Activity 1:

1. **Simple Example of Newton's 2nd law:** Consider a particle of mass m being acted on by a constant force $\vec{F}_0$ pointing in the $\hat{x}$-direction:

$$\vec{F}_0 = F_0\hat{x}$$

At time $t = 0$, the particle has velocity v_0 and position x_0. Find:

(a) a(t)

$$F_0\hat{x} = m\vec{a}$$

$$\boxed{\vec{a} = \frac{F_0}{m}\hat{x}}$$

(b) v(t)

$$F_0 = m\frac{d}{dt}\dot{x} \qquad \int_0^t \frac{F_0}{m}\,dt' = \int_0^t \frac{d}{dt'}\dot{x}\,dt'$$

$$\frac{F_0}{m}t'\Big|_0^t = \dot{x}(t')\Big|_0^t$$

$$\frac{F_0}{m}t = \dot{x}(t) - \dot{x}(0)$$

$$\boxed{\frac{F_0}{m}t + v_0 = v(t)} \qquad \vec{v}(t) = v(t)\hat{x}$$

(c) x(t)

$$\frac{F_0}{m}t + v_0 = \frac{d}{dt}x$$

$$\int_0^t \left(\frac{F_0}{m}t' + v_0\right)dt' = \int_0^t \frac{d}{dt'}x\,dt'$$

$$\left[\frac{F_0}{m}\frac{1}{2}(t')^2 + v_0t'\right]_0^t = x(t')\Big|_0^t = x(t) - x(0)$$

let $x_0 = x(0)$

$$\boxed{\frac{1}{2}\frac{F_0}{m}t^2 + v_0t + x_0 = x(t)}$$

Kinematic eq. for a constant force (or, a constant acceleration)

2. **Newton's 3rd Law and Conservation of Momentum.** Newton's 3^{rd} law states:

> *If object 1 exerts a force $\vec{F}_{21}$ on object 2, then object 2 always exerts an* equal and opposite *reaction force $\vec{F}_{12}$ on object 1:*

$$\vec{F}_{12} = -\vec{F}_{21}$$

Consider a system of N particles, each of which exerts a force on every other, see Fig. 1.

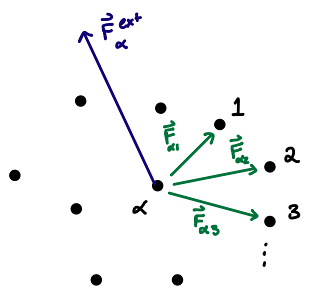


Figure 1: An N-particle system, which forces $\vec{F}_{ij}$ acting between all particles in the system (on particle i by particle j), and an additional external force $\vec{F}^{\text{ext}}$ which acts on the i^{th} particle with force $\vec{F}_i^{\text{ext}}$.

Consider one particle in the N-particle system, labelled "α," identified near the center of Fig. 1. The total force acting on α is:

$$\vec{F}_\alpha^{\text{total}} = \vec{F}_\alpha^{\text{ext}} + \vec{F}_{\alpha 1} + \vec{F}_{\alpha_2} + ... + \vec{F}_{\alpha N}$$

(a) What is the instantaneous change of momentum of particle α, or in other words, what is $\dot{\vec{p}}_\alpha$?

$$\dot{\vec{p}}_\alpha = m\vec{a}_\alpha = \vec{F}_\alpha^{total}$$

$$= \vec{F}^{ext}_\alpha + \sum_{i \neq \alpha} \vec{F}_{\alpha i}$$

(b) Let $\vec{P}$ be the total momentum of the system:

$$\vec{P} = \sum_{\alpha=1}^{N} \vec{p}_\alpha$$

Consider $\dot{\vec{P}}$, the first time derivative of $\vec{P}$. Show that the following is true:

$$\dot{\vec{P}} = \vec{F}^{\text{ext}}_{\text{total}}$$

Hint: Use Newton's third law.

$$\dot{\vec{P}} = \sum_{\alpha=1}^{N} \dot{\vec{p}}_\alpha = \sum_{\alpha=1}^{N}\left(\vec{F}^{ext}_\alpha + \sum_{i\neq\alpha}\vec{F}_{\alpha i}\right) = \vec{F}^{ext}_{total} + \sum_{\alpha=1}^{N}\sum_{i\neq\alpha}\vec{F}_{\alpha i}$$

$$\begin{aligned}\sum_{\alpha=1}^{N}\sum_{i\neq\alpha}\vec{F}_{\alpha i} = {} & \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1N} \\ & + \vec{F}_{21} + \vec{F}_{23} + \dots + \vec{F}_{2N} \\ & + \\ & \vdots \\ & + \vec{F}_{N1} + \vec{F}_{N2} + \dots\end{aligned}$$

$$= \underbrace{\vec{F}_{12} + \vec{F}_{21}}_{=0} + \underbrace{\vec{F}_{13} + \vec{F}_{31}}_{=0} + \dots + \underbrace{\vec{F}_{1N} + \vec{F}_{N}}_{=0} = 0$$

$$\Rightarrow \boxed{\dot{\vec{P}} = \vec{F}^{ext}_{total} + 0}$$

(c) If the net external force $\vec{F}^{\text{ext}}_{\text{total}} = 0$, what can we say about the total momentum $\vec{P}$ of the system?

$\vec{P}$ is constant

3. **Newton's 2nd Law in Cartesian coordinates.** In Cartesian coordinates, we can write Newton's 2nd law,

$$\vec{F} = m\vec{a} = m\ddot{\vec{r}},$$

in component for as:

$$F_x = m\ddot{x}$$
$$F_y = m\ddot{y}$$
$$F_z = m\ddot{z}$$

Consider a golfer hitting a golf ball. The ball has an initial speed v_0 at an angle θ above the ground, see Fig. 2.

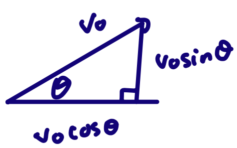
(a) Find $\vec{r}(t)$.

$F_x = 0 = m\ddot{x}$

$\Rightarrow \quad x(t) = x_0 + v_{0x} t$

Let $x_0 = y_0 = 0$

$$\boxed{x(t) = v_0 \cos\theta \, t}$$



$F_y = -mg = m\ddot{y}$

$$\int_0^t -g\,dt' = \int_0^t \frac{d\dot{y}}{dt'}\,dt'$$

$$-gt'\Big|_0^t = \dot{y}(t')\Big|_0^t$$

$$-gt = \dot{y}(t) - v_{y0}$$

$$\int_0^t -gt'\,dt' + \int_0^t v_{y0}\,dt' = \int_0^t \dot{y}(t')\,dt'$$

$$-g\,\frac{1}{2}(t')^2\Big|_0^t + v_{y0}t'\Big|_0^t = y(t')\Big|_0^t$$

$$-\frac{1}{2}gt^2 + v_{0y}t = y(t) - y(0)$$

$$\boxed{y(t) = -\frac{1}{2}gt^2 + v_0\sin\theta\, t}$$

(b) Find the time of flight of the golfball. Assume the ground is completely flat.

$$y(t) = 0 = -\frac{1}{2}gt^2 + v_0\sin\theta\, t$$

$$v_0\sin\theta = \frac{1}{2}gt \quad \Rightarrow \quad \boxed{t_f = \frac{2v_0\sin\theta}{g}}$$

(c) Find the range of the golfball.

$$x(t_f) = v_0\cos\theta\,\frac{2v_0\sin\theta}{g} = \boxed{\frac{2v_0^2\sin\theta\cos\theta}{g}}$$

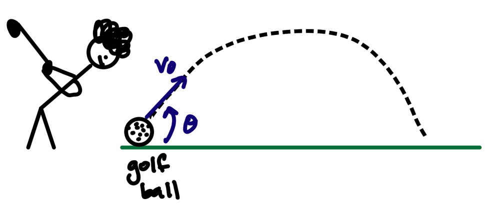


Figure 2