1)

$$\dot{x}_1 = x_1 - x_1^3 + x_2$$

$$V_1 = \frac{x_1^2}{2} \quad \dot{V}_1 = x_1 \dot{x}_1 = x_1^2 - x_1^4 + x_2 x_1$$

$$\Rightarrow \cancel{2x_2} - x_1 \qquad \Rightarrow u = -x_1$$

$$\Rightarrow \quad \dot{x}_1 = -x_1^3 + z_1 \qquad z_1 = -x_1 + x_2$$

$$\dot{z}_1 = -\dot{x}_1 + \dot{x}_2$$

$$= -x_1^3 + z_1 + u$$

$$V = \frac{x_1^2}{2} + \frac{z_1^2}{2}$$

$$\dot{V} = x_1 \dot{x}_1 + z_1 \dot{z}_1$$

$$= -x_1^4 + z_1 x_1 - z_1 x_1^3 + z_1^2 + z_1 u$$

$$\Rightarrow u = x_1^3 - 2z_1 - x_1 - K z_1 \qquad K > 0$$

$$\Rightarrow \boxed{\cancel{u = x_1^3 - 2x_2}}$$

$$u = x_1^3 + 2x_1 - 2x_2 + K(x_1 - x_2)$$

$$\boxed{u = x_1^3 + K(x_1 - x_2) + x_2} \qquad K > 0$$

2) $\dot{x}_1 = x_1^2 x_2$

$\dot{x}_2 = u$

$V_1 = \frac{x_1^2}{2} \quad \dot{V}_1 = x_1 \dot{x}_1 = x_1^3 u$

$u_s = -x_1$

$\therefore \quad \dot{x}_1 = x_1^2(-x_1 + z_1)$

$z_1 = x_2 + x_1$

$\dot{z}_1 = \dot{x}_2 + x_1^2(-x_1 + z_1)$

$= u - x_1^3 + z_1 x_1^2$

$V = \frac{1}{2} x_1^2 + \frac{1}{2} z_1^2$

$\dot{V} = x_1(-x_1^3 + z_1 x_1^2) + z_1(u - x_1^3 + z_1 x_1^2)$

$= -x_1^4 + z_1 x_1^3 - z_1 x_1^3 + z_1^2 x_1^2 + z_1 u$

$\cancel{-z_1 x_1^2}$

$u = -z_1 x_1^2 - x_1^3 - x_1^2 - z_1$

$= -(x_1 + x_2) x_1^2 - x_1^3 - x_1^2 - (x_1 + x_2)$

a) a) $E = \cos(x_1) - 1 + \frac{x_2^2}{2}$

$\dot{E} = -\sin x_1 \dot{x}_1 + x_2 \dot{x}_2$

$= -\sin x_1 x_2 + x_2(\sin x_1)$ $(u = 0)$

$\boxed{\dot{E} = 0}$

b) $V_p = \frac{E^2}{2}$ $\dot{V}_p = E\dot{E}$

$= E \cdot (\dot{E}_{\cancel{u=0}} + \dot{E}_{u \neq 0})$

$\dot{E} = -\sin x_1 x_2 + x_2 \sin x_1 - x_2 \cos x_1 u$

$\dot{E} = -x_2 \cos x_1 u$

$\dot{V}_p = E(-x_2 \cos x_1 u)$

c) $\tilde{E} = 1 + \cos(x_1) + \frac{x_2^2}{2}$

~~$\dot{E} = \sin$~~ let $x_1 = z_1 + \pi$

(2) $\tilde{E} = 1 - \cos z_1 + \frac{x_2^2}{2}$

$\dot{\tilde{E}} = \sin z_1 \dot{z}_1 + x_2 \dot{x}_2$

$= \sin z_1 x_2 + x_2 (-\sin z_1 + \cos z_1 u)$

$= x_2 \cos z_1 u$

$= x_2 \cos z_1 (E\, x_2 \cos z_1)$

$= -x_2^2 \cos^2 z_1 \, E$

$= -x_2^2 \cos^2 z_1 \, E$

$E < 0$ when $\cos x_1 - 1 + \frac{x_2^2}{2}$

lets Define $D = \{ |x_1| < \pi/4 \,,\; x_2 < \}$

so $\{ \dot{V} = 0$ only when $x = 0$ so.

the origin is unstable

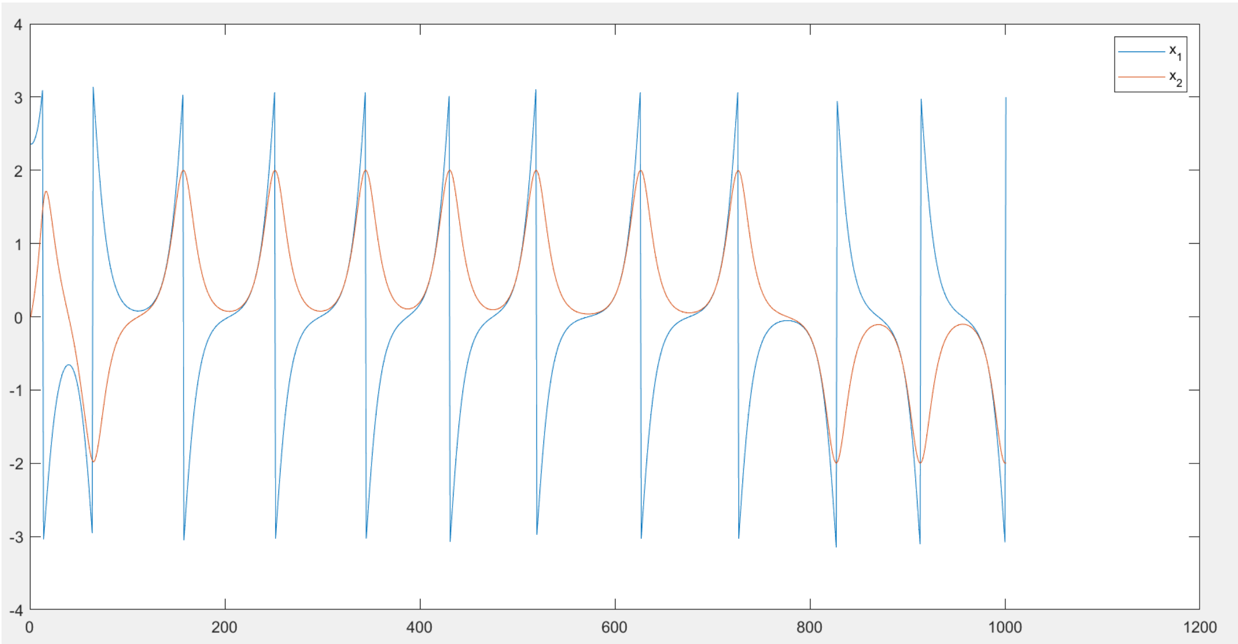
5) a) $$V = V_p + \frac{\sigma}{2} x_4^2$$

2) $$\dot{V} = \dot{V}_p + \cancel{2}\frac{\sigma}{\cancel{2}} x_4 \dot{x}_4$$

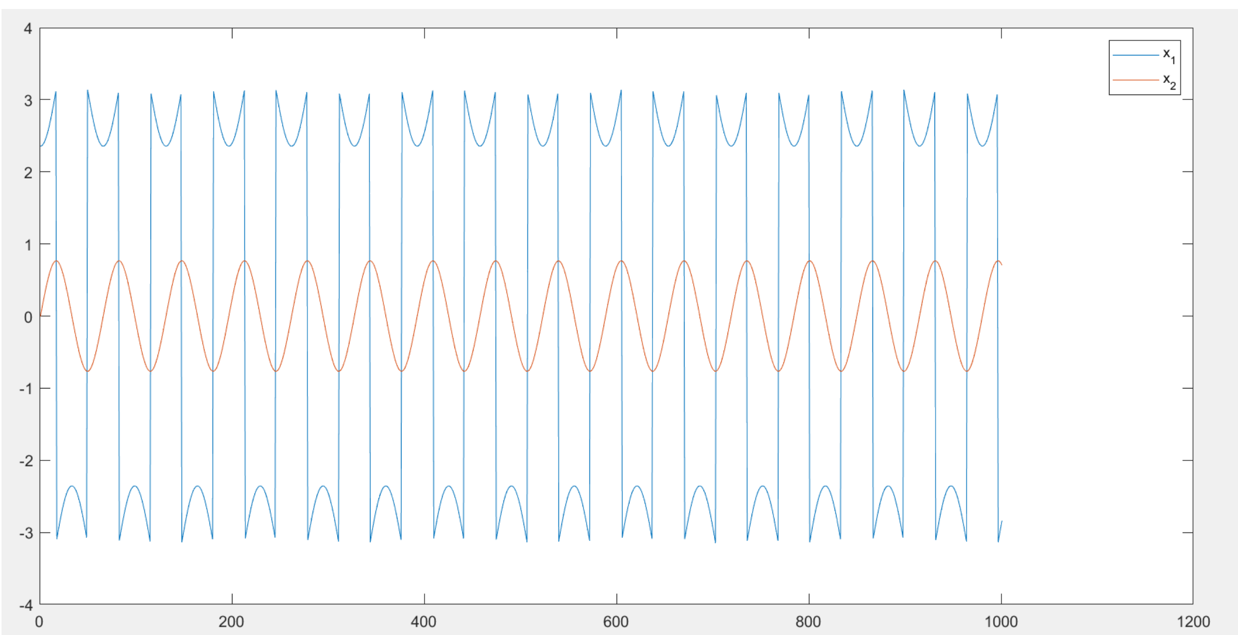
$$= - E x_2 \cos x_1 u + \sigma x_4 u$$

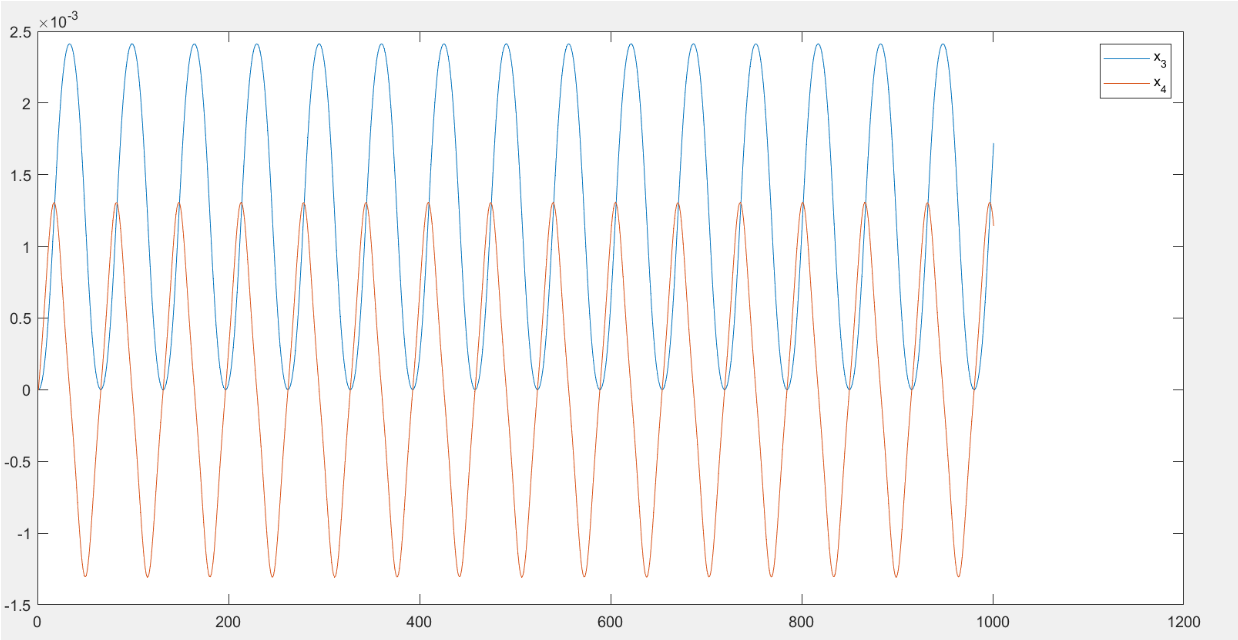
$$= -(E x_2 \cos x_1 - \sigma x_4) u$$

Question 4:



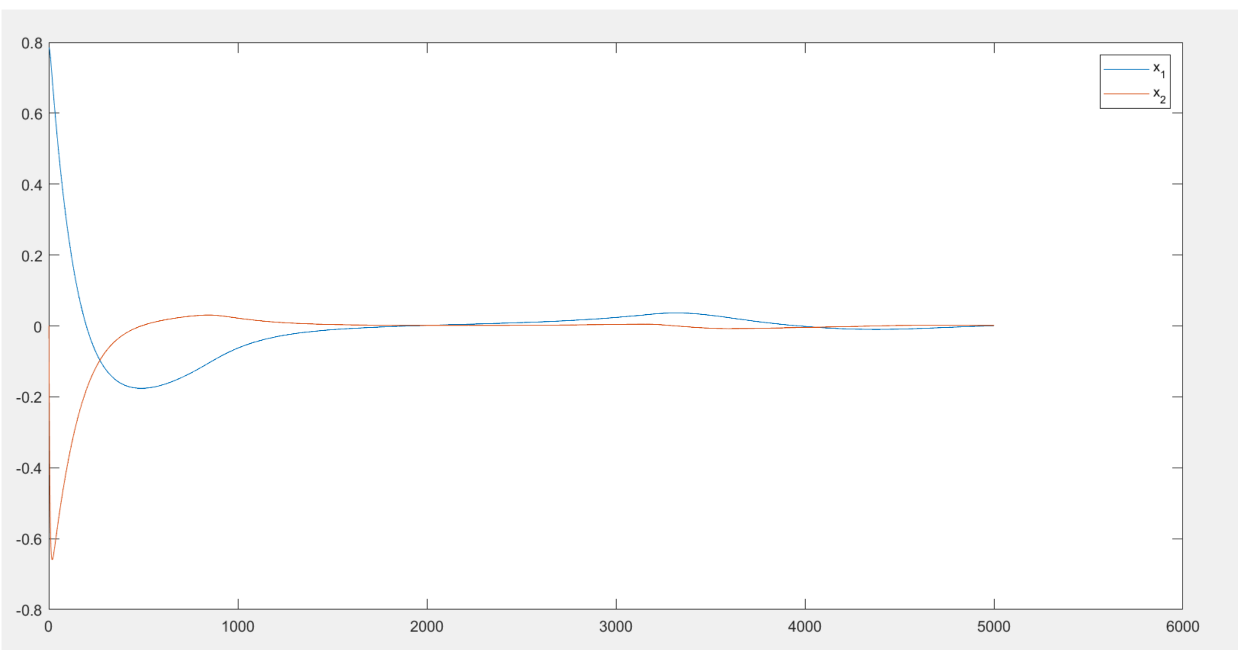
Question 5:
Part b



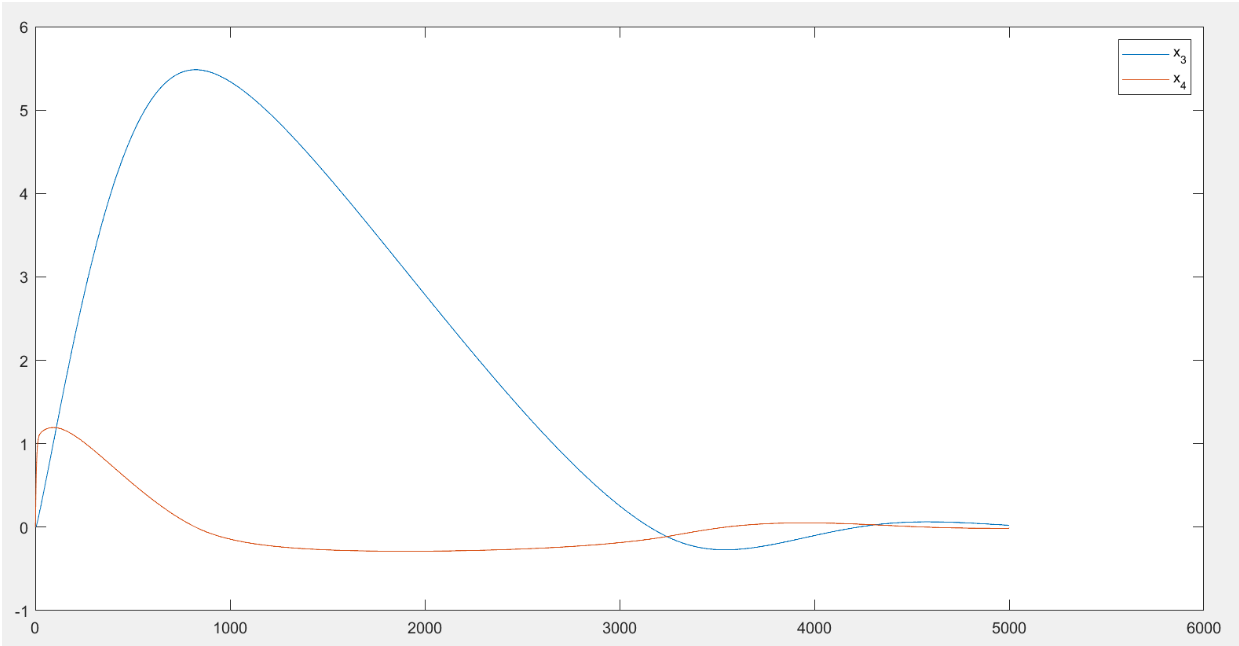


×10-3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
200
400
600
800
1000
1200
x3
x4

Part C:



0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
1000
2000
3000
4000
5000
6000
x1
x2



The switching happens when the energy value goes less than 0.1 which is similar to the previous controller where a positive definite function has a limiting value.