1)

$$\dot{x}_1 = x_1 - x_1^3 + x_2$$

$$V_1 = \frac{x_1^2}{2} = x_1\dot{x}_1 = x_1^2 - x_1^4 + x_1 x_2$$

$\Rightarrow$ $\cancel{z}$ $-x_1$ $\quad z u = -x_1$

$\Rightarrow \quad \dot{x}_1 = -x_1^3 + z_1 \qquad z_1 = -x_1 + x_2$

$$\dot{z}_1 = -\dot{x}_1 + \dot{x}_2$$

$$= -x_1^3 + z_1 + u$$

$$V = \frac{x_1^2}{2} + \frac{z_1^2}{2}$$

$$= x_1\dot{x}_1 + z_1\dot{z}_1$$

$$= -x_1^4 + z_1 x_1 - z_1 x_1^3 + z_1^2 + z_1 u$$

$\Rightarrow \quad u = x_1^3 - 2z_1 - x_1 - Kz_1 \qquad K>0$

$\Rightarrow$ $\boxed{\cancel{u = 2x_1^3 - 2x_1 x_2}}$

$$u = x_1^3 + 2x_1 - 2x_2 + K(x_1 - x_2)$$

$$\boxed{u = x_1^3 + K(x_1 - x_2) + x_2} \qquad K>0$$

2) $\dot{x}_1 = x_1^2 x_2$

$\dot{x}_2 = u$

$V_1 = \frac{x_1^2}{2}$, $\dot{V}_1 = x_1 \dot{x}_1 = x_1^3 u$

$u = -x_1$

$\Rightarrow \dot{x}_1 = x_1^2(-x_1 + z_1)$

$z_1 = x_2 + x_1$

$\dot{z}_1 = \dot{x}_2 + x_1^2(-x_1 + z_1)$

$= u - x_1^3 + z_1 x_1^2$

$V = \frac{1}{2} x_1^2 + \frac{1}{2} z_1^2$

$\dot{V} = x_1(-x_1^3 + z_1 x_1^2) + z_1(u - x_1^3 + z_1 x_1^2)$

$= -x_1^4 + z_1 x_1^3 - z_1 x_1^2 + z_1^2 x_1^2 + z_1 u$

$-\cancel{z_1 x_1^2}$

$u = -z_1 x_1^2 - x_1^3 - x_1^2 - z_1$

$= -(x_1 + x_2) x_1^2 - x_1^3 - x_1^2 - (x_1 + x_2)$

a) a) $E = \cos(x_1) - 1 + \frac{x_2^2}{2}$

$\dot{E} = -\sin x_1 \dot{x}_1 + x_2 \dot{x}_2$

$= -\sin x_1 \, x_2 + x_2 (\sin x_1)$ $(u = 0)$

$\boxed{\dot{E} = 0}$

b) $V_p = \frac{E^2}{2}$ $\dot{V}_p = E\dot{E}$

$=$ ~~[illegible]~~

$= E \cdot (\dot{E}_{\cancel{u=0}} + \dot{E}_{u \neq 0})$

$\dot{E} = -\sin x_1 x_2 + x_2 \sin x_1 - x_2 \cos x_1 u$

$\dot{E} = -x_2 \cos x_1 u$

$\dot{V}_p = E(-x_2 \cos x_1 u)$

c) $E = 1 + \cos(x_1) + \frac{x_2^2}{2}$

$\dot{E} = \cancel{\sin}$ let $x_1 = z_1 + \pi$

2) $\tilde{E} = 1 - \cos z_1 + \frac{x_2^2}{2}$

$\dot{\tilde{E}} = \sin z_1 \dot{z}_1 + x_2 \dot{x}_2$

$= \sin z_1 x_2 + x_2 (-\sin z_1 + \cos z_1 u)$

$= x_2 \cos z_1 u$

$= x_2 \cos z_1 (E x_2 \cos z_1)$

$= -x_2^2 \cos^2 z_1 E$

$= -x_2^2 \cos^2 z_1 E$

$E < 0$ when $\cos x_1 - 1 + \frac{x_2^2}{2}$

lets Define $D = \{ |x_1| < \pi/4 , x_2 < \}$

so $\dot{V} = 0$ only when $x = 0$ so.

the origin is unstable

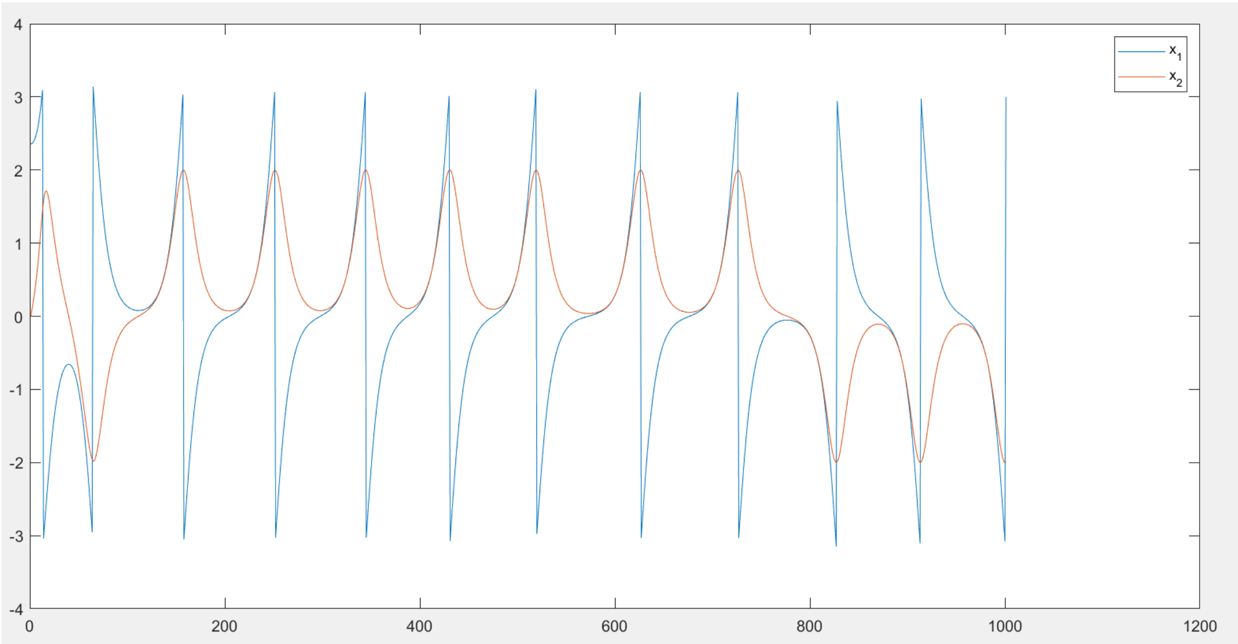
5) a) $V = V_p + \frac{\sigma}{2} x_4^2$

2) $\dot{V} = \dot{V}_p + \cancel{2}\frac{\sigma}{\cancel{2}} x_4 \dot{x}_4$

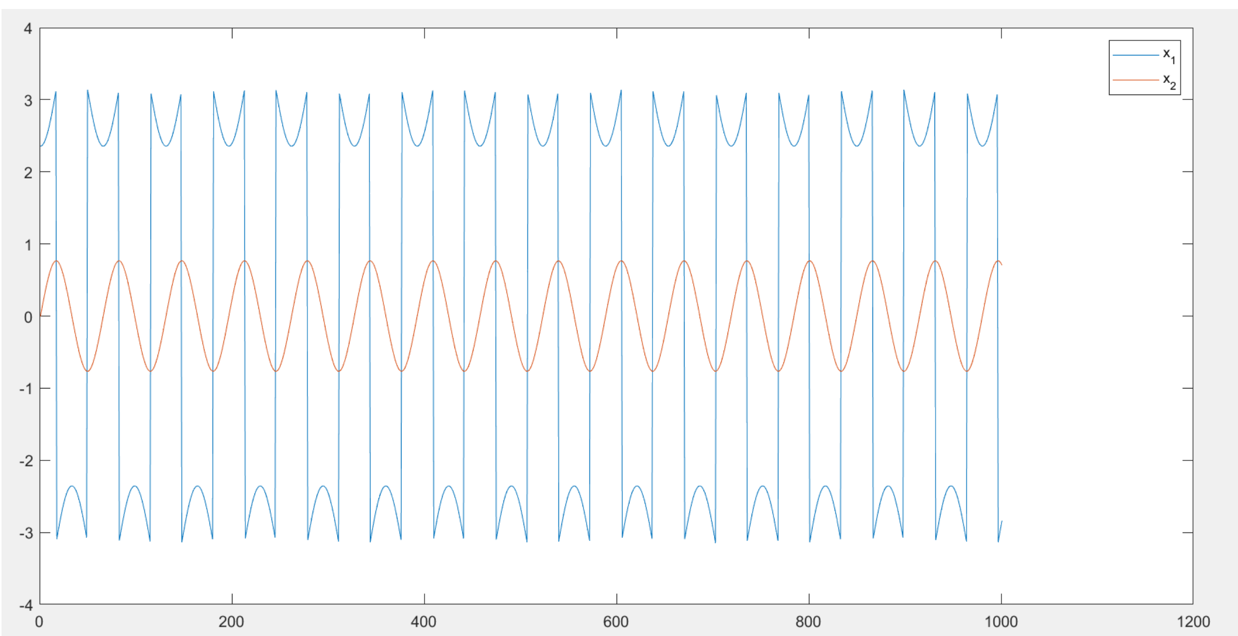
$= -E x_2 \cos x_1 u + \sigma x_4 u$

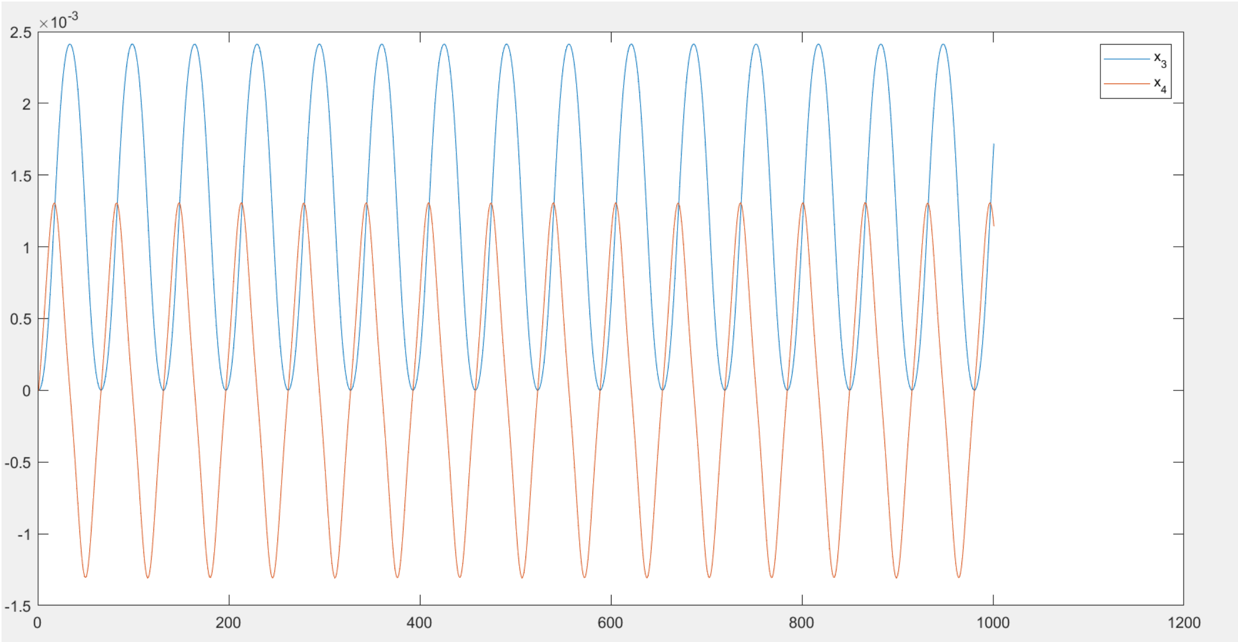
$= -(E x_2 \cos x_1 - \sigma x_4) u$

Question 4:

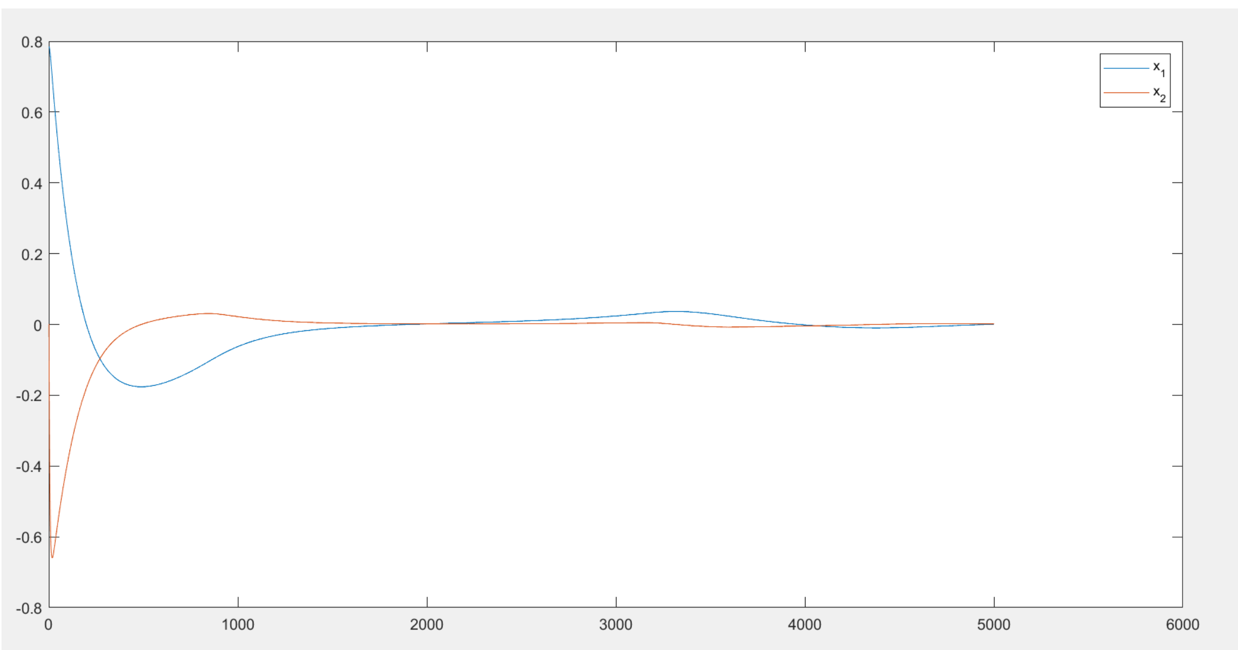


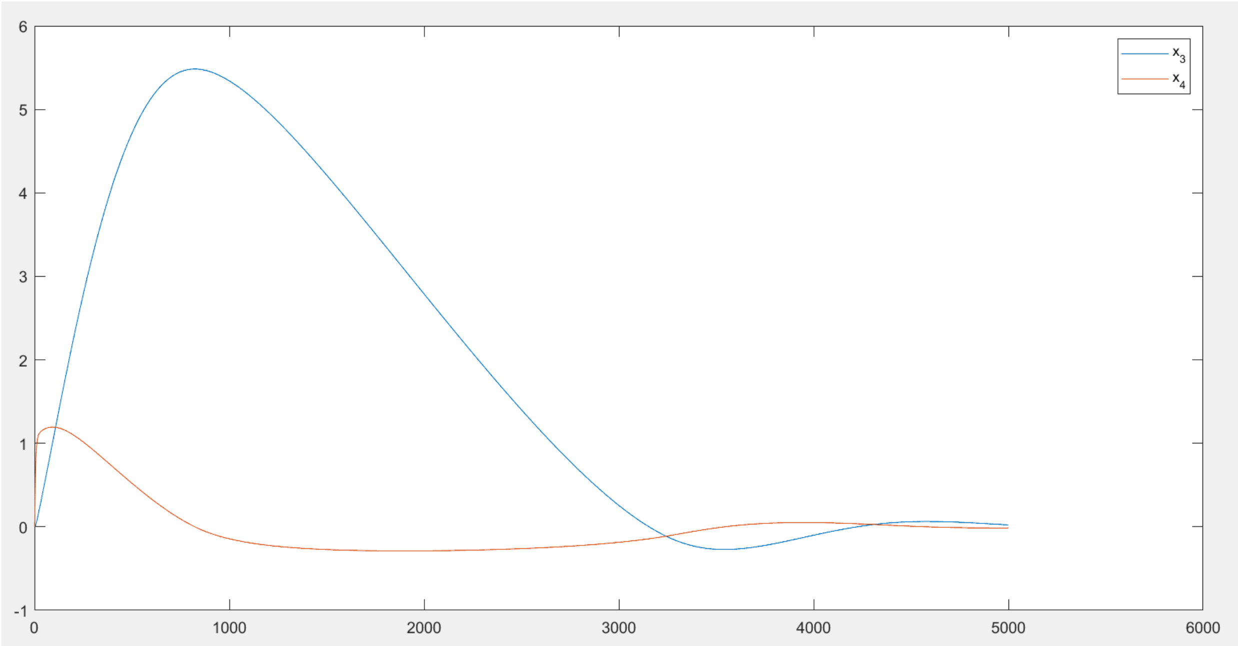
Question 5:
Part b





Part C:





The switching happens when the energy value goes less than 0.1 which is similar to the previous controller where a positive definite function has a limiting value.

Code:

Question 4:

```
z0 = [3*pi/4; 0];
t = 0:0.1:100;

[t_out,z] = ode45(@(t,z) pend_dyn(t,z), t, z0);
z_out = wrapToPi(z(:,1));

% plot(z_out)


plot(z_out(:))
hold on
plot(z(:,2))
legend('x_1','x_2');


function z_dot = pend_dyn(t,z_in)
z = z_in;
```

```
E = cos(z(1))-1 + ((z(2)*z(2))/2);

u = E*z(2)*cos(z(1));

z_dot = [z(2) ; sin(z(1))-(cos(z(1))*u)];

end
```

Question 5:

```
z0 = [3*pi/4; 0;0;0];
t = 0:0.1:100;

[t_out,z] = ode45(@(t,z) swing_pend(t,z), t, z0);
z_out = wrapToPi(z(:,1));


plot(z_out(:))
hold on
plot(z(:,2))
legend('x_1','x_2');
figure();
plot(z(:,3))
hold on
plot(z(:,4))
legend('x_3','x_4');

function op = better_dyn(t,z)

sigma = 1000;
%
% E = cos(z(1))-1 + ((z(2)*z(2))/2);
% u = (E*z(2)*cos(z(1))) - (sigma*z(4));

k = [19.3000, 22.9750, 1.5900, 5.5250];
k1 = k(1);
k2 = k(2);
k3 = k(3);
k4 = k(4);

u = ((k1*z(1) + k2*z(2))/cos(z(1))) + (k3*z(3)/(1+abs(z(3)))) + (k4*z(4)/(1+abs(z(4))));
```

```
op = [z(2);sin(z(1))-(cos(z(1))*u);z(4);u];
end


t = 0:0.01:50;
global threshold;
x_init = pi/4;
threshold = 0.1;

x0 = [x_init;0;0;0]; % 0.69 is the limit
result = val_lyaponov(x0);
tend = 0;
xend = x0;
x_tot = [];
if(result==1)
   Opt    = odeset('Events', @myEvent);
   [t_lin,x_lin] = ode45(@(t,x) swing_pend(t,x), t, x0, Opt); % output of linear system
   tend = t_lin(end);
   xend = x_lin(end,:);
   x_tot = [x_tot;x_lin];
end

result = val_lyaponov(xend);
if(result==0)
   x0_new = [xend(1);xend(2);xend(3);xend(4)];
   [t_fin,x_fin] = ode45(@(t,x) better_dyn(t,x), t, x0_new); % output of linear system
   x_tot = [x_tot;x_fin(:,1:4)];
end

plot(x_tot(:,1))
hold on
plot(x_tot(:,2))
legend('x_1','x_2');
figure();
plot(x_tot(:,3))
hold on
plot(x_tot(:,4))
legend('x_3','x_4');

% figure();
% plot(x_fin(:,5));
% hold on;
% plot(x_fin(:,6))
```

```
% legend('zeta_1','zeta_2');


function [value, isterminal, direction] = myEvent(t, x)
    value      = val_lyaponov(x);
    isterminal = 1;   % Stop the integration
    direction  = -1;
end

function result = val_lyaponov(z)
    global threshold;
    lyaponov = cos(z(1))-1 + ((z(2)*z(2))/2);
    result =1;
    if(lyaponov<threshold)
        result = 0;
    end
end
```