5) a)

$$f(\xi) = \begin{bmatrix} \dot{\xi_1} \\ \xi_2 \end{bmatrix} = \begin{bmatrix} \xi_2 \\ \dfrac{-a_1 \xi_1}{1 + K_1 |\xi_1|} - \dfrac{a_2 \xi_2}{1 + K_2 |\xi_2|} \end{bmatrix}$$

To linearise system

$$\frac{\partial f(1)}{\partial \xi_1} = 0 \qquad \frac{\partial f(1)}{\partial \xi_2} = 1$$

$$\frac{\partial f(2)}{\partial \xi_2} = -a_2 \left(\frac{(1 + K_2 |\xi_2|) - K_2 \, \text{sgn}(\xi_2) \, \xi_2}{(1 + K_2 |\xi_2|)^2} \right)$$

$$= \frac{-a_2}{(1 + K_2 |\xi_2|)^2}$$

So the variables are differentriable as

$$|\xi_2| \to 0 \qquad \text{as} \qquad \xi \to 0.$$

2) $\lim\limits_{\phi \xi_2 \to 0} \dfrac{f(2)}{\partial \xi_2} = -a_2$

Similarly $\lim\limits_{\xi_1 \to 0} \dfrac{\partial f(2)}{\partial \xi_1} = -a_1$

$\Rightarrow$ $A = \begin{bmatrix} 0 & 1 \\ -\alpha_1 & -\alpha_2 \end{bmatrix}$

char eq^n is $\qquad \lambda(\lambda + \alpha_2) + \alpha_1 = 0.$

$$\lambda^2 + \alpha_2 \lambda + \alpha_1 = 0.$$

$\Rightarrow$ roots are $\rightarrow \qquad \dfrac{-\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_1}}{2}$

~~roots will be either~~ are or .

real part of roots will always be -ve as

$\left(\alpha_i > 0 \right)$

f. System $\qquad \dot{\xi} = A\xi$ and the eigen values are -ve

$\| \xi(t) \leq \| \xi \|$

So the origin is exponentially stable (locally)

near the origin

b) $\quad V(\xi) = \alpha_1 \int_0^{\xi_1} \dfrac{\sigma}{1 + k_1 |m|} dm + \dfrac{\xi_2^2}{2}$

$\dot{V}(\xi) = \left(\alpha_1 \dfrac{\xi_1}{1 + k_1 |\xi_1|} \right) \dot{\xi}_1 + \xi_2 \dot{\xi}_2$

$= \dfrac{\alpha_1 \xi_1 \xi_2}{1 + k_1 |\xi_1|} + \xi_2 \left(\dfrac{-\alpha_1 \xi_1}{1 + k_1 |\xi_1|} - \dfrac{\alpha_2 \xi_2}{1 + k_2 |\xi_2|} \right)$

$= -\dfrac{\alpha_2 \xi_2^2}{1 + k_2 |\xi_2|}$

$\dot{V}(\xi) \leq 0 \qquad \rightarrow \text{ semi definite}$

$\xi \quad \text{if} \quad \text{wie look } \dot{V}(\xi) = 0$

$\Rightarrow \qquad D = \{ \xi \mid \xi_2 = 0 \}$

$f(m) = 0 = \begin{bmatrix} \xi_2 \\ \dfrac{-\alpha_1 \xi_1}{1 + k_1 |\xi_1|} - \dfrac{\alpha_2 \xi_2}{1 + k_2 |\xi_2|} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \qquad \dot{\xi}_2 = 0$

$$\frac{-\alpha_1 \xi_1}{1 + k_1 |\xi_1|} = 0 \Rightarrow \xi_1 = 0$$

Only pt in invariant set is $\xi = 0$

So. origin is globally Asymptotically Stable.

5) c)

$$\sup \left| \frac{\alpha_1 \, \xi_1}{1 + k_1 |\xi_1|} + \frac{\alpha_2 \, \xi_2}{1 + k_2 |\xi_2|} \right| \le \varepsilon$$

we know that

$$\sup \left| \frac{\alpha_1 \, \xi_1}{1 + k_1 |\xi_1|} + \frac{\alpha_2 \, \xi_2}{1 + k_2 |\xi_2|} \right| \le \sup \left| \frac{\alpha_1 \, \xi_1}{1 + k_1 |\xi_1|} \right| + \sup \left| \frac{\alpha_2 \, \xi_2}{1 + k_2 |\xi_2|} \right|$$

$$\le \sup \left| \frac{\alpha_1}{\frac{1}{|\xi_1|} + k_1} \right| + \sup \left| \frac{\alpha_2}{\frac{1}{|\xi_2|} + k_2} \right|$$

$$\le \underset{(\varepsilon/2)}{\left| \frac{\alpha_1}{k_1} \right|} + \underset{(\varepsilon/2)}{\left| \frac{\alpha_2}{k_2} \right|}$$

∴

$$\sup \left| \frac{\alpha_1 \, \xi_1}{1 + k_1 |\xi_1|} + \frac{\alpha_2 \, \xi_2}{1 + k_2 |\xi_2|} \right| \le \varepsilon$$

4)   let $z = y - \bar{y}$ $y = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$

let $\dfrac{\partial y}{\partial t} = g(t, y)$

$\Rightarrow$ $\dfrac{\partial z}{\partial t} = g(t, y) - \dot{\bar{y}}(t, y)$

         $= g(t, z + \bar{y}) - g(t, \bar{y})$

if $\dot{\bar{y}} \to 0$ the problem will be ~~linear~~ analysed linearly

$\&$

     $\dfrac{\partial z}{\partial t} \to (A + Bk)z$ $\&$ $eig(A + Bk) < 0$

    z is   exponentially stable