Physics 3221 Spring 2026: Practice Midterm 1 Problem:

Grade:

1. You Found Me!

When exploring the wilderness, you find a suspiciously placed boulder at the bottom of an icy hill. On the other side of the hill, there's a hole, see Fig. 1. You hit the boulder so that it has an initial speed v_0 sliding up the incline.

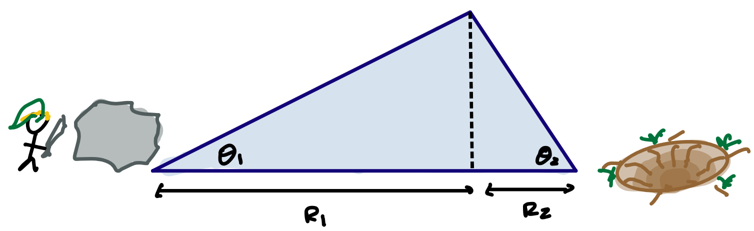

Figure 1: The icy, frictionless hill. The boulder has mass m, and the gravitational acceleration in this wilderness is g.

(a) Find the time t_* it takes for the boulder to get to the top of the hill in terms of v_0, θ_1, R_1, and g.

(b) What is the minimum initial velocity, v_0^{min}, required for the boulder to make it to the top of the hill? Put your answer in terms of g, θ_1 and R_1.

Grade:

(c) Assuming $v_0 = v_0^{\min}$, how much time does it take for the boulder to make it to the bottom of the other side of the hill? Put your answer in terms of $v_0, \theta_1, R_1, g, \theta_2$, and R_2.

(d) Assuming $v_0 < v_0^{\min}$, how much time does it take for the boulder to return back to its original spot? Put your answer in terms of v_0, θ_1, R_1, and g.

Physics 3221 Spring 2026: Practice Midterm 1 Problem:

Grade:

2. Air resistance

Consider the same setup shown in Fig. 1. You hit the boulder so that it has an initial speed v_0 sliding up the incline. This time, the boulder experiences air resistance:

$$\vec{F}_{\text{drag}} = -f(v)\hat{v}$$

$$f(v) = bv + cv^2$$

(a) Assuming $b \gg cv$, find an implicit expression for the time t_* it takes for the boulder to get to the top of the hill in terms of v_0, θ_1, R_1, b, m, and g.

(b) Assuming $b \gg cv$, find an implicit expression for the minimum initial velocity, v_0^{min}, required for the boulder to make it to the top of the hill? Put your answer in terms of g, θ_1, R_1, b, m, and v_0^{min}.

(c) Assuming $b \ll cv$, find an implicit expression for the time t_* it takes for the boulder to get to the top of the hill in terms of v_0, θ_1, R_1, c, m, and g.

(d) Assuming $b \gg cv$ and $v_0 = v_0^{\min}$, how much time does it take for the boulder to fall from the top of the hill down to the bottom of the other side of the hill? Find an implicit expression in terms of v_0, m, b, g, θ_2, and R_2.

(e) Assuming $b \gg cv$ and $v_0 < v_0^{\min}$, how much time does it take for the boulder to return back to its original spot? Put your answer as an implicit expression for t in terms of v_0, θ_1, R_1, m, b, and g.

Grade:

3. Rolling Ball

Now consider a uniform solid disk of mass M and radius R rolling **without** slipping down an incline, as shown in Fig. 2.

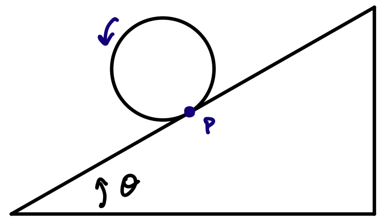

Figure 2: The disk rolls without slipping with initial angular velocity ω. This means the initial translational velocity of the CM is $v = R\omega$.

(a) Find the total torque about point P. The moment of inertia for a disk about a point on its circumference is $\frac{3}{2}MR^2$.

(b) Find the total torque about the CM. The moment of inertia for a disk about its CM is $\frac{1}{2}MR^2$.

(c) Find the linear acceleration of the disk.

Physics 3221 Spring 2026: Practice Midterm 1 Problem: Grade:

4. Zonai Device

You come across a device that hovers in place. Upon examining it, you realize that it's a rocket expelling fuel vertically downwards to counteract the force of gravity. The rocket has initial mass m_0 and expels fuel at a velocity $v_{\rm ex}$. The rocket can only afford to burn 90% of its mass as fuel.

(a) How long can the rocket hover?

(b) If you increase $v_{\rm ex}$ but you require that the rocket must continue to hover in place, what happens to $\frac{dm}{dt}$?

(c) If you double $v_{\rm ex}$ but you require that the rocket must continue to hover in place, how long can the rocket hover compared to your answer in part (a)?

(d) What happens to the rocket if you double $\frac{dm}{dt}$ but keep $v_{\rm ex}$ the same? Why?

Grade:

5. Doing Work on a Charged Ball

A charged ball of mass M slides on a table, constrained to follow the path from point A to point B, as depicted in Fig. 3. This ball has charge $+q$ and is subject to an electric field (the magnetic field is zero):

$$\vec{E} = \frac{1}{2}\kappa y^2 \hat{x} + \kappa x y \hat{y}\,,$$

where κ is a constant with units of $(\mathrm{m}\cdot\mathrm{kg})/(\mathrm{C}\cdot\mathrm{s}^2)$. The ball only experiences the Lorentz force from the $\vec{E}$ field and possibly friction while sliding on the horizontal table (plus gravity and the normal force, which cancel in the vertical direction, while the normal force does no work in the horizontal plane of the table).

(a) Find the work done by the Lorentz force along the path shown in Fig. 3.

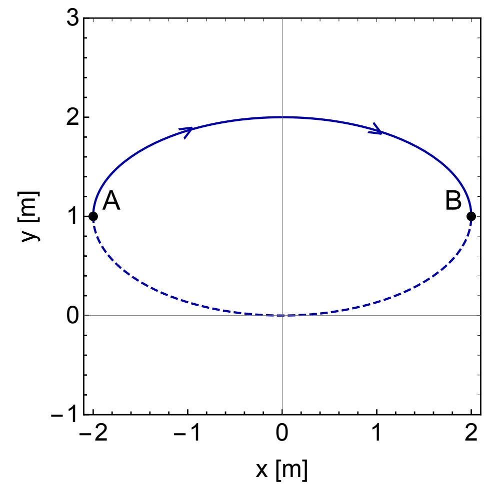

Figure 3: The path along which the ball slides. Distances are measured in units of meters (m).

(c) The ball is released from point A from rest. By the the time gets to point B, the speed of the ball is $v_B = \sqrt{q\kappa/M}$. Given v_B, did the ball lose energy to friction?

Formulas (just for the Practice Midterm)

Newton's 2nd Law

$$\vec{F} = m\ddot{\vec{r}}$$

$$\text{(Cartesian)} \begin{cases} F_x = m\ddot{x} \\ F_y = m\ddot{y} \\ F_z = m\ddot{z} \end{cases}$$

$$\text{(Polar)} \begin{cases} F_r = m(\ddot{r} - r\dot{\phi}^2) \\ F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \end{cases}$$

$$\text{(Cylindrical Polar)} \begin{cases} F_\rho = m(\ddot{\rho} - \rho\dot{\phi}^2) \\ F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) \\ F_z = m\ddot{z} \end{cases}$$

Coordinate Systems

$$\begin{aligned} \text{Cartesian:} \quad \vec{r} &= x\hat{x} + y\hat{y} + z\hat{z} \\ d\vec{r} &= dx\hat{x} + dy\hat{y} + dz\hat{z} \end{aligned}$$

$$\begin{aligned} \text{Cylindrical :} \quad \vec{r} &= \rho\hat{\rho} + z\hat{z} \\ d\vec{r} &= d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z} \end{aligned}$$

$$\begin{aligned} \text{Spherical:} \quad \vec{r} &= r\hat{r} \\ d\vec{r} &= dr\hat{r} + rd\theta\,\hat{\theta} + r^2\sin\theta\, d\phi\,\hat{\phi} \end{aligned}$$

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Momentum and Force

$$\vec{p} = m\vec{v}$$
$$\dot{\vec{P}} = \vec{F}^{\text{ext}}$$

Lorentz Force Law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Angular Momentum and Torque

$$\vec{l} = \vec{r} \times \vec{p}$$
$$\dot{\vec{L}} = \vec{\Gamma}^{\text{ext}}$$
$$\vec{\Gamma} = \vec{r} \times \vec{F}$$
$$L = I\omega$$

Work

$$W = \int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r}\,') \cdot d\vec{r}\,'$$

Conservative forces

$$U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r}\,') \cdot d\vec{r}\,'$$
$$F = -\vec{\nabla}U$$
$$\vec{\nabla} \times \vec{F} = 0$$

Useful Integrals

$$\int \frac{du}{u} = \ln(u)$$
$$\int \frac{dx}{1 + (x/a)^2} = a\tan^{-1}\left(\frac{x}{a}\right)$$
$$\int \frac{dx}{1 - (x/a)^2} = a\tanh^{-1}\left(\frac{x}{a}\right)$$