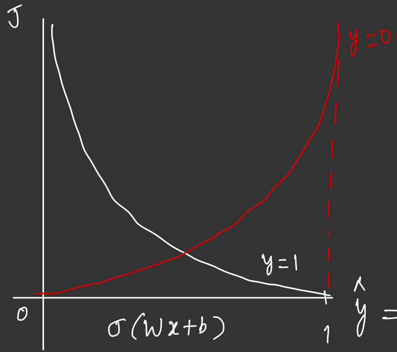
MLIS-2 : Problem Sheet 1

Q1.

$$J(\hat{y}, y) = \begin{cases} -\log \hat{y} & \text{if } y = 1 \\ -\log(1-\hat{y}) & \text{if } y = 0 \end{cases}$$



$$\hat{y} = \sigma(wx+b)$$

BCE LOSS

$$P(y|x) = \hat{y}^{y}(1-\hat{y})^{1-y}$$

if $y = 1$

$$P(y|x) = \hat{y}$$

if $y = 0$

$$P(y|x) = 1 - \hat{y}$$

Maximize

$$\log P(y|x) = y \log \hat{y} + (1-y)\log(1-\hat{y})$$

Minimize

$$L_{CE}(\hat{y}, y) = -\log P(y|x) = -[y \log \hat{y} + (1-y)\log(1-\hat{y})]$$

or

$$L_{CE}(\hat{y}, y) = -[y \log \sigma(w.x+b) + (1-y)\log(1-\sigma(wx+b))]$$

$$J = \frac{1}{N}\sum -[y \log \sigma(wx+b) + (1-y)\log(1-\sigma(wx+b))]$$

here, $$\frac{\partial J}{\partial W} = \frac{\partial J}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial W}$$

$$\frac{\partial J}{\partial \hat{y}} = -y \cdot \frac{1}{\hat{y}} + (1-y) \cdot \frac{1}{1-\hat{y}}$$

$$= \frac{-y}{\hat{y}} + \frac{1-y}{1-\hat{y}}$$

$$\frac{\partial \hat{y}}{\partial W} = \hat{y}(1-\hat{y})x$$

$$\therefore \frac{\partial J}{\partial W} = \left(\frac{-y}{\hat{y}} + \frac{1-y}{1-\hat{y}}\right)\hat{y}(1-\hat{y})x$$

$$= [-y(1-\hat{y}) + \hat{y}(1-y)]x$$

$$= (-y + \cancel{y\hat{y}} + \hat{y} - \cancel{y\hat{y}})x$$

In vector form

$$\frac{\partial J}{\partial W} = \nabla_W J = \frac{\sum(\hat{y} - y)x^T}{N}$$