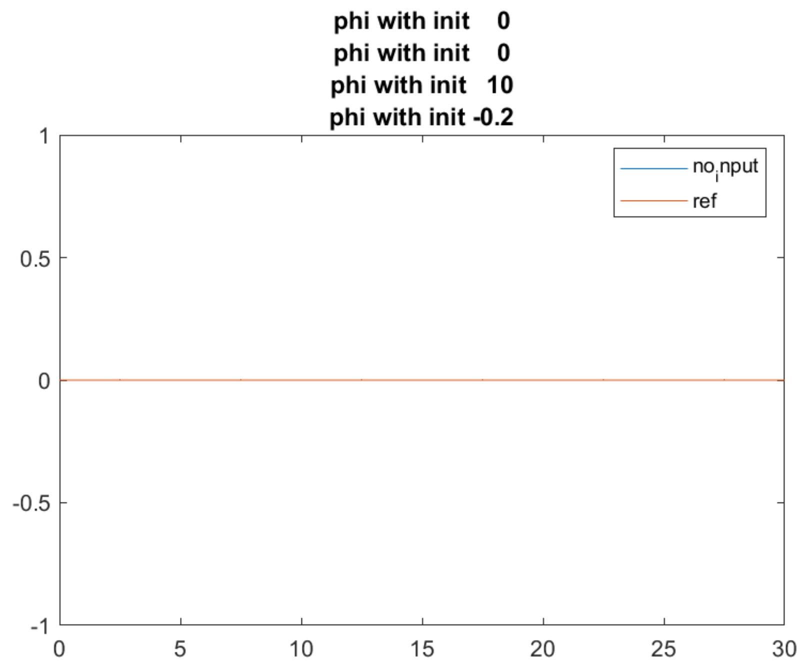
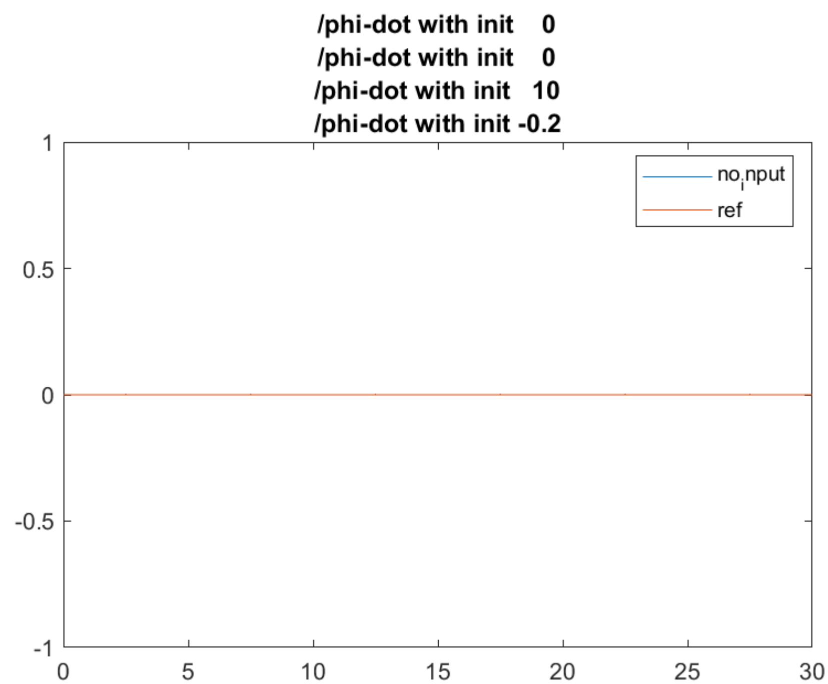
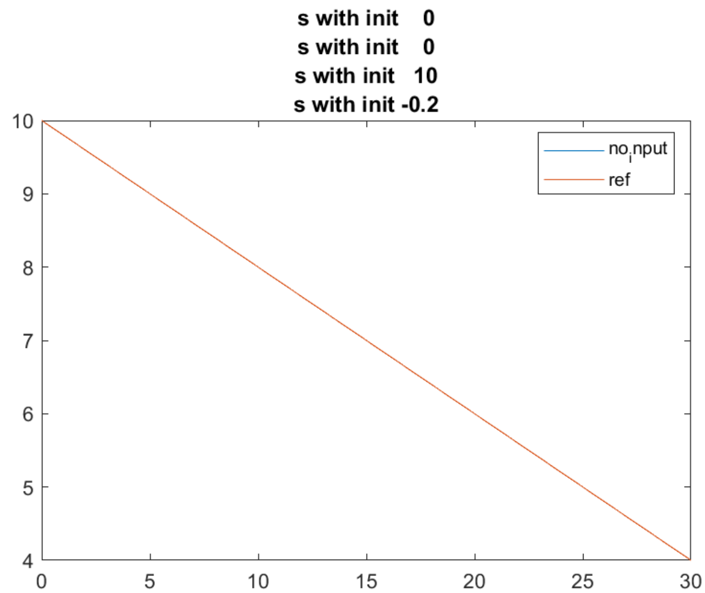
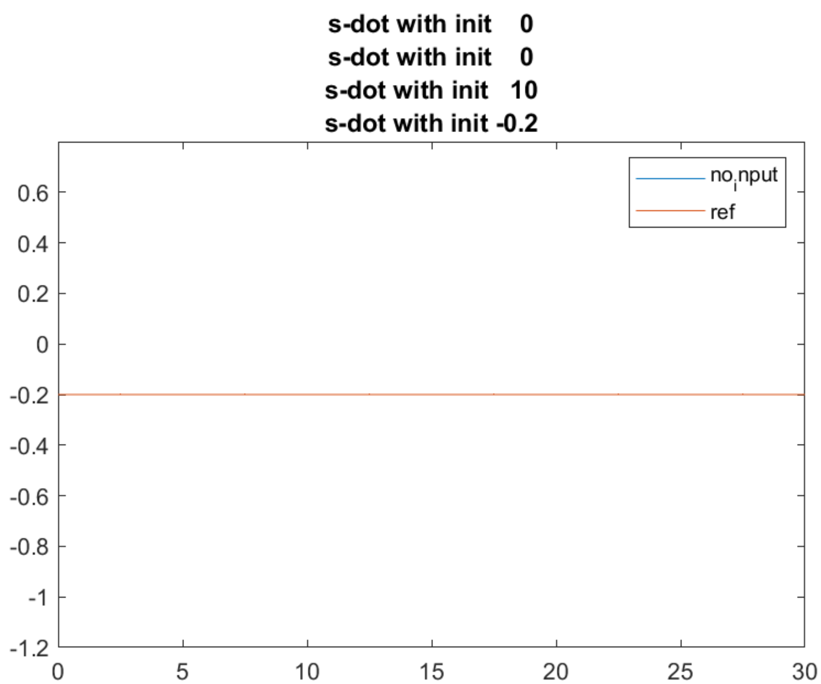
Problem 1:

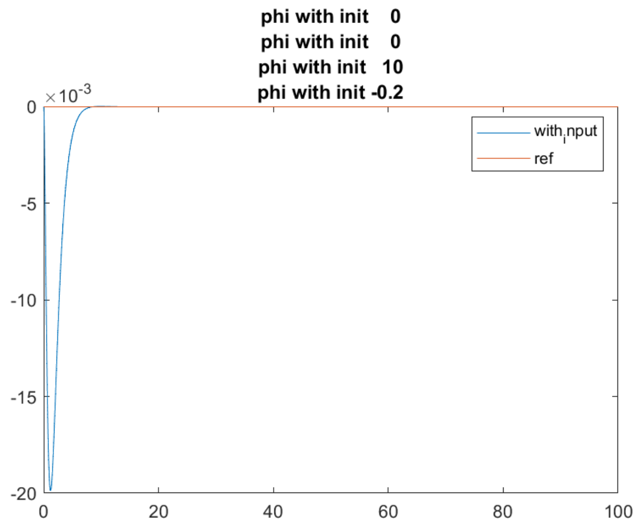

phi with init 0
phi with init 0
phi with init 10
phi with init -0.2
no_input
ref
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30

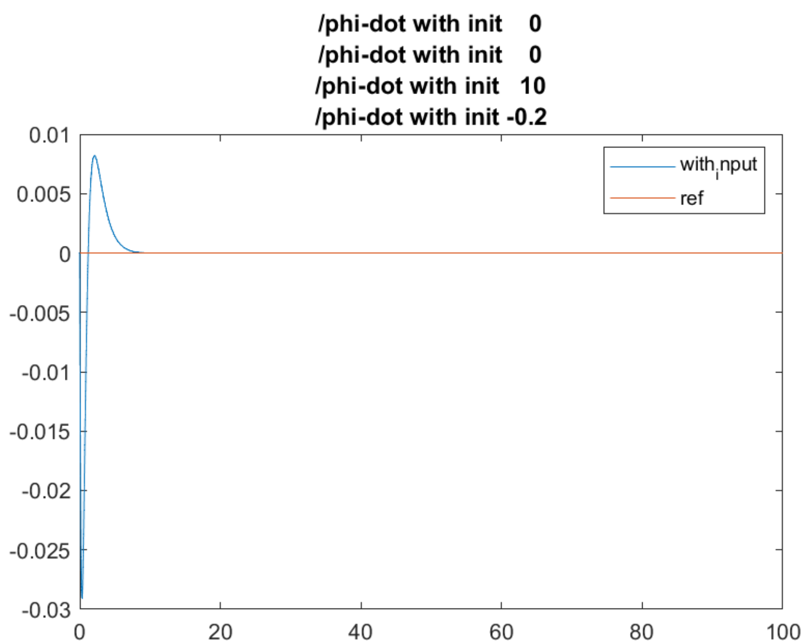

/phi-dot with init 0
/phi-dot with init 0
/phi-dot with init 10
/phi-dot with init -0.2
no_input
ref
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30

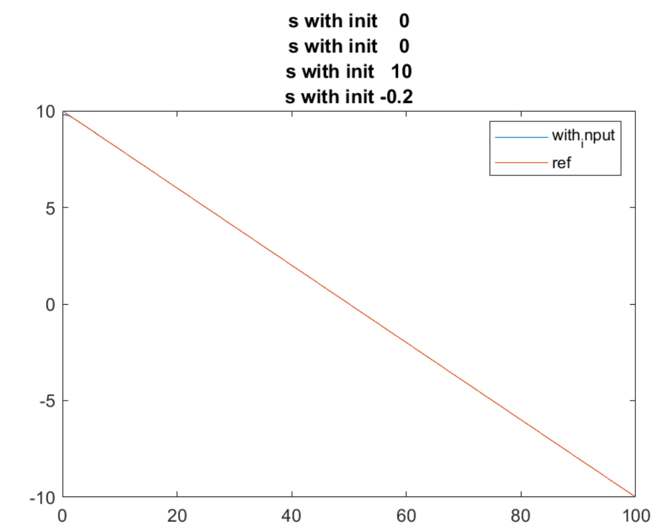

s with init 0
s with init 0
s with init 10
s with init -0.2
no_input
ref
10
9
8
7
6
5
4
0
5
10
15
20
25
30

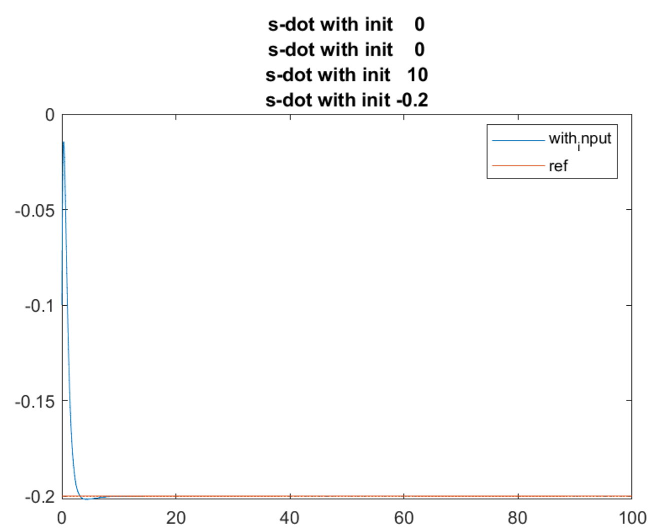

s-dot with init 0
s-dot with init 0
s-dot with init 10
s-dot with init -0.2
no_input
ref
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
5
10
15
20
25
30

Problem 2:

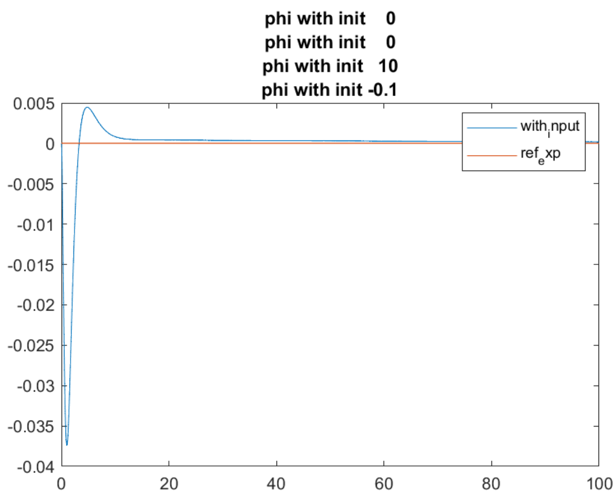

phi with init 0
phi with init 0
phi with init 10
phi with init -0.2
×10^-3
with_input
ref
0
-5
-10
-15
-20
0
20
40
60
80
100


/phi-dot with init 0
/phi-dot with init 0
/phi-dot with init 10
/phi-dot with init -0.2
with_input
ref
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
0
20
40
60
80
100

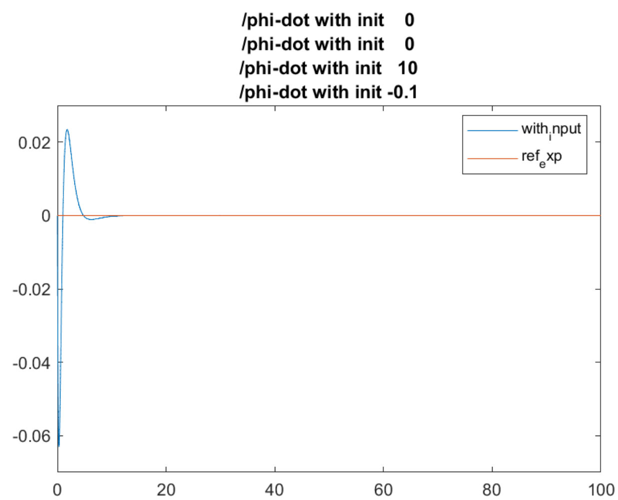

s with init 0
s with init 0
s with init 10
s with init -0.2
with_input
ref
10
5
0
-5
-10
0
20
40
60
80
100

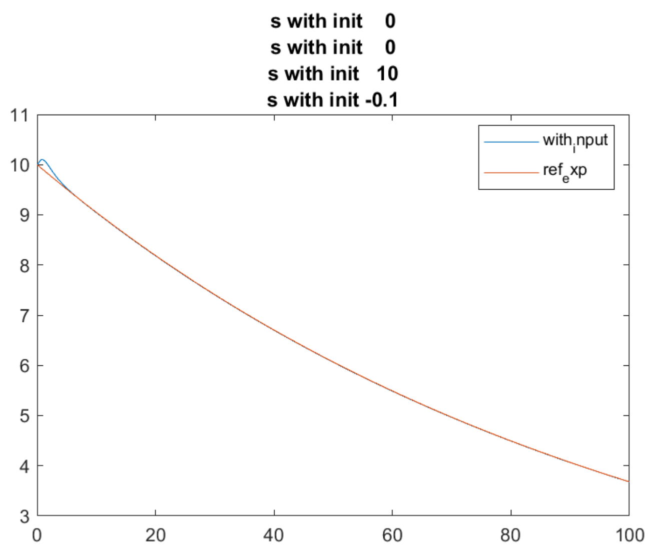

s-dot with init 0
s-dot with init 0
s-dot with init 10
s-dot with init -0.2
with_input
ref
0
-0.05
-0.1
-0.15
-0.2
0
20
40
60
80
100

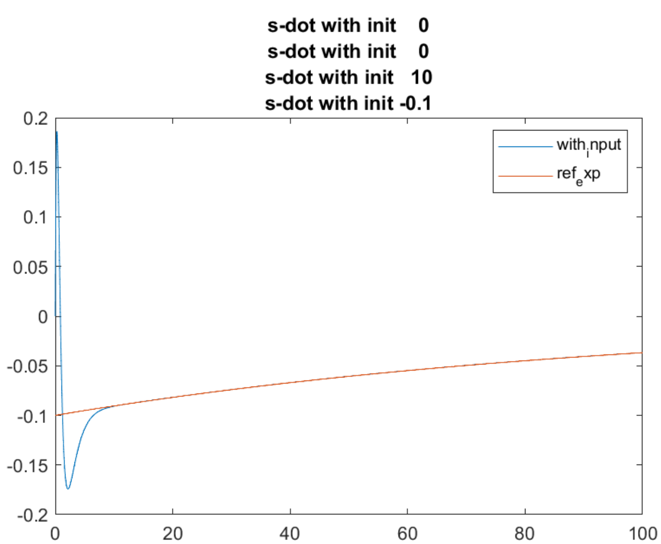
Problem 3:



phi with init 0
phi with init 0
phi with init 10
phi with init -0.1
with_input
ref_exp
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
-0.035
-0.04
0
20
40
60
80
100


/phi-dot with init 0
/phi-dot with init 0
/phi-dot with init 10
/phi-dot with init -0.1
with_input
ref_exp
0.02
0
-0.02
-0.04
-0.06
0
20
40
60
80
100


s with init 0
s with init 0
s with init 10
s with init -0.1
with_input
ref_exp
11
10
9
8
7
6
5
4
3
0
20
40
60
80
100


s-dot with init 0
s-dot with init 0
s-dot with init 10
s-dot with init -0.1
with_input
ref_exp
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
20
40
60
80
100

5) a) $f(\xi) = \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \xi_2 \\ \dfrac{-\alpha_1 \xi_1}{1 + K_1 |\xi_1|} - \dfrac{\alpha_2 \xi_2}{1 + K_2 |\xi_2|} \end{bmatrix}$

To linearise system

$$\frac{\partial f(1)}{\partial \xi_1} = 0 \qquad \frac{\partial f(1)}{\partial \xi_2} = 1$$

$$\frac{\partial f(2)}{\partial \xi_2} = -\alpha_2 \left(\frac{(1 + K_2 |\xi_2|) - K_2 \, \mathrm{sgn}(\xi_2) \, \xi_2}{(1 + K_2 |\xi_2|)^2} \right)$$

$$= \frac{-\alpha_2}{(1 + K_2 |\xi_2|)^2}$$

So the variables are differentiable as

$|\xi_2| \to 0$ as $\xi \to 0$

2) $$\lim_{\xi_2 \to 0} \frac{\partial f(2)}{\partial \xi_2} = -\alpha_2$$

Similarly $$\lim_{\xi_1 \to 0} \frac{\partial f(2)}{\partial \xi_1} = -\alpha_1$$

$\Rightarrow A = \begin{bmatrix} 0 & 1 \\ -\alpha_1 & -\alpha_2 \end{bmatrix}$

Char eq^n is $\quad x(x+\alpha_2) + \alpha_1 = 0.$

$$x^2 + \alpha_2 x + \alpha_1 = 0.$$

$\Rightarrow$ roots are : $\quad \dfrac{-\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_1}}{2}$

~~roots will be either are~~ or

real part of roots will always be -ve as $(\alpha_2 > 0)$

$\therefore$ System $\dot{\xi} = A\xi$ and the eigen values are -ve

$\|\dot{\xi}\| \cancel{\leq \|A\xi\|}$

So the origin is exponentially stable (locally) near the origin

b) $V(\xi) = \alpha_1 \int_0^{\xi_1} \frac{\eta}{1+k_1|\eta|} d\eta + \frac{\xi_2^2}{2}$

$\dot{V}(\xi) = \left(\alpha_1 \frac{\xi_1}{1+k_1|\xi_1|}\right) \dot{\xi}_1 + \xi_2 \dot{\xi}_2$

$= \frac{\alpha_1 \xi_1 \xi_2}{1+k_1|\xi_1|} + \xi_2 \left(\frac{-\alpha_1 \xi_1}{1+k_1|\xi_1|} - \frac{\alpha_2 \xi_2}{1+k_2|\xi_2|} \right)$

$= - \frac{\alpha_2 \xi_2^2}{1+k_2|\xi_2|}$

$\dot{V}(\xi) \leq 0 \rightarrow$ semi definite

& if we look $\dot{V}(\xi) = 0$

$\Rightarrow \quad D = \{ \xi \mid \xi_2 = 0 \}$

$f(x) = 0 = \begin{bmatrix} \xi_2 \\ \frac{-\alpha_1 \xi_1}{1+k_1|\xi_1|} - \frac{\alpha_2 \xi_2}{1+k_2|\xi_2|} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad \xi_2' = 0$

$$\frac{-\alpha_1 \xi_1}{1 + K_1|\xi_1|} = 0 \Rightarrow \xi_1 = 0$$

Only pt in invariant set is $\xi = 0$

So. origin is globally Asymptotically stable.

5) c) $$\sup \left| \frac{\alpha_1 \xi_1}{1+K_1|\xi_1|} + \frac{\alpha_2 \xi_2}{1+K_2|\xi_2|} \right| \le \varepsilon$$

we know that

$$\sup \left| \frac{\alpha_1 \xi_1}{1+K_1|\xi_1|} + \frac{\alpha_2 \xi_2}{1+K_2|\xi_2|} \right| \le \sup \left| \frac{\alpha_1 \xi_1}{1+K_1|\xi_1|} \right| + \sup \left| \frac{\alpha_2 \xi_2}{1+K_2|\xi_2|} \right|$$

$$\le \sup \left| \frac{\alpha_1}{\frac{1}{|\xi_1|} + K_1} \right| + \sup \left| \frac{\alpha_2}{\frac{1}{|\xi_2|} + K_2} \right|$$

$$\le \left| \frac{\alpha_1}{K_1} \right| + \left| \frac{\alpha_2}{K_2} \right|$$

$(\xi/2)$ $\qquad$ $(\xi/2)$

$\therefore$

$$\sup \left| \frac{\alpha_1 \xi_1}{1+K_1|\xi_1|} + \frac{\alpha_2 \xi_2}{1+K_2|\xi_2|} \right| \le \xi$$

u) let $z = y - \bar{y}$ $y = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$

let $\frac{\partial y}{\partial t} = g(t, y)$

$\Rightarrow \frac{\partial \cancel{z}}{\partial t} = g(t, y) - \dot{\bar{y}}(t, y)$

$= g(t, z + \bar{y}) - g(t, \bar{y})$

if $\dot{\bar{y}} \to 0$ the problem will be ~~linear~~ analysed linearly

& $\frac{\partial z}{\partial t} \to (A + BK)z$ & $\text{eig}(A + BK) < 0$

So. z is exponentially stable