AEROSP 584
Homework 1

Due Tueday, September 20, by 11:59 PM.

Instructions:

1. Please check out and follow the homework preparation and uploading guidelines posted in the Section “Assignment Policies and Specification” of the syllabus, especially with regard to how to submit if you are working on a team (optional for Homework 1).

2. Attach all your Matlab and Simulink code in a single zip file, along with a single PDF file with your answers to the homework problems; please include in the PDF a copy-paste of your code.

3. In this homework, you will be using MATLAB’s ode45. A corresponding MATLAB file with an example has been posted in the course Canvas page; please follow this link to find it.

4. Please read and follow the honor code guidelines in the Section “Additional Policies” of the syllabus.

NOTE: The dynamics notes refers to the “Geometry, kinematics, statics, and dynamics” book in the Canvas “Useful Books” file.

Problem 1 (10 points). Let $\vec{x}(t)$ be a physical vector and let $\mathrm{F_A}$ be a frame. Show that, if $|\vec{x}(t)|$ is constant, then $\vec{x}(t)$ and $\overset{\mathrm{A}\bullet}{\vec{x}}(t)$ are mutually orthogonal. (Note: This is in the notes somewhere, but try to do it on your own.)

Problem 2 (10 points). Derive double transport by differentiating transport. (Note: This is in the notes somewhere, but try to do it on your own.)

Problem 3 (10 points). Problem 4.17.11 in the dynamics notes.

Problem 4 (10 points). Problem 4.18.1 in the dynamics notes.

Problem 5 (10 points). Problem 7.12.1 in the dynamics notes.

Problem 6 (10 points). Problem 7.12.6 in the dynamics notes.

Problem 7 (30 points). This problem studies the relationship between the Euler-angle derivatives and the angular velocity vector. Let frame $\mathrm{F_D}$ be obtained by applying a 3-2-1 sequence of Euler-angle rotations to frame $\mathrm{F_A}$, where Ψ, Θ, and Φ denote the azimuth, elevation, and bank angles, respectively. The frames $\mathrm{F_D}$ and $\mathrm{F_A}$ are thus related by

$$\mathrm{F_A} \xrightarrow[3]{\Psi} \mathrm{F_B} \xrightarrow[2]{\Theta} \mathrm{F_C} \xrightarrow[1]{\Phi} \mathrm{F_D}. \tag{1}$$

Let the angular velocity of $\mathrm{F_D}$ relative to $\mathrm{F_A}$ evaluated at $\mathrm{F_D}$ $\vec{\omega}_{\mathrm{D/A}}\big|_{\mathrm{D}}$ be given by

$$\vec{\omega}_{\mathrm{D/A}}\Big|_{\mathrm{D}} = \begin{bmatrix} \cos 2t \\ \cos 2t \\ 0.025t \end{bmatrix}, \tag{2}$$

where t is the time in seconds. For all $t \in \{0, 0.01, 0.02, \ldots, 9.98, 9.99, 10\}$ s, you'll derive the corresponding Euler angles $\Phi(t), \Theta(t), \Psi(t)$, and the components from the orientation matrix $\mathcal{O}_{\mathrm{D/A}}(t)$ in the case where $\Phi(0) = \Theta(0) = \Psi(0) = 0$ rad. You'll need to use ode45. An example has been posted in the course Canvas page.

a) Integrate equation (4.10.10) from the dynamics notes to obtain $\Phi(t), \Theta(t), \Psi(t)$, and use those angles to compute $\mathcal{O}_{\mathrm{D/A}}(t)$ (see equations (2.13.32) and (2.15.3)). Plot the components of $\vec{\omega}_{\mathrm{D/A}}\big|_{\mathrm{D}}$ versus time, and plot $\Phi(t), \Theta(t), \Psi(t)$ versus time in another figure. Finally, plot all components from the orientation matrix $\mathcal{O}_{\mathrm{D/A}}(t)$ versus time in a 3-by-3 figure grid using the subplot function in another figure.

b) Integrate Poisson's equation (4.3.19) from the dynamics notes to obtain $\mathcal{O}_{\mathrm{D/A}}(t)$. Plot all components from the orientation matrix $\mathcal{O}_{\mathrm{D/A}}(t)$ obtained here (regular blue line) and the ones obtained in a) (dashed orange line) versus time in a 3-by-3 figure grid using the subplot function. Do both coincide? **NOTE:** The example in the Canvas page shows how to integrate a matrix differential equation.

c) Use the orientation matrices $\mathcal{O}_{\mathrm{D/A}}(t)$ obtained in a) and b) to obtain two sets of Euler angles $\Phi(t), \Theta(t), \Psi(t)$ by using asin, acos and atan2 functions. Plot $\Phi(t), \Theta(t), \Psi(t)$ versus time. Use regular blue lines for the Euler angles obtained in a), dashed orange lines for the angles obtained here using $\mathcal{O}_{\mathrm{D/A}}(t)$ obtained in a) and dash-dot yellow lines for the angles obtained here using $\mathcal{O}_{\mathrm{D/A}}(t)$ obtained in b). Do all Euler angles coincide?