1) $y_1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ $\quad y_2 = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}$ $\quad y_3 = \begin{bmatrix} 4 \\ -4 \\ 6 \end{bmatrix}$

$$\boxed{K_1 = y_1}$$

$$K_2 = y_2 - \frac{\langle y_1, y_2 \rangle}{\|y_1\|^2} y_1$$

$$\langle y_1, y_2 \rangle = (-2 \cdot (0) + 4 - 1)$$
$$= 3$$

$$\|y_1\|^2 = (-2)^2 + (1)^2 + (1)^2 = 6.$$

$$K_2 = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \frac{3}{6} \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$\cancel{K_2 = \begin{bmatrix} 2/6 \\ 23/6 \\ -7/6 \end{bmatrix}} \qquad \boxed{K_2 = \begin{bmatrix} 1 \\ 7/2 \\ -3/2 \end{bmatrix}}$$

$$k_3 = y_3 - \left(\frac{\langle y_1, y_3\rangle}{\langle y_1, y_1\rangle} \cdot y_1\right) - \left(\frac{\langle y_2 y_3\rangle}{\langle y_2 y_2\rangle} y_2\right)$$

$$\langle y_1, y_3\rangle = -2 \cdot 4 - 4 + 6 = -6$$

$$\langle y_2 y_3\rangle = 0 \cdot 4 - 4 \cdot 4 - 1 \cdot 6 = -22$$

$$\langle y_1, y_1\rangle = 6$$

$$\langle y_2, y_2\rangle = 0 + 16 + 1 = 17$$

$$y_3 - \left(\frac{\langle y_1 y_3\rangle}{\langle y_1 y_1\rangle} y_1\right) = y_3 + y_1 = \begin{bmatrix} 4 \\ -4 \\ 6 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} = p'$$

$$p' - \frac{\langle y_2 y_3\rangle}{\langle y_2 y_2\rangle} y_2 = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} + \frac{22}{17}\begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 37/17 \\ 97/17 \end{bmatrix}$$

Q4) $x = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$

$y_1 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$ ; $y_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

let $\hat{x}$ be the sol$^n$
then
$\langle (x - \hat{x}), y_i \rangle = 0$

$\Rightarrow \langle x, y_i \rangle = \langle \hat{x}, y_i \rangle$ let $\hat{x} = \alpha_1 y_1 + \alpha_2 y_2$

$\Rightarrow \alpha_i \langle y_i, y_i \rangle = \langle x, y_i \rangle$

$\langle x, y_1 \rangle = \text{trace}(x^T y_1)$

$= \text{tr}\left( \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \right)$

$= \text{tr}\left( \begin{bmatrix} 4 & 0 \\ -1 & 0 \end{bmatrix} \right) = 4$

$\langle x, y_2 \rangle = \text{trace}(x^T y_2)$

$= \text{trace}\left( \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)$

$= \text{tr}\left( \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \right) = 1$

$$\langle y_1, y_1\rangle = \mathrm{tr}\left(\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}\right)$$

$$= \mathrm{tr}\left(\begin{bmatrix}5 & 0\\ 0 & 0\end{bmatrix}\right) = 5$$

$$\langle y_2, y_2\rangle = \mathrm{tr}\left(\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right)$$

$$= \mathrm{tr}\left(\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\right) = 4$$

$$\alpha_1 = \frac{\langle x, y_1\rangle}{\langle y_1, y_1\rangle} = \frac{4}{5} \qquad \alpha_2 = \frac{\langle x, y_2\rangle}{\langle y_2, y_2\rangle} = \frac{1}{4}$$

$$\hat{x} = \alpha_1 y_1 + \alpha_2 y_2$$

$$= \frac{4}{5}\begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

$$\boxed{\hat{x} = \begin{bmatrix}21/20 & 1/4\\ 37/20 & 1/4\end{bmatrix}}$$

8) $\hat{x} = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3$

$x = e^t$

$y_1 = 1$ $\quad y_2 = t$ $\quad y_3 = \frac{1}{2}(3t^2 - 1)$

$\langle x - \hat{x}, y_i \rangle = 0$

$\langle \hat{x}, y_i \rangle = \langle x, y_i \rangle$ $\quad \hat{x} = \sum \alpha_i y_i$

c) $\alpha_i = \dfrac{\langle x, y_i \rangle}{\langle y_i, y_i \rangle}$

$$\langle x, y_1 \rangle = \int_{-1}^{1} e^t \cdot dt = [e^t]_{-1}^{1} = \frac{e^2 - 1}{e} = e - e^{-1}$$

$$\langle x, y_2 \rangle = \int_{-1}^{1} te^t \cdot dt = [te^t - e^t]_{-1}^{1}$$

$$= [e^1 - e^1] - [-e^{-1} - e^{-1}]$$

$$= 2e^{-1}$$

$$\langle x, y_3 \rangle = \frac{1}{2}\left[\int_{-1}^{1} e^t \cdot 3t^2 - \int_{-1}^{1} e^t\right]$$

$$= \frac{1}{2}\left[3\int_{-1}^{1} t^2 e^t - \int_{-1}^{1} e^t\right]$$

$$\langle x, y_3 \rangle = \frac{1}{2}\left[3\left[t^2e^t - 2te^t + 2e^t\right] - e^t\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[3t^2e^t - 6te^t + 5e^t\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[(3e - 6e + 5e) - (3e^{-1} + 6e^{-1} + 5e^{-1})\right]$$

$$= \frac{1}{2}\left[2e - 14e^{-1}\right] = e - 7e^{-1}$$

$$\langle y_1, y_1 \rangle = \int_{-1}^{1} 1.1.dt = 2$$

$$\langle y_2, y_2 \rangle = \int_{-1}^{1} t.t.dt = \left[\frac{t^3}{3}\right]_{-1}^{1} = 2/3$$

$$\langle y_3, y_3 \rangle = \int_{-1}^{1}\left(\frac{3t^2 - 1}{2}\right)^2 = \cancel{\left[\frac{t^3}{2}\right]_{-1}^{1} - \left[\frac{t}{2}\right]_{-1}^{1}}$$

$$= \cancel{1 - 1} = 0.$$

$$= \frac{2}{4}\int_0^1 (3t^2 - 1)^2 = \frac{1}{2}\int_0^1 (9t^4 - 6t^2 + 1)$$

$$= \frac{1}{2}\left(\frac{9}{5} - \frac{6}{3} + 1\right) = 0.4$$

$$\alpha_1 = \frac{e - e^{-1}}{2}$$

$$\alpha_2 = \frac{2e^{-1}}{(2/3)} = 3e^{-1}$$

$$\alpha_3 = \frac{10}{4}\left(e - 7e^{-1}\right) = \frac{5}{2}\left(e - 7e^{-1}\right)$$

$$\hat{x} = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3$$

wher $y_1 = 1$

$y_2 = t$

$$y_3 = \frac{(3t^2 - 1)}{2}$$

5)

$$\delta = \inf_{y \in M} \|x-y\| \le \left\| \frac{x-m_1}{2} + \frac{x-m_2}{2} \right\|$$

let $m_1$ & $m_2$ be 2 sol$^n$

$\|$ where

$$\|x-m_1\| = \delta \quad \& \quad \|x-m_2\| = \delta$$

$$\Rightarrow \left\| \frac{x-m_1}{2} + \frac{x-m_2}{2} \right\| \le \left\| \frac{x-m_1}{2} \right\| + \left\| \frac{x-m_2}{2} \right\|$$

$$\le \frac{\delta}{2} + \frac{\delta}{2}$$

$$\le \delta$$

$$\Rightarrow \delta \le \left\| \frac{x-m_1}{2} + \frac{x-m_2}{2} \right\| \le \delta$$

$$\Rightarrow \left\| \frac{x-m_1}{2} + \frac{x-m_2}{2} \right\| = \delta$$

$$= \left\| \frac{x-m_1}{2} \right\| + \left\| \frac{x-m_2}{2} \right\|$$

$\Rightarrow$ by def$^n$

$$(x-m_1) = \alpha (x-m_2)$$

taking norm

$$\|x - m_1\| = \|\alpha\| \, \|x - m_2\|$$

$$\Rightarrow \|\alpha\| = 1 \quad \Rightarrow \alpha = \pm 1$$

$$\Rightarrow x - m_1 = 1\,(x - m_2) \quad = \quad m_1 = m_2$$

$$(x - m_1) = -(x - m_2) \Rightarrow m_2 = 2x - m_1$$

$\Rightarrow$ ~~ad~~ $\Rightarrow 2x - m_1 \in M$

& distance b/w

both m & n is ~~not~~ ze[illegible]

$\therefore$ only sol$^n$ is $\boxed{m_1 = m_2}$

6)

a) $||x|| = |x_1| + |x_2|$

So let $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$||x+y||_1 = ||\begin{bmatrix} 1 \\ 1 \end{bmatrix}||_1 = 2$

$||x||_1 + ||y||_1 = 1 + 1 = 2$

but $\boxed{x \neq \alpha y}$

$\therefore$

c) $||x||_\infty = \max\{|x_1|, |x_2|\}$

$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$||x+y||_\infty = ||\begin{bmatrix} 2 \\ 1 \end{bmatrix}||_\infty = 2$

$||x||_\infty = ||\begin{bmatrix} 1 \\ 0 \end{bmatrix}||_\infty = 1$

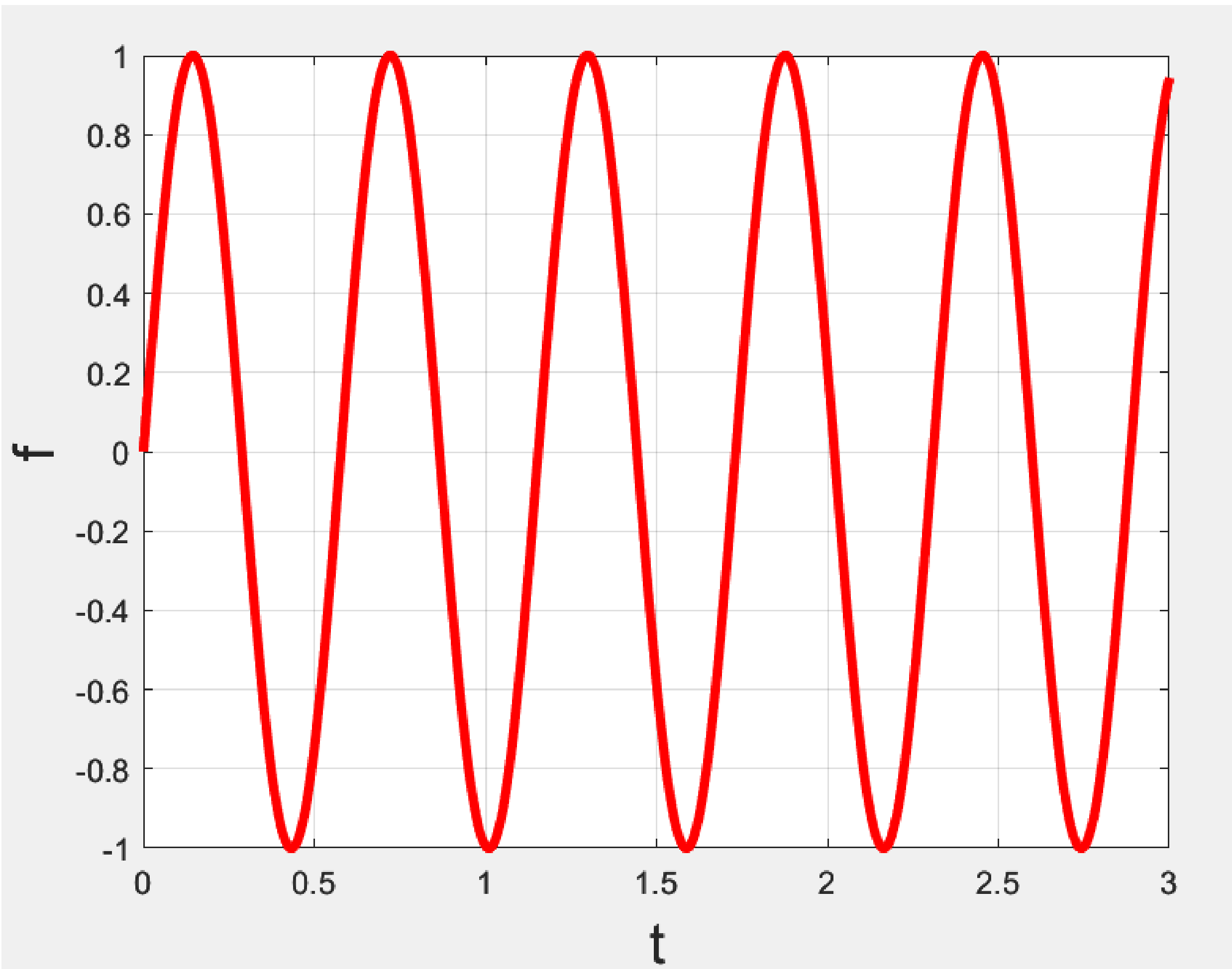
$||y||_\infty = ||\begin{bmatrix} 1 \\ 1 \end{bmatrix}||_\infty = 1$

$||x+y||_\infty = ||x||_\infty + ||y||_\infty$ but $\boxed{x \neq \alpha y}$

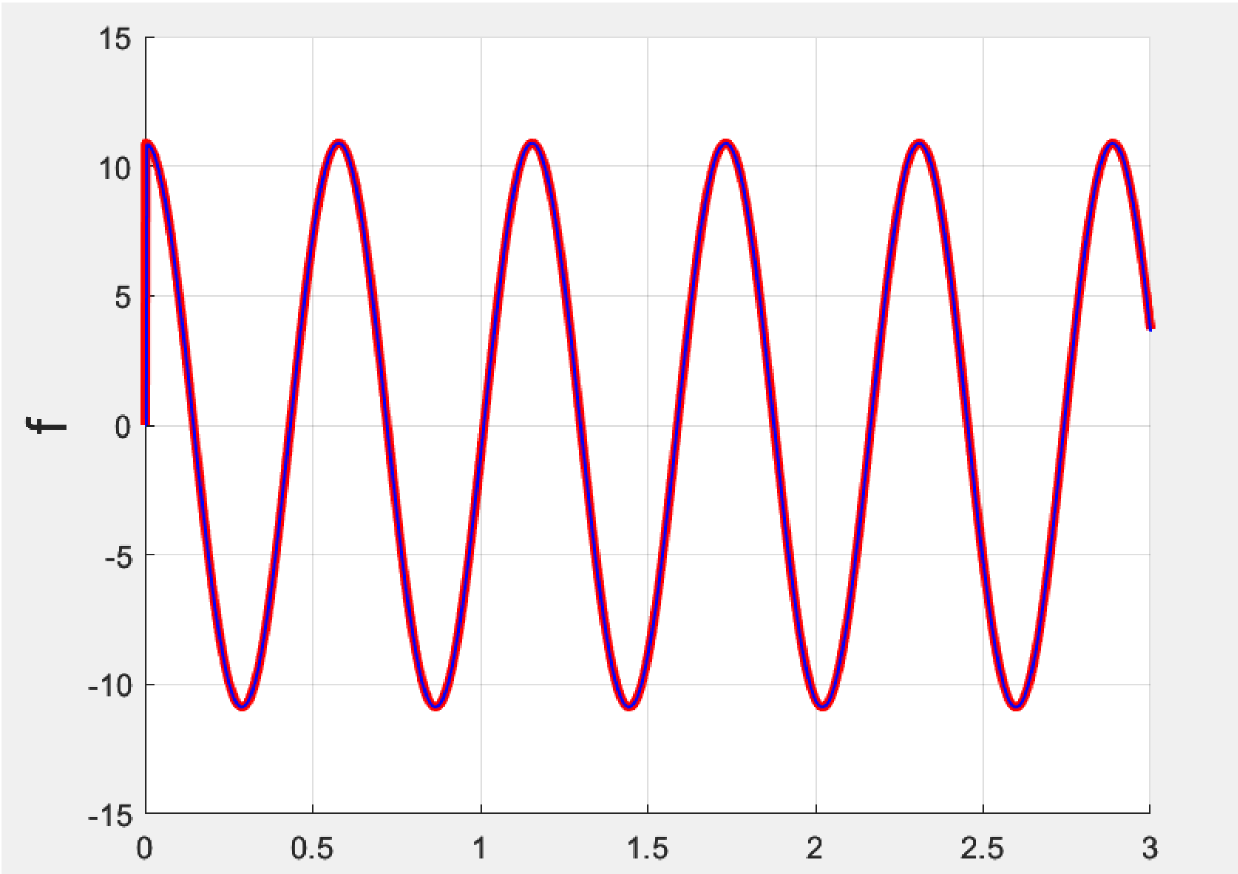
**Homework 6**

## Problem 2

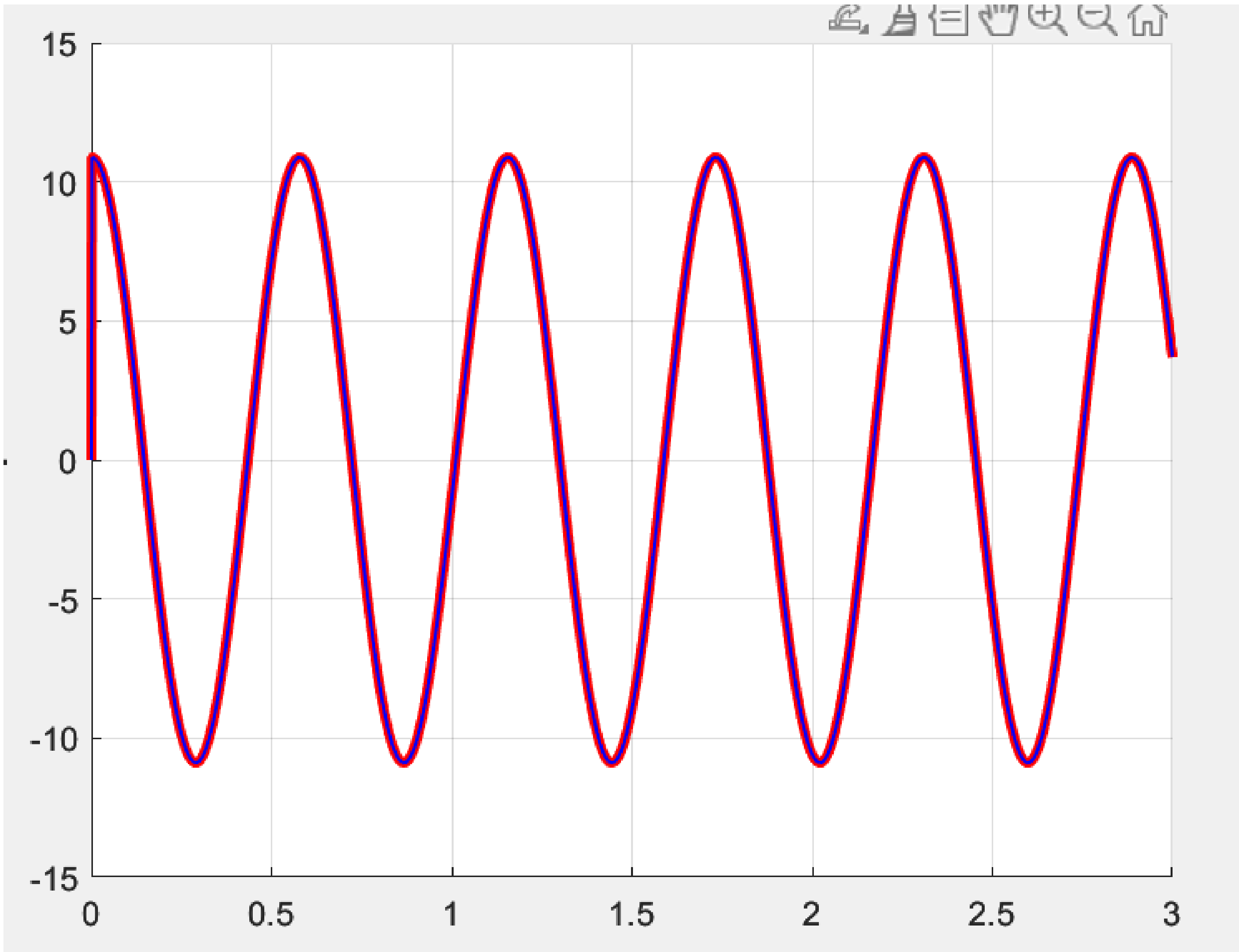
**Original Function:**



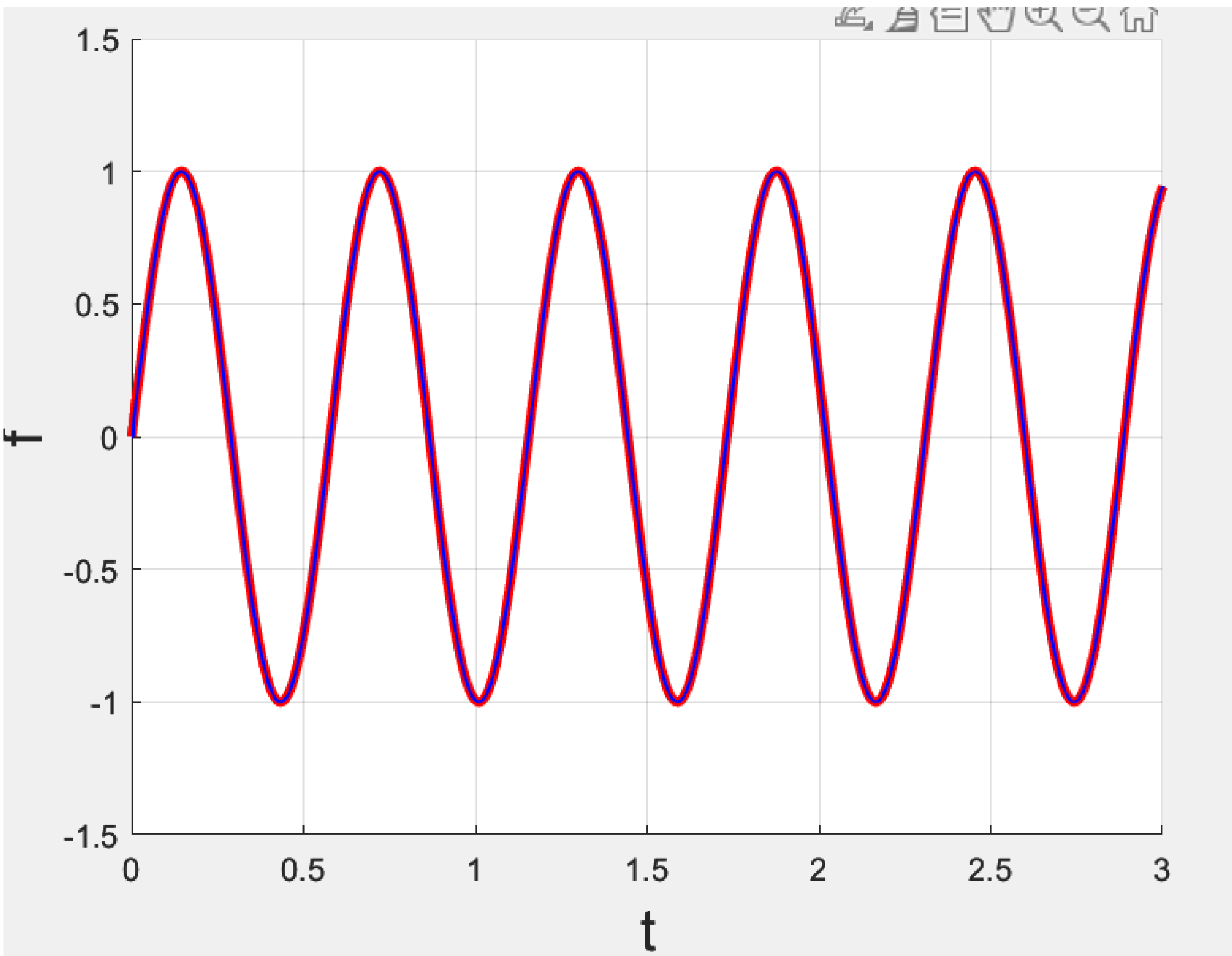
Naive derivative  vs original derivative



Regression derivative vs original



Regression vs original



## Code:

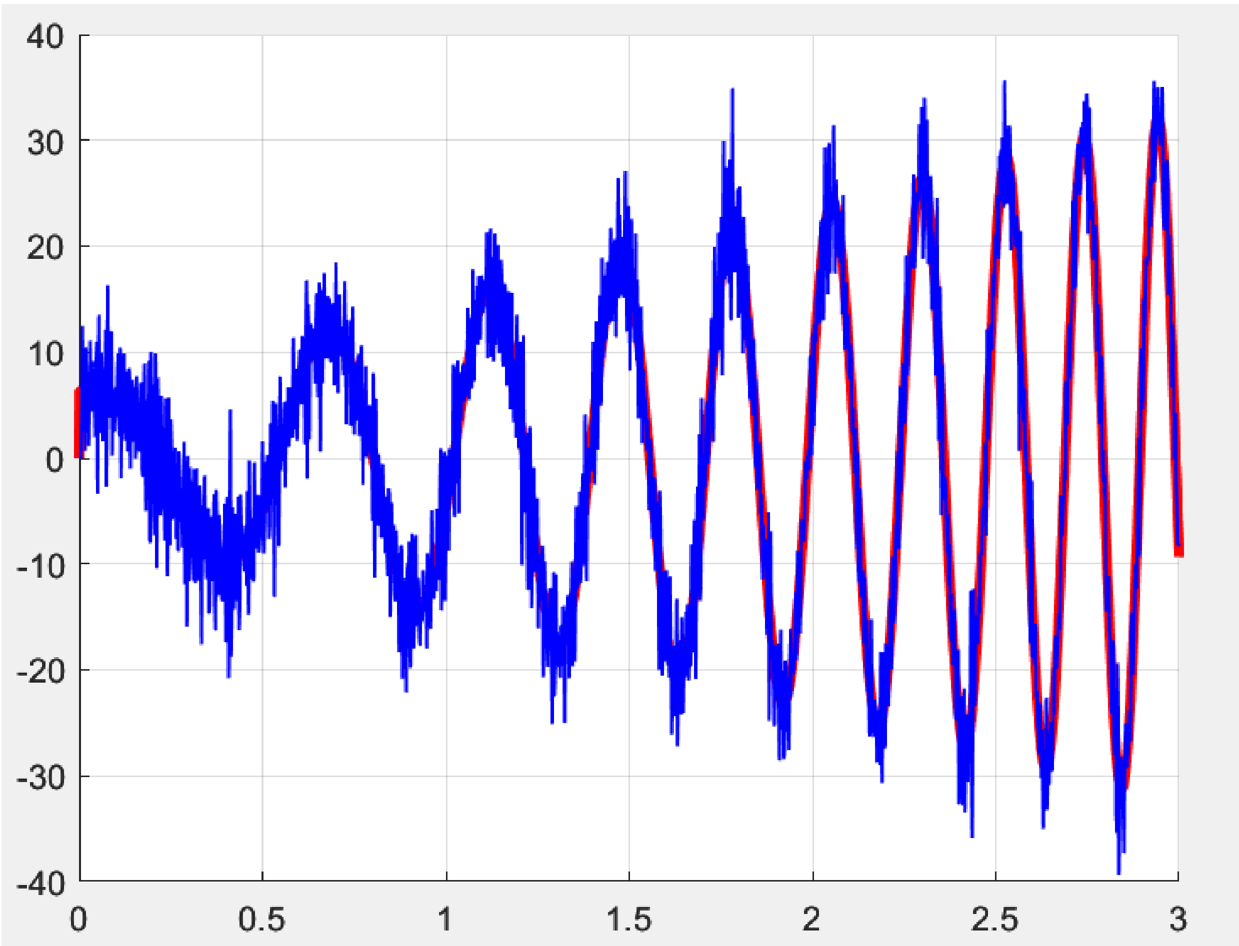
```
load ./data/DataHW06_Prob2
fig1 = figure();
plot(t,y,'r','linewidth',3);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
dy_dt = naive_der(y,t);
[y_regress,dy_dt_regress] = do_regress(y,t);
fig2 = figure();
hold on
plot(t,dy,'r','linewidth',3);
plot(t,dy_dt,'b','linewidth',1);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
hold off
fig3 = figure();
hold on
plot(t,y,'r','linewidth',3);
plot(t,y_regress,'b','linewidth',1);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
hold off
fig4 = figure();
hold on
plot(t,dy,'r','linewidth',3);
plot(t,dy_dt_regress,'b','linewidth',1);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
hold off

function [y_regress,dy_dt_regress] = do_regress(y,t)
dy_dt_regress = zeros(1, length(y));
y_regress = zeros(1, length(y));
for i = 4 : length(y)
   y_window = [y(i - 3) ;y(i - 2); y(i - 1); y(i)];
   moving_window = [t(i-3); t(i-2); t(i-1); t(i)];
   A = [ones(4,1), moving_window, moving_window.^2];
   alpha_hat = inv(A' * A)*A'*y_window;
   dy_dt_regress(i) = alpha_hat(2) + 2*alpha_hat(3) * t(i);
   y_regress(i) = alpha_hat(1) + alpha_hat(2)*(t(i)) + alpha_hat(3) *
(t(i)^2);
end
end
```

```
function dy_dt = naive_der(y,t)
%NAOVE_DER Summary of this function goes here
%   Detailed explanation goes here
dy_dt = t;
for i=2:length(t)
   dy_dt(i) = ((y(i)-y(i-1))/(t(i)-t(i-1)));
end
end
```

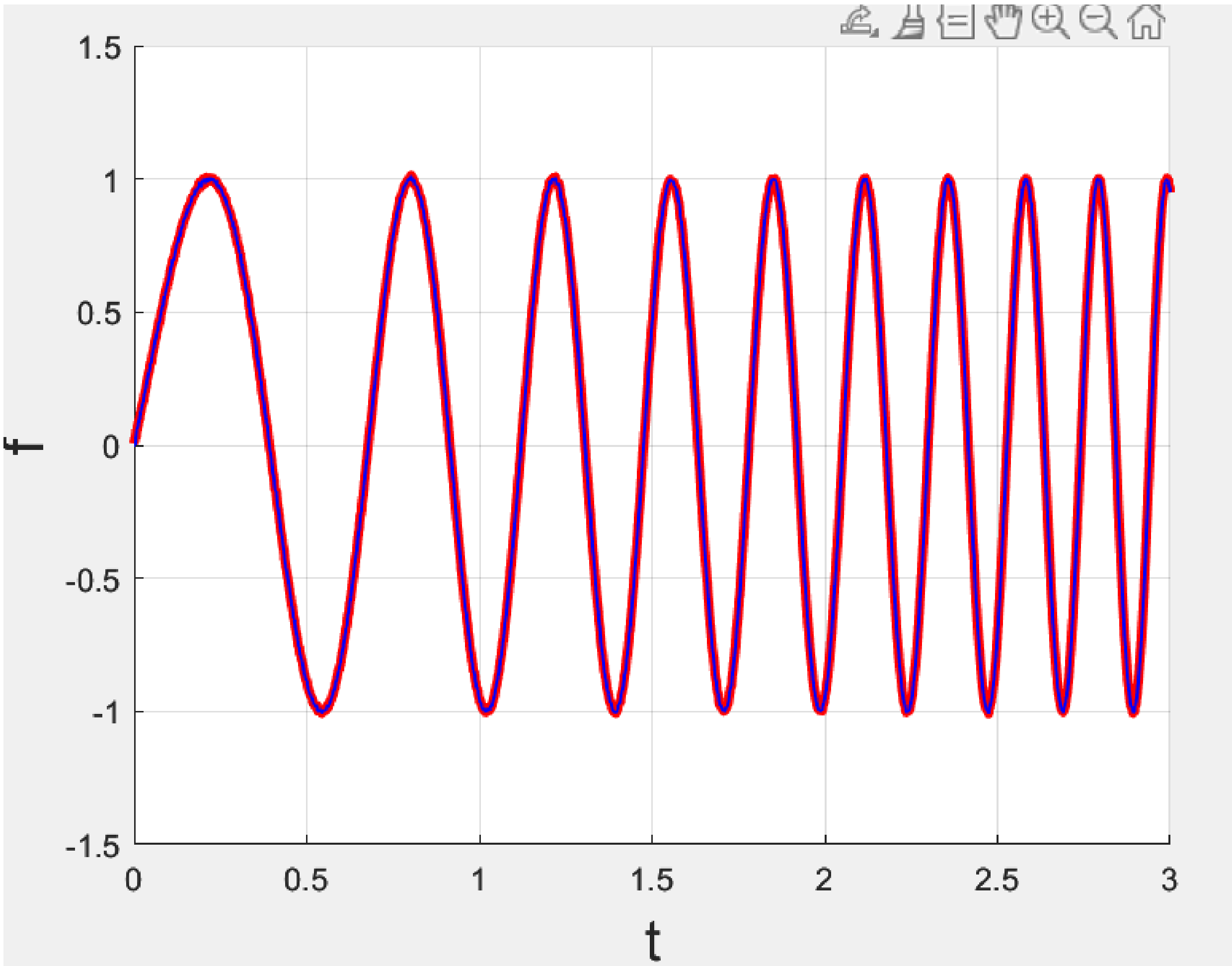
**Question 3:**

**RMSE value is  3.9919**

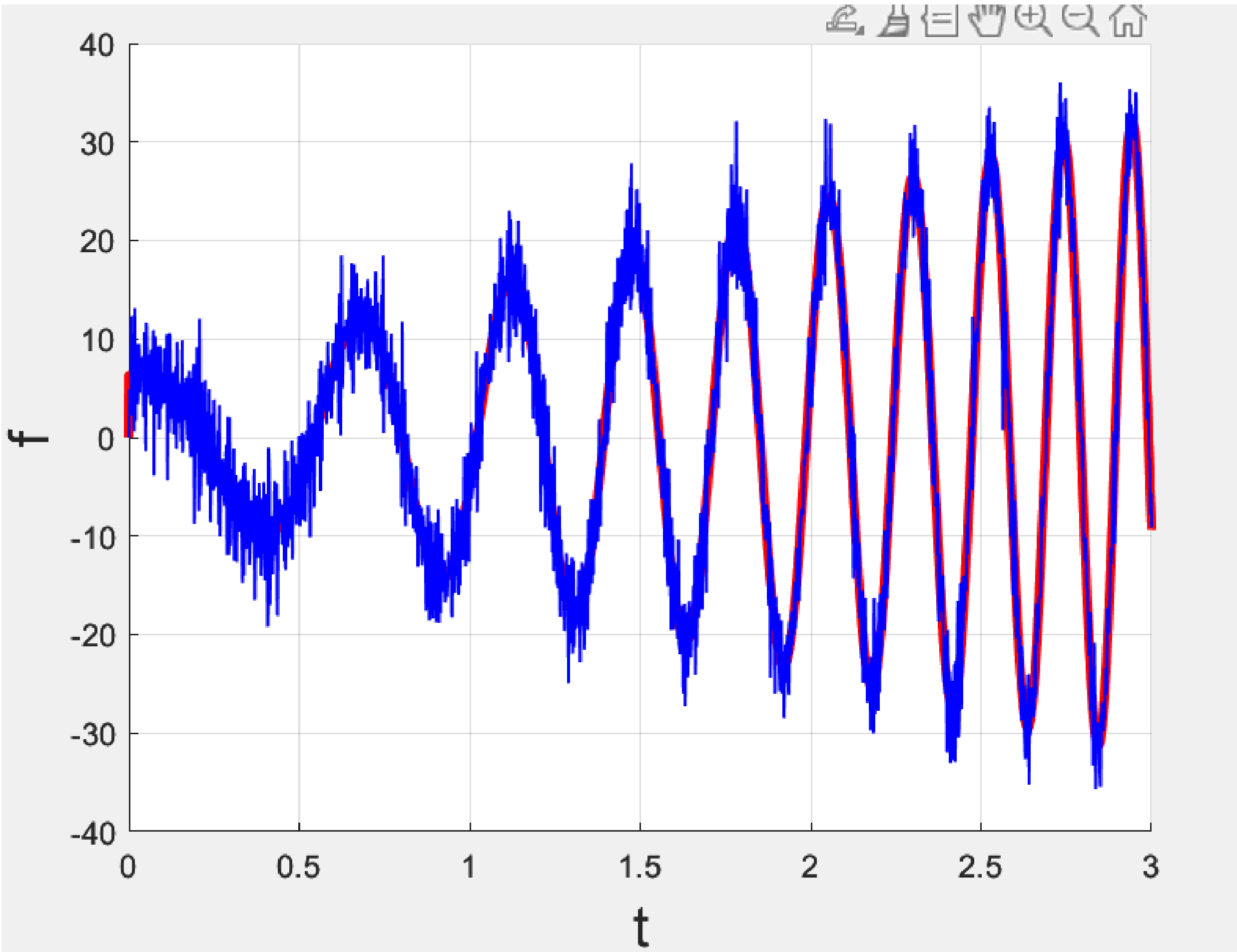
**Regression derivative vs Original derivative**



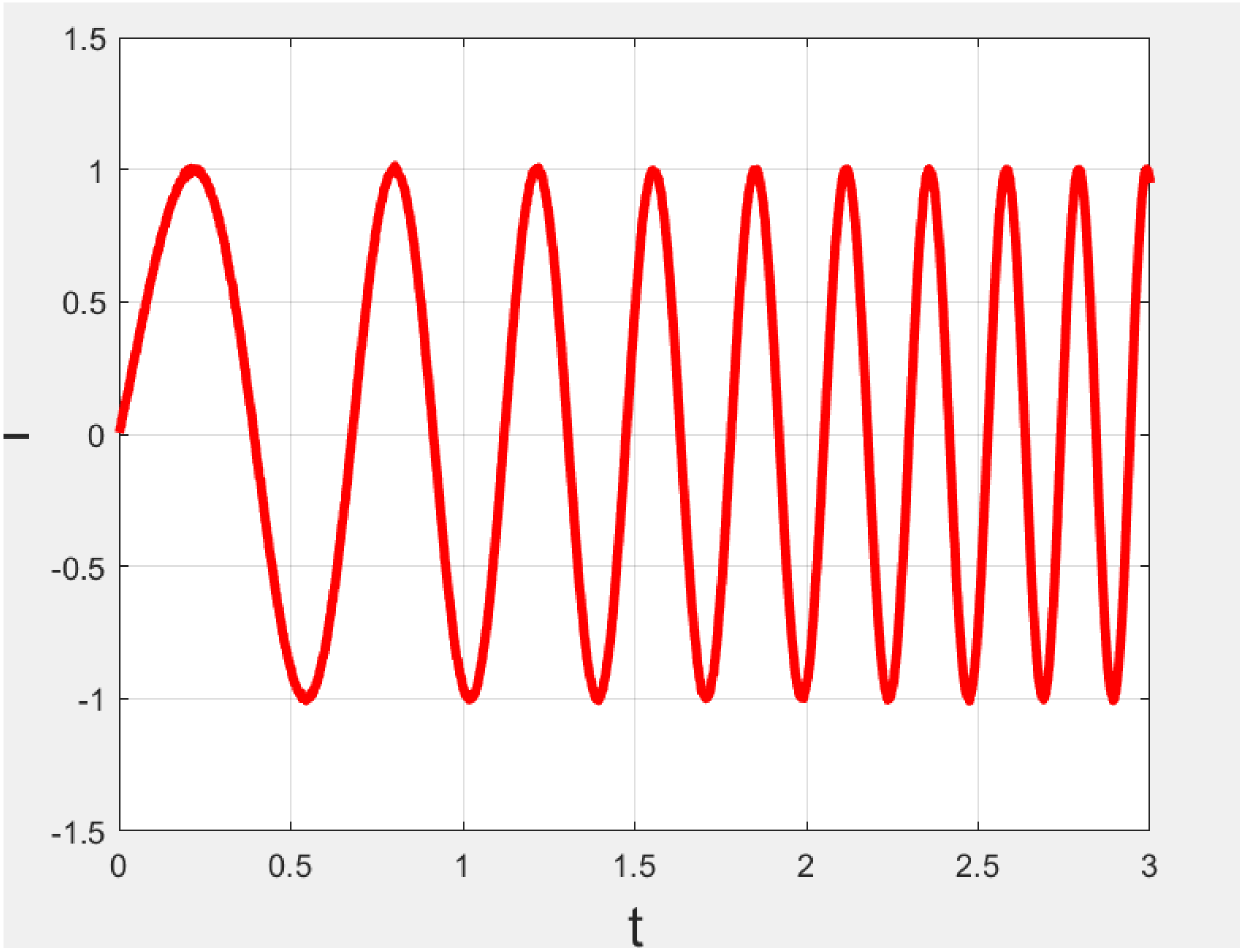

**Regression vs function**




**Naive derivative vs original derivative**




Original Function


1.5
1
0.5
0
-0.5
-1
-1.5
I
0
0.5
1
1.5
2
2.5
3
t

Code:

```
load ./data/DataHW06_Prob3
fig1 = figure();
plot(t,y,'r','linewidth',3);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
dy_dt = naive_der(y,t);
[y_regress,dy_dt_regress, rmse_err] = do_regress(y,t, dy);
fig2 = figure();
hold on
plot(t,dy,'r','linewidth',3);
plot(t,dy_dt,'b','linewidth',1);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
hold off
fig3 = figure();
hold on
plot(t,y,'r','linewidth',3);
plot(t,y_regress,'b','linewidth',1);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
hold off
fig4 = figure();
hold on
plot(t,dy,'r','linewidth',3);
plot(t,dy_dt_regress,'b','linewidth',1);
grid on
xlabel('t','FontSize',18);
ylabel('f','FontSize',18);
hold off

function dy_dt = naive_der(y,t)
%NAOVE_DER Summary of this function goes here
%   Detailed explanation goes here
dy_dt = t;
for i=2:length(t)
   dy_dt(i) = ((y(i)-y(i-1))/(t(i)-t(i-1)));
end
end

function [y_regress,dy_dt_regress, err] = do_regress(y,t, dy)
dy_dt_regress = zeros(1, length(y));
y_regress = zeros(1, length(y));
err = 0;
```

```
count = 0;
for i = 4 : length(y)
   y_window = [y(i - 3) ;y(i - 2); y(i - 1); y(i)];
   moving_window = [t(i-3); t(i-2); t(i-1); t(i)];
   A = [ones(4,1), moving_window, moving_window.^2];
   alpha_hat = inv(A' * A)*A'*y_window;
   dy_dt_regress(i) = alpha_hat(2) + 2*alpha_hat(3) * t(i);
   y_regress(i) = alpha_hat(1) + alpha_hat(2)*(t(i)) + alpha_hat(3) * (t(i)^2);
   err = err + (dy_dt_regress(i) - dy(i))^2;
   count = count + 1;
end
err = sqrt(err/count);
end
```

## Question 7

Code:

```
% Initialize values
A = diag([0.5, 1, 1, 0.5, 1]);
B = [3, 0, 2, 0, 1]';
C = 0.25;
D = B';
%% Function call
A_inv = inv(A);
output_matrix = matrix_lemma(A, B, C, D, A_inv)
req_ans = inv(A+(B*C*D))

function output_matrix = matrix_lemma(A, B, C, D, A_inv)
C_inv = inv(C);
E = inv((C_inv + (D*A_inv*B)));
output_matrix = A_inv - (A_inv*B*E*D*A_inv);
end
```

req_ans =

```
    0.6667         0   -0.4444         0   -0.2222
         0    1.0000         0         0         0
   -0.4444         0    0.8519         0   -0.0741
         0         0         0    2.0000         0
   -0.2222         0   -0.0741         0    0.9630
```

output_matrix =

```
    0.6667         0   -0.4444         0   -0.2222
         0    1.0000         0         0         0
   -0.4444         0    0.8519         0   -0.0741
         0         0         0    2.0000         0
   -0.2222         0   -0.0741         0    0.9630
```