

WILEY SERIES IN PROBABILITY AND STATISTICS



THIRD EDITION

AN INTRODUCTION TO PROBABILITY AND STATISTICS

VIJAY K. ROHATGI
A. K. MD. EHSANES SALEH

WILEY

AN INTRODUCTION TO PROBABILITY AND STATISTICS

WILEY SERIES IN PROBABILITY AND STATISTICS

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To Bina and Shahidara.

CONTENTS

PREFACE TO THE THIRD EDITION	xiii
PREFACE TO THE SECOND EDITION	xv
PREFACE TO THE FIRST EDITION	xvii
ACKNOWLEDGMENTS	xix
ENUMERATION OF THEOREMS AND REFERENCES	xxi
1 Probability	1
1.1 Introduction, 1	
1.2 Sample Space, 2	
1.3 Probability Axioms, 7	
1.4 Combinatorics: Probability on Finite Sample Spaces, 20	
1.5 Conditional Probability and Bayes Theorem, 26	
1.6 Independence of Events, 31	
2 Random Variables and Their Probability Distributions	39
2.1 Introduction, 39	
2.2 Random Variables, 39	
2.3 Probability Distribution of a Random Variable, 42	
2.4 Discrete and Continuous Random Variables, 47	
2.5 Functions of a Random Variable, 55	

3 Moments and Generating Functions	67
3.1 Introduction, 67	
3.2 Moments of a Distribution Function, 67	
3.3 Generating Functions, 83	
3.4 Some Moment Inequalities, 93	
4 Multiple Random Variables	99
4.1 Introduction, 99	
4.2 Multiple Random Variables, 99	
4.3 Independent Random Variables, 114	
4.4 Functions of Several Random Variables, 123	
4.5 Covariance, Correlation and Moments, 143	
4.6 Conditional Expectation, 157	
4.7 Order Statistics and Their Distributions, 164	
5 Some Special Distributions	173
5.1 Introduction, 173	
5.2 Some Discrete Distributions, 173	
5.2.1 Degenerate Distribution, 173	
5.2.2 Two-Point Distribution, 174	
5.2.3 Uniform Distribution on n Points, 175	
5.2.4 Binomial Distribution, 176	
5.2.5 Negative Binomial Distribution (Pascal or Waiting Time Distribution), 178	
5.2.6 Hypergeometric Distribution, 183	
5.2.7 Negative Hypergeometric Distribution, 185	
5.2.8 Poisson Distribution, 186	
5.2.9 Multinomial Distribution, 189	
5.2.10 Multivariate Hypergeometric Distribution, 192	
5.2.11 Multivariate Negative Binomial Distribution, 192	
5.3 Some Continuous Distributions, 196	
5.3.1 Uniform Distribution (Rectangular Distribution), 199	
5.3.2 Gamma Distribution, 202	
5.3.3 Beta Distribution, 210	
5.3.4 Cauchy Distribution, 213	
5.3.5 Normal Distribution (the Gaussian Law), 216	
5.3.6 Some Other Continuous Distributions, 222	
5.4 Bivariate and Multivariate Normal Distributions, 228	
5.5 Exponential Family of Distributions, 240	
6 Sample Statistics and Their Distributions	245
6.1 Introduction, 245	
6.2 Random Sampling, 246	
6.3 Sample Characteristics and Their Distributions, 249	

6.4	Chi-Square, t -, and F -Distributions: Exact Sampling Distributions,	262
6.5	Distribution of (\bar{X}, S^2) in Sampling from a Normal Population,	271
6.6	Sampling from a Bivariate Normal Distribution,	276
7	Basic Asymptotics: Large Sample Theory	285
7.1	Introduction,	285
7.2	Modes of Convergence,	285
7.3	Weak Law of Large Numbers,	302
7.4	Strong Law of Large Numbers,	308
7.5	Limiting Moment Generating Functions,	316
7.6	Central Limit Theorem,	321
7.7	Large Sample Theory,	331
8	Parametric Point Estimation	337
8.1	Introduction,	337
8.2	Problem of Point Estimation,	338
8.3	Sufficiency, Completeness and Ancillarity,	342
8.4	Unbiased Estimation,	359
8.5	Unbiased Estimation (<i>Continued</i>): A Lower Bound for the Variance of An Estimator,	372
8.6	Substitution Principle (Method of Moments),	386
8.7	Maximum Likelihood Estimators,	388
8.8	Bayes and Minimax Estimation,	401
8.9	Principle of Equivariance,	418
9	Neyman–Pearson Theory of Testing of Hypotheses	429
9.1	Introduction,	429
9.2	Some Fundamental Notions of Hypotheses Testing,	429
9.3	Neyman–Pearson Lemma,	438
9.4	Families with Monotone Likelihood Ratio,	446
9.5	Unbiased and Invariant Tests,	453
9.6	Locally Most Powerful Tests,	459
10	Some Further Results on Hypotheses Testing	463
10.1	Introduction,	463
10.2	Generalized Likelihood Ratio Tests,	463
10.3	Chi-Square Tests,	472
10.4	t -Tests,	484
10.5	F -Tests,	489
10.6	Bayes and Minimax Procedures,	491

11 Confidence Estimation	499
11.1 Introduction, 499	
11.2 Some Fundamental Notions of Confidence Estimation, 499	
11.3 Methods of Finding Confidence Intervals, 504	
11.4 Shortest-Length Confidence Intervals, 517	
11.5 Unbiased and Equivariant Confidence Intervals, 523	
11.6 Resampling: Bootstrap Method, 530	
12 General Linear Hypothesis	535
12.1 Introduction, 535	
12.2 General Linear Hypothesis, 535	
12.3 Regression Analysis, 543	
12.3.1 Multiple Linear Regression, 543	
12.3.2 Logistic and Poisson Regression, 551	
12.4 One-Way Analysis of Variance, 554	
12.5 Two-Way Analysis of Variance with One Observation Per Cell, 560	
12.6 Two-Way Analysis of Variance with Interaction, 566	
13 Nonparametric Statistical Inference	575
13.1 Introduction, 575	
13.2 <i>U</i> -Statistics, 576	
13.3 Some Single-Sample Problems, 584	
13.3.1 Goodness-of-Fit Problem, 584	
13.3.2 Problem of Location, 590	
13.4 Some Two-Sample Problems, 599	
13.4.1 Median Test, 601	
13.4.2 Kolmogorov–Smirnov Test, 602	
13.4.3 The Mann–Whitney–Wilcoxon Test, 604	
13.5 Tests of Independence, 608	
13.5.1 Chi-square Test of Independence—Contingency Tables, 608	
13.5.2 Kendall’s Tau, 611	
13.5.3 Spearman’s Rank Correlation Coefficient, 614	
13.6 Some Applications of Order Statistics, 619	
13.7 Robustness, 625	
13.7.1 Effect of Deviations from Model Assumptions on Some Parametric Procedures, 625	
13.7.2 Some Robust Procedures, 631	
FREQUENTLY USED SYMBOLS AND ABBREVIATIONS	637
REFERENCES	641

CONTENTS	xi
STATISTICAL TABLES	647
ANSWERS TO SELECTED PROBLEMS	667
AUTHOR INDEX	677
SUBJECT INDEX	679

PREFACE TO THE THIRD EDITION

The *Third Edition* contains some new material. More specifically, the chapter on large sample theory has been reorganized, repositioned, and re-titled in recognition of the growing role of asymptotic statistics. In Chapter 12 on General Linear Hypothesis, the section on regression analysis has been greatly expanded to include multiple regression and logistic and Poisson regression.

Some more problems and remarks have been added to illustrate the material covered. The basic character of the book, however, remains the same as enunciated in the Preface to the first edition. It remains a solid introduction to first-year graduate students or advanced seniors in mathematics and statistics as well as a reference to students and researchers in other sciences.

We are grateful to the readers for their comments on this book over the past 40 years and would welcome any questions, comments, and suggestions. You can communicate with Vijay K. Rohatgi at vrohatg@bgsu.edu and with A. K. Md. Ehsanes Saleh at esaleh@math.carleton.ca.

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