

(i) Syllabus, see Canvas

Supplemental material at: rachoutz.github.io/rachel-houtz/mech1

Grade Breakdown: HW : 30% → Prepares you for tests

Quizzes: 10% → Easy, makes you read ahead of lecture

M1 : 17.5% } 35% } To pass this class you

M2 : 17.5% } must be ready for exams

Final : 25% → 25%

(ii) Class structure:

Mondays: Review last week (first ~10 min)

Lecture

Wednesdays: Quiz on reading (first ~10 min)

Lecture

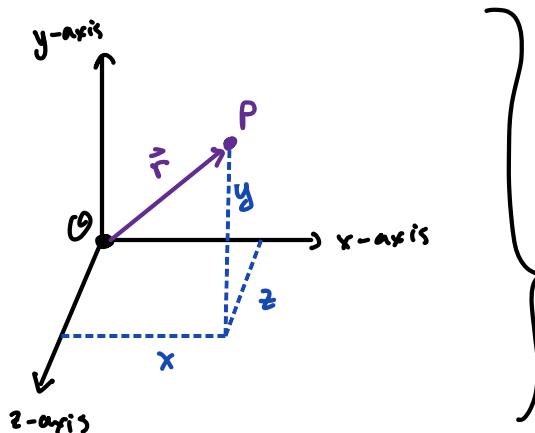
Fridays: Group Activities:

- practice problems
- derivations
- examples

}
 not dissimilar to what I do for you in lecture :)

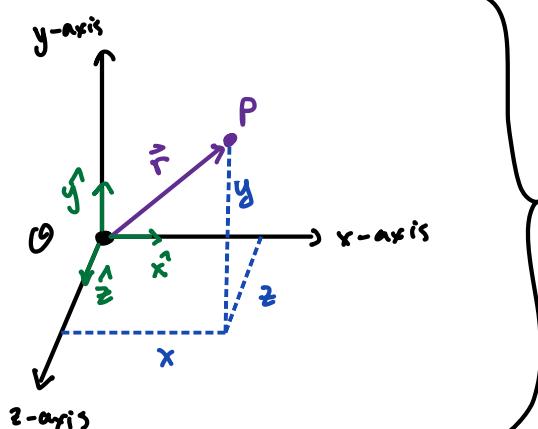
I. Vectors

You will see a few different notations for vectors in this course:



- $\vec{r} = (x, y, z)$
- $\vec{r} = (r_1, r_2, r_3)$
- $\vec{r} = (r_x, r_y, r_z)$

$\hat{x}, \hat{y}, \hat{z}$ are unit vectors.
Unit vectors have length = 1



- $\vec{r} = x \hat{x} + y \hat{y} + z \hat{z}$
- $\vec{r} = r_1 \hat{e}_1 + r_2 \hat{e}_2 + r_3 \hat{e}_3 = \sum_{i=1}^3 r_i \hat{e}_i$
 $= r_1 \hat{e}_1 + r_2 \hat{e}_2 + r_3 \hat{e}_3$
- $\vec{r} = r_1 \hat{x}_1 + r_2 \hat{x}_2 + r_3 \hat{x}_3 = \sum_{i=1}^3 r_i \hat{x}_i$
 $\hat{x} = x_1 \hat{x}_1 + x_2 \hat{x}_2 + x_3 \hat{x}_3 = \sum_{i=1}^3 x_i \hat{x}_i$

Einstein notation : When an index is repeated twice in a single term, it's implied that the index should be summed over.

So we can shorten $\sum_{i=1}^3 r_i x_i$ to $r_i x_i$

In summary $\sum_{n=1}^{n_{\max}} a_n b_n = \sum_{n=1}^{n_{\max}} a_n b_n$

only if "n" is used twice

II. Vector Operations

- Addition : Let $\vec{r} = (r_x, r_y, r_z)$ and $\vec{s} = (s_x, s_y, s_z)$

$$\text{then } \vec{r} + \vec{s} = (r_x + s_x, r_y + s_y, r_z + s_z)$$

Very natural in this notation:

$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}, \quad \vec{s} = s_x \hat{x} + s_y \hat{y} + s_z \hat{z}$$

$$\vec{r} + \vec{s} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z} + s_x \hat{x} + s_y \hat{y} + s_z \hat{z}$$

$$= (r_x + s_x) \hat{x} + (r_y + s_y) \hat{y} + (r_z + s_z) \hat{z}$$

- Multiplication : (scalar) \times (vector)

$$(\text{scalar})(\text{vector}) = \text{vector}$$

$$c \vec{r} = c(r_x \hat{x} + r_y \hat{y} + r_z \hat{z}) = c r_x \hat{x} + c r_y \hat{y} + c r_z \hat{z} = (c r_x, c r_y, c r_z)$$

↑ ↑
scalar vector

- Multiplication : dot product (AKA scalar product)

$$(\text{vector}) \cdot (\text{vector}) = \text{scalar}$$

$$\vec{r} \cdot \vec{s} = r_x s_x + r_y s_y + r_z s_z = \sum_{i=1}^3 r_i s_i = r_i s_i$$

Einstein
notation

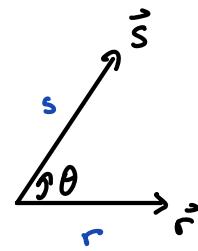
Magnitude: The length of \vec{r} is denoted by:

$$|\vec{r}| = r = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{\sum_{i=1}^3 r_i^2} = \sqrt{r_i r_i} = \sqrt{\vec{r} \cdot \vec{r}}$$

$$\vec{r} \cdot \vec{r} = r^2 \quad (\text{sometimes written } \vec{r}^2)$$

More properties:

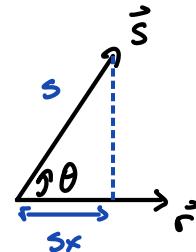
$$\vec{r} \cdot \vec{s} = \sum_{i=1}^3 r_i s_i = rs \cos \theta$$



Choose axes st. $\vec{r} = (r, 0, 0)$

$$\text{then } \vec{s} = (s_x, s_y, s_z)$$

$$\vec{r} \cdot \vec{s} = rs_x + 0s_y + 0s_z = rs_x$$



$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{s_x}{s}$$

$$\Rightarrow s_x = s \cos \theta$$

$$\vec{r} \cdot \vec{s} = rs_x = rs \cos \theta$$



- Multiplication: cross product (AKA vector product)

(vector) \times (vector) = vector

$$\vec{p} = \vec{r} \times \vec{s} : \quad \begin{aligned} p_1 &= r_2 s_3 - r_3 s_2 \\ p_2 &= r_3 s_1 - r_1 s_3 \\ p_3 &= r_1 s_2 - r_2 s_1 \end{aligned} \quad \left. \right\} \text{There's clearly a pattern here...}$$

Levi-Civita Symbol (AKA the totally antisymmetric tensor)

$$\epsilon_{ijk} = \begin{cases} 0 & \text{if } i=j \text{ or } j=k \text{ or } i=k \\ +1 & \text{if } (ijk) = (123), (231), (312) \Rightarrow \text{even permutations of } (123) \\ -1 & \text{if } (ijk) = (321), (213), (132) \Rightarrow \text{odd permutations of } (123) \end{cases}$$

$123 \xrightarrow{\text{swp with one line}} 132 \Rightarrow 1 \text{ swap} = \text{odd}$

$123 \xrightarrow{\text{swp}} 152 \xrightarrow{\text{swp}} 312 \Rightarrow 2 \text{ swaps} = \text{even}$

$$\vec{p} = \vec{r} \times \vec{s} \quad \text{or} \quad p_i = \epsilon_{ijk} r_j s_k \quad (\text{Einstein notation}) = \sum_{j=1}^3 \sum_{k=1}^3 \epsilon_{ijk} r_j s_k$$

$$p_1 = \epsilon_{ijk} r_j s_k = \epsilon_{111} r_1 s_1 + \epsilon_{112} r_1 s_2 + \epsilon_{113} r_1 s_3$$

$$+ \epsilon_{121} r_2 s_1 + \epsilon_{122} r_2 s_2 + \epsilon_{123} r_2 s_3$$

$$+ \epsilon_{131} r_3 s_1 + \epsilon_{132} r_3 s_2 + \epsilon_{133} r_3 s_3$$

$$= \cancel{\epsilon_{111} r_1 s_1} + \cancel{\epsilon_{112} r_1 s_2} + \cancel{\epsilon_{113} r_1 s_3}$$

$$+ \cancel{\epsilon_{121} r_2 s_1} + \cancel{\epsilon_{122} r_2 s_2} + \cancel{\epsilon_{123} r_2 s_3}$$

$$+ \cancel{\epsilon_{131} r_3 s_1} + \cancel{\epsilon_{132} r_3 s_2} + \cancel{\epsilon_{133} r_3 s_3}$$

$$= \underbrace{\epsilon_{123} s_2 r_3}_{(+1)} + \underbrace{\epsilon_{132} s_3 r_2}_{(-1)} = s_2 r_3 - s_3 r_2 \quad \checkmark$$

$$p_2 = \epsilon_{2jk} r_j s_k = \epsilon_{213} r_1 s_3 + \epsilon_{231} r_3 s_1 = -r_1 s_3 + r_3 s_1 \quad \checkmark$$

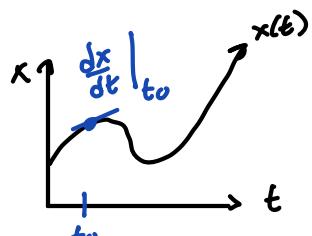
$$p_3 = \epsilon_{3jk} r_j s_k = \epsilon_{312} r_1 s_2 + \epsilon_{321} r_2 s_1 = r_1 s_2 - r_2 s_1 \quad \checkmark$$

$p_i = \epsilon_{ijk} r_j s_k \Rightarrow$ very condensed notation

$$(\vec{r} \times \vec{s})_i = \epsilon_{ijk} r_j s_k$$

• vector differentiation

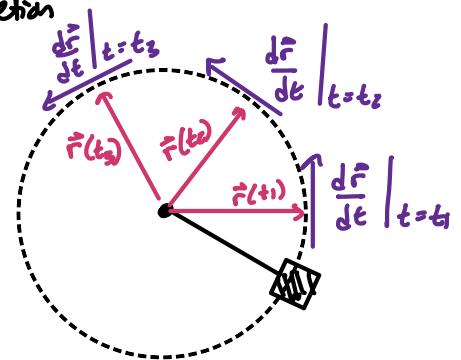
$$\frac{dx}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$



$$\frac{d\vec{r}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} \quad , \text{ but } \vec{r} \text{ has magnitude \& direction}$$

Example: consider a mass attached to a string that swings in constant circular motion:

\vec{r} has the same magnitude as t changes, but $\vec{r} \neq 0$ because \vec{r} changes direction



Rules for vector differentiation:

$$\frac{d}{dt} (\vec{r} + \vec{s}) = \frac{d\vec{r}}{dt} + \frac{d\vec{s}}{dt}$$

$$\frac{d}{dt} (c\vec{r}) = \frac{dc}{dt} \vec{r} + c \frac{d\vec{r}}{dt} \quad (\text{product rule})$$

$$\vec{v} = \frac{d\vec{r}}{dt} : \quad \vec{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z}$$

$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$$

$\frac{d\vec{r}}{dt} = \frac{d\vec{r}_x}{dt} \hat{x} + \frac{d\vec{r}_y}{dt} \hat{y} + \frac{d\vec{r}_z}{dt} \hat{z} \rightarrow$ this is because in Cartesian coordinates, \hat{x} , \hat{y} , and \hat{z} are fixed & time-independent

$$\Rightarrow v_x = \frac{dr_x}{dt}, \quad v_y = \frac{dr_y}{dt}, \quad v_z = \frac{dr_z}{dt}$$

II. Newton's Laws

Newton's First Law : In the absence of forces, a particle moves with constant velocity \vec{v} .

Newton's 2nd Law : For any particle of mass m , the net force \vec{F} on that particle is always

$$\vec{F} = m\vec{a}$$

↑ acceleration

$$\vec{a} = \frac{d\vec{r}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\vec{a} = \ddot{\vec{v}} = \ddot{\vec{r}} \quad \text{dot notation indicates time derivatives}$$

$$\text{momentum: } \vec{p} = m\vec{v} \Rightarrow \dot{\vec{p}} = m\dot{\vec{v}} = m\vec{a} = \vec{F}$$

$$\Rightarrow \vec{F} = \dot{\vec{p}} \quad (\text{another version of Newton's 2nd law})$$