

Quiz 5.4

1. Test whether the following force is path-independent using $\vec{\nabla} \times \vec{F}$:

$$\vec{F}_{\text{quiz}} = -y\hat{x}. \quad (F_{\text{quiz}})_x = -y \quad (1)$$

You can use:

$$\vec{\nabla} \times \vec{A} = \hat{x} \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) + \hat{y} \left(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \right) + \hat{z} \left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \right) \quad (2)$$

$$\begin{aligned} \vec{\nabla} \times \vec{F}_{\text{quiz}} &= \hat{x} (0 - 0) + \hat{y} \left(\frac{\partial}{\partial z} (-y) - 0 \right) + \hat{z} \left(0 - \frac{\partial}{\partial y} (-y) \right) \\ &= 0\hat{x} + 0\hat{y} + \hat{z}(+1) \\ &= \hat{z} \neq 0 \quad \Rightarrow \text{not path-independent} \end{aligned}$$

2. Can you define a potential energy associated with \vec{F}_{quiz} ?

No

3. The potential energy of a particle is given by $U = Axy^2 + B \sin Cz$, where A , B , and C are constants. What is the corresponding force? You can use:

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \quad (3)$$

$$\vec{F} = -\vec{\nabla} U = -\frac{\partial U}{\partial x} \hat{x} - \frac{\partial U}{\partial y} \hat{y} - \frac{\partial U}{\partial z} \hat{z}$$

$$\frac{\partial U}{\partial x} = Ay^2$$

$$\frac{\partial U}{\partial y} = 2Axy$$

$$\frac{\partial U}{\partial z} = BC \cos(Cz)$$

\Rightarrow

$$\vec{F} = -Ay\hat{x} - 2Axy\hat{y} - BC \cos(Cz)\hat{z}$$