Q find the value of T(2) for the recurrence relation $T(n) = 3T(n-1) + 12n$, given that $T(0) = 5$.

$T(n) = 3T(n-1) + 12n$

$T(0) = 5$

$T(1) = 3T(1-1) + 12 \times (1)$

$= 3T(0) + 12$

$= 3 \times 5 + 12 = 15 + 12 = 27$

$T(2) = 3T(2-1) + 12 \times (2)$

$= 3T(1) + \cancel{12 \times 2}\ 24$

$= 3 \times 27 + 24$

$\boxed{T(2) = 81 + 24 = 105}$

Q2 Given a recurrence relation, solve it using the substitution method.

~~Q~~ 9) $T(n) = T(n-1) + c$ — 1

$T(n-1) = T((n-1)-1) + c$
$= T(n-2) + c$ — 2

$T(n-2) = T(n-3) + c$ — 3

$\vdots$

k times

~~$T(n-k) = T(n-k) + c$~~

~~$n-k=0$~~ ~~$T(n-1) = T(n$~~ $\boxed{\cancel{n = k}}$

$T(T(n-3)+c)+c$

$= T$

$T(n) = (T(n-2) + c) + c$
$= T(n-2) + 2c$
$= T(n-3) + 3c$
$\vdots$
$T(n-k) + kc$

$n - k = 1$

$\boxed{k = n - 1}$

$= T(n - n + 1) + (n-1)c$
$= T(1) + (n-1)c$
$= 1 + n - c = n - c$

$\boxed{O(n)} \simeq$

g assumed for all question 1 ~~where it~~ if $n = 1$

$\cancel{Q2 \quad T(n) = 2T(n/2) + n \quad — (1)}$

$\cancel{T(n/2) = 2T(n/2^2) + n/2 \quad — (2)}$

$\cancel{T(n/2^2) = 2T(n/2^3) + n/2^2 \quad — (3)}$

$\cancel{= 2(2(2T(n/2^3) + n/2^2) + n/2}$

$\cancel{= 4T(n/2^3) + n/2^2) + n/2}$

Q2) $T(n) = 2T(n/2) + n \quad — (1)$

$T(n/2) = 2T(n/4) + n/2 \quad — (2)$

$T(n/4) = 2T(n/8) + n/4 \quad — (3)$

eq (2) put in (1)

$T(n) = 2(2T(n/4) + n/2) + n$

$= 2(2(2T(n/8) + n/4) + n/2) + n$

$= 2(4T(n/8) + 2n/4 + n/2$

$= 8T(n/8 + \cancel{4}.n/\cancel{4} + \cancel{2}n/\cancel{2} + n$

$= 2^3 T(n/2^3) + 3n$

$\boxed{2^k T(n/2^k) + kn} \rightarrow k^{th}$ time

$n/2^k = 1$

~~taking log both side~~

~~log~~ $n = 2^k$

taking log both side

$\log n = k \log_2 2$

$\boxed{\log n = k}$

$\cancel{2^{\log n} T(n/2^{\log n}) + n \log n}$

$\cancel{2^{\log n} T(1) + n \log n}$

$2^k T(1) + kn$

$n * 1 + n \log n$

$n + n \log$

$O(n \log n)$ Ans

Q1 $T(n) = 2T(n/2) + c$ — (1)

$T(n/2) = 2T(n/4) + c$ — (2)

$T(n/4) = 2T(n/8) + c$ — (3)

equ (2) put in eq (1)

$T(n) = 2(2T(n/4) + c) + c$

$T(n) = 2(2(2T(n/8) + c) + c) + c$

$= 2(4T(n/8) + 2c + c) + c$

$= 8T(n/8) + 6c + c$

$= 8T(n/8) + 7c$

$= 2^3 T(n/2^3) + (2^3 - 1)c$

$= 2^k T(n/2^k) + (2^k - 1)c$

$n/2^k = 1$

$n = 2^k$

taking log both side

$\log n = k \log_2 2$

$\boxed{\log n = k}$

$2^k * 1 + (2^k - 1)c$

$n + (n-1)c$

$n + nc - c$

$\simeq O(n)$ Ans

Q2 $T(n) = T(n/2) + c$ — (1)

$T(n/2) = T(n/4) + c$ — (2)

$T(n/4) = T(n/8) + c$ — (3)

$T(n/8) = T(n/16) + c$ — (4)

eq (2) put into eq (1)

$T(n) = (T(n/4) + c) + c$

$= T(n/4) + 2c$

$T(n) = T(n/8) + 3c$

$= T(n/2^3) + 3c$

$= T(n/2^k) + kc$

$n/2^k = 1$

$\boxed{n = 2^k}$

$\log n = k \log_2 2$

$\boxed{\log n = k}$

$= T(n/2^k) + kC$

$= \cancel{T(n/n)}$

$= 1 + \log n . C$

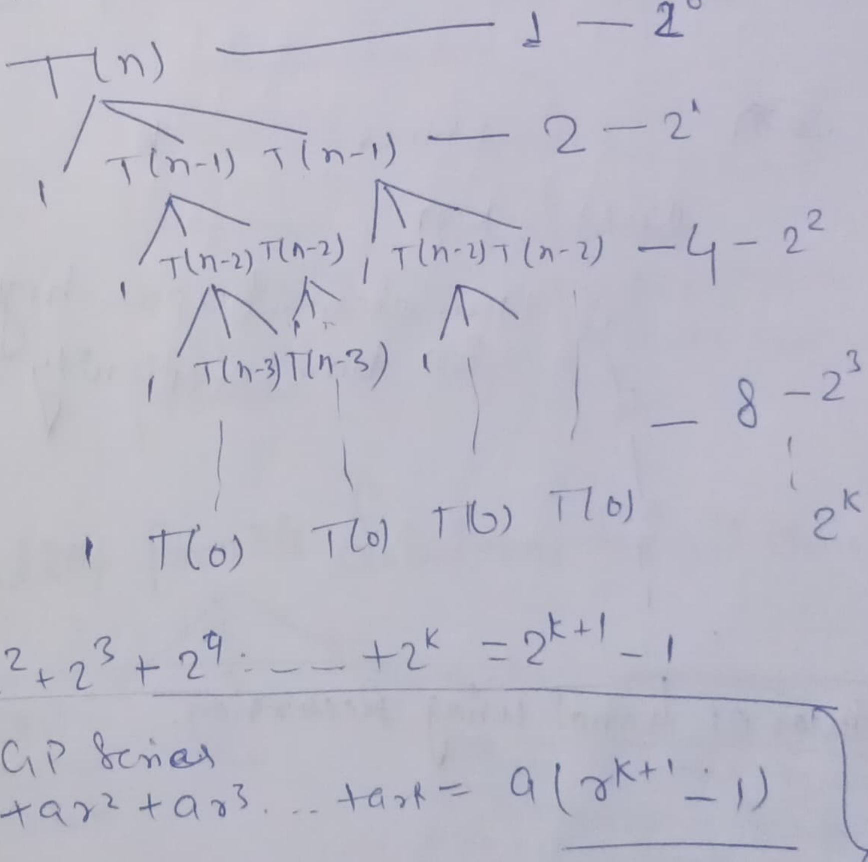
$= O(\log n)$ Ans

Q3 Given a recurrence relation, solve it using the recursive ~~tree~~ tree approach:

(a) $T(n) = 2T(n-1) + 1$

Sol

$$T(n) = \begin{cases} 1 & n = 0 \\ 2T(n-1) + 1 & n > 0 \end{cases}$$



$2^0 + 2^1 + 2^2 + 2^3 + 2^4 \ldots + 2^k = 2^{k+1} - 1$

Note :- It is a GP Series

$a + ar + ar^2 + ar^3 \ldots + ar^k = \dfrac{a(r^{k+1} - 1)}{r - 1}$

$a = 1$

$r = \dfrac{2^1}{2^0} = \dfrac{2}{1} = 2$

So $= \dfrac{2^{k+1} - 1}{2 - 1} = \dfrac{2^{k+1} - 1}{1}$

Assume $n - k = 0$

$\boxed{k = n}$

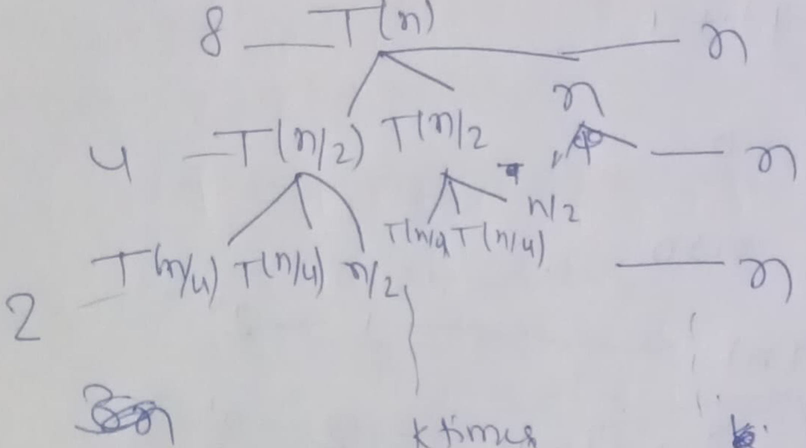
$= 2^{n+1} - 1 \approxeq O(2^n)$

Ans

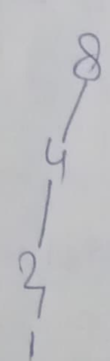
Q6 $T(n) = 2T(n/2) + n$

let

$$\begin{cases} 1 & n == 0 \\ 2T(n/2) + n & n > 0 \end{cases}$$



Cost = kn



It is depend upon hight of the tree so time complexity = logn Ans

Tower of hanoi using Recursion.