Activity 4

1. Time to review! Draw free body diagrams on top of the following figures, and write down Newton's second law (or conservation of momentum) in terms of components of your chosen coordinate system. Write them as differential equations that describe the equation of motion. In each case, outline your method of solving the differential equation and obtaining the kinematic equations.

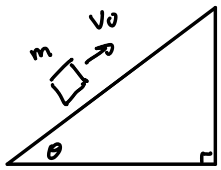

Figure 1: A block of mass m slides up a hill that makes angle θ with the horizontal. Gravitational acceleration is g and gravity points downwards.

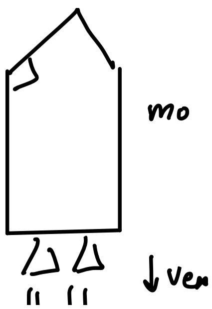

Figure 2: A rocket shoots upwards. It starts with initial mass m_0 and expels fuel at a speed v_{ex}. Gravitational acceleration is g and gravity points downwards.

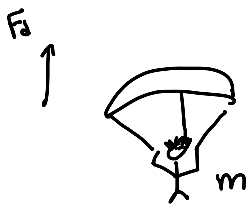

Figure 3: A parachuter of mass m falls with drag force $\vec{F} = -cv^2\hat{v}$. Gravitational acceleration is g and gravity points downwards.

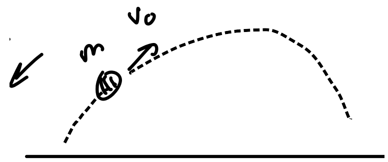

Figure 4: A projectile of mass m is shot with speed v_0 and experiences a drag force $\vec{F}_d = -b\vec{v}$ and gravity pointing downwards.

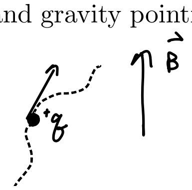

Figure 5: A charge $+q$ moves in a magnetic field $\vec{B} = B\hat{z}$ with an initial velocity $\vec{v}_0$. There is no gravity.

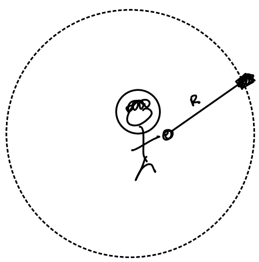

Figure 6: An astronaut swings a mass in a circle. There is no gravity.

2. The position of the center of mass, $\vec{R}_{\text{CM}}$ and the moment of inertia I of a system of N particles are defined as:

$$\vec{R}_{\text{CM}} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha \qquad\qquad I = \sum_{\alpha}^{N} m_\alpha \rho_\alpha^2\,, \tag{1}$$

where ρ is the distance of particle α from the axis of rotation. Find the moment of inertia of the uniform cone shown in Fig. 7.

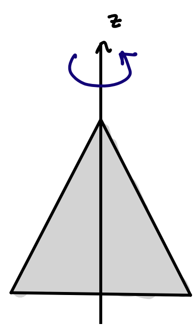

Figure 7: The cone rotating about the z-axis. It has height h and radius b at the base

3. Find the work done from point A to point B along the three paths shown in Fig. 8. Use:

$$W(A \to B) = \int_A^B \vec{F} \cdot d\vec{r} \tag{2}$$

$$\vec{F} = xy^2\hat{x} + y^3\hat{y} \tag{3}$$

Useful integral:

$$\int xe^x dx = (x-1)e^x + C \tag{4}$$

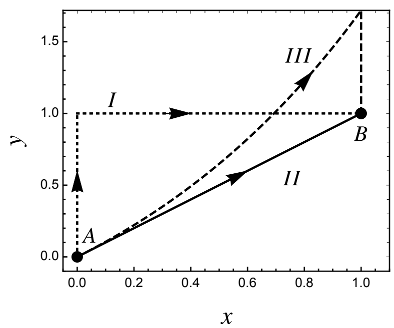

Figure 8: The paths the force acts along. Path II is defined by $y = x$ and path III is defined by $y = e^x - 1$