

2) given $\frac{d(\alpha(t))}{dt} = f(\alpha(t))$

$$\therefore g(\tau) = \alpha(\tau/a)$$

$$\frac{dg(\tau)}{d\tau} = \frac{d}{d\tau} (\alpha(\tau/a))$$

applying ~~root~~ $a\tau/b = t$

$$= \frac{d(\alpha(t))}{dt} \frac{dt}{d(\tau/b)} \frac{d\tau}{dt}$$

$$= \frac{1}{a} f(\alpha(t))$$

$$= \frac{1}{a} f(\alpha(\tau/a))$$

$$\boxed{\frac{dg(\tau)}{d\tau} = \frac{1}{a} f(g(\tau))}$$

b) applying $\frac{d}{dt} \ln y(t)$ rule

$$\Rightarrow \frac{1}{y(t)} \alpha' = -1$$

$$\Rightarrow \frac{dy(t)}{d(\tau(t)x)} = (-1) \cdot \frac{f(y(t))}{\frac{b}{T_b}} \quad | \cdot T_b$$

$$\boxed{\frac{dy(t)}{d(\tau(t)x)} = -\frac{f(y(t))}{\frac{b}{T_b}}} \quad \text{principio}$$

$$3b \quad (+) b \quad \cancel{+ b} =$$

$$c) \quad \text{given} \quad \frac{d\alpha(t)}{dt} = 3\alpha(t)$$

$$\Rightarrow f(\alpha(t)) = 3\alpha(t)$$

$$\Rightarrow \frac{dy(t)}{d(\tau(t)x)} = -3y(t)$$

$$\boxed{\frac{dy(t)}{d(\tau(t)x)} = -\frac{3y(t)}{\frac{b}{T_b}}}$$

4) a) $m = m^4$ mindestens für (d)

sobald der Delta, Mittenpunkt 2NT

sollte m^4 sein (Hälfte der Summe)

oder negativ (mittenpunkt)

$$\left(\frac{4}{m^3} \right)^{m(t)} \underset{t \rightarrow 0}{\sim} t$$

$$\text{Sollte Mittenpunkt } \frac{1}{m^3} = t/4 \text{ werden}$$

$$(t - \text{sollt. } 0) = n \cdot m^3 = n \cdot 1 / (1 - t/4)$$

$$m(t) = \sqrt[3]{1 / (1 - t/4)} \quad \text{will go to}$$

~~sollte Mittenpunkt~~ ~~hätte es sein~~ sollt ∞ an $t = 4$

oder sollte es nicht ∞ sein?

0.98762

oder das ist es nicht mehr? oder

oder irgendwas? ~~0.98762~~ 0+

$$4) b) \quad \dot{\alpha} = \alpha' \sin \alpha$$

This sequence will always be inside a compact set W around the equilibrium point

$$\text{where } W = \left(\theta + \frac{\pi}{2}, \theta + \frac{\pi}{2} \right)$$

where $\theta \rightarrow$ equilibrium point

$$= n\pi$$

$$(n \in \mathbb{Z}) \quad (n = 0, \pm 1, \pm 2, \dots)$$

stop then

$\forall \delta > 0$ the sequence won't escape the set

so, it doesn't have finite time escape.

And from Theorem 3.3 it will converge \rightarrow ~~escapes~~ equilibrium point

5) a) Yes, the function is locally Lipschitz continuous

& the Lipschitz constant can be

given by

$$\frac{|f(x) - f(y)|}{|x - y|} \leq L$$

& L can be given by $\max |f'(x)|$
where $x \in (m, n)$

if $x \in (m_1, m_2)$

$$f'(x) = 2x + 3x^2 \operatorname{sgn}(x)$$

$$\max_{x \in (m_1, m_2)} |f'(x)| = \begin{cases} |2m_2 + 3m_2^2 \operatorname{sgn}(m_2)| & \text{if } |m_2| > |m_1| \\ |2m_1 + 3m_1^2 \operatorname{sgn}(m_1)| & \text{if } |m_1| > |m_2| \end{cases}$$

$$= |2m_1 + 3m_1^2 \operatorname{sgn}(m_1)| \quad (m_1 > m_2)$$

exists for

b) b. d. 2 This is not Lipschitz continuous (a) (c)
because the function is not piecewise
continuous

because the function is not piecewise
continuous
it has discontinuity at

$$m \geq 0 \quad f(m) = \begin{cases} (e^x)^m - (e^y)^m & m \geq 0 \\ -\cos m & m < 0 \end{cases}$$

$(e^x)^m$ and $-\cos m$

This can also be verified

If we take $x = -d$, $y = -d$
where $d \rightarrow 0^+$

then $\cos x \approx \cos d \approx 1$

$$(e^{-d})^d \approx \cos y + (\cos(-d))^d \approx 1$$

$$f(d) \approx 1 \quad f(-d) \approx -1$$

$$\frac{\|f(d) - f(-d)\|}{\|d - (-d)\|} \approx \frac{2}{2\|d\|} \text{ as } d \rightarrow 0.$$

The value shoots to ∞ .

∴ not Lipschitz continuous

c) Yes, It is locally Lipschitz. (b)

similar to Part (a), with some steps

$$L = \max_{n \in \{m_1, m_2\}} (|f'(c_n)|)$$
$$|f'(c_n)| = \frac{2n}{2\sqrt{n^2+1}} = \begin{cases} \frac{m_1}{\sqrt{m_1^2+1}} \\ \frac{m_2}{\sqrt{m_2^2+1}} \end{cases}$$

$\therefore L = \text{Max of } |f'(c_n)|$ if

$|f'(c_n)| = (m) \text{ for } n \in \{m_1, m_2\}$
if $f'(c_n)$ has range from $(-1, 1)$

$$\therefore L = 1 \quad \forall n \in \mathbb{N}$$

\therefore We can also say that this is globally Lipschitz

d) No, this is not locally Lipschitz

Because the function is discontinuous

at $x = -1$ (discontinuous)

$$f(x) = \begin{cases} x^2 & |x| \leq 1 \\ x & |x| > 1 \end{cases}$$

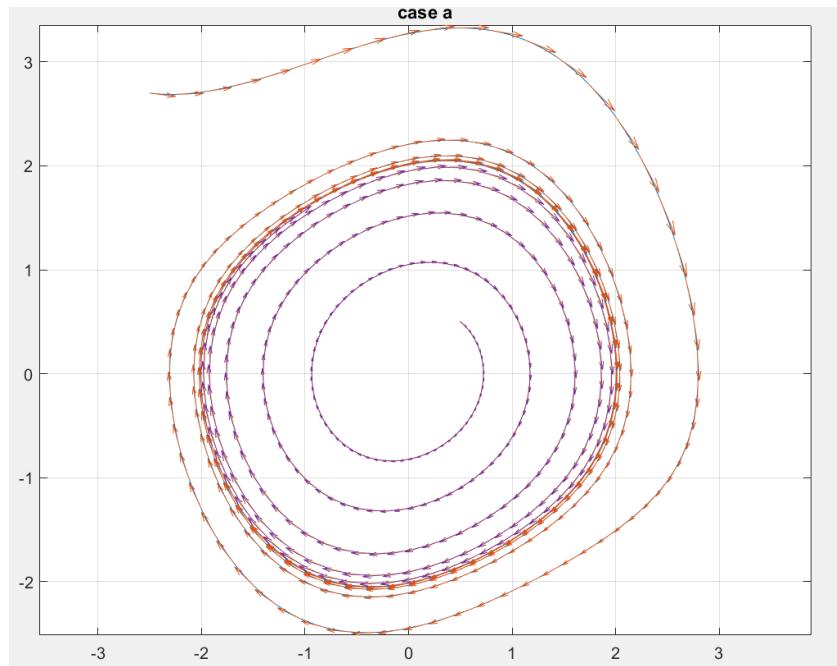
$\therefore \text{at } \lim_{m \rightarrow 1^-} f(m) = -1$
 $\lim_{m \rightarrow 1^+} f(m) = 1$

function is not continuous at $x = 1$

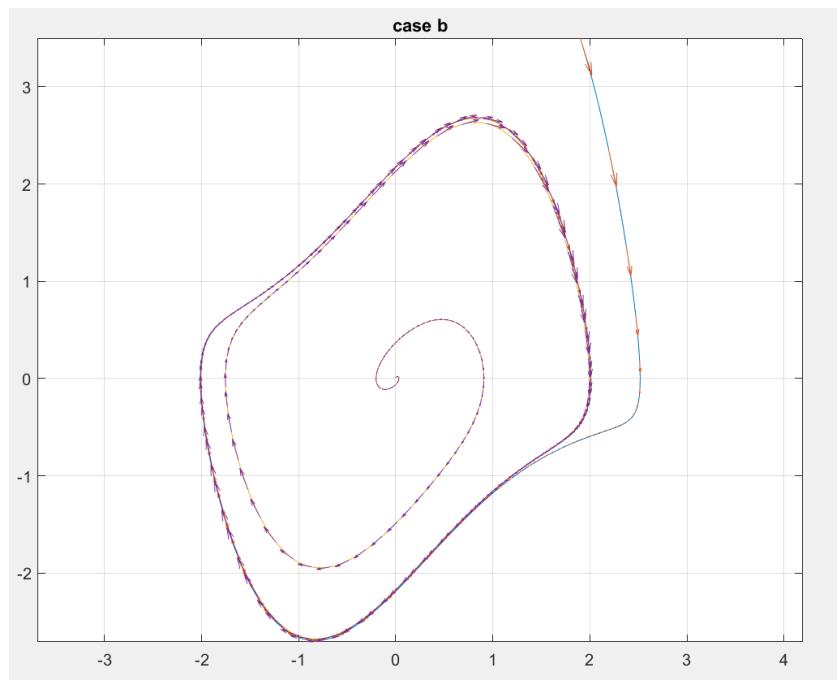
Homework 1

Question 1:

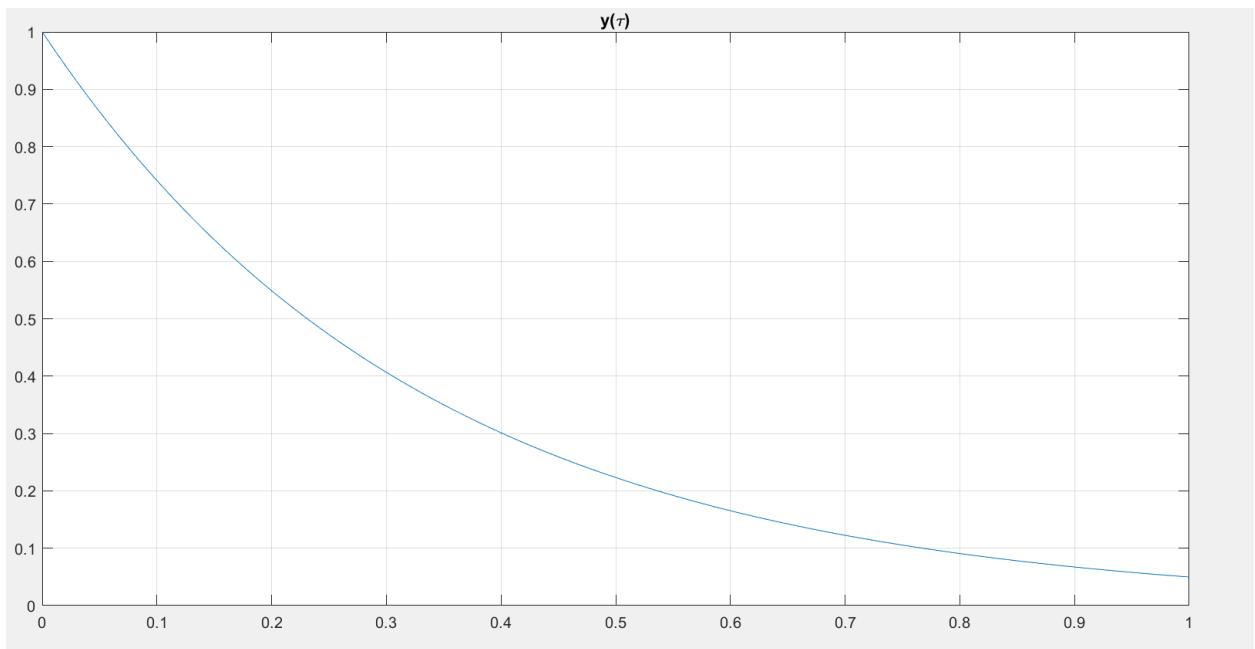
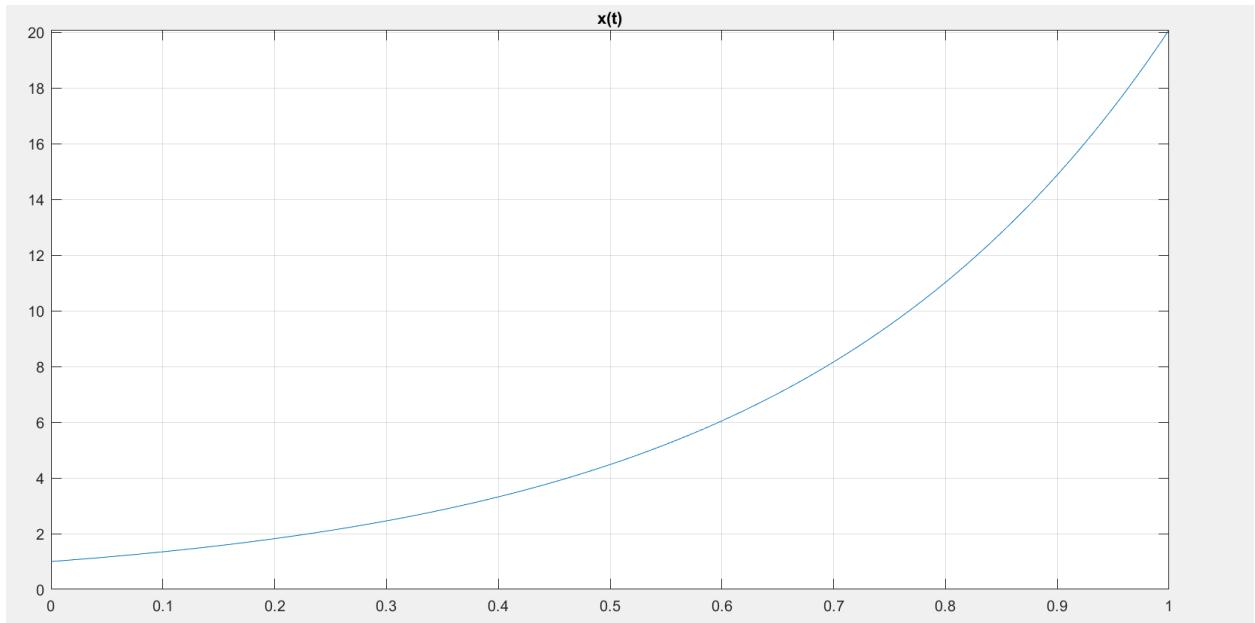
Case (a):



Case (b):

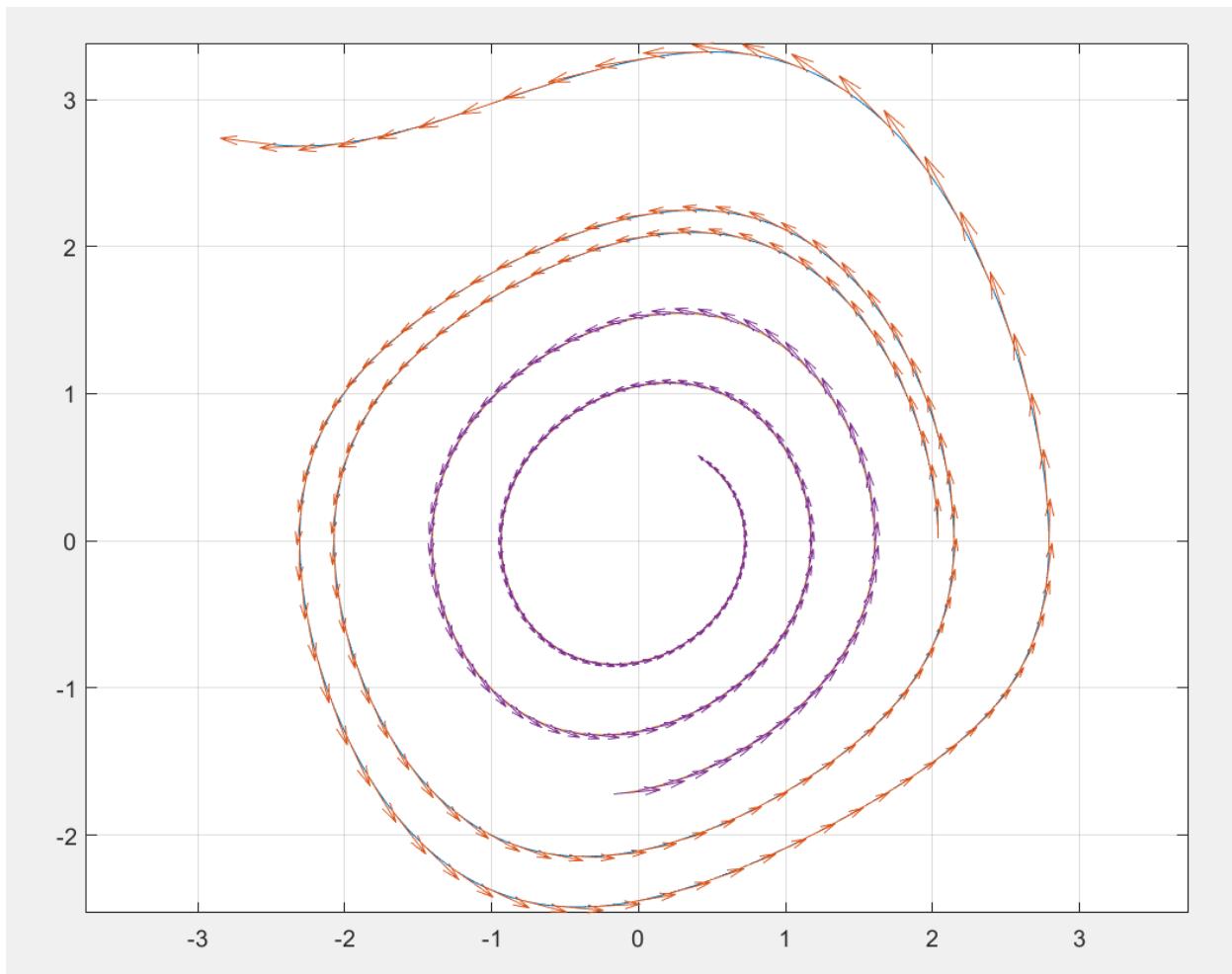


Question 2:



Question 3:

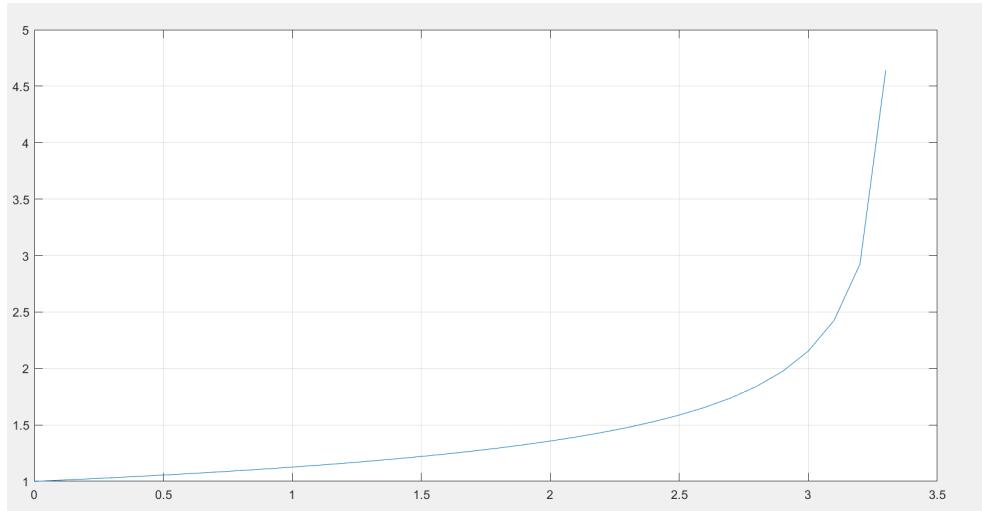
The plot is similar to question q with the direction reversed in the opposite direction. This is because the derivative is negative of the one in question 1.



Question 4:

Case a:

The ode stops near $t = 4$



Case b:

ODE approaching next nearest equilibrium point

