Lecture 1 Jan 21, 2026 Rachel Houtz

(i) Syllabus, see Canvas

Supplemental material at: rahoutz.github.io/rachel-houtz/mech1

Grade Breakdown:

HW	30 %	$\rightarrow$ Prepares you for tests	
Quizzes	10 %	$\rightarrow$ Easy, makes you read ahead of lecture	
M1	17.5 %	35 %	To pass this class you must be ready for exams
M2	17.5 %		
Final	25 %	$\rightarrow$ 25 %	

(ii) Class structure:

Mondays: Review last week (first ~10 min)
Lecture

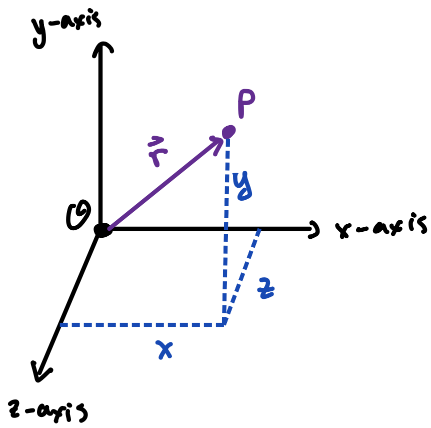
Wednesdays: Quiz on reading (first ~10 min)
Lecture

Fridays: Group Activities:
- practice problems
- derivations
- examples

not dissimilar to what I do for you in lecture :)

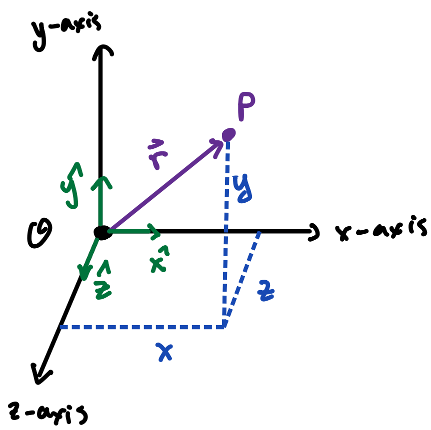
I. Vectors

You will see a few different notations for vectors in this course:



- $\vec{r} = (x, y, z)$
- $\vec{r} = (r_1, r_2, r_3)$
- $\vec{r} = (r_x, r_y, r_z)$

$\hat{x}, \hat{y}, \hat{z}$ are unit vectors.
Unit vectors have length = 1



- $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$
- $\vec{r} = r_1\hat{e}_1 + r_2\hat{e}_2 + r_3\hat{e}_3 = \sum_{i=1}^{3} r_i \hat{e}_i$

$= r_1\vec{e}_1 + r_2\vec{e}_2 + r_3\vec{e}_3$

- $\vec{r} = r_1\hat{x}_1 + r_2\hat{x}_2 + r_3\hat{x}_3 = \sum_{i=1}^{3} r_i \hat{x}_i$

$\vec{x} = x_1\hat{x}_1 + x_2\hat{x}_2 + x_3\hat{x}_3 = \sum_{i=1}^{3} x_i \hat{x}_i$

Einstein notation: When an index is repeated twice in a single term, it's implied that the index should be summed over.

So we can shorten $\sum_{i=1}^{3} r_i \hat{x}_i$ to $r_i \hat{x}_i$

In summary $a_n b_n = \sum_{n=1}^{n_{max}} a_n b_n$

only if "n" is used <u>twice</u>

II. Vector Operations

• <u>Addition</u>: Let $\vec{r} = (r_1, r_2, r_3)$ and $\vec{s} = (s_1, s_2, s_3)$

then $\vec{r} + \vec{s} = (r_1+s_1,\ r_2+s_2,\ r_3+s_3)$

Very natural in this notation:

$$\vec{r} = r_1\hat{x} + r_2\hat{y} + r_3\hat{z} \quad , \quad \vec{s} = s_1\hat{x} + s_2\hat{y} + s_3\hat{z}$$

$$\vec{r}+\vec{s} = r_1\hat{x} + r_2\hat{y} + r_3\hat{z} + s_1\hat{x} + s_2\hat{y} + s_3\hat{z}$$

$$= (r_1+s_1)\hat{x} + (r_2+s_2)\hat{y} + (r_3+s_3)\hat{z}$$

• <u>Multiplication: (scalar) x (vector)</u>

(scalar)(vector) = vector

$$\underset{\text{scalar}}{c}\ \underset{\text{vector}}{\vec{r}} = c(r_x\hat{x} + r_y\hat{y} + r_z\hat{z}) = cr_x\hat{x} + cr_y\hat{y} + cr_z\hat{z} = (cr_x, cr_y, cr_z)$$

• <u>Multiplication: dot product</u> (AKA scalar product)

(vector) • (vector) = scalar

$$\vec{r}\cdot\vec{s} = r_1s_1 + r_2s_2 + r_3s_3 = \sum_{i=1}^{3} r_i s_i = \underbrace{r_i s_i}_{\text{Einstein notation}}$$

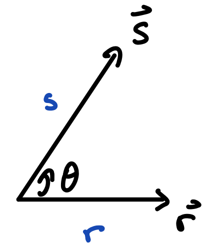
Magnitude: The length of $\vec{r}$ is denoted by:

$$|\vec{r}| = r = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{\sum_{i=1}^{3} r_i r_i} = \sqrt{r_i r_i} = \sqrt{\vec{r}\cdot\vec{r}}$$

$\vec{r}\cdot\vec{r} = r^2$ (sometimes written $\vec{r}^{\,2}$)

More properties:

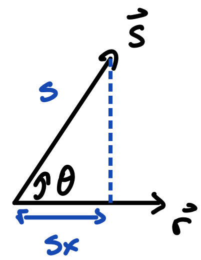
$$\vec{r}\cdot\vec{s} = \sum_{i=1}^{3} r_i s_i = rs\cos\theta$$



Choose axes s.t. $\vec{r} = (r, 0, 0)$

then $\vec{s} = (s_x, s_y, s_z)$

$$\vec{r}\cdot\vec{s} = rs_x + 0s_y + 0s_z = rs_x$$



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{s_x}{s}$$

$$\Rightarrow s_x = s\cos\theta$$

$$\vec{r}\cdot\vec{s} = rs_x = rs\cos\theta$$

☑

- <u>Multiplication: cross product</u> (AKA vector product)

(vector) × (vector) = vector

$\vec{p} = \vec{r}\times\vec{s}$:

$$\left.\begin{aligned} p_1 &= r_2 s_3 - r_3 s_2 \\ p_2 &= r_3 s_1 - r_1 s_3 \\ p_3 &= r_1 s_2 - r_2 s_1 \end{aligned}\right\}$$ There's clearly a pattern here...

<u>Levi-Civita Symbol</u> (AKA the totally antisymmetric tensor)

$$\varepsilon_{ijk} = \begin{cases} 0 & \text{if } i=j \text{ or } j=k \text{ or } i=k \\ +1 & \text{if } (ijk) = (123), (231), (312) \Rightarrow \text{even permutations of } (123) \\ -1 & \text{if } (ijk) = (321), (213), (132) \Rightarrow \text{odd permutations of } (123) \end{cases}$$

123 ⟶ 132 ⇒ 1 swap = odd

switch one time

123 → 132 ⟶ 312 ⇒ 2 swaps = even

$\vec{p} = \vec{r} \times \vec{s}$ or $p_i = \varepsilon_{ijk} r_j s_k$ (Einstein notation) $= \sum_{j=1}^{3} \sum_{k=1}^{3} \varepsilon_{ijk} r_j s_k$

$$\begin{aligned}
p_1 = \varepsilon_{1jk} r_j s_k &= \varepsilon_{111} r_1 s_1 + \varepsilon_{112} r_1 s_2 + \varepsilon_{113} r_1 s_3 \\
&+ \varepsilon_{121} r_2 s_1 + \varepsilon_{122} r_2 s_2 + \varepsilon_{123} r_2 s_3 \\
&+ \varepsilon_{131} r_3 s_1 + \varepsilon_{132} r_3 s_2 + \varepsilon_{133} r_3 s_3 \\
&= \cancel{\varepsilon_{111} r_1 s_1}^{\,i=j=k} + \cancel{\varepsilon_{112} r_1 s_2}^{\,i=j} + \cancel{\varepsilon_{113} r_1 s_3}^{\,i=j} \\
&+ \cancel{\varepsilon_{121} r_2 s_1}^{\,i=k} + \cancel{\varepsilon_{122} r_2 s_2}^{\,j=k} + \varepsilon_{123} r_2 s_3 \\
&+ \cancel{\varepsilon_{131} r_3 s_1}^{\,i=k} + \varepsilon_{132} r_3 s_2 + \cancel{\varepsilon_{133} r_3 s_3}^{\,j=k} \\
&= \underbrace{\varepsilon_{123}}_{(+1)} s_2 r_3 + \underbrace{\varepsilon_{132}}_{(-1)} r_3 s_2 = s_2 r_3 - s_3 r_2 \quad \checkmark
\end{aligned}$$

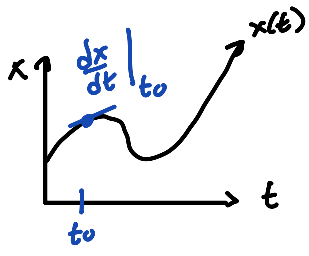
$$p_2 = \varepsilon_{2jk} r_j s_k = \varepsilon_{213} r_1 s_3 + \varepsilon_{231} r_3 s_1 = -r_1 s_3 + r_3 s_1 \quad \checkmark$$

$$p_3 = \varepsilon_{3jk} r_j s_k = \varepsilon_{312} r_1 s_2 + \varepsilon_{321} r_2 s_1 = r_1 s_2 - r_2 s_1 \quad \checkmark$$

$p_i = \varepsilon_{ijk} r_j s_k \Rightarrow$ very condensed notation

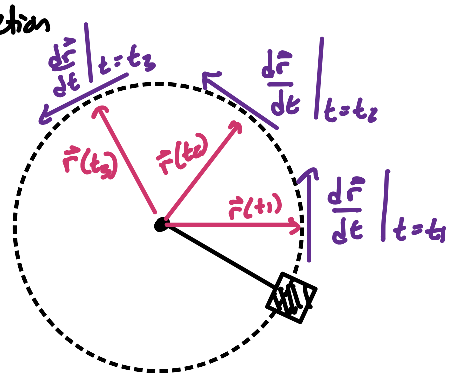
$(\vec{r} \times \vec{s})_i = \varepsilon_{ijk} r_j s_k$

- <u>vector differentiation</u>



$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$, but $\vec{r}$ has magnitude & direction



Example: consider a mass attached to a string that swings in constant circular motion:

$\vec{r}$ has the same magnitude as t changes, but $\Delta \vec{r} \neq 0$ because $\vec{r}$ changes direction

Rules for vector differentiation:

$$\frac{d}{dt}(\vec{r}+\vec{s}) = \frac{d\vec{r}}{dt} + \frac{d\vec{s}}{dt}$$

$$\frac{d}{dt}(c\vec{r}) = \frac{dc}{dt}\vec{r} + c\frac{d\vec{r}}{dt} \qquad \text{(product rule)}$$

$$\vec{v} = \frac{d\vec{r}}{dt} \quad : \quad \vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

$$\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z}$$

$$\frac{d\vec{r}}{dt} = \frac{dr_x}{dt}\hat{x} + \frac{dr_y}{dt}\hat{y} + \frac{dr_z}{dt}\hat{z}$$

→ this is because in Cartesian coordinates, $\hat{x}$, $\hat{y}$, and $\hat{z}$ are fixed & time-independent

$$\Rightarrow \quad v_x = \frac{dr_x}{dt}, \quad v_y = \frac{dr_y}{dt}, \quad v_z = \frac{dr_z}{dt}$$

II. Newton's Laws

Newton's First Law: In the absence of forces, a particle moves with constant velocity $\vec{v}$.

Newton's 2nd Law: For any particle of mass m, the net force $\vec{F}$ on that particle is always

$$\vec{F} = m\vec{a}$$

acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2}{dt^2}\vec{r}$$

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$$

dot notation indicates time derivatives

momentum: $\vec{p} = m\vec{v} \Rightarrow \dot{\vec{p}} = m\dot{\vec{v}} = m\vec{a} = \vec{F}$

$$\Rightarrow \vec{F} = \dot{\vec{p}}$$

(another version of Newton's 2nd law)

Simple Example of Newton's 2^{nd} Law:

Consider a particle being acted on by a constant force $\vec{F}_0$ in the $\hat{x}$-direction

$$\vec{F} = m\vec{a} \Rightarrow F_0\,\hat{x} = ma\,\hat{x}$$

$$F_0 = ma = m\ddot{x}$$

$$F_0 = m\frac{d}{dt}\dot{x}$$

$$\int_0^t \frac{F_0}{m}\,dt' = \int_0^t \frac{d}{dt'}\dot{x}\,dt'$$

$$\frac{F_0}{m}t'\Big|_0^t = \dot{x}(t')\Big|_0^t$$

$$\frac{F_0}{m}t = \dot{x}(t) - \dot{x}(0)$$ Let $\dot{x}(0) = v_0$

$$\frac{F_0}{m}t + v_0 = \frac{d}{dt}x$$

$$\int_0^t \left(\frac{F_0}{m}t' + v_0\right)dt' = \int_0^t \frac{d}{dt'}x\,dt'$$

$$\left[\frac{F_0}{m}\frac{1}{2}(t')^2 + v_0 t'\right]_0^t = x(t')\Big|_0^t = x(t) - x(0)$$

Let $x_0 = x(0)$

$$\boxed{\frac{1}{2}\frac{F_0}{m}t^2 + v_0 t + x_0 = x(t)}$$

Kinematic eq. for a constant force (or, a constant acceleration)

Newton's 3^{rd} Law:

If object 1 exerts a force $\vec{F}_{21}$ on object 2,

then object 2 exerts an equal and opposite reaction force $\vec{F}_{12}$ on object 1:

$$\vec{F}_{12} = -\vec{F}_{21}$$

This leads to the principle of conservation of momentum.