Image formation

- Pinhole camera model

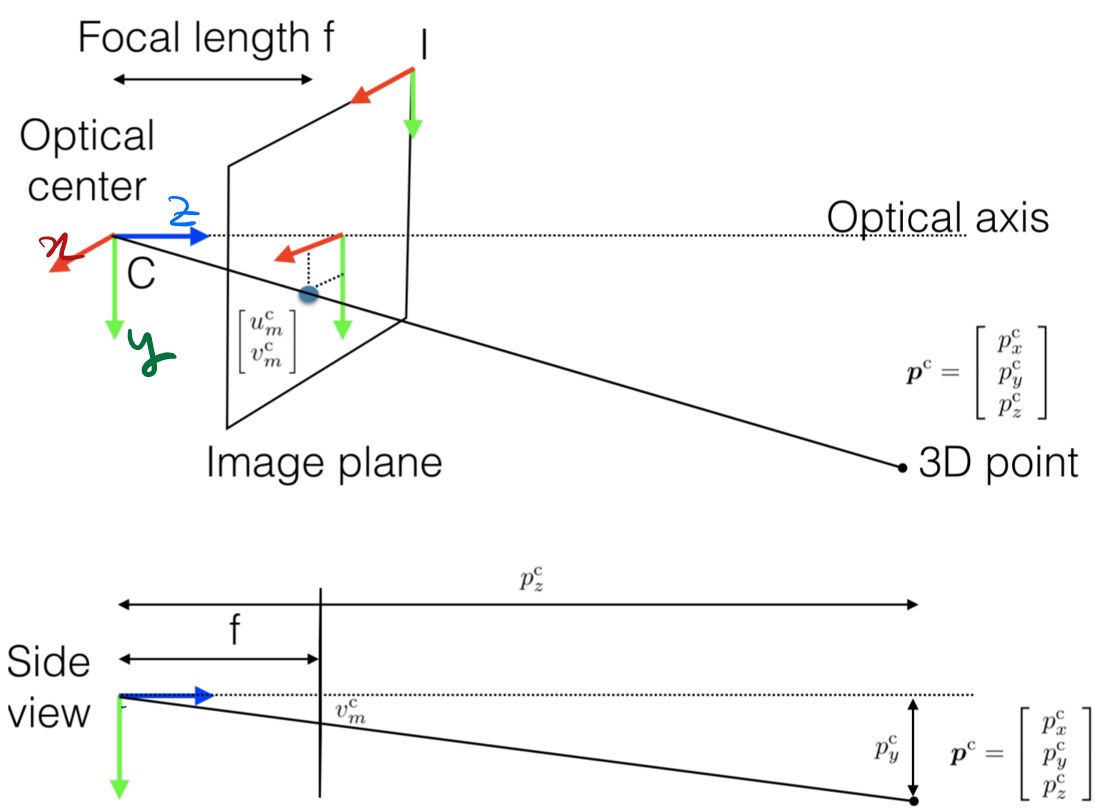


Figure 1: Pinhole Model.

GOAL compute pixel projection $\begin{pmatrix} u_m^c \\ v_m^c \end{pmatrix}$ given a 3D point p^c, and focal length f.

SIMILAR TRIANGLES

$$\frac{f}{P_z^c} = \frac{V_m^c}{P_y^c} \Rightarrow V_m^c = f \cdot \frac{P_y^c}{P_z^c}$$

$$\frac{f}{P_z^c} = \frac{U_m^c}{P_x^c} \Rightarrow U_m^c = f \, \frac{P_x^c}{P_z^c}$$

$$P_z^c = P_z^c$$

$$P_z^c \begin{bmatrix} U_m^c \\ V_m^c \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} P_x^c \\ P_y^c \\ P_z^c \end{bmatrix}}_{= \, P^c}$$

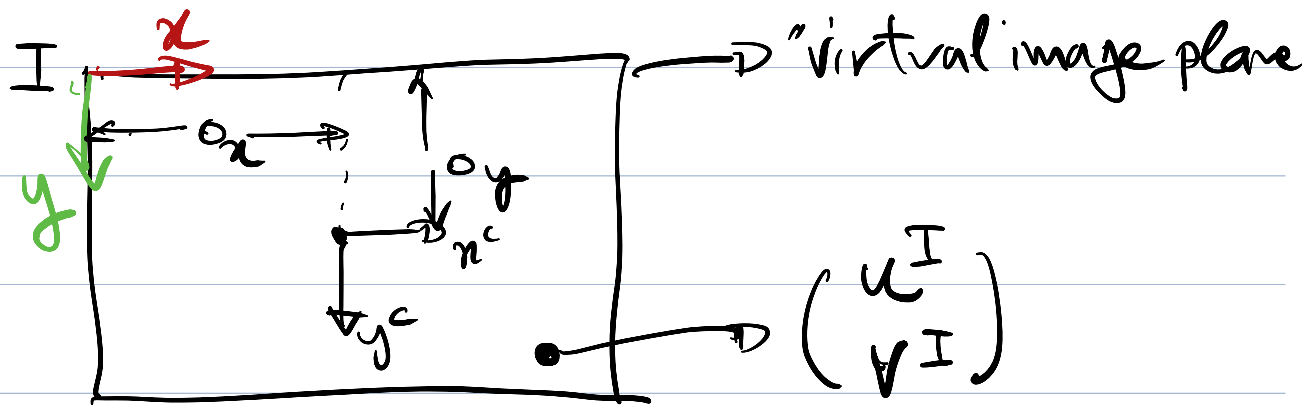
given $\tilde{P}_c = \begin{bmatrix} P^c \\ 1 \end{bmatrix}$

$$\Leftarrow P^c = \left[I_{3\times3} , 0_{3\times1} \right] \tilde{P}^c$$

$\|$

$$p_z^c \begin{bmatrix} u_m^c \\ v_m^c \\ 1 \end{bmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \tilde{p}^c$$

CONVERSION TO PIXEL & TO THE I frame



s_x , s_y : pixels/meter across the x and y dimensions.

$$\left.\begin{aligned} u^I &= s_x u_m^c + o_x \\ v^I &= s_y v_m^c + o_y \end{aligned}\right\} \text{expression in pixels w.r.t. } I \text{ frames.}$$

$$\begin{pmatrix} u^I \\ v^I \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_m^c \\ v_m^c \\ 1 \end{pmatrix}$$

<u>Overall:</u>

$$p_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & & \\ & f & \\ & & \end{bmatrix} \cdot$$

$$\cdot \left[I_3 \; 0_{3\times 1} \right] \tilde{p}^c$$

$$= \underbrace{\begin{pmatrix} s_x f & 0 & o_x \\ 0 & s_y f & o_y \\ 0 & 0 & 1 \end{pmatrix}}_{K \equiv} \cdot$$

"intrinsic camera matrix" $\leftarrow K \equiv$

$$\cdot \underbrace{\left[I_3 \, , \, 0_{3\times 1} \right]}\tilde{p}^c$$

$|||$

canonical projection $\leftarrow \Pi_0$

It remains to express $\tilde{P}_c$ in a world frame W.

$$\tilde{p}^c = T^c_w \, \tilde{p}^w = \begin{pmatrix} R^c_w , & t^c_w \\ 0_{3\times1} , & 1 \end{pmatrix} \tilde{p}_w$$

$$\Rightarrow P_z^c \begin{bmatrix} u^I \\ v^I \\ 1 \end{bmatrix} = K \, [I_3 , 0_{3\times1}] \, T^c_w \, \tilde{p}^w$$

$$= \underbrace{K \cdot [R^c_w , t^c_w]}_{\equiv \Pi \,\to\, \text{projection matrix}} \tilde{p}^w$$

$$= \Pi \tilde{p}^w = \begin{bmatrix} [\Pi \tilde{p}^w]_1 \\ [\Pi \tilde{p}^w]_2 \\ [\Pi \tilde{p}^w]_3 \end{bmatrix}$$

- Now, given $\tilde{p}^w$, we can find u^I, and v^I as follows:

$$u^I = \frac{[\Pi \tilde{p}^w]_1}{[\Pi \tilde{p}^w]_3}$$

$$v^I = \frac{[\Pi \tilde{p}^w]_2}{[\Pi \tilde{p}^w]_3}$$

- In the following lectures: given multiple u^I, v^I we will find $\tilde{p}^w$