1.1 $\vec b = \hat x + \hat y$ $\Rightarrow$ $\vec b + \vec c = 2\hat x + \hat y + \hat z$ ☑

$\vec c = \hat x + \hat z$

$5\vec b + 2\vec c = 5\hat x + 5\hat y + 2\hat x + 2\hat z = 7\hat x + 5\hat y + 2\hat z$ ☑

$\vec b \cdot \vec c = (1)(1) + (1)(0) + (0)(1) = 1 + 0 + 0 = 1$ ☑

$$\vec b \times \vec c = \varepsilon_{ijk} b_j c_k \hat x_i = \varepsilon_{123}(1)(1)\hat x + \varepsilon_{132}(0)(0)\hat x + \varepsilon_{213}(1)(1)\hat y + \varepsilon_{231}(0)(1)\hat y + \varepsilon_{312}(1)(0)\hat z + \varepsilon_{321}(1)(1)\hat z$$

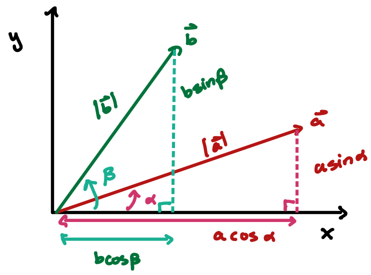
$$= (1)(1)(1)\hat x + 0\hat x + (-1)(1)(1)\hat y + 0\hat y + 0\hat z + (-1)(1)(1)\hat z$$

$$= \hat x - \hat y - \hat z$$ ☑

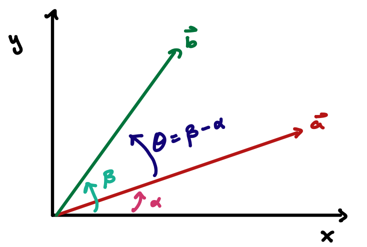
1.22 Vectors $\vec a$ and $\vec b$ lie in x-y plane & make ∡'s α and β with the x-axis.

a) Prove

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



$\vec a \cdot \vec b = a_x b_x + a_y b_y = a\cos\alpha\, b\cos\beta + a\sin\alpha\, b\sin\alpha$



$\vec a \cdot \vec b = ab\cos\theta = ab\cos(\beta - \alpha)$

$\vec a \cdot \vec b = \vec a \cdot \vec b$

$ab\cos(\beta - \alpha) = ab\cos\alpha\cos\beta + ab\sin\alpha\sin\beta$

$\boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$

Note that $\cos(-x) = x$

so $\cos(\beta - \alpha) = \cos(\alpha - \beta)$

b) Prove $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\vec a \times \vec b = \varepsilon_{ijk} a_j b_k \hat x_i = \varepsilon_{123} a_y (\cancel{0}) \hat x + \varepsilon_{132} (\cancel{0}) b_y \hat x + \varepsilon_{213} a_x (\cancel{0}) \hat y + \varepsilon_{231} (\cancel{0}) b_x \hat y + \varepsilon_{312} a_x b_y \hat z + \varepsilon_{321} a_y b_x \hat z$$

$$= (1)\, a\cos\alpha\, b\sin\beta\, \hat z + (-1)\, a\sin\alpha\, b\cos\beta\, \hat z$$

$$= ab(\cos\alpha\sin\beta - \sin\alpha\cos\beta)\hat z$$

$\vec a \times \vec b = ab\sin\theta\, \hat z = ab\sin(\beta - \alpha)\hat z = -ab\sin(\alpha - \beta)\hat z$

use the RH rule to determine direction of $\vec a \times \vec b$

$\vec a \times \vec b = \vec a \times \vec b$

$-ab\sin(\alpha - \beta) = ab(\cos\alpha\sin\beta - \sin\alpha\cos\beta)\hat z$

$\boxed{\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta}$

1.28 Prove $\dot{\vec{P}} = \vec{F}_{ext}^{total}$ for a system of $N=3$ particles.

[net force on particle i)

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_1^{ext}$$
$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_2^{ext}$$
$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_3^{ext}$$

$$\dot{\vec{p}}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_1^{ext}$$
$$\dot{\vec{p}}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_2^{ext}$$
$$\dot{\vec{p}}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_3^{ext}$$

Thank goodness for the copy & paste feature on my tablet !! :)

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$$
$$\dot{\vec{P}} = \dot{\vec{p}}_1 + \dot{\vec{p}}_2 + \dot{\vec{p}}_3$$

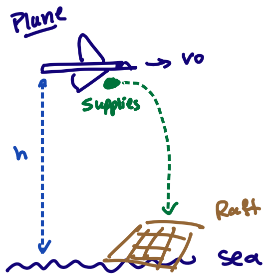
$$\dot{\vec{P}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_1^{ext} + \vec{F}_{21} + \vec{F}_{23} + \vec{F}_2^{ext} + \vec{F}_{31} + \vec{F}_{32} + \vec{F}_3^{ext}$$

$$\dot{\vec{P}} = \vec{F}_{12} + \vec{F}_{21} + \vec{F}_{13} + \vec{F}_{31} + \vec{F}_{23} + \vec{F}_{32} + \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext}$$

$$\dot{\vec{P}} = \underbrace{\vec{F}_{12} - \vec{F}_{12}}_{=0} + \underbrace{\vec{F}_{13} - \vec{F}_{13}}_{=0} + \underbrace{\vec{F}_{23} - \vec{F}_{23}}_{=0} + \underbrace{\vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext}}_{= \vec{F}_{ext}^{total}}$$

$$\boxed{\dot{\vec{P}} = \vec{F}_{ext}^{total}}$$

1.36



a) Find $\vec{r}(t)$ for the bundle of supplies

$$\vec{F} = m\ddot{\vec{r}} \Rightarrow F_x = 0 = m\ddot{x}$$

$$0 = \int_0^t m \frac{d}{dt'}\dot{x}(t')\,dt' = m\dot{x}(t')\Big|_0^t = m\dot{x}(t) - m\dot{x}(0)$$

$\dot{x}(0) = v_0$ since bundle starts off on the plane

$$0 = m\dot{x} - mv_0$$

$$v_0 = \dot{x}$$

$$\int_0^t v_0\,dt' = \int_0^t \frac{d}{dt'}x(t')\,dt'$$

$$v_0 t'\Big|_0^t = x(t')\Big|_0^t$$

$$v_0 t = x(t) - x(0)$$

let $x=0$ at $t=0$

$$\boxed{v_0 t = x(t)}$$

$$F_y = -mg = m\ddot{y}$$

$$-g = \frac{d}{dt}\dot{y}$$

$$\int_0^t -g\,dt' = \int_0^t \frac{d}{dt'}\dot{y}(t')\,dt'$$

$$-gt = \dot{y}(t')\Big|_0^t$$

$$-gt = \dot{y}(t) - \dot{y}(0)$$

Note that $\dot{y}(0) = 0$

$$-gt = \dot{y}(t)$$

$$\int_0^t -gt'\,dt' = \int_0^t \frac{d}{dt'}y(t')\,dt'$$

$$-\tfrac{1}{2}g(t')^2\Big|_0^t = y(t')\Big|_0^t$$

$$-\tfrac{1}{2}gt^2 = y(t) - y(0) \qquad y(0) = h$$

$$\boxed{y(t) = h - \tfrac{1}{2}gt^2}$$

$$\Rightarrow \boxed{\vec{r}(t) = v_0 t\,\hat{x} + \left(h - \tfrac{1}{2}gt^2\right)\hat{y}}$$

(b) How far from the raft should the plane be when it drops supplies?

$v_0 = 50$ m/s, $h = 100$ m, use $g \approx 10$ m/s

First, find time of flight.

$$y(t) = h - \tfrac{1}{2}gt^2$$

$$0 = h - \tfrac{1}{2}gt_f^2$$ (the raft is at $y=0$)

$$\boxed{\sqrt{\frac{2h}{g}} = t_f}$$

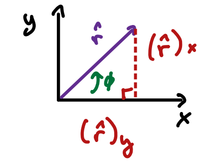
Next, find horizontal distance covered

$$x(t_f) = v_0 t_f = \boxed{v_0\sqrt{\frac{2h}{g}} = x_f}$$

$$x_f = (50\text{ m/s})\sqrt{\frac{2(100\text{ m})}{10\text{ m/s}^2}} = (50\text{ m/s})\sqrt{20\text{ s}^2} \approx \boxed{220\text{ m}}$$

$\Rightarrow$ This is how far away the raft should be

1.43 a) Prove $\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ and find $\hat{\phi}$



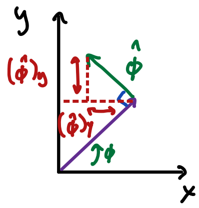
$$\hat{r} = (\hat{r})_x\,\hat{x} + (\hat{r})_y\,\hat{y}$$

$$\hat{r} = |\hat{r}|\cos\phi\,\hat{x} + |\hat{r}|\sin\phi\,\hat{y} \qquad \text{(from diagram)}$$

We know $|\hat{r}| = 1$

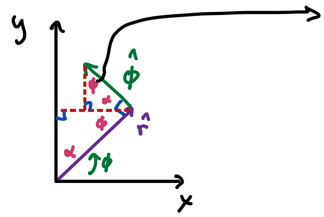
$$\Rightarrow \hat{r} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$

$\hat{\phi}$ is $\perp$ $\hat{r}$ by definition



let $\alpha = \frac{\pi}{2} - \phi$

(or $90° - \phi$)



from this angle, one can see

$$\cos\phi = +\frac{(\hat{\phi})_y}{|\hat{\phi}|} = (\hat{\phi})_y$$

$$\sin\phi = -\frac{(\hat{\phi})_x}{|\hat{\phi}|} = -(\hat{\phi})_x$$

(Because $\hat{\phi}$ is pointing in the $-\hat{x}$ direction)

$$\hat{\phi} = (\hat{\phi})_x\,\hat{x} + (\hat{\phi})_y\,\hat{y} = \boxed{-\sin\phi\,\hat{x} + \cos\phi\,\hat{y} = \hat{\phi}}$$

(b) Prove $\frac{d\hat{r}}{dt} = \dot{\phi}\,\hat{\phi}$ and $\frac{d\hat{\phi}}{dt} = -\dot{\phi}\,\hat{r}$

$$\frac{d\hat{r}}{dt} = \frac{d}{dt}\left(\cos\phi\,\hat{x} + \sin\phi\,\hat{y}\right) = -\sin\phi\,\dot{\phi}\,\hat{x} + \cos\phi\,\cancel{\frac{d\hat{x}}{dt}}^{\,0} + \cos\phi\,\dot{\phi}\,\hat{y} + \sin\phi\,\cancel{\frac{d\hat{y}}{dt}}^{\,0}\Big)$$

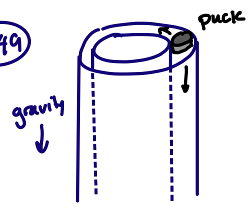
Because $\hat{x}$ and $\hat{y}$ are constant

$$\frac{d\hat{r}}{dt} = \dot{\phi}\underbrace{\left(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}\right)}_{=\hat{\phi}} = \dot{\phi}\,\hat{\phi} \quad ☑$$

$$\frac{d\hat{\phi}}{dt} = \frac{d}{dt}\left(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}\right) = -\cos\phi\,\dot{\phi}\,\hat{x} - \sin\phi\,\dot{\phi}\,\hat{y} = -\dot{\phi}\underbrace{\left(\cos\phi\,\hat{x} + \sin\phi\,\hat{y}\right)}_{\hat{r}}$$

$$\frac{d\hat{\phi}}{dt} = -\dot{\phi}\,\hat{r} \quad ☑$$

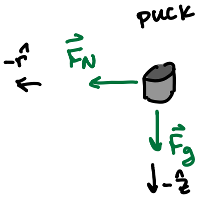
1.49



Write down and solve $\vec{F} = m\vec{a}$ for the puck in cyllindrical coordinates.

$$\vec{F} = m\ddot{\vec{r}} \quad \text{and} \quad \ddot{\vec{r}} = \underbrace{(\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi}}_{\text{This is eqn (1.47) in book (except use } \rho \text{ instead of } r)} + \underbrace{\ddot{z}\hat{z}}_{\text{From Eq. (1.34) in book}}$$

$$\vec{F} = F_\rho \hat{\rho} + F_\phi \hat{\phi} + F_z \hat{z}$$



(no forces in $\hat{\phi}$ - direction)

In the ρ- direction:

$$F_\rho = m(\ddot{\rho} - \rho\dot{\phi}^2)$$

We know $\rho = R$ because the puck is always between the concentric cyllinders $\Rightarrow$ $\dot{\rho} = 0$ and $\ddot{\rho} = 0$

$$F_N = -mR\dot{\phi}^2$$

don't know this, but we don't really need it since we know $\boxed{\rho(t) = R}$

In the ϕ- direction: $F_\phi = m(\rho\ddot{\phi} + 2\underbrace{\dot{\rho}}_{\dot{\rho}=0}\dot{\phi}) = mR\ddot{\phi}$

There are no forces in ϕ- direction, so $F_\phi = 0$ $\Rightarrow$ $0 = mR\ddot{\phi} = \cancel{mR}\dfrac{d\dot{\phi}}{dt}$

$$0 = \int_0^t \frac{d\dot{\phi}}{dt'}dt' = [\dot{\phi}(t) - \dot{\phi}(0)]$$

Let $\dot{\phi}(0) = \omega$ (I could call it ω_0, but since $\ddot{\phi} = 0$, we know $\dot{\phi} = \text{const}$, so let's just say $\omega_0 = \omega$)

$$0 = \dot{\phi}(t) - \omega_0 = \frac{d\phi}{dt} - \omega$$

$$0 = \int_0^t \frac{d\phi}{dt'}dt' - \int_0^t \omega\, dt'$$

$$0 = [\phi(t) - \phi(0)] - \omega t$$

$$0 = \phi(t) - \phi_0 - \omega t$$

$$\boxed{\phi(t) = \omega t + \phi_0}$$

In the $\hat{z}$- direction:

$$F_z = m\ddot{z} \qquad F_z = -mg \quad \Rightarrow \quad -mg = m\ddot{z}$$

$$-g = \frac{d\dot{z}}{dt}$$

$$\int_0^t -g\, dt' = \int_{\dot{z}_0}^{\dot{z}} d\dot{z}' \qquad \text{let } v_{z0} \equiv \dot{z}(t=0)$$

$$-gt = \dot{z} - v_{z0}$$

$$\int_0^t \left(-gt' + v_{z0}\right) dt' = \int_{z_0}^{z} dz'$$

$$-\tfrac{1}{2}gt^2 + v_{z0}t = z - z_0$$

$$\Rightarrow \boxed{z(t) = z_0 + v_{z0}t - \tfrac{1}{2}gt^2}$$

To summarize:

$$\boxed{\begin{array}{lll} F_N = -mR\dot{\phi}^2 = -mR\omega^2 & & \rho(t) = R \\ 0 = mR\ddot{\phi} & \Rightarrow & \phi(t) = \omega t + \phi_0 \\ -mg = m\ddot{z} & \Rightarrow & z(t) = z_0 + v_{z0}t - \tfrac{1}{2}gt^2 \end{array}}$$

(2.5) Assume a projectile subject to linear $\vec{F}_d$ is thrown down with v_{y0} s.t. $|v_{y0}| > v_{ter}$.

Find $v_y(t)$ and plot v_y vs. t for $|v_{y0}| = 2v_{ter}$.

$$y(t) = (v_{y0} + v_{ter})\,\tau\,(1 - e^{-t/\tau}) - v_{ter}t \qquad \text{(Eq. 2.36)}$$

where $\tau = m/b$

$$v_y(t) = +(v_{y0} + v_{ter})\,e^{-t/\tau} - v_{ter}$$

$$v_y(t) = v_{y0}e^{-t/\tau} - v_{ter}(1 - e^{-t/\tau})$$

(like 2.31, except some signs are different because for Eq. 2.31, Taylor was using $\rightarrow +x$, $\downarrow +y$ as his coordinate system)

We know $v_{y0} < 0$ and $\underbrace{v_{y0} < -v_{ter}}$

v_{y0} is more negative than $-v_{ter}$ (and remember $v_{ter} \equiv \frac{mg}{b} > 0$)