1) F.S

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_{<T>} x(t) e^{-jk\omega_0 t} \, dt$$

Properties

1) Linearity $\Rightarrow$ $\alpha x(t) + \beta y(t) \xrightarrow{FS} \alpha a_k + \beta b_k$

2) Timeshift $\Rightarrow$ $x(t-t_0) \xrightarrow{FS} a_k e^{-jk\omega_0 t_0}$

3) Freq shift $\Rightarrow$ $e^{jL\omega_0 t} x(t) \xrightarrow{FS} a_{k-L}$

4) Convolution $\Rightarrow$ $x(t) * y(t) \xrightarrow{FS} a_k \cdot b_k \cdot T$

5) Multiplication $\Rightarrow$ $x(t) \cdot y(t) \xrightarrow{FS} a_k * b_k$

6) Differentiation $\Rightarrow$ $\frac{d}{dt} x(t) \xrightarrow{FS} jk\omega_0 a_k$

7) Scaling $\Rightarrow$ $x(at) \xrightarrow{FS} a_k$ (No change)

8) Time Reversal $\Rightarrow$ $x(-t) \xrightarrow{FS} a_{-k}$

9) Conjugate $\Rightarrow$ $x^*(t) = \sum a^*_{-k} e^{jk\omega_0 t}$

if $x(t) \rightarrow$ real $\Rightarrow$ $a_k = a^*_{-k}$

if $x(t) \rightarrow$ imag $\Rightarrow$ $a_k = -a^*_{-k}$

Parsevals' Theorem:-

$$P = \frac{1}{T} \int_{<T>} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

F.T

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Properties:-

1) Linearity $\Rightarrow$ $ax(t) + by(t) \xleftrightarrow{FT} aX(\omega) + bY(\omega)$

2) Time shifting $\Rightarrow$ $x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(\omega)$

3) Freq shifting $\Rightarrow$ $x(t) e^{j\omega_c t} \xleftrightarrow{FT} X(\omega-\omega_c)$

4) Time scaling $\Rightarrow$ $x(at) \xleftrightarrow{FT} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

5) Differentiation $\Rightarrow$ $\frac{dx(t)}{dt} \xleftrightarrow{FT} j\omega X(\omega)$

$$-jt\, x(t) \xleftrightarrow{FT} \frac{dX(\omega)}{dt}$$

6) Convolution $\Rightarrow$ $x(t) * h(t) \xleftrightarrow{FT} X(\omega) \cdot H(\omega)$

7) conjugate $\Rightarrow$ $x^*(t) \xleftrightarrow{FT} X^*(-\omega)$

if $x(t)$ is real $\Rightarrow$ $X^*(\omega) = X(-\omega)$

Trigonometric Fourier Series:-

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \quad ; t_1 < t < t_1 + T$$

$$\boxed{a_0 = \frac{1}{T} \int_{t_1}^{t_1+T} x(t) dt} \quad \boxed{a_n = \frac{2}{T} \int_{t_1}^{t_1+T} x(t) \cos n\omega_0 t\, dt} \quad n = 1, 2, 3 \ldots$$

$$\boxed{b_n = \frac{2}{T} \int_{t_1}^{t_1+T} x(t) \sin n\omega_0 t\, dt} \quad n = 1, 2, 3 \ldots$$

Duality:-

$$x(t) \xleftrightarrow{FT} X(\omega)$$

$$X(t) \xleftrightarrow{FT} 2\pi x(-\omega) = x(-f)$$

$$* \boxed{X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)}$$

Hilbert Transform:-

$$\hat{x}(t) = HT(x(t)) = x(t) * \frac{1}{\pi t}$$

$$\hat{X}(f) = X(f) \cdot (-j \, \mathrm{sgn} f)$$

$$\hat{x}(f) = \begin{cases} -j X(f) & ; f > 0 \\ j X(f) & ; f < 0 \end{cases}$$

Properties:-

1) Linearity $\Rightarrow$ $HT\{\alpha x(t) + \beta y(t)\} = \alpha \hat{x}(t) + \beta \hat{y}(t)$

2) Timeshift $\Rightarrow$ $HT\{x(t-t_0)\} = \hat{x}(t-t_0)$

3) conjugate $\rightarrow$ $\{\hat{X}(f)\}^* = -j \, \mathrm{sgn} f \cdot X^*(f) = -jX^*(f) ; f > 0$

$$= j X^*(f) ; f < 0$$

4) Derivative $\Rightarrow$ $\frac{d}{dt} HT\{g(t)\} = HT\{g'(t)\}$

5) Convolution $\Rightarrow$ $HT\{x(t) * y(t)\} = \hat{x}(t) * y(t) = \hat{y}(t) * x(t)$

Parseval's Theorem:- $P = \int X(f) X^*(f) df$

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Laplace Transform:

$s = \sigma + j\omega$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad (\text{Bilateral})$$

rational $X(s) = \dfrac{a_0 (s-z_1) \dots (s-z_m)}{b_0 (s-p_1) \dots (s-p_n)}$

$z_1, z_2, \dots z_m \Rightarrow$ zeroes

$p_1, p_2, \dots p_n \Rightarrow$ poles

* Poles lie outside of R.O.C (Region of Convergence)

$$x(t) = \frac{1}{2\pi j} \int_{\sigma-\infty}^{\sigma+\infty} X(s) e^{st} ds$$

$$e^{-at} u(t) \xleftrightarrow{LT} \frac{1}{s+a}, \quad Re\{s\} > -a$$

$$-e^{-at} u(-t) \xleftrightarrow{LT} \frac{1}{s+a}, \quad Re\{s\} < -a$$

Properties:-

i) Linearity $\Rightarrow a x_1(t) + b x_2(t) \xleftrightarrow{LT} a X_1(s) + b X_2(s)$

$R' \supset R_1 \cap R_2$

ii) Timeshift $\Rightarrow x(t-t_0) \xleftrightarrow{LT} e^{-st_0} X(s)$; $R' = R$

iii) s-shift $\Rightarrow e^{s_0 t} x(t) \xleftrightarrow{LT} X(s-s_0)$; $R' = R + Re(s_0)$

iv) scaling $\Rightarrow x(at) \xleftrightarrow{LT} \frac{1}{|a|} X\left(\frac{s}{a}\right)$; $R' = aR$

v) Time Reversal $\Rightarrow x(-t) \xleftrightarrow{LT} X(-s)$; $R' = -R$

vi) Differentiation:-

$$\frac{dx(t)}{dt} \xleftrightarrow{LT} sX(s) \; ; \; R' \supset R$$

$$-tx(t) \xleftrightarrow{LT} \frac{dX(s)}{dt} \; ; \; R' = R$$

vii) Conjugate: $\Rightarrow$ $x^*(t) \xleftrightarrow{LT} X^*(s^*) \; ; \; R' = R$

viii) Integration $\Rightarrow$ $\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{LT} \frac{1}{s} X(s) \; ; \; R' = R \cap (Re\{s\} > 0)$

ix) Convolution $\Rightarrow$ $x_1(t) * x_2(t) \xleftrightarrow{LT} X_1(s) X_2(s) \; ; \; R' \supset R_1 \cap R_2$

DTFS:-

$$x(n) = \sum_{k=0}^{N-1} a_k e^{jk\omega_0 n} = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n} \Rightarrow \text{synthesis eq}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-jk\frac{2\pi}{N}n} \Rightarrow \text{Analysis eq}$$

DTFT

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega \Rightarrow \text{synthesis eq}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \Rightarrow \text{Analysis}$$

Condition for existence of DTFT $\Rightarrow$ $\sum_{n=-\infty}^{\infty} |x(n)|^2 < \infty$

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Properties of DTFT :-

1) Linearity $\Rightarrow$ $a x_1(n) + b x_2(n) \xleftrightarrow{DTFT} a X_1(\omega) + b X_2(\omega)$

2) Time shift $\Rightarrow$ $x(n-K) \xleftrightarrow{DTFT} e^{-j\omega K} X(\omega)$

3) Time reversal $\Rightarrow$ $x(-n) \xleftrightarrow{DTFT} X(-\omega)$

4) Convolution $\Rightarrow$ $x_1(n) * x_2(n) \xleftrightarrow{DTFT} X_1(\omega) X_2(\omega)$

5) Freq shift $\Rightarrow$ $e^{j\omega_0 n} x(n) \xleftrightarrow{DTFT} X(\omega - \omega_0)$

6) Modulation theorem $\Rightarrow$ $x(n) \cos \omega_0 n \xleftrightarrow{DTFT} \frac{1}{2}\left[X(\omega + \omega_0) + X(\omega - \omega_0)\right]$

7) Multiplication $\Rightarrow$ $x_1(n)\, x_2(n) \xleftrightarrow{DTFT} \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\lambda) X_2(\omega - \lambda)\, d\lambda$

8) Differentiation $\Rightarrow$ $n x(n) \xleftrightarrow{DTFT} j \frac{d X(\omega)}{d\omega}$

PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting	$e^{jM\omega_0 t} = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	$x^*(t)$	a^*_{-k}
Time Reversal	$x(-t)$	a_{-k}
Time Scaling	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution	$\int_T x(\tau) y(t - \tau) d\tau$	$T a_k b_k$
Multiplication	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation	$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T} a_k$
Integration	$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{Re}\{a_k\} = \mathcal{Re}\{a_{-k}\} \\ \mathcal{Im}\{a_k\} = -\mathcal{Im}\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x(t)$ real and even	a_k real and even
Real and Odd Signals	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e(t) = \mathcal{Ev}\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{Od}\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\mathcal{Re}\{a_k\}$ $j\mathcal{Im}\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Properties of the Fourier Transform

Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
	$x_1(t)$	$X_1(\omega)$
	$x_2(t)$	$X_2(\omega)$
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(\omega) + a_2 X_2(\omega)$
Time shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\pi X(0)\delta(\omega) + \frac{1}{j\omega} X(\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(\omega) X_2(\omega)$
Multiplication	$x_1(t) x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Real signal	$x(t) = x_e(t) + x_o(t)$	$X(\omega) = A(\omega) + jB(\omega)$
		$X(-\omega) = X^*(\omega)$
Even component	$x_e(t)$	$\operatorname{Re}\{X(\omega)\} = A(\omega)$
Odd component	$x_o(t)$	$j \operatorname{Im}\{X(\omega)\} = jB(\omega)$

Parseval's relations

$$\int_{-\infty}^{\infty} x_1(\lambda) X_2(\lambda)\, d\lambda = \int_{-\infty}^{\infty} X_1(\lambda) x_2(\lambda)\, d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t) x_2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\omega) X_2(-\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

Common Fourier Transforms Pairs

$x(t)$	$X(\omega)$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$
$u(-t)$	$\pi\delta(\omega)-\dfrac{1}{j\omega}$
$e^{-at}u(t), a>0$	$\dfrac{1}{j\omega+a}$
$te^{-at}u(t), a>0$	$\dfrac{1}{(j\omega+a)^2}$
$e^{-a\|t\|}, a>0$	$\dfrac{2a}{a^2+\omega^2}$
$\dfrac{1}{a^2+t^2}$	$e^{-a\|\omega\|}$
$e^{-at^2}, a>0$	$\sqrt{\dfrac{\pi}{a}}\,e^{-\omega^2/4a}$
$p_a(t)=\begin{cases}1 & \|t\|<a\\ 0 & \|t\|>a\end{cases}$	$2a\dfrac{\sin\omega a}{\omega a}$
$\dfrac{\sin at}{\pi t}$	$p_a(\omega)=\begin{cases}1 & \|\omega\|<a\\ 0 & \|\omega\|>a\end{cases}$
$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$
$\sum_{k=-\infty}^{\infty}\delta(t-kT)$	$\omega_0\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0), \omega_0=\dfrac{2\pi}{T}$

Hilbert transform pairs.

Time-domain signal	Hilbert transform
$g(t)$	$\hat{g}(t)$
$a_1 g_1(t) + a_2 g_2(t);\ a_1, a_2 \in \mathbb{C}$	$a_1 \hat{g}_1(t) + a_2 \hat{g}_2(t)$
$h(t - t_0)$	$\hat{h}(t - t_0)$
$h(at);\ a \neq 0$	$\operatorname{sgn}(a)\hat{h}(at)$
$\frac{\mathrm{d}}{\mathrm{d}t} h(t)$	$\frac{\mathrm{d}}{\mathrm{d}t} \hat{h}(t)$
$\delta(t)$	$\frac{1}{\pi t}$
$e^{\mathrm{j}t}$	$-\mathrm{j}e^{\mathrm{j}t}$
$e^{-\mathrm{j}t}$	$\mathrm{j}e^{-\mathrm{j}t}$
$\cos(t)$	$\sin(t)$
$\operatorname{rect}(t)$	$\frac{1}{\pi} \ln \lvert (2t+1)/(2t-1) \rvert$
$\operatorname{sinc}(t)$	$\frac{\pi t}{2} \operatorname{sinc}^2(t/2) = \sin(\pi t/2) \operatorname{sinc}(t/2)$
$1/(1+t^2)$	$t/(1+t^2)$

PROPERTIES OF THE LAPLACE TRANSFORM

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{R}e\{s\} > 0\}$

Initial- and Final-Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \to \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t \longrightarrow \infty} x(t) = \lim_{s \longrightarrow 0} sX(s)$$

LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13	$[e^{-\alpha t}\cos \omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$

Property	Periodic signal	Fourier series coefficients
	$\left.\begin{array}{l} x[n] \\ y[n] \end{array}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{array}{l} a_k \\ b_k \end{array}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time shift	$x[n-n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shift	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m] & \text{if } n \text{ is a multiple of } m \\ 0 & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k \left(\begin{array}{l}\text{viewed as} \\ \text{periodic with} \\ \text{period } mN\end{array}\right)$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	Na_kb_k
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

Properties of the Fourier Transform for Discrete-Time Signals

Property	Time Domain	Frequency Domain
Notation	$x(n)$	$X(\omega)$
	$x_1(n)$	$X_1(\omega)$
	$x_2(n)$	$X_2(\omega)$
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time shifting	$x(n-k)$	$e^{-j\omega k}X(\omega)$
Time reversal	$x(-n)$	$X(-\omega)$
Convolution	$x_1(n) * x_2(n)$	$X_1(\omega)X_2(\omega)$
Correlation	$r_{x_1x_2}(l) = x_1(l) * x_2(-l)$	$S_{x_1x_2}(\omega) = X_1(\omega)X_2(-\omega)$ $= X_1(\omega)X_2^*(\omega)$ [if $x_2(n)$ is real]
Wiener–Khintchine theorem	$r_{xx}(l)$	$S_{xx}(\omega)$
Frequency shifting	$e^{j\omega_0 n}x(n)$	$X(\omega - \omega_0)$
Modulation	$x(n)\cos\omega_0 n$	$\frac{1}{2}X(\omega+\omega_0) + \frac{1}{2}X(\omega-\omega_0)$
Multiplication	$x_1(n)x_2(n)$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\lambda)X_2(\omega-\lambda)d\lambda$
Differentiation in the frequency domain	$nx(n)$	$j\frac{dX(\omega)}{d\omega}$
Conjugation	$x^*(n)$	$X^*(-\omega)$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} x_1(n)x_2^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\omega)X_2^*(\omega)d\omega$	

BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$

Signal	Fourier Transform
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & N_1 < \lvert n\rvert \le N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
$a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$x[n] \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π
$\delta[n]$	1
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega - 2\pi k)$
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$(n+1)a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$

Property	DTFS	CTFS	DTFT	CTFT
Synthesis	$x[n] = \sum_{k=<N>} a_k e^{jk\Omega_0 n}$	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$
Analysis	$a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n}$	$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$	$X(e^{j\Omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\Omega n}$	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
Linearity	$\alpha x[n] + \beta y[n] \leftrightarrow \alpha a_k + \beta b_k$	$\alpha x(t) + \beta y(t) \leftrightarrow \alpha a_k + \beta b_k$	$\alpha x[n] + \beta y[n] \leftrightarrow \alpha X(e^{j\Omega}) + \beta Y(e^{j\Omega})$	$\alpha x(t) + \beta y(t) \leftrightarrow \alpha X(j\omega) + \beta Y(j\omega)$
Time Shifting	$x[n-n_0] \leftrightarrow a_k e^{-j2\pi n_0 k/N}$	$x(t-t_0) \leftrightarrow a_k e^{-jk\omega_0 t_0}$	$x[n-n_0] \leftrightarrow e^{-j\Omega n_0}X(e^{j\Omega})$	$x(t-t_0) \leftrightarrow e^{-j\omega t_0}X(j\omega)$
Frequency Shift	$x[n]e^{j2\pi mn/N} \leftrightarrow a_{k-m}$	$x(t)e^{jm\omega_0 t} \leftrightarrow a_{k-m}$	$x[n]e^{j\Omega_0 n} \leftrightarrow X(e^{j(\Omega-\Omega_0)n})$	$x(t)e^{j\omega_0 t} \leftrightarrow X(j(\omega-\omega_0))$
Conjugation	$x^*[n] \leftrightarrow a^*_{-k}$	$x^*(t) \leftrightarrow a^*_{-k}$	$x^*[n] \leftrightarrow X^*(e^{-j\Omega})$	$x^*(t) \leftrightarrow X^*(-j\omega)$
Time Reversal	$x[-n] \leftrightarrow a_{-k}$	$x(-t) \leftrightarrow a_{-k}$	$x[-n] \leftrightarrow X(e^{-j\Omega})$	$x(-t) \leftrightarrow X(-j\omega)$
Convolution	$\sum_{r=0}^{N-1} x[r]y[n-r] \leftrightarrow Na_k b_k$	$\int_T x(\tau)y(t-\tau)d\tau \leftrightarrow Ta_k b_k$	$x[n] * y[n] \leftrightarrow X(e^{j\Omega})Y(e^{j\Omega})$	$x(t) * y(t) \leftrightarrow X(j\omega)Y(j\omega)$
Multiplication	$x[n]y[n] \leftrightarrow \sum_{r=0}^{N-1} a_r b_{k-r}$	$x(t)y(t) \leftrightarrow a_k * b_k$	$x[n]y[n] \leftrightarrow \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\Omega-\theta)})d\theta$	$x(t)y(t) \leftrightarrow \frac{1}{2\pi}X(j\omega) * Y(j\omega)$
First Difference/ Derivative	$x[n] - x[n-1] \leftrightarrow (1-e^{-j2\pi k/N})a_k$	$\frac{dx(t)}{dt} \leftrightarrow jk\omega_0 a_k$	$x[n] - x[n-1] \leftrightarrow (1-e^{-j\Omega})X(e^{j\Omega})$	$\frac{dx(t)}{dt} \leftrightarrow j\omega X(j\omega)$
Running Sum/ Integration	$\sum_{k=-\infty}^{n} x[k] \leftrightarrow \frac{a_k}{1-e^{-j2\pi k/N}}$	$\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{a_k}{jk\omega_0}$	$\sum_{k=-\infty}^{n} x[k] \leftrightarrow \frac{X(e^{j\Omega})}{1-e^{-j\Omega}} + \pi X(e^{j0})\delta(\Omega)$	$\int_{-\infty}^{t} x(\tau)d\tau \leftrightarrow \frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$
Parseval's Relation	$\frac{1}{N}\sum_{n=0}^{N-1} \|x[n]\|^2 = \sum_{k=0}^{N-1} \|a_k\|^2$	$\frac{1}{T}\int_T \|x(t)\|^2 dt = \sum_{k=-\infty}^{\infty} \|a_k\|^2$	$\sum_{n=-\infty}^{\infty} \|x[n]\|^2 = \frac{1}{2\pi}\int_{2\pi} \|X(e^{j\Omega})\|^2 d\Omega$	$\int_{-\infty}^{\infty} \|x(t)\|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \|X(j\omega)\|^2 d\omega$
Real and even signals	Real and even in frequency domain			
Real and odd signals	Purely imaginary and odd in frequency domain			