Activity 6

1. In your reading, you saw that

$$\vec{\nabla}_1 U(\vec{r}_1 - \vec{r}_2) = -\vec{\nabla}_2 U(\vec{r}_1 - \vec{r}_2)\,, \qquad \text{where} \qquad \vec{\nabla}_i f = \hat{x}\frac{\partial f}{\partial x_i} + \hat{y}\frac{\partial f}{\partial y_i} + \hat{z}\frac{\partial f}{\partial z_i} \tag{1}$$

   (a) Show $\frac{\partial}{\partial x_1} f(x_1 - x_2) = -\frac{\partial}{\partial x_2} f(x_1 - x_2)$

   (b) Generalize this to argue that Eq. (1) is true.

2. **Energy of Interaction of Two Particles**. Consider the gravitational force between two isolated masses:

$$\vec{F}_{12} = -\frac{Gm_1 m_2}{|\vec{r}_1 - \vec{r}_2|^3}\,(\vec{r}_1 - \vec{r}_2) \tag{2}$$

   (a) Does $\vec{F}_{12}$ depend on the position of the origin from which $\vec{r}_1$ and $\vec{r}_2$ are measured? Explain why or why not.

(b) The total work on the system is given by:

$$W = d\vec{r}_1 \cdot \vec{F}_{12} + d\vec{r}_2 \cdot \vec{F}_{21} \,. \tag{3}$$

Show that the total work on the system can be written as

$$W = -dU \,, \tag{4}$$

where recall the chain rule, $dU = d\vec{r} \cdot \vec{\nabla} U(\vec{r})$

(c) Show that for an elastic collision, $\Delta T = 0$. Use the fact that an elastic collision assumes the particles interact only via a conservative force, and that their potential energy of interaction $U(\vec{r}_1 - \vec{r}_2) \to 0$ as their separation goes to infinity.

3. As you saw in the reading, for an N-particle system,

$$T = \sum_\alpha T_\alpha = \sum_\alpha \frac{1}{2} m_\alpha v_\alpha^2 \quad \text{and} \quad U = U^{\text{int}} + U^{\text{ext}}\,, \quad \text{where} \quad U^{\text{int}} = \sum_\alpha \sum_{\beta>\alpha} U_{\alpha\beta}(|\vec{r}_\alpha - \vec{r}_\beta|)$$

(a) If these N particles form a rigid body, how does $|\vec{r}_\alpha - \vec{r}_\beta|$ change with time? In other words, from an initial to a final state, what is $\Delta(|\vec{r}_\alpha - \vec{r}_\beta|)$?

(b) Given this, what is ΔU_{int} for a rigid body?

(c) Write down the change in total mechanical energy ΔE for a rigid body.