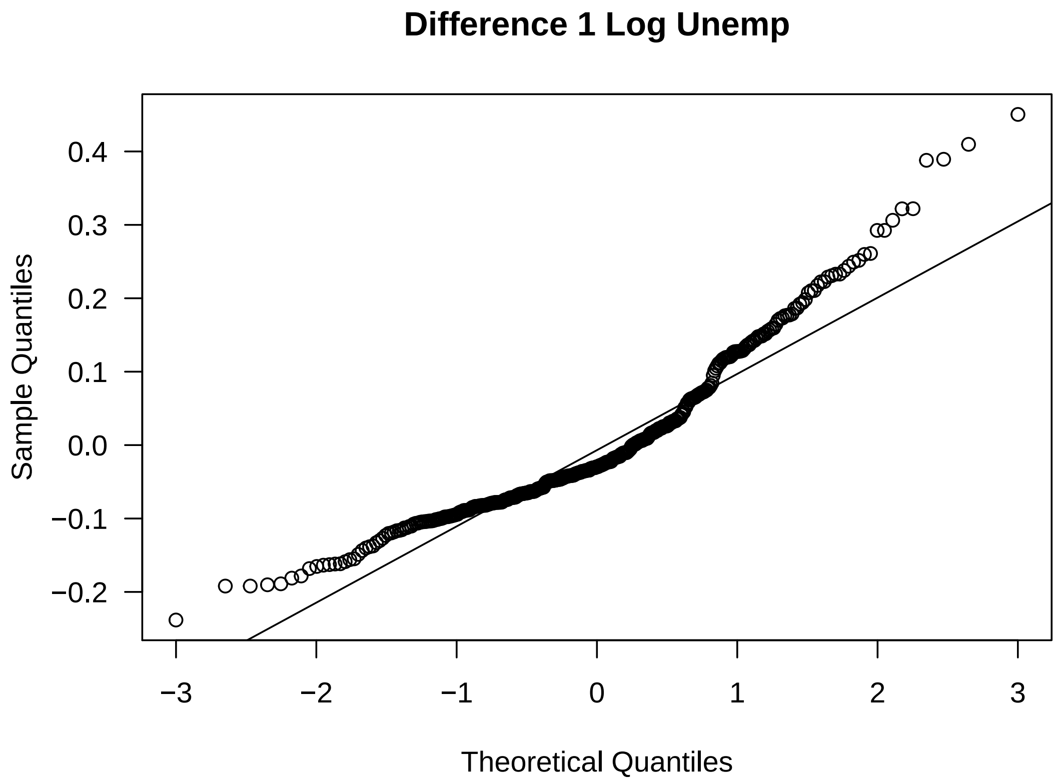


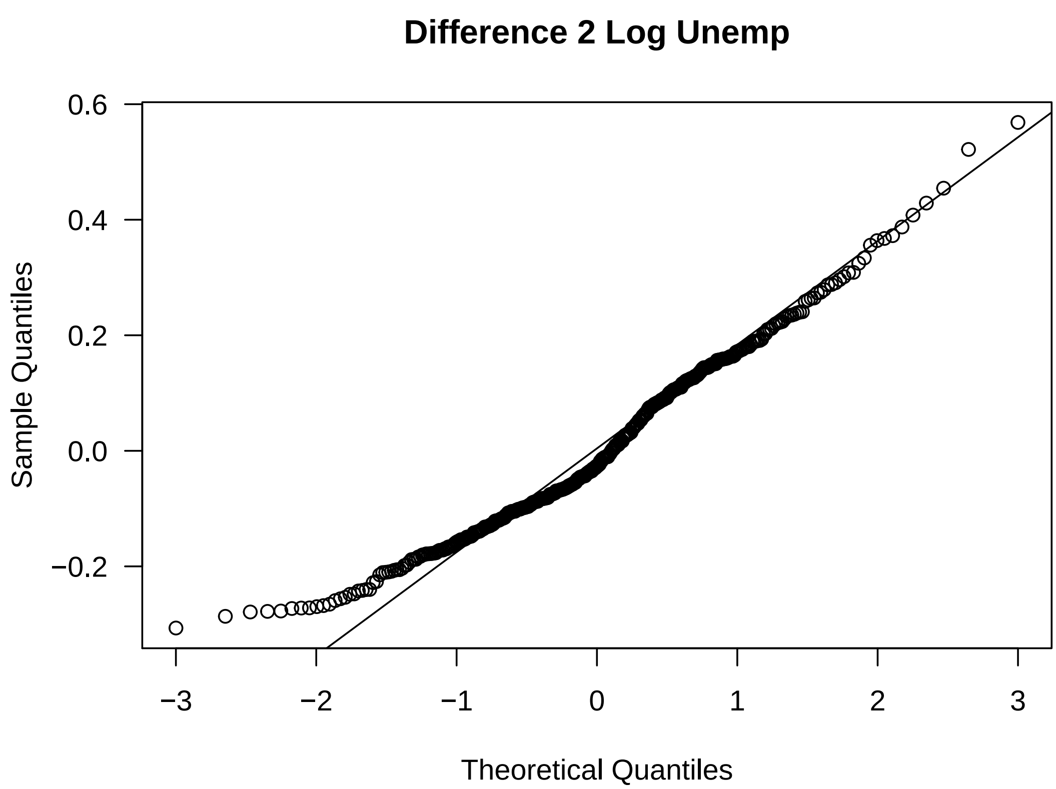
```r
eacf(dlunemp)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 o o x o x x x x x x o  x  o  x
## 1 x o x o x o x x x x o  x  x  o
## 2 x x o x x o o x o o o  x  x  x
## 3 x x x x x o o x o o o  x  x  x
## 4 x x o x o x o o o o o  x  o  x
## 5 x x o x o x o o o o o  x  o  x
## 6 x x x o o x o o o o o  x  o  x
## 7 x x o x x o x o o o o  x  o  o
```

```r
eacf(ddlunemp)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x o x x o x x x x  x  x  x
## 1 x x x x x o o o x x x  x  x  x
## 2 x o x o x o o o o o o  x  x  x
## 3 x o o o o x o o o o o  x  x  o
## 4 x x o o o x o o o o o  x  x  x
## 5 x x o o o x o x o o o  x  x  o
## 6 x x o o o x x o o o o  x  x  x
## 7 x o o o x x o x o o o  x  o  x
```


Difference 1 Log Unemp
Sample Quantiles
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−3
−2
−1
0
1
2
3
Theoretical Quantiles


Difference 2 Log Unemp
Sample Quantiles
0.6
0.4
0.2
0.0
−0.2
−3
−2
−1
0
1
2
3
Theoretical Quantiles

```r
fit1 <- Arima(lunemp, order=c(1, 1, 1))
fit1
```

```
## Series: lunemp
## ARIMA(1,1,1)
##
## Coefficients:
##           ar1     ma1
##       -0.7592  0.8157
## s.e.   0.0952  0.0796
##
## sigma^2 estimated as 0.01289:  log likelihood=281.71
## AIC=-557.42   AICc=-557.35   BIC=-545.67
```

```r
fit2 <- Arima(lunemp, order=c(0, 1, 3))
fit2
```

```
## Series: lunemp
## ARIMA(0,1,3)
##
## Coefficients:
##           ma1     ma2      ma3
##       -0.0079  0.0277  -0.3629
## s.e.   0.0470  0.0506   0.0481
##
## sigma^2 estimated as 0.01177:  log likelihood=298.85
## AIC=-589.7   AICc=-589.59   BIC=-574.03
```

```r
fit3 <- Arima(lunemp, order=c(0, 2, 1))
fit3
```

```
## Series: lunemp
## ARIMA(0,2,1)
##
## Coefficients:
##          ma1
##       -1.000
## s.e.   0.007
##
## sigma^2 estimated as 0.01298:  log likelihood=276.19
## AIC=-548.39   AICc=-548.36   BIC=-540.56
```