COMPUTATIONAL STATISTICS ASSIGNMENT

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Problem 1 : Principal Component Regression

Answers:

```
BEFORE PCA IN LAST 100 DATA POINTS TEST DATA
```

```
> before
                y
  [1,]  -29.58514
  [2,]  -74.98697
  [3,]  -57.90908
  [4,]  -49.40902
  [5,]  -63.06587
  [6,]  -95.96224
  [7,]  -87.91644
  [8,]  -48.82905
  [9,]  -63.29259
 [10,]  -62.49860
 [11,] -101.43948
 [12,]  -55.52751
 [13,]  -73.71066
 [14,]  -42.06903
 [15,]  -88.75302
 [16,]  -62.42009
 [17,]  -81.00464
 [18,]  -67.70099
 [19,]  -83.79788
 [20,]  -68.44751
 [21,]  -56.38498
 [22,]  -91.29007
 [23,]  -58.76799
 [24,]  -38.81030
 [25,]  -66.56304
 [26,]  -68.02344
 [27,]  -50.59879
 [28,]  -52.78113
 [29,]  -66.29301
 [30,]  -67.56520
 [31,]  -49.61230
 [32,]  -63.07641
 [33,]  -57.15544
 [34,]  -50.69719
```

```
 [35,]  -55.26011
 [36,]  -55.31588
 [37,]  -55.00962
 [38,]  -72.53351
 [39,]  -63.51185
 [40,]  -63.34615
 [41,]  -55.42431
 [42,]  -48.78935
 [43,]  -72.11494
 [44,]  -81.44972
 [45,]  -62.19493
 [46,]  -82.56678
 [47,]  -55.54706
 [48,]  -87.42165
 [49,]  -70.69117
 [50,]  -73.08905
 [51,]  -68.53888
 [52,]  -61.83007
 [53,]  -66.73719
 [54,]  -82.77291
 [55,]  -49.50351
 [56,]  -76.90581
 [57,]  -57.13495
 [58,]  -55.15680
 [59,]  -63.75467
 [60,]  -49.23410
 [61,]  -84.78451
 [62,]  -66.83434
 [63,]  -56.34492
 [64,]  -41.03712
 [65,]  -77.32193
 [66,]  -71.11376
 [67,]  -79.77172
 [68,]  -52.26303
 [69,]  -58.03937
 [70,]  -70.91842
 [71,]  -49.25612
 [72,]  -55.68030
 [73,]  -60.08965
 [74,]  -61.05283
 [75,]  -73.36631
 [76,]  -92.63440
 [77,]  -45.42577
 [78,]  -98.09780
 [79,]  -77.48607
 [80,]  -48.44011
 [81,]  -69.59899
 [82,]  -61.81468
```

```
 [83,]  -81.65876
 [84,]  -48.46088
 [85,]  -33.27077
 [86,]  -85.34737
 [87,]  -59.02557
 [88,]  -56.16094
 [89,]  -64.38483
 [90,]  -67.71166
 [91,]  -57.97806
 [92,]  -67.65507
 [93,]  -83.06038
 [94,]  -90.20698
 [95,]  -67.34685
 [96,]  -31.55206
 [97,]  -82.79161
 [98,]  -81.28438
 [99,]  -61.83914
[100,]  -65.05404
```

AFTER PCA IN LAST 100 DATA POINTS TEST DATA

```
> after
                y
  [1,]  -30.34441
  [2,]  -74.50488
  [3,]  -58.02604
  [4,]  -49.71689
  [5,]  -63.21423
  [6,]  -95.41354
  [7,]  -86.81427
  [8,]  -49.11836
  [9,]  -62.78480
 [10,]  -62.49982
 [11,] -100.59212
 [12,]  -55.69888
 [13,]  -73.95451
 [14,]  -43.07117
 [15,]  -87.82233
 [16,]  -62.58515
 [17,]  -80.33632
 [18,]  -66.84678
 [19,]  -83.20964
 [20,]  -68.54223
 [21,]  -57.15720
 [22,]  -90.84295
 [23,]  -58.73239
 [24,]  -39.92120
 [25,]  -66.99185
```

```
 [26,]  -67.28959
 [27,]  -50.34699
 [28,]  -52.79308
 [29,]  -65.30810
 [30,]  -67.30659
 [31,]  -49.10263
 [32,]  -62.91084
 [33,]  -57.68099
 [34,]  -51.08544
 [35,]  -55.77871
 [36,]  -55.48919
 [37,]  -55.13877
 [38,]  -71.98648
 [39,]  -63.41935
 [40,]  -63.28072
 [41,]  -55.12821
 [42,]  -49.12467
 [43,]  -72.22892
 [44,]  -80.87356
 [45,]  -62.15756
 [46,]  -82.34566
 [47,]  -56.17953
 [48,]  -86.86995
 [49,]  -70.36362
 [50,]  -72.61110
 [51,]  -68.81426
 [52,]  -62.00572
 [53,]  -67.20486
 [54,]  -82.41622
 [55,]  -49.83151
 [56,]  -76.72601
 [57,]  -57.19517
 [58,]  -55.87102
 [59,]  -63.87187
 [60,]  -49.81014
 [61,]  -84.09124
 [62,]  -67.09987
 [63,]  -56.52025
 [64,]  -41.74589
 [65,]  -76.26949
 [66,]  -70.48629
 [67,]  -79.44955
 [68,]  -52.35052
 [69,]  -58.58133
 [70,]  -71.10631
 [71,]  -49.61473
 [72,]  -55.39126
 [73,]  -60.16111
```

```
 [74,]  -61.16418
 [75,]  -73.14781
 [76,]  -91.40847
 [77,]  -46.06258
 [78,]  -96.01739
 [79,]  -77.33386
 [80,]  -49.55355
 [81,]  -69.60571
 [82,]  -62.26433
 [83,]  -81.98924
 [84,]  -49.05402
 [85,]  -34.25242
 [86,]  -84.79683
 [87,]  -59.09573
 [88,]  -56.25972
 [89,]  -63.92828
 [90,]  -67.76024
 [91,]  -58.53899
 [92,]  -67.58493
 [93,]  -82.57345
 [94,]  -89.60528
 [95,]  -67.73741
 [96,]  -32.33692
 [97,]  -82.44122
 [98,]  -81.24603
 [99,]  -62.00686
[100,]  -64.98664
```

Graphs: None

Code:

```
library(MASS)
x0 <- matrix(1 , nrow=100, ncol=1 )
x1 <- read.csv("x1train.csv")
x2 <- read.csv("x2train.csv")
x3 <- read.csv("x3train.csv")
x4 <- read.csv("x4train.csv")
y <-  read.csv("ytrain.csv")
xx0 <- matrix(1 , nrow=100, ncol=1 )
xx1 <- read.csv("x1test.csv")
xx2 <- read.csv("x2test.csv")
xx3 <- read.csv("x3test.csv")
xx4 <- read.csv("x4test.csv")
```

```
d <- data.frame( x0, x1 , x2 , x3 , x4 )
XX <- data.frame( xx0, xx1 , xx2 , xx3 , xx4 )
XX <- data.matrix(XX)

X=data.matrix(d, rownames.force = NA)
xx <- X
Y=data.matrix(y)

estimator <- ((ginv(t(xx)%*%xx))%*%t(xx))%*%Y

matrix(data=d)
X=data.matrix(d, rownames.force = NA)
dim(Y)
e=(ginv((t(X) %*% X)) %*% t(X) %*% Y)
before<-XX%*%e
print(e)

q<-svd(X)
#We will use the first three parameters as they are explaining 98% of
the variance in the model

rr<-q$v[,c(1,2,3)]
w<-X%*%rr
estimator <- ((ginv(t(w)%*%w))%*%t(w))%*%Y
after<-XX%*%rr%*%estimator
```

Problem 2 : Recursive Least Squares Regression

Answers:

Estimator after each iteration (without forgetting factor)

```
              Y
[1,] -12.5988988
[2,]   0.3496447
[3,]  -0.7252545
              Y
[1,] -12.6457218
[2,]   0.3512952
[3,]  -0.7286488
              Y
```

```
[1,] -12.6610732
[2,]   0.3507754
[3,]  -0.7293641
               Y
[1,] -12.6707892
[2,]   0.3505018
[3,]  -0.7294149
               Y
[1,] -12.6789856
[2,]   0.3513503
[3,]  -0.7294277
               Y
[1,] -12.6610796
[2,]   0.3514999
[3,]  -0.7307829
               Y
[1,] -12.6121340
[2,]   0.3489205
[3,]  -0.7354518
               Y
[1,] -12.6196520
[2,]   0.3480754
[3,]  -0.7357627
               Y
[1,] -12.6227093
[2,]   0.3482746
[3,]  -0.7359187
               Y
[1,] -12.6497223
[2,]   0.3510586
[3,]  -0.7361123
               Y
[1,] -12.6794329
[2,]   0.3519030
[3,]  -0.7364692
               Y
[1,] -12.6893365
[2,]   0.3533533
[3,]  -0.7357555
               Y
[1,] -12.6958837
[2,]   0.3530101
[3,]  -0.7351384
               Y
[1,] -12.6962396
[2,]   0.3530377
[3,]  -0.7351074
               Y
```

```
[1,] -12.7123377
[2,]   0.3538064
[3,]  -0.7339430
               Y
[1,] -12.6765379
[2,]   0.3574456
[3,]  -0.7323162
               Y
[1,] -12.6473669
[2,]   0.3549073
[3,]  -0.7317830
               Y
[1,] -12.6746031
[2,]   0.3576160
[3,]  -0.7304408
               Y
[1,] -12.6692821
[2,]   0.3574337
[3,]  -0.7299843
               Y
[1,] -12.6459771
[2,]   0.3569413
[3,]  -0.7315367
               Y
[1,] -12.6799366
[2,]   0.3567841
[3,]  -0.7274371
               Y
[1,] -12.6808347
[2,]   0.3567918
[3,]  -0.7274118
               Y
[1,] -12.6873175
[2,]   0.3568047
[3,]  -0.7279847
               Y
[1,] -12.7025625
[2,]   0.3583828
[3,]  -0.7259442
               Y
[1,] -12.7285107
[2,]   0.3579901
[3,]  -0.7279689
               Y
[1,] -12.7125439
[2,]   0.3574619
[3,]  -0.7266058
               Y
```

```
[1,] -12.7059695
[2,]   0.3579124
[3,]  -0.7267709
               Y
[1,] -12.7480086
[2,]   0.3585919
[3,]  -0.7284715
               Y
[1,] -12.7248720
[2,]   0.3584804
[3,]  -0.7310331
               Y
[1,] -12.7733986
[2,]   0.3545170
[3,]  -0.7344681
               Y
[1,] -12.8007530
[2,]   0.3565940
[3,]  -0.7363903
               Y
[1,] -12.8256380
[2,]   0.3563124
[3,]  -0.7370570
               Y
[1,] -12.8095592
[2,]   0.3569197
[3,]  -0.7361119
               Y
[1,] -12.7903902
[2,]   0.3571318
[3,]  -0.7369658
               Y
[1,] -12.8015307
[2,]   0.3571366
[3,]  -0.7365108
               Y
[1,] -12.7875099
[2,]   0.3574493
[3,]  -0.7355837
               Y
[1,] -12.8459906
[2,]   0.3609544
[3,]  -0.7406004
               Y
[1,] -12.8547818
[2,]   0.3600135
[3,]  -0.7413575
               Y
```

```
[1,] -12.8493553
[2,]   0.3597833
[3,]  -0.7418741
              Y
[1,] -12.8534587
[2,]   0.3594937
[3,]  -0.7421804
              Y
[1,] -12.8509742
[2,]   0.3593926
[3,]  -0.7419673
              Y
[1,] -12.8275554
[2,]   0.3576118
[3,]  -0.7421610
              Y
[1,] -12.8251863
[2,]   0.3578377
[3,]  -0.7421678
              Y
[1,] -12.8240257
[2,]   0.3578801
[3,]  -0.7420635
              Y
[1,] -12.8403948
[2,]   0.3581679
[3,]  -0.7412919
              Y
[1,] -12.8680845
[2,]   0.3548873
[3,]  -0.7409782
              Y
[1,] -12.9038737
[2,]   0.3558674
[3,]  -0.7429782
              Y
[1,] -12.8940425
[2,]   0.3553989
[3,]  -0.7441960
              Y
[1,] -12.8809675
[2,]   0.3555151
[3,]  -0.7453169
              Y
[1,] -12.8763574
[2,]   0.3556935
[3,]  -0.7452006
```

Estimator after each iteration forgetting factor = 0.99)

```
               Y
[1,] -12.6233565
[2,]   0.3505069
[3,]  -0.7270275
               Y
[1,] -12.6315113
[2,]   0.3502307
[3,]  -0.7274075
               Y
[1,] -12.6366384
[2,]   0.3500863
[3,]  -0.7274343
               Y
[1,] -12.6414011
[2,]   0.3505794
[3,]  -0.7274417
               Y
[1,] -12.6322456
[2,]   0.3506559
[3,]  -0.7281347
               Y
[1,] -12.6070178
[2,]   0.3493264
[3,]  -0.7305411
               Y
[1,] -12.6111171
[2,]   0.3488656
[3,]  -0.7307106
               Y
[1,] -12.6134405
[2,]   0.3490170
[3,]  -0.7308292
               Y
[1,] -12.6275642
[2,]   0.3504726
[3,]  -0.7309304
               Y
[1,] -12.6429473
[2,]   0.3509098
[3,]  -0.7311152
               Y
[1,] -12.6481547
[2,]   0.3516723
[3,]  -0.7307399
               Y
```

```
[1,] -12.6509767
[2,]   0.3515244
[3,]  -0.7304740
               Y
[1,] -12.6511007
[2,]   0.3515341
[3,]  -0.7304631
               Y
[1,] -12.6592396
[2,]   0.3519227
[3,]  -0.7298744
               Y
[1,] -12.6408618
[2,]   0.3537908
[3,]  -0.7290394
               Y
[1,] -12.6268895
[2,]   0.3525751
[3,]  -0.7287839
               Y
[1,] -12.6410733
[2,]   0.3539857
[3,]  -0.7280850
               Y
[1,] -12.6396639
[2,]   0.3539374
[3,]  -0.7279641
               Y
[1,] -12.6278730
[2,]   0.3536882
[3,]  -0.7287495
               Y
[1,] -12.6449685
[2,]   0.3536091
[3,]  -0.7266857
               Y
[1,] -12.6456326
[2,]   0.3536148
[3,]  -0.7266670
               Y
[1,] -12.6496802
[2,]   0.3536229
[3,]  -0.7270247
               Y
[1,] -12.6577672
[2,]   0.3544600
[3,]  -0.7259423
               Y
```

```
[1,] -12.6715259
[2,]   0.3542518
[3,]  -0.7270158
              Y
[1,] -12.6643666
[2,]   0.3540149
[3,]  -0.7264046
              Y
[1,] -12.6609841
[2,]   0.3542467
[3,]  -0.7264896
              Y
[1,] -12.6828339
[2,]   0.3545999
[3,]  -0.7273735
              Y
[1,] -12.6711724
[2,]   0.3545437
[3,]  -0.7286646
              Y
[1,] -12.6963920
[2,]   0.3524839
[3,]  -0.7304498
              Y
[1,] -12.7119092
[2,]   0.3536621
[3,]  -0.7315402
              Y
[1,] -12.7252096
[2,]   0.3535116
[3,]  -0.7318965
              Y
[1,] -12.7180719
[2,]   0.3537812
[3,]  -0.7314769
             Y
[1,] -12.708507
[2,]   0.353887
[3,]  -0.731903
              Y
[1,] -12.7142910
[2,]   0.3538895
[3,]  -0.7316668
              Y
[1,] -12.7082127
[2,]   0.3540251
[3,]  -0.7312649
              Y
```

```
[1,] -12.7399527
[2,]   0.3559275
[3,]  -0.7339877
               Y
[1,] -12.7453494
[2,]   0.3553498
[3,]  -0.7344524
               Y
[1,] -12.7425993
[2,]   0.3552332
[3,]  -0.7347142
               Y
[1,] -12.7457802
[2,]   0.3550087
[3,]  -0.7349516
               Y
[1,] -12.7465162
[2,]   0.3550386
[3,]  -0.7350147
               Y
[1,] -12.7354842
[2,]   0.3541997
[3,]  -0.7351060
               Y
[1,] -12.7341484
[2,]   0.3543271
[3,]  -0.7351098
               Y
[1,] -12.7349450
[2,]   0.3542980
[3,]  -0.7351814
               Y
[1,] -12.7433509
[2,]   0.3544457
[3,]  -0.7347852
               Y
[1,] -12.7572668
[2,]   0.3527970
[3,]  -0.7346275
               Y
[1,] -12.7766310
[2,]   0.3533273
[3,]  -0.7357096
               Y
[1,] -12.7714512
[2,]   0.3530805
[3,]  -0.7363513
               Y
```

```
[1,] -12.7646153
[2,]   0.3531412
[3,]  -0.7369373
               Y
[1,] -12.7629512
[2,]   0.3532056
[3,]  -0.7368953
```

Estimator after each iteration forgetting factor = 0.95)

```
               Y
[1,] -12.8923514
[2,]   0.3568090
[3,]  -0.7448459
               Y
[1,] -12.8945319
[2,]   0.3567157
[3,]  -0.7449409
               Y
[1,] -12.8964285
[2,]   0.3566478
[3,]  -0.7449415
               Y
[1,] -12.8964563
[2,]   0.3566513
[3,]  -0.7449414
               Y
[1,] -12.8913611
[2,]   0.3567294
[3,]  -0.7453140
               Y
[1,] -12.8770450
[2,]   0.3560858
[3,]  -0.7466838
               Y
[1,] -12.8787171
[2,]   0.3558936
[3,]  -0.7467396
               Y
[1,] -12.8766735
[2,]   0.3557310
[3,]  -0.7466266
               Y
[1,] -12.8812121
[2,]   0.3563007
[3,]  -0.7466384
               Y
```

```
[1,] -12.8878322
[2,]   0.3564899
[3,]  -0.7466872
               Y
[1,] -12.8897409
[2,]   0.3568038
[3,]  -0.7465307
               Y
[1,] -12.8922619
[2,]   0.3566679
[3,]  -0.7463206
               Y
[1,] -12.8921215
[2,]   0.3566572
[3,]  -0.7463331
               Y
[1,] -12.8968382
[2,]   0.3568679
[3,]  -0.7459837
               Y
[1,] -12.8851684
[2,]   0.3581601
[3,]  -0.7455287
               Y
[1,] -12.8764498
[2,]   0.3572342
[3,]  -0.7454264
               Y
[1,] -12.8837869
[2,]   0.3580339
[3,]  -0.7450034
               Y
[1,] -12.8799077
[2,]   0.3578600
[3,]  -0.7446462
               Y
[1,] -12.8724251
[2,]   0.3577214
[3,]  -0.7451464
               Y
[1,] -12.8847369
[2,]   0.3576189
[3,]  -0.7438332
               Y
[1,] -12.8847394
[2,]   0.3576189
[3,]  -0.7438332
               Y
```

```
[1,] -12.8836153
[2,]   0.3576154
[3,]  -0.7437273
               Y
[1,] -12.8891444
[2,]   0.3581497
[3,]  -0.7430296
               Y
[1,] -12.8939136
[2,]   0.3580626
[3,]  -0.7434151
               Y
[1,] -12.8872803
[2,]   0.3577918
[3,]  -0.7428020
               Y
[1,] -12.8851665
[2,]   0.3579411
[3,]  -0.7428615
               Y
[1,] -12.8961987
[2,]   0.3581379
[3,]  -0.7432831
               Y
[1,] -12.8886048
[2,]   0.3581312
[3,]  -0.7440693
               Y
[1,] -12.9018487
[2,]   0.3569386
[3,]  -0.7449877
               Y
[1,] -12.9081052
[2,]   0.3575120
[3,]  -0.7454626
               Y
[1,] -12.9152411
[2,]   0.3574181
[3,]  -0.7456308
               Y
[1,] -12.9094760
[2,]   0.3576589
[3,]  -0.7452996
               Y
[1,] -12.9030945
[2,]   0.3577465
[3,]  -0.7455953
               Y
```

```
[1,] -12.9067291
[2,]   0.3577388
[3,]  -0.7454384
               Y
[1,] -12.9014350
[2,]   0.3578715
[3,]  -0.7450899
               Y
[1,] -12.9174737
[2,]   0.3589936
[3,]  -0.7465787
               Y
[1,] -12.9193649
[2,]   0.3587804
[3,]  -0.7467379
               Y
[1,] -12.9176308
[2,]   0.3587120
[3,]  -0.7469027
               Y
[1,] -12.9182926
[2,]   0.3586616
[3,]  -0.7469517
               Y
[1,] -12.9169370
[2,]   0.3586017
[3,]  -0.7468274
               Y
[1,] -12.9092512
[2,]   0.3579864
[3,]  -0.7469116
               Y
[1,] -12.9082359
[2,]   0.3580844
[3,]  -0.7469163
               Y
[1,] -12.9071383
[2,]   0.3581290
[3,]  -0.7468159
               Y
[1,] -12.9125916
[2,]   0.3582126
[3,]  -0.7465493
               Y
[1,] -12.9227028
[2,]   0.3570306
[3,]  -0.7464266
               Y
```

```
[1,] -12.9337580
[2,]   0.3573386
[3,]  -0.7470591
              Y
[1,] -12.9304556
[2,]   0.3571929
[3,]  -0.7474641
              Y
[1,] -12.9258544
[2,]   0.3572478
[3,]  -0.7478523
              Y
[1,] -12.9241199
[2,]   0.3573207
[3,]  -0.7478121
```

Estimator after each iteration forgetting factor = 0.2)

```
              Y
[1,] -12.7907345
[2,]   0.3545482
[3,]  -0.7375165
              Y
[1,] -12.7959814
[2,]   0.3543287
[3,]  -0.7377471
              Y
[1,] -12.7999064
[2,]   0.3541914
[3,]  -0.7377505
              Y
[1,] -12.8013657
[2,]   0.3543699
[3,]  -0.7377456
              Y
[1,] -12.7921426
[2,]   0.3545039
[3,]  -0.7384230
              Y
[1,] -12.7669062
[2,]   0.3533457
[3,]  -0.7408394
              Y
[1,] -12.7703346
[2,]   0.3529521
[3,]  -0.7409570
```

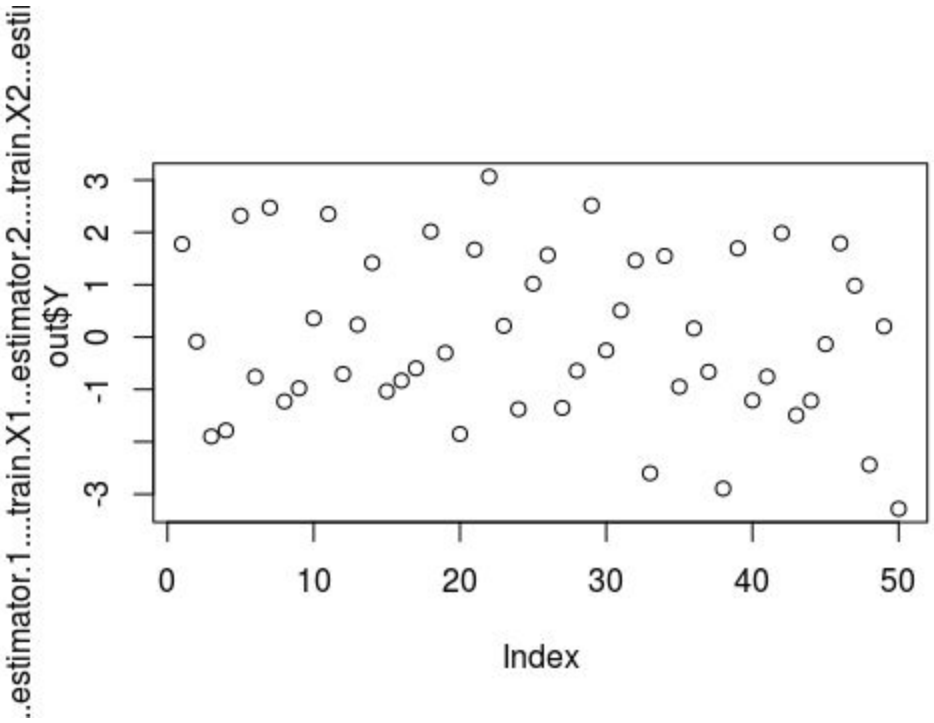
```
                Y
[1,] -12.7688314
[2,]   0.3528354
[3,]  -0.7408748
                Y
[1,] -12.7784950
[2,]   0.3540202
[3,]  -0.7409047
                Y
[1,] -12.7913071
[2,]   0.3543866
[3,]  -0.7410059
                Y
[1,] -12.7952541
[2,]   0.3550267
[3,]  -0.7406867
                Y
[1,] -12.7992356
[2,]   0.3548130
[3,]  -0.7403505
                Y
[1,] -12.7991390
[2,]   0.3548057
[3,]  -0.7403591
                Y
[1,] -12.8075250
[2,]   0.3551836
[3,]  -0.7397392
                Y
[1,] -12.7878665
[2,]   0.3573423
[3,]  -0.7389572
                Y
[1,] -12.7733843
[2,]   0.3558404
[3,]  -0.7387761
                Y
[1,] -12.7867386
[2,]   0.3572806
[3,]  -0.7380189
                Y
[1,] -12.7820267
[2,]   0.3570762
[3,]  -0.7375891
                Y
[1,] -12.7690096
[2,]   0.3568307
[3,]  -0.7384592
```

```
                Y
[1,] -12.7898732
[2,]   0.3566663
[3,]  -0.7362038
                Y
[1,] -12.7901034
[2,]   0.3566679
[3,]  -0.7361963
                Y
[1,] -12.7904938
[2,]   0.3566691
[3,]  -0.7362328
                Y
[1,] -12.7997996
[2,]   0.3575758
[3,]  -0.7350508
                Y
[1,] -12.8102750
[2,]   0.3573886
[3,]  -0.7358938
                Y
[1,] -12.8005845
[2,]   0.3570033
[3,]  -0.7350075
                Y
[1,] -12.7968226
[2,]   0.3572680
[3,]  -0.7351122
                Y
[1,] -12.8173896
[2,]   0.3576307
[3,]  -0.7359039
                Y
[1,] -12.8038632
[2,]   0.3576121
[3,]  -0.7373152
                Y
[1,] -12.8283688
[2,]   0.3554305
[3,]  -0.7390202
                Y
[1,] -12.8408854
[2,]   0.3565500
[3,]  -0.7399599
                Y
[1,] -12.8540011
[2,]   0.3563805
[3,]  -0.7402746
```

```
                Y
[1,] -12.8448203
[2,]   0.3567593
[3,]  -0.7397454
                Y
[1,] -12.8337416
[2,]   0.3569075
[3,]  -0.7402564
                Y
[1,] -12.8401234
[2,]   0.3568962
[3,]  -0.7399826
                Y
[1,] -12.8319160
[2,]   0.3570990
[3,]  -0.7394419
                Y
[1,] -12.8614246
[2,]   0.3591217
[3,]  -0.7421513
                Y
[1,] -12.8657832
[2,]   0.3586334
[3,]  -0.7425194
                Y
[1,] -12.8626628
[2,]   0.3585091
[3,]  -0.7428160
                Y
[1,] -12.8646718
[2,]   0.3583576
[3,]  -0.7429650
                Y
[1,] -12.8632460
[2,]   0.3582953
[3,]  -0.7428355
                Y
[1,] -12.8501855
[2,]   0.3572567
[3,]  -0.7429740
                Y
[1,] -12.8485778
[2,]   0.3574117
[3,]  -0.7429810
                Y
[1,] -12.8476307
[2,]   0.3574497
[3,]  -0.7428945
```

```
                Y
[1,] -12.8571766
[2,]   0.3575989
[3,]  -0.7424299
                Y
[1,] -12.8744097
[2,]   0.3555807
[3,]  -0.7422227
                Y
[1,] -12.8943674
[2,]   0.3561354
[3,]  -0.7433610
                Y
[1,] -12.8884805
[2,]   0.3558729
[3,]  -0.7440840
                Y
[1,] -12.8804315
[2,]   0.3559655
[3,]  -0.7447646
                Y
[1,] -12.8777243
[2,]   0.3560782
[3,]  -0.7447010
```

Graphs:



Training Target values

Code:

```
library(MASS)

train <- read.csv("Problem1_Input_Training.csv")
out <- read.csv("Problem1_Output_Training.csv")

x0 <- matrix(1 , nrow=50, ncol=1 )

d <- data.frame(x0,train$X1,train$X2)
df <- d
y <- data.frame(out)

xx <- as.matrix(d)
y <- as.matrix(out)

estimator <- ((ginv(t(xx)%*%xx))%*%t(xx))%*%(y)
```

```
dd <-
data.frame(x0*estimator[1]+train$X1*estimator[2]+train$X2*estimator[3
])
write.csv(dd,"abc.csv")

plot(dd$x0...estimator.1....train.X1...estimator.2....train.X2...esti
mator.3.-out$Y)


forget_factor <- 0.2   #To be added

real_time <- read.csv("Problem1_Input_Test.csv")
real_output <- read.csv("Problem1_Output_Test.csv")
dd <- data.frame(x0,real_time$X1,real_time$X2)
dd <- as.matrix(dd)

Mn <- ginv(t(xx)%*%(xx))
i=1
xx <- as.matrix(df)
Mn <- ginv(t(xx)%*%(xx))
d <- data.frame(x0,real_time$X1,real_time$X2)
mx <- as.matrix(d[i,])
estimator <- estimator
+((Mn-(Mn%*%t(mx)%*%mx%*%Mn)/(1+mx%*%Mn%*%t(mx))[1])%*%t(mx))*(real_o
utput[i,1]-mx%*%estimator)[1]
df <- rbind(df,c(1,real_time$X1[i],real_time$X2[i]))
print(estimator)
xn <- rbind(xx[1:50,],dd[1,])

for(i in 2:50)
{
  xx <- as.matrix(df)
  Mn <- ginv(forget_factor*t(xn)%*%(xn)+t(xx)%*%xx)
  d <- data.frame(x0,real_time$X1,real_time$X2)
  mx <- as.matrix(d[i,])
  estimator <- estimator
+((Mn-(Mn%*%t(mx)%*%mx%*%Mn)/(1+mx%*%Mn%*%t(mx))[1])%*%t(mx))*(real_o
utput[i,1]-mx%*%estimator)[1]
  df <- rbind(df,c(1,real_time$X1[i],real_time$X2[i]))
  xn <- rbind(xn,dd[i,])
  print(estimator)
}
```

Problem 3 : Recursive Least Squares Regression

Answers:

Estimator after each iteration (without forgetting factor)

```
              Y
[1,] -11.9651919
[2,]  -0.0168799
[3,]  -0.7623284
              Y
[1,] -11.9853679
[2,]  -0.0161687
[3,]  -0.7637910
              Y
[1,] -12.01643309
[2,]  -0.01722063
[3,]  -0.76523832
              Y
[1,] -12.03158175
[2,]  -0.01764726
[3,]  -0.76531766
              Y
[1,] -12.07341752
[2,]  -0.01331602
[3,]  -0.76538288
              Y
[1,] -12.06085026
[2,]  -0.01321105
[3,]  -0.76633401
              Y
[1,] -12.0439477
[2,]  -0.0141018
[3,]  -0.7679463
              Y
[1,] -12.01085711
[2,]  -0.01038198
[3,]  -0.76657807
              Y
[1,] -12.027281256
[2,]  -0.009311902
[3,]  -0.767416095
              Y
[1,] -11.98931347
```

```
[2,]  -0.01322493
[3,]  -0.76714395
                Y
[1,] -11.97517121
[2,]  -0.01362686
[3,]  -0.76697406
                Y
[1,] -12.00844377
[2,]  -0.00875439
[3,]  -0.76457624
                Y
[1,] -11.980084501
[2,]  -0.007267774
[3,]  -0.767249170
                Y
[1,] -11.992210568
[2,]  -0.006324388
[3,]  -0.766190866
                Y
[1,] -11.989957403
[2,]  -0.006431978
[3,]  -0.766353838
                Y
[1,] -12.03030527
[2,]  -0.01053341
[3,]  -0.76818726
                Y
[1,] -12.01541715
[2,]  -0.01182888
[3,]  -0.76791509
                Y
[1,] -12.01281516
[2,]  -0.01208766
[3,]  -0.76804331
                Y
[1,] -12.03314827
[2,]  -0.01139097
[3,]  -0.76978764
                Y
[1,] -12.07625358
[2,]  -0.01048018
[3,]  -0.76691627
                Y
[1,] -12.05862026
[2,]  -0.01039854
[3,]  -0.76904497
                Y
[1,] -12.06716287
```

```
[2,]  -0.01032503
[3,]  -0.76880463
                Y
[1,] -12.05156191
[2,]  -0.01035616
[3,]  -0.76742614
                 Y
[1,] -12.057812984
[2,]  -0.009709085
[3,]  -0.766589457
                Y
[1,] -12.11855999
[2,]  -0.01062846
[3,]  -0.77132941
                 Y
[1,] -12.138690838
[2,]  -0.009962454
[3,]  -0.773047934
                 Y
[1,] -12.131169593
[2,]  -0.009447043
[3,]  -0.773236830
                 Y
[1,] -12.141804702
[2,]  -0.009275144
[3,]  -0.773667050
                Y
[1,] -12.11514881
[2,]  -0.00940361
[3,]  -0.77661824
                Y
[1,] -12.15000301
[2,]  -0.01225028
[3,]  -0.77908543
                Y
[1,] -12.10111922
[2,]  -0.01596186
[3,]  -0.77565035
                Y
[1,] -12.09089540
[2,]  -0.01584617
[3,]  -0.77537646
                Y
[1,] -12.12678753
[2,]  -0.01720183
[3,]  -0.77748612
                Y
[1,] -12.15193930
```

```
[2,]  -0.01748017
[3,]  -0.77636575
                Y
[1,] -12.18798523
[2,]  -0.01746471
[3,]  -0.77489360
                Y
[1,] -12.18448798
[2,]  -0.01738672
[3,]  -0.77466235
                Y
[1,] -12.20569943
[2,]  -0.01611537
[3,]  -0.77648198
                Y
[1,] -12.21346137
[2,]  -0.01694613
[3,]  -0.77715035
                Y
[1,] -12.20909382
[2,]  -0.01713136
[3,]  -0.77756616
                Y
[1,] -12.23575667
[2,]  -0.01901343
[3,]  -0.77955635
                Y
[1,] -12.22987842
[2,]  -0.01925251
[3,]  -0.77905226
                Y
[1,] -12.22494392
[2,]  -0.01962773
[3,]  -0.77909308
                Y
[1,] -12.20035397
[2,]  -0.01728292
[3,]  -0.77916317
                Y
[1,] -12.23181529
[2,]  -0.01843262
[3,]  -0.78199051
                Y
[1,] -12.19367389
[2,]  -0.01910312
[3,]  -0.78378845
               Y
[1,] -12.1573573
```

```
[2,]  -0.0148004
[3,]  -0.7841998
                Y
[1,] -12.13797135
[2,]  -0.01533131
[3,]  -0.78311652
                Y
[1,] -12.15113702
[2,]  -0.01470391
[3,]  -0.78148560
                Y
[1,] -12.16393432
[2,]  -0.01481759
[3,]  -0.78038848
                Y
[1,] -12.15603200
[2,]  -0.01451178
[3,]  -0.78018903
```

Estimator after each iteration forgetting factor = 0.99)

```
              Y
[1,] -12.1386617
[2,]  -0.0146673
[3,]  -0.7783137
                Y
[1,] -12.14743366
[2,]  -0.01501543
[3,]  -0.77870668
                Y
[1,] -12.15224734
[2,]  -0.01517498
[3,]  -0.77871664
                Y
[1,] -12.16420808
[2,]  -0.01378084
[3,]  -0.77869294
                Y
[1,] -12.16021078
[2,]  -0.01372972
[3,]  -0.77898927
                Y
[1,] -12.15465222
[2,]  -0.01399614
[3,]  -0.77952189
                Y
[1,] -12.14121958
```

```
[2,]  -0.01245966
[3,]  -0.77903371
                Y
[1,] -12.14466372
[2,]  -0.01220583
[3,]  -0.77921784
                Y
[1,] -12.13153288
[2,]  -0.01373546
[3,]  -0.77916240
                Y
[1,] -12.12570373
[2,]  -0.01390215
[3,]  -0.77910963
                Y
[1,] -12.13740970
[2,]  -0.01205953
[3,]  -0.77819267
                Y
[1,] -12.12641629
[2,]  -0.01147442
[3,]  -0.77914899
                Y
[1,] -12.13133750
[2,]  -0.01109719
[3,]  -0.77871411
                Y
[1,] -12.13078045
[2,]  -0.01112277
[3,]  -0.77875508
                Y
[1,] -12.14412847
[2,]  -0.01255966
[3,]  -0.77930891
               Y
[1,] -12.1383604
[2,]  -0.0131277
[3,]  -0.7792271
              Y
[1,] -12.1373696
[2,]  -0.0132321
[3,]  -0.7792812
                Y
[1,] -12.14206850
[2,]  -0.01304211
[3,]  -0.77970151
                Y
[1,] -12.15898251
```

```
[2,]  -0.01271108
[3,]  -0.77857157
                Y
[1,] -12.15232143
[2,]  -0.01266508
[3,]  -0.77931382
                Y
[1,] -12.15554115
[2,]  -0.01264083
[3,]  -0.77921259
                Y
[1,] -12.14808928
[2,]  -0.01266021
[3,]  -0.77852847
                Y
[1,] -12.15117600
[2,]  -0.01235404
[3,]  -0.77813056
                Y
[1,] -12.17104283
[2,]  -0.01269214
[3,]  -0.77971440
                Y
[1,] -12.17700244
[2,]  -0.01246807
[3,]  -0.78024758
                Y
[1,] -12.17404562
[2,]  -0.01226156
[3,]  -0.78032751
                Y
[1,] -12.17754087
[2,]  -0.01220148
[3,]  -0.78046414
                Y
[1,] -12.16689433
[2,]  -0.01222721
[3,]  -0.78159431
                Y
[1,] -12.17920167
[2,]  -0.01329571
[3,]  -0.78245596
               Y
[1,] -12.1610641
[2,]  -0.0148370
[3,]  -0.7811238
                Y
[1,] -12.15676820
```

```
[2,]  -0.01478368
[3,]  -0.78101702
                Y
[1,] -12.16978025
[2,]  -0.01530648
[3,]  -0.78177200
                Y
[1,] -12.17989548
[2,]  -0.01543446
[3,]  -0.78131008
                Y
[1,] -12.19436922
[2,]  -0.01545011
[3,]  -0.78069786
               Y
[1,] -12.1925868
[2,]  -0.0154074
[3,]  -0.7805803
               Y
[1,] -12.1996744
[2,]  -0.0149418
[3,]  -0.7812167
               Y
[1,] -12.2025110
[2,]  -0.0152553
[3,]  -0.7814579
                Y
[1,] -12.20105092
[2,]  -0.01531467
[3,]  -0.78159679
                Y
[1,] -12.21148916
[2,]  -0.01608646
[3,]  -0.78237264
               Y
[1,] -12.2088785
[2,]  -0.0161979
[3,]  -0.7821399
                Y
[1,] -12.20685916
[2,]  -0.01635622
[3,]  -0.78215983
                Y
[1,] -12.19715596
[2,]  -0.01542367
[3,]  -0.78219766
                Y
[1,] -12.20939579
```

```
[2,]  -0.01590061
[3,]  -0.78330978
               Y
[1,] -12.19413551
[2,]  -0.01614887
[3,]  -0.78404537
               Y
[1,] -12.17902376
[2,]  -0.01437262
[3,]  -0.78422371
              Y
[1,] -12.1713113
[2,]  -0.0145859
[3,]  -0.7837867
               Y
[1,] -12.17670731
[2,]  -0.01433989
[3,]  -0.78312202
               Y
[1,] -12.18209264
[2,]  -0.01439732
[3,]  -0.78266456
               Y
[1,] -12.17879026
[2,]  -0.01426298
[3,]  -0.78258512
```

Estimator after each iteration forgetting factor = 0.95)

```
              Y
[1,] -12.17906505
[2,]  -0.01425731
[3,]  -0.78265316
               Y
[1,] -12.18536570
[2,]  -0.01452676
[3,]  -0.78292758
               Y
[1,] -12.18899499
[2,]  -0.01465668
[3,]  -0.78292882
               Y
[1,] -12.19734647
[2,]  -0.01361265
[3,]  -0.78289595
               Y
```

```
[1,] -12.19431122
[2,]  -0.01356615
[3,]  -0.78311790
                Y
[1,] -12.19006629
[2,]  -0.01375699
[3,]  -0.78352405
                Y
[1,] -12.17908230
[2,]  -0.01249454
[3,]  -0.78315753
               Y
[1,] -12.1811026
[2,]  -0.0123338
[3,]  -0.7832692
                Y
[1,] -12.17122788
[2,]  -0.01357338
[3,]  -0.78324358
                Y
[1,] -12.16653501
[2,]  -0.01370752
[3,]  -0.78320894
                Y
[1,] -12.17547627
[2,]  -0.01223748
[3,]  -0.78247554
                Y
[1,] -12.16650247
[2,]  -0.01175382
[3,]  -0.78322334
                Y
[1,] -12.17053777
[2,]  -0.01144798
[3,]  -0.78286482
               Y
[1,] -12.1701658
[2,]  -0.0114646
[3,]  -0.7828924
                Y
[1,] -12.18055177
[2,]  -0.01261462
[3,]  -0.78329727
                Y
[1,] -12.17597567
[2,]  -0.01310059
[3,]  -0.78324355
                Y
```

```
[1,] -12.17517490
[2,]  -0.01318787
[3,]  -0.78328971
                Y
[1,] -12.17808905
[2,]  -0.01305728
[3,]  -0.78355807
                Y
[1,] -12.19203595
[2,]  -0.01279891
[3,]  -0.78262573
                Y
[1,] -12.18657865
[2,]  -0.01275345
[3,]  -0.78320781
                Y
[1,] -12.18918907
[2,]  -0.01273583
[3,]  -0.78312090
                Y
[1,] -12.18291957
[2,]  -0.01275502
[3,]  -0.78253038
                Y
[1,] -12.18564399
[2,]  -0.01249176
[3,]  -0.78218661
                Y
[1,] -12.20052550
[2,]  -0.01276352
[3,]  -0.78338958
                Y
[1,] -12.20461145
[2,]  -0.01259669
[3,]  -0.78376727
                Y
[1,] -12.20219673
[2,]  -0.01242619
[3,]  -0.78383532
                Y
[1,] -12.20474460
[2,]  -0.01238073
[3,]  -0.78393268
                Y
[1,] -12.19584006
[2,]  -0.01238862
[3,]  -0.78485452
                Y
```

```
[1,] -12.20537816
[2,]  -0.01324748
[3,]  -0.78551597
                Y
[1,] -12.19097783
[2,]  -0.01456724
[3,]  -0.78442300
                Y
[1,] -12.18731452
[2,]  -0.01451902
[3,]  -0.78433661
                Y
[1,] -12.19756004
[2,]  -0.01494696
[3,]  -0.78492527
                Y
[1,] -12.20607072
[2,]  -0.01506379
[3,]  -0.78453092
                Y
[1,] -12.21820423
[2,]  -0.01508956
[3,]  -0.78400695
                Y
[1,] -12.21646665
[2,]  -0.01504599
[3,]  -0.78389257
                Y
[1,] -12.22167102
[2,]  -0.01468188
[3,]  -0.78437570
               Y
[1,] -12.2236795
[2,]  -0.0149083
[3,]  -0.7845447
                Y
[1,] -12.22255748
[2,]  -0.01495253
[3,]  -0.78465133
                Y
[1,] -12.23092503
[2,]  -0.01558972
[3,]  -0.78527068
                Y
[1,] -12.22848690
[2,]  -0.01569748
[3,]  -0.78504727
                Y
```

```
[1,] -12.22674001
[2,]  -0.01583734
[3,]  -0.78506640
               Y
[1,] -12.21849810
[2,]  -0.01504136
[3,]  -0.78510438
               Y
[1,] -12.22831422
[2,]  -0.01544027
[3,]  -0.78600240
               Y
[1,] -12.21548160
[2,]  -0.01563697
[3,]  -0.78662975
               Y
[1,] -12.20246184
[2,]  -0.01411486
[3,]  -0.78678769
               Y
[1,] -12.19589250
[2,]  -0.01429789
[3,]  -0.78641183
               Y
[1,] -12.20061078
[2,]  -0.01408981
[3,]  -0.78583321
               Y
[1,] -12.20534875
[2,]  -0.01414629
[3,]  -0.78543350
               Y
[1,] -12.20244287
[2,]  -0.01402411
[3,]  -0.78536609
```

Estimator after each iteration forgetting factor = 0.2)

```
              Y
[1,] -11.9822218
[2,]  -0.0162796
[3,]  -0.7635629
               Y
[1,] -12.00832285
[2,]  -0.01716343
[3,]  -0.76477898
```

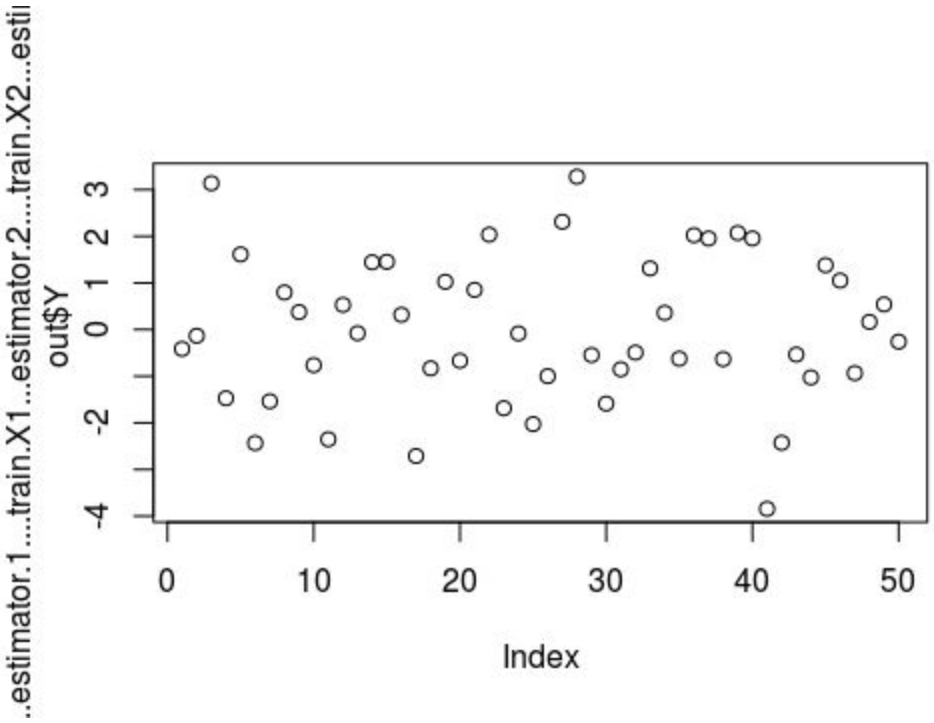
```
                Y
[1,] -12.02110220
[2,]  -0.01752334
[3,]  -0.76484590
                Y
[1,] -12.05650243
[2,]  -0.01385837
[3,]  -0.76490109
               Y
[1,] -12.0460430
[2,]  -0.0137710
[3,]  -0.7656927
                Y
[1,] -12.03192539
[2,]  -0.01451498
[3,]  -0.76703937
                Y
[1,] -12.00411378
[2,]  -0.01138859
[3,]  -0.76588938
                Y
[1,] -12.01857396
[2,]  -0.01044647
[3,]  -0.76662720
                Y
[1,] -11.98718501
[2,]  -0.01368147
[3,]  -0.76640221
                Y
[1,] -11.97545070
[2,]  -0.01401496
[3,]  -0.76626124
                 Y
[1,] -12.003524071
[2,]  -0.009903869
[3,]  -0.764238107
                 Y
[1,] -11.979537033
[2,]  -0.008646448
[3,]  -0.766498944
                 Y
[1,] -11.989743202
[2,]  -0.007852427
[3,]  -0.765608200
                 Y
[1,] -11.987893032
[2,]  -0.007940774
[3,]  -0.765742022
```

```
                Y
[1,] -12.02139765
[2,]  -0.01134658
[3,]  -0.76726448
                Y
[1,] -12.00929342
[2,]  -0.01239981
[3,]  -0.76704321
                Y
[1,] -12.00720325
[2,]  -0.01260769
[3,]  -0.76714621
                Y
[1,] -12.02484669
[2,]  -0.01200316
[3,]  -0.76865980
                Y
[1,] -12.06084915
[2,]  -0.01124244
[3,]  -0.76626157
                Y
[1,] -12.04604509
[2,]  -0.01117391
[3,]  -0.76804872
               Y
[1,] -12.0532386
[2,]  -0.0111120
[3,]  -0.7678463
                Y
[1,] -12.04074975
[2,]  -0.01113693
[3,]  -0.76674283
               Y
[1,] -12.0460468
[2,]  -0.0105886
[3,]  -0.7660338
                Y
[1,] -12.09736787
[2,]  -0.01136532
[3,]  -0.77003831
                Y
[1,] -12.11511712
[2,]  -0.01077811
[3,]  -0.77155352
                Y
[1,] -12.10875990
[2,]  -0.01034246
[3,]  -0.77171318
```

```
                 Y
[1,] -12.11813139
[2,]  -0.01019099
[3,]  -0.77209229
                 Y
[1,] -12.09569377
[2,]  -0.01029913
[3,]  -0.77457646
                 Y
[1,] -12.12544044
[2,]  -0.01272864
[3,]  -0.77668210
               Y
[1,] -12.0852561
[2,]  -0.0157797
[3,]  -0.7738583
                 Y
[1,] -12.07696298
[2,]  -0.01568586
[3,]  -0.77363616
               Y
[1,] -12.1074165
[2,]  -0.0168361
[3,]  -0.7754262
                 Y
[1,] -12.12840257
[2,]  -0.01706834
[3,]  -0.77449135
                 Y
[1,] -12.15855649
[2,]  -0.01705541
[3,]  -0.77325983
                 Y
[1,] -12.15623346
[2,]  -0.01700361
[3,]  -0.77310623
                 Y
[1,] -12.17466437
[2,]  -0.01589892
[3,]  -0.77468733
                 Y
[1,] -12.18201255
[2,]  -0.01668539
[3,]  -0.77532007
               Y
[1,] -12.1783294
[2,]  -0.0168416
[3,]  -0.7756707
```

```
                Y
[1,] -12.20139902
[2,]  -0.01847003
[3,]  -0.77739270
                Y
[1,] -12.19721690
[2,]  -0.01864012
[3,]  -0.77703407
                Y
[1,] -12.19329014
[2,]  -0.01893872
[3,]  -0.77706655
                Y
[1,] -12.17299501
[2,]  -0.01700345
[3,]  -0.77712440
                Y
[1,] -12.20008395
[2,]  -0.01799337
[3,]  -0.77955881
                Y
[1,] -12.16826014
[2,]  -0.01855281
[3,]  -0.78105894
                Y
[1,] -12.13790140
[2,]  -0.01495596
[3,]  -0.78140284
                Y
[1,] -12.12221937
[2,]  -0.01538543
[3,]  -0.78052651
                Y
[1,] -12.13292970
[2,]  -0.01487504
[3,]  -0.77919975
                Y
[1,] -12.14343396
[2,]  -0.01496834
[3,]  -0.77829921
                Y
[1,] -12.13705885
[2,]  -0.01472163
[3,]  -0.77813831
```

Graphs:



Training Target Variable

Code:

```
library(MASS)

train <- read.csv("Problem1_Input_Training.csv")
out <- read.csv("Problem2_Output_Training.csv")

x0 <- matrix(1 , nrow=50, ncol=1 )

d <- data.frame(x0,train$X1,train$X2)
df <- d
y <- data.frame(out)

xx <- as.matrix(d)
y <- as.matrix(out)

estimator <- ((ginv(t(xx)%*%xx))%*%t(xx))%*%(y)
```

```
dd <-
data.frame(x0*estimator[1]+train$X1*estimator[2]+train$X2*estimator[3
])
write.csv(dd,"abc.csv")

plot(dd$x0...estimator.1....train.X1...estimator.2....train.X2...esti
mator.3.-out$Y)



forget_factor <- 0.2  #To be added

real_time <- read.csv("Problem1_Input_Test.csv")
real_output <- read.csv("Problem2_Output_Test.csv")
dd <- data.frame(x0,real_time$X1,real_time$X2)
dd <- as.matrix(dd)

Mn <- ginv(t(xx)%*%(xx))
i=1
xx <- as.matrix(df)
Mn <- ginv(t(xx)%*%(xx))
d <- data.frame(x0,real_time$X1,real_time$X2)
mx <- as.matrix(d[i,])
estimator <- estimator
+((Mn-(Mn%*%t(mx)%*%mx%*%Mn)/(1+mx%*%Mn%*%t(mx))[1])%*%t(mx))*(real_o
utput[i,1]-mx%*%estimator)[1]
df <- rbind(df,c(1,real_time$X1[i],real_time$X2[i]))
print(estimator)
xn <- rbind(xx[1:50,],dd[1,])

for(i in 2:50)
{
  xx <- as.matrix(df)
  Mn <- ginv(forget_factor*t(xn)%*%(xn)+t(xx)%*%xx)
  d <- data.frame(x0,real_time$X1,real_time$X2)
  mx <- as.matrix(d[i,])
  estimator <- estimator
+((Mn-(Mn%*%t(mx)%*%mx%*%Mn)/(1+mx%*%Mn%*%t(mx))[1])%*%t(mx))*(real_o
utput[i,1]-mx%*%estimator)[1]
  df <- rbind(df,c(1,real_time$X1[i],real_time$X2[i]))
  xn <- rbind(xn,dd[i,])
  print(estimator)
}
```

Problem 4 : Locally Weighted Regression

Answers:

Each Predicted Value for Heaviside Unit Step Function

```
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
        Y
-2.630204
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
```

Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0.7860604
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
-2.645184
Y
0
Y
0
Y
0
Y
0

Y
0
Y
0
Y
0
Y
0
Y
0
Y
0
Y
0

Each Predicted Value for y = exp(-di^2/2)

Y
-5.311127
Y
-34.2156
Y
-14.72224
Y
-2.927352
Y
-25.24385
Y
3.492806
Y
-3.218871
Y
-8.668442
Y
-30.20424
Y
-26.47651
Y
-16.36044
Y
-15.20313
Y
5.478978
Y
-2.091285
Y
-6.922793

Y
-12.04756
Y
-24.03737
Y
-11.73891
Y
-36.6496
Y
-5.057906
Y
4.068868
Y
-14.02117
Y
-30.66851
Y
-3.061566
Y
-16.49828
Y
-36.24089
Y
3.661228
Y
-19.74716
Y
3.176758
Y
-16.54649
Y
-38.93757
Y
-13.97023
Y
-21.94231
Y
-10.32949
Y
-10.61046
Y
-27.52245
Y
-37.5633
Y
-21.9764
Y
-3.230242

```
         Y
-22.53454
         Y
-35.13161
         Y
0.1307241
         Y
-2.869354
         Y
-22.27302
         Y
-6.054649
         Y
0.1081085
         Y
-28.59431
        Y
-18.6208
       Y
75.4623
         Y
-15.30552
```

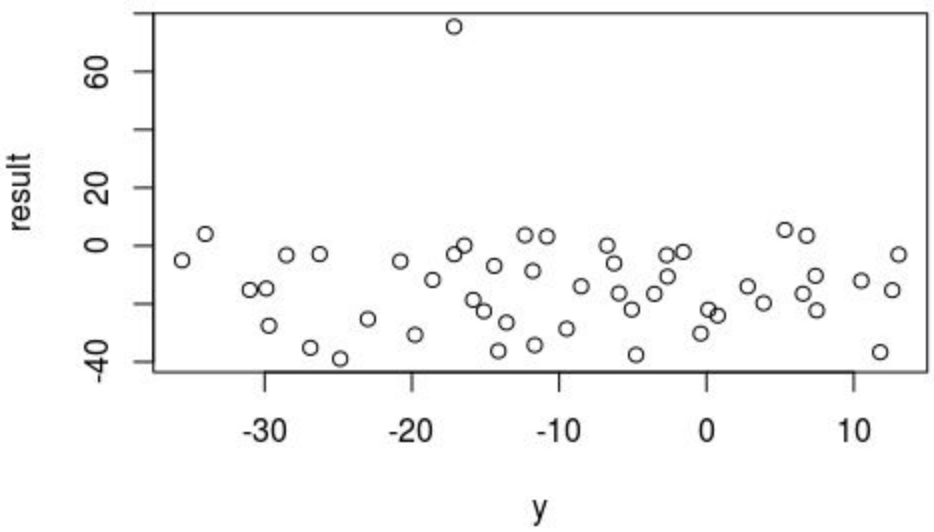
Each Predicted Value for y = { 10 if di=0 , 1/di otherwise }

```
       Y
-10.47076
         Y
-33.31735
         Y
-15.94794
         Y
-8.310207
        Y
-22.3931
        Y
1.587149
          Y
-0.7506627
         Y
-5.513174
         Y
-30.15696
        Y
-23.6999
```

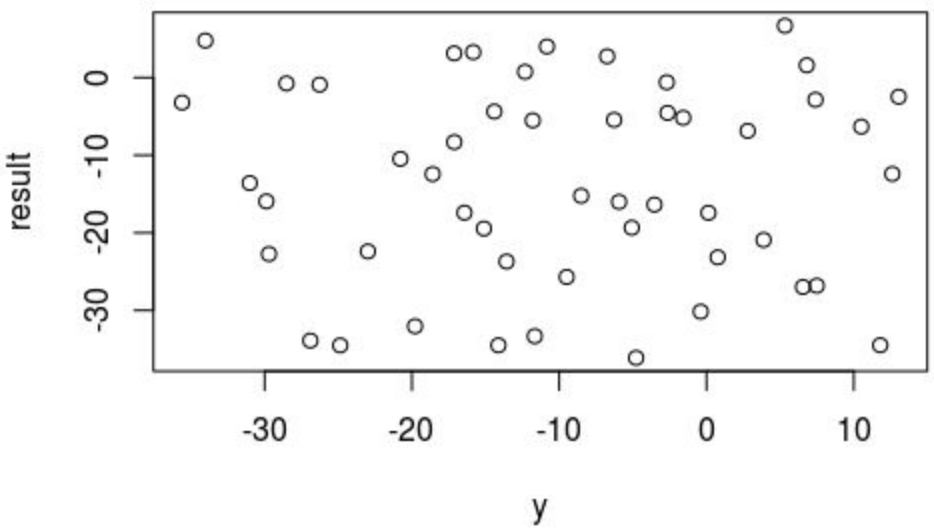
Y
-16.01439
Y
-13.59222
Y
6.688057
Y
-5.176768
Y
-4.371394
Y
-6.318124
Y
-23.1556
Y
-12.43519
Y
-34.47836
Y
-3.215557
Y
4.76911
Y
-6.863953
Y
-32.03467
Y
-2.465819
Y
-26.97728
Y
-34.49285
Y
0.7468434
Y
-20.91777
Y
4.004062
Y
-16.3682
Y
-34.5014
Y
-15.23251
Y
-19.33996
Y
-2.853138

```
         Y
-4.548527
         Y
-22.73254
         Y
-36.10941
         Y
-17.42033
         Y
-0.603457
       Y
-19.4511
         Y
-33.89007
         Y
-17.40599
          Y
-0.9112036
         Y
-26.81468
         Y
-5.446286
       Y
2.741454
         Y
-25.68753
       Y
3.281219
       Y
3.126776
         Y
-12.41273
```

Graphs:



Predicted vs Observed Plot (y = exp(-di^2/2))



Predicted vs Observed Plot
for y = { 10 if di=0 , 1/di otherwise }}

Code:

```
train <- read.csv("Problem1_Input_Training.csv")
out <- read.csv("Problem1_Output_Training.csv")
df<- read.csv("Problem1_Input_Training.csv")
real_time_input <- read.csv("Problem1_Input_Test.csv")
y<- read.csv("Problem3_Output_Training.csv")
real_time_output <- read.csv("Problem3_Output_Test.csv")

vi<-diag(50)

x0 <- matrix(1 , nrow=50, ncol=1 )

d <- data.frame(x0,train$X1,train$X2)
df <- d
y <- data.frame(out)

xx <- as.matrix(d)
y <- as.matrix(out)

estimator <- ((ginv(t(xx)%*%xx))%*%t(xx))%*%(y)

dd <-
data.frame(x0*estimator[1]+train$X1*estimator[2]+train$X2*estimator[3
])
write.csv(dd,"abc.csv")

plot(dd$x0...estimator.1....train.X1...estimator.2....train.X2...esti
mator.3.-out$Y) #Shows residual are normally distributed


#Part 1
#Heaviside Unit Step Function
for(i in 1:50)
{
  rr<-diag(50)
  for(j in 1:50)
  {

if(((train$X1[j]-real_time_input$X1[i])^2+(train$X2[j]-real_time_inpu
t$X2[i])^2)<1)
      rr[j,j]=1
```

```
    else
      rr[j,j]=0;
  }
  estimator <-
((ginv(t(xx)%*%ginv(rr)%*%xx))%*%t(xx))%*%ginv(rr)%*%(y)

print(estimator[1,1]*1+estimator[2,1]*real_time_input$X1[i]+estimator
[3,1]*real_time_input$X2[i])
}
#Part 2
#wi=e^-di^2/2
for(i in 1:50)
{
  rr<-diag(50)
  for(j in 1:50)
  {

x<-exp((-((train$X1[j]-real_time_input$X1[i])^2+(train$X2[j]-real_tim
e_input$X2[i])^2))/2)
      rr[j,j]<-x
  }
  estimator <-
((ginv(t(xx)%*%ginv(rr)%*%xx))%*%t(xx))%*%ginv(rr)%*%(y)

print(estimator[1,1]*1+estimator[2,1]*real_time_input$X1[i]+estimator
[3,1]*real_time_input$X2[i])
}
#Part 3
#Inverse Function
for(i in 1:50)
{
  rr<-diag(50)
  for(j in 1:50)
  {

if((train$X1[j]-real_time_input$X1[i])^2+(train$X2[j]-real_time_input
$X2[i])^2==0){
      rr[j,j]=10
    }
    else{

x<-sqrt((train$X1[j]-real_time_input$X1[i])^2+(train$X2[j]-real_time_
input$X2[i])^2)
```

```
      rr[j,j]=1/x;
    }
  }
  estimator <-
((ginv(t(xx)%*%ginv(rr)%*%xx))%*%t(xx))%*%ginv(rr)%*%(y)

print(estimator[1,1]*1+estimator[2,1]*real_time_input$X1[i]+estimator
[3,1]*real_time_input$X2[i])
}
```