$\int x^n dx = \frac{x^{n+1}}{n+1}$ $\int \frac{1}{x} \cdot dx = \log(x)$ $\frac{d}{dx} x^n = n \cdot x^{n-1}$

NAME Integrals

1st kind → $\int_0^\infty \cos x \, dx$ ⇒ Value of Integral, $a, b = \infty$

2nd kind → $\int_0^1 \frac{1}{x} dx$ ⇒ $f(x)$ is not bounded ; 3rd kind = 1+2

$$\int_{-\infty}^{\infty} f(x)\, dx = \lim_{\substack{R_1 \to \infty \\ R_2 \to \infty}} \int_{-R_1}^{R_2} f(x)\, dx$$

$$\int_a^b f(x) \cdot dx = \lim_{\substack{\epsilon_1 \to 0^+ \\ \epsilon_2 \to 0^+}} \int_{a+\epsilon_1}^{b-\epsilon_2} f(x) \cdot dx$$

Test - Integral (i) :-

$$\int_a^\infty \frac{1}{x^p} dx = \ln\left(\frac{R}{a}\right) \quad p = 1$$

$$\frac{1}{1-p}\left[\frac{1}{R^{p-1}} - \frac{1}{a^{p-1}}\right] \quad p \neq 1$$

$p \leq 1$ → divergent

$p > 1$ → converging $\left[\frac{1}{p-1} \cdot \frac{1}{a^{p-1}}\right]$

Test Integral II :-

$$\int_a^b \frac{1}{(x-a)^p} dx$$

$p < 1$ ⇒ convergent $\frac{1}{1-p} \cdot \frac{1}{(b-a)^{p-1}}$

$p \geq 1$ ⇒ divergent

Gamma and Beta :-

$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$; $m>0, n>0$ ⇒ $B(m,n) = B(n,m)$

$\Gamma(n) = \int_0^\infty e^{-x} \cdot x^{n-1} dx \quad (n>0)$ ⇒ $\boxed{\Gamma(n+1) = n \cdot \Gamma(n)}$

$\boxed{\Gamma(n) = \frac{\Gamma(n+1)}{n} \quad \Gamma(1) = 1}$

$\boxed{\Gamma(n+1) = n!}$ $\boxed{\Gamma(1/2) = \sqrt{\pi}}$

$$\boxed{\Gamma(z) \cdot \Gamma(1-z) = \frac{\pi}{\sin(\pi z)}}$$

$$\boxed{B(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}}$$

$y = f(x)$ ⇒ $f'(x) > 0$ ⇒ Increasing

$f'(x) < 0$ ⇒ Decreasing

$f''(x) > 0$ ⇒ concave up.

$f''(x) < 0$ ⇒ concave down

$f''(x) = 0$ ⇒ Point of Inflection.

$f(x) = \frac{x^n}{x^m}$ ⇒

If $m > n$ ⇒ $y = 0$ is horizontal asy

$m = n$ ⇒ Ratio of leading coeff

$m < n$ no horizon. asy.

$f(x) = e^{kx}$

$y = 0$ hori

No vert.

oblique asy :-

if $n = m+1$ $\frac{n}{m}$

$f(x) = \frac{x^2+1}{x^2-1} = x+1 + \frac{2}{x-1}$

$y = x+1$ is an oblique asym.

$\lim_{x \to \infty} \frac{2}{x-1} \to 0$

curvature :- $K = \left|\frac{d\psi}{ds}\right|$ $y = f(x)$ $\frac{d\psi}{ds} = \frac{d\psi}{dx} \Big/ \frac{ds}{dx}$

$$K = \frac{f''(x)}{[1+(f'(x))^2]^{3/2}} \Rightarrow f''(x) = \frac{d}{dx}(f'(x)) \Rightarrow f'(x) = \frac{dy}{dx}f$$

↑
curvature

$\frac{df}{dx}$, $\frac{d^2f}{dx^2}$

Radius of curvature $= \frac{1}{K}$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{ds}{d\psi} = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u} \Big\} \text{ assingment}$$

limit exist or not
$x=0, y=0$
along x-axis
$x = y$
$x = y^2$

Total Derivative (chain Rule) :-

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$u = f(x,y)$
$x = \phi(t)$; $y = \psi(t)$
$u = f(\phi(t), \psi(t))$
$\frac{du}{dt}$

speial case

$z = f(x,y)$; $y = g(x) \rightarrow z = f(x, g(x))$

$$\frac{dz}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dx}$$

did book

Implicit differentiation :-

$f(x,y) = 0$

$0 = f_x + f_y \frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = -\frac{f_x}{f_y}} \quad f_x \rightarrow \frac{\partial f}{\partial x}, \quad f_y \rightarrow \frac{\partial f}{\partial y}$$

$y = f(x) \rightarrow$ Explicity
$\frac{dy}{dx}$

$f(x,y) = 0 \rightarrow$ implicit
$\frac{dy}{dx}$

$$J\left(\frac{u,v}{x,y}\right) = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

maxima, minima, saddle :-

critical points $\Rightarrow \frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$

H = Hessian Matrix

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

$D = f_{xx}\cdot f_{yy} - f_{xy}^2$ (discriminant)

let $(a,b) \rightarrow$ critical point

If $D>0$ and $f_{xx}(a,b)>0$ then (a,b) minima

$D>0$ and $f_{xx}(a,b)<0$ then maxima

$D<0$ then (a,b) saddle point

$f'(x) = 0$

$f''(x)<0$ ↓ maxima

$f''(x)>0$ ↓ minima

eg

ROLL NUMBER

Ravindra Bharathi School

DATE:

NAME

CLASS

SUBJECT

Formative Assessment / Summative Assessment / Annual Examinations

Invigilator

Marks

Signature of the Parent :

Remarks of the Examiner, if any

$g(x, y) = c$

$f(x, y) = x + y$

Lagrange's Method :-

$g(x, y) = x^2 + y^2 = 1$

$f_x = 1, \; f_y = 1$

$\left.\begin{array}{l} \dfrac{\partial h}{\partial x} - \lambda \dfrac{\partial g}{\partial x} = 0 \\ \dfrac{\partial h}{\partial y} - \lambda \dfrac{\partial g}{\partial y} = 0 \end{array}\right\}$ find (x, y)

$g_x = 2x, \; g_y = 2y$

$1 = \lambda(2x)$

$1 = \lambda(2y) \Rightarrow x = y$

$x^2 + y^2 = 1$

$\cancel{x = \pm\sqrt{2}}$

$x = \pm\sqrt{1/2}$

vector- calculus

Directional Derivative = $D_u f$

$\vec{u} = (u_1, u_2)$

$D_u f = \frac{\partial f}{\partial x} u_1 + \frac{\partial f}{\partial y} u_2$

Gradient

$D_u f = (\text{grad } f)\vec{u}$

$\text{grad } f = \nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j$

$|\text{grad } f| = \sqrt{f_x^2 + f_y^2}$

Divergence:

$F(x, y, z) = P\hat{i} + Q\hat{j} + R\hat{k} \Rightarrow \nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

Divergence $F = \nabla F$

$= \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

curl

(-) Rotating clock wise

(0) No Rotating

$\text{curl } F = \vec{\nabla} \times \vec{F}$

$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$

Laplacian operator:-

$\Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

consider $\vec{F}$

If $\vec{\nabla} \times \vec{F} = \vec{0} \Rightarrow \vec{F}$ Irrational(or) curl free

If $\vec{\nabla} \cdot \vec{F} = 0$ then $\vec{F}$ solenoidal divergence free

If $\vec{F} = \vec{\nabla}\phi$ $\vec{F}$ conservative and ϕ is scalar pot. of $\vec{F}$

$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$s \rightarrow$ Length of the curve

$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

Line Integral :-

work done $= \vec{F} \cdot d\vec{r}$

$\vec{F} = F_x i + F_y j + F_z k$

$d\vec{r} = dx\, i + dy\, j + dz\, k$

$\int \vec{F} \cdot d\vec{r} = \int F_x\, dx + F_y\, dy + F_z\, dz$

$\vec{F} = y^2 z^3 \hat{i} + 2xyz^3 \hat{j} + 3xy^2z^2 \hat{k}$

c_2 $x = t^2, \quad y = t, \quad z = t^2$

$(0,0,0) \to (1,1,1)$

$\int_{c_2} y^2 z^3\, dx + 2xyz^3\, dy + 3xy^2z^2\, dz$

$dx = 2t\,dt \qquad dy = dt \qquad dz = 2t\,dt$

Green's Theorem :

$$\oint_c f(x,y)\, dx + g(x,y)\, dy = \iint \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dx\, dy$$