Office Hours 3

1. **Practice solving first order dif. eq.'s.** Solve the following differential equations. Assume $v_{\text{ex}}, k, \omega, b$ are constants, and m_0, v_0, v_{x_0}, and v_{y_0} are m, v, v_x, and v_y at time $t = 0$, respectively.

   (a) $m dv = -v_{\text{ex}} dm$

   (b) $(m_0 - kt) \frac{dv}{dt} = -kv_{\text{ex}} - (m_0 - kt)\, g$

   (c) $(m_0 - kt) \frac{dv}{dt} = -kv_{\text{ex}} - bv$

   (d) $\left\{ \dot{v}_x = \omega^2 v_y \ , \ \dot{v}_y = -\omega^2 v_x \right\}$

2. **Practice integrating.** Evaluate the following integrals (same as last time, feel free to skip this problem if you feel confident with this material).

   (a) $\displaystyle\int dx$

   (b) $\displaystyle\int dt$

   (c) $\displaystyle\int dv_y$

   (d) $\displaystyle\int x\, dx$

   (e) $\displaystyle\int \frac{1}{x^2}\, dx$

   (f) $\displaystyle\int \left(at + bt^2 + ct^3 + dt^{-2} + ft^{-3}\right)\, dt$

   (g) $\displaystyle\int \frac{1}{x}\, dx$

   (h) $\displaystyle\int e^x\, dx$

   (i) $\displaystyle\int e^{Ax}\, dx$

   (j) $\displaystyle\int e^{Ax+B}\, dx$

   (k) $\displaystyle\int \frac{1}{Ax}\, dx$

   (l) $\displaystyle\int \frac{1}{Ax + B}\, dx$

   (m) $\displaystyle\int \cos(v)\, dv$

   (n) $\displaystyle\int \cos(5x)\, dx$

   (o) $\displaystyle\int x \cos(x)\, dx$

   (p) $\displaystyle\int (Ax + B) e^{Cx+D}\, dx$

3. **Practice with complex numbers.** For each of the following w, z, compute $z + w,\ z - w,\ zw,\ z/w,\ |w|$, and $|z|$.

   (a) $w = e^{-20\pi i/7}$, $z = -7 - 2i$

   (b) $w = 4e^{-2\pi i}$, $z = 9 + 9i$

   (c) $w = -2 + 3i$, $z = 6e^{20\pi i}$

   (d) $w = -7 + 7i$, $z = -2 - 5i$

   (e) $w = 8e^{-14\pi i/9}$, $z = 5e^{-12\pi i/7}$

   (f) $w = 5 - 4i$, $z = -3 - 9i$

4. Consider a rocket with an initial speed v_0 and mass m_0 that undergoes the following stages:

   - The rocket expels fuel in the $-\hat{x}$ direction at a rate of v_{ex} until it has lost one third of its mass
   - The rocket stops expelling fuel for t_{rest} seconds
   - The rocket expels fuel in the $+\hat{x}$ direction at a rate of v_{ex} until it has lost two thirds of its mass

   Calculate $v(t)$ and $m(t)$ for this rocket.

5. Calculate the CM for the following objects:

   (a) A cone of mass M, radius R, and height h with **non**-uniform density $\rho(z) = \alpha z$ where $\alpha = 4M/(\pi R^2 h^2)$.

   (b) A cube of mass M and uniform density with sides of length l, with one corner at the origin.

   (c) A cube of mass M and density $\rho = \alpha x^2 y z^3$ with sides of length l, with one corner at the origin. Here, $\alpha = 24M/l^9$.

   (d) A cylinder of mass M, radius R and length L. The density is $\rho = \alpha s$, where s is the distance from the z-axis, and $\alpha = 3M/(2L\pi R^3)$.

   (e) A cone of mass M, base radius R and height h. The density is $\rho = \alpha s$, where s is the distance from the z-axis, and $\alpha = 6M/(h\pi R^3)$.