EECS 562/AE 551 - Nonlinear Systems and Control
HW #1

Due on Thursday, January 12th, 2023
By 11:59pm, on Canvas

Read Chapters 1 and 2 of Khalil on your own; we begin the course with Chapter 3. This HW set is a short easy assignment that you should be able to complete on your own.

Note: Hints for the problems are given on a separate page. If you do not wish to consult the hints, simply delete that page from the PDF file!

1. (20 points) Use your favorite ODE simulation package to verify the phase portraits in Figure 2.19 (a) and (b). Turn in a plot of your simulation. You do not need to turn in your MATLAB code or whatever package you use. Done

2. (20 points) (Change of time scale) Consider the differential equation

chills OK

$$\frac{dx(t)}{dt} = f(x(t)) \qquad (*)$$

   (a) Suppose that $x(t)$ is a solution of (*). Define $\tau = at$, $a \neq 0$, and note that $t = \frac{\tau}{a}$. Define $y(\tau) = x(\frac{\tau}{a})$ and determine the differential equation satisfied by $y(\tau)$.

   (b) Specialize your result to $\tau = -t$.

   (c) Apply the result of (b) to $\frac{dx(t)}{dt} = 3x(t)$, and sketch the solutions of the equations for $x(t)$ and $y(\tau)$.

3. (20 points) Construct the phase portrait of

$$\begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= x_1 - 0.2(1 - x_1^2)x_2. \end{aligned}$$

Where to find points..rest ok

   Remark: Compare the system with Example 2.6 in the textbook and see if you relate the phase portrait of the above system and the one in Example 2.6.

4. (20 points) Lookup finite escape time in our textbook.

   (a) Use simulation to verify that the scalar differential equation $\dot{x} = x^4$ has a finite escape time from $x(0) = 1$. It is OPTIONAL to also give an analytical proof.

yes

   (b) Would you expect the scalar differential equation $\dot{x} = x^4 \sin(x)$ to have a finite escape time for some initial condition $x_0 \in \mathbb{R}$? Or, will solutions conditions exist on $0 \leq t < \infty$ for all initial conditions x_0? no finite time escape: Theorem 3.3: it is locally lipschitz and stays within certain radius for sure

5. (20 points) Which of the functions $f : \mathbb{R} \to \mathbb{R}$ given below are locally Lipschitz continuous? Find the Lipschitz constant also, if applicable.

Initial guess: YNYN

   (a) $f(x) = x^2 + |x^3|$ Its convex so pakka

   (b) $f(x) = \text{sign}(x)\cos(x)$

Disc, :(

   (c) $f(x) = \sqrt{x^2 + 1}$ same here bro

   (d) $f(x) = x\ \text{sat}(x^4)$

Disc. :(

Hints

Hints for Problem 1: I assume that most of you will use MATLAB. You can use `SIMULINK` or one of the ODE tools, such as `ODE45`. If you have not done this before, I also assume that you can use the web to figure out how to integrate an ODE in MATLAB. Try searching on *using matlab to simulate an ode.* In MATLAB, you can type

```
>> help ode45
```

In the Inverted Pendulum Project, you will have to simulate a nonlinear feedback system.

Hints for Problem 2: Apply the chain rule: $\frac{dy(\tau)}{d\tau} = \frac{d}{d\tau}x(\frac{\tau}{a})$, etc. If you have forgotten the chain rule, re-learn it!

Hints for Problem 3: The answer is Fig. 2.19 (a) with the arrows reversed. Your solution should make it obvious why this is the case.

Hints for Problem 4: For part (a), it is enough to simulate and find out; I am not expecting you to compute an analytical solution. For part (b), think about equilibrium points.