EECS 560: Segway Example

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1 Introduction

EECS 560 is not a feedback design course [1]. It is a linear systems analysis course. It is just as valuable for signal processing students as for feedback control students. In EECS 560, we learn about state variable models, controllability, observability, stabilizability, observer (i.e., estimator) design, minimality, internal and external stability, separation principle, and so forth.

As a vehicle for illustrating many of these ideas, it will be convenient to have an example that is familiar to most of you, not totally trivial, and yet not too difficult either. For this, I have chosen the Segway. The presentation of the model uses facts from mechanics. Don't let that worry you; knowledge of mechanics will not be necessary on any exams or HW sets.

The only things you really need to do are:

- Look at Fig. 1 to understand the coordinates being used to model the Segway.
- Check out (9) to see the choice of state variables in the model.
- Skip to (12) to see a linear state variable model of the Segway.
- You can either stop here or run some simulations and animations of the Segway system that I have prepared for you; see Sec. 2.

The material in Sections 3 and 4 is for those who have had a course in Lagrangian mechanics and wish to understand better how to derive the model from scratch. It is not needed anywhere in EECS 560. You can just accept the model given in (12).

Remark: The method of Lagrange is used widely in robotics, so, if by chance you are interested in working in robotics, you should take a dynamics course (or physics course) and learn this method. Personally, because I am an EE, I had not even heard of Lagrange's method of deriving models for mechanical systems when I took the equivalent of EECS 560, so don't feel bad. I learned it my second year in grad school in a physics course. Our ME Dept. has a course called Intermediate Dynamics that covers the material, I believe.

2 MATLAB Files Available

In the C-Tools Resource Folder MATLAB, you will find the file `Segway.zip`. When you unzip it, you will find several files. The file `Segway_sim.m` is the master file. At a MATLAB prompt, just type the file name

[1] EECS 560 considers neither time domain performance criteria (such as over shoot, settling time), nor robustness margins (to deal with model uncertainty), nor realistic nonlinearities. EECS 565 will formulate a pretty complete 'Control Problem' for linear systems, and EECS 562 will give you tools for dealing with nonlinearities. These are both offered in the Winter term.

to obtain a simulation of the Segway system with a linear state variable feedback controller (a kind of controller we will study this term; you are NOT expected to know it already). Relevant plots are made and an animation is provided.

Things you can change easily:

- The initial condition is set at the very top of the file `Segway_sim.m`; the simulation duration is also there.
- You can switch between the full nonlinear model of the Segway and the linear model by opening the file `Segway_ODE45.m` and looking for the relevant `if` statement. You can even compare several different controllers:
  1. Basic linear feedback controller from EECS 560
  2. A more advanced linear feedback design from EECS 565
  3. A nonlinear controller from EECS 562

You are NOT obliged to run any of these for your HW set. However, I think you will find the simulations fun and informative. At the very least, you can see how a linear controller behaves on an interesting nonlinear system.

By the way, my animation file is rather klutzy. If someone wishes to clean that up and share it with me, that would be great!

3 Nonlinear Model of a Planar Segway

Consider the planar Segway system shown in Figure 1, along with the indicated coordinates, $q = (q_1, q_2)' = (\phi, \theta)'$, where ϕ is the (absolute) angle of the pendulum with respect to the vertical, θ is the relative angle of the wheel with respect to the pendulum, and the $'$ denotes transpose. In both cases, we take clockwise to be positive. The system has one actuator u that acts at the axis connecting the inverted pendulum to the wheel; consequently, θ is the actuated variable and the angle of the pendulum is indirectly regulated through the angle of the wheel with respect to the pendulum.

If we assume that there is no slipping between the wheel and the ground, then a model of the system can be easily derived using Lagrangian mechanics. Denote the Lagrangian by $L = K - V$, where K is the kinetic energy and V is the potential energy. The method of Lagrange yields[2]

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = \begin{cases} 0 & k = 1 \\ u & k = 2 \end{cases}, \tag{1}$$

with u taking values in $\mathbb{R}$.

For simple mechanical systems like the Segway, the kinetic energy is quadratic in the velocities, $K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$, with $D(q)$ positive definite[3]. Moreover, the potential energy is independent of the velocities and thus has the form $V = V(q)$. It follows that (1) can be written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Gamma u, \tag{2}$$

[2] You are not responsible for knowing this method; it is a nice method for deriving the models of electromechanical systems.

[3] The fact that D is positive definite follows from facts in mechanics. For us, the thing to note is that because D is positive definite, it is invertible and its diagonal entries are strictly greater than zero.

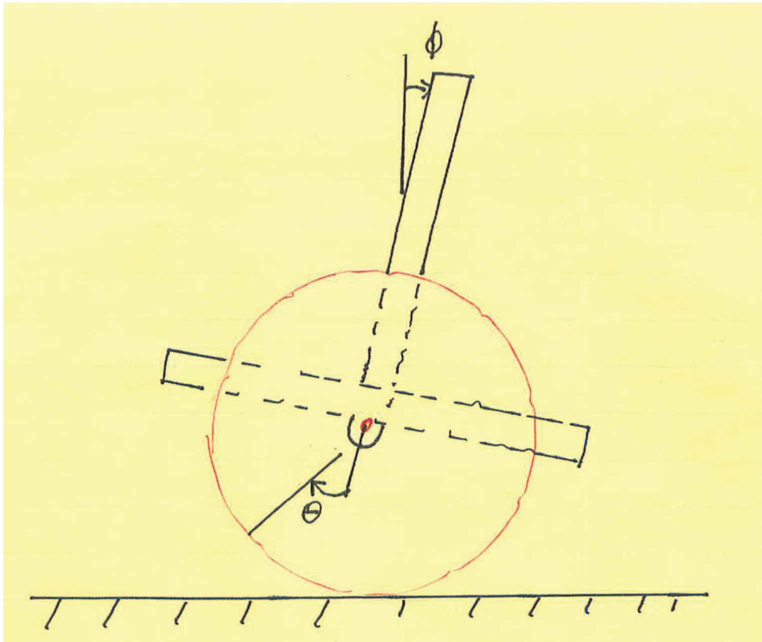

Figure 1: Crude drawing of a planar Segway.

where $\Gamma = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The 2×2 matrices D and C and the 2×1 vector G are computed in the symbolic file `symb_model_planar_segway.m` (available in the file `Segway.zip`) in case you are curious. I repeat that in this course you do not need to know how to derive the model.

Remark: The standard way to put the mechanical model (2) in state variable form is to select the states as $x = (q; \dot{q})$ so that

$$\dot{x} = f(x) + g(x)u, \tag{3}$$

where

$$f(x) = f(q, \dot{q}) = \begin{bmatrix} \dot{q} \\ -D^{-1}(q)\left(C(q, \dot{q})\dot{q} + G(q)\right) \end{bmatrix} \tag{4}$$

and

$$g(x) = g(q) = \begin{bmatrix} 0 \\ D^{-1}(q)\Gamma \end{bmatrix}. \tag{5}$$

We will do something similar with the linear approximation of the system.

Yes, state variable models can be linear or nonlinear! In this course, we only analyze linear, models. Essentially all practical systems are nonlinear in one way or another. Fortunately, linear approximations often work extremely well in practice. Models of the form (3) are analyzed in EECS 562. We will not be studying such models in EECS 560. Once again, I am just giving background information on the model; you are free to ignore it.

4 Linear Model of a Planar Segway

Equilibrium points of a mechanical model are solutions corresponding to constant position and zero velocity. For the Segway, these correspond to ϕ equal to 0 or π and θ equal to any constant value. For obvious reasons, we will consider the upright equilibrium point with $q_e = (\phi, \theta)' = (0, 0)'$. The linearized form of (2) about the equilibrium point $(q_e, 0)'$ can be shown to be[4]

$$D(q_e)\ddot{q} + \left.\frac{\partial G(q)}{\partial q}\right|_{q_e} q = \Gamma u, \tag{6}$$

where, for particular choices of parameters,

$$D(q_e) = \begin{bmatrix} 16.733 & 7.1667 \\ 7.1667 & 3.6667 \end{bmatrix} \tag{7}$$

and

$$\left.\frac{\partial G(q)}{\partial q}\right|_{q_e} = \begin{bmatrix} -7 & 0 \\ 0 & 0 \end{bmatrix}. \tag{8}$$

Choosing state variables as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \phi \\ \theta \\ \dot{\phi} \\ \dot{\theta} \end{bmatrix}, \tag{9}$$

results in

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}, \tag{10}$$

and

$$\frac{d}{dt}\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} = -D^{-1}(q_e)\left.\frac{\partial G(q)}{\partial q}\right|_{q_e}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D^{-1}(q_e)\Gamma u. \tag{11}$$

Putting these last two equations together gives the linear state variable model

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 2.568 & 0 & 0 & 0 \\ -5.020 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -0.7172 \\ 1.6744 \end{bmatrix} u. \tag{12}$$

At this point, one should also define which variables are to be considered the outputs of the model. One could assume all states are measured, giving

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x, \tag{13}$$

or only the two angles ϕ and θ are measured,

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x, \tag{14}$$

or, for example, the angle of the wheel with respect to the inertial frame, which would be $\phi + \theta$

$$y = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} x. \tag{15}$$

[4] The proof of this fact is not needed in the course and hence not given.