1)

$$\dot{x}_1 = -x_1 - x_2 + \frac{x_1 x_2}{12}$$

$$\dot{x}_2 = u$$

$$f(x) = \begin{bmatrix} -x_1 - x_2 + \frac{x_1 x_2}{12} \\ 0 \end{bmatrix} \qquad g(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

WKT $(x \neq 0 \;\; L_g V(x) = 0) \Rightarrow L_f(V(x)) < 0.$

$\Rightarrow$ $L_g V(x) = 0$ is our domain

$$L_g \dot{V}(x) = \frac{\partial V}{\partial x} g(x) = 2x^T P g(x) = 2(x_1 \;\; x_2)\begin{bmatrix} 2 & 1.5 \\ 1.5 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boxed{L_g V(x) = 3x_1 + 6x_2}$$

$L_g V(x) = 0 \Rightarrow \boxed{x_1 = -2x_2}$

ii)

$L_f V(x) < 0 \Rightarrow 2x^T P f(x)$

$$\Rightarrow [x_1 \;\; x_2]\begin{bmatrix} 2 & 1.5 \\ 1.5 & 3 \end{bmatrix}\begin{bmatrix} -x_1 - x_2 + \frac{x_1 x_2}{12} \\ 0 \end{bmatrix}$$

$$= (2x_1 + 1.5x_2)\left(-x_1 - x_2 + \frac{x_1 x_2}{12}\right)$$

$x_1 = -2x_2$

$$\Rightarrow = -2.5x_2\left(x_2 - \frac{x_2^2}{6}\right) = \left(\frac{x_2}{6} - 1\right)x_2^2 \leq 0$$

$\Rightarrow$ $x_2 < 6$ & ~~x_1~~ $x_1 > -12$

$\therefore$ Openset is $D = \{ x \in R^2 \mid x_1 > -12 \;\&\; x_2 < 6 \}$

b) we can use sontags formula here

for control input over this Domain.

$$u = \begin{cases} \dfrac{-L_f V(x) + \sqrt{(L_f V(x))^2 + (L_g V(x))^4}}{L_g V(x)}, & L_g V(x) \neq 0 \\ 0, & L_g V(x) = 0 \end{cases}$$

2)

$$\dot{x}_1 = x_2$$

$$v_1(x_1) = \frac{x_1^2}{2} = x_1 \dot{x}_1$$

$\Rightarrow$ let $\phi_1(x_1) = -x_1$

$$\Rightarrow \dot{x}_1 = -x_1 + (x_2 + x_1).$$

$$z_1 = x_2 + x_1$$

$$\Rightarrow \dot{z}_1 = \dot{x}_2 + \dot{x}_1$$

$$= x_3 - x_1 + z_1$$

$$\dot{v}_2(x_1, z_1) = x_1(-x_1 + z_1) + z_1(\dot{z}_1)$$

$$= -x_1^2 + z_1 x_1 + \cancel{z_1(\ldots - x_1 + z_1)}$$

$$+ z_1 \phi_2$$

$$\Rightarrow \phi_2 = -x_1 - z_1$$

$$\Rightarrow \dot{z}_1 = (-x_1 - z_1) + \underbrace{(x_3 + 2z_1)}_{z_2} = -x_1 - z_1 + z_2$$

$$\Rightarrow z_2 = x_3 + 2z_1$$

$$\dot{z}_2 = \dot{x}_3 + 2\dot{z}_1$$

$$\dot{V}(x_1, z_1, z_2) = x_1 \dot{x}_1 + z_1 \dot{z}_1 + z_2 \dot{z}_2$$

$$= -x_1^2 + z_1 x_1 - z_1^2 - z_1 x_1 + z_1 z_2$$

$$+ z_2(\dot{x}_3 + 2\dot{z}_1)$$

$$\dot{V} = -x_1^2 - z_1^2 + z_1 z_2$$

$$+ z_2 u + 2 z_2 (-x_1 - \cancel{z_1} + z_2)$$

$$\dot{V} = -x_1^2 - z_1^2 + z_2 u + (-2 z_2 x_1 - z_1 z_2 + 2 z_2^2)$$

$\Rightarrow$ for liner setting

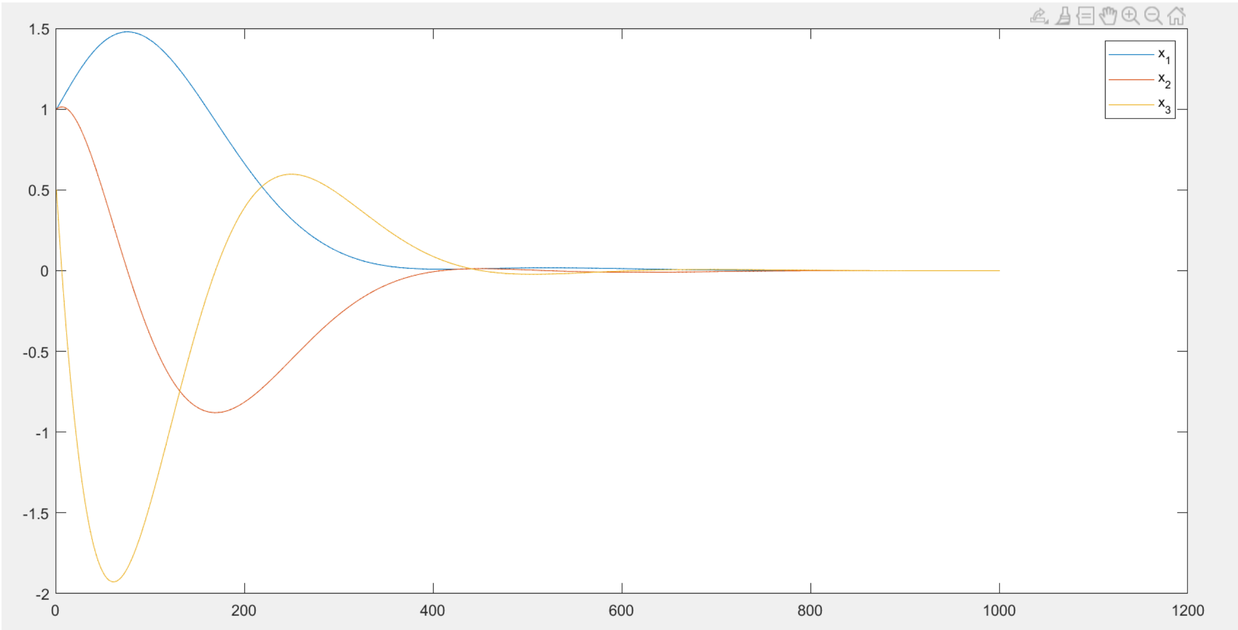
$$\boxed{u = -3 z_2 + z_1 + 2 x_1}$$

for non linear setting

$$\boxed{u = -z_2^3 - 3 z_2 + z_1 + 2 x_1}$$

for that $\dot{V} = -x_1^2 - z_1^2 - z_2^2 - z_2^4$

Question 3:

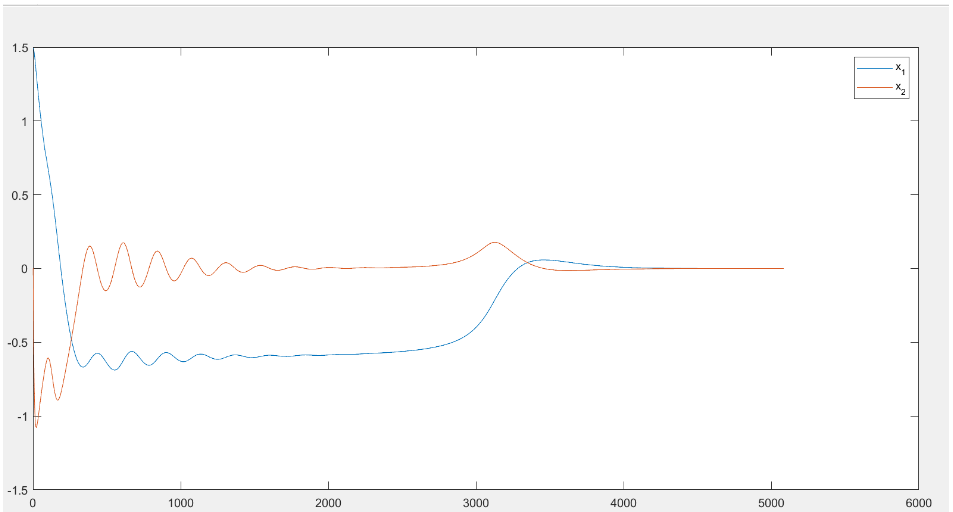


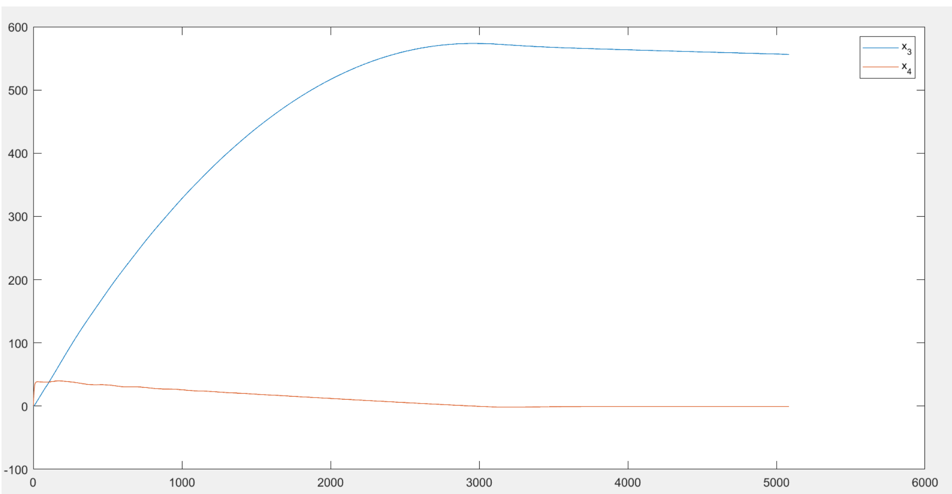
Question 4:

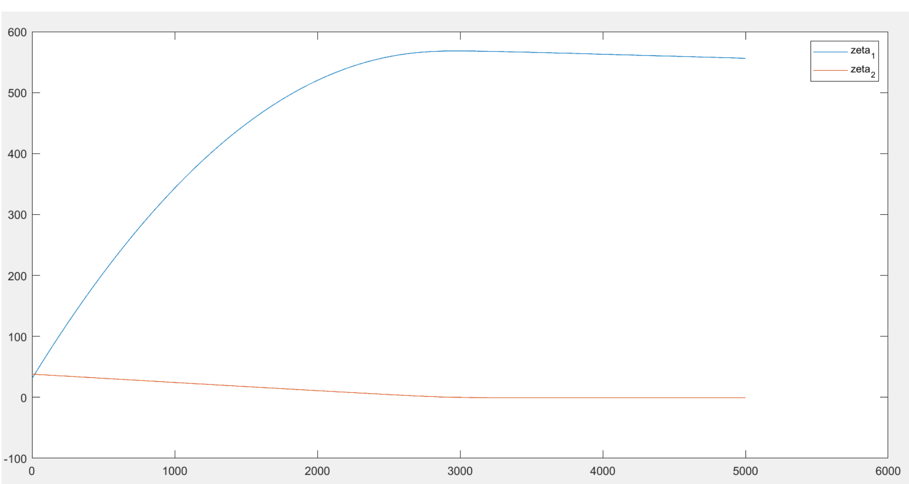
Maximum initialization is |theta| < pi/2
The values used for tuning are:
Alpha1 = alpha2 = 0.5
K1 = K2 = 1
Threshold = 0.6

Although the gains make the system stabilize this is not a practical solution. If we see the graph of x_3 it goes to a value of almost 200. This is because to make sure that the system stabilizes before it goes to an unstable state. More practical bound can be determined when there are limitations of system actuation capacity.And the controller doesn't try to

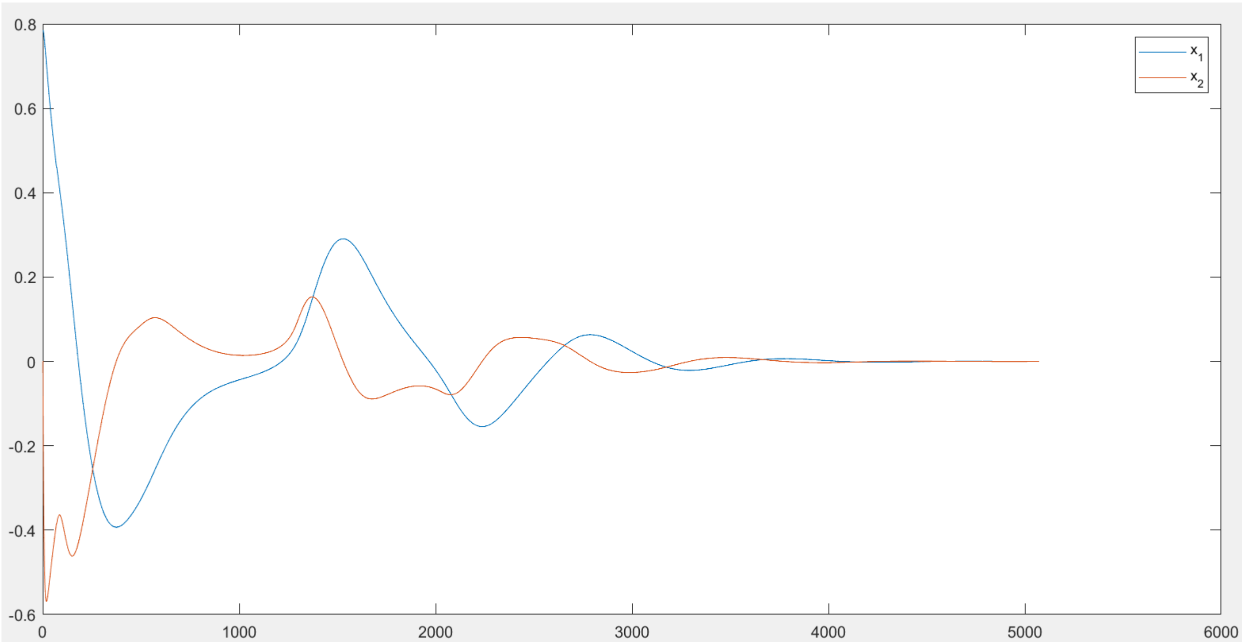
Below is tested value for **pi/2.1:**


1.5
1
0.5
0
-0.5
-1
-1.5
0
1000
2000
3000
4000
5000
6000
x_1
x_2

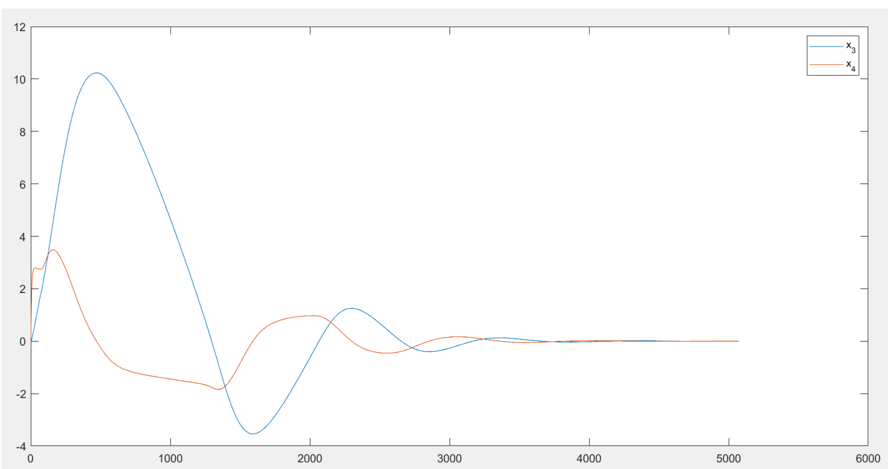

600
500
400
300
200
100
0
-100
0
1000
2000
3000
4000
5000
6000
x_3
x_4


600
500
400
300
200
100
0
-100
0
1000
2000
3000
4000
5000
6000
$zeta_1$
$zeta_2$

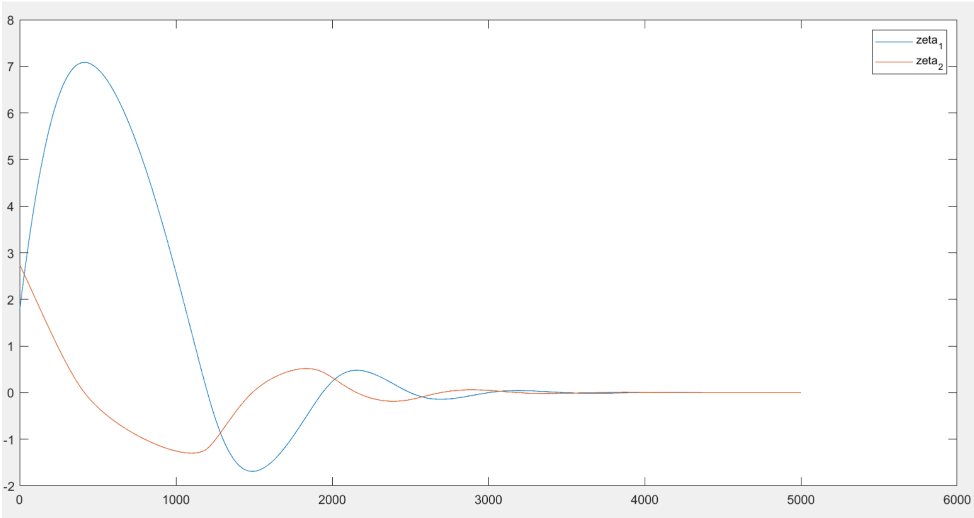
Case: pi/4:



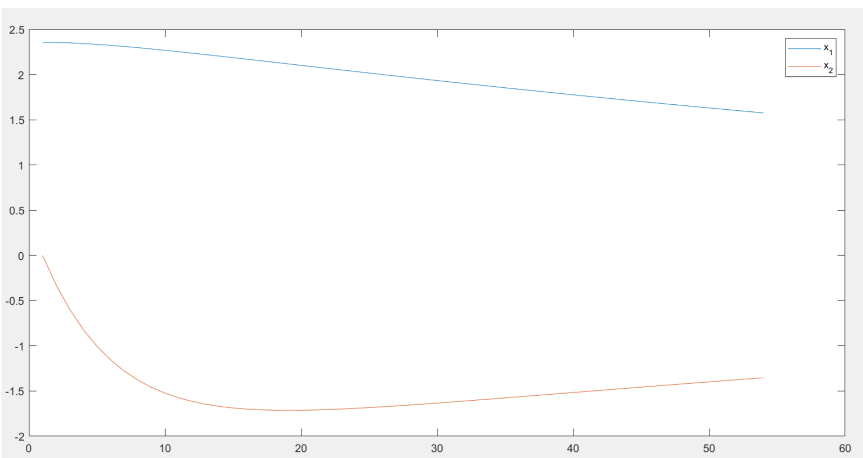
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
1000
2000
3000
4000
5000
6000
x_1
x_2

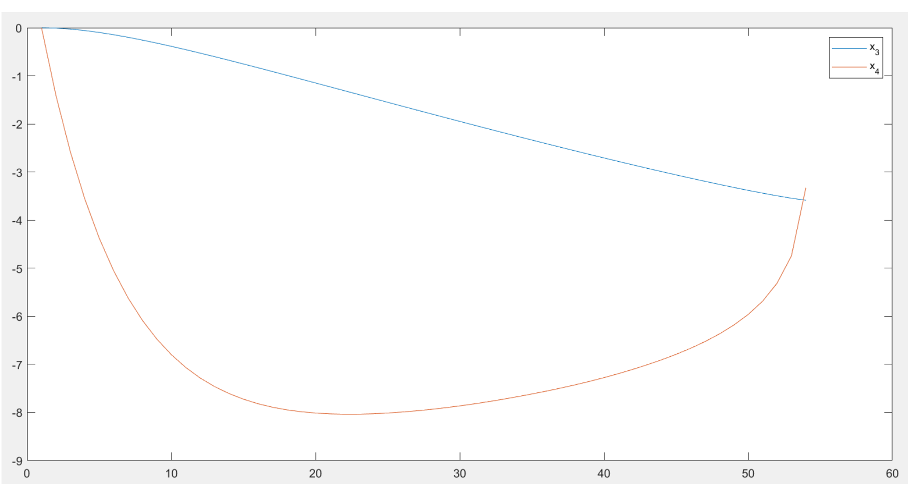


12
10
8
6
4
2
0
-2
-4
0
1000
2000
3000
4000
5000
6000
x_3
x_4



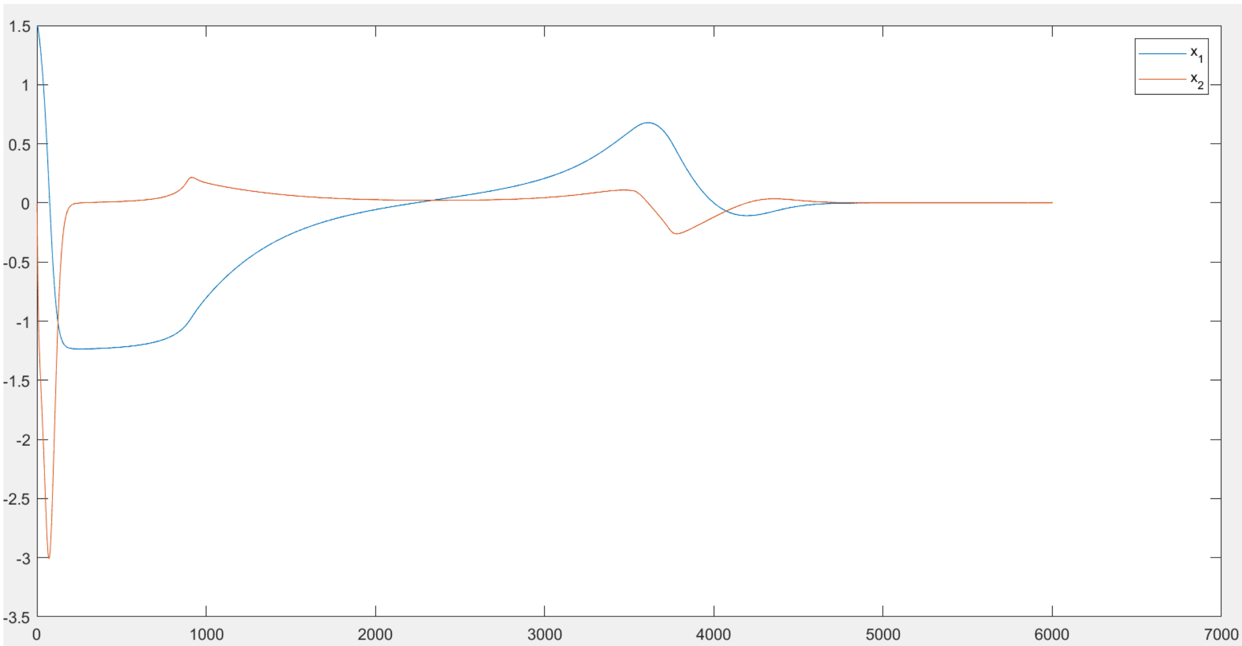
Fail Case: 3*pi/4

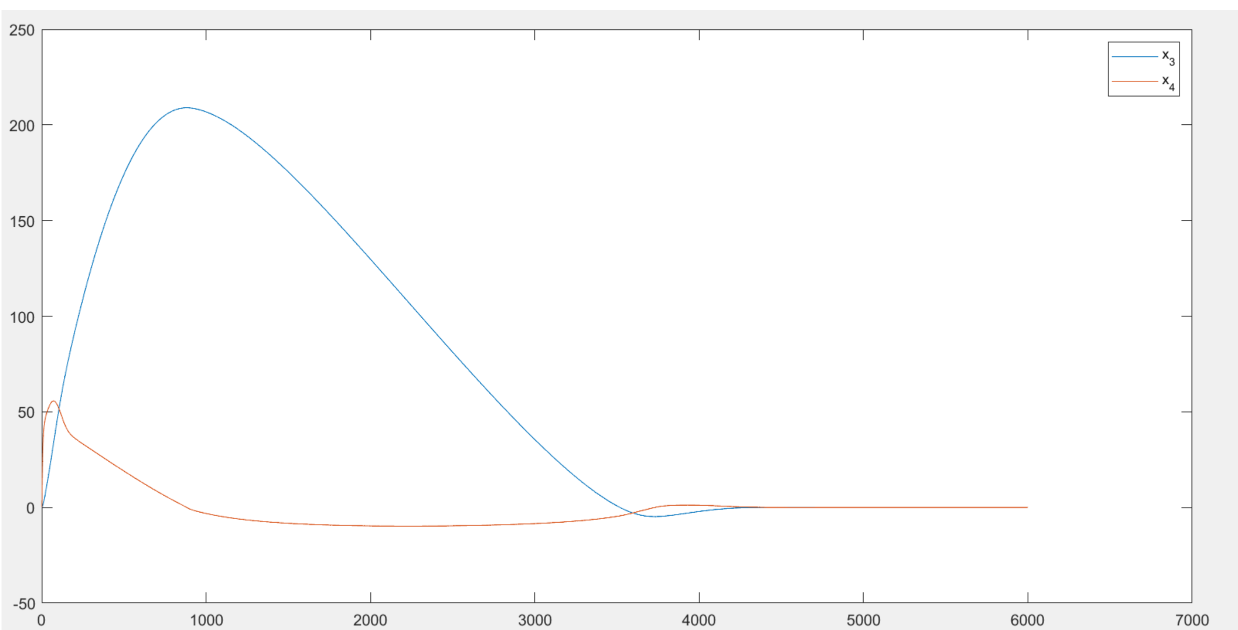




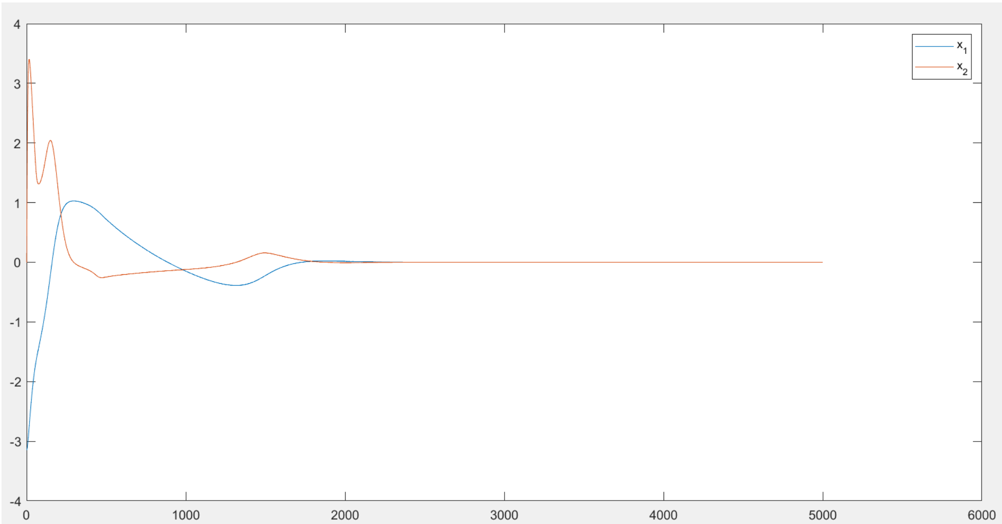
Question 5:

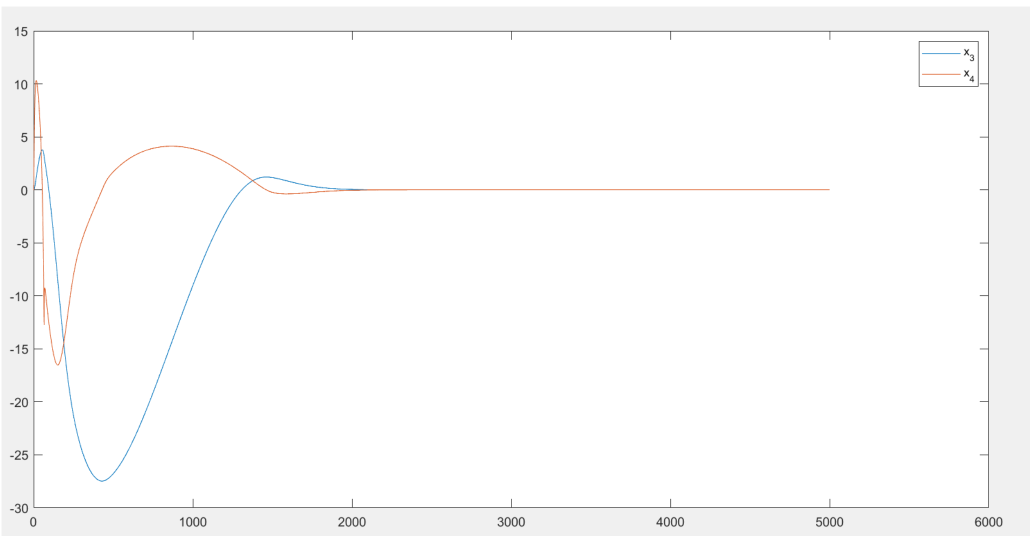
The maximum value is |theta| < pi/2 and |theta|>pi/2
Gains used are: K1, K2 = [0.05;0.1];
The below shows the simulation of **pi/2.1**



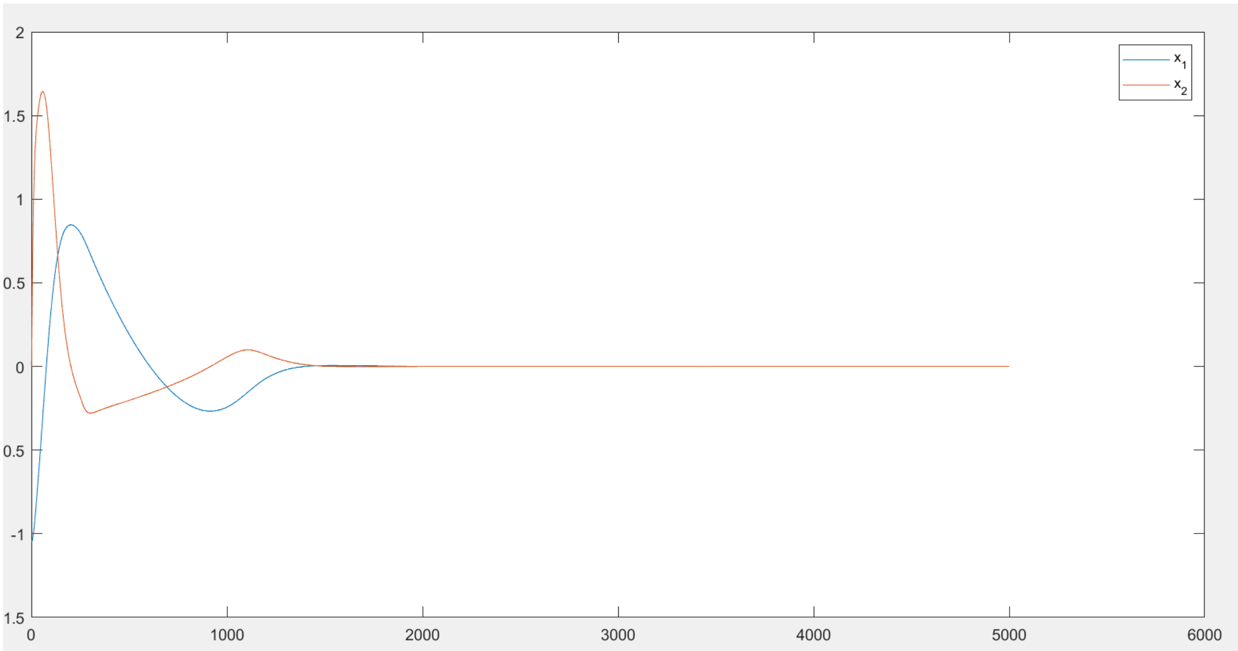


Simulation for pi:

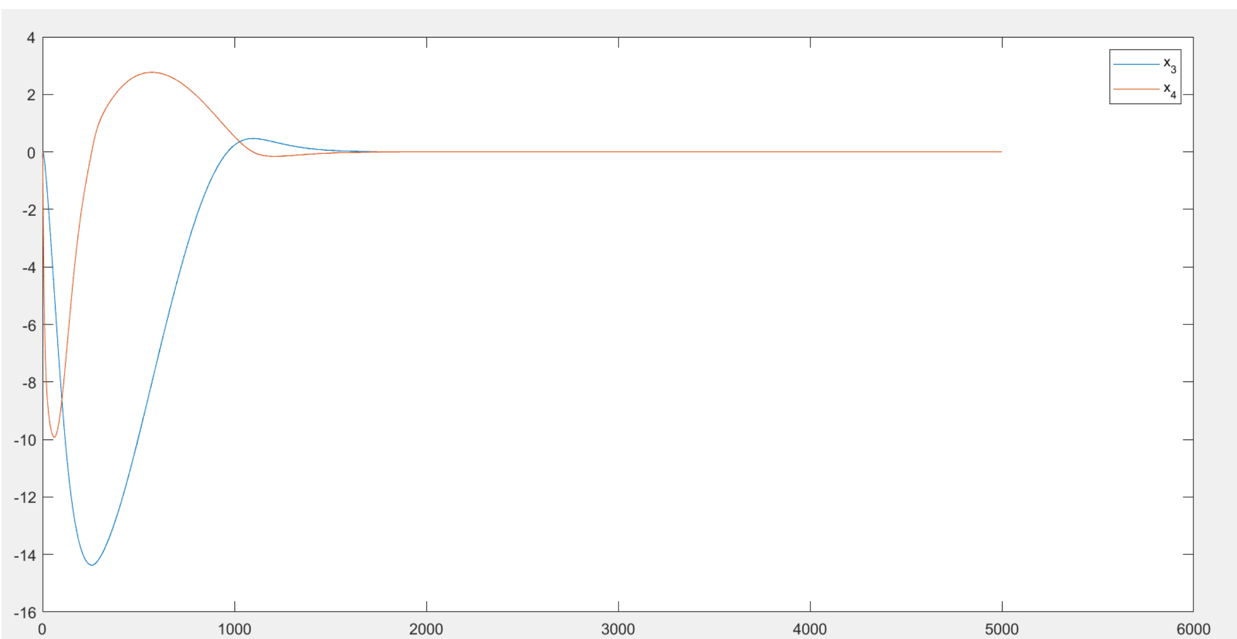




Case pi/3:



x1
x2
2
1.5
1
0.5
0
0.5
-1
1.5
0
1000
2000
3000
4000
5000
6000



x3
x4
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
0
1000
2000
3000
4000
5000
6000

Fail case: Simulation for pi/2: