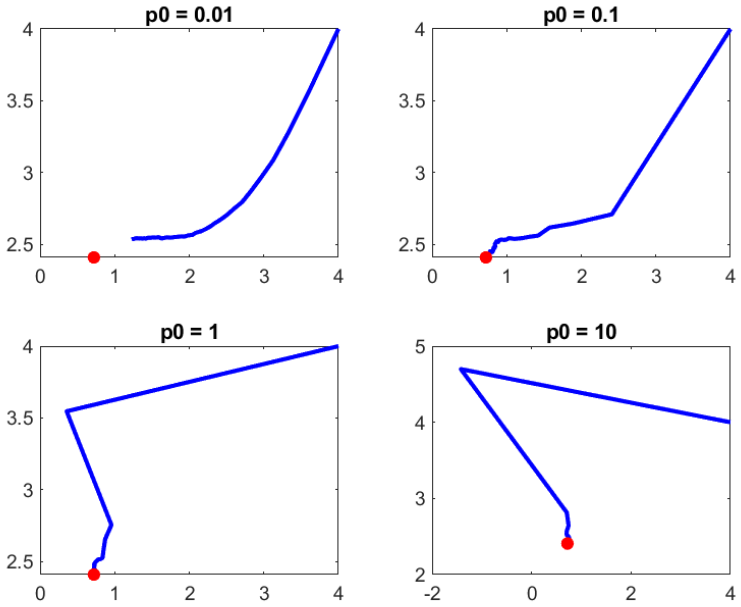
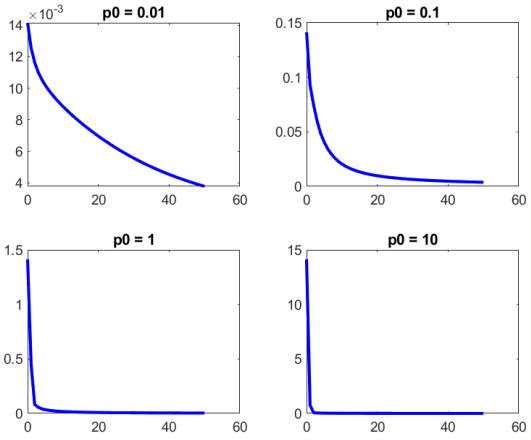
Homework 4

Question 1: Part b

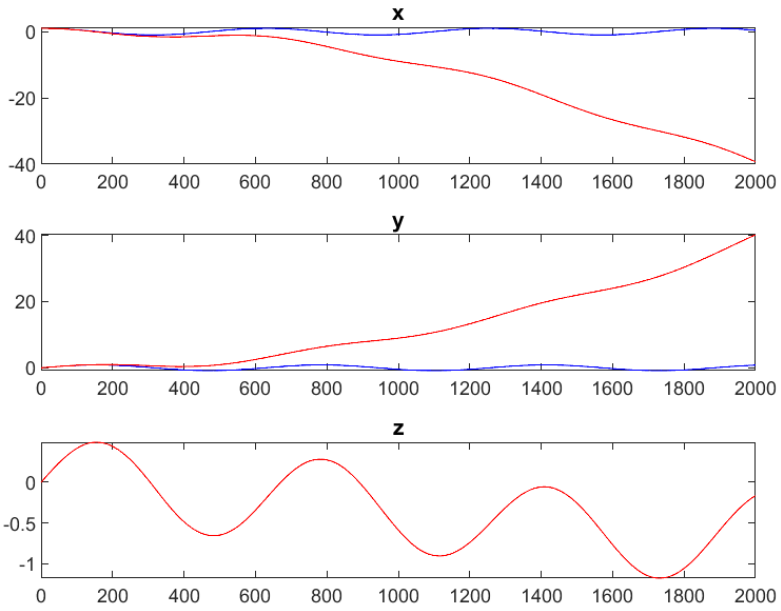
Trajectories:



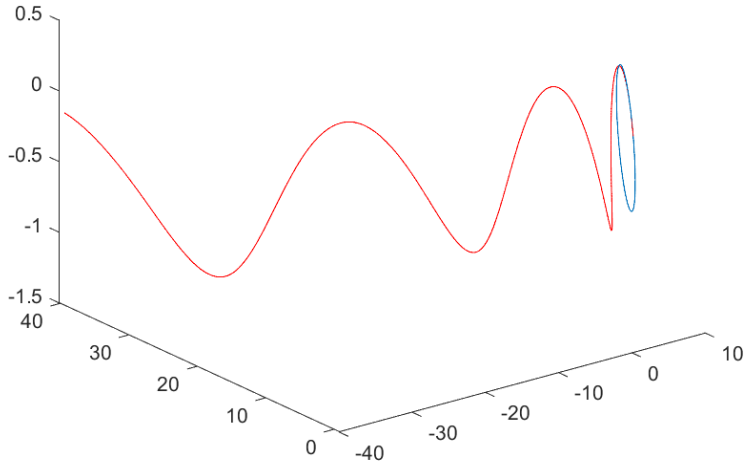
Norm:



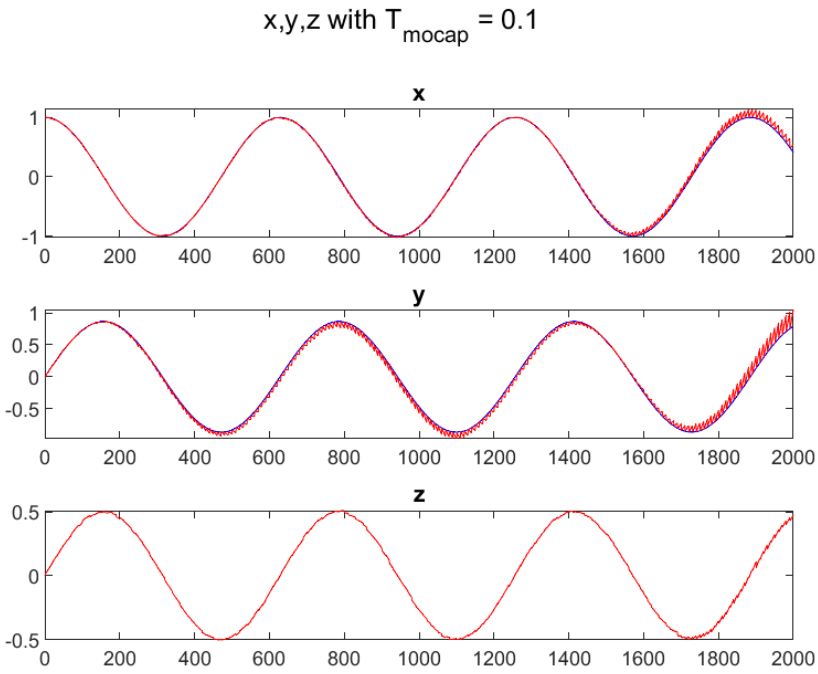
Question 2, Part a:

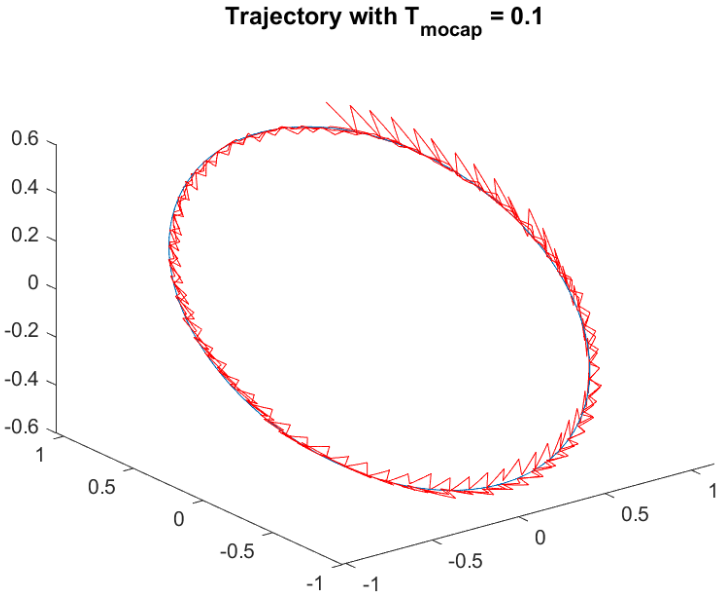


Trajectory with Inertial Nav Only

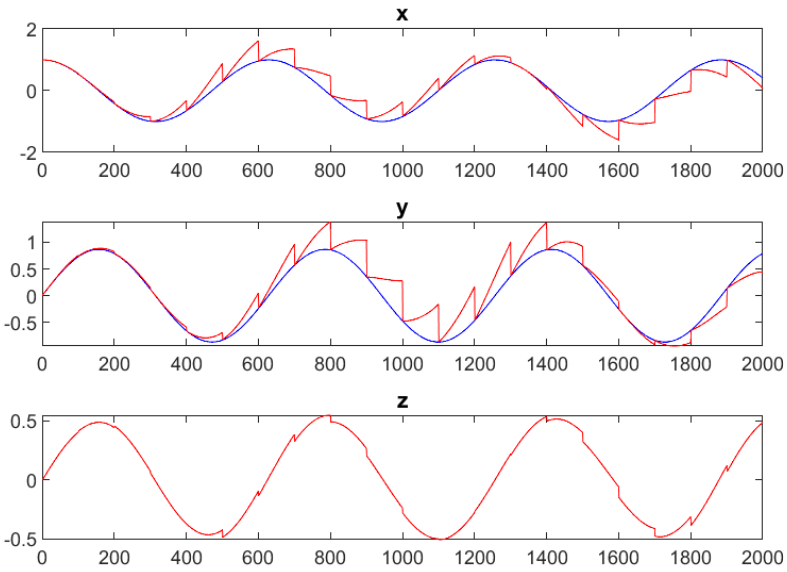


Question 2: Part b:

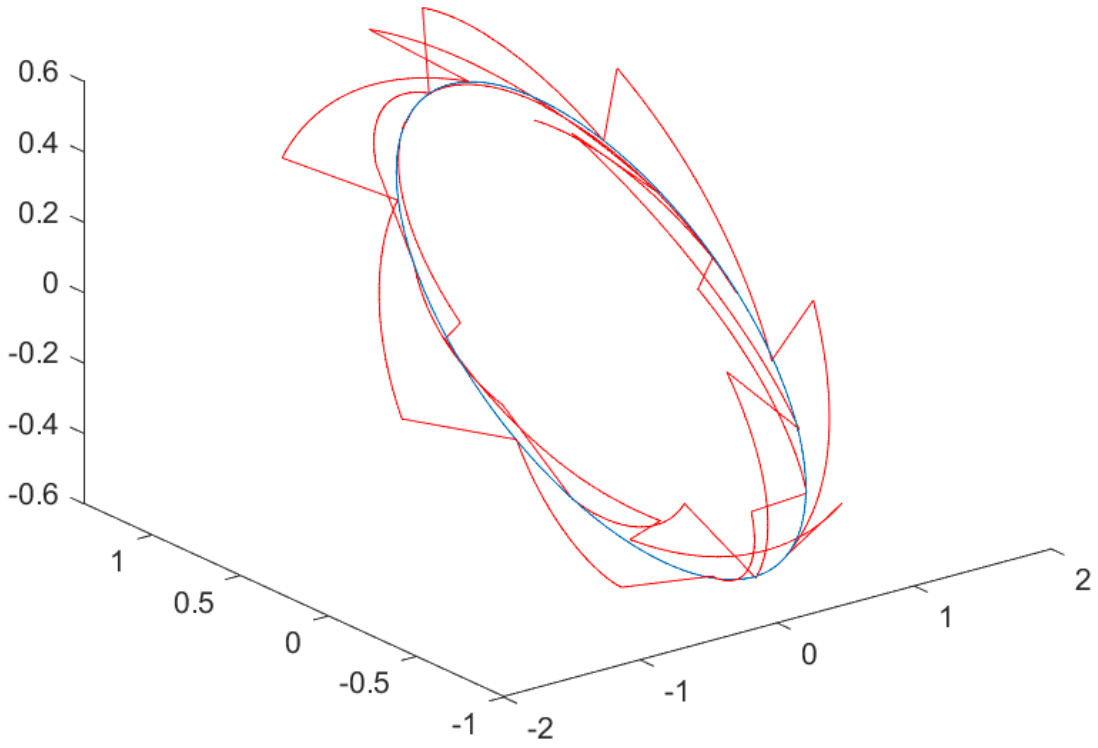




x,y,z with T_{mocap} = 1



Trajectory with T_{mocap} = 1



Codes:

Question 1:

```
P0 =[0.01,0.1,1,10];
for_posx = zeros(4,51);
for_posy = zeros(4,51);
forb = zeros(4,51);
x1 = [0;0];
x2 = [5;5];
x3 = [2.5;0];
xx = [0.7212;2.4080];
r1 = sqrt((xx-x1)'*(xx-x1));
r2 = sqrt((xx-x2)'*(xx-x2));
r3 = sqrt((xx-x3)'*(xx-x3));

for j=1:4
   p0 = P0(j);
   D = 0.1*eye(3);
   x_k = [4;4];
   P_k = p0*eye(2);
   R = 0.1*eye(3);
   for_posx(j,1) = x_k(1);
   for_posy(j,1) = x_k(2);
   forb(j,1) = norm(P_k,'fro');
   for i=1:50
      A_k = eye(3);
      C_k = get_c(x1,x2,x3,x_k);
      w_k = randn(3,1);
      y_k = [r1;r2;r3] + D*w_k;
      g_k = get_dists(x1,x2,x3,x_k(1:2));
      K_k = P_k*C_k'*inv((C_k*P_k*C_k') + R);
      x_k_new = x_k + (K_k*(y_k - g_k));
      P_k_new = P_k - (K_k*C_k*P_k);

      x_k = x_k_new;
      P_k = P_k_new;
      for_posx(j,i+1) = x_k(1);
      for_posy(j,i+1) = x_k(2);
      forb(j,i+1) = norm(P_k,'fro');
   end
end
```

```
fig1 = figure();
for k=1:4
   subplot(2,2,k)
   plot(for_posx(k,:),for_posy(k,:),'b', 'LineWidth',2);
   hold on;
   scatter(xx(1),xx(2),'r','fill');
   hold off;
   title("p0 = "+P0(k));
end

saveas(fig1,'plot.png')

fig2 = figure();
for k=1:4
   subplot(2,2,k)
   plot(0:50,forb(k,:),'b', 'LineWidth',2);
   title("p0 = "+P0(k));
end
saveas(fig2,'norm.png')

function C_k = get_c(x1,x2,x3,r)
r_dash_1 = sqrt((r(1)-x1(1))^2 + (r(2)-x1(2))^2);
r_dash_2 = sqrt((r(1)-x2(1))^2 + (r(2)-x2(2))^2);
r_dash_3 = sqrt((r(1)-x2(1))^2 + (r(2)-x3(2))^2);
C_k = [
      (r(1)-x1(1))/r_dash_1,(r(2)-x1(2))/r_dash_1;
      (r(1)-x2(1))/r_dash_2,(r(2)-x2(2))/r_dash_2;
      (r(1)-x3(1))/r_dash_3,(r(2)-x3(2))/r_dash_3;
       ];
end

function r = get_dists(x1,x2,x3,xx)

r1 = sqrt((xx-x1)'*(xx-x1));
r2 = sqrt((xx-x2)'*(xx-x2));
r3 = sqrt((xx-x3)'*(xx-x3));
r = [r1;r2;r3];
end
```

Question 2

Part a

```
load('../rcwA.mat')
prev = rcwA_Ts_0_01;
now = rcwA_Ts_0_01;

T = 0.01;
g =  9.80665;
phi = pi/6;

x_k = [1;0;0;0;cos(phi);sin(phi)];
I = eye(3);
A = [I, T*I;
     0*I, I];
B = [((T^2))*I;
    2*T*I
    ];
Ori_k = [
        1, 0, 0;
        0, cos(phi), sin(phi);
        0, -sin(phi), cos(phi);
        ];
g_a = [0;0;-g];

D1 = diag([0.1,0.1,0.1]);
D2 = diag([0.1,0.1,0.1]);

for i =1:2000
    k = i-1;
    w1 = randn(3,1);
    w2 = randn(3,1);
    omega_k = [0;0;1] + D1*w1;
    a_k = [(-1-(g*(sin(phi)*sin(k*T))));-g*sin(phi)*cos(k*T);-g*cos(phi)] + D2*w2;
    omega_cross = get_om_cross(omega_k);

    E_k = expm(-T*omega_cross);
    x_new = (A*x_k) + 0.5*(B*((Ori_k'*a_k)-g_a));
    Ori_new = E_k*Ori_k;

    x_k = x_new;
    Ori_k = Ori_new;
```

```
    now(:,i+1) =x_k(1:3);
end

fig1 = figure();
subplot(3,1,1)
plot(0:2000,prev(1,:),'b');
hold on
plot(0:2000,now(1,:),'r');
title("x", 'FontSize',10);
hold off
subplot(3,1,2)
plot(0:2000,prev(2,:),'b');
hold on
plot(0:2000,now(2,:),'r');
title("y", 'FontSize',10);
hold off
subplot(3,1,3)
plot(0:2000,prev(3,:),'b');
hold off
plot(0:2000,now(3,:),'r');
title("z", 'FontSize',10);
hold off
saveas(fig1,'Inertial_nav.png')

fig2 = figure();
plot3(prev(1,:), prev(2,:),prev(3,:));
hold on
plot3(now(1,:), now(2,:),now(3,:),'r');
hold off
title("Trajectory with Inertial Nav Only", 'FontSize',12);
saveas(fig2,'Inertial_nav_traj.png')

function omega_cross = get_om_cross(omega)
omega_cross = [
            0, -omega(3), omega(2);
            omega(3), 0, -omega(1);
            -omega(2), omega(1), 0;
            ];
end
```

Part b:

```
load('../rcwA.mat')
prev = rcwA_Ts_0_01;
now = rcwA_Ts_0_01;
samp = [0.1,1];
for tt = 1:2
    T = 0.01;
    g =  9.80665;
    phi = pi/6;

    x_k = [1;0;0;0;cos(phi);sin(phi)];
    I = eye(3);
    A = [I, T*I;
         0*I, I];
    B = [((T^2))*I;
        2*T*I
        ];
    Ori_k = [
            1, 0, 0;
            0, cos(phi), sin(phi);
            0, -sin(phi), cos(phi);
            ];
    g_a = [0;0;-g];

    D1 = diag([0.1,0.1,0.1]);
    D2 = diag([0.1,0.1,0.1]);
    D3 = diag([0.005,0.005,0.005]);

    R = 0.001*I;
    Q = 10*eye(6);
    P_k = 10*eye(6);
    T_mocap = samp(tt);
    rat = T_mocap/T;

    for i =1:2000
        k = i-1;

        w1 = randn(3,1);
        w2 = randn(3,1);
        w3 = randn(3,1);

        if(mod(i,rat)==0)
            C_k = [I, 0*I];
        else
            C_k = [0*I, 0*I];
```

```
    end


    omega_k = [0;0;1] + D1*w1;
    a_k = [(-1-(g*(sin(phi)*sin(k*T))));-g*sin(phi)*cos(k*T);-g*cos(phi)] + D2*w2;
    omega_cross = get_om_cross(omega_k);
    E_k = expm(-T*omega_cross);
    y_k = prev(:,i+1) + D3*w3;

    %D = B*Ori_k;
    %Q = D*D';

    x_new = (A*x_k) + 0.5*(B*((Ori_k'*a_k)-g_a));
    Ori_new = E_k*Ori_k;

    P_k1_k = A*P_k*A' + Q;
    K_k = P_k1_k*C_k'*(inv((C_k*P_k1_k*C_k') + R));

    x_k_new = x_new + (K_k*(y_k - (C_k*x_new)));
    P_k_new = P_k1_k - (K_k*C_k*P_k1_k);


    Ori_k = Ori_new;
    x_k = x_k_new;
    now(:,i+1) =x_k(1:3);
end

fig1 = figure();
subplot(3,1,1)
plot(0:2000,prev(1,:),'b');
hold on
plot(0:2000,now(1,:),'r');
title("x", 'FontSize',10);
hold off
subplot(3,1,2)
plot(0:2000,prev(2,:),'b');
hold on
plot(0:2000,now(2,:),'r');
title("y", 'FontSize',10);
hold off
subplot(3,1,3)
plot(0:2000,prev(3,:),'b');
hold off
plot(0:2000,now(3,:),'r');
```

```
    title("z", 'FontSize',10);
    hold off
    sgtitle("x,y,z with T_{mocap} = " + samp(tt), 'FontSize',12);
    saveas(fig1,samp(tt)+".png")

    fig2 = figure();
    plot3(prev(1,:), prev(2,:),prev(3,:));
    hold on
    plot3(now(1,:), now(2,:),now(3,:),'r');
    hold off
    title("Trajectory with T_{mocap} = " + samp(tt), 'FontSize',12);
    saveas(fig2,samp(tt)+"_traj.png")
end

function omega_cross = get_om_cross(omega)
omega_cross = [
            0, -omega(3), omega(2);
            omega(3), 0, -omega(1);
            -omega(2), omega(1), 0;
            ];
end
```

System

$$X_{k+1} = X_k$$

$$Y_{k+1} = g(X_{k+1}) + D_2 w_{k+1}$$

where $g(X_{k+1}) = \begin{bmatrix} \sqrt{(x_1 - x_k)^2 + (y_1 - y_k)^2} \\ \sqrt{(x_2 - x_k)^2 + (y_2 - y_k)^2} \\ \sqrt{(x_3 - x_k)^2 + (y_3 - y_k)^2} \end{bmatrix}$

Filter

$$\hat{X}_{k+1|k} = \hat{X}_{k|k} \triangleq \hat{X}_k$$

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_k \left(Y_{k+1} - g(\hat{X}_{k+1|k}) \right)$$

$$P_{k+1|k} = P_{k|k} \qquad (Q = 0)$$

$$K_k = P_{k+1|k} C^T_{k+1} \left(C_{k+1} P_{k+1|k} C^T_{k+1} + R_{k+1} \right)^{-1}$$

$$P_{k+1|k+1} = P_{k+1|k} - K_k C_{k+1} P_{k+1|k}$$

where $C_{k+1} = \dfrac{\partial g(k+1)}{\partial X_{k+1}}$

row $C_i = \dfrac{1}{g(i)_{k+1}} \left[(\hat{x}_{k+1} - x_i)(\hat{y}_{k+1} - y_i) \right]$