Rahul_Goyal_main Usage and Description

ME 326 Winter 2018 - Laboratory Assignment #6

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Description: This script simulates the motion of a crank-rocker. Afterwards, it animates the crank-rocker by using the simulation data and compares input power to time.

Required Files:

- MyPosIC.m - This file contains a function that represents the error for a set of given initial angular positions of the links of the crank-rocker. It returns the error with an input of link lengths, the angular position of link 2, and a guess for the angular positions of link 3 and link 4.
- Simulator.slx - This file uses Simulink to double integrate part of a MATLAB Function Block which describes the accelerations and forces of the simulation (only the accelerations are integrated). It outputs the positions as xout, the velocities as vout, the accelerations as aout, the forces as Fout, and the times as tout with inputs of the MATLAB function, initial conditions, and final condition.
- link_solver.m - This file contains a function that represents the accelerations and forces of the simulation. It returns x with an input of u.

Called Functions

- MyPosIC

Still To Do:

- Done!

Contents

Problem Statement

Choose a set of link lengths within the rocker crank constraint to design a mechanism. Assume the input crank, R2, moves at a constant angular velocity. Develop the kinematic and kinetic equations of motion. Produce an animation of the rocker-crank linkage and determine the input power as a function of time.

Reset

The following was used while debugging.

```
close all;
clear all;
clc;
```

Declare Global Variables

The following declares global variables.

```
global image t_step;
```

Set Values

The following is used to easily change the lengths of the links, the initial angular position of link 2, and the angular velocity of link 2. (A Grashof mechanism has the constraint R1 + R2 <= R3 + R4).

```
r = [4, 2, 5, 4];               % Length of links 1, 2, 3, 4 (m)
t2_0 = deg2rad(45);             % Angular position initial of link 2 (rad)
tdot_2 = 1;                     % Angular velocity of link 2 (rad/s)
```

Given Values

The following assigns values given by the problem statement to variables.

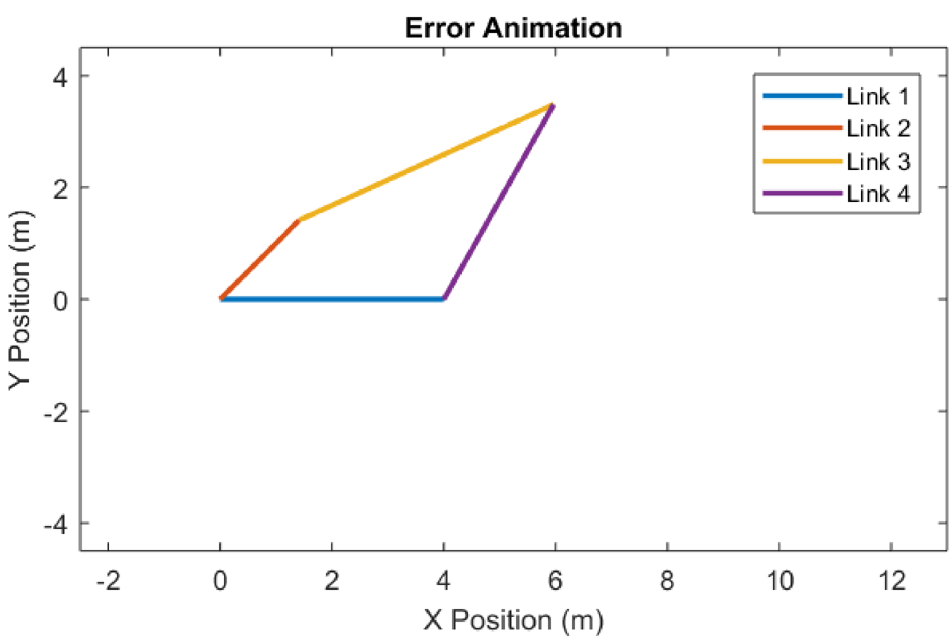
```
t2_f = t2_0+4*pi;               % Angular position final of link  (rad)
```

Position Initial Conditions

The following sets the position initial conditions of the crank-rocker.

```
t3_0 = 0;                       % Angular position initial of link 3 (rad) [GUESS]
t4_0 = 0;                       % Angular position initial of link 4 (rad) [GUESS]
% Set up an anonymous function for fminsearch
minimize = @(x) MyPosIC(r, t2_0, x);
% Minimize the error in the initial angular positions of link 3 and link 4
t_0 = fminsearch(minimize, [t3_0, t4_0], optimset('TolFun', 1e-6));
% Plot labeling (last frame)
title('Error Animation');
xlabel('X Position (m)');
ylabel('Y Position (m)');
legend('Link 1', 'Link 2', 'Link 3', 'Link 4');

% Easy access to...
t3_0 = t_0(1);                  % Angular position initial of link 3 (rad)
t4_0 = t_0(2);                  % Angular position initial of link 4 (rad)
```

```
% Simplicity and Compactness of Notation
c2_0 = cos(t2_0);
s2_0 = sin(t2_0);
c3_0 = cos(t3_0);
s3_0 = sin(t3_0);
c4_0 = cos(t4_0);
s4_0 = sin(t4_0);

x2_0 = r(2)/2*c2_0;              % COM[x] initial of link 2 (m)
y2_0 = r(2)/2*s2_0;              % COM[y] initial of link 2 (m)
x3_0 = r(2)*c2_0 + r(3)/2*c3_0; % COM[x] initial of link 3 (m)
y3_0 = r(2)*s2_0 + r(3)/2*s3_0; % COM[y] initial of link 3 (m)
x4_0 = r(1) + r(4)/2*c4_0;      % COM[x] initial of link 4 (m)
y4_0 = r(4)/2*s4_0;              % COM[y] initial of link 4 (m)

% Position Initial Conditions Matrix
x_0 = [t3_0, t4_0, x2_0, y2_0, x3_0, y3_0, x4_0, y4_0];
```



Velocity Initial Conditions

The following sets the velocity initial conditions of the crank-rocker.

```
A = [-r(3)*s3_0, r(4)*s4_0
     r(3)*c3_0, -r(4)*c4_0];
b = [r(2)*s2_0*tdot_2
     -r(2)*c2_0*tdot_2];
w_0 = A \ b;

% Easy access to...
tdot3_0 = w_0(1);               % Angular velocity initial of link 3 (rad/s)
tdot4_0 = w_0(2);               % Angular velocity initial of link 4 (rad/s)
```

```
% Velocity_G[x] initial of link 2
xdot2_0 = -r(2)/2*s2_0*tdot_2;
% Velocity_G[y] initial of link 2
ydot2_0 = r(2)/2*c2_0*tdot_2;
% Velocity_G[x] initial of link 3
xdot3_0 = -r(2)*s2_0*tdot_2 - r(3)/2*s3_0*tdot3_0;
% Velocity_G[y] initial of link 3
ydot3_0 = r(2)*c2_0*tdot_2 + r(3)/2*c3_0*tdot3_0;
% Velocity_G[x] initial of link 4
xdot4_0 = -r(4)/2*s4_0*tdot4_0;
% Velocity_G[y] initial of link 4
ydot4_0 = r(4)/2*c4_0*tdot4_0;

% Velocity Initial Conditions Matrix
v_0 = [tdot3_0, tdot4_0, xdot2_0, ydot2_0, xdot3_0, ydot3_0, xdot4_0, ydot4_0];
```

Simulate the Crank-Rocker Using Simulink

The following calls the Simulink file Simulator.slx, which outputs the positions as xout, the velocities as vout, the accelerations as aout, the forces as Fout, and the times as tout with with link_solver.m as the input for the MATLAB Fuction, tdot_2, t2_0, v_0, and x_0 as the inputs for the initial conditions, and t2_f as the input for the final conditions.

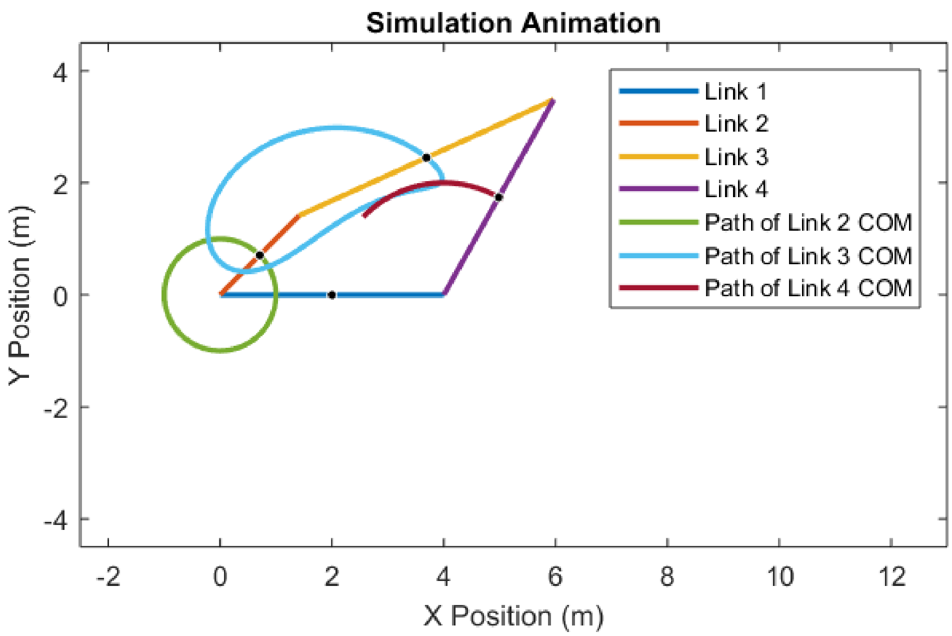
```
sim('Simulator.slx');
```

Simulation Animation

The following animates the crank-rocker by using the simulation data.

```
% Cartesian Coordinates of Link 1
r1_x = [0, r(1)];
r1_y = [0, 0];
% Cartesian Coordinates of COM of Link 1
x_1 = (r1_x(end)-r1_y(1))/2;
y_1 = (r1_y(end)-r1_y(2))/2;

% Easy access to...
t_2 = t2_0 + tdot_2*tout;       % Angular positions of link 2 (rad)
t_3 = xout(:, 1);               % Angular positions of link 3 (rad)
t_4 = xout(:, 2);               % Angular positions of link 4 (rad)
x_2 = xout(:, 3);               % COMs[x] of link 2 (m)
y_2 = xout(:, 4);

% COMs[y] of link 2 (m)
x_3 = xout(:, 5);               % COMs[x] of link 3 (m)
y_3 = xout(:, 6);               % COMs[y] of link 3 (m)
x_4 = xout(:, 7);               % COMs[x] of link 4 (m)
y_4 = xout(:, 8);               % COMs[y] of link 4 (m)

for t = 1:length(tout)

    % Cartesian Coordinates of Link 2
    r2_x = [0, r(2)*cos(t_2(t))];
    r2_y = [0, r(2)*sin(t_2(t))];
    % Cartesian Coordinates of Link 3
    r3_x = [r2_x(end), r2_x(end) + r(3)*cos(t_3(t))];
    r3_y = [r2_y(end), r2_y(end) + r(3)*sin(t_3(t))];
    % Cartesian Coordinates of Link 4
    r4_x = [r1_x(end), r1_x(end) + r(4)*cos(t_4(t))];
```

```
    r4_y = [r1_y(end), r1_y(end) + r(4)*sin(t_4(t))];

    % Plot the links, COMs, COM paths
    plot(r1_x, r1_y, ...              % Link 1
         r2_x, r2_y, ...              % Link 2
         r3_x, r3_y, ...              % Link 3
         r4_x, r4_y, ...              % Link 4
         x_2(1:t), y_2(1:t), ...      % Path of link 2 COM
         x_3(1:t), y_3(1:t), ...      % Path of link 3 COM
         x_4(1:t), y_4(1:t), ...      % Path of link 4 COM
         'LineWidth', 2);             % Line Properties
    % COM of Link 1
    viscircles([x_1, y_1], 0.025, 'Color', 'k');
    % COM of Link 2
    viscircles([x_2(t), y_2(t)], 0.025, 'Color', 'k');
    % COM of Link 3
    viscircles([x_3(t), y_3(t)], 0.025, 'Color', 'k');
    % COM of Link 4
    viscircles([x_4(t), y_4(t)], 0.025, 'Color', 'k');
    % Keep the frame consistent
    axis equal;
    axis([-r(2)-0.5, r(1)+r(4)+5, -r(4)-0.5, r(4)+0.5]);

    % Calculate the time step and pause accordingly
    if t ~= length(tout)              % Prevent index error
        % Calculate the time step (s) and store for later use
        t_step(length(t_step)+1) = tout(t+1) - tout(t);
%         pause(t_step(t));             % Assume negligible processing time
    else
        t_step(length(t_step)+1) = 0;
    end

    % Convert the plot frame to an image and store for later use
    image{length(image)+1} = frame2im(getframe(1));

end

% Plot labeling (last frame)
title('Simulation Animation');
xlabel('X Position (m)');
ylabel('Y Position (m)');
legend('Link 1', 'Link 2', 'Link 3', 'Link 4', ...
       'Path of Link 2 COM', 'Path of Link 3 COM', 'Path of Link 4 COM');
```



Export as GIF

The following exports the animation as an animated GIF.

```
file_name = 'CrankRockerAnimation.gif';
for i = 1:length(image)

    % Convert the RGB image to an indexed image
    [A, map] = rgb2ind(image{i}, 256);

    % If first iteration, also run setup code
    if i == 1
        imwrite(A, map, file_name, ...
                'LoopCount', inf, ...
                'DelayTime', t_step(i));

    % Else, append images
    else
        imwrite(A, map, file_name, ...
                'WriteMode', 'append', ...
                'DelayTime', t_step(i));

    end

end
```

Input Power vs. Time

The following plots the input power as a function of time.

The input power can be defined by the input torque times the angular velocity. The area under the curve represents energy because energy is equivalent to the integral of power with respect to time.

The graph of the input power resembles the graph of the input torque, however this is only because the angular velocity of link 2 is constant. By visual observation, the areas under the curve sum to zero. Therefore, energy is conserved over a cycle.

```
T = Fout(:, 9);                 % Input torque (Nm)
P = T*tdot_2;                   % Input power (W)

% Plot
area(tout, P);
title('Input Power vs. Time');
xlabel({'Time (s)'
        ''
        % Figure label
        '\bfFigure 1: \rmInput Power vs. Time'});
ylabel('Input Power (W)');

% Find the area under the curve
A = trapz(tout, P);             % Trapezoidal numerical integration
fprintf("The area under the curve is approximately: ");
fprintf(num2str(A));
fprintf(" J.");
```

```
The area under the curve is approximately: 0.011162 J.
```

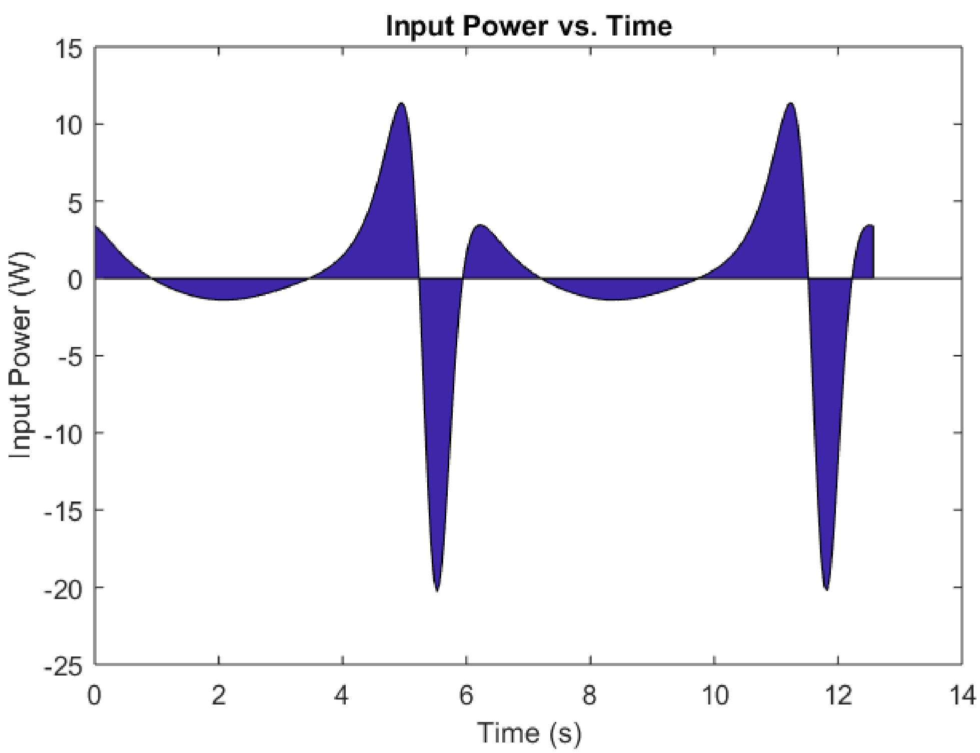


Figure 1: Input Power vs. Time

Contents


```
function [E] = MyPosIC(r, t_2, x)
```

Declare Global Variables

The following declares global variables.

```
global image t_step;
```

Solved Values

See attached file for hand calculations.

```
% Easy access to...
t_3 = x(1);                     % Angular position of link 3
t_4 = x(2);                     % Angular position of link 4

% Find the Error
e_x = r(1) + r(4)*cos(t_4) - r(2)*cos(t_2) - r(3)*cos(t_3);
e_y = r(4)*sin(t_4) - r(2)*sin(t_2) - r(3)*sin(t_3);
E = hypot(e_x, e_y);
```

Error Animation

The following animates the error in the angular positions of the links of the crank-rocker.

```
% Cartesian Coordinates of Link 1
r1_x = [0, r(1)];
r1_y = [0, 0];
% Cartesian Coordinates of Link 2
r2_x = [0, r(2)*cos(t_2)];
r2_y = [0, r(2)*sin(t_2)];
% Cartesian Coordinates of Link 3
r3_x = [r2_x(end), r2_x(end) + r(3)*cos(t_3)];
r3_y = [r2_y(end), r2_y(end) + r(3)*sin(t_3)];
% Cartesian Coordinates of Link 4
r4_x = [r1_x(end), r1_x(end) + r(4)*cos(t_4)];
r4_y = [r1_y(end), r1_y(end) + r(4)*sin(t_4)];

% Plot the links
plot(r1_x, r1_y, ...            % Link 1
     r2_x, r2_y, ...            % Link 2
     r3_x, r3_y, ...            % Link 3
     r4_x, r4_y, ...            % Link 4
     'LineWidth', 2);           % Line Properties
% Keep the frame consistent
axis equal;
axis([-r(2)-0.5, r(1)+r(4)+5, -r(4)-0.5, r(4)+0.5]);
```

```
% Store the time step for later use
t_step(length(t_step)+1) = 0.0001;
% pause(t_step(end));             % Pause for humans

% Convert the plot frame to an image and store for later use
image{length(image)+1} = frame2im(getframe(1));
```

```
end
```

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Contents

Called Functions

Set Values

The following is used to easily change the lengths and masses of the links. (A Grashof mechanism has the constraint R1 + R2 <= R3 + R4).

```
r = [4, 2, 5, 4];               % Length of links 1, 2, 3, 4 (m)
m_2 = 1;                        % Mass of link 2 (kg)
m_3 = 1;                        % Mass of link 3 (kg)
m_4 = 1;                        % Mass of link 4 (kg)
```

Solved Values

The following assigns values derived and/or solved from the given values to variables. See the attached file for hand calculations.

```
% Easy access to...
t_2 = u(1);                     % Angular position of link 2 (rad)
tdot_2 = u(2);                  % Angular velocity of link 2 (rad/s)
t_3 = u(3);                     % Angular position of link 3 (rad)
t_4 = u(4);                     % Angular position of link 4 (rad)
tdot_3 = u(11);                 % Angular velocity of link 3 (rad/s)
tdot_4 = u(12);                 % Angular velocity of link 4 (rad/s)

% Simplicity and Compactness of Notation
c_2 = cos(t_2);
s_2 = sin(t_2);
c_3 = cos(t_3);
s_3 = sin(t_3);
c_4 = cos(t_4);
s_4 = sin(t_4);

I_3 = 1/12*m_3*r(3)^2;          % Moment of inertia of link 3 (kg*m^2)
I_4 = 1/12*m_4*r(4)^2;          % Moment of inertia of link 4 (kg*m^2)

A = [-r(3)*s_3, r(4)*s_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     r(3)*c_3, -r(4)*c_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     r(3)/2*s_3, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     -r(3)/2*c_3, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     0, r(4)/2*s_4, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     0, -r(4)/2*c_4, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0;
     0, 0, m_2, 0, 0, 0, 0, 0, -1, 0, -1, 0, 0, 0, 0, 0, 0;
     0, 0, 0, m_2, 0, 0, 0, 0, 0, -1, 0, -1, 0, 0, 0, 0, 0;
     0, 0, 0, 0, 0, 0, 0, 0, r(2)/2*s_2, -r(2)/2*c_2, -r(2)/2*s_2, r(2)/2*c_2, 0, 0, 0, 0, -1;
     0, 0, 0, 0, m_3, 0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0, 0;
     0, 0, 0, 0, 0, m_3, 0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0;
```

```
     I_3, 0, 0, 0, 0, 0, 0, 0, r(3)/2*s_3, -r(3)/2*c_3, 0, 0, 0, 0, r(3)/2*s_3, -r(3)/2*c_3, 0;
     0, 0, 0, 0, 0, 0, m_4, 0, 0, 0, 0, 0, -1, 0, 1, 0, 0;
     0, 0, 0, 0, 0, 0, 0, m_4, 0, 0, 0, 0, 0, -1, 0, 1, 0;
     0, I_4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -r(4)/2*s_4, r(4)/2*c_4, -r(4)/2*s_4, r(4)/2*c_4, 0];

b = [-r(4)*c_4*tdot_4^2 + r(2)*c_2*tdot_2^2 + r(3)*c_3*tdot_3^2;
     -r(4)*s_4*tdot_4^2 + r(2)*s_2*tdot_2^2 + r(3)*s_3*tdot_3^2;
     -r(2)/2*c_2*tdot_2^2;
     -r(2)/2*s_2*tdot_2^2;
     -r(2)*c_2*tdot_2^2 - r(3)/2*c_3*tdot_3^2;
     -r(2)*s_2*tdot_2^2 - r(3)/2*s_3*tdot_3^2;
     -r(4)/2*c_4*tdot_4^2;
     -r(4)/2*s_4*tdot_4^2;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0];
```

```
Not enough input arguments.

Error in link_solver (line 22)
t_2 = u(1);                     % Angular position of link 2 (rad)
```

Solve for x

Solve for x using mldivide.

```
x = A \ b;
```

```
end
```

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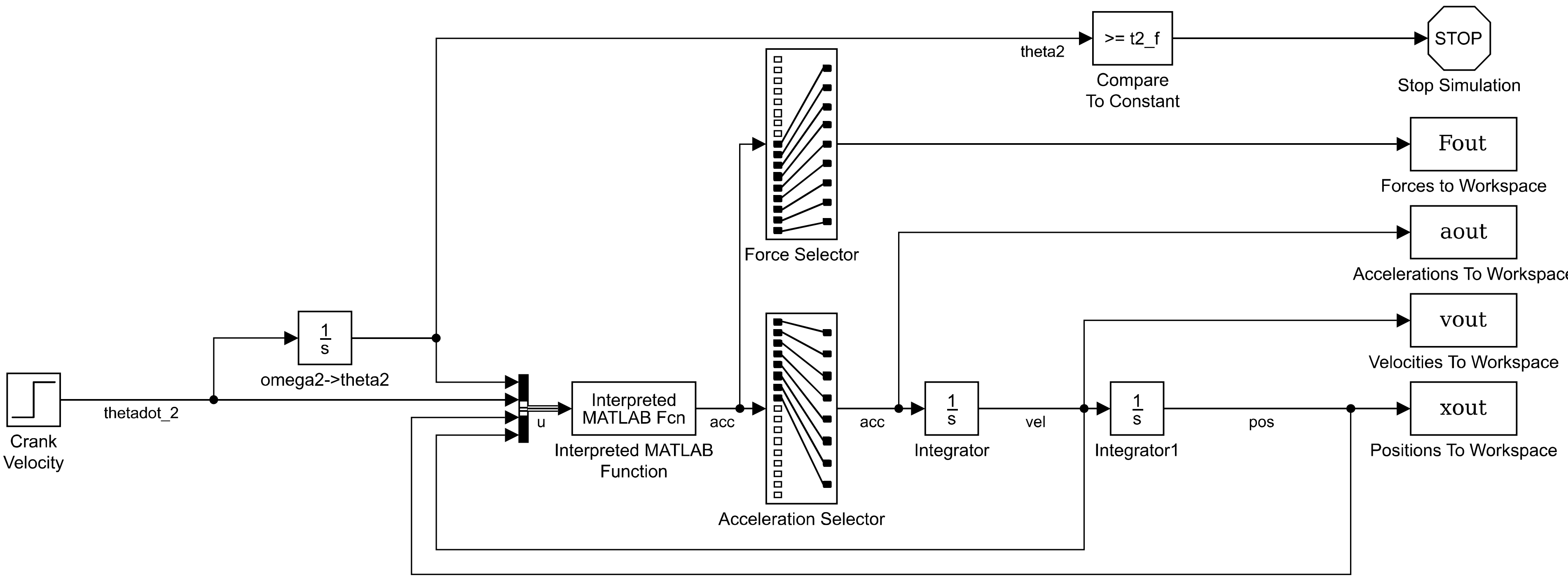
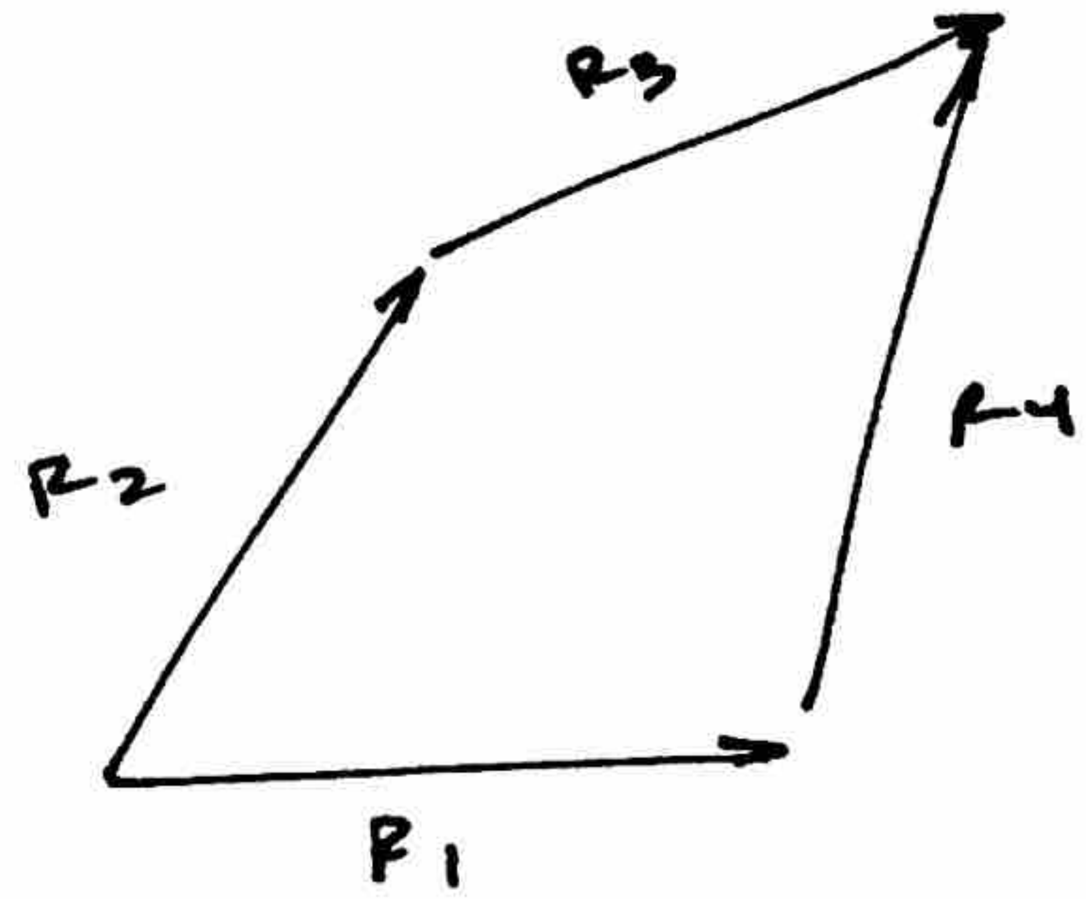


>= t2_f
STOP
theta2
Compare
To Constant
Stop Simulation
Fout
Forces to Workspace
Force Selector
aout
Accelerations To Workspace
vout
Velocities To Workspace
1
s
omega2->theta2
Crank
Velocity
thetadot_2
u
Interpreted
MATLAB Fcn
Interpreted MATLAB
Function
acc
Acceleration Selector
acc
1
s
Integrator
vel
1
s
Integrator1
pos
xout
Positions To Workspace

LAB 6
HAND CALCULATIONS



$$R_2 + R_3 = R_1 + R_4$$

$$0 = R_1 + R_4 - R_2 - R_3$$

① $\hat{i}: 0 = R_1 + R_4 C_4 - R_2 C_2 - R_3 C_3$

② $\hat{j}: 0 = R_4 S_4 - R_2 S_2 - R_3 S_3$

$$\frac{d}{dt}\begin{pmatrix} ① \\ ② \end{pmatrix} \quad \begin{aligned} 0 &= -R_4 S_4 \dot{\theta}_4 + R_2 S_2 \dot{\theta}_2 + R_3 S_3 \dot{\theta}_3 \\ 0 &= R_4 C_4 \dot{\theta}_4 - R_2 C_2 \dot{\theta}_2 - R_3 C_3 \dot{\theta}_3 \end{aligned}$$

$$\frac{d}{dt}\begin{pmatrix} \dot{①} \\ \dot{②} \end{pmatrix} \quad \begin{aligned} 0 &= -R_4 C_4 \dot{\theta}_4^2 - R_4 S_4 \ddot{\theta}_4 + R_2 C_2 \dot{\theta}_2^2 + R_3 C_3 \dot{\theta}_3^2 + R_3 S_3 \ddot{\theta}_3 \\ 0 &= -R_4 S_4 \dot{\theta}_4^2 + R_4 C_4 \ddot{\theta}_4 + R_2 S_2 \dot{\theta}_2^2 + R_3 S_3 \dot{\theta}_3^2 - R_3 C_3 \ddot{\theta}_3 \end{aligned}$$

$\ddot{①}$ $\quad R_4 S_4 \ddot{\theta}_4 - R_3 S_3 \ddot{\theta}_3 = -R_4 C_4 \dot{\theta}_4^2 + R_2 C_2 \dot{\theta}_2^2 + R_3 C_3 \dot{\theta}_3^2$

$\ddot{②}$ $\quad -R_4 C_4 \ddot{\theta}_4 + R_3 C_3 \ddot{\theta}_3 = -R_4 S_4 \dot{\theta}_4^2 + R_2 S_2 \dot{\theta}_2^2 + R_3 S_3 \dot{\theta}_3^2$

$$(3)\quad \hat{i}: \bar{x}_2 = \tfrac{1}{2} R_2 C_2$$
$$(4)\quad \hat{j}: \bar{y}_2 = \tfrac{1}{2} R_2 S_2$$

$$\frac{d}{dt}\begin{pmatrix} (3) \\ (4) \end{pmatrix} \quad \begin{aligned} \dot{\bar{x}}_2 &= -\tfrac{1}{2} R_2 S_2 \dot{\theta}_2 \\ \dot{\bar{y}}_2 &= \tfrac{1}{2} R_2 C_2 \dot{\theta}_2 \end{aligned}$$

$$\frac{d}{dt}\begin{pmatrix} (\dot{3}) \\ (\dot{4}) \end{pmatrix} \quad \begin{aligned} \ddot{\bar{x}}_2 &= -\tfrac{1}{2} R_2 C_2 \dot{\theta}_2^2 \\ \ddot{\bar{y}}_2 &= -\tfrac{1}{2} R_2 S_2 \dot{\theta}_2^2 \end{aligned}$$

$$(\ddot{3})\quad \ddot{\bar{x}}_2 = -\tfrac{1}{2} R_2 C_2 \dot{\theta}_2^2$$
$$(\ddot{4})\quad \ddot{\bar{y}}_2 = -\tfrac{1}{2} R_2 S_2 \dot{\theta}_2^2$$

$$(5)\quad \hat{i}: \bar{x}_3 = R_2 C_2 + \tfrac{1}{2} R_3 C_3$$
$$(6)\quad \hat{j}: \bar{y}_3 = R_2 S_2 + \tfrac{1}{2} R_3 S_3$$

$$\frac{d}{dt}\begin{pmatrix} (5) \\ (6) \end{pmatrix} \quad \begin{aligned} \dot{\bar{x}}_3 &= -R_2 S_2 \dot{\theta}_2 - \tfrac{1}{2} R_3 S_3 \dot{\theta}_3 \\ \dot{\bar{y}}_3 &= R_2 C_2 \dot{\theta}_2 + \tfrac{1}{2} R_3 C_3 \dot{\theta}_3 \end{aligned}$$

$$\frac{d}{dt}\begin{pmatrix} (\dot{5}) \\ (\dot{6}) \end{pmatrix} \quad \begin{aligned} \ddot{\bar{x}}_3 &= -R_2 C_2 \dot{\theta}_2^2 - \tfrac{1}{2} R_3 S_3 \dot{\theta}_3^2 - \tfrac{1}{2} R_3 S_3 \ddot{\theta}_3 \\ \ddot{\bar{y}}_3 &= -R_2 S_2 \dot{\theta}_2^2 - \tfrac{1}{2} R_3 S_3 \dot{\theta}_3^2 + \tfrac{1}{2} R_3 C_3 \ddot{\theta}_3 \end{aligned}$$

$$(\ddot{5})\quad \tfrac{1}{2} R_3 S_3 \ddot{\theta}_3 + \ddot{\bar{x}}_3 = -R_2 C_2 \dot{\theta}_2^2 - \tfrac{1}{2} R_3 S_3 \dot{\theta}_3^2$$
$$(\ddot{6})\quad -\tfrac{1}{2} R_3 S_3 + \ddot{\bar{y}}_3 = -R_2 S_2 \dot{\theta}_2^2 - \tfrac{1}{2} R_3 S_3 \dot{\theta}_3^2$$

$$(7)\quad \bar{x}_4 = R_1 + \tfrac{1}{2} R_4 C_4$$
$$(8)\quad \bar{y}_4 = \tfrac{1}{2} R_4 S_4$$

$$\frac{d}{dt}\begin{pmatrix} (7) \\ (8) \end{pmatrix} \quad \begin{aligned} \dot{\bar{x}}_4 &= \tfrac{1}{2} R_4 S_4 \dot{\theta}_4 \\ \dot{\bar{y}}_4 &= \tfrac{1}{2} R_4 C_4 \dot{\theta}_4 \end{aligned}$$

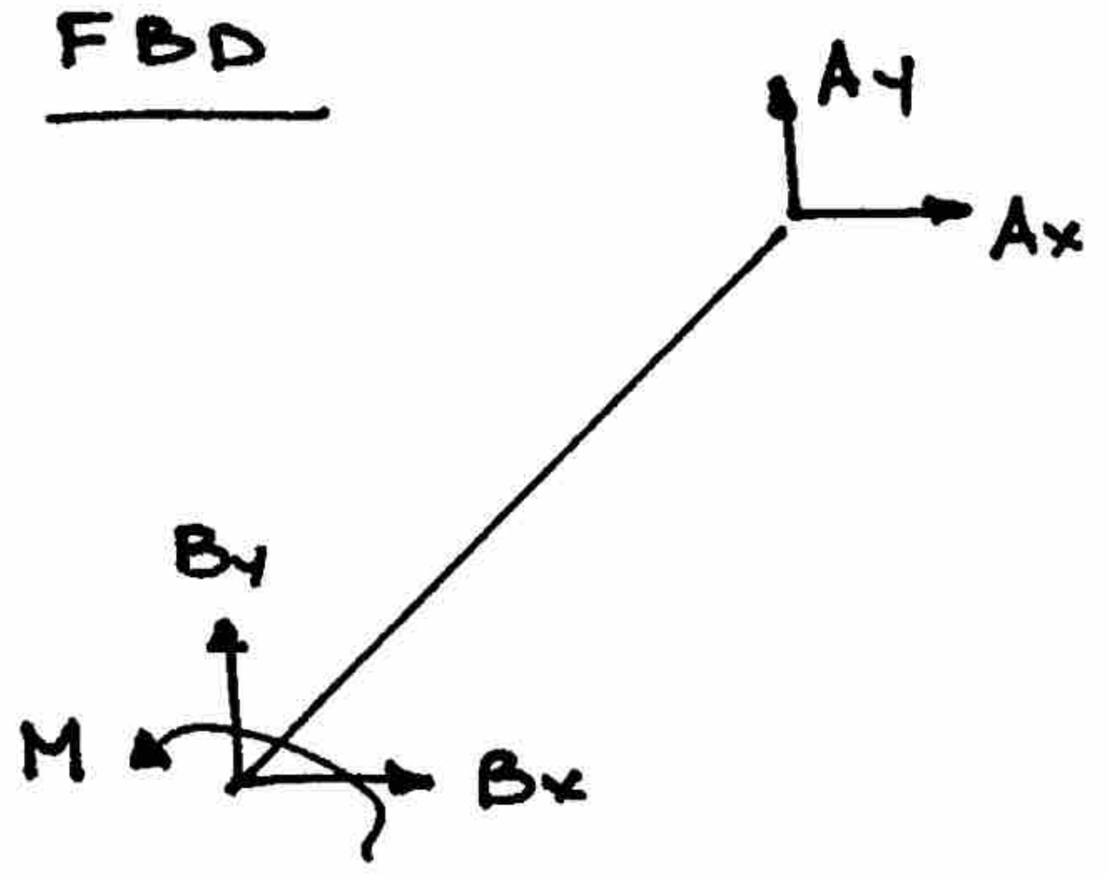
$$\frac{d}{dt}\begin{pmatrix} (\dot{9}) \\ (\dot{8}) \end{pmatrix} \quad \begin{aligned} \ddot{\bar{x}}_4 &= -\tfrac{1}{2} R_4 C_4 \dot{\theta}_4^2 - \tfrac{1}{2} R_4 S_4 \ddot{\theta}_4 \\ \ddot{\bar{y}}_4 &= -\tfrac{1}{2} R_4 S_4 \dot{\theta}_4^2 + \tfrac{1}{2} R_4 S_4 \ddot{\theta}_4 \end{aligned}$$

$$(\ddot{7})\quad \tfrac{1}{2} R_4 S_4 \ddot{\theta}_4 + \ddot{\bar{x}}_4 = -\tfrac{1}{2} R_4 C_4 \dot{\theta}_4^2$$

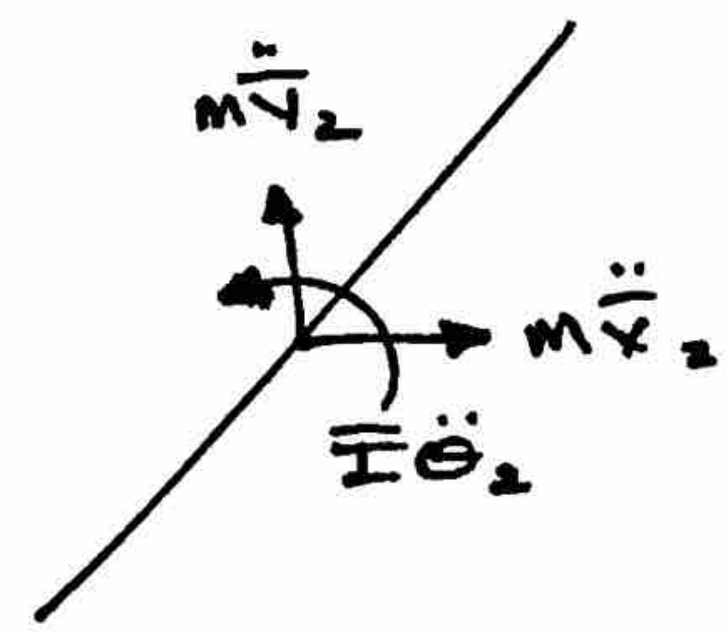
$$(\ddot{8})\quad -\tfrac{1}{2} R_4 C_4 \ddot{\theta}_4 + \ddot{\bar{y}}_4 = -\tfrac{1}{2} R_4 S_4 \dot{\theta}_4^2$$

LINK 2

FBD



KD



(9) $\Sigma F_x: A_x + B_x = m_2\ddot{X}_2$

(10) $\Sigma F_y: A_y + B_y = m_2\ddot{Y}_2$

(11) $\Sigma M: M - A_x \frac{R_2}{2}S_2 + A_y\frac{R_2}{2}C_2 + B_x\frac{R_2}{2}S_2 - B_y\frac{R_2}{2}C_2 = \bar{I}\ddot{\theta}_2$

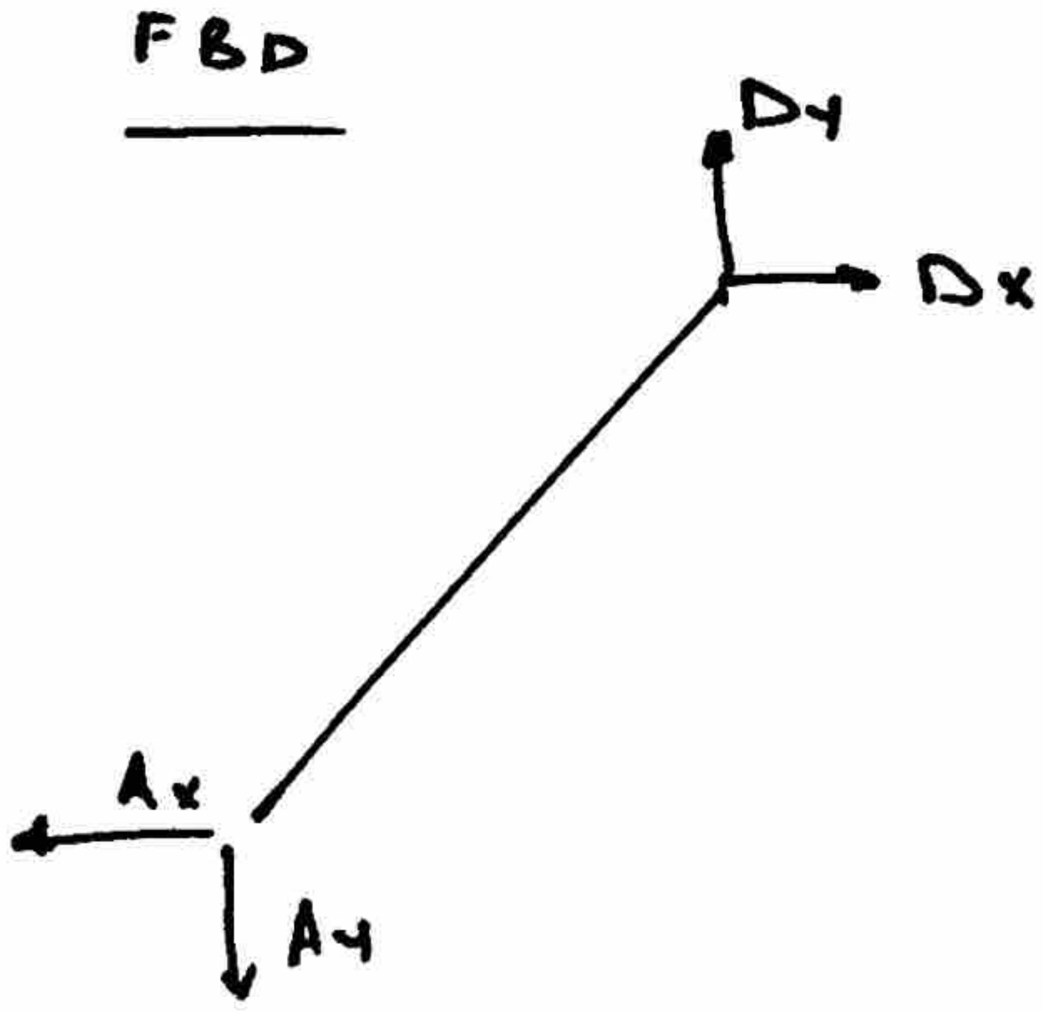
(9) $m_2\ddot{X}_2 - A_x - B_x = 0$

(10) $m_2\ddot{Y}_2 - A_y - B_y = 0$

(11) $\frac{1}{2}R_2S_2A_x - \frac{1}{2}R_2C_2A_y - \frac{1}{2}R_2S_2B_x + \frac{1}{2}R_2C_2B_y - M = 0$

LINK 3

FBD



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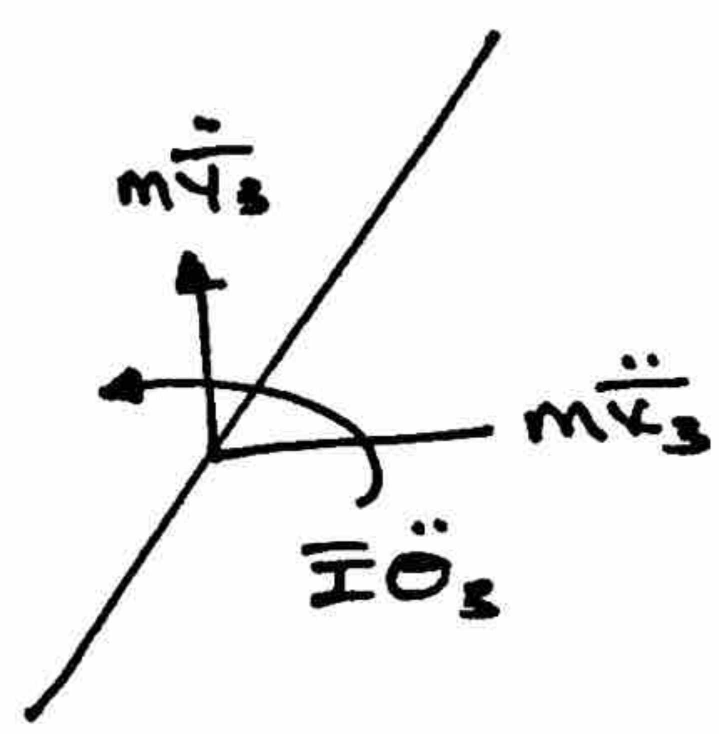


(12) $\Sigma F_x: -A_x + D_x = m_3\ddot{X}$

(13) $\Sigma F_y: -A_y + D_y = m\ddot{Y}$

(14) $\Sigma M: -A_x\frac{R_2}{2}S_3 + A_y\frac{R_3}{2}C_3 - D_x\frac{R_2}{2}S_3 + D_y\frac{R_3}{2}C_3 = \bar{I}\ddot{\theta}_3$

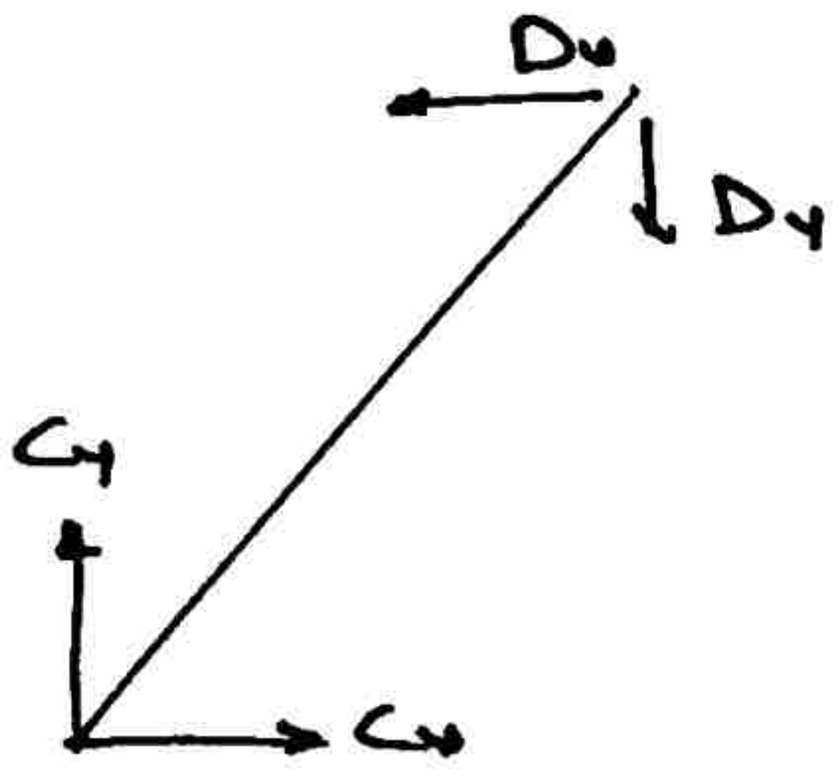
(12) $m_3\ddot{X} + A_x - D_x = 0$

(13) $m_3\ddot{Y} + A_y - D_y = 0$

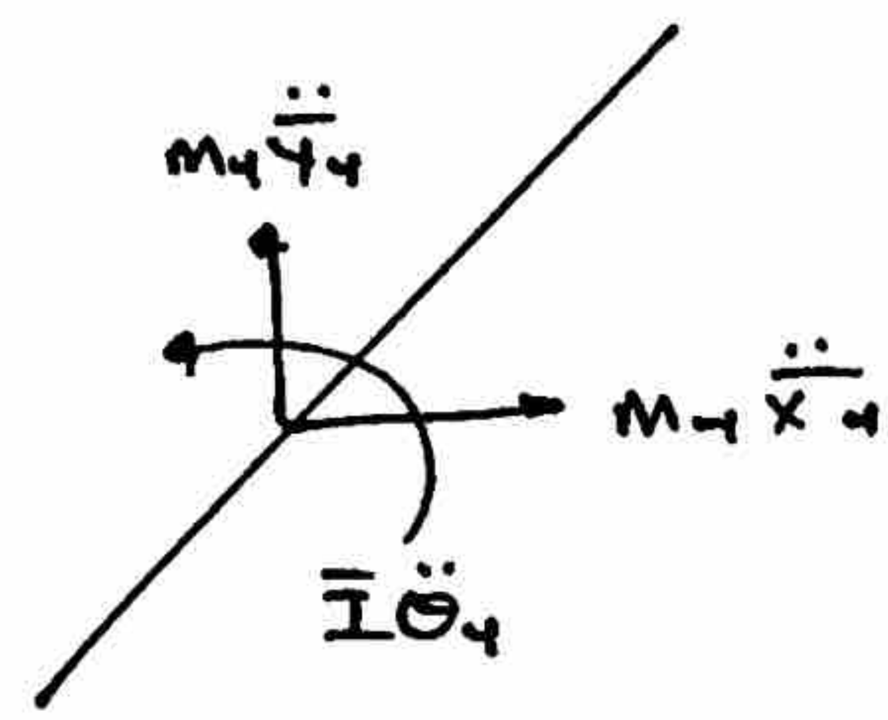
(14) $\bar{I}_3\ddot{\theta}_3 + \frac{1}{2}R_3S_3A_x - \frac{1}{2}R_3C_3A_y + \frac{1}{2}R_3S_3D_x - \frac{1}{2}R_3C_3D_y$

LINK 4

FBD



KD



(15) $\Sigma F_x: C_x - D_x = M_4 \ddot{X}_4$

(16) $\Sigma F_y: C_y - D_y = M_4 \ddot{Y}_4$

(17) $\Sigma M: C_x \frac{R_4}{2} S - C_y \frac{R_4}{2} C_4 + D_x \frac{R_4}{2} S_4 - D_y \frac{R_4}{2} C_4 = \bar{I} \ddot{\theta}_4$

(15) $M_4 \ddot{X}_4 - C_x + D_x = 0$

(16) $M_4 \ddot{Y}_4 - C_y + D_y = 0$

(17) $\bar{I} \ddot{\theta}_4 - \frac{1}{2} R_4 S_4 C_x + \frac{1}{2} R_4 C_4 C_y - \frac{1}{2} R_4 S_4 D_x + \frac{1}{2} R_4 C_4 D_y = 0$