Newton's first law: There exist an inertial frame.

(Not provable).

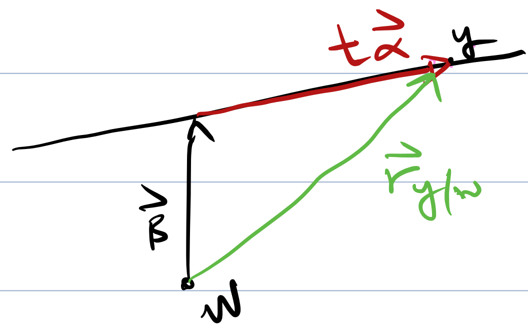
Definition F_A is an inertial frame if for all unforced particles y and w,

$$\overset{A\cdot\cdot}{\vec{r}}_{y/w} = 0.$$

That is, $\overset{A\cdot}{\vec{r}}_{y/w} = \vec{\alpha}$ and $\vec{r}_{y/w} = t\vec{\alpha} + \vec{\beta}$

where $\overset{A\cdot}{\vec{\alpha}} = 0$ and $\overset{A\cdot}{\vec{\beta}} = 0$

$\Rightarrow$ relative motion of $\vec{\alpha}$ and $\vec{\beta}$ is a line.



Fact If Newton's first law is true, then distant stars approximate an inertial frame, since distant stars appear as unforced particles moving along lines

Fact Let F_A be an inertial frame and F_B be a frame. Then, F_B is an inertial frame if and only if $\vec{\omega}_{B/A} = 0$.

Newton's Second Law (for a particle)

Let F_A be an inertial frame; y be a particle with mass m; $\vec{f}_y$ be the force acting on y; and w be an unforced particle.

Then:

$$m\,\vec{a}_{y/w/A} = \vec{f}_y$$

Note: If $\vec{f}_y = 0$, then

$$\vec{a}_{y/w/A} = 0,$$

which is true due to the fact that F_A is an inertial frame.

To state the 2nd law for multiple particles let's define the notions of <u>body</u> and <u>center of mass</u>

<u>Definition</u> A <u>body</u> is a collection of particles. It may or may not be rigid.

<u>Definition</u> Let B be a body composed of particles $y_1, \ldots, y_\ell$ with masses $m_1, \ldots, m_\ell$, and let w be a point. Then, the <u>center of mass</u> c of B is the point:

$$\vec{r}_{c/w} \triangleq \frac{1}{m_B} \sum_{i=1}^{\ell} m_i \vec{r}_{y_i/w},$$

where $m_B \triangleq \sum_{i=1}^{\ell} m_i$ is the total mass of B.

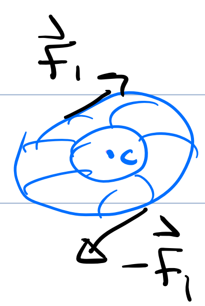
<u>Fact</u>. Let B be the above body. Also, let F_A be an inertial frame; $\vec{f}_{y_i}$ be the force applied to y_i; and w be an unforced particle. Then,

$$m_B \, \vec{a}_{c/w}|_A = \vec{f}_B \qquad (*)$$

where $\vec{f}_B \triangleq \sum_{i=1}^{\ell} \vec{f}_i$

<u>Note</u>: It's like all forces are applied at a particle with mass m_B at c.

<u>Note</u> If B is a donut, there is nothing at c!



To state the laws for angular acceleration, we revisit the notions of angular momentum.

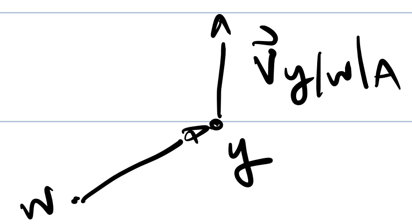
Let y be a particle with mass m;

w be a point;

F_A be a frame.

Then, the angular momentum is:

$$\vec{H}_{y/w/A} \triangleq \vec{r}_{y/w} \times m\vec{v}_{y/w/A}$$



For a body $B = \{y_1, \dots, y_\ell\}$:

$$\vec{H}_{B/w/A} \triangleq \sum_{i=1}^{\ell} \vec{H}_{y_i/w/A}$$

Definition: Assume $\vec{f}_{y_i}$ is applied to y_i. Then, the moment (torque) on B relative to w is:

$$\vec{M}_{B/w} \triangleq \sum_{i=1}^{\ell} \vec{r}_{y_i/w} \times \vec{f}_{y_i}$$

Newton's 2nd law for rotation

Assume F_A is inertial, and w has zero inertial acceleration. Then,

$$\overset{A\cdot}{\vec{H}}_{B/w/A} = \vec{M}_{B/w} \qquad (c \text{ or } w)$$

Fact Assuming B is rigid with physical inertia matrix relative to c:

$$\vec{J}_{B/C} \, {}^{B}\dot{\vec{\omega}}_{B/A} + \vec{\omega}_{B/A} \times \vec{J}_{B/C} \, \vec{\omega}_{B/A} = \vec{M}_{B/C}$$

This is Euler's equation (generalization of $\tau = I a$)