Office Hours 2

1. **Practice separating variables.** Take the following differential equations and separate variables. Set up an integral with bounds of integration, but don't solve the integral. Assume $A, B, v_{x_0}, v_{y_0}, F_0, g, m, b$, and c are constants.

   (a) $A = \frac{dx}{dt}$

   (b) $\frac{Ax}{Bt} = \frac{dx}{dt}$

   (c) $\frac{At}{Bx} = \frac{dx}{dt}$

   (d) $Ax + B = \frac{dx}{dt}$

   (e) $A + Bt = \frac{dx}{dt}$

   (f) $v_{x_0} = \frac{dx}{dt}$

   (g) $v_{y_0} - gt = \frac{dy}{dt}$

   (h) $F_0 = m\frac{dv}{dt}$

   (i) $m\frac{dv_y}{dt} = mg$

   (j) $m\ddot{y} = mg$

   (k) $m\frac{dv}{dt} = -bv$

   (l) $m\frac{dv_y}{dt} = -bv_y - mg$

   (m) $m\frac{dv_x}{dt} = -c^2 v_x^2$

   (n) $m\dot{v}_y = cv_y^2 - mg$

   (o) $ma_x = -bv_x$

2. **Practice integrating.**Evaluate the following integrals.

   (a) $\int dx$

   (b) $\int dt$

   (c) $\int dv_y$

   (d) $\int x\,dx$

   (e) $\int \frac{1}{x^2}\,dx$

   (f) $\int \left(at + bt^2 + ct^3 + dt^{-2} + ft^{-3}\right)\,dt$

   (g) $\int \frac{1}{x}\,dx$

   (h) $\int e^x\,dx$

   (i) $\int e^{Ax}\,dx$

   (j) $\int e^{Ax+B}\,dx$

   (k) $\int \frac{1}{Ax}\,dx$

   (l) $\int \frac{1}{Ax+B}\,dx$

   (m) $\int \cos(v)\,dv$

   (n) $\int \cos(5x)\,dx$

   (o) $\int x\cos(x)\,dx$

   (p) $\int (Ax+B)e^{Cx+D}\,dx$

3. Consider a projectile with mass m and an initial velocity $\vec{v}_0$, subject to gravity and a linear drag force:

$$\vec{F}_{\text{drag}} = -b\vec{v} \tag{1}$$

   (a) Find the terminal velocity of this particle
   (b) Find the horizontal motion $x(t)$ and the vertical motion $y(t)$.
   (c) Find the maximum height of the projectile
   (d) Find the range of the projectile
   (e) Find the height of the projectile in the limit where $b \to 0$. Compare the two cases ($b \neq 0$ and $b = 0$).
   (f) Find the range of the projectile in the limit where $b \to 0$. Compare the two cases ($b \neq 0$ and $b = 0$).

4. Consider a projectile with mass m and an initial velocity $v_0\hat{y}$ where $v_0 > 0$. The projectile is subject to gravity and a quadratic drag force:

$$\vec{F}_{\text{drag}} = -cv^2\hat{v} \tag{2}$$

   (a) Find the the vertical motion $y(t)$ before the projectile reaches its maximum height
   (b) Find the maximum height of the projectile
   (c) Find the height of the projectile in the limit where $c \to 0$. Compare the two cases ($c \neq 0$ and $c = 0$).

5. Consider a projectile with mass m and an initial velocity $v_0\hat{x}$ where $v_0 > 0$. The projectile is subject to either a linear or a quadratic drag force:

$$\vec{F}_{\text{drag}} = -b\vec{v} \tag{3}$$

$$\vec{F}_{\text{drag}} = -cv^2\hat{v} \tag{4}$$

   (a) Find the the horizontal motion $x(t)$ in both cases
   (b) Plot $x(t)$ in both cases