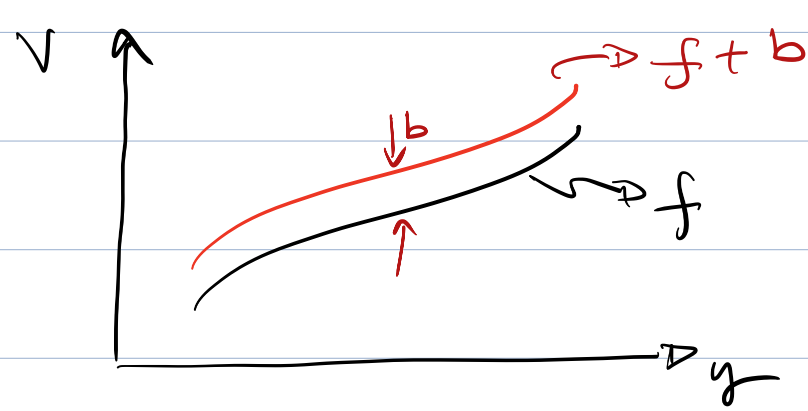
Dynamic Sensor Specifications

i.e., specifications that involve time.

- Drift (growing/changing bias)
- Latency (being delayed)
- Frequency response
- Rate saturation
- Hysteresis (done last lecture)

Drift

Drift is the rate of an increasing bias:



b = bias : • typically unknown (but could be estimated!)

• can be constant

• can be time-varying

Example: Suppose a clock is initially exact, but it loses 3 sec/month.

$\Rightarrow$ After 1 month, its bias b is 3 sec.

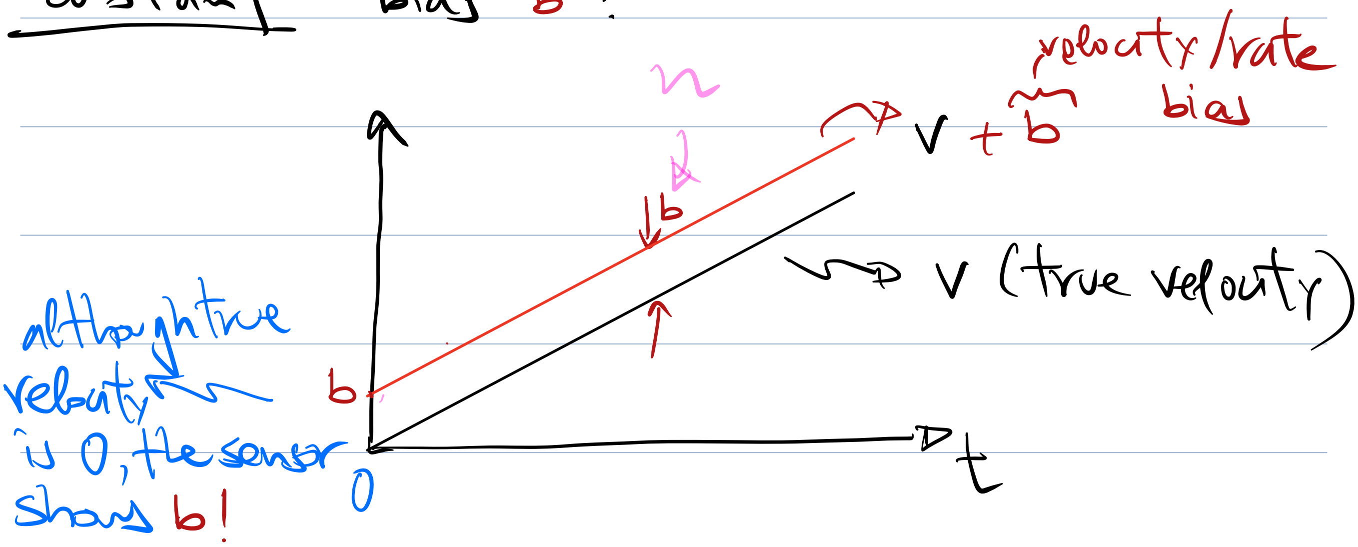
After 2 months, its bias b is 6 sec.

$\Rightarrow$ Drift = rate of change of bias

= sec/month

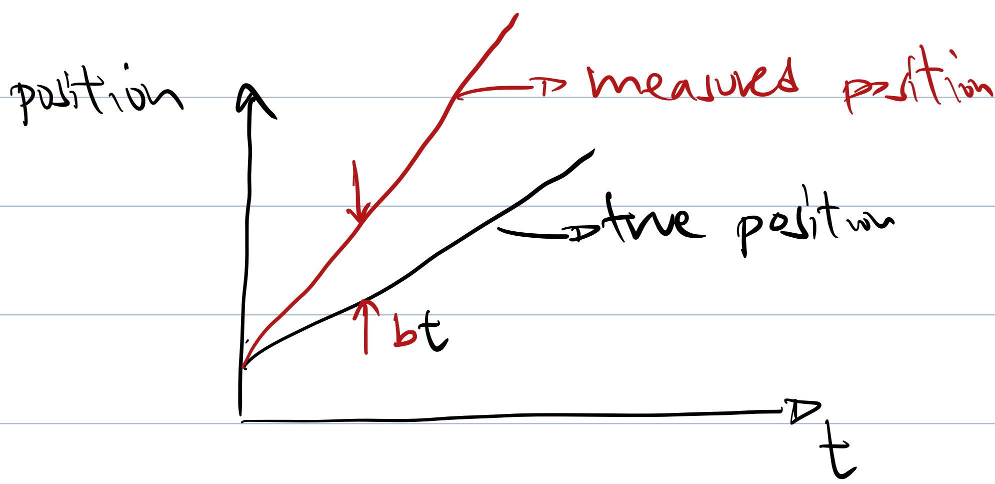
Example (Rate bias causing position drift):

Suppose we measure velocity with a <u>constant</u> bias b:



$$\text{Position}(t) = \int_0^t (v+b)\,d\tau = \underbrace{\int_0^t v\,d\tau}_{\text{true position}} + \underbrace{bt}$$

position error grows <u>linearly</u> in time due to b!



bt is drift

Summary of drift

Drift is a time-dependent bias. When we integrate the sensor output:

- constant bias b $\Rightarrow$ Linearly growing error (bt)

- constant drift $\Rightarrow$ Quadratically growing error ($b\frac{t^2}{2}$)

Ways to counter drift

- Estimate b and subtract it out!

(but estimate $\hat{b}$ of b will not be exact, in general!)

What causes bias?

- Inexact calibration of sensing device
- Non-repetable initial setpoint
  - electrical : temperature
  - mechanical : friction, hysteresis.

POLL 1

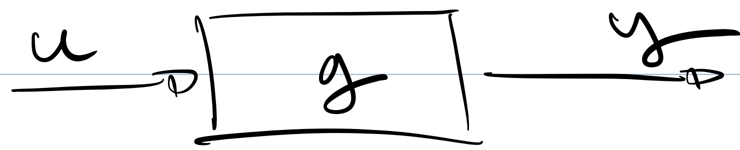
Latency

Latency is lateness/delay of information. Affected by:

- Sample rate
  (e.g., 10 μsec sample rate)
- Speed to access memory
  (e.g. 50 nsec)

Note Control systems (e.g, aircrafts, missiles) are <u>very</u> sensitive to latency

Frequency response (of a system g)



- $y(t) = \int_{t_0}^{t} g(t,\tau)\, u(\tau)\, d\tau \quad (*)$

current time

impulse response at t when an impulse was applies at time τ.

- $\hat{y}(s) = \hat{g}(s)\, \hat{u}(s)$ after taking the laplace transform in $(*)$

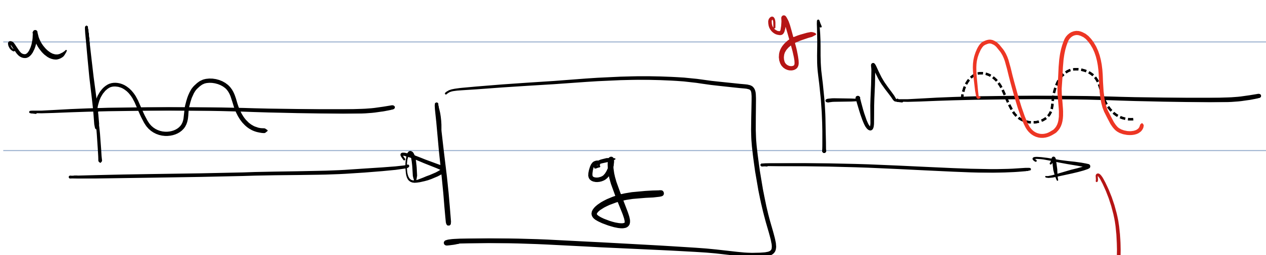
[Laplace transform maps (*) to frequency domain:

$$\mathcal{L}\{y\}(s) = \int_0^\infty y(t)e^{-st}\,dt$$

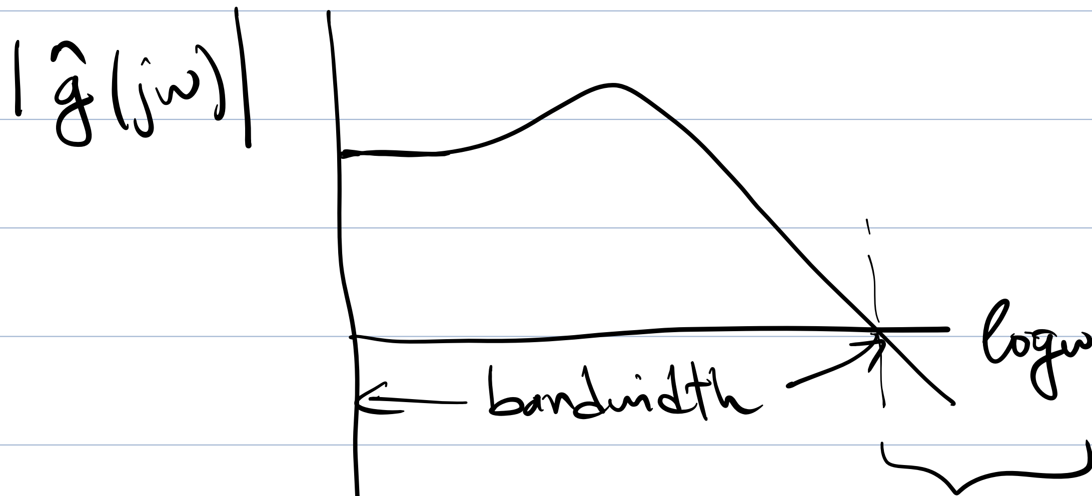
where $s = \sigma + j\omega$]

reals

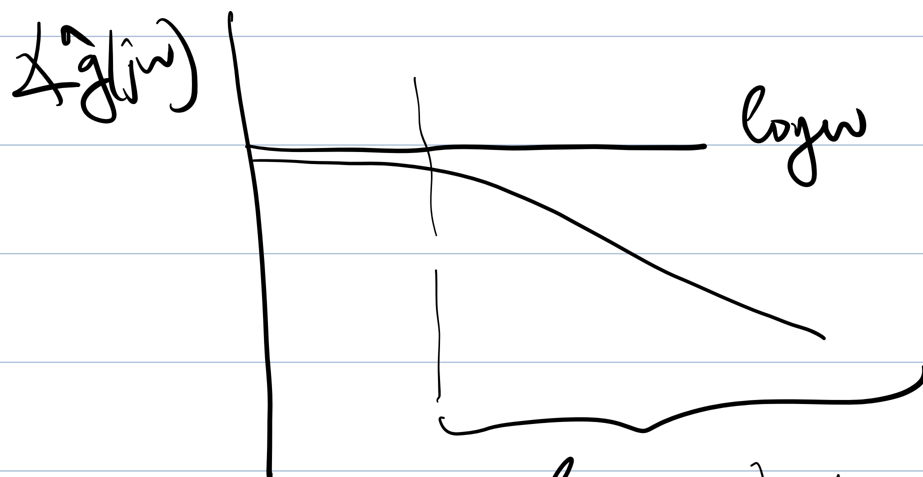
$\Rightarrow$ $\hat{g}(s)$ captures magnitude and frequency change of $u(t)$ to $y(t)$.



g changed u in both magnitude and freq.

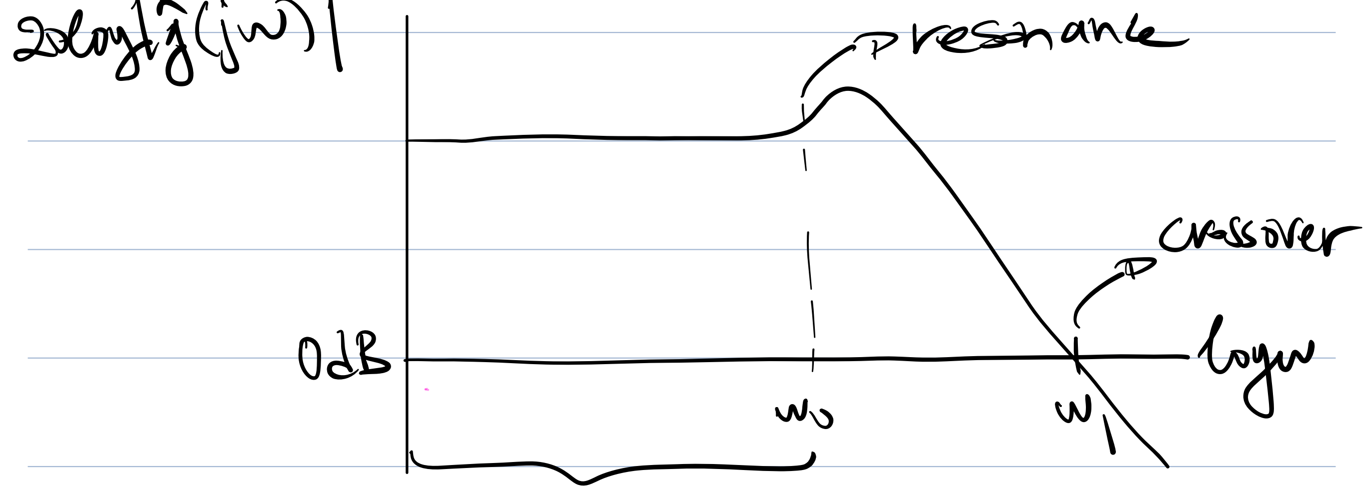


$\hat{g}$ weakens
(dissappears
higher freq)



large phase
shift at high freq.

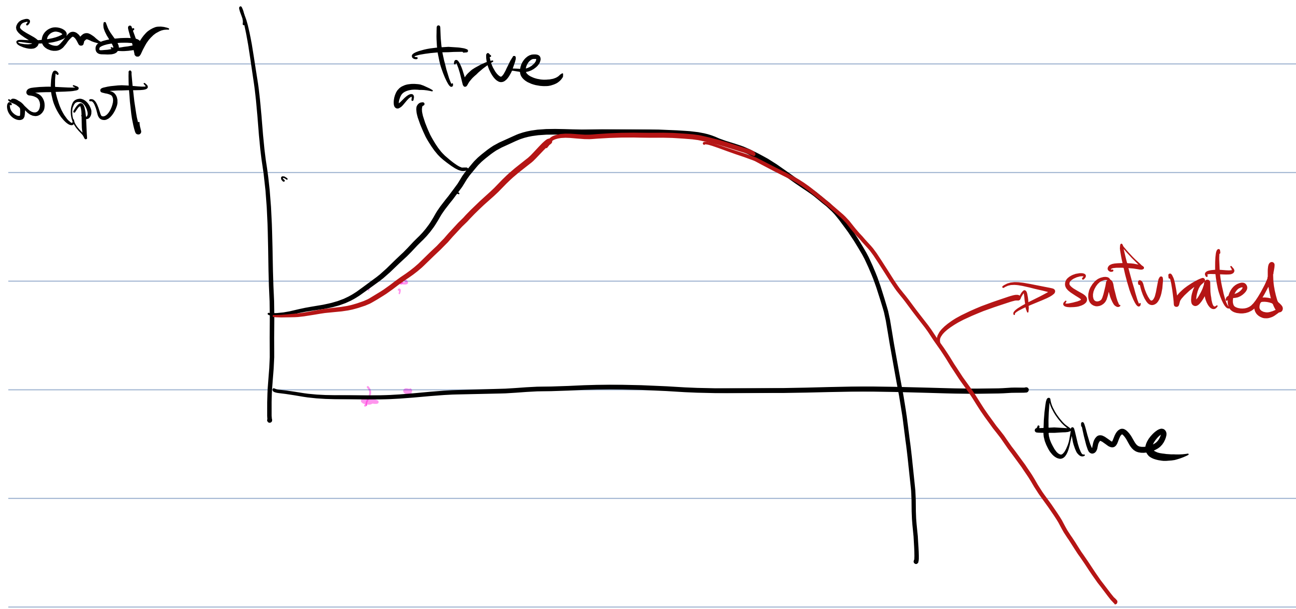
Frequency response of a sensor



useful bandwidth
(doesn't distort magnitude, but only amplifies it <u>uniformly</u> for all $\omega \in [0, \omega_0]$)

Rate saturation

Rate saturation means that the rate of change of the sensor output is bounded



Rate saturation is a highly nonlinear bandwidth constraint.