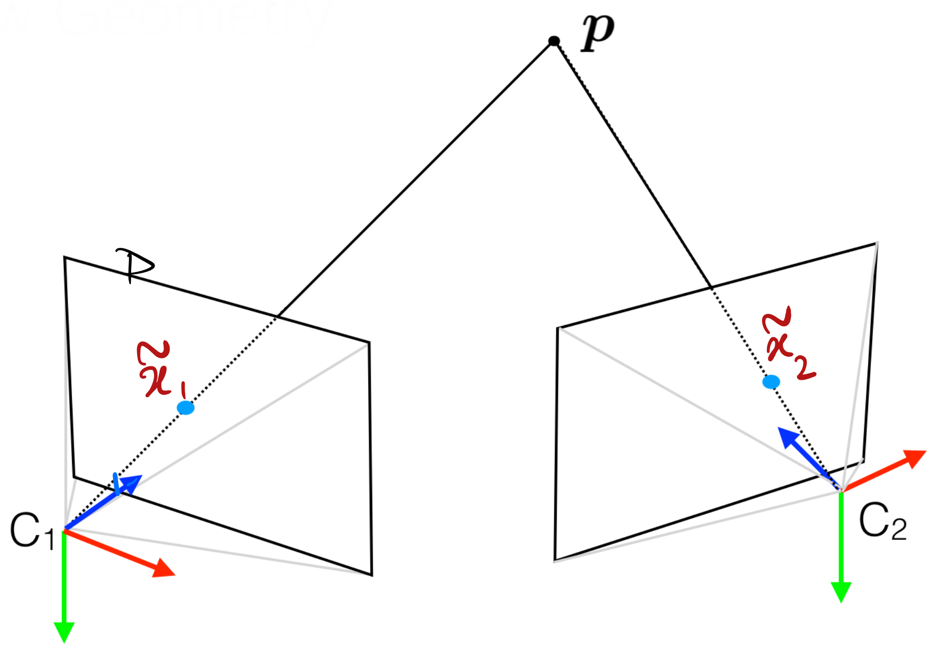
2-view geometry

Question: Given point correspondences, compute the relative pose $T^{c_1}_{c_2}$ (up to scale) between the cameras c_1 and c_2



(1) $$p_z^{c_1}\ \tilde{\boldsymbol{x}}_1 = \boldsymbol{K}_1 \left[\boldsymbol{R}^{c_1}_{w}\ \boldsymbol{t}^{c_1}_{w}\right] \tilde{\boldsymbol{p}}^{w} \qquad p_z^{c_2}\ \tilde{\boldsymbol{x}}_2 = \boldsymbol{K}_2 \left[\boldsymbol{R}^{c_2}_{w}\ \boldsymbol{t}^{c_2}_{w}\right] \tilde{\boldsymbol{p}}^{w}$$

2 Steps

- Compute the Essential matrix
- Retrive pose from essential matrix

Assume that $C_1 = W$ $\Rightarrow$

$$\underbrace{p_z^{c_1}}_{\triangleq d_1} \tilde{x}_1 = K_1 [I_{3\times3}\ 0_{3\times1}] \tilde{p}^{c_1} = K_1 p^{c_1}$$

$$\underbrace{p_z^{c_2}}_{\triangleq d_2} \tilde{x}_2 = K_2 [\underbrace{R_{c_1}^{c_2}}_{\triangleq R}\ \underbrace{t_{c_1}^{c_2}}_{\triangleq t}] \cdot \tilde{p}^{c_1}$$

$$d_1 \underbrace{K_1^{-1} \tilde{x}_1}_{\triangleq \tilde{y}_1} = p^{c_1}$$

$$d_2 \underbrace{K_2^{-1} \tilde{x}_2}_{\triangleq \tilde{y}_2} = [R\ t] \tilde{p}^{c_1} = R p^{c_1} + t$$

$\Rightarrow$

$$\boxed{d_2 \tilde{y}_2 = d_1 R \tilde{y}_1 + t}$$

$[t]_x y = t \times y$

- Premultiply with $[t]_x$:

  [recall: $[t]_x t = 0$]

$$d_2 [t]_x \tilde{y}_2 = d_1 [t]_x R \tilde{y}_1 + [t]_x t \qquad (\circledast)$$

- Premultiply with $\tilde{y}_2^T$:

[recall: $\tilde{y}_2^T [t]_\times \tilde{y}_2 = 0$]

$$\circledast \Rightarrow 0 = d_1 \tilde{y}_2^T \underbrace{[t]_\times R}_{\triangleq E = [t]_\times R} \tilde{y}_1$$

Essential matrix

Equivalently:

$$\tilde{y}_2^T E \tilde{y}_1 = 0$$

Epipolar constraint.

We scale the equation such that

$$\|t\| = 1.$$

Property of E: Any essential matrix has singular values $\{\|t\|, \|t\|, 0\}$.

Estimatimating E from point correspondences

$$\tilde{y}_{2,k}^T \, E \, \tilde{y}_{1,k} = 0, \quad k = 1, \dots, N$$

Equivalently,

$$a_k^T e = 0, \quad k = 1, \dots, N$$

where $e = \mathrm{vec}(E)$ (9×1)

(see a 2D example of the derivation on the last page)

Equivalently:

$$\begin{bmatrix} a_1^T e \\ \vdots \\ a_N^T e \end{bmatrix} = 0 \Rightarrow \overbrace{\begin{bmatrix} a_1^T \\ \vdots \\ a_N^T \end{bmatrix}}^{\triangleq A} e = 0$$

$$\underset{N\times 9}{A}\ \underset{9\times 1}{e} = 0 \qquad \circledast$$

Assume $N=9$

If $N=8$ and A has rank 8

$\Rightarrow \circledast$ has unique non-zero solution

- <u>Example</u> of going from $\tilde{y}_{2,k}^T E \tilde{y}_{1,k} = 0$ to $a_k^T e = 0$.

For simplicity, assume a "2D-case" where $y_{i,k}$ is 2x1 (instead of 3x1) and E is 2x2 (instead of 3x3):

$$\tilde{y}_{2,k}^T E \tilde{y}_{1,k} =$$

$$[y_2^1, y_2^2] \begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} \begin{bmatrix} y_1^1 \\ y_1^2 \end{bmatrix}$$

$$= [y_2^1 e_1 + y_2^2 e_3 , y_2^1 e_2 + y_2^2 e_4] \begin{bmatrix} y_1^1 \\ y_1^2 \end{bmatrix}$$

$$= y_1^1 [y_2^1 e_1 + y_2^2 e_3] + y_1^2 [y_2^1 e_2 + y_2^2 e_4]$$

$$= \underbrace{\left[y_1^1 y_2^1,\ y_1^2 y_2^1,\ y_1^1 y_2^2,\ y_1^2 y_2^2\right]}_{=a_k^T} \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}}_{e}$$