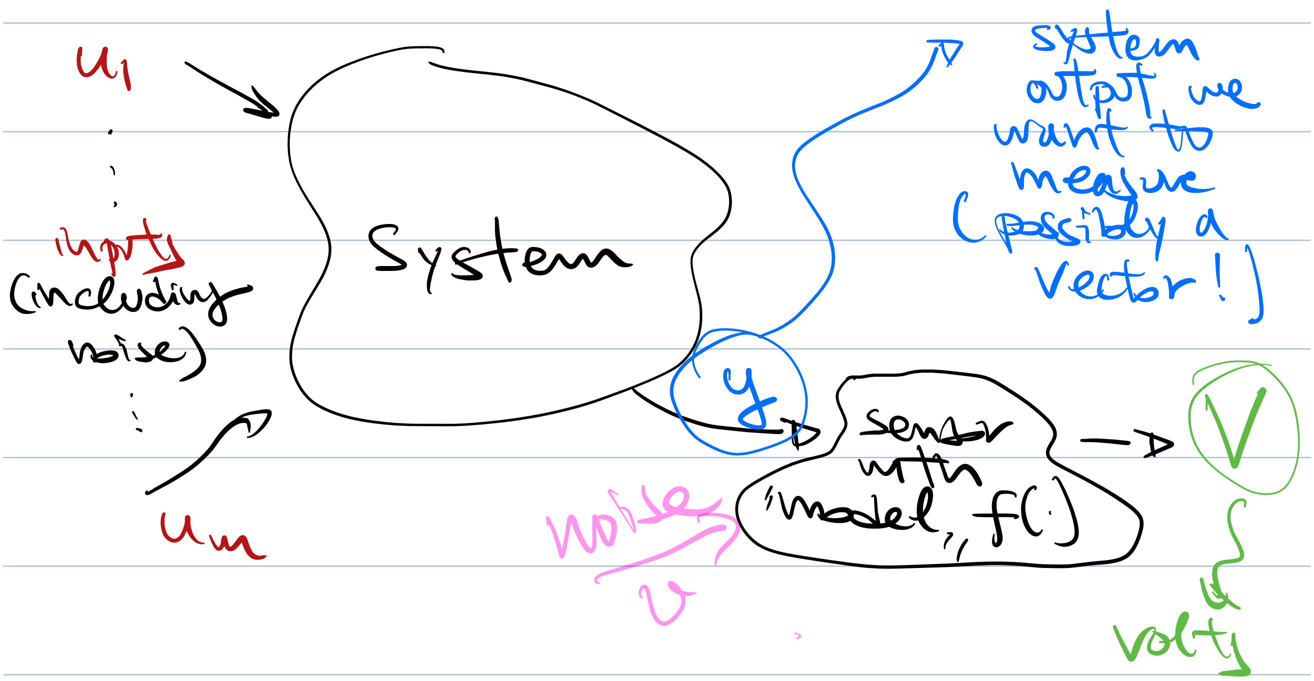
Static sensor specifications



Sensor model: $V = f(y) + v$ (*)

sensor curve

unknown

Assume a calibration curve $\hat{f}$.
Then, (*) can written as:

$$V = \hat{f}(y) + v + f(y) - \hat{f}(y)$$

$$\underbrace{\qquad\qquad\qquad}_{\triangleq\ \hat{v}}$$

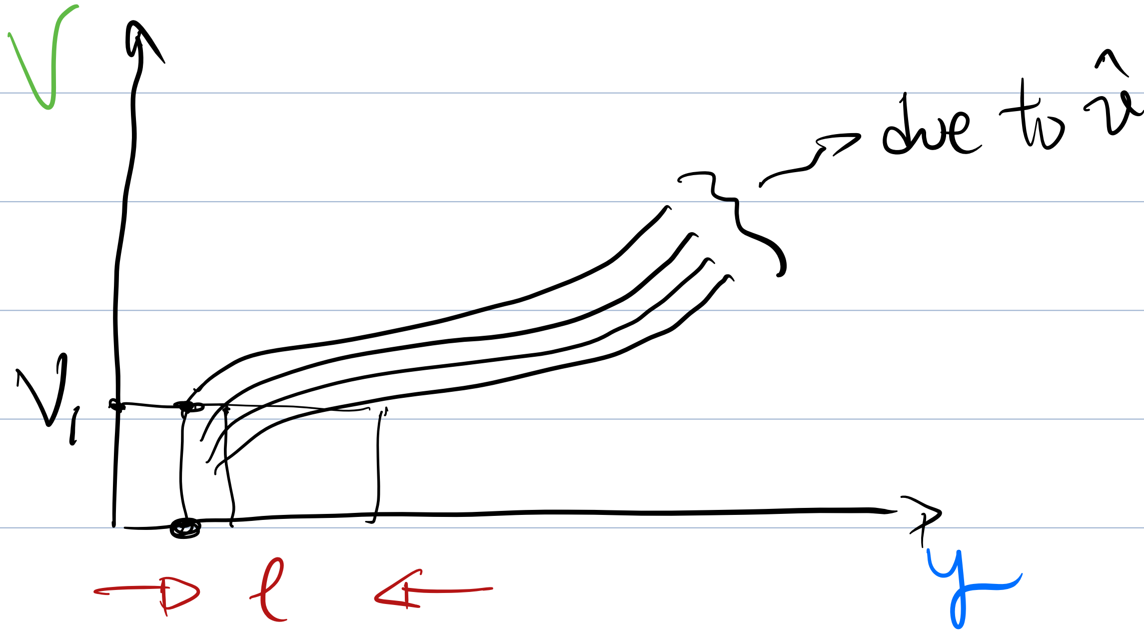
calibration error in the "output" (V-domain)

can be treated as noise!

$$\Longrightarrow \quad V = \hat{f}(y) + \hat{v}. \quad (**)$$

$$\Longrightarrow \quad y = \hat{f}^{-1}(V - \hat{v})$$

Since $\hat{v}$ is random, measurement V may be different each time the input is y:

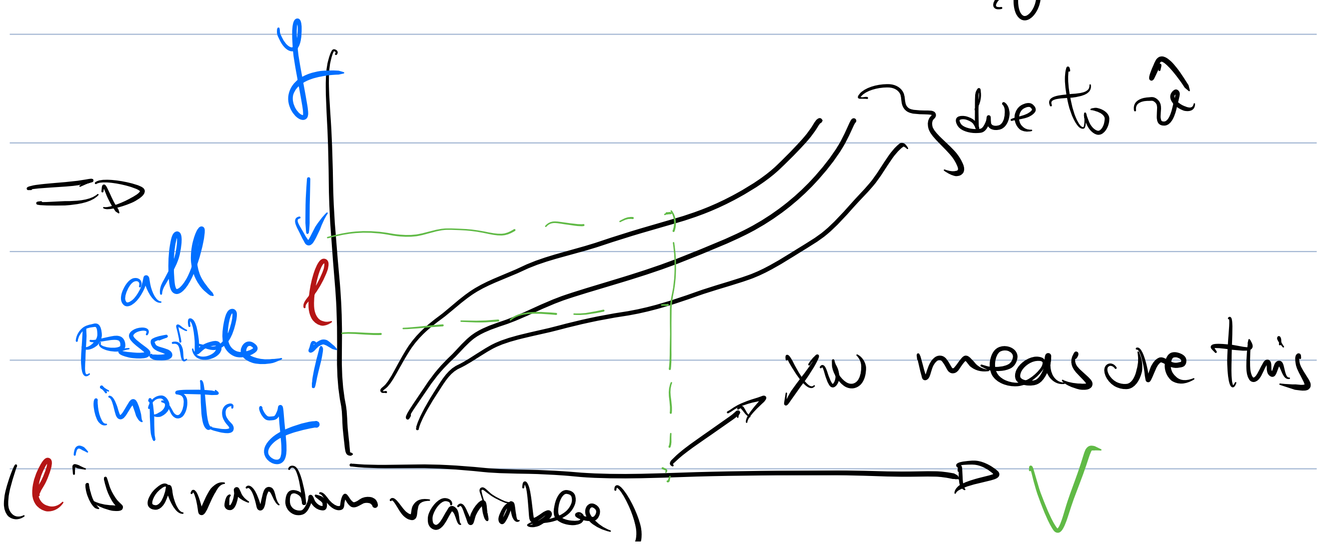


Similarly with respect to

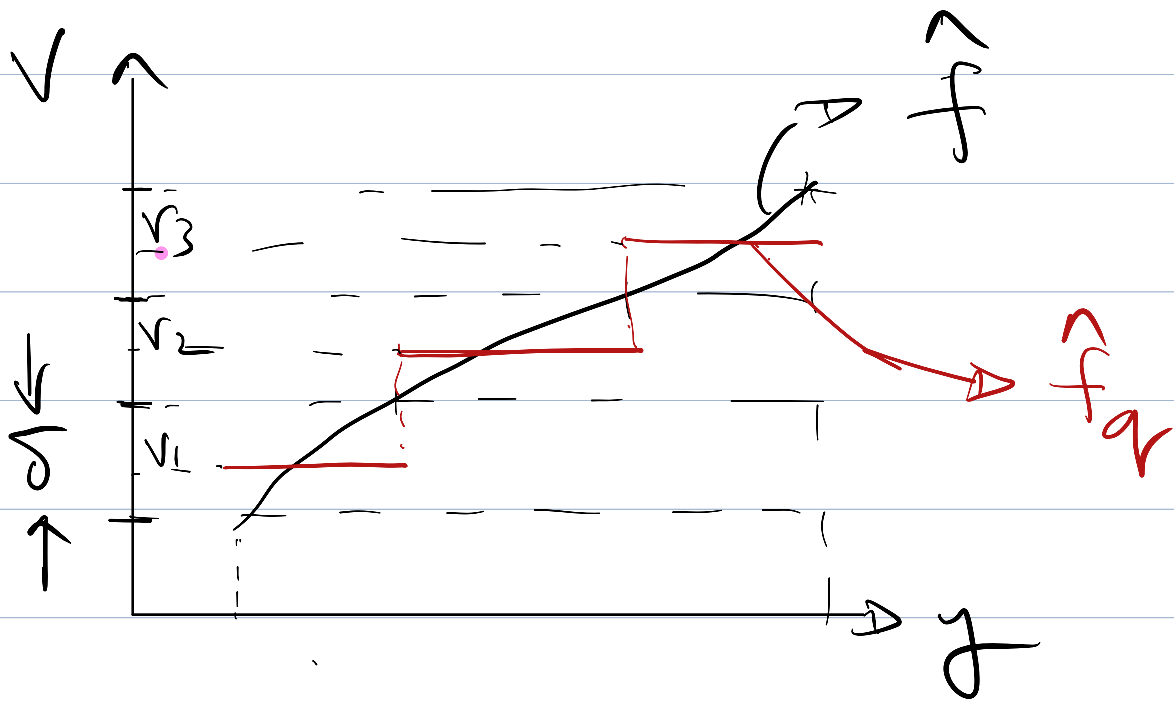
$$y = \hat{f}^{-1}(\underbrace{V - \hat{v}})$$

↳ this becomes random due to $\hat{v}$

⟹



Quantization has a similar effect as noise (not random!)

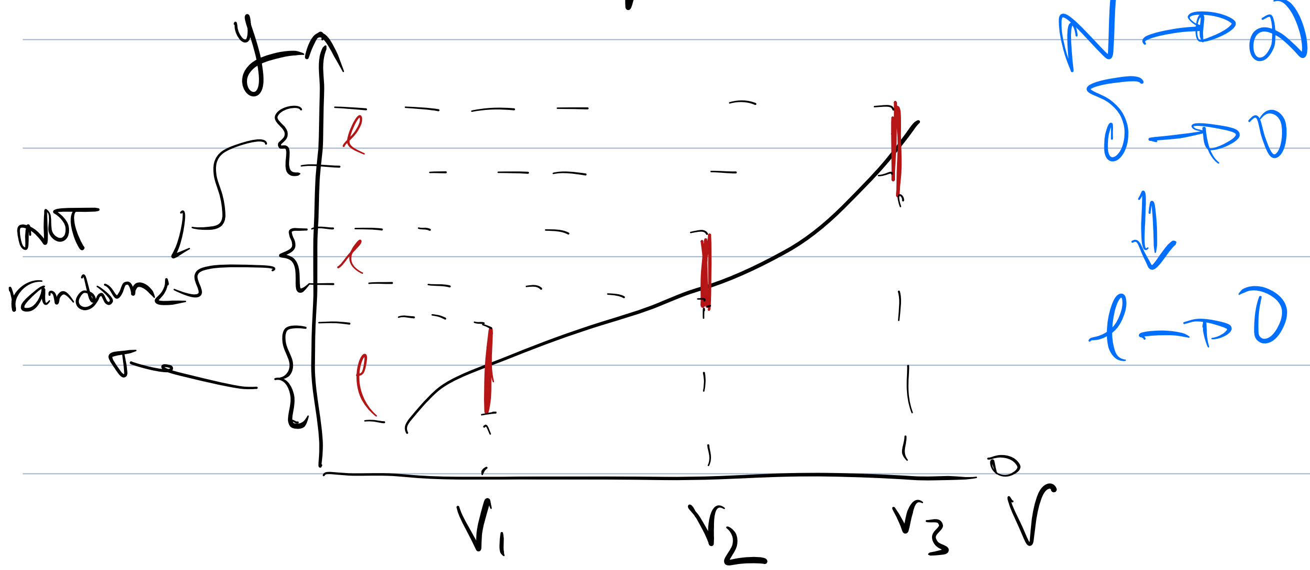


$$V = \hat{f}(y) + \hat{u}, \quad \text{where } |\hat{u}| \leq \delta/2$$

not random!

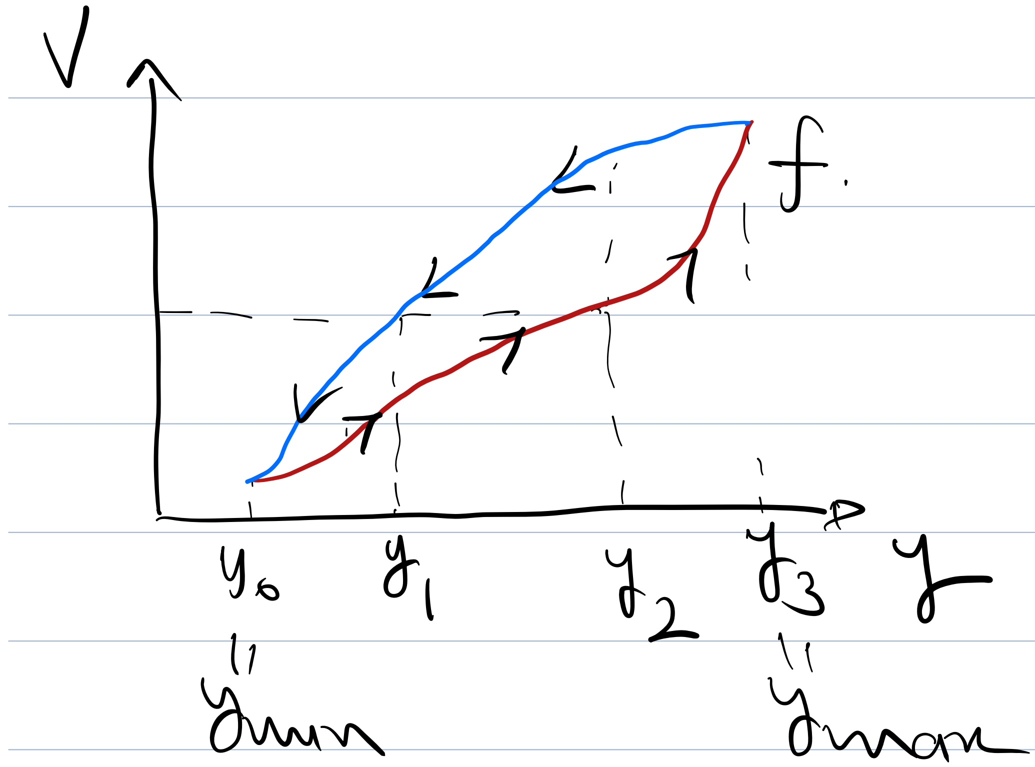
instead: repeatable!

Looking at $\hat{f}_q^{-1}$:



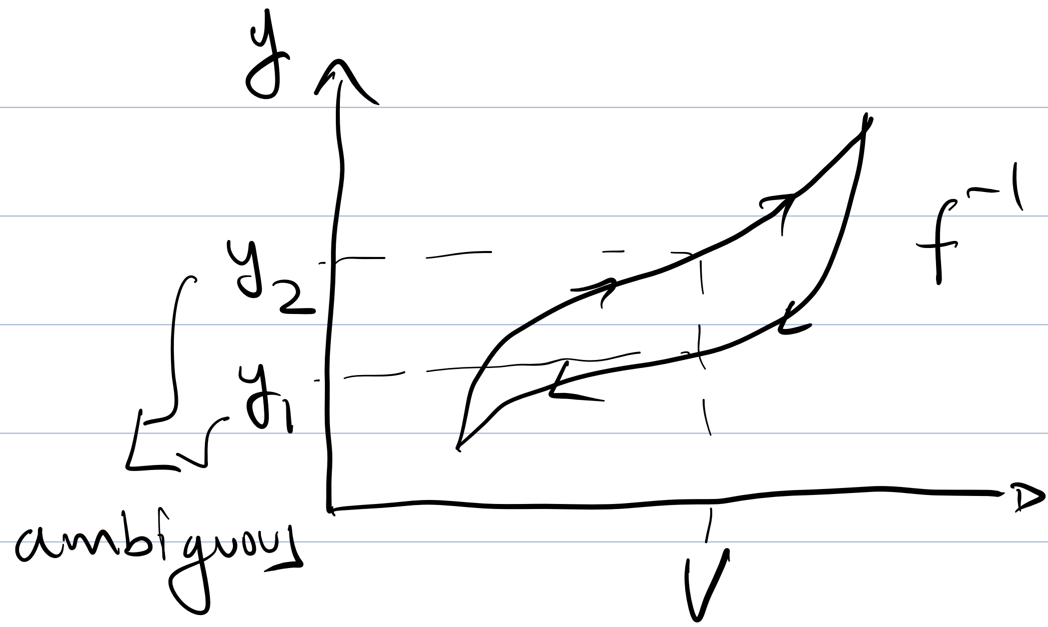
Still, poor precision (y cannot be determined unambiguously)

Another source of poor precision can be "hysteris":

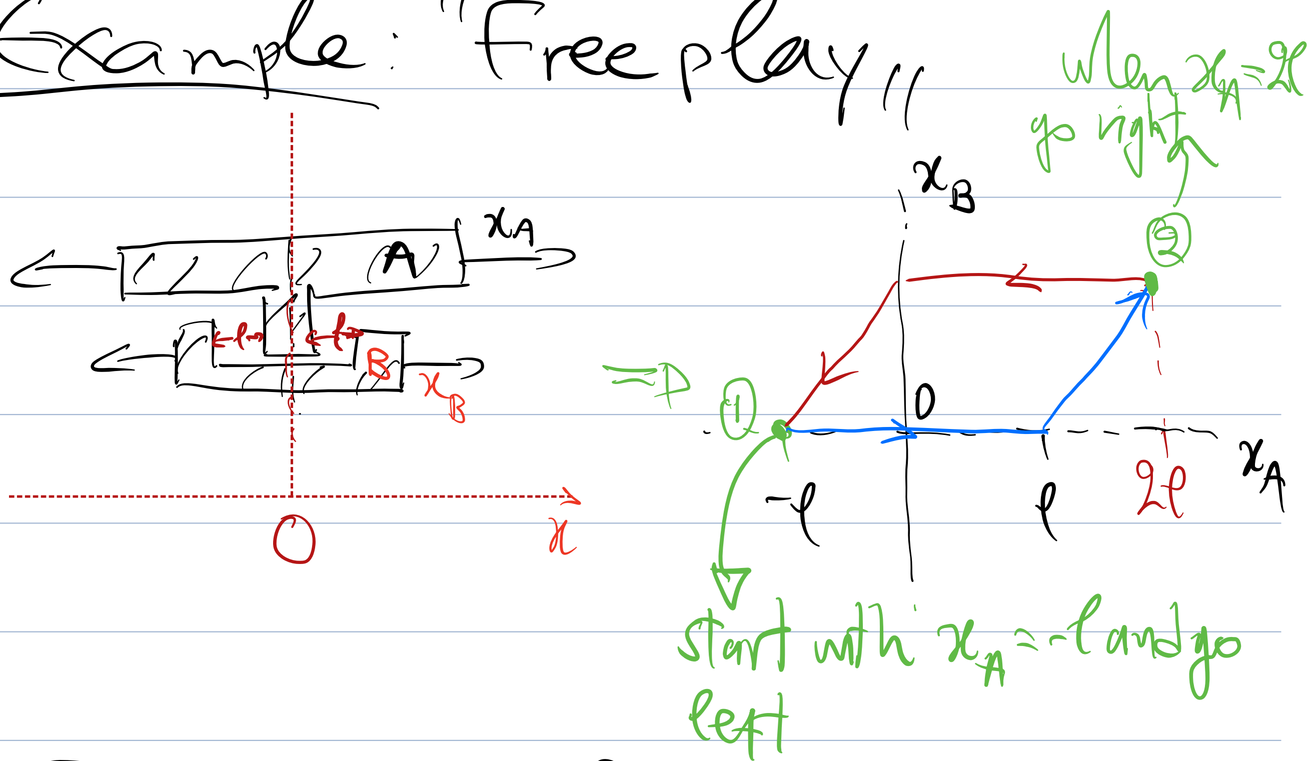


If we increase y from y_0 to y_3, and the decrease it from y_3 to y_0, we get the "double" sensor curve above.

The f^{-1} will also be hysteretic:



Example: "Free play"



Summary on Precision

(Input) Precision is the ability

to unambiguously determine y from V

Precision is degraded by $\hat{f}$ that:

- has flat regions
- is multivalued

3 cases:

1) noise $\Rightarrow$ multivalued $\hat{f}$
2) quantization $\Rightarrow$ $\hat{f}$ with flat regions
3) hysteresis $\Rightarrow$ multivalued $\hat{f}$

POLL 1

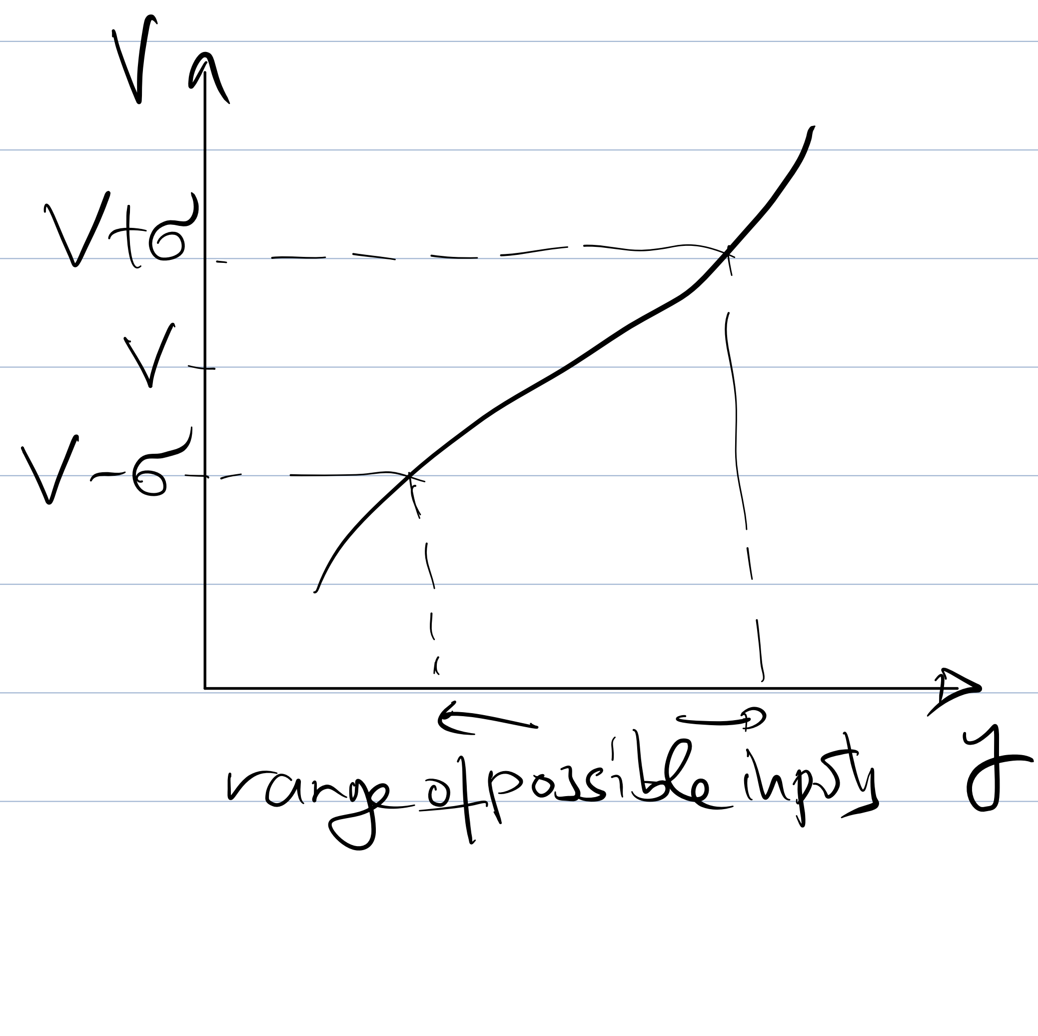
Noise models

- Deterministic noise model

$$V = \hat{f}(y) + v \Rightarrow \hat{f}(y) = V - v$$

$v \to$ noise

Assume v is bounded: $|v| \leq \sigma$:



Oftentimes, $\sum_{k=1}^{\infty} v_k = 0$

↳ k-th time we are taking the same measurement

with probability 1.

That is, v_k is a sample of a random variable with mean zero.

Random variable

A random variable v has a probability density function

$p(v)$:

- $p(v) \geq 0 \quad \forall v \in \mathbb{R}$
- $\int_{-\infty}^{\infty} p(v)\, dv = 1$

- Probability for v to be in $[a, b]$:

$$\mathbb{P}[a \leq v \leq b] = \int_a^b p(v)\, dv$$

- Mean (average) of v:

$$\mathbb{E}(v) = \int_{-\infty}^{+\infty} v\, p(v)\, dv \triangleq \mu$$

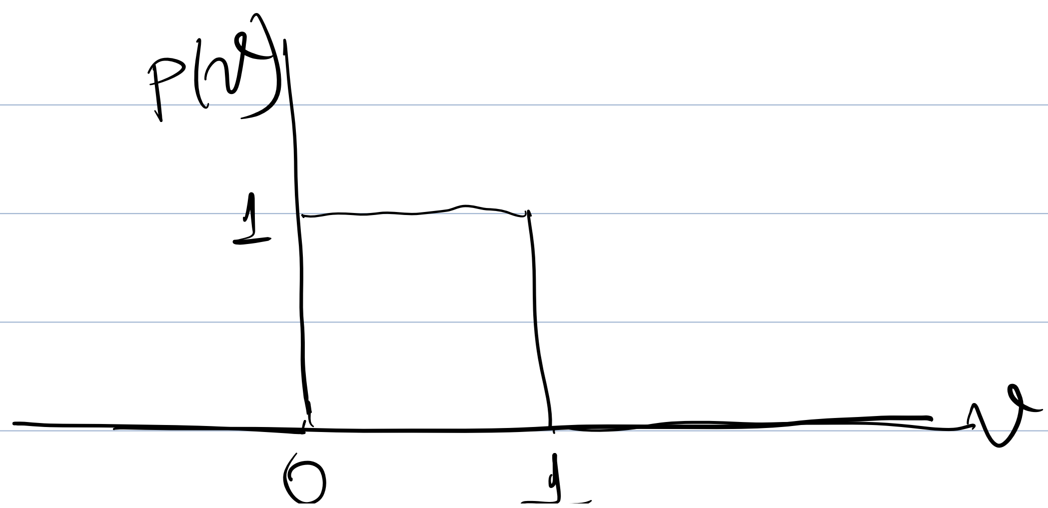
- if v was discrete: $\mathbb{E}(v) = \sum_{k=-\infty}^{\infty} v_k P_k$
- Variance of v:

$$\mathbb{E}\left[(v-\mu)^2\right] = \int_{-\infty}^{+\infty} (v-\mu)^2 p(v)\,dv$$

$$\triangleq \sigma^2$$

Examples

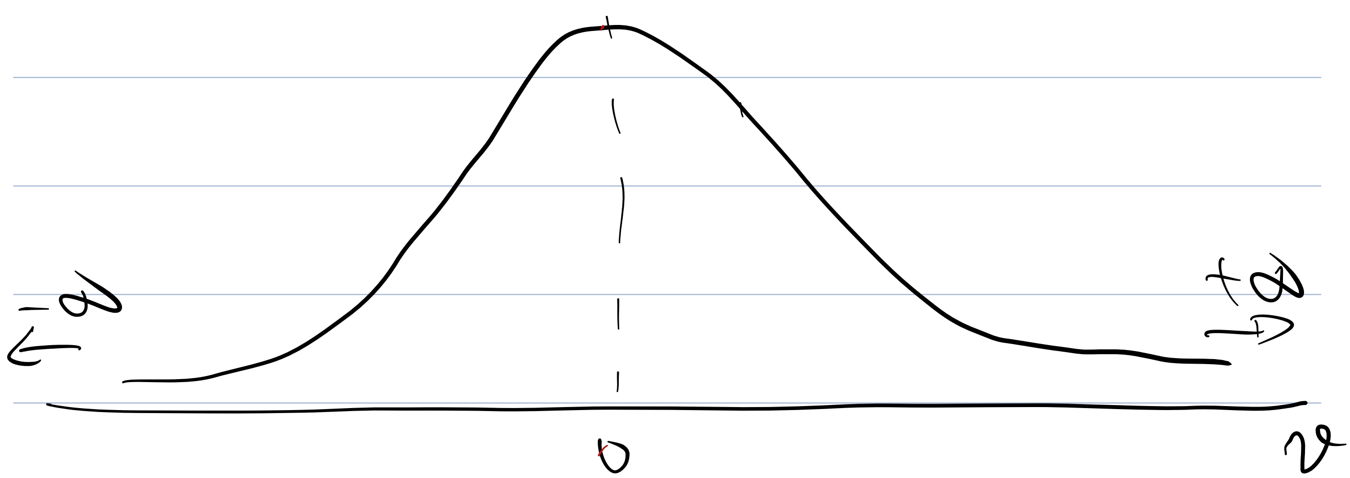
- Uniform density



All values of v in $[0,1]$ have "equal" probability.

$$\Rightarrow \boxed{\begin{aligned} \mu &= 1/2 \\ \sigma^2 &= 1/12 \end{aligned}}$$

· <u>Gaussian density</u>



$$p(v) = \frac{1}{\sqrt{2\pi}} e^{-v^2/2}$$

$$\Rightarrow \quad \mu = 0$$
$$\sigma^2 = 1.$$

Properties of random variables

Let:

- v is a random variable
- v_k is a sample of v

Then:

$$\mu = \mathbb{E}(v)$$
$$\sigma^2 = \mathbb{E}\left[(v-\mu)^2\right]$$

For large n:

$$\mu_n \triangleq \frac{1}{n} \sum_{k=1}^{n} v_k \simeq \mu$$

$$\sigma_n^2 \triangleq \frac{1}{n-1} \sum_{i=1}^{n} (v_k - \mu_k)^2 \approx \sigma^2$$

With probability 1:

$$\boxed{\mu_n \to \mu, \quad \sigma_n \to \sigma, \quad \text{for } n \to \infty}$$

Note: if $\mu = 0$, then we say the noise is unbiased.