# Office Hours 5

1. Use the following relationship between force and potential energy

$$\vec{F} = -\vec{\nabla}U = -\hat{x}\frac{\partial U}{\partial x} - \hat{y}\frac{\partial U}{\partial y} - \hat{z}\frac{\partial U}{\partial z} = -\hat{x}_i\frac{\partial U}{\partial x_i} = -\hat{x}_i\partial_i U \tag{1}$$

   to find $\vec{F}$ for the following potential energy functions:

   (a) $U = \cos^2 x + \sin^3 y + \tan\left(\frac{z}{y}\right)$

   (b) $U = x\left(y\ln(z^2)\right)^3$

   (c) $U = \frac{y}{z}e^{1/x}$

   (d) $U = 1/r^3$

   (e) $U = \sin\phi\tan\theta e^{1/r}$

   (f) $U = \phi\sin r$

2. Find $\vec{\nabla}\times F$ for the following forces. Are they conservative? If so, find $U(\vec{r})$ and check that $\vec{\nabla}U = -\vec{F}$.

   (a) $F = x\hat{x} + y\hat{y} + z\hat{z}$

   (b) $F = x\hat{y} + z\hat{y} + x\hat{z}$

   (c) $F = z\hat{y} + y\hat{z}$

   (d) $F = z\hat{y} - y\hat{z}$

   (e) $\vec{F} = r\hat{r}$

   (f) $\vec{F} = r\sin\phi\hat{\phi} + z\hat{r} + r\hat{z}$

   (g) $\vec{F} = \tan(r\cos(\theta\sin\phi))\hat{r} + r\sin\theta\hat{\theta} + r\sin\phi\hat{\phi}$

   (h) $\vec{F} = f(r)\hat{r}$

   (i) $\vec{F} = f(\theta)\hat{r}$

   (j) $\vec{F} = f(z)\hat{\phi}$