Assignment: Derivatives, Critical Points, and Gradient Descent

Problem Statement:

Question 1: First Order Derivative

Consider the function $f(x) = 3x^2 + 5x + 2$

a) Find the first-order derivative $f'(x)$.

b) Determine the critical points of $f(x)$ by setting $f'(x) = 0$ and solving for x .

c) Use the first-order derivative test to classify each critical point as a local minimum, local maximum, or neither.

Question 2: Second Order Derivative

Continuing from Question 1, let $f(x) = 3x^2 + 5x + 2$

a) Find the second-order derivative $f''(x)$.

b) Evaluate $f''(x)$ at the critical points found in Question 1.

c) Use the second-order derivative test to determine whether each critical point is a local minimum, local maximum, or neither.

Question 3: Chain Rule

Consider the functions $g(u) = u^3$ and $h(x) = 2x - 1$, where u is a function of x, i.e., $u = h(x)$.

a) Find $g'(u)$ and $h'(x)$

b) Apply the chain rule to find $\frac{dg}{dx}$.

Question 4: Gradient Descent

A machine learning model has a cost function $J(\theta) = \theta^2 - 4\theta + 5$, where θ is the model parameter.

a) Find the first-order partial derivative $\frac{\partial J}{\partial \theta}$

b) Apply the gradient descent update rule: $\theta_{new} = \theta_{old} - \alpha\frac{\partial J}{\partial \theta}$, where α is the learning rate (assume $\alpha = 0.1$)

c) Explain how the gradient descent process helps in finding the minimum of the cost function.

Guidelines:

- Show all your steps and calculations.
- Clearly state any assumptions made during the calculations.
- Use appropriate mathematical notation and symbols.
- Double-check your answers for correctness.

Step-by-Step Approach:

For Questions 1 and 2:
- Calculate the first-order derivative of the given function.
- Identify critical points by setting the first-order derivative equal to zero and solving for x.
- Apply the first-order derivative test to classify critical points as local minima, local maxima, or neither.
- Calculate the second-order derivative of the given function.
- Evaluate the second-order derivative at critical points.
- Use the second-order derivative test to classify critical points.

For Question 3:
- Calculate the $g'(u)$ and $h'(x)$
- Apply the chain rule to find $\frac{dg}{dx}$

For Question 4:
- Calculate the first-order partial derivative of the cost function.
- Apply the gradient descent update rule with a specified learning rate α
- Explain the intuition behind how gradient descent helps in minimizing the cost function.

Good luck with your assignment!