Data Structures & Algorithms

Data Structure :-

* Data structure is a particular way of storing and organizing data in a computer so that it can be used efficiently.

classification of data structure :

* There are two major classification of data structure :-
  1. Linear data structure (link list, stack, Queue)
  2. Non-linear data structure (Tree, graph)

Algorithms :-

* Sequence of steps to solve any given problem
* Algorithms have a definite beginning and a definite end.
* A finite no of steps.

Characteristic of data structure :

Correctness : Data structure implementation should implement its interface correctly.

Time Complexity :- Running time or the execution time of operations of data structure must be as small as possible.

Space complexity :

Need for data structure :

Searching data

Need to manage processor speed.

Execution time cases :

Worst Case

Average Case

Best Case

Arrays

* An array is a finite collection of similar elements stored in adjacent memory locations.
* By finite, we mean that there are specific no. of elements in an array.
* By similar, we mean that all the elements in an array are of same type.
e.g., any array may contain all integers or all characters but not both.
* an array consisting of n number of elements is referenced using an index that varies from 0 to (n-1).
* The elements of A[n] containing n elements are denoted by A[0], A[1] --- A[n-1],
where 0 is the lower bound.
n-1 is the upper bound.
* In general, the lowest index of an array is called its lower bound and highest index of an array is called its upper bound.
* The number of elements in the array is called its range.

* Thus, an array is a set of pairs of an index & a value. For each index, there is a value associated with it.

a[0]	a[1]	a[2]	a[3]	a[4]	a[5]	a[6]
34	1	5	-6	12	9	2

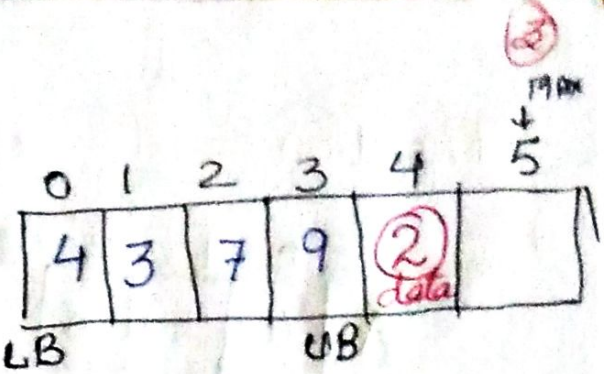
* An array is further categorized as a one-dimensional array & multi-dimensional array.
* A multi-dimensional array can be a 2-D array, 3-D array, 4-D array etc. Whether an array is a 1-D or 2-D can be judged by the syntax used to declare the array.

A[5] :- A 1-D array holding 5 elements.
A[2][5] :- A 2-D array with 2 rows & 5 columns holding 10 elements.
A[2][5][3] :- A 3-D array with two 2-D array each of which is having 5 rows and 3 columns., thus holding totally 30 elements.

Operations on Array :-

* Insertion
* Deletion
* traversal
* Search

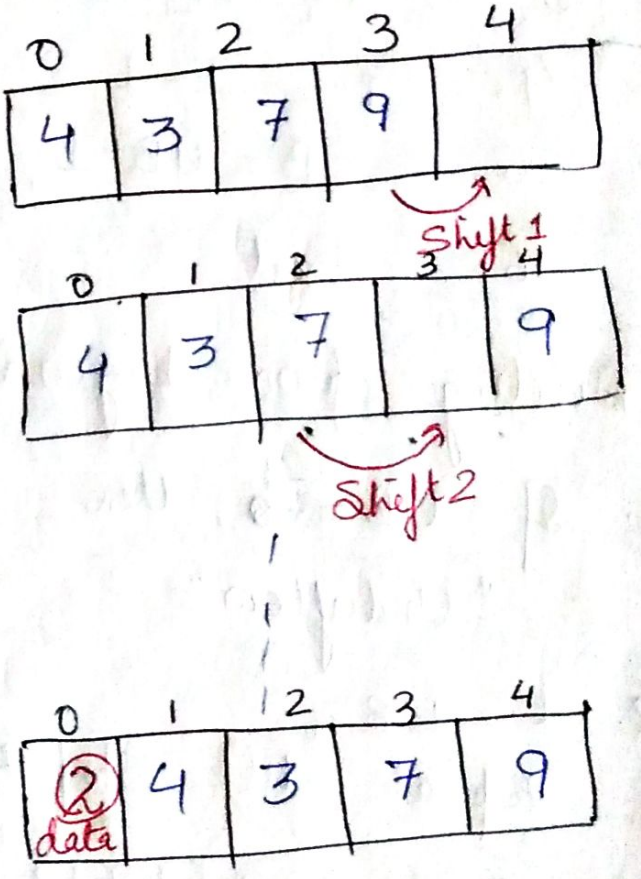
Insertion at End :-



Let A be an array of size N.

1. If UB = ~~MAX-1~~ N-1
   then "overflow"
2. Read data or value to be inserted
3. Set UB = UB + 1
4. A[UB] = data
5. Stop or Exit

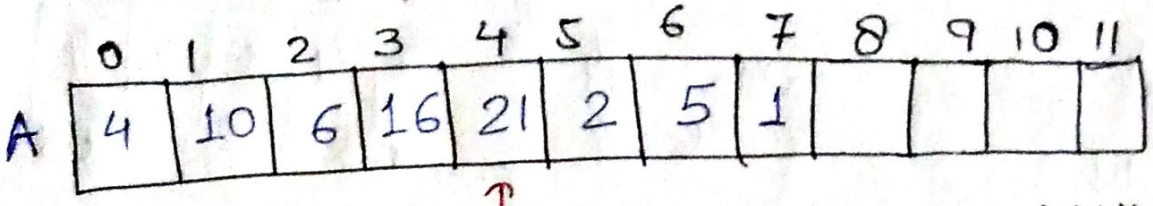
Insertion at Beginning :-



Let A be an array of size N.

1. If UB = ~~MAX-1~~ N-1
   then "overflow"
2. Else Read data
3. ~~UB~~ K = UB
4. Repeat step (5)
   while K ≥ LB
5. A[K+1] = A[K]
6. K = K-1
7. A[LB] = data
8. Stop

Insertion at specific location :-



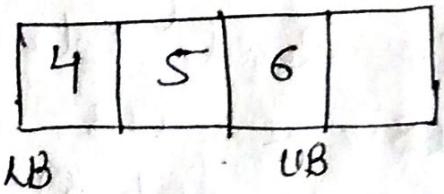
MAX = 11
UB = 7
loc = 4
data = 100

7 to 4 ~~Shifted~~ → 8 to 5
UB to loc (UB+1) to (loc+1)

1. Initialize a counter I = ~~MAX~~ UB
2. Repeat step 4 while (I >= ~~loc~~)
3. A [I+1] = A [I]
4. I = I - 1
5. UB = UB + 1, A [loc] = data
6. Exit

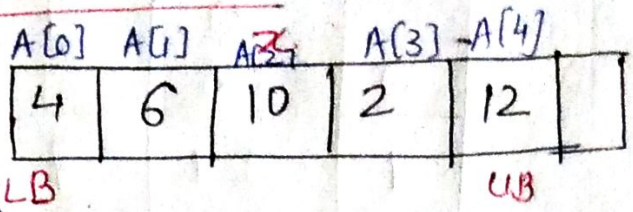
Deleting from beginning

1. If UB = ~~0~~ -1, then "Underflow"



2. K = LB
3. Repeat till Step 5 while K < UB
4. A [K] = A [K+1]
5. K = K + 1
6. A [UB] = NULL
7. UB = UB - 1
8. Exit

Deletion of a particular element :-



1. If UB = -1, "Underflow"
2. Read data
3. K = LB
4. Repeat step (5) while A[K] ≠ data
5. K = K + 1
6. Repeat step (7) while K < UB
7. A[K] = A[K+1]
8. K = K+1

Representation of 2-D arays in memory :-

* A 2-D array is a collection of elements placed in m rows & n columns.
* The syntax used to declare a 2-D array includes two subscripts, of which one specifies the no. of rows & the other specifies the no. of columns of an array.
* e.g; A[3][4] is a 2-D array containing 3 rows & 4 columns. & A[0][2] is an element placed at 0th row & 2nd column.

* 2-D array is also called a matrix.

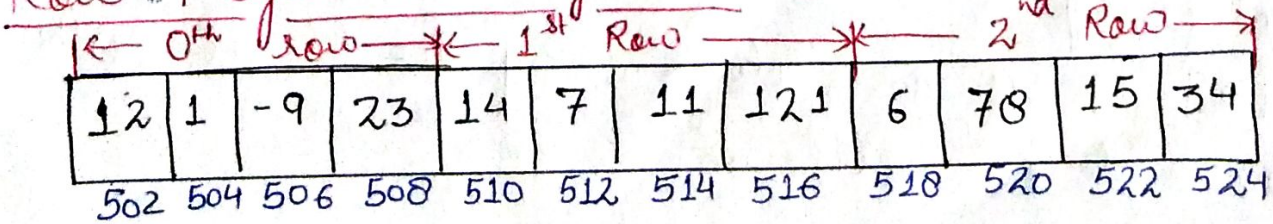
	0	1	2	3
0	12	1	-9	23
1	14	7	11	121
2	6	78	15	34

Row Major & column Major Arrangement:

* Row & columns of a matrix are only a matter of imagination. when a matrix gets stored in memory all elements of it are stored linearly.
* Since computer's memory can only be viewed as consecutive units of memory.
* This leads to two possible arrangements
  - → Row-Major arrangements
  - → column-Major arrangements

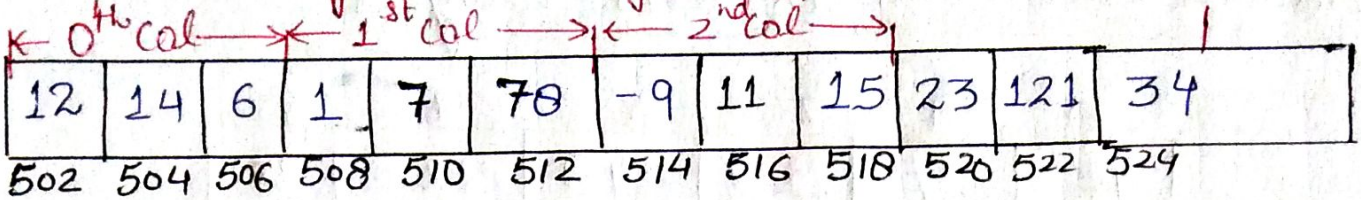
```
int A[3][4] = {
                {12, 1, -9, 23},
                {14, 7, 11, 121},
                {6, 78, 15, 34},
              };
```

Row Major Arrangement :-



+ UB = UB - 1

Column-Major Arrangement :-



* Since the array elements are stored in adjacent memory locations, we can access any element of the array once we know the base address of the array & no. of rows & columns present in the array.
* e.g., if base address of the array is 502 & we wish to refer the element 121, then the calculation involved as follows:-

Row-Major Arrangement :-

* Element 121 is present at A[1][3].

Hence location of 121 would be

$= 502 + 1 * 4 + 3 = 516$

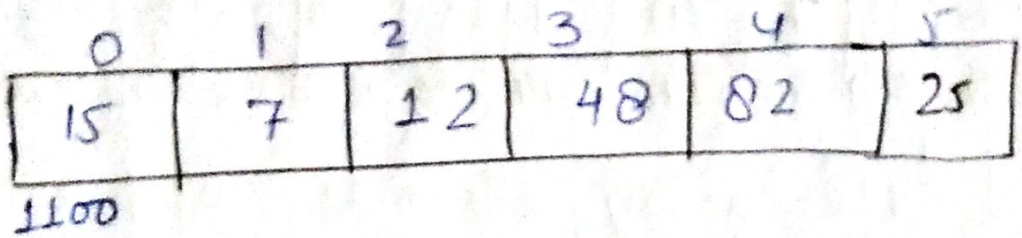
In general, for an array A[m][n] the address of element A[i][j] would be

Baseaddress + i * n + j

Ily. for column major arrangement

Base address + j * m + i

Address Calculation in 1-D array :-



address of $A[i] = B + W * (i - LB)$

B → Base address of array
W → Storage size of one element
I → Subscript of element whose address is to be found
LB → Lower Bound of subscript, if not specified assume 0 (zero)

Question :- Given base address of an array B[1300 .... 1900] as 1020 & size of each element is 2 bytes. Find address of B[1700]

the given values are $B = 1020$, $W = 2$, $LB = 1300$, $I = 1700$

Ans :- 1020

Address of A[i][j] = B + W * [N * (I - Lr) + (J - Lc)]
{Row Major order}

Address of A[i][j] = B + W * [(I - Lr) + M * (J - Lc)]
{column Major order}

where,

B → Base Address

I → Row Subscript of element whose add. is to be found.

J → column " " " " " " "

W → Storage Size of one element

Lr → Lower limit / start row index, if not given assume zero.

Lc → Lower limit / start column index, if not given assume zero

M → No. of rows of given matrix

N → No. of columns " " "

Note:- Usually no. of rows & columns of a matrix are given like A[20][30] etc. but if not given as A[Lr ---- Ur][Lc ... Uc] then the no. of rows & columns are calculated using :

No. of rows (M) = (Ur - Lr) + 1

No. of cols (N) = (Uc - Lc) + 1

Examples :- X[-15 --- 10, 15 --- 40] requires one byte of storage. If beginning location is 1500, Determine the location of X[15][20].

Solution :- M = [10 - (-15)] + 1 = 26

N = [40 - 15] + 1 = 26

(i) Column Major Calculation :-

B = 1500, w = 1, I = 15, J = 20, Lr = -15, Lc = 15, M = 26

Address of A[I][J] = B + w * [(I - Lr) + M * (J - Lc)]

= 1500 + 1 * [(15 - (-15)) + 26 * (20 - 15)]

= 1500 + 1 * [30 + 26 * 5]

= 1500 + 1 * [160]

= 1660

Stacks :-

* The linear data structures as array and a linked list allow us to insert & delete an element at any place in the list.
* However, sometimes it is required to permit the addition or deletion of elements only at one end that is either beginning or end.
* Stack and Queues are two types of data structures in which addition or deletion of an element is done at end, rather than middle.
* A stack is a data structure in which addition of a new element or deletion of an existing element always takes place at the same end.
* This end is often known as top of stack.
* e.g., A stack of plates in cafeteria in where every new plate is added to the stack is added at top.
* similarly, every new plate taken off the stack is also from top.

→ When an item is added to the stack, the operation is called push and when an item is removed from the stack, the operation is called pop.

→ Stack is also called as last-in-first-out (LIFO) list.

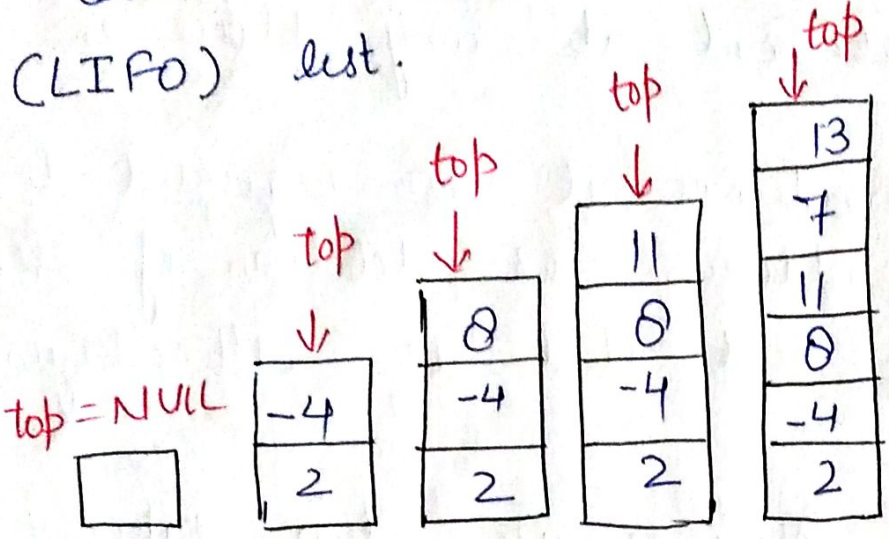


fig :- Representation of Stack after inserting elements

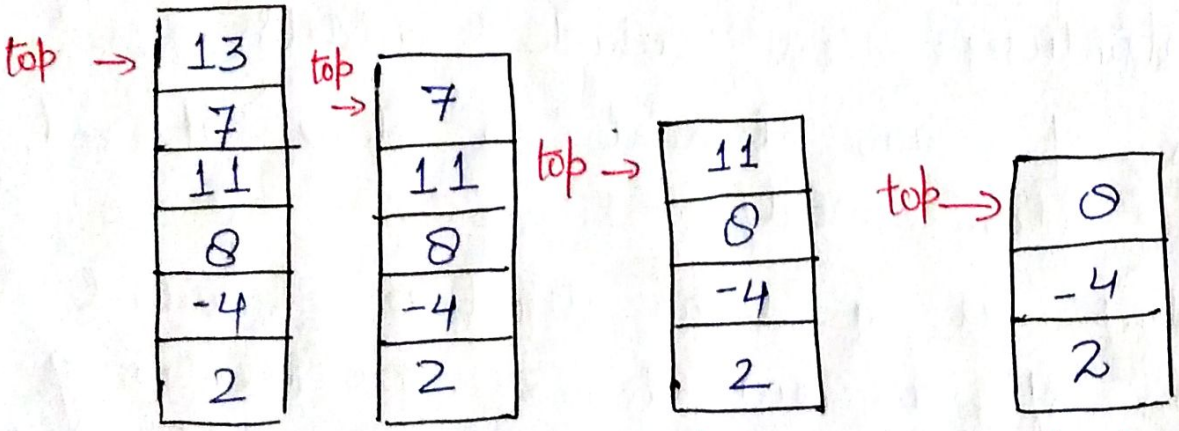


fig :- Representation of Stack after deletion

Operations on Stack :-

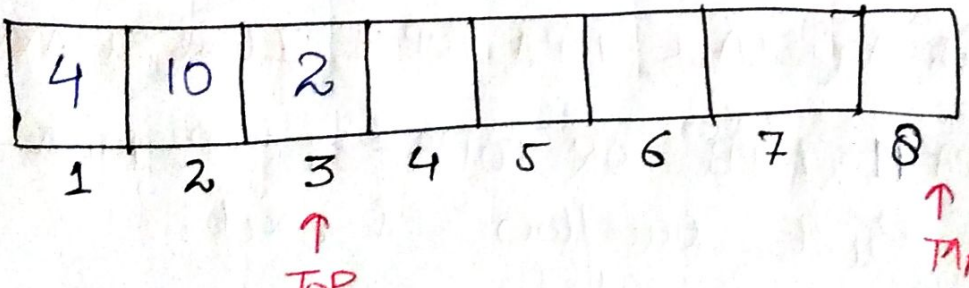
Stack is generally implemented with two basic operations

* Push
* Pop

Representation of Stack as Array :-

* Stack contain an ordered collection of elements. An array is used to store ordered list of elements., Hence it would be very easy to manage a stack if we represent it using an array.

* Each of our Stacks will be maintained by
→ a linear array STACK; → a pointer variable TOP, which contains the location of the top element of the stack.
→ A variable MAXSTK which gives the maximum no. of elements that can be held by the stack.
→ The condition TOP = 0 or TOP = NULL will indicate that the stack is empty.



PUSH (STACK, TOP, MAXSTK, ITEM)

(This algo. pushes an item onto a stack)

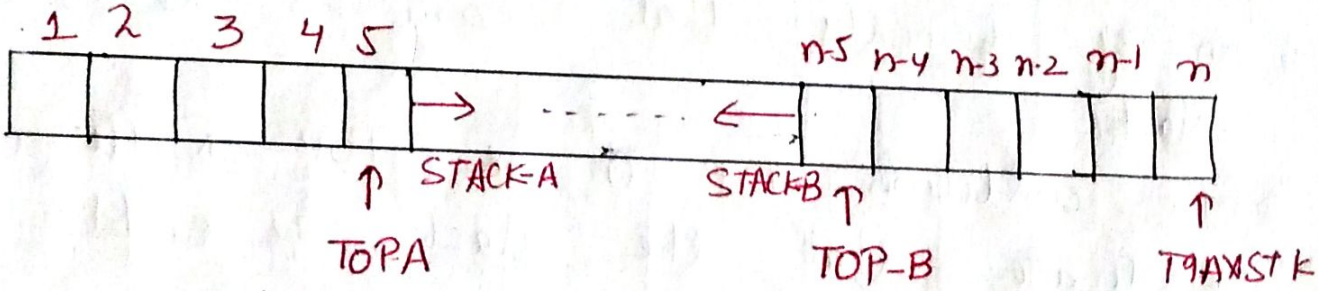
1. If TOP = MAXSTK then Print Overflow & Return
   [stack already filled]
2. Set TOP = TOP + 1 [Increase Top by 1]
3. Set STACK [TOP] = ITEM [Insert item]
4. Return.

POP (STACK, TOP, ITEM)

This algo. deletes the top element of STACK & assigns it to the variable ITEM.

1. If TOP = 0 then Print UNDERFLOW & Return
   [Stack has an item to be removed?]
2. Set ITEM = STACK [TOP]. [Assigns TOP element to ITEM]
3. Set TOP = TOP - 1 [decreases TOP by 1]
4. Return

Multiple Stack Implementation using Single Array :-



Algo. Push-A (STACK, TOPA, TOPB, Item, MAXSTK)

1. If (TOPA + 1 = TOPB OR TOPB == 1 OR TOPA = MAXSTK then Print "Overflow" & Exit.
2. TOPA = TOPA + 1
3. STACK [TOPA] = Item
4. Exit

Algo PushB (STACK, TOPA, TOPB, item, MAXSTK)

1. if (TOPA+1 = TOPB OR TopA = MAXSTK OR TOPB = 1)
   Print overflow & exit
2. if (TOPB = 0) then TOPB = MAXSTK
3. Else TOPB = TOPB - 1
4. Set STACK[TOPB] = Item
5. Exit

Algo POPA (STACK, TOPA, Item)

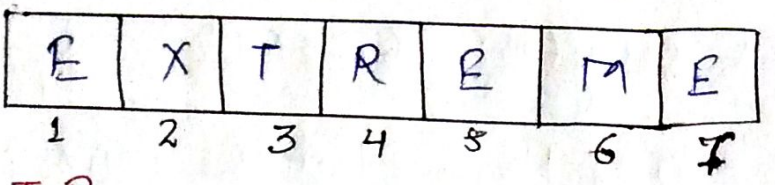
1. If (TOPA = 0) then Print "Underflow" Exit.
2. Else item = STACK[TOPA]
3. TopA = TOPA - 1
4. Exit

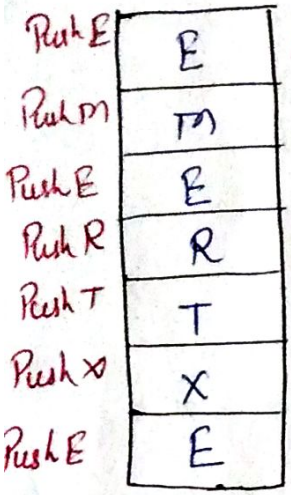
Algo POP B (STACK, TOPB, Item, MAXSTK)

1. If TOPB = 0 Print "Underflow" & exit
2. Else { Item = STACK[TOPB]
3. if (TOPB = MAXSTK) then TOPB = 0 }
4. Else TOPB = TOPB + 1
5. Exit

Stack Applications

① Reversing a String





Algo :- 1. Traverse the list and push all its values onto a stack.

2. Traverse the list from TOP element & POP a value & connect them in reverse order.

② Factorial Calculation :-

Recursion :- When a function calls itself.

```
main(){
    fun();
}
fun(){
    fun()
}
```

direct recursion

```
main(){
    a();
}
a(){
    b();
}
b(){
    a();
}
```

Indirect recursion

e.g. factorial of a number

$4! = 4 * 3!$ } Recursive condition $[n * (n-1)!]$

$3! * 2!$

$2 * 1!$

$1 * 0!$

1 } Base Condition / Stop condition

```
#include <stdio.h>
int fact (int n) {
    if (n == 0)
        return 1;
    else
        return (n * fact (n-1));
}

void main()
{
    int num, result;
    printf("Enter a no");
    scanf("%d", &num);
    result = fact(num);
    printf("%d", result);
}
```

LIFO (Stack)

main()

Executed last → fact(3) ← Called first

3 * fact(2)

2 * fact(1)

1 * fact(0)

1

Polish Notation

* The process of writing the operators of an expression either before their operands or after them is called Polish Notation.

* The fundamental property is that the order in which the operations are to be performed is completely determined by the positions of the operators and operands in the expressions.

* One never needs parenthesis when writing expression in Polish Notation. These are classified in three categories :-

  (i) Infix
  (ii) Prefix
  (iii) Postfix

* Infix :- When operators exist between two operands, the expression is called Infix.

* Prefix :- When operators are written before their operands, the expression is called prefix polish notation.

* Postfix :- When the operators come after their operands the resulting expression is called the reverse polish notation or Postfix notation.

e.g., $z + (y * x - (w / v \uparrow u) * t) * s$

By Inspection & Hand :-

$\Rightarrow z + (y * x - (w / v u \uparrow) * t) * s$ ①

$\Rightarrow z + (y * x - w v u \uparrow / * t) * s$ ②

$\Rightarrow z + (y x * - w v u \uparrow / t *) * s$ ③

$\Rightarrow z + y x * w v u \uparrow / t * - * s$ ④

$\Rightarrow z + y x * w v u \uparrow / t * - s *$ ⑤

$\Rightarrow z y x * w v u \uparrow / t * - s * +$ ⑥

Stack Implementation of Infix to Postfix

Label No.	Symbol Scanned	Stack	Expression
1.	z	(	z
2.	+	(+	z
3.	(	(+ (	z
4.	y	(+ (	z y
5.	*	(+ (*	z y
6.	x	(+ (*	z y x
7.	−	(+ (−	z y x *
8.	(	(+ (− (	z y x *
9.	ω	(+ (− (	z y x * ω
10.	/	(+ (− (/	z y x * ω
11.	v	(+ (− (/	z y x * ω v
12.	↑	(+ (− (/ ↑	z y x * ω v
13.	u	(+ (− (/ ↑	z y x * ω v u
14.	)	(+ (−	z y x * ω v u ↑ /
15.	*	(+ (− *	z y x * ω v u ↑ /
16.	t	(+ (− *	z y x * ω v u ↑ / t
17.	)	(+	z y x * ω v u ↑ / t * −
18.	*	(+ *	z y x * ω v u ↑ / t * −
19.	s	(+ *	z y x * ω v u ↑ / t * − s
20.	)		z y x * ω v u ↑ / t * − s * +

Postfix Expression

Algo for converting Infix Expression To Postfix

Postfix (Q, P) :-

Q is an arithmetic expression within Infix

P is an equivalent postfix.

1. Push "(" onto stack & add ")" to the end of Q.
2. Scan Q from L to R & repeat till (6) for each element of Q until the stack is empty.
3. If an operand is encountered, push it onto P.
4. If a left paranthesis is encountered, push it onto Stack.
5. If an operator is encountered then
   (a) Add it to stack.
   (b) Repeatedly Pop from Stack & add P each operator which has the same precedence as or higher than operator.
6. If the right parenthesis is encountered :-
   (a) Repeatedly Pop from Stack & add it to P each operator until a left parenthesis is encountered.
   (b) Remove left parenthesis.
7. Exit.

Evaluating the Postfix Expression

Example :- P :- 5, 6, 2, +, *, 12, 4, /, -,)
↓ Sentinel

Symbol Scanned	Stack
5	5
6	5, 6
2	5, 6, 2
+	5, 8
*	40
12	40, 12
4	40, 12, 4
/	40, 3
-	37

Algorithm :-

→ This algo. finds the value of an arithmetic expression P written in postfix notation

→ This algo. uses Stack to hold operands to evaluate P.

→ (1) Add a right parenthesis ")" at the end of P.

(2) Scan P from L to R & repeat steps 3 & 4.

~~(2) If an operand is encountered until ")"~~

for each element of P until ")" is encountered.

(3) If an operand is encountered, put it on Stack.

(4) If an operator is encountered, then

(A) Remove the top two elements of Stack, where A is the top element & B is the next-to-top element.

(B) Evaluate B ⊗ A.

(C) Place the result of (B) on the Stack

[End of step ② loop]

⑤ Set value equal to the top element on Stack.

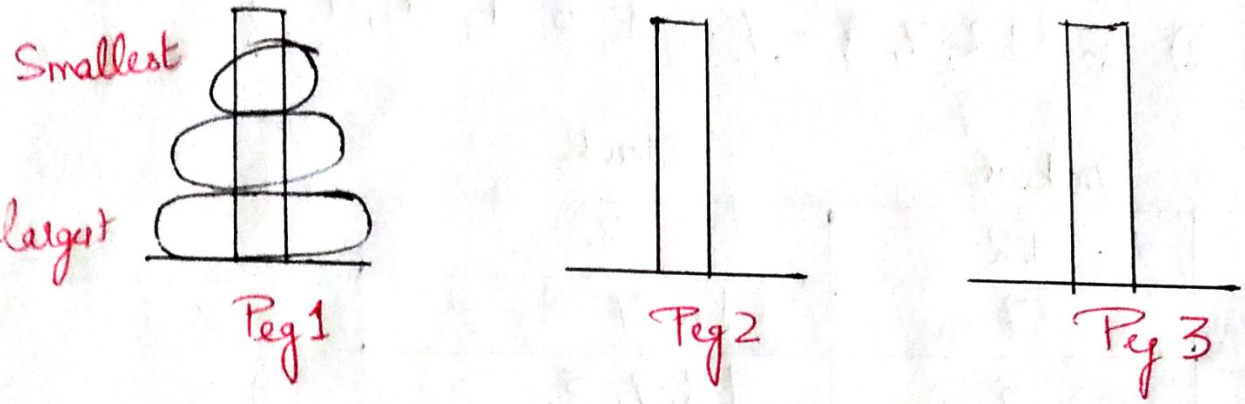
⑥ Exit.

Example ② :- 12, 7, 3, −, /, 2, 1, 5, +, *, +, } Sentinel

Symbol	Stack
12	12
7	12, 7
3	12, 7, 3
−	12, 4
/	3
2	3, 2
1	3, 2, 1
5	3, 2, 1, 5
+	3, 2, 6
*	3, 12
+	15

Tower Of Hanoi :-

* Tower of Hanoi is a mathematical puzzle which consists of three towers (pegs) and more than one rings is as depicted -



* These Rings are of different sizes & stacked upon in an ascending order i.e., smaller one is placed over the larger one.

Note :- There are other variations of the puzzle where the no. of disks increase, but the tower count remain the same.

Rules :- The mission is to move all the disks to some tower without violating the sequence of arrangement. A few rules are :-

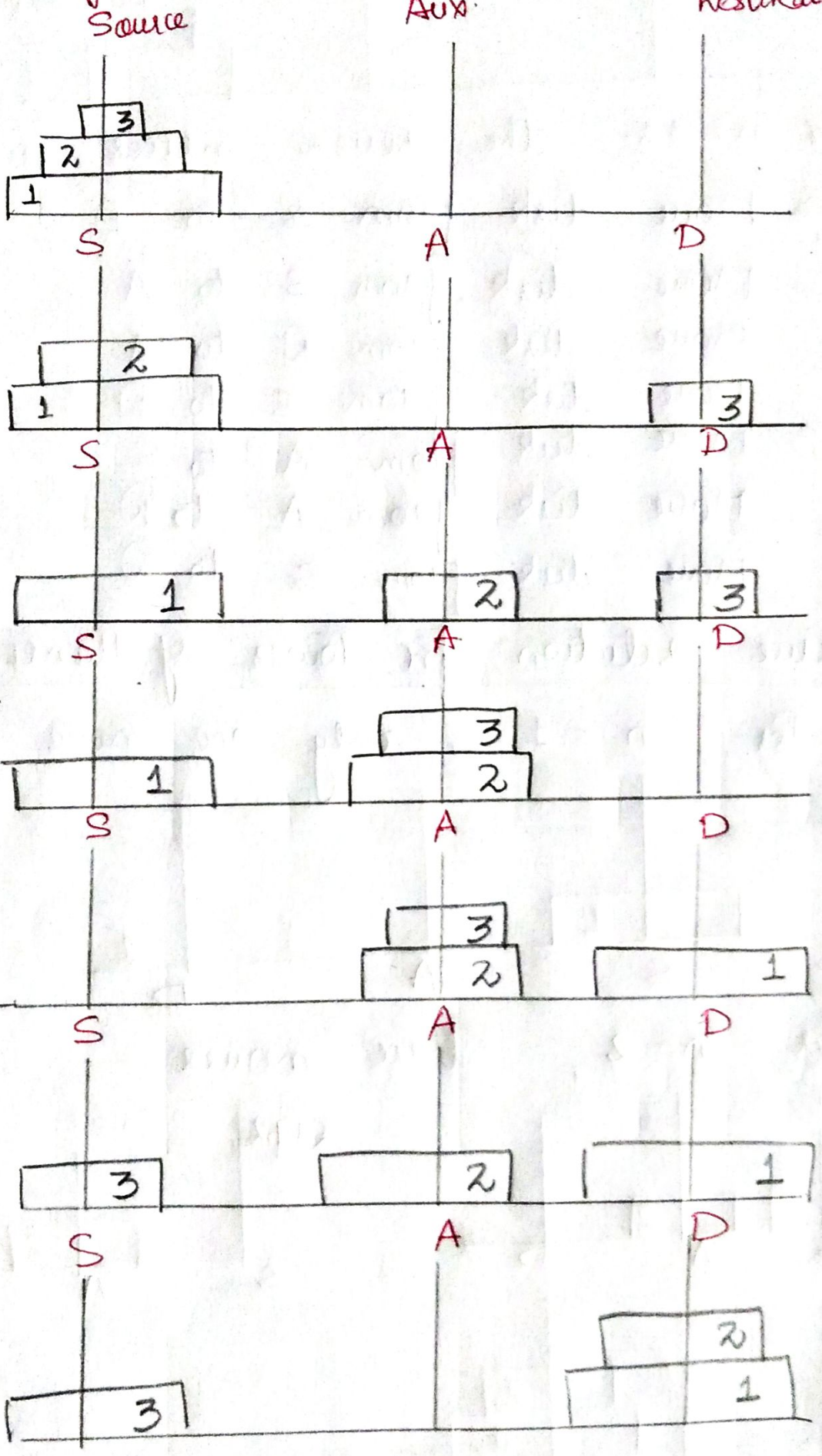
→ Only one disk can be moved among the towers at any given time.

→ Only the top disk can be removed.

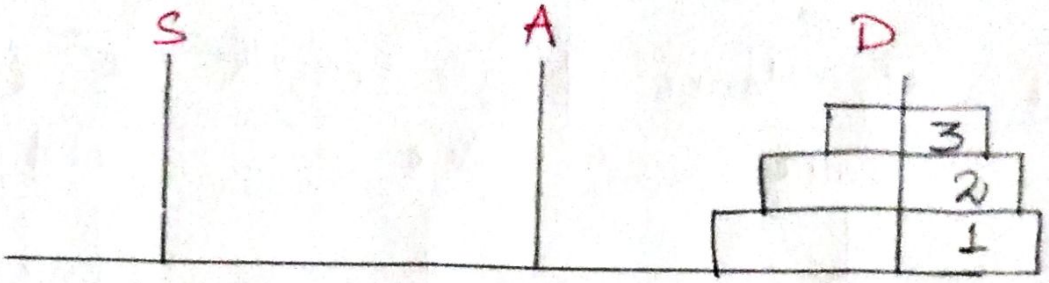
→ NO large disk can be placed over the small disk.

Tower of Hanoi puzzle with 3 disks

Problem:



Step 7 :-

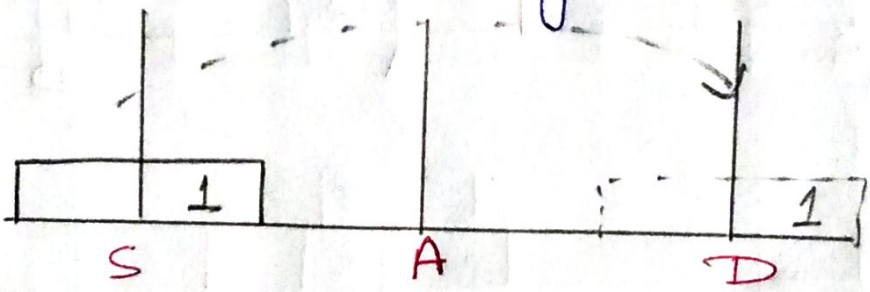


Thus for $n = 3$:- the seven moves will be;

→ Move disk from S to D
Move disk from S to A
Move disk from D to A
Move disk from S to D
Move disk from A to S
Move disk from A to D
Move disk from S to D

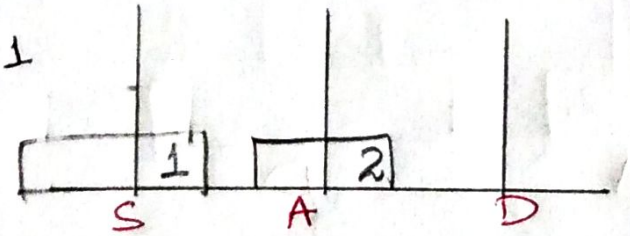
Recursive solution for Tower of Hanoi Problem :-

* For $n = 1$, only one move

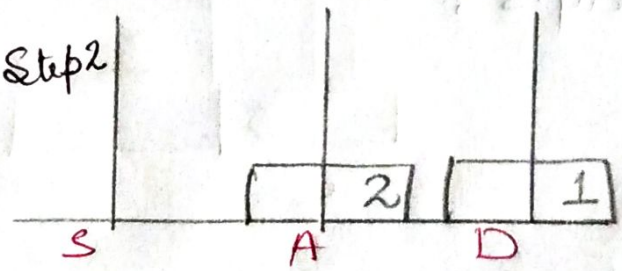


* For $n = 2$, three moves

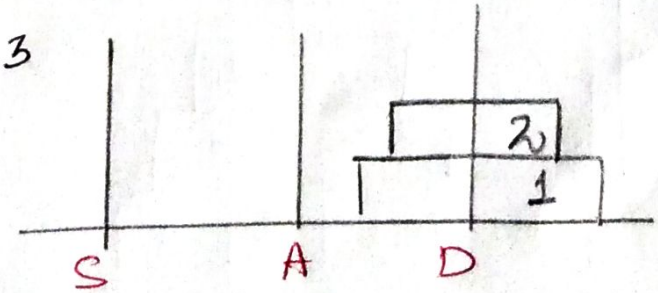
Step 1



Step 2



Step 3



* Rather than finding a separate solution for each value of n, we'll use the technique of recursion to develop a general solution.

* First we observe that the solution to the Towers of Hanoi problem for $n > 1$ disks may be reduced to the following subproblems:-

1) move top (n-1) disks from S to A.
2) move top disk from S to D.
3) move top (n-1) disks from A to D.

TOWER (N, S, A, D)

1) if N = 1 then
   print ("move disk from S to D").
2) Else TOWER (N-1, S, D, A)
3) print ("move disk from S to D")
4) TOWER (N-1, A, S, D)

TOWER (4, S, A, D)