Quiz ~~5~~ 4

1. Test whether the following force is path-independent using $\vec{\nabla} \times \vec{F}$:

$$\vec{F}_{\text{quiz}} = -y\hat{x}\,. \tag{1}$$

$(F_{quiz})_x = -y$

$(F_{quiz})_y = (F_{quiz})_z = 0$

You can use:

$$\vec{\nabla} \times \vec{A} = \hat{x}\left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right) + \hat{y}\left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right) + \hat{z}\left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right) \tag{2}$$

$$\vec{\nabla} \times \vec{F}_{quiz} = \hat{x}\,(0 - 0) + \hat{y}\left(\cancel{\frac{\partial}{\partial z}(-y)}^{\,0} - 0\right) + \hat{z}\left(0 - \frac{\partial}{\partial y}(-y)\right)$$

$$= 0\hat{x} + 0\hat{y} + \hat{z}\,(+1)$$

$$= \hat{z} \neq 0 \quad \Rightarrow \text{ not path-independent}$$

2. Can you define a potential energy associated with $\vec{F}_{\text{quiz}}$?

No

3. The potential energy of a particle is given by $U = Axy^2 + B\sin Cz$, where $A, B,$ and C are constants. What is the corresponding force? You can use:

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \tag{3}$$

$$\vec{F} = -\vec{\nabla} U = -\frac{\partial U}{\partial x}\hat{x} - \frac{\partial U}{\partial y}\hat{y} - \frac{\partial U}{\partial z}\hat{z}$$

$$\frac{\partial U}{\partial x} = Ay^2$$

$$\frac{\partial U}{\partial y} = 2Axy \qquad \Rightarrow \qquad \boxed{\vec{F} = -Ay^2\hat{x} - 2Axy\,\hat{y} - BC\cos(Cz)\,\hat{z}}$$

$$\frac{\partial U}{\partial z} = BC\cos(Cz)$$