Recitation #1 - 01/31/2023

→ Problems 1.10 & 1.13 (Lira)

→ Gibb's phase rule

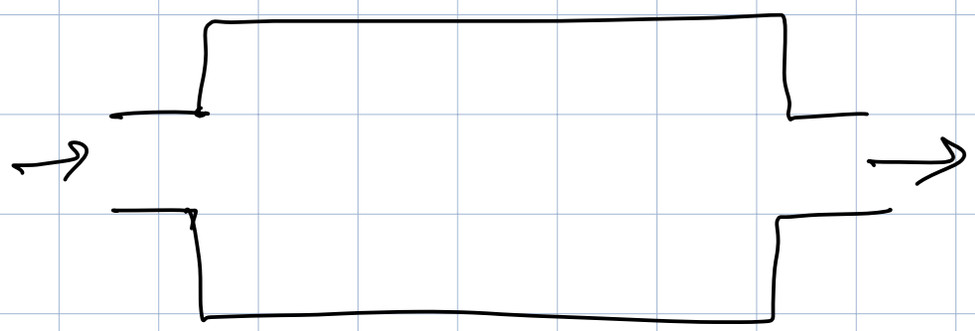
→ use of steam tables & interpolation.

Key points to remember for solving most Problems

1) Draw your system

eg:

gas inlet Parameters



gas outlet Parameters.

$\dot{m}$ = _

T = _

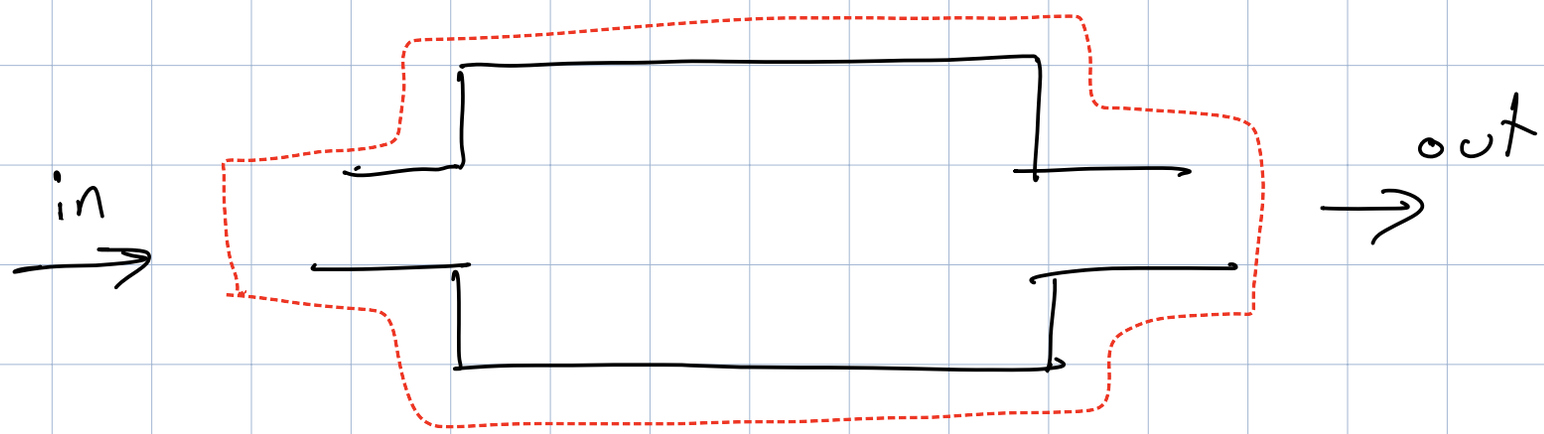
X = _

$\dot{m}$ = _

T = _

X = _

2) Define your control volume



General balance ⇒ in − out + net generation = Accumulation.

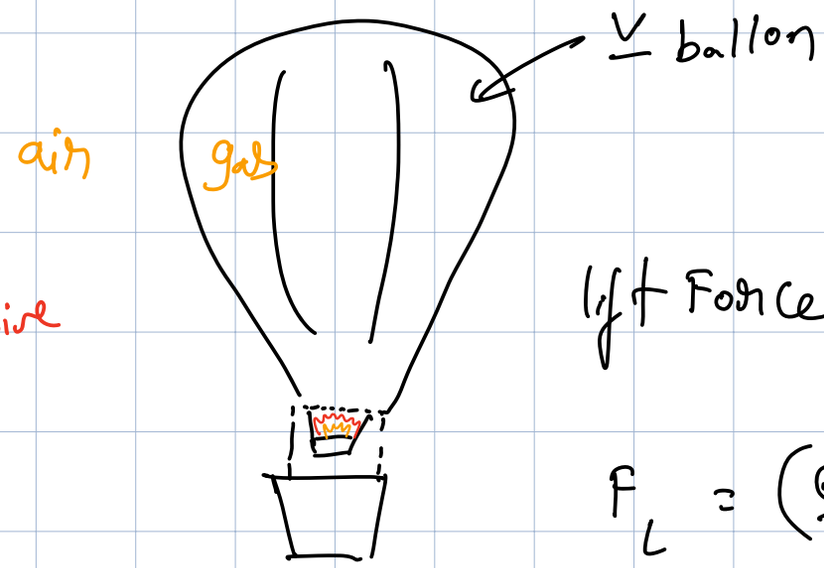
Applies to mass & energy

3) State your assumptions as appropriate

eg :-

Ideal gas
Steady-state operation
Isothermal.

1.10



V means extensive property

lift Force (kg) ?

$$F_L = (\rho_{air} - \rho_{gas})\, V_{balloon}$$

Given:

air	gas
$T_{air} = 25\,^{\circ}C$	$T_{gas} = 100\,^{\circ}C$
$P_{air} = 1\,atm$	$P_{gas} = 1\,atm$
$MW_{air} = 29\ \frac{g}{mol}$	$MW_{gas} = 32\ \frac{g}{mol}$

intensive properties - the property does not depend on the amount of substance.

eg: Temp, density.

extensive properties - depends on amount of substance.

eg: volume, mass, length.

$$F_L = (\rho_{air} - \rho_{gas})\, V_{balloon}$$

we need to find these two

Assume the gas & air are ideal gases

$$\therefore \quad PV = nRT$$

$$\because \quad n = \frac{m}{MW}$$

$$\therefore \quad PV = \frac{m}{MW} \cdot RT$$

$$\therefore \quad \frac{P \cdot MW}{RT} = \frac{m}{V} = \rho$$

$$\therefore \quad \rho = \frac{P \cdot MW}{RT}$$

$$\rho_{air} = \frac{P \cdot MW}{RT} = \frac{1 \times 29}{8.205 \times 10^{-5} \times (273.15 + 25)}$$

$\cancel{atm}$ $\quad \frac{g}{\cancel{mol}}$

$\frac{m^3 \cancel{atm}}{\cancel{mol} \cdot \cancel{K}}$ $\quad \cancel{K}$

You may note these R values on your cheat sheet

R-values

8.314	$J\,K^{-1}\,mol^{-1}$
8.314	$m^3\,Pa\,K^{-1}\,mol^{-1}$
8.314	$L\,Pa\,K^{-1}\,mol^{-1}$
0.083	$L\,bar\,K^{-1}\,mol^{-1}$
8.205×10^{-5}	$m^3\,atm\,K^{-1}\,mol^{-1}$

$$\rho_{air} = 1185.45 \ g/m^3$$

$$\therefore \ \rho_{air} = 1.18 \ kg/m^3$$

Now,

$$\rho_{gas} = \frac{P \cdot MW}{RT} = \frac{1 \ \cancel{atm} \times 32 \ \frac{g}{\cancel{mol}}}{8.205 \times 10^{-5} \ \frac{m^3 \ \cancel{atm}}{\cancel{mol} \ \cancel{K}} \times (273.15 + 100) \ \cancel{K}}$$

$$\therefore \ \rho_{gas} = 1.04 \ kg/m^3$$

<u>check</u>: $\rho_{air} > \rho_{gas} \Rightarrow$ driving force for lifting balloon

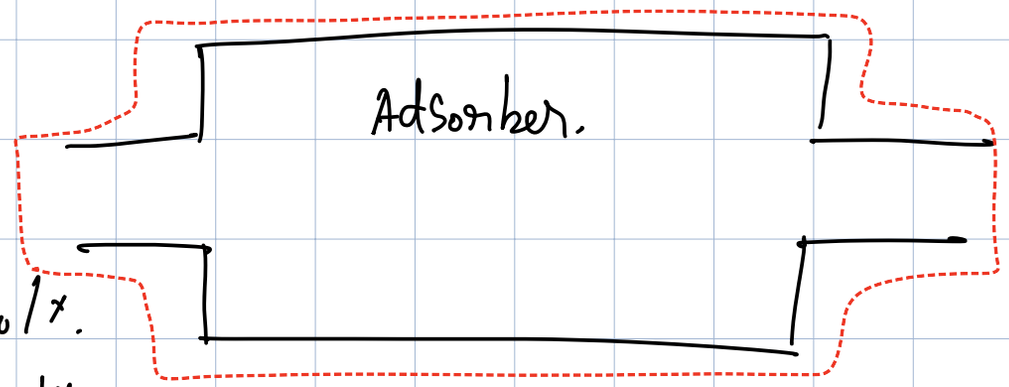
$$F_{lift} = (\rho_{air} - \rho_{gas})\, V_{balon}$$

$$= (1.18 - 1.04) \times 1.5 \times 10^3$$

$$\frac{kg}{m^3} \times m^3$$

$$\boxed{F_{lift} = 210 \text{ kgf}}$$

1.13



CO_2 : 20 mol%

Air : 80 mol%

$\dot{V}_{in}$ = 1 m^3/min

P_{in} = 1 bar

T_{in} = 360 K

gas $\dot{m}_{in}$ & $\dot{m}_{out}$?

CO_2 : inlet $\times 0.02$ (20 mol%)

$\dot{V}$: volumetric flowrate

$\dot{m}$: mass flowrate.

☆ quantities with a dot over it are time derivatives (i.e per unit time)

Assume : ideal gas

$$PV = nRT$$

$$\therefore \quad n = \frac{PV}{RT}$$

$$\therefore \quad \dot{n} = \frac{P\dot{V}}{RT}$$

$$\dot{n}_{in} = \frac{P\dot{V}_{in}}{RT} = \frac{\overset{\cancel{bar}}{1} \times \overset{\cancel{m^3}/min}{1}}{\underset{\frac{\cancel{L}\cdot\cancel{bar}}{\cancel{K}\,mol}}{0.083} \times \underset{\cancel{K}}{360}} \times \overset{\cancel{L}/\cancel{m^3}}{1000}$$

$$\dot{n}_{in} = 33.47 \ mol/min$$

$\dot{n}_{in}$

80% Air — 26.78 mol.

20% CO_2 — 6.69 mol.

$$m_{in} = \dot{n}_{CO_2,in} \times MW_{CO_2} + \dot{n}_{Air,in} \times MW_{Air}.$$

$MW_{CO_2} = 44$ g/mol.

$MW_{Air} = 28.84$ g/mol

79% N_2 — 21% O_2

M.W 28 — 32 g/mol.

$\therefore M.W_{air} = 0.79 \times 28 + 0.21 \times 32$

$= 28.84 \ g/mol.$

$\dot{m}_{in} = \underset{mol/min}{6.69} \times \underset{\frac{g}{mol}}{44} + 26.78 \times 28.84$

$\therefore \dot{m}_{in} = 1066.69 \ g/min$

$\therefore \dot{m}_{in} = 1.07 \ kg/min.$

outlet

$\dot{n}_{in}$

80% Air

26.78 mol.

20% CO_2

6.69 mol.

$\dot{n}_{out}$

Air

26.78 mol

CO_2

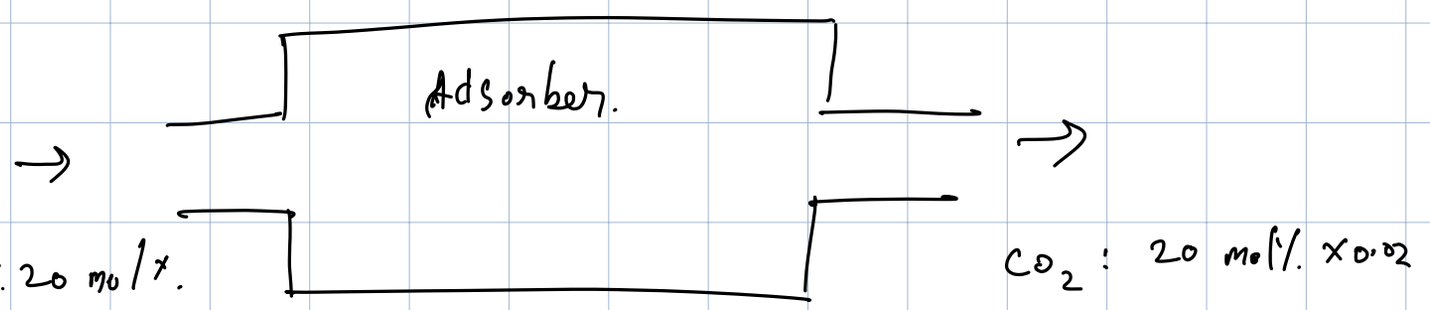
6.69×0.02

$= 0.1338 \ mol.$

$$\dot{M}_{out} = \overbrace{26.78 \times 28.84}^{Air} + \overbrace{0.134 \times 44}^{CO_2}$$

$$\dot{M}_{out} = 0.78 \ kg/min$$

b



$\dot{V}_{in}$ = ~~1 m³/min~~

P_{in} = 1 bar

T_{in} = ~~350 K~~ 320 K

Find $\dot{V}_{out}$?

$\dot{n}_{in/out}$ remains the same

$\dot{n}_{out} \Rightarrow$

$$\dot{n}_{out}$$

Air: 26.78 mol

CO_2: $6.69 \times 0.02 = 0.1338$ mol

$\therefore \dot{n}_{out} = 26.91$ mol/min

$\because$ ideal

$$\bar{V}_{out} = \frac{n_{out} R T}{P} = \frac{26.91 \times 0.083 \times 320}{1}$$

$\cancel{mol}/min \quad \frac{L\ \cancel{bar}}{\cancel{mol}\ \cancel{K}} \quad \cancel{K}$

$\cancel{bar}$

$= 714.7$ L/min

$\bar{V}_{out} = 0.71$ m^3/min

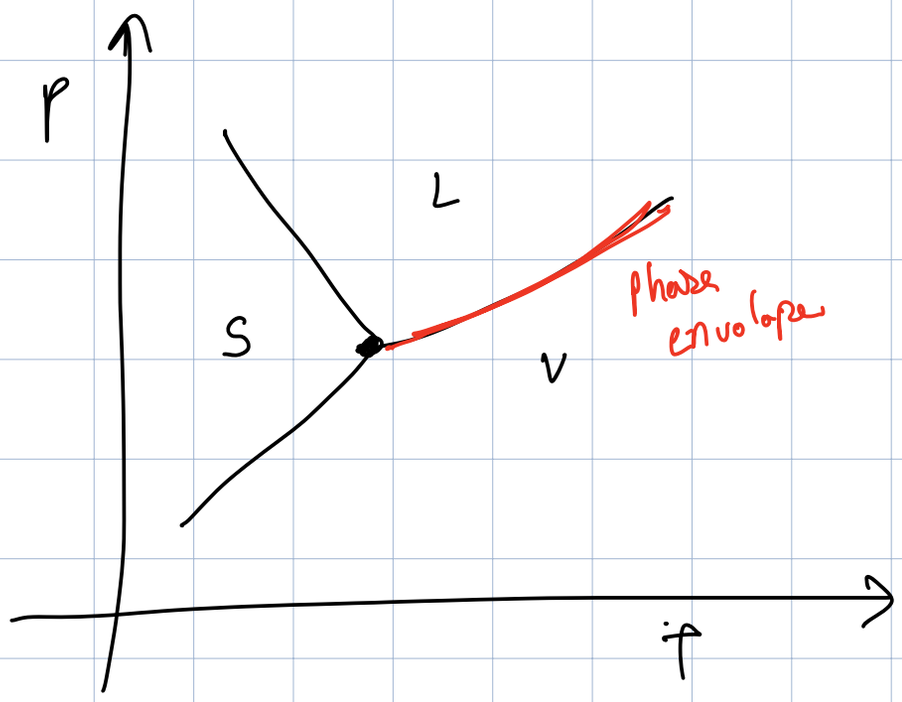
Phase diagrams

&

Reading steam tables

Gibbs phase rule & Phase diagrams

1 component = water

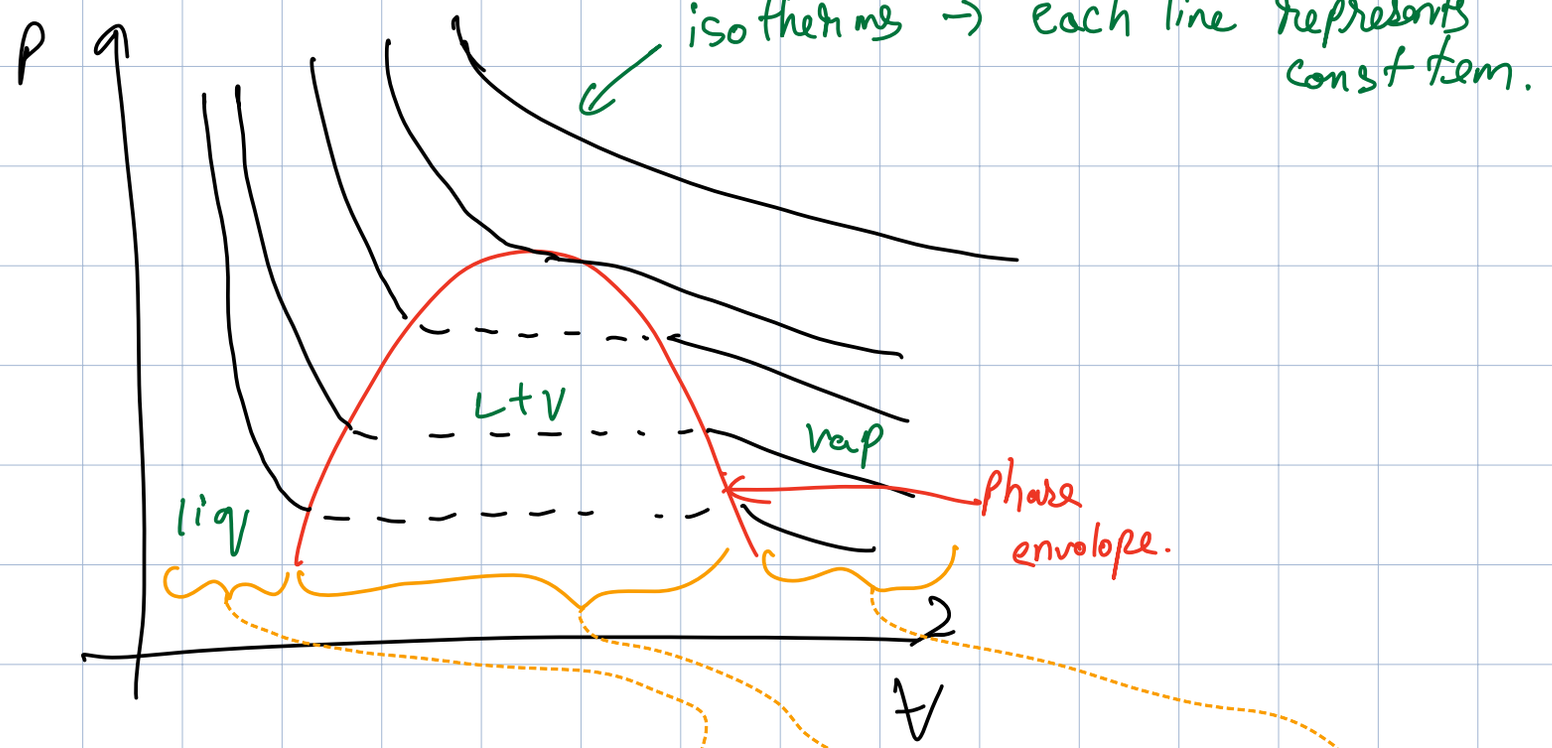


$$F = C - P + 2$$

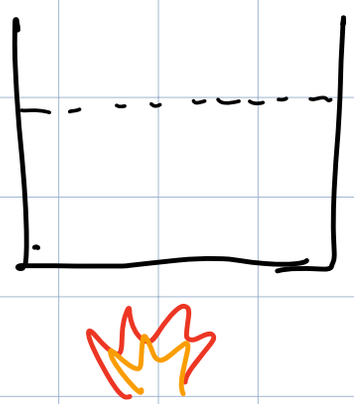
$P = 1$: $F = 1 - 1 + 2 = 2$

$P = 2$: $F = 1 - 2 + 2 = 1$

$P = 3$: $F = 1 - 3 + 2 = 0$



Eg: water boiling at 1atm.



$25°C \rightarrow 100°C \rightarrow 100°C \rightarrow 101°C$ (above)

liquid throughout (Sensible heating)

Additional heat vaporizes the water (latent heat)

super heated steam.

Steam tables

Look for thermodynamic properties of water with:

$$T = 180°C$$

$$P = 0.7 \text{ MPa}.$$

1) Find corresponding temp or pressure in the saturated tables

→ $T^{sat} = 180°C$ gives $P = 1.002$ MPa
(Table I)

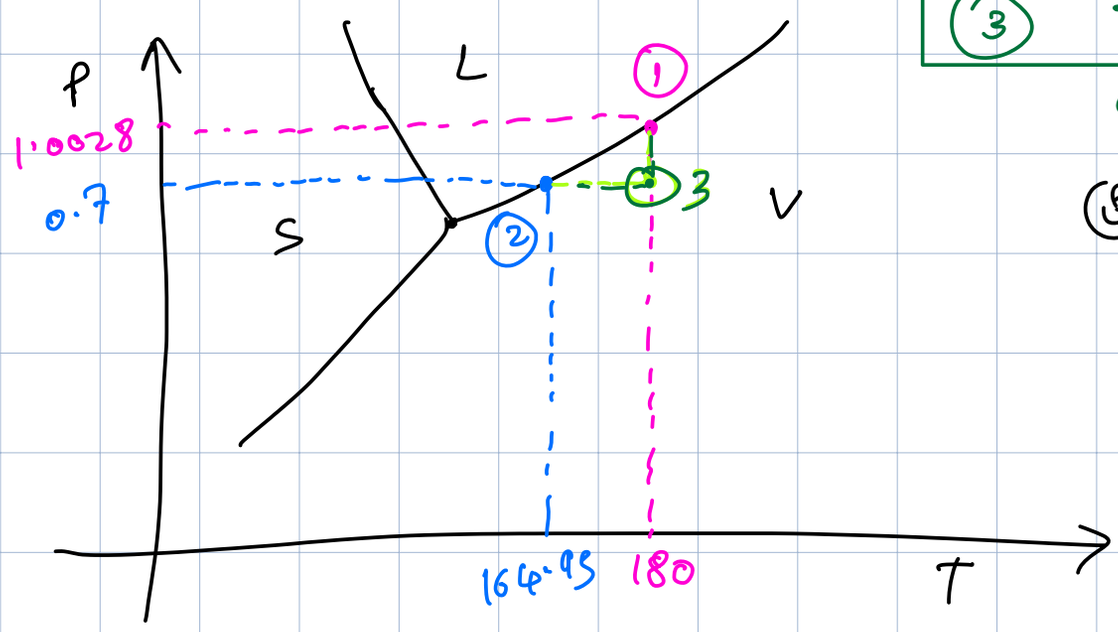
→ $P^{sat} = 0.7$ MPa gives $T = 164.95°C$
(Table II)

→ Neither has both the desired T & P

2) Find saturated steam value on the graph.
(saturated means the value lies along the phase env.)

① $T = 180^\circ C$, $P = 1.0028\ MPa$ → P too high

② $T = 164.95^\circ C$, $P = 0.7\ MPa$ → T too low.

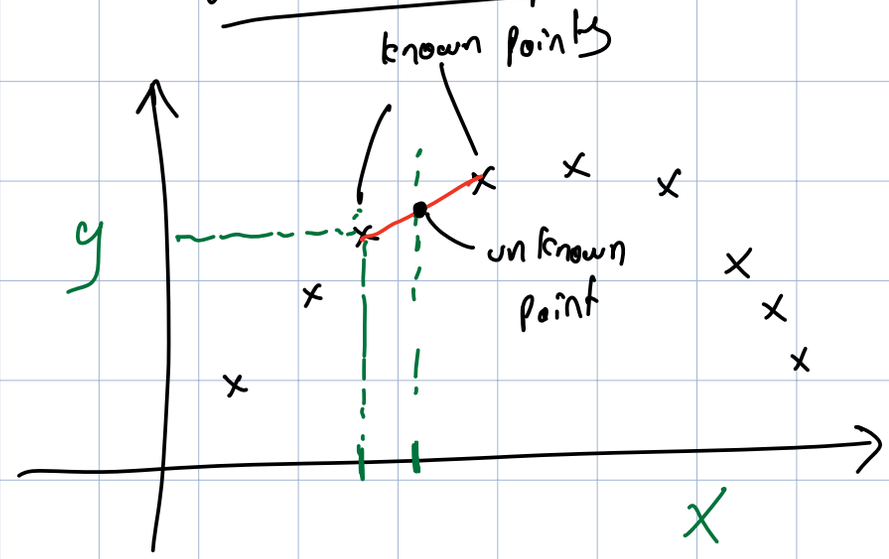


$\boxed{③\quad T = 180^\circ C,\ P = 0.7\ MPa}$
desired parameters

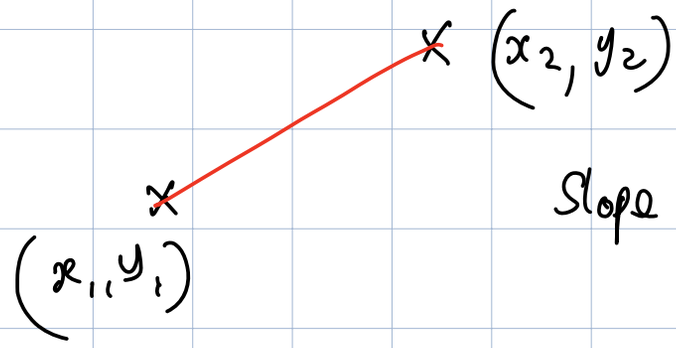
③ in vapour region
so look in Table III
Superheated steam.

but exact values not listed for condition 3

Requires Interpolation



* interpolation is effectively finding two bounding points close together and assuming the line connecting them is linear in that region.



Slope of line $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation of the line in two point form:

$$y - y_2 = m(x - x_2)$$

$$y = m(x - x_2) + y_2$$

$$\boxed{y = y_2 - m(x - x_2)}$$

Back to finding the value : $T = 180°C$, $P = 0.7 MPa$

let us try to find $\hat{u}$

there isn't a section in this table for 0.7 MPa but there is for 0.6 MPa and 0.8 MPa, also these sections dont have values for 180°C

values we have.

P(MPa)	T^{sat}_1 (x_1)	T_2 (x_2)	T_3	$\hat{u}_1$ (kJ/kg) (y_1)	$\hat{u}_2$ (y_2)	u_3
0.6	158.8	200	180	2566.8	2639.3	need to find (1) —
0.7	180	180	180	—	—	
0.8	170.4	200	180	2576	2631	need to find. — (2)

(first we interpolate to 180°C)

1) For 0.6 MPa -

interpolation formula $\Rightarrow$

$$y = m(x - x_2) + y_2$$

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2$$

$$\hat{u}_{180} = \frac{\hat{u}_2 - \hat{u}_1}{T_2 - T_1^{sat}}(T_3 - T_2) + \hat{u}_2$$

$$\hat{u}_{180} = \frac{2639.3 - 2566.8}{200 - 158.8}(180 - 200) + 2639.3$$

1 @ 0.6 MPa $\hat{u}_{180} = 2604.10$ kJ/kg

(2) For 0.8 MPa

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2$$

$$\hat{u}_{180} = \frac{\hat{u}_2 - \hat{u}_1}{T_2 - T_1^{sat}}(T_3 - T_2) + \hat{u}_2$$

$$\hat{u}_{180} = \frac{2631 - 2576}{200 - 170.4}(180 - 200) + 2631$$

(a) 0.8 MPa $\hat{u}_{180} = 2593.83$ kJ/kg

(2)

3) Now for 0.7

rewriting the table for temperature interpolation:

P(MPa)	T^{sat}_1 (x_1)	T^{sat}_2 (x_2)	T_3	$\hat{u}$ (kJ/kg)$_1$ (y_1)	$\hat{u}_2$ (y_2)	u_3
0.6	158.8	200	180	2566.8	2639.3	2604.1
0.7	180	180		–	–	need to find
0.8	170.4	200	180	2576	2631	2593.83

table for pressure interpolation:

	P (MPa)	T_3	$\hat{u}_3$	
x_1	0.6	180	2604.1	y_1
	0.7	180	need to find.	
x_2	0.8	180	2593.8 kJ/kg	y_2

$$y = m(x - x_2) + y_2$$

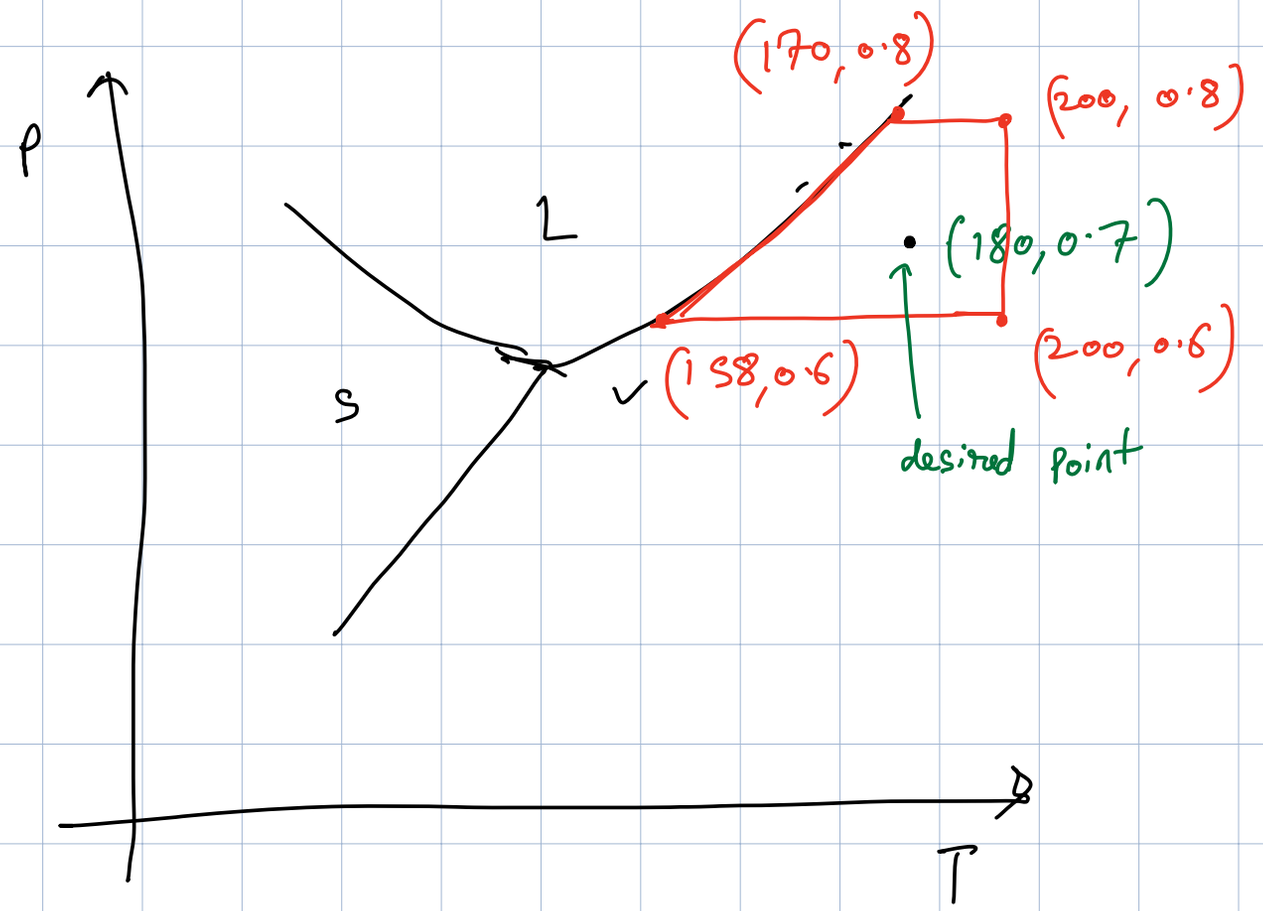
$$y = \frac{2593.8 - 2604.1}{0.8 - 0.6}(0.7 - 0.8) + 2593.8$$

$$y = 2598.95$$

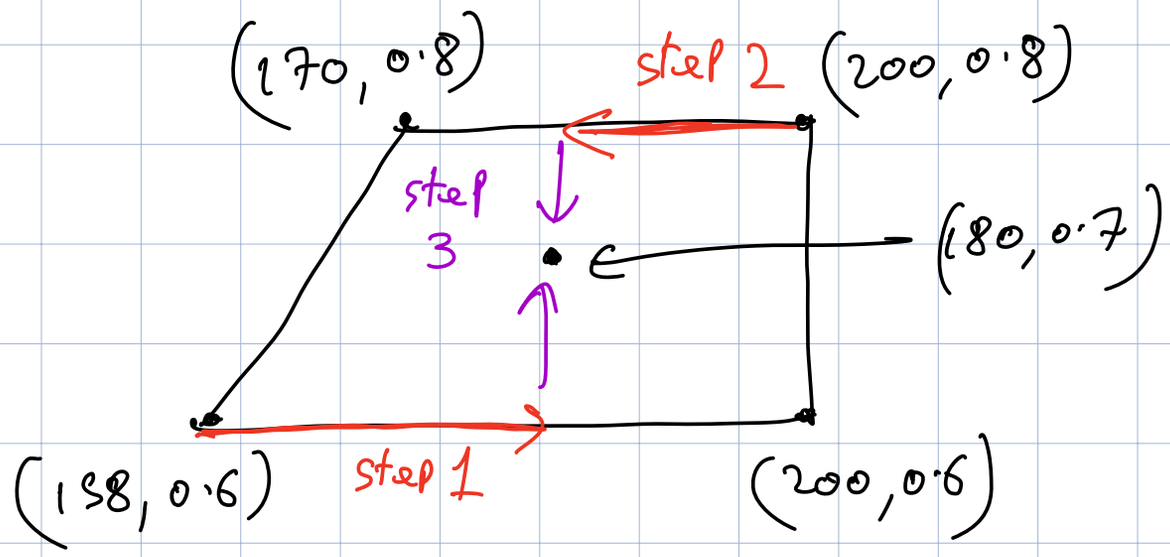
$\hat{u}_{180}$ @ 0.7 MPa. = 2598.95 kJ/kg

This is the thermodynamic property at the desired condition.

Graphical representation of the interpolation:



Redrawing the trapazoid and Showing the interpolation steps



I strongly recommend you guys to create a tool on excel or matlab which will help you do the interpolation. It will be useful for homeworks since you will have to alot

of interpolation.

* (The computer tool will not be permitted during the exam)