Physics 3221 Spring 2026: Midterm 1

Grade:

1. A Falling Rocket (20pts)

Consider a rocket suspended in air, see Fig. 1. At time $t = 0$, the rocket has an initial speed $v_0 = 0$ and initial mass m_0. The rocket begins ejecting fuel from its tank at a constant speed v_{ex} relative to the rocket's speed, decreasing the mass $m(t)$ of the rocket over time with a rate $\dot{m} = -k$, where k is constant. The rocket is subject to the gravitational force $\vec{F}_g = -mg\hat{y}$.

(a) *(2 pts)* Find $m(t)$.

(b) *(4 pts)* Find $\frac{dP}{dt}$ for the rocket, including the rocket's fuel.

Hint: $dP = (P + dP) - P$ *and* $P = p_{rocket} + p_{fuel}$.

(c) *(5 pts)* Using $\frac{d\vec{P}}{dt} = \vec{F}_{\text{ext}}$, write a differential equation for the rocket's velocity in the y-direction. Put your final differential equation in terms of v, t, v_{ex}, g, m_0, and k.

(Note: you can use the variable $v \equiv v_y$ *as all motion occurs only in the* y*-direction.)*

(d) *(5 pts)* Find $v(t)$ in terms of v_{ex}, m_0, k, and g.

(e) *(2 pts)* Assume v_{ex} is small enough that the rocket is falling right after it is released from rest. Find the t at which the rocket's acceleration becomes zero.

(f) *(2 pts)* Find the maximum v_{ex} such that the rocket is falling right after it is released from rest. *(Hint: Find* v_{ex} *such that the rocket's acceleration is never zero for* $t > 0$*).*

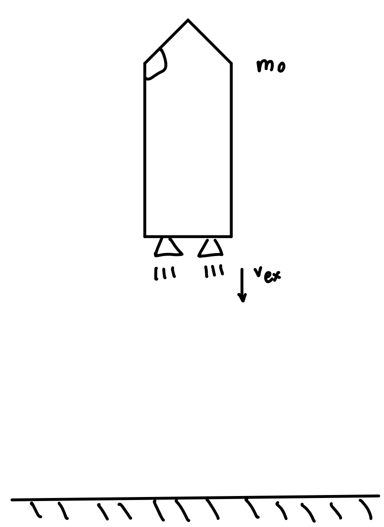

Figure 1: The falling rocket

2. Drag on a Train (20pts)

Consider a train on frictionless tracks, but subject to a drag force due to air resistance, see Fig. 2. The train has a mass m and an initial speed v_0. The drag force is given by $\vec{F} = -b\vec{v}$, where b is the drag coefficient and has units kg/s. There are no other forces acting on the train.

(a) *(6 points)* Find the velocity $v(t)$ in terms of v_0, m, b, and t.

(b) *(6 points)* Find the position $x(t)$ in terms of v_0, m, b, and t. Assume $x(0) = 0$.

(c) *(4 points)* Find the work done on the train when it travels a distance d from the starting position. Leave your answer in terms of v_0, m, b, and d.

(d) *(4 points)* Find the change in kinetic energy of the train from time $t = 0$ to time t_d, where $x(t_d) = d$. Leave your answer in terms of v_0, m, b, and d.

(e) *(4 points)* The train car is filled with various materials, such that the train's density is given by

$$\rho(x, y, z) = \alpha x(h - z)\,,$$

where the x-axis points in the direction of the initial velocity, and the z-axis points upwards away from the ground. The point $x = y = 0$ is the bottom, left, front corner of the train car. The train has a length l in the x-direction, width w in the y-direction pointing into the page, and height h in the z-direction. Find the center of mass of the train. Remember that $dm = \rho(x, y, z)dV$, which in Cartesian coordinates is $dm = \rho(x, y, z)dxdydz$. Leave your answer in terms of α, m, l, w, and h.

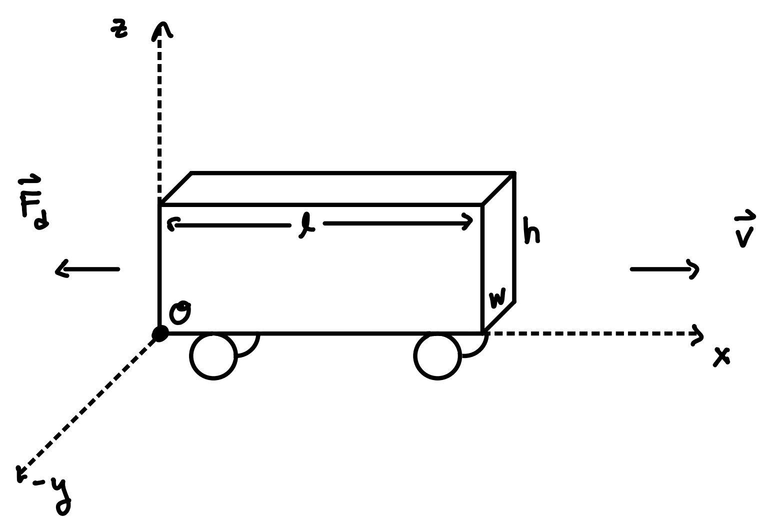

Figure 2: A train subject to a drag force

Grade:

3. Charged Puck (20pts)

A charged puck of mass m and charge $+q$ is confined to move between two concentric cylinders, so that its center of mass is always a distance R away from the axis of the cylinder, see Fig. 3. The charge experiences a magnetic field $\vec{B}$. Ignore gravity.

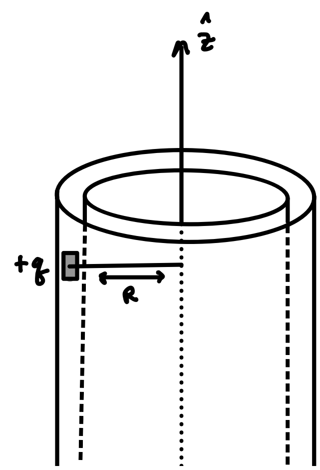

Figure 3: A puck confined to move between two concentric cylinders such that its distance from the z-axis is fixed to be $\rho = R$.

(a) *(6 points)* Using the Lorentz Force law $\vec{F} = q\vec{v} \times \vec{B}$, write down a set of differential equations that describe the motion of the puck in cylindrical polar coordinates. In this coordinate system:

$$\vec{v} = v_\rho \hat{\rho} + v_\phi \hat{\phi} + v_z \hat{z} = v_\rho \hat{\rho} + \rho\dot{\phi}\hat{\phi} + v_z \hat{z}$$
$$\vec{B} = B_\rho \hat{\rho} + B_\phi \hat{\phi} + B_z \hat{z}$$

Note that ρ in cylindrical coordinates is analogous to r in polar coordinates.

(b) *(6 points)* Simplify the differential equations found in part (a) using the fact that $\rho = R =$ constant in cylindrical coordinates. Write these as a system of coupled first order differential equations for $v_z = \dot{z}$ and $v_\phi = R\dot{\phi}$.

(c) *(3 points)* Which component(s) of $\vec{B}$ impact the motion of the puck?

(d) *(5 points)* What is the torque on the puck about the axis of the cylinder? Leave your answer in terms of B_ρ, R, q, v_z, and v_ϕ. Is the angular momentum of the puck conserved?

Formulas (just for the Midterm)

Newton's 2nd Law

$$\vec{F} = m\ddot{\vec{r}}$$

$$\text{(Cartesian)} \begin{cases} F_x = m\ddot{x} \\ F_y = m\ddot{y} \\ F_z = m\ddot{z} \end{cases}$$

$$\text{(Polar)} \begin{cases} F_r = m(\ddot{r} - r\dot{\phi}^2) \\ F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) \end{cases}$$

$$\text{(Cylindrical Polar)} \begin{cases} F_\rho = m(\ddot{\rho} - \rho\dot{\phi}^2) \\ F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) \\ F_z = m\ddot{z} \end{cases}$$

Position vector

$$\text{(Cartesian)}\ \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\text{(Polar)}\ \vec{r} = r\hat{r}$$

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Momentum and Force

$$\vec{p} = m\vec{v}$$

$$\frac{d\vec{P}}{dt} = \vec{F}^{\text{ext}}$$

Angular Momentum and Torque

$$\vec{l} = \vec{r} \times \vec{p}$$

$$\dot{\vec{L}} = \vec{\Gamma}^{\text{ext}}$$

$$\vec{\Gamma} = \vec{r} \times \vec{F}$$

$$L = I\omega$$

$$I = \sum_{\alpha}^{N} m_\alpha r_\alpha^2$$

Center of Mass

$$\vec{R}_{\text{CM}} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha$$

$$\vec{R}_{\text{CM}} = \frac{1}{M}\int \vec{r}\,dm$$

Useful Integrals

$$\int \frac{du}{u} = \ln(u) + c$$

Work

$$W = \int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r}\,') \cdot d\vec{r}\,'$$