Thermo

Recitation #3 (02/14/2023)

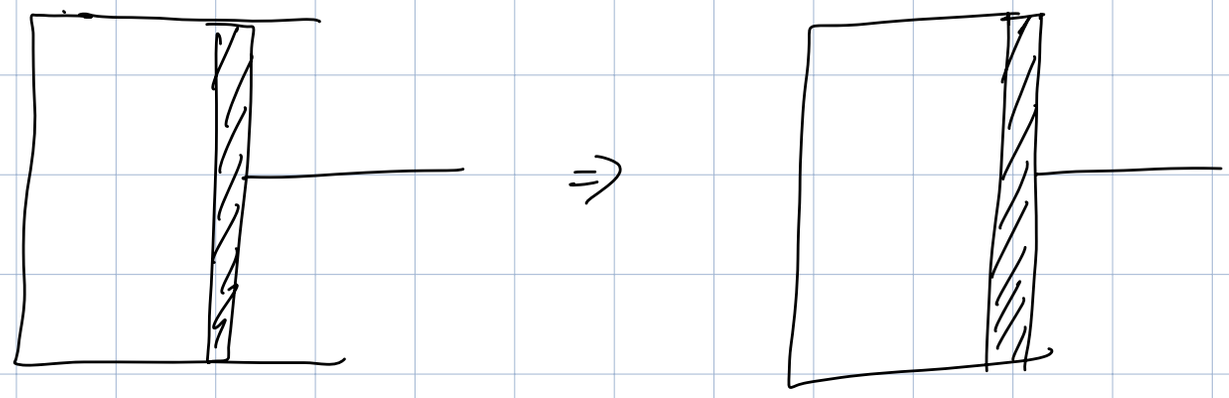
Q1

initial

$n = 1$ mol

$T_i = 500K$

$P_i = 0.6 MPa$



final

$n = 1$ mol

$T_2 = ?$

$P_2 = 0.1 MPa$

Find: ΔU, ΔH, Q, W_{EC}

⇒ **Assumptions:**

Ideal gas

Closed system

Rigid boundaries

(a) <u>isothermal</u> : $\therefore \Delta T = 0$

$\therefore T_1 = T_2 = 500\ K$

we know, $\Delta U = C_V \cancel{\Delta T}^{0} = 0$

& $\Delta H = C_p \cancel{\Delta T}^{0} = 0$

Also, $\because \Delta U = Q + W_{EC}$

$\therefore Q = -W_{EC}$

Now, for W_{EC} :

$$\underline{W}_{EC} = -\int_{\underline{V_1}}^{\underline{V_2}} P\,dV = nRT \int_{\underline{V_1}}^{\underline{V_2}} \frac{d\underline{V}}{\underline{V}}$$

$P = \frac{nRT}{V}$

$$\therefore \quad \underline{W}_{EC} = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

we know, $\frac{V_2}{V_1} = \frac{P_1}{P_2}$ (This correlation was derived in last recitation class)

$$\therefore \quad \underline{W}_{EC} = -nRT \ln\left(\frac{P_1}{P_2}\right)$$

$$= -1 \times 8.314 \times 500 \times \ln\left(\frac{0.6}{0.1}\right)$$

$$\cancel{mol} \quad \frac{J}{\cancel{mol}\,\cancel{K}} \quad \cancel{K}$$

$$\therefore \quad \underline{W}_{EC} = -7448.34\ J = -Q$$

(b) <u>Isochoric</u> : Const. Volume

$$\Delta V = 0$$

$$\therefore \; W_{EC} = -\int p\,\cancel{dV}^{\,0} = 0$$

$$\because \; \Delta U = Q + \cancel{W_{EC}}^{\,0}$$

$$\therefore \; \Delta U = Q$$

Now, $\Delta U = C_v \underbrace{\Delta T}_{(T_2 - T_1)}$

$$\therefore \; \Delta U = \frac{3R}{2}(83.3 - 500)$$

$$\therefore \; \Delta U = -5196.67 \; J/mol$$

$$\therefore \; \Delta \underline{U} = n\,\Delta U = 1 \times \Delta U = -5196.67 \; J$$

$$PV = nRT$$

$$\therefore \; \frac{P_1}{T_1} = \underbrace{\frac{nR}{V}}_{\text{Const.}} = \frac{P_2}{T_2}$$

$$\therefore \; T_2 = \frac{P_2 T_1}{P_1} = \frac{0.1 \times 500}{0.6}$$

$$T_2 = 83.3 \, K$$

$\therefore \quad Q = \Delta U = -5196.67 \text{ J/mol}$

$$\Delta H = n C_p \Delta T = 1 \times \frac{5R}{2} \times (-416.6)$$

$$\Delta H = -8661.11 \text{ J}$$

(e) <u>Adiabatic</u> : $Q = 0\ J$

$\because \Delta U = \cancel{Q}^{\,0} + W_{EC}$

$\therefore \Delta U = W_{EC}$

$\therefore n C_v \Delta T = -P \Delta V$

taking limit & integrating.

$$\therefore \int n C_v \, dT = -\int P \, dV$$

$$\therefore n C_v \int dT = -nR \int \frac{T \, dV}{V}$$

$$\therefore n C_v \int_{T_1}^{T_2} \frac{dT}{T} = -nR \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\therefore \ln\left(\frac{T_2}{T_1}\right)^{C_v/R} = \ln\left(\frac{V_1}{V_2}\right)$$

$$\therefore \left(\frac{T_2}{T_1}\right)^{C_v/R} = \frac{V_1}{V_2}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_v}$$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{C_v/C_p}$$

(derived in last recitation class)

$$\therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{\cancel{C_v}} \cdot \frac{\cancel{C_v}}{C_p}}$$

$$\therefore \quad T_2 = \left(\frac{P_2}{P_1}\right)^{R/C_p} \cdot T_1$$

$$= \left(\frac{0.1}{0.6}\right)^{2/5} \times 500$$

$$\therefore \quad T_2 = 244.18 \text{ K}$$

$$\therefore \quad \Delta T = 244.18 - 500$$

$$\Delta T = -255.8 \text{ K}$$

$$\therefore \quad \underline{\Delta U} = n C_v \Delta T$$

$$= 1 \times \frac{3R}{2} (-255.8)$$

$$\therefore \quad \underline{\Delta U} = -3190 \text{ J} = W_{EC}$$

$$\underline{\Delta H} = n C_p \Delta T$$

$$= 1 \times \frac{5R}{2} (-255.8)$$

$$\underline{\Delta H} = -5317 \text{ J}$$

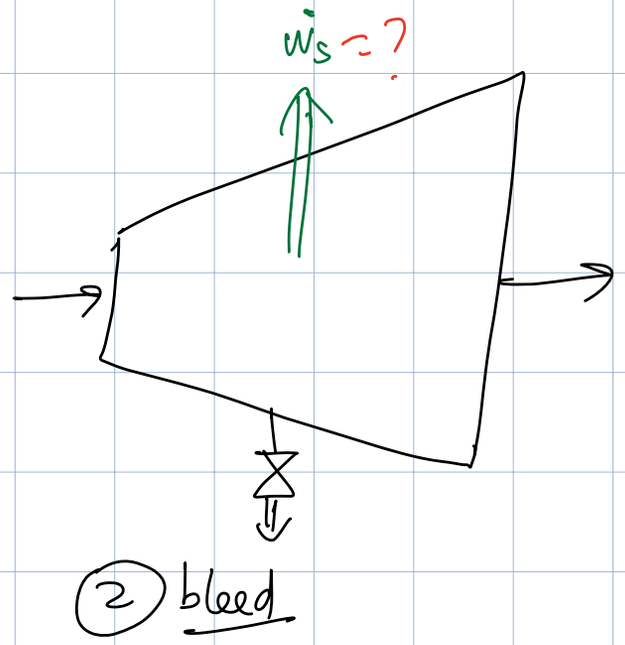
Q2

(1) in

$P_1 = 15 MPa$

$T_1 = 600 °C$

$\dot{m}_1 = 100 kg/s$



(3) out

$P_3 = 75 kPa$

$P_3 = 0.075 MPa$

$q = 90\%$

$\dot{m}_3 = ?$

$P_2 = 2 MPa$

$T_2 = 350 °C$

$\dot{m}_2 = 20 kg/s$

=> Assumption: Adiabatic → $\dot{Q} = 0$

steady state

open system => $\dot{W}_{EC} = 0$

Mass balance

$$\sum \dot{m}_{out} = \sum \dot{m}_{in}$$

$$\dot{m}_3 + \dot{m}_2 = \dot{m}_1$$

$$\therefore \quad \dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

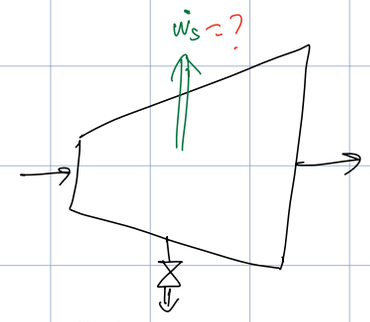
$$= 100 - 20$$

$$\therefore \quad \dot{m}_3 = 80 \text{ kg/s}$$

Energy balance

$$0 = \dot{Q} + \dot{W}_s + \dot{m}_1 H_1 - (\dot{m}_1 H_3 + \dot{m}_2 H_2)$$

$$\therefore \quad \dot{W}_s = \dot{m}_3 H_3 + \dot{m}_2 H_2 - \dot{m}_1 H_1$$



① in

$P_1 = 15 \text{ MPa}$

$T_1 = 600\,^\circ C$

$\dot{m}_1 = 100 \text{ kg/s}$

② bleed

$P_2 = 2 \text{ MPa}$

$T_2 = 350\,^\circ C$

$\dot{m}_2 = 20 \text{ kg/s}$

③ out

$P_3 = 75 \text{ kPa}$

$P_3 = 0.075 \text{ MPa}$

$q = 90\%$

$\dot{m}_3 = ?$

① Superheated steam (Table III)

$H_1 = 3583.1 \text{ kJ/kg}$

② Superheated.

$H_2 = 3137.7 \text{ kJ/kg}$

(3)

	P (MPa)	H^L (kJ/kg)		ΔH^{vap} (kJ/kg)	
x_1	0.07	376.75	y_1	2282.67	y_1
x	0.075	___	y	___	y
x_2	0.08	391.71	y_2	2273.47	y_2

For H^L @ 0.075 MPa. (use y in green)

interpolation formular: $y = m(x - x_1) + y_2$, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

(For interpolation review refer recitation #1 notes)

$$\therefore y = \frac{391.71 - 376.75}{0.08 - 0.07}(0.075 - 0.08) + 391.71$$

$\therefore y = 384.23 \text{ kJ/kg}$

$\therefore H^L @ 0.075 \text{ mPa} = 384.23 \text{ kJ/kg}$

Now, For ΔH^{vap} @ 0.075 mPa (use y in pink)

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2$$

$$y = \frac{2273.47 - 2282.67}{0.08 - 0.07} \times (0.075 - 0.08) + 2273.47$$

$\therefore y = 2278.07 \text{ kJ/kg}$

$\therefore \Delta H^{vap} @ 0.075 \text{ mPa} = 2278.07 \text{ kJ/kg}$

	P (MPa)	H^L (kJ/kg)		ΔH^{vap} (kJ/kg)	
x_1	0.07	376.75	y_1	2282.67	y_1
x	0.075	384.23	y	2278.07	y
x_2	0.08	391.71	y_2	2273.47	y_2

$$H_3 = H^L + q \cdot \Delta H^{vap} = H^L + q\left(H^V - H^L\right)$$

$$\therefore H_3 = 384.73 + 0.9\left(2278.07\right) = 2434.49 \text{ kJ/kg}$$

$$\therefore \quad \dot{W}_s = \dot{m}_3 H_3 + \dot{m}_2 H_2 - \dot{m}_1 H_1$$

$$= 80 \times 2434.5 + 20 \times 3137.7 - 100 \times 3583.1$$

$$\therefore \quad \dot{W}_s = -100{,}796.56 \text{ kJ/s} = -100.8 \text{ MW}$$

$$H_3 = (1-q) H^L + q H^V$$

$$H_3 = H^L - H^L q + q H^V$$

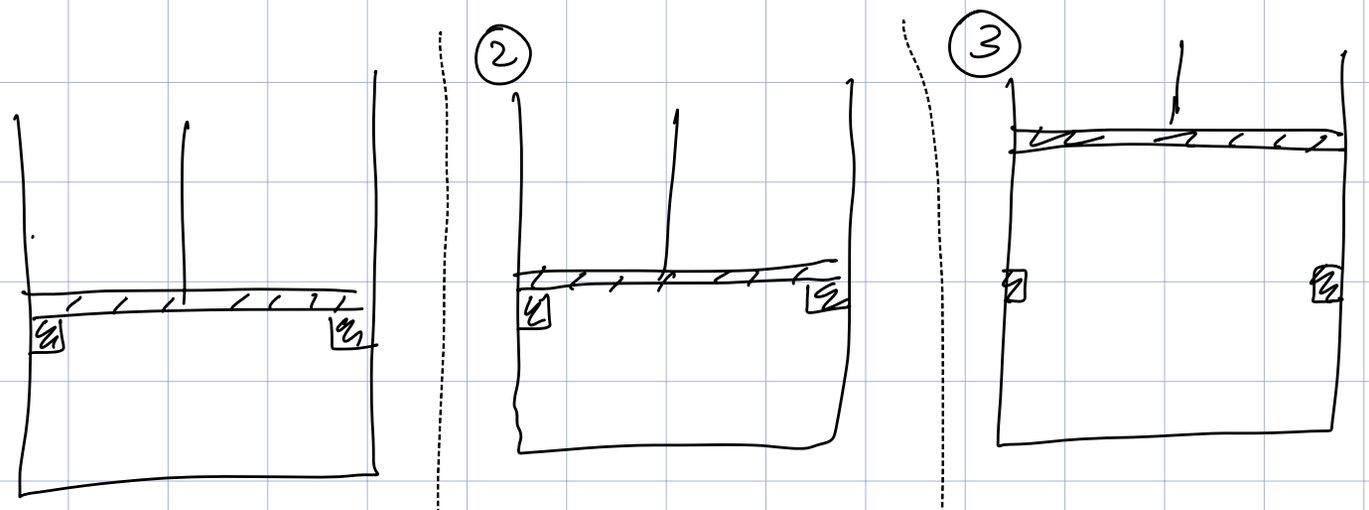
$$H_3 = H^L + q(H^V - H^L)$$

$$H_3 = H^L + q \Delta H^{vap}$$

$$\therefore \quad H_3 = (1-q) H^L + q H^V = H^L + q \Delta H^{vap}$$

both these equation mean the same & are thus valid.

3/



$V = 0.5\ m^3$

$P_1 = 200\ kPa$

$q = 100\%$

$V_2 = V_1$

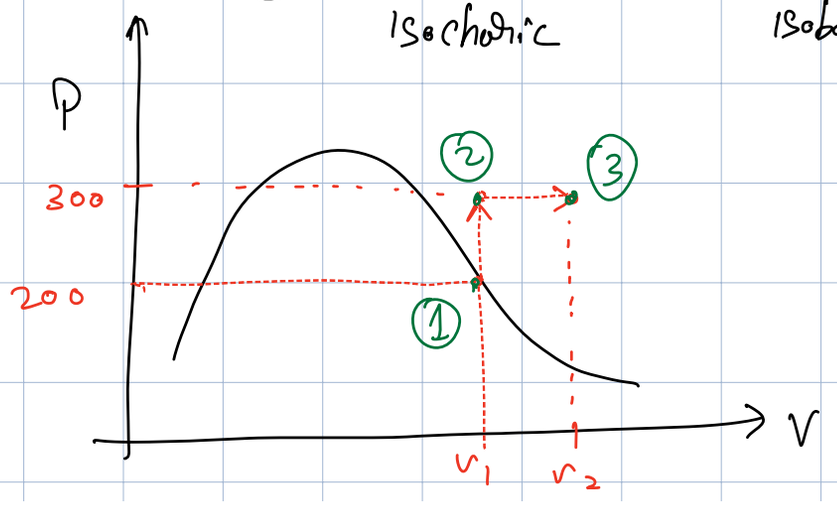
$P_2 = 300\ kPa$

$V_3 = 2V_1$

$P_3 = 300\ kPa$

$T_3 = ?$

$$1 \xrightarrow[\text{isochoric}]{} 2 \xrightarrow[\text{isobaric}]{} 3$$



Assume : closed system, $\Delta m = 0$

Reversible expansion, $W_s = 0$

$\therefore W = W_{EC}$

Energy balance

$1 \rightarrow 2$

$$\Delta U_{12} = Q_{12} + W_{EC,12}$$

$$\Delta U_{12} = Q_{12}$$

$2 \rightarrow 3$

$$\Delta U_{23} = Q_{23} + W_{EC\,23} \leftarrow W_E$$

$1 \rightarrow 3$

$$\Delta U_{13} = \underbrace{Q_{12} + Q_{23}}_{Q_T} + W_{EC}$$

$$\therefore \Delta U_{13} = (U_3 - \cancel{U_2} + \cancel{U_2} - U_1)$$

$$Q_T = \Delta U_{13} - W_{EC}$$

T^{sat} @ $0.2\,MPa = 120.21\,°C$

Mass of gas in ① $= m = \dfrac{V_1}{v^v} = \dfrac{0.5\,m^3}{0.8857\,m^3/kg}$

$\therefore m = 0.5645\,kg$

$u_1 = u^v = 2529.09\,kJ/kg$

$u_3 = ?$

$$\underline{W}_{EC} = -\int_{V_2}^{V_1} P\,dV = -P\,\Delta V_{23} = -300 \times 10^3 (1 - 0.5)$$

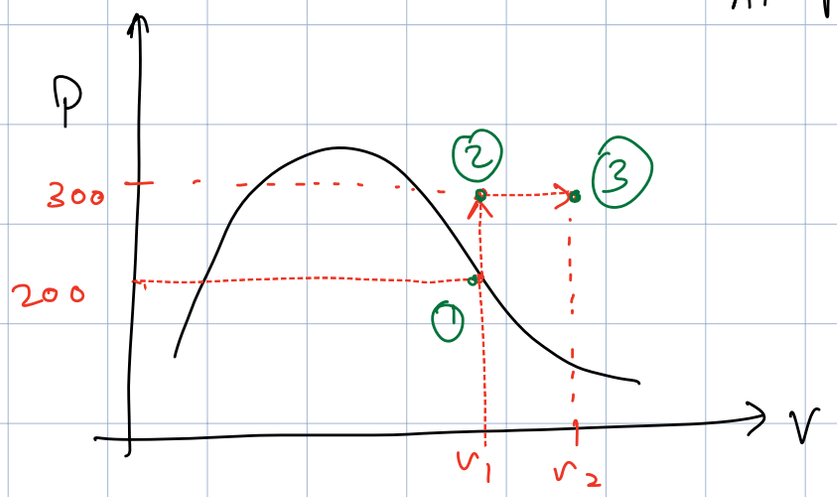
$\underline{W}_{EC} = -150\,kJ$

$$\therefore \quad Q_T = m\,\Delta u_{13} - W_{ec}$$

$u_3 - u_1$

need to find.

Find U_3



At point 3 ⇒ we have Superheated steam.

$$V_3 = 2V_1 = 1.7714 \ m^3/kg$$

Now finding the temperatures which have a v^v value closer to this

	$T(°C)$	$V\ (m^3/kg)$	
y_1	850	1.7271	x_1
y		1.7714	x ← we need to find T for which V has this value.
y_2	900	1.8042	x_2

interpolation:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2$$

$$= \frac{900 - 850}{1.8042 - 1.7271}(1.7714 - 1.8042) + 900$$

$$y = 878.73\ °C$$

$$T_3 = 878.73\ °C$$

Now, we need to find v_3 at T_3:

	T (°C)	v (m^3/kg)	u	
x_1	850	1.7271	3759.3	y_1
x	878.73	1.7714	3814.86	y ← need value of u_3 at $T_3 = 878.73°C$
x_2	900	1.8042	3856.0	y_2

interpolation:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2$$

$$\therefore \quad y = \frac{3856 - 3759.3}{900 - 850}(878.73 - 900) + 3856.0$$

$$\therefore \quad y = 3814.86 \ kJ/kg$$

$$\therefore \quad u_3 = 3814.86 \ kJ/kg$$

Now,

$$Q_T = m\,\Delta u_{13} - \underline{w}_{EC}$$

$$= 0.5645\,(3814.9 - 259.1) - (-150)$$

$$\therefore Q_T = 875.85 \text{ kJ}$$

Q4

$T_1 = 450°C = 723.15\ K$ $\xrightarrow{\text{state change}}$ $T_2 = 150°C = 423.15K$

$P_1 = 3.5\ MPa$ $\qquad$ $P_2 = 0.3\ MPa$

Superheated. $\qquad$ Superheated.

(a) Steam table

(1) $V_1 = 0.0920\ m^3/kg$

$U_1 = 3016.1\ kJ/kg$

$H_1 = 3338.0\ kJ/kg$

(2) $V_2 = 0.6340\ m^3/kg$

$U_2 = 2571.0\ kJ/kg$

$H_2 = 2761.2\ kJ/kg$

$\therefore \quad \Delta V = 0.542\ m^3/kg$

$\Delta U = -445.1\ kJ/kg$

$\Delta H = -576.8\ kJ/kg$

(b) <u>Ideal gas assumption</u>

$$V_1 = \frac{RT_1}{P_1} \quad , \quad V_2 = \frac{RT_2}{P_2}$$

$$\therefore \Delta V = V_2 - V_1 = R\left(\frac{T_2}{P_2} - \frac{T_1}{P_1}\right)$$

$$\therefore \Delta V = 8.314\left(\frac{423.15}{0.3} - \frac{723.15}{3.5}\right)$$

$$\therefore \Delta V = 0.010091 \ m^3/mol$$

$$\because M_w = 0.018 \ kg/mol.$$

$$\therefore \Delta V = \frac{0.010091}{M_w} = 0.556 \ m^3/kg$$

(# 2.58 %)

w.r.t steam table.

$$C_p^{1.4} = 4.04\,R \quad J/mol\,K$$

$$\therefore C_p^{1.4} = \frac{4.04}{0.018}\,R \quad J/kg\cdot K$$

$$\Delta T = 300\,K$$

$$\therefore \Delta H = C_p\,\Delta T$$

$$\Delta H = -559.81\ kJ/kg$$

(-2.94%)

$$C_v^{1.4} = \frac{3.04}{0.018}\,R$$

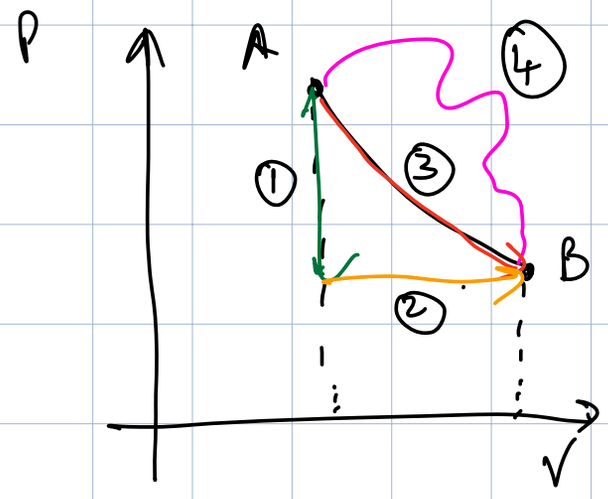
$$\therefore \Delta U = C_v^{1.4}\,\Delta T$$

$$\therefore \Delta U = -421.24\ \frac{kJ}{kg}$$

(-5.35%)

☆ Imp Points

→ state v/s Path variables



Path	State
Q & W	U, V, H, P, T, S ...

Wec - To go from A → B

(3) → isothermal, $\Delta T = 0$

$$W_{ec} = -nRT \ln \frac{V_2}{V_1}$$

(1) + (2) → $W_1 = 0 + W_2 = -P\Delta V$

(1) → isochoric, $\Delta V = 0$

(2) → isobaric

→ Steam tables

H.W 2
Q1(a)

initial

$T_i = 0°C$

$P_i = 1\ MPa$

H_i

Final

$T_f = 250°C$

$P_f = 0.4\ MPa$

H_f

Find ΔH
↓
$H_f - H_i$

H_f ⇒ Table II → At 0.4 MPa

$T_{sat} = 143°C$

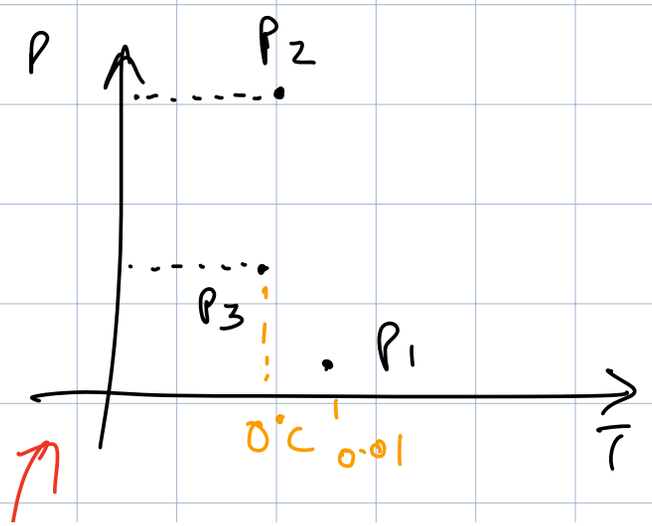
∴ at T_f it is steam.

∴ table III → $H_f = 2964.5\ kJ/kg$

$H_i \rightarrow$ Table II $\rightarrow$ @1 MPa

$T^{sat} = 179\,°C$

		T_1	H^L	
x_1	$P_1 = 0.000612$	0.01	0	y_1
x_2	$P_2 = 5$ MPa	0	5	y_2
x	$P_3 = 1$ MPa	0		y



$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2$$

$$= \frac{5 - 0}{5 - 0.000612}(1 - 5) + 5$$

* we assume that all three points are in a straight line.

$$H_i = 0.999 \text{ kJ/mol.}$$

$$\Delta H = H_f - H_i$$