

$$1) \quad \|\alpha\|_0 \leq \max_{i=1}^n |\alpha_i| = \sum_{i=1}^n |\alpha_i| = \|\alpha\|_1$$

not true sc. (2 others are true)

$$\|\alpha\|_1 = \left( \sum_{i=1}^n |\alpha_i| \right) \leq \sum_{i=1}^n \max(|\alpha_i|)$$

$$= n \|\alpha\|_\infty$$

$$\|\alpha\|_2 = \sqrt{\sum_{i=1}^n \alpha_i^2} \leq \sqrt{\sum_{i=1}^n \max(\alpha_i^2)} \\ \leq \sqrt{n} \|\alpha\|_\infty$$

$$\leq \|\alpha\|_\infty \sqrt{n}$$

Similarly  $\|\alpha\|_2 = \sqrt{\sum_{i=1}^n \alpha_i^2} \geq \sqrt{\alpha_{\min}}$

$$\|\alpha\|_2 \geq \|\alpha\|_\infty$$

$$\|\alpha\|_2^2 \leq \left( \sum_{i=1}^n |\alpha_i|^2 \right)^2 \leq \max(|\alpha_1|^2, \sum_{i=1}^n |\alpha_i|^2)$$

$$\|\alpha\|_2^2 \leq \|\alpha\|_\infty \|\alpha\|_1$$

$$\text{as } \|\alpha\|_\infty < \|\alpha\|_2$$

$$\therefore \|\alpha\|_2 \leq \|\alpha\|_1$$

$$\|x\| = \sup_{i \in I} \|x_i\|_1 \leq \sqrt{n} \|x\|_2$$

we can use induction

$$\text{If } \left( \sum_{i=1}^{n-1} \|x_i\|_1 \right)^2 \leq (n-1) \left( \sum_{i=1}^{n-1} x_i^2 \right)$$

$$\Rightarrow \left( \sum_{i=1}^n \|x_i\|_1 \right)^2 = \left( \sum_{i=1}^{n-1} \|x_i\|_1 \right)^2 + 2\|x_n\|_1 \left( \sum_{i=1}^{n-1} \|x_i\|_1 \right) + \|x_n\|^2$$

$$\begin{aligned} \|\bar{x}\|_1 &\geq \\ &\leq (n-1) \sum_{i=1}^{n-1} x_i^2 \\ \|\bar{x}\|_1 &\geq \sum_{i=1}^{n-1} \|x_i\|_1 + \sum_{i=1}^{n-1} \|x_i\|_1^2 + \|x_n\|^2 \\ \|\bar{x}\|_1 &< \sum_{i=1}^{n-1} \|x_i\|_1 + \|\bar{x}\|_1 \\ &\leq n \sum_{i=1}^{n-1} x_i^2 \\ \|\bar{x}\|_1 &\leq \|\bar{x}\|_1 \end{aligned}$$

$$\text{so } \sum_{i=1}^n \|x_i\|_1 \leq \|\bar{x}\|_1 \sqrt{n} \|\bar{x}\|_2$$

$$\|\bar{x}\|_1 \leq \|\bar{x}\|_1$$

$$\|\bar{x}\|_1 > \|\bar{x}\|_1 \text{ so}$$

$$\|\bar{x}\|_1 \geq \|\bar{x}\|_1$$

2) If  $f$  is a continuous function (2)  
 $\epsilon \rightarrow$  for some  $\delta$  s.t if  
if  $\forall \delta > 0 \exists \delta_0$  s.t  
such that  $|x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon$   
such that  $|f(x_n) - f(y)| < \epsilon$

as  $\lim_{n \rightarrow \infty} x_n = x_0$  os  $\delta$ .  
 $\forall \delta > 0 \exists n_0 \in \mathbb{N} \text{ s.t. } N \leq n_0$   
 $\exists \delta > 0 \forall n > n_0 \text{ s.t. } |x_n - x_0| < \delta$

$\Rightarrow$   $\forall n (\geq n_0) \text{ s.t. } n \in \mathbb{N}$

( $\forall \delta > 0 \exists \delta_0$  s.t  $|f(x_n) - f(x_0)| < \epsilon$ )

then  $\lim_{n \rightarrow \infty} f(x_n) = f(x_0)$

Q.E.D.

3) a) ~~subset~~ bounded & s.t.  $\lim_{n \rightarrow \infty} a_n = \bar{a}$

$\forall \epsilon > 0 \exists N \in \mathbb{N} \text{ s.t. } \forall n > N$

$|a - a_n| \leq \epsilon$

$\Rightarrow \{a_n\}$  is a Cauchy sequence

b) as  $a_n \in S$

$\forall \epsilon > 0 \exists m \in S \text{ s.t. }$

such that  $\forall n > m \quad |a_n - m| \leq \epsilon$

$\Rightarrow |a_n - m| \leq \epsilon$

$\Rightarrow d(a_n, S) \geq 0$

$\Rightarrow \text{Im}(\bar{a}) \subseteq \text{Cl}(S)$  (closure of S)

for closed set  $S \in \mathcal{F}$  (given in question)

$\Rightarrow \overline{\bigcup_{a \in S}}$

$$4) a) \ddot{x} = -x^2 \quad (\text{for } t_0, \dot{x} = \dot{x}_0)$$

The sol' doesn't exist for all time

$$(t + p\pi) + (t - p\pi) = 2$$

$$\frac{\dot{x}}{x^2} = -1 \quad \text{at } t_0$$

but  $\frac{dx}{dt}$  is not defined next

$$\frac{d}{dt} \frac{1}{x} = \frac{1}{x^2} \quad t_0$$

Solutions are continuous when values are chosen equal to starting values

$$b) \text{similar to 3) second kind} \quad \frac{dx}{dt} = -dt$$

$$\Rightarrow [-3x^2]_{t_0}^t = [-t]_{t_0}^t$$

$$\Rightarrow 3x^2 - 3x_0^2 = (t - t_0)$$

$$x^2 = \frac{x_0^2}{3x_0(t-t_0) + 1}$$

denominator is always > 0

$\therefore$  sol' always exist

$$c) \quad i = \alpha \cos(\pi r) \quad r \in [0, 1] \quad (\text{p.v.})$$

satisfies w.c.t. consider interval  $[l, u]$

$$S = (\alpha_{\text{eq}} - \delta, \alpha_{\text{eq}} + \delta)$$

$$\text{if } \alpha_0 \in S \quad \frac{\alpha}{r}$$

then derivative at  $r = \alpha_{\text{eq}} - \delta$

$$\frac{\alpha}{r} \Big|_{r_1} \cdot \frac{d}{dr} > 0.$$

$$\begin{array}{c} \alpha \\ \downarrow \\ \alpha_0 \\ \alpha_1 \end{array} \quad \alpha < 0 \\ \alpha_1 = \text{measured}$$

$\Rightarrow$  or after contraction mapping principle  
Lipschitz continuous locally

q.e.d. Doesn't have infinite time (d)

Riccati

$$\begin{cases} \dot{x} = f(t, x) \\ x(t_0) = x_0 \end{cases} \quad \text{has unique solution}$$

$$(t_0 - \delta) \quad t_0 - \delta < t < t_0 + \delta$$

$$\begin{array}{c} \overbrace{t_0}^{\text{initial}} \\ \downarrow \\ t_0 - \delta \quad t_0 + \delta \end{array}$$

as points are non-interacting  
time points  $t_0$  ?

$$4) d) \quad \begin{aligned} \dot{x}_1 &= 1.2 \sin x_2 \\ \dot{x}_2 &= -x_1 - x_2 \end{aligned} \quad \left. \begin{array}{l} \text{is stable} \\ \text{asymptotic} \end{array} \right\} \quad \text{(b)}$$

$$(1.81) < 1 \quad [1, 1.81] \times [0, 2]$$

$\therefore$  globally Lipschitz

So the sol' exists at all time

$$(st-t) \leq \frac{P_{T_0}^2}{C_{\alpha}} \cdot P_{T_0-t}$$

$$P_{T_0-t} \leq (st-t)^{\frac{1}{2}} P_{T_0}^2$$

$$(st-t)^{\frac{1}{2}} P_{T_0}^2 \rightarrow 0$$

5)

q) for contraction mapping in  $C(\mathbb{C}, \mathbb{C})$

$$\|f(x_n) - f(x_m)\| \leq Ld \|x_n - x_m\|$$

$Ld \leq 1$  is sufficient (and)

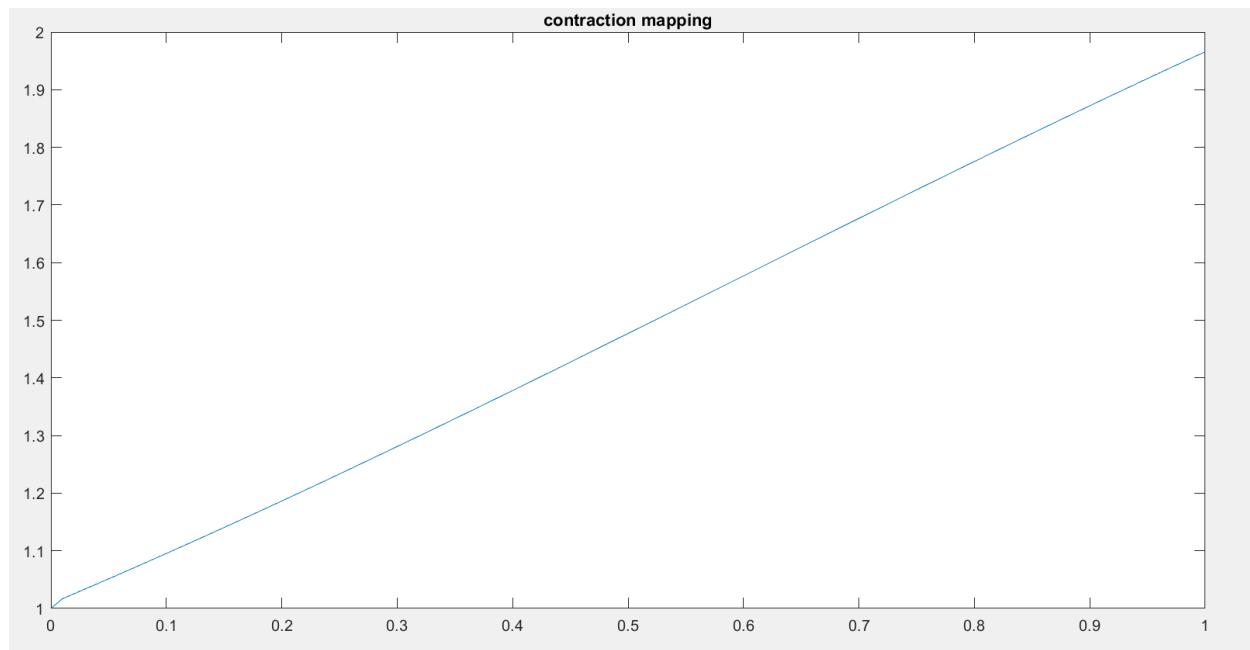
a)  $L=1$  (globally Lipschitz)

v)  $d \leq 1$

$\therefore \boxed{d \leq 1}$

Both 5b and 5c plots are almost identical.

Problem 5b:



Problem 5c:

