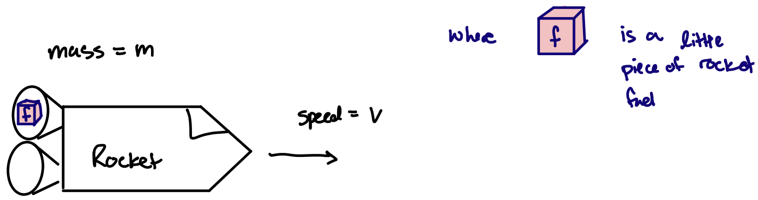
Activity 3

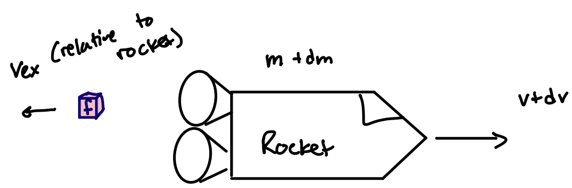
1. Consider a rocket and a fuel system that starts off with mass m and speed v. The rocket ejects fuel with a speed v_{ex} relative to the rocket. **There are no external forces acting on the rocket.**



   (a) What is the change in total momentum of the system?

   (b) At time t_i, the rocket begins ejecting fuel. Find $\vec{P}_i$ for the full rocket and fuel system right before any fuel is ejected.

2. Now consider how the system evolves after an infinitesimally small increment of time to $t_f = t_i + dt$.



   The rocket's new velocity is $v + dv$, and its mass is $m + dm$.

   (a) We know $m_{\text{fuel}} + m_{\text{rocket}} = m$. Find m_{fuel}.

   (b) Find $\vec{P}_f$ and time t_f for the whole system (fuel + rocket)

   (c) Use conservation of momentum to find a relationship between m, dm, dv, and v_{ex}.

   (d) Integrate this relationship to find v as a function of m. Let $m(t = 0) = m_0$ and $v(t = 0) = v_0$.

3. The position of the center of mass (CM) of a system of N particles is defined as:

$$\vec{R} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha \tag{1}$$

(a) Find the position of the center of mass of three particles with positions $\vec{r}_1 = (1, 1, 0)$, $\vec{r}_2 = (1, -1, 0)$, $\vec{r}_3 = (0, 0, 3)$ and masses $m_1 = m_2 = m$ and $m_3 = 10m$.

(b) Sometimes one needs to find the CM for an extended massive object, rather than a collection of particles. Rewrite Eq. (1) above as an integral formula. Imagine each individual particle with mass m_α gets shrunk down to an infinitesimally small piece of the object with mass dm.

(c) Now imagine the object has constant density $\rho = m/V$. Rewrite the above integral in terms of dV instead of dm. Write an expression for just the z-component of $\vec{R}$ in Cartesian coordinates.

(d) Find the position of the CM of uniform solid cone. The cone lines up with the z-axis, the cone has a height h, radius r, the tip of the cone is positioned at the origin.