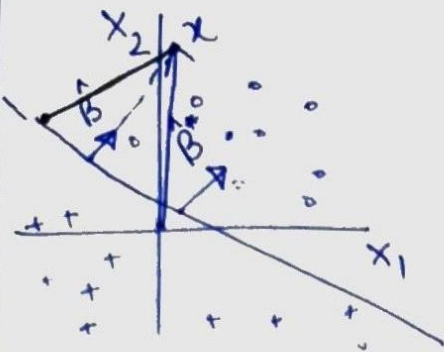


# Modelling a separating hyperplane



$$f(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 = 0 = L$$

$$= \beta_0 + \beta^T x = 0$$

Properties:

$$1) \beta^T(x_1 - x_2) = 0 \Rightarrow \beta^* = \frac{\beta}{\|\beta\|}$$

$$2) \beta^T x_0 = -\beta_0 \quad \forall x_0 \in L \Rightarrow$$

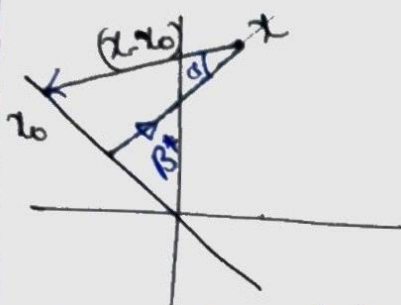
$$3) \beta^T(x - x_0) = (\beta^T x - \beta^T x_0)$$

$$= \frac{1}{\|\beta\|} (\beta^T x + \beta_0) = \frac{f(x)}{\|\beta\|}$$

$$= \frac{f(x)}{\|\beta^*(x)\|}$$

$$f'(x) = \beta$$

(Distance of  $x$  from hyperplane)  
signed distance



$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|}$$

$$\frac{(x - x_0) \cdot \beta^*}{\|\beta^*\| \cdot \|x - x_0\|} = \cos \alpha$$

## Perceptron Learning Algorithm

Minimize distance of misclassified points to decision boundary.

$$y_i = 1, \text{ is misclassified} \Rightarrow x_i^T \beta + \beta_0 < 0$$

$$y_i = -1, \text{ is misclassified} \Rightarrow x_i^T \beta + \beta_0 > 0$$

Assumption

$$\rightarrow \begin{cases} y_i = 1 \Rightarrow x_i^T \beta + \beta_0 > 0 \\ y_i = -1 \Rightarrow x_i^T \beta + \beta_0 < 0 \end{cases}$$

True classifier

$$y_i (x_i^T \beta + \beta_0) < 0, \quad y_i = 1 \quad y_i = -1 \quad \text{Misclassified}$$

$$- y_i (x_i^T \beta + \beta_0) > 0, \quad y_i = 1 \quad \text{for sample } (x_i) > 0 \quad y_i = -1$$

For all samples

$$\sum y_i (x_i^T \beta + \beta_0) \rightarrow \text{Minimise}$$

[1]

## Lecture 20

$$D(\beta, \beta_0) = - \sum_{i \in M} \gamma_i (x_i^T \beta + \beta_0), \quad (M - \text{samples are misclassified})$$

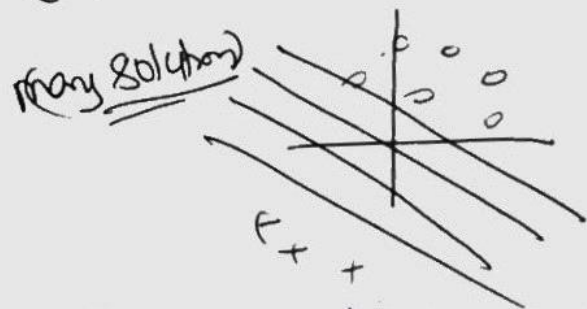
(Minimize  $M \rightarrow 0$ ) our motive

$$\frac{\partial D}{\partial \beta} = - \sum_i \gamma_i (x_i), \quad \frac{\partial D}{\partial \beta_0} = \sum_i \gamma_i$$

$$\begin{bmatrix} \beta \\ \beta_0 \end{bmatrix} = \begin{bmatrix} \beta \\ \beta_0 \end{bmatrix} - (\alpha) \begin{bmatrix} - \sum_i (\gamma_i x_i) \\ \sum \gamma_i \end{bmatrix} \rightarrow \text{[vectors]}$$

— (problems)

- Linearly separable: then Converge to some solution.
- Can take along time if gap bet two classes are very less



— Not linearly separable  $\rightarrow$  it enters in (loop) ( $\infty$ )