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# Homework 3 solutions

(2.14) An object w/ mass $m$ moves along x-axis and feels a force $F(v) = -F_0 e^{v/V}$

a) Find $v(t)$ using $v(t=0) = v_0 > 0$

$$\vec{F} = m\vec{a} \Rightarrow F(v) = m\ddot{x}$$

$$-F_0 e^{v/V} = m\dot{v} = m\frac{dv}{dt}$$

$$-F_0\,dt = m e^{-v/V}\,dv$$

$$-F_0 \int_0^t dt' = m \int_{v_0}^{v} e^{-v'/V}\,dv'$$

$$-F_0 t = m\left(\frac{1}{-1/V} e^{-v'/V}\right)\Big|_{v_0}^{v}$$

$$-\frac{F_0}{m} t = -V\left(e^{-v/V} - e^{-v_0/V}\right)$$

$$\frac{F_0}{mV} t = e^{-v/V} - e^{-v_0/V}$$

$$e^{-v_0/V} + \frac{F_0}{mV} t = e^{-v/V}$$

$$\ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right) = -v/V$$

$$\boxed{v(t) = -V \ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right)}$$

b) At what time does object come to rest?

At what time is $v(t) = 0$?

$$0 = \cancel{-V} \ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right) \qquad \ln(1) = 0$$

$$1 = e^{-v_0/V} + \frac{F_0 t}{mV}$$

$$\boxed{\frac{mV}{F_0}\left(1 - e^{-v_0/V}\right) = t_{rest}}$$

c) Find $x(t)$ and find $x$ when $v(t)=0$

$$v(t) = -V \ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right)$$

$$\frac{dx}{dt} = -V \ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right)$$

$$\int_{x_0}^{x} dx' = \int_0^t -V \ln\left(e^{-v_0/V} + \frac{F_0 t'}{mV}\right) dt'$$

$$x'\Big|_{x_0}^{x} = -V\left[\frac{\left(e^{-v_0/V} + \frac{F_0 t'}{mV}\right)\ln\left(e^{-v_0/V} + \frac{F_0 t'}{mV}\right)}{\frac{F_0}{mV}} - t'\right]_0^t$$

$$x - x_0 = -V\left[\frac{mV}{F_0}\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right)\ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right) - t - \frac{mV}{F_0} e^{-v_0/V} \ln\left(e^{-v_0/V}\right) + 0\right]$$

$$x - x_0 = -\frac{mV^2}{F_0}\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right)\ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right) + Vt + \frac{mV^2}{F_0} e^{-v_0/V}\left(\frac{-v_0}{V}\right)$$

$$x = -\frac{mV^2}{F_0}\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right)\ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right) + Vt - \frac{mVv_0}{F_0} e^{-v_0/V} + x_0$$

Choose $x_0 = 0$ and simplify

$$x = Vt - \frac{mV^2}{F_0}\left[\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right)\ln\left(e^{-v_0/V} + \frac{F_0 t}{mV}\right) + \frac{v_0}{V} e^{-v_0/V}\right]$$

When $t = t_{rest}$ :

$$x(t_{rest}) = \frac{mV^2}{F_0}\left(1 - e^{-v_0/V}\right) - \frac{mV^2}{F_0}\left[\left(e^{-v_0/V} + 1 - e^{-v_0/V}\right)\ln(1) + \frac{v_0}{V} e^{-v_0/V}\right]$$

$$x(t_{rest}) = \frac{mV^2}{F_0}\left(1 - e^{-v_0/V}\right) - \frac{mVv_0}{F_0} e^{-v_0/V}$$

$$\boxed{x(t_{rest}) = \frac{mV^2}{F_0}\left(1 - \left(1 + \frac{v_0}{V}\right)e^{-v_0/V}\right)}$$

2.19 Consider the projectile in a linear medium

$$x(t) = v_{xo}\,\tau(1-e^{-t/\tau})$$
$$y(t) = (v_{yo}-v_{ter})\,\tau(1-e^{-t/\tau}) - v_{ter}\,\tau$$

Eqn. 2.36 & 2.37 in the book

a) Assuming no air resistance, write down $x(t)$, $y(t)$ and eliminate $t$ to find $y(x)$

$$\vec{F} = m\vec{a}$$
$$-mg\hat{y} = m\ddot{x}\,\hat{x} + m\ddot{y}\,\hat{y}$$

$$\Rightarrow 0 = m\ddot{x} = m\frac{d\dot{x}}{dt}$$ (divide both sides by m)

$$0 = \frac{d\dot{x}}{dt}$$

$$0 = \int_0^t \frac{d\dot{x}(t')}{dt} = \dot{x}(t')\Big|_0^t = \dot{x}(t) - v_{xo}$$

$$v_{xo} = \frac{dx}{dt}$$

$$\int_0^t v_{xo}\,dt' = \int_0^t \frac{dx(t')}{dt'}$$

$$v_{xo}\,t'\Big|_0^t = x(t')\Big|_0^t$$

$$v_{xo}t = x(t) - x_o$$

$$x(t) = v_{xo}t + x_o$$

(you could just write this down directly & still get full credit)

$$-mg = m\ddot{y} = m\frac{d\dot{y}}{dt}$$

$$-g = \frac{d\dot{y}}{dt}$$

$$\int_0^t -g\,dt' = \int_0^t \frac{d\dot{y}(t')}{dt'}$$

$$-gt'\Big|_0^t = \dot{y}(t')\Big|_0^t = \dot{y}(t) - v_{yo}$$

$$-gt = \dot{y}(t) - v_{yo}$$

$$\int_0^t (v_{yo} - gt')\,dt' = \int_0^t \frac{dy(t')}{dt'}$$

$$\left[v_{yo}t' - \tfrac{1}{2}g(t')^2\right]_0^t = y(t')\Big|_0^t$$

$$v_{yo}t - \tfrac{1}{2}gt^2 = y(t) - y_o$$

$$y(t) = y_o + v_{yo}t - \tfrac{1}{2}gt^2$$

(You can also set $x_o = y_o = 0$ and get full credit)

Eliminate $t$: $x = v_{xo}t + x_o$

$$\frac{x-x_o}{v_{xo}} = t \quad \Rightarrow \text{plug into } y(t) \Rightarrow \quad y = y_o + v_{yo}\left(\frac{x-x_o}{v_{xo}}\right) - \tfrac{1}{2}g\left(\frac{x-x_o}{v_{xo}}\right)^2$$

$$y(x) = y_o + \frac{v_{yo}}{v_{xo}}x - \frac{v_{yo}x_o}{v_{xo}} - \frac{g}{2v_{xo}^2}(x^2 - 2x_ox + x_o^2)$$

$$= y_o + \frac{v_{yo}}{v_{xo}}x - \frac{v_{yo}x_o}{v_{xo}} - \frac{gx^2}{2v_{xo}^2} + \frac{gx_o}{v_{xo}^2}x - \frac{gx_o^2}{2v_{xo}^2}$$

$$y(x) = \frac{-g}{2v_{xo}^2}x^2 + \frac{v_{yo}v_{xo} + gx_o}{v_{xo}^2}x - \frac{v_{yo}v_{xo} + gx_o}{v_{xo}^2}x_o + y_o$$

$$\boxed{y(x) = \frac{-g}{2v_{xo}^2}x^2 + \frac{v_{yo}v_{xo} + gx_o}{v_{xo}^2}(x - x_o) + y_o}$$

If $x_o = 0$, $y_o = 0$:

$$\boxed{y(x) = \frac{-g}{2v_{xo}^2}x^2 + \frac{v_{yo}}{v_{xo}}x}$$

b) The trajectory in the presence of a drag force is:

$$y = \frac{v_{y0} + v_{ter}}{v_{x0}} x + v_{ter} \tau \ln\left(1 - \frac{x}{v_{x0}\tau}\right)$$

Show this reduces to part (a) when $\vec{F}_d$ is turned off $(\tau \text{ \& } v_{term} = g\tau \to \infty)$

$$\ln(1-\varepsilon) = -\varepsilon - \frac{1}{2}\varepsilon^2 + \mathcal{O}(\varepsilon^3) \qquad \text{When } \tau \to \infty, \ \frac{x}{v_{x0}\tau} \to 0$$

$$\ln\left(1 - \frac{x}{v_{x0}\tau}\right) = \frac{-x}{v_{x0}\tau} - \frac{1}{2}\frac{x^2}{v_{x0}^2\tau^2} + \mathcal{O}\left(\frac{1}{\tau^3}\right)$$

$$y = \frac{v_{y0}}{v_{x0}}x + \frac{v_{ter}}{v_{x0}}x - \frac{v_{ter}\tau x}{v_{x0}\tau} - \frac{1}{2}\frac{v_{ter}\tau x^2}{v_{x0}^2\tau^2} + \mathcal{O}\left(\frac{v_{ter}\tau}{\tau^3}\right)$$

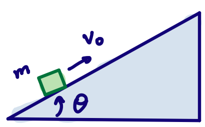
$$y = \frac{v_{y0}}{v_{x0}}x + \cancel{\frac{v_{ter}}{v_{x0}}x} - \cancel{\frac{v_{ter}\, x}{v_{x0}}} - \frac{1}{2}\frac{v_{ter}\, x^2}{v_{x0}^2\tau} + \mathcal{O}\left(\frac{v_{ter}}{\tau^2}\right)$$

$$y = \frac{v_{y0}}{v_{x0}}x - \frac{1}{2}\frac{(g\tau)x^2}{v_{x0}^2\tau} + \mathcal{O}\left(\frac{g\tau}{\tau^2}\right)$$

$$y = \frac{v_{y0}}{v_{x0}}x - \frac{1}{2}\frac{g\, x^2}{v_{x0}^2} + \mathcal{O}\left(\frac{g}{\tau}\right)$$

$$\boxed{\lim_{\tau\to\infty} y = \frac{v_{y0}}{v_{x0}} - \frac{1}{2}\frac{g x^2}{v_{x0}^2} = \text{answer to part (a)}}$$

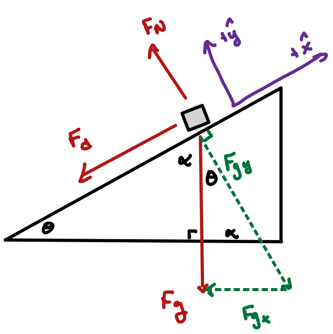
2.27




air resistance $f(v) = cv^2$

Find $\vec{v}(t)$ and how long the puck's journey to its maximum height takes

Choose $+\hat{x}$ to lie $\parallel$ to incline, $+\hat{y}$ to lie $\perp$ to incline




let $\alpha + \theta = \pi/2 \Rightarrow \vec{F}_g = -mg\sin\theta\,\hat{x} - mg\cos\theta\,\hat{y}$

$$\vec{F}_d = -cv^2\hat{v} = -c|v|\vec{v} = -c|v_x|v_x\hat{x}$$

$$\vec{F}_{total} = -c|v_x|v_x\hat{x} - mg\sin\theta\,\hat{x} - mg\cos\theta\,\hat{y} + F_N\hat{y}$$

There is no motion in $\hat{y}$-direction ($\vec{F}_N$ & $F_{gy}$ cancel)

In the $\hat{x}$-direction:

$$m\ddot{x} = m\dot{v}_x = -c\,|v_x|\,v_x \;-mg\sin\theta$$

$$\frac{dv_x}{dt} = -c/m\,|v_x|\,v_x - g\sin\theta = -g\sin\theta\left(\frac{c}{mg\sin\theta}\,|v_x|\,v_x + 1\right) \qquad \text{let} \quad k^2 = \frac{c}{mg\sin\theta}$$

$$\underbrace{\int_{v_0}^{v_x} \frac{dv_x'}{k^2|v_x'|\,v_x' + 1}} = \int_0^t -g\sin\theta\, dt'$$

This is tricky to integrate.
$|v_x| = \sqrt{v_x^2}$ = a mess.

To simplify, split this integral up into two pieces

We know the mass starts with $v_x = v_0 > 0$, meaning that at first, $v_x > 0$ and $|v_x| = v_x$

At some time (which we will call $t_1$), the mass reaches its highest point on the incline. Then it turns around and heads back. Its speed is now in the $-\hat{x}$ direction

So we can say: when $t < t_1$, $v_x > 0 \Rightarrow |v_x| = v_x$

when $t > t_1$, $v_x < 0 \Rightarrow |v_x| = -v_x$

So we can split up the integral:

$$\int_{v_0}^{v_x} \frac{dv_x'}{k^2|v_x'|\,v_x' + 1} = \int_{v_0}^{v_x(t_1)} \frac{dv_x'}{k^2(v_x')^2 + 1} + \int_{v_x(t_1)}^{v_x(t)} \frac{dv_x'}{-k^2(v_x')^2 + 1}$$

(Assuming $t > t_1$. If $t < t_1$, then we only keep the first term)

To be very careful, we can say:

$$\int_{v_0}^{v_x} \frac{dv_x'}{k^2|v_x'|\,v_x' + 1} = \begin{cases} \displaystyle\int_{v_0}^{v_x(t_1)} \frac{dv_x'}{k^2(v_x')^2 + 1} + \int_{v_x(t_1)}^{v_x(t)} \frac{dv_x'}{-k^2(v_x')^2 + 1}, & t > t_1 \quad \text{case I} \\[2ex] \displaystyle\int_{v_0}^{v_x(t)} \frac{dv_x'}{k^2(v_x')^2 + 1}, & t \le t_1 \quad \text{case II} \end{cases}$$

We can simplify this further. We know that at $t = t_1$, the block is at is maximum, and its velocity $= 0$

$\Rightarrow v_x(t_1) = 0$ (by our definition of $t_1$)

Let's find $v(t)$ for each case. Start with case I, where we assume the mass reaches its highest point and turns around:

$$\int_{v_0}^{0} \frac{dv_x'}{k^2(v_x')^2+1} + \int_{0}^{v_x} \frac{dv_x'}{-k^2(v_x')^2+1} = -g\sin\theta \int_0^t dt' \qquad (t > t_1)$$

$$\frac{1}{k}\underbrace{\tan^{-1}(kv_x')}_{\text{Normal tangent}}\Big|_{v_0}^{0} + \frac{1}{k}\underbrace{\tanh^{-1}(kv_x')}_{\text{Hyperbolic tangent}}\Big|_{0}^{v_x} = -gt\sin\theta$$

$$0 - \frac{1}{k}\tan^{-1}(kv_0) + \frac{1}{k}\tanh^{-1}(kv_x) - 0 = -gt\sin\theta$$

$$\tanh^{-1}(kv_x) = \tan^{-1}(kv_0) - gk\sin\theta\, t$$

$$v_x = \frac{1}{k}\tanh\left(\tan^{-1}(kv_0) - gk\sin\theta\, t\right) \qquad \text{for case I, } t > t_1$$

Now consider case II, where the block hasn't yet reached its maximum trajectory

$$\int_{v_0}^{v_x} \frac{dv_x'}{k^2(v_x')^2+1} = -g\sin\theta \int_0^t dt'$$

$$\frac{1}{k}\tan^{-1}(kv_x')\Big|_{v_0}^{v_x} = -g\sin\theta\, t'\Big|_0^t$$

$$\frac{1}{k}\tan^{-1}(kv_x) - \frac{1}{k}\tan^{-1}(kv_0) = -g\sin\theta\, t$$

$$v_x = \frac{1}{k}\tan\left(\tan^{-1}(kv_0) - g\sin\theta\, t\right) \qquad \text{for case II, when } t < t_1$$

We can find $t_1$ as well. $v_x(t_1) = 0$ by definition.

<u>case II</u>

$$0 = \frac{1}{k}\tan\left(\tan^{-1}(kv_0) - gk\sin\theta\, t_1\right)$$

$$0 = \tan^{-1}(kv_0) - gk\sin\theta\, t_1$$

$$\frac{\tan^{-1}(kv_0)}{gk\sin\theta} = t_1$$

<u>case I</u>

$$0 = \frac{1}{k}\tanh\left(\tan^{-1}(kv_0) - g\sin\theta\, t_1\right)$$

$$0 = \tan^{-1}(kv_0) - g\sin\theta\, t_1$$

$$\frac{\tan^{-1}(kv_0)}{g\sin\theta} = t_1 \qquad \Rightarrow \text{ same answer (which it should be)}$$

In summary:

$$v_x(t) = \begin{cases} \frac{1}{k}\tanh\left(\tan^{-1}(kv_0) - gk\sin\theta\, t\right) & t > \frac{\tan^{-1}(kv_0)}{g\sin\theta} \\ \underbrace{\frac{1}{k}\tan\left(\tan^{-1}(kv_0) - gk\sin\theta\, t\right)}_{\text{all you need for full credit}} & t \le \frac{\tan^{-1}(kv_0)}{g\sin\theta} \end{cases}$$

& $k^2 = \frac{c}{mg\sin\theta}$

where $t = \frac{\tan^{-1}(kv_0)}{kg\sin\theta} = \frac{\tan^{-1}\left(\sqrt{\frac{c}{mg\sin\theta}}\, v_0\right)}{\sqrt{cg\sin\theta/m}}$ is the time it takes for puck to reach its maximum

2.45 (a) Using $e^{i\theta} = \cos\theta + i\sin\theta$, prove any $z = x+iy$ can be written as $z = re^{i\theta}$

$$z = x+iy = r\cos\theta + ir\sin\theta = re^{i\theta}$$

polar coordinates

(b) Rewrite $z = 3+4i$

$$r = \sqrt{x^2+y^2} = \sqrt{9+16} = 5$$

$$3 = r\cos\theta = 5\cos\theta$$

$$3/5 = \cos\theta$$

$$\Rightarrow \boxed{z = 5e^{i\cos^{-1}(3/5)}}$$

(c) Rewrite $z = 2e^{-i\pi/3}$

$$x = r\cos\theta = 2\cos(-\pi/3) = 1$$

$$y = r\sin\theta = 2\sin(-\pi/3) = -\sqrt{3}$$

$$\Rightarrow \boxed{z = 1 - i\sqrt{3}}$$

2.47 Compute $z+w$, $z-w$, $zw$, $z/w$

a) $z = 6+8i$, $w = 3-4i$

$z = 10e^{i\tan^{-1}(4/3)}$ $\quad w = 5e^{-i\tan^{-1}(4/3)}$

$z+w = 9+4i$

$z-w = 3+12i$

$zw = 50$

$$z/w = \frac{6+8i}{3-4i}\cdot\frac{3+4i}{3+4i} = \frac{18+24i+24i-32}{9+16} = \frac{-14+48i}{25}$$

b) $z = 8e^{i\pi/3}$ $\quad w = 4e^{i\pi/6}$

$z = 4+4\sqrt{3}i$ $\quad w = 2\sqrt{3}+2i$

$z+w = 4+2\sqrt{3} + (4\sqrt{3}+2)i$

$z-w = 4-2\sqrt{3} + (4\sqrt{3}-2)i$

$zw = 4(8)e^{i\pi/3+i\pi/6} = 32e^{i\pi/2}$

$z/w = 4/8\, e^{i\pi/3 - i\pi/6} = 2e^{i\pi/6}$

(2.49) Consider $z = e^{i\theta} = \cos\theta + i\sin\theta$

(a) Prove $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$

$$z^2 = e^{2i\theta} = \cos 2\theta + i\sin 2\theta \qquad z^2 = (\cos\theta + i\sin\theta)^2 = \cos^2\theta - \sin^2\theta + 2i\cos\theta\sin\theta$$

$$\mathrm{Re}(z^2) = \mathrm{Re}(z^2) \qquad \mathrm{Im}(z^2) = \mathrm{Im}(z^2)$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \qquad \sin 2\theta = 2\cos\theta\sin\theta \quad \checkmark$$

(b) Find $\cos 3\theta$ and $\sin 3\theta$

$$z^3 = e^{3i\theta} = \cos 3\theta + i\sin 3\theta$$

using $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$z^3 = (\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$$
$$= \cos^3\theta - 3\cos\theta\sin^2\theta + i\left(3\cos^2\theta\sin\theta - \sin^3\theta\right)$$

$$\mathrm{Re}(z^3) = \mathrm{Re}(z^3) \qquad \mathrm{Im}(z^3) = \mathrm{Im}(z^3)$$

$$\boxed{\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta} \qquad \boxed{\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta}$$

(2.55) Particle of charge $+q$ feels $\vec{E} = E\hat{y}$ and $\vec{B} = B\hat{z}$ with initial $\vec{v} = v_{x0}\hat{x}$

(a) write down EOM's

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = q(E\hat{y} + \varepsilon_{ijk} v_j B_k \hat{x}_i) = q(E\hat{y} + v_y B\hat{x} - v_x B\hat{y}) = qBv_y\hat{x} + (qE - qBv_x)\hat{y} = m\vec{a}$$

$$\boxed{\begin{aligned} m\ddot{x} &= qBv_y \\ m\ddot{y} &= qE - qBv_x \\ m\ddot{z} &= 0 \end{aligned}}$$

Since $\ddot{z} = 0 \Rightarrow \dot{z} = v_{z0}$
$\Rightarrow z = v_{z0}t + z_0$
$\Rightarrow z(t) = 0 \quad \checkmark$

But $\vec{v}_0$ is in the x-direction $\Rightarrow v_{z0} = 0$
The initial position is @ origin $\Rightarrow z_0 = 0$

(b) Show $\exists\ v_{x0} = v_{dr}$ s.t. the particle is undeflected

No deflection $\Rightarrow \dot{v}_y = \dot{v}_x = 0 \Rightarrow m\ddot{x} = m\ddot{y} = 0$

$\Rightarrow 0 = qE - qBv_x$ and $\dot{v}_x = 0 \Rightarrow 0 = qBv_y \Rightarrow v_y = 0$
$\Rightarrow E = Bv_x$

$$\boxed{\vec{v}_{dr} = E/B\,\hat{x}}$$

c) Solve EOM's

$$\dot{v}_x = qB/m\, v_y \qquad \text{let } \eta = v_x + iv_y$$
$$\dot{v}_y = qE/m - qB/m\, v_x \qquad \text{let } \frac{qB}{m} = \omega_B,\ \frac{qE}{m} = \omega_E$$

$$\Rightarrow \quad \dot\eta = \dot v_x + i\dot v_y = \omega_B v_y + i\omega_E - i\omega_B v_x$$

$$\frac{1}{-i} = +i$$

$$\dot\eta = i\omega_E - i\omega_B(v_x + iv_y) = i\omega_E - i\omega_B\eta$$

$$\frac{d\eta}{dt} = -i\omega_B\left(-\omega_E/\omega_B + \eta\right)$$

$$\int_{\eta_0}^{\eta} \frac{d\eta'}{\eta' - \omega_E/\omega_B} = -i\omega_B \int_0^t dt'$$

$$\ln\left(\eta' - \omega_E/\omega_B\right)\Big|_{\eta_0}^{\eta} = -i\omega_B t'\Big|_0^t \qquad \omega_E/\omega_B = E/B = v_{dr}$$

$$\ln\left(\frac{\eta - v_{dr}}{\eta_0 - v_{dr}}\right) = -i\omega_B t$$

$$\frac{\eta - v_{dr}}{\eta_0 - v_{dr}} = e^{-i\omega_B t}$$

$$\eta = (\eta_0 - v_{dr})e^{-i\omega_B t} + v_{dr} \qquad \eta_0 = v_{0x} + i\cancel{v_y}^{\,0} = v_{0x} \equiv v_0$$

$$\eta = (v_0 - v_{dr})\cos\omega_B t - i(v_0 - v_{dr})\sin\omega_B t + v_{dr} = v_x + iv_y$$

$$v_x = (v_0 - v_{dr})\cos\omega_B t + v_{dr} \quad \Rightarrow \quad \boxed{v_x = (v_{0x} - v_{dr})\cos\frac{qB}{m}t + v_{dr}}$$

$$v_y = -(v_0 - v_{dr})\sin\omega_B t \quad \Rightarrow \quad \boxed{v_y = -(v_{0x} - v_{dr})\sin\frac{qB}{m}t}$$

d)

$$\int_0^t v_x(t')\,dt' = \int_0^t \left[(v_0 - v_{dr})\cos\omega_B t' + v_{dr}\right]dt'$$

$$x(t) - x_0 = \left[\frac{v_0 - v_{dr}}{\omega_B}\sin\omega_B t' + v_{dr}t'\right]_0^t$$

$$x(t) = \frac{v_0 - v_{dr}}{\omega_B}\sin\omega_B t + v_{dr}t + x_0$$

$$\boxed{x(t) = \frac{v_0 - v_{dr}}{qB/m}\sin\left(\frac{qB}{m}t\right) + v_{dr}t + x_0}$$

$$\int_0^t v_y(t')\,dt' = -\int_0^t (v_0 - v_{dr})\sin\omega_B t'\,dt'$$

$$y(t) - y_0 = \left[\frac{v_0 - v_{dr}}{\omega_B}\cos\omega_B t'\right]_0^t$$

$$y(t) = \frac{v_0 - v_{dr}}{\omega_B}\left(\cos\omega_B t - 1\right) + y_0$$

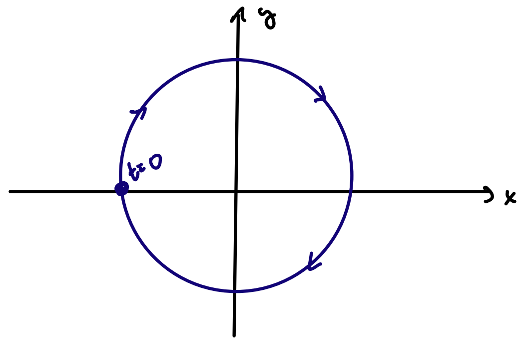
$$\boxed{y(t) = \frac{v_0 - v_{dr}}{qB/m}\left[\cos\left(\frac{qB}{m}t\right) - 1\right] + y_0}$$

Sketch: consider $z(t) = x(t) + iy(t) = \frac{v_0 - v_{dr}}{\omega_B}\left[\sin\omega_B t + i\cos\omega t\right] + v_{dr}t + x_0 + iy_0$

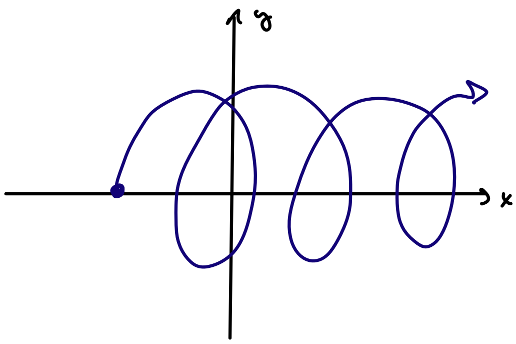
$$= \frac{v_0 - v_{dr}}{\omega_B}(i)\,e^{-i\omega_B t} + v_{dr}t + x_0 + iy_0$$

$$= \underbrace{\frac{v_0 - v_{dr}}{\omega_B}e^{i(\pi - \omega_B t)}}_{\text{Circular motion with radius } \frac{v_0 - v_{dr}}{\omega_B}} + \underbrace{v_{dr}t}_{\text{Constant speed in } +x \text{ direction}} + \underbrace{x_0 + iy_0}_{\text{initial position}}$$

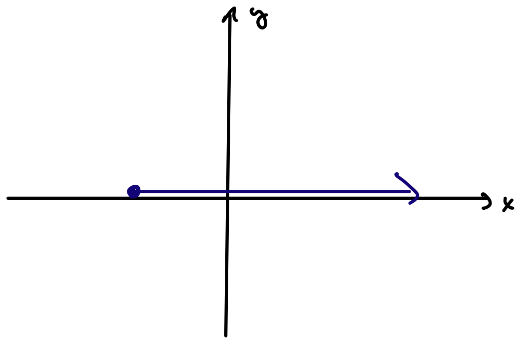
In the limit $v_{dr} \to 0$, $\xi(t) = \frac{v_0}{\omega_B} e^{i(\pi - \omega_B t)} + x_0 + i y_0$ let $(x_0, y_0) = (0, 0)$




If $V_{dr} > 0$, this circle moves




In limit $v_0 \to v_{dr}$: $\xi(t) = + v_{dr} t$




$V_{x0} = V_{dr}$