

Rahul_Goyal_main Usage and Description

ME 326 Winter 2018 - Laboratory Assignment #4

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Description: This script simulates the motion of a bowling ball. It detects when the ball stops slipping and adjusts the acting friction accordingly. Additionally, it produces an animated GIF of the simulation in which the bowling ball changes its color from blue to green when it stops slipping.

Required Files:

- Integrator.slx - This file uses Simlunk to integrate a MATLAB Function Block which describes the equations of motion. It outputs x as $xout$ (and the corresponding \dot{x} and time values as \dot{x} and $tout$, respectively) with inputs of the MATLAB function and initial conditions.
- BowlingBallEOM.m - This file contains a function that represents the equations of motion for the simulation. It returns \dot{x} with an input of x .
- plot_lane.m - This file contains a function that generates a properly dimensioned plot of a lane including all ten pins.

Called Functions

- [plot_lane](#)

Still To Do:

- Done!

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Problem Statement

Consider a bowling ball thrown with an initial angular velocity w_0 and an initial linear velocity v_0 from the instant it makes contact with an alley lane. The velocity at the contact point, v_C , is due to the velocity of the center of mass of the bowling ball, v_B , and due to the relative velocity, $v_{C/B}$, caused by the angular velocity of the bowling ball. The surface of the bowling alley will be defined as the xy -plane, where the y -axis extends down the alley and the x -axis extends toward the right gutter. The direction of the contact velocity is what determines the direction of the friction force. Angle θ is defined as the angle between the positive extension of the x -axis and the contact velocity vector. Therefore, when $\theta = \pi/2$ the ball will be traveling straight down the alley toward the pins.

The friction present between the ball and the smooth wooden lane is best approximated by a Coulomb friction model. That is $F = \mu_k N$ if the ball is slipping and $F \leq \mu_s N$ if the ball is rolling without slip. In each regime the friction force opposes the relative velocity (or impending relative velocity) at the contact point. In order to develop equations of motion, the magnitude and direction of the coulomb friction force must be computed, and then put into component form in the xy plane.

The bowling ball is modeled as a sphere of uniform density that weighs 15 lb and has a diameter of 8.5 in. The kinetic friction coefficient, μ_k , between the ball and surface is assumed to be 0.12 and the static friction coefficient, μ_s , between the ball and surface is assumed to be 0.14.

Reset

The following was used while debugging.

```
close all;
clear all;
clc;
```

Initial Conditions

The following sets the initial conditions of the bowling ball.

```
% Initial Conditions
x_0 = [1.5;           % Velocity[x] of ball (ft/s)
      30;             % Velocity[y] of ball (ft/s)
      0;              % Angular velocity[x] of ball (rad/s)
      -25;            % Angular velocity[y] of ball (rad/s)
      0;              % Displacement[x] of ball (ft)
      0];             % Displacement[y] of ball (ft)
```

Simulate the Bowling Ball Using Simulink

The following calls the Simulink file Integrator.slx, which outputs x as xout (and the corresponding xdot and time values as xdot and tout, respectively) with BowlingBallEOM.m as the input for the MATLAB Function and x_0 as the input for the initial conditions.

```
sim('Integrator');
```

Animation

The following animates the motion of the bowling ball. It plots the lane and the bowling ball (color dependent on whether the bowling ball is slipping). Additionally, it stores the frames of the animation (converted to images) for exporting them as an animated GIF afterwards.

```
% Calculate when the bowling ball stops slipping
for t = 1:length(tout)

    % Acceleration[linear[x, y], angular[x, y]]
    a = xdot(t, 1:4);           % (ft/s, ft/s, rad/s^2, rad/s^2)

    % If no slip, set bowling ball color to green
    if a == 0
        % Set start of no-slip[x]
        x0_noslip = num2str(xout(t, 5));
        % Set start of no-slip[y]
        y0_noslip = num2str(xout(t, 6));
```

```

        break
    end

end

% Format start of no-slip position as text
txt = char('The bowling ball', ...
    'stops slipping at:', ...
    ['(', x0_noslip, ', ', y0_noslip, ') ft.']);

% Plot the bowling ball on the lane
for t = 1:length(tout)

    % If the bowling ball has traveled the lane length, finish
    lane_length = 62+(10+3/16)/12; % Lane length (ft)
    if xout(t, 6) > lane_length
        break;
    end

    % Plot the Lane
    plot_lane(); % Plot the lane
    ylim([0, lane_length]); % Set y-axis limits to lane length
    axis equal; % Scale x-axis and y-axis equally

    % Plot the Bowling Ball
    d = 8.5; % Given diameter (in)
    r = (d/2)/12; % Solved radius (ft)
    x = xout(t, 5); % X position of ball's center (ft)
    y = xout(t, 6); % Y position of ball's center (ft)

    % Acceleration[linear[x, y], angular[x, y]]
    a = xdot(t, 1:4); % (ft/s, ft/s, rad/s^2, rad/s^2)

    % If no slip, set bowling ball color to green
    if a == 0
        color = 'g';
    % Else, set bowling ball color to blue
    else
        color = 'b';
    end

    % Draw the bowling ball
    viscircles([x, y], r, 'Color', color);

    % Calculate the time step and pause accordingly
    if t ~= length(tout) % Prevent index error
        % Calculate the time step (s) and store for later use
        t_step(t) = tout(t+1) - tout(t);
        pause(t_step(t)); % Assume negligible processing time
    end

    % Format Plot
    title('Bowling Ball Animation');

```

```

xlabel({'X Position (ft)'
      ''
      % Figure label
      '\bFigure 1: \rmBowling Ball Animation'});
ylabel('Y Position (ft)');
text(5, 10, txt);

% Convert the plot frame to an image and store for later use
image{t} = frame2im(getframe(1));

end

```

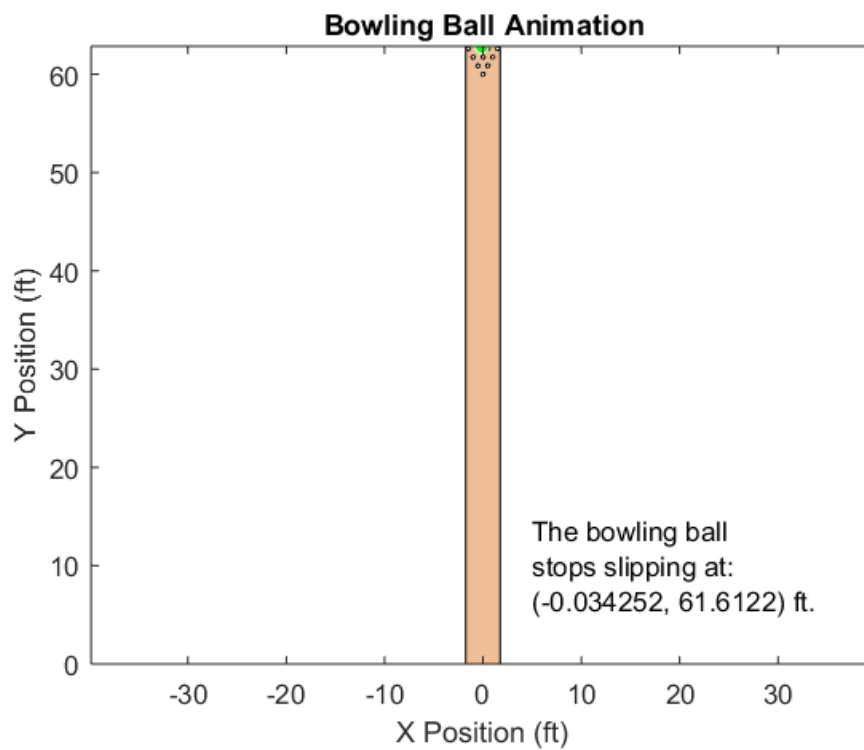


Figure 1: Bowling Ball Animation

Export as GIF

The following exports the animation as an animated GIF.

```

file_name = 'BowlingBallAnimation.gif';
for i = 1:length(image)

    % Convert the RGB image to an indexed image
    [A, map] = rgb2ind(image{i}, 256);

    % If first iteration, also run setup code
    if i == 1
        imwrite(A, map, file_name, ...
                'LoopCount', inf, ...
                'DelayTime', t_step(i));

    % Else, append images
    else

```

```

imwrite(A, map, file_name, ...
        'WriteMode', 'append', ...
        'DelayTime', t_step(i));

```

```

end

```

```

end

```

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```

function [ ] = plot_lane( )
%PLOT_LANE Plot the outline of a bowling lane with pins
% PLOT_LANE() plots the shape of a bowling lane with a wood colored
% background and all ten bowling pins as seen from the top. The pins
% appear as circles filled in with white. The dimensions of the lane
% conform to the standard for bowling alleys. That is the lane has
% dimensions 42" W by 62'10-3/16"L. The bowling pins have a maximum
% diameter of 4.766" as is standard. The pins are arranged in a
% triangular grid, with 12" between centers. The center of the head-pin
% is 60' from the beginning of the lane.

% For speed the constants necessary to plot the lane and pins are stored as
% persistent data.
persistent lane_length single_pin pin_array lane_array

% Generate the persistent data only if the data has not already been
% created
if isempty(pin_array)
    % Lane length in units of feet.
    lane_length = 62+(10+3/16)/12;

    % Array of data points forming a circle to represent a single pin.
    % Units are also in feet.
    single_pin = 4.766/2/12*[cos(0:pi/50:2*pi)' sin(0:pi/50:2*pi)'];

    % Cell array containing all 10 pins translated to their location
    % relative to the head-pin. This allows them each to be plotted
    % individually.
    pin_array = {single_pin+[ 0.0 0.0 ]; %Pin 1
                  single_pin+[-0.5 sqrt(3)/2 ]; %Pin 2
                  single_pin+[ 0.5 sqrt(3)/2 ]; %Pin 3
                  single_pin+[-1.0 sqrt(3) ]; %Pin 4
                  single_pin+[ 0.0 sqrt(3) ]; %Pin 5
                  single_pin+[ 1.0 sqrt(3) ]; %Pin 6
                  single_pin+[-1.5 3*sqrt(3)/2]; %Pin 7
                  single_pin+[-0.5 3*sqrt(3)/2]; %Pin 8
                  single_pin+[+0.5 3*sqrt(3)/2]; %Pin 9
                  single_pin+[+1.5 3*sqrt(3)/2]; %Pin 10

    % Array of points forming the outline of the lane.
    lane_array = [-21/12 0
                  -21/12 lane_length
                  21/12 lane_length
                  21/12 0
                  -21/12 0 ];
end

```

```
% This variable determines whether or not hold should be turned back on
% after the lane is done plotting.
hold_state = ishold;

% Plot the bowling lane itself and set the color to appear as wood.
fill(lane_array(:,1),lane_array(:,2),[240/255 190/255 150/255]);

% Plot the pins one by one as white circles translated to their location on
% the lane.
hold on;
for n=1:length(pin_array)
    fill(pin_array{n}(:,1),pin_array{n}(:,2)+60,'white');
end

% If needed, re-enable the plot hold so that further data (like the ball)
% may be plotted.
if ~hold_state
    hold off;
end

end
```

Contents

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- [Given Values](#)
- [Solve for the Friction Force](#)
- [Solve for \$\dot{x}\$](#)

Called Functions

Given Values

The following assigns values given by the problem statement to variables. (The value of the gravitational constant is assumed).

```
% Given Values
w = 15;                % Weight of bowling ball (lb)
d = 8.5;               % Diameter of bowling ball (in)
mu_k = 0.12;           % Kinetic friction coefficient (unitless)
mu_s = 0.14;           % Static friction coefficient (unitless)
g = 32.174;            % Gravitational constant (ft/s^2)

% Solved Values
m = w/g;               % Mass of bowling ball (slug)
r = (d/2)/12;          % Radius of bowling ball (ft)
I = 2/5*m*r^2;         % Moment of inertia (slug*ft^2)
```

Solve for the Friction Force

To find the friction force, the following first checks whether the bowling ball is slipping by evaluating the velocity of the contact point. A contact point velocity of 0 corresponds to a ball that rolls without slip; otherwise, the ball is slipping. A deadband is used to avoid chatter. See the attached file for hand calculations.

```
% Easy access to variables
v_x = x(1);            % Easy access to v_x (ft/s)
v_y = x(2);            % Easy access to v_y (ft/s)
omega_x = x(3);         % Easy access to omega_x (rad/s)
omega_y = x(4);         % Easy access to omega_y (rad/s)

% Velocity at contact point
vc_x = v_x - omega_y*r; % Velocity at contact point[x] (ft/s)
vc_y = v_y + omega_x*r; % Velocity at contact point[y] (ft/s)

% If no slip, set friction force to 0
deadband = 0.02;
if abs(vc_x) < deadband && abs(vc_y) < deadband

    F_x = 0;            % Friction force[x] (lb)
    F_y = 0;            % Friction force[y] (lb)

% Else, set friction force to kinetic friction
else

    N = w;              % Normal force (lb)

    theta = atan2(vc_y, vc_x); % Theta (rad)

    F_x = -mu_k*N*cos(theta); % Friction force[x] (lb)
```

```
F_y = -mu_k*N*sin(theta);      % Friction force[y] (lb)

end
```

Not enough input arguments.

Error in BowlingBallEOM (line 32)

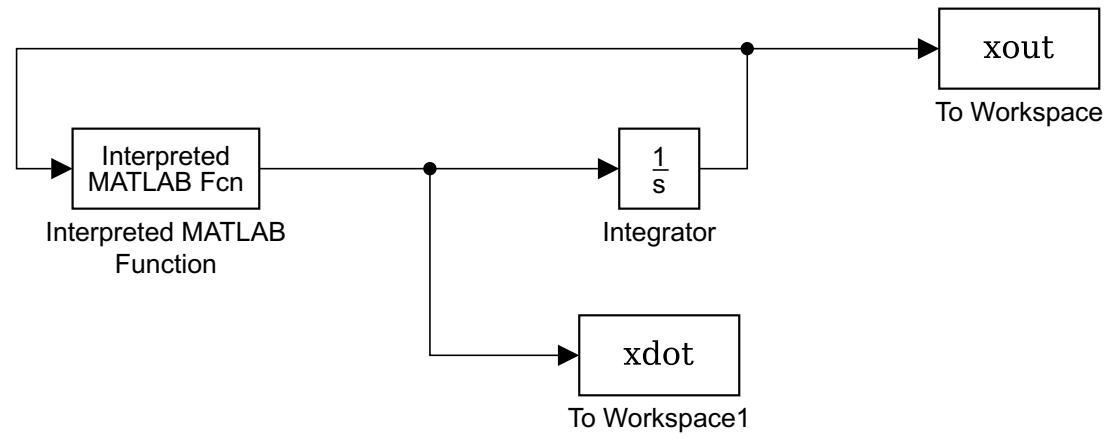
v_x = x(1); % Easy access to v_x (ft/s)

Solve for xdot

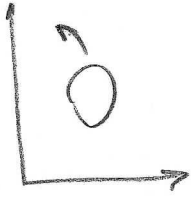
See the attached file for hand calculations.

```
xdot = [F_x/m;
        F_y/m;
        F_y*r/I;
        -F_x*r/I;
        v_x;
        v_y];
```

```
end
```

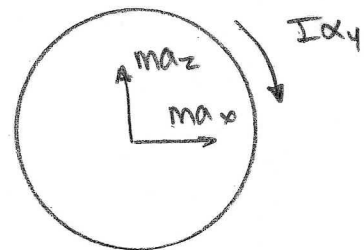
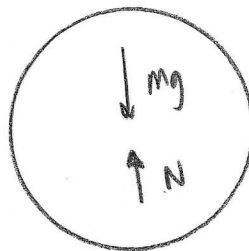
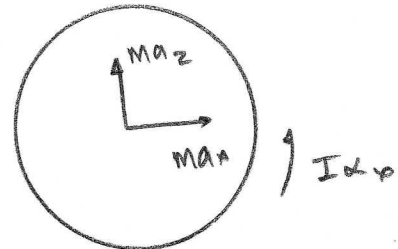
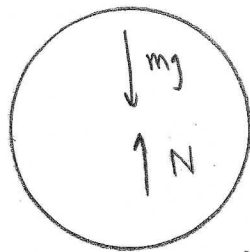
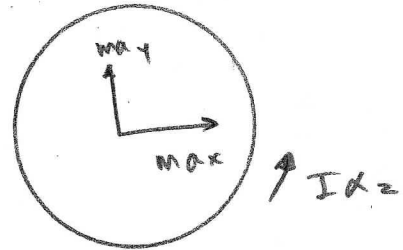
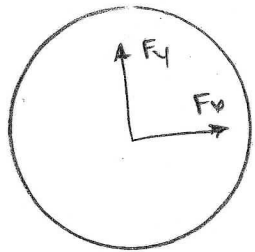



LAB 4 HAND CALCULATIONS RANVUL GOYAL



FBDs

KDS



$$\textcircled{1} \sum F_x = ma_x$$

$$\frac{F_x}{m} = a_x$$

$$\frac{F_y}{m} = a_y$$

$$\textcircled{2} \sum F_z = m \overset{0}{a_z}$$

$$N = mg$$

$$\sum M_y = \bar{I} \alpha_y$$

$$\alpha_y = \frac{F_x \cdot r}{\bar{I}}$$

$$\textcircled{3} \sum F_z = m \overset{0}{a_z}$$

$$N = mg$$

$$\sum M_y = \bar{I} \alpha_y$$

$$\alpha_y = -\frac{F_x \cdot r}{\bar{I}}$$

LAB 4 HAND CALCULATIONS

CONTINUED

$$\vec{v} = \begin{bmatrix} v_x \\ v_y \\ \omega \\ r \\ \theta \end{bmatrix}$$

$$\frac{d}{dt} \vec{v} = \begin{bmatrix} F_x/m \\ F_y/m \\ F_y r/I \\ -F_x r/I \\ v_x \\ v_y \end{bmatrix}$$

$$\vec{v}_c = \vec{v}_B + \vec{v}_{c/B}$$

$$\vec{v}_c = (v_x \hat{i} + v_y \hat{j}) + (\omega_x \hat{i} + \omega_y \hat{j}) \times -r \hat{k}$$

$$\vec{v}_{cx} = (v_x - \omega_y \cdot r) \hat{i}$$

$$\vec{v}_{cy} = (v_y + \omega_x \cdot r) \hat{j}$$

$$\theta = \arctan\left(\frac{v_{cy}}{v_{cx}}\right)$$

When slipping

$$|F| = \mu_k N$$

$$\vec{F}_x = \mu_k N \cos \theta \hat{i}$$

$$\vec{F}_y = \mu_k N \sin \theta \hat{j}$$

When not slipping

$$|F| = 0$$

$$\vec{F}_x = \vec{0}$$

$$\vec{F}_y = \vec{0}$$