Solutions
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Quiz 1

1. Here is Newton's second law in polar coordinates:

$$\vec{F} = m\left(\ddot{r} - r\dot{\phi}^2\right)\hat{r} + m(r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\phi} \tag{1}$$

Simplify these equations for the case of circular motion (hint: r is constant).

$r =$ constant : $\Rightarrow r = R$
$\dot{r} = 0$
$\ddot{r} = 0$

$\Rightarrow \vec{F} = -mr\dot{\phi}^2\hat{r} + mr\ddot{\phi}\hat{\phi}$

2. Drag force depends most directly on *(select one)*:

(a) position
(b) velocity
(c) acceleration

$\vec{f} = -f(v)\hat{v}$

where $f(v) = f_{lin} + f_{quad} = bv + cv^2$

3. The following differential equation will commonly appear in problems with drag force. Find $v(t)$ by solving this differential equation. Use $v(t=0) = v_0$.

$$\dot{v} = -kv \tag{2}$$

$\frac{dv}{dt} = -kv$

$\frac{dv}{v} = -k\,dt$

$\int_{v_0}^{v} \frac{dv'}{v'} = \int_0^t -k\,dt'$

$\ln(v')\Big|_{v_0}^{v} = -kt'\Big|_0^t$

$\ln\left(\frac{v}{v_0}\right) = -kt$

$\frac{v}{v_0} = e^{-kt}$

$\boxed{v = v_0 e^{-kt}}$ ☑

$\Rightarrow$ OR you can just realize that v must be an exponential because $\frac{d}{dt}(e^{at}) = ae^{at}$

$\underbrace{\frac{d}{dt}(f) = af}$

taking a derivative of an exponent pulls out a constant, but leave the exponent the same.

$\Rightarrow$ Guess $v = Ae^{bt}$

$\dot{v} = Abe^{bt}$

$\dot{v} = -kv = -k(Ae^{bt})$

$\dot{v} = -kAe^{bt}$

Equate these two:

$Abe^{bt} = -kAe^{bt}$

$\Rightarrow \boxed{b = -k}$

$v = Ae^{-kt}$

$v(t=0) = v_0 = Ae^0 = A$

$\Rightarrow \boxed{v_0 = A}$

$\Rightarrow v = v_0 e^{-kt}$ ☑