Question 5:

Part b:

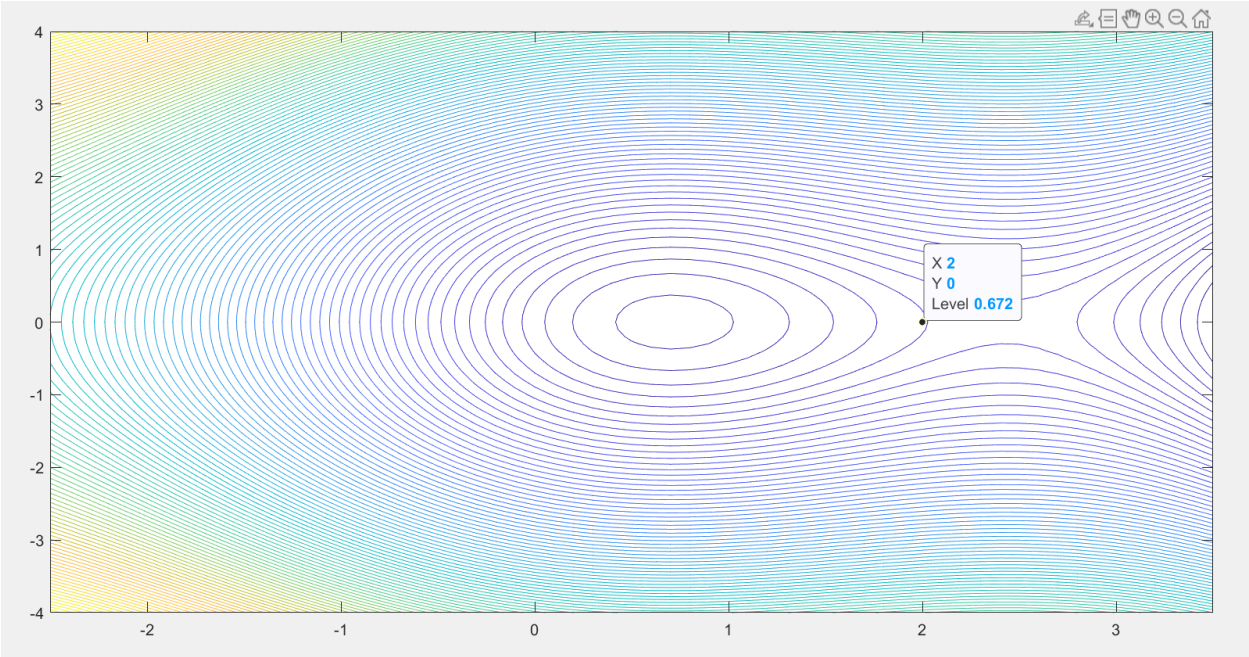
From the plot of the Lyapunov functions we can see that, the in defined region the contour

V(x) < 0.672 ; can be safely assumed as the region of attraction

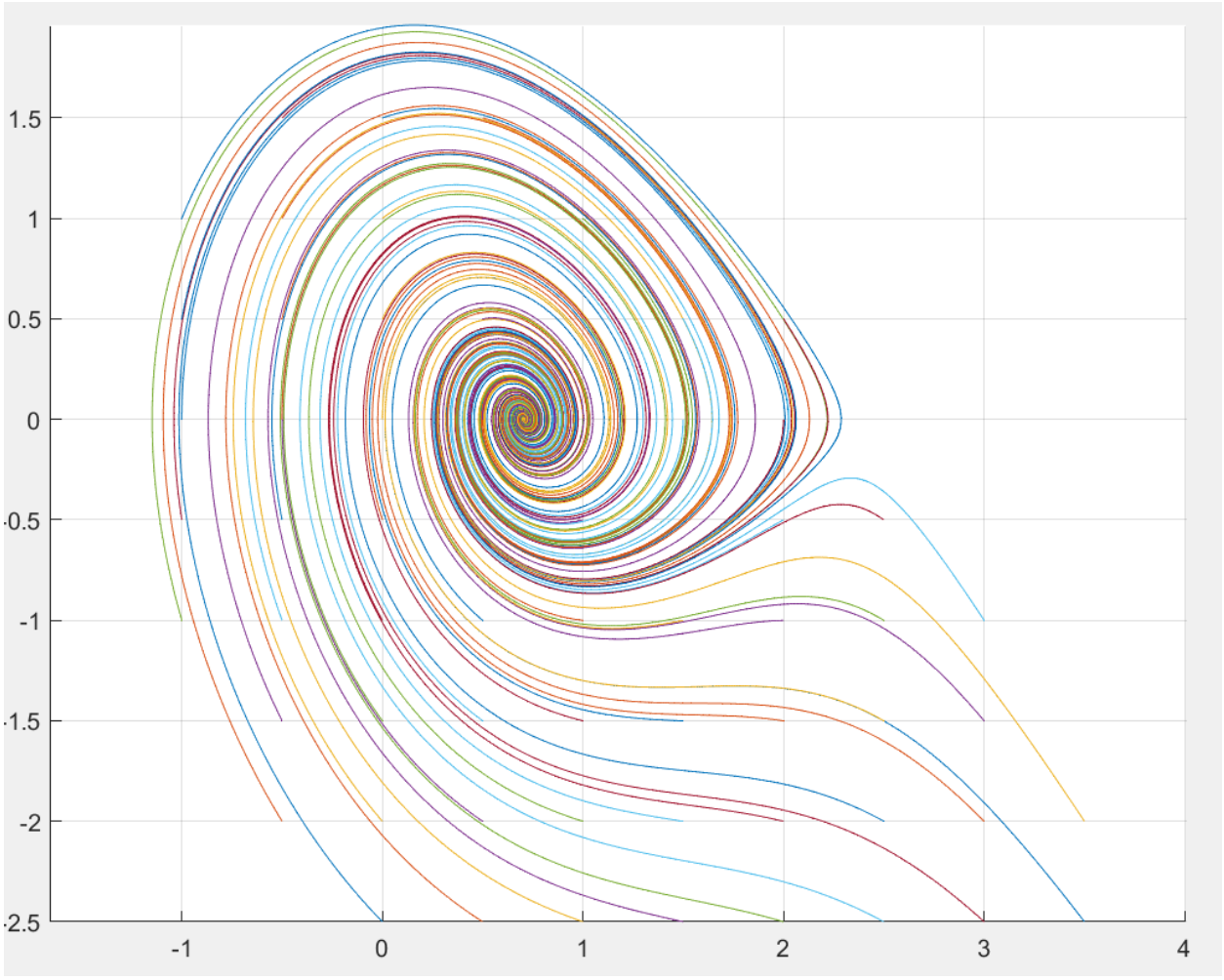
For **part C,** the **maximum value of lyapunov** obtained from **numerical analysis is 0.672**

This is because as we saw that

- the derivative is negative semi definite over the whole domain
- [0.707,0] is the only point in invariant set
- And v(x) =0.672 is the maximum compact set found in the domain through the plot



But if we observe the actual region of attraction its different as shown below:



Code:

```
%% Sub section 1
epsilon = 0.2;
samp = 10;

t = 0:0.01:30;

optPos = odeset('RelTol', 1e-6, 'AbsTol',1e-6);

x = -2.5:0.5:3.5;
y = -2.5:0.5:3.5;
[X,Y] = meshgrid(x,y);
```

```
ind = size(X);

fig = figure();
for i=1:ind(1)
   for j=1:ind(2)

      x0 = [X(i,j);Y(i,j)];
      [tP, x_new] = ode45(@(t,x)dyn(t,x), t, x0, optPos);
      if((abs(x_new(end,1)-0.707)<0.1) && (abs(x_new(end,2))<0.1))
         hold on
         plot(x_new(:,1),x_new(:,2))
      end
      % quiver(x_new(1:samp:end,1),x_new(1:samp:end,2))

   end
end
hold off
axis equal
grid on



function x_dot = dyn(t,x)
x_dot = [x(2); (-2*sin(x(1)))- (0.5*x(2)) + 1.3];
end

—----------------------------------------------------------------



x = -2.5:0.1:3.5;
y = -4:0.1:4;
[X,Y] = meshgrid(x,y);

xs = 0.707;
ys = 0;

X_dot = Y;

F = ((0.5*(Y.*Y) -2*(cos(X)-cos(xs)) - 1.3*(X-xs)));

G = (((Y.*(-2*sin(X)- 0.5*Y + 1.3)) +2*(sin(X).*Y) - 1.3*(Y)));

R = sqrt((X-xs).^2 + (Y-ys).^2);
```

```
figure();
contour(X,Y,F,100);
```