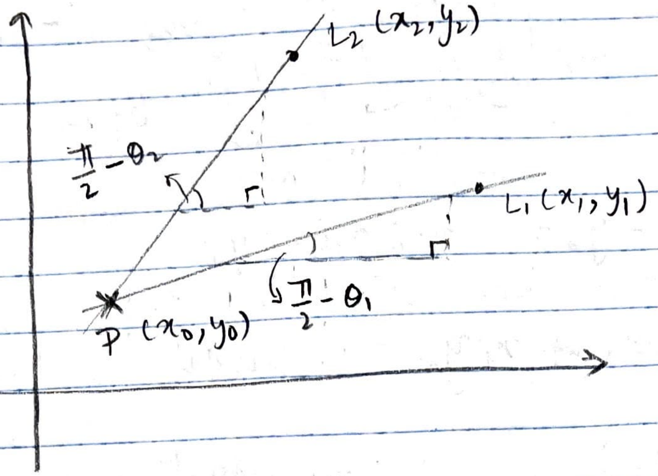
AE 567

Homework 3



Since, $P(x_0, y_0)$ and $L_i (x_i, y_i)$ lie on the same ray. $(y = m_i x + b_i)$ — ⓐ

The co-ordinates satisfy the equation of line

Putting $L_i(x_i, y_i)$ in ray equation

$$y_i = m_i x_i + b_i \quad -(1)$$

Putting $P(x_0, y_0)$ in ray equation

$$y_0 = m_i x_0 + b_i \quad -(2)$$

Subtracting (1) from (2)

$$y_0 - y_i = m_i (x_0 - x_i)$$

$$\Rightarrow \boxed{m_i = \frac{y_0 - y_i}{x_0 - x_i} = \tan\left(\frac{\pi}{2} - \theta_i\right)} \quad -(3)$$

$\because$ slope is $\tan\theta$

Substituting (3) in (1)

$$y_i = \left(\frac{y_0 - y_i}{x_0 - x_i}\right) x_i + b_i$$

$$b_i = \frac{y_i x_0 - \cancel{y_i x_i} + y_0 x_i + \cancel{x_i y_i}}{x_0 - x_i}$$

$$\Rightarrow \boxed{b_i = \frac{y_i x_0 - y_0 x_i}{x_0 - x_i}} \quad - (4)$$

Further, using (3)

$$m_i x_0 - m_i x_i = y_0 - y_i$$

$$m_i x_0 - y_0 = m_i x_i - y_i$$

$$\Rightarrow \boxed{\begin{bmatrix} m_i & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} m_i x_i - y_i \end{bmatrix}} \quad - (5)$$

using (4)

$$(b_i - y_i) x_0 + x_i y_0 = b_i x_i$$

$$\boxed{\begin{bmatrix} (b_i - y_i) & x_i \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} b_i x_i \end{bmatrix}} \quad - (6)$$

Combining (5) and (6)

we get

$$\begin{bmatrix} m_i & -1 \\ b_i - y_i & x_i \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} m_i x_i - y_i \\ b_i x_i \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} m_i & -1 \\ b_i - y_i & x_i \end{bmatrix}^{-1} \begin{bmatrix} m_i x_i - y_i \\ b_i x_i \end{bmatrix}} \quad - (7)$$

Hence proved!

b) Consider the slope the ray be

$$T_i = \tan(\pi/2 - \theta_i) = m_i$$

So, $L_1(x_1, y_1)$ and $P(x_0, y_0)$ both satisfy the equation of ray (a)

$$y_0 = T_1 x_0 + b_1 \quad - (8)$$

$$y_1 = T_1 x_1 + b_1 \quad - (9)$$

from (8) and (9)

$$T_1 x_0 - y_0 = T_1 x_1 - y_1$$

Or, $$\boxed{\begin{bmatrix} T_1 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} T_1 x_1 - y_1 \end{bmatrix}} \quad - (10)$$

Since $L_2(x_2, y_2)$ and $P(x_0, y_0)$ lie on the same ray, we get.

$$y_0 = T_2 x_0 + b_2 \quad -(11)$$

$$y_2 = T_2 x_2 + b_2 \quad -(12)$$

From (11) and (12)

$$T_2 x_0 - y_0 = T_2 x_2 - y_2$$

$$\boxed{\begin{bmatrix} T_2 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} T_2 x_2 - y_2 \end{bmatrix}} \quad -(13)$$

Combining (10) & (13)

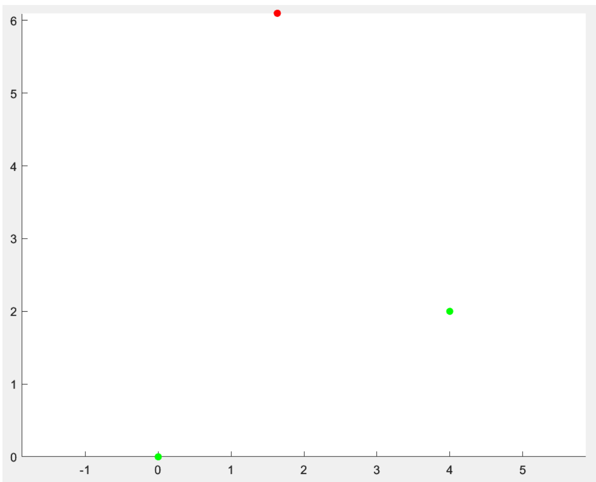
$$\boxed{\begin{bmatrix} T_1 & -1 \\ T_2 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} T_1 x_1 - y_1 \\ T_2 x_2 - y_2 \end{bmatrix}}$$

Hence proved!

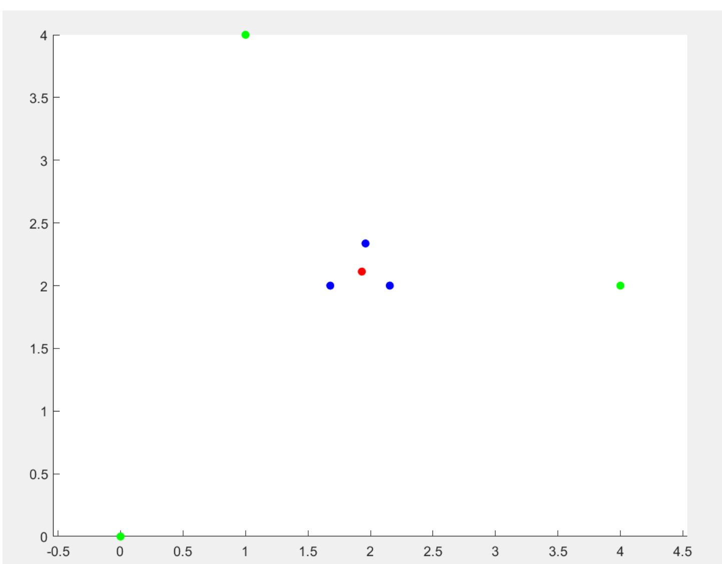
Plots:

Question 1:

Part c: Solution : [`1.6358;6.0980]`



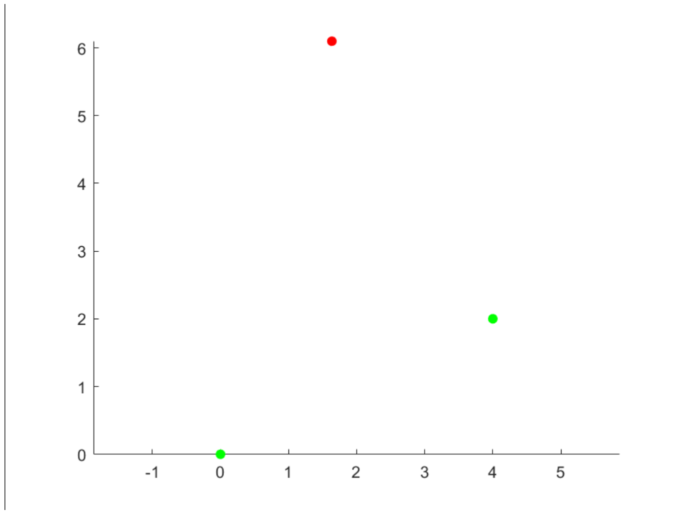
Part d

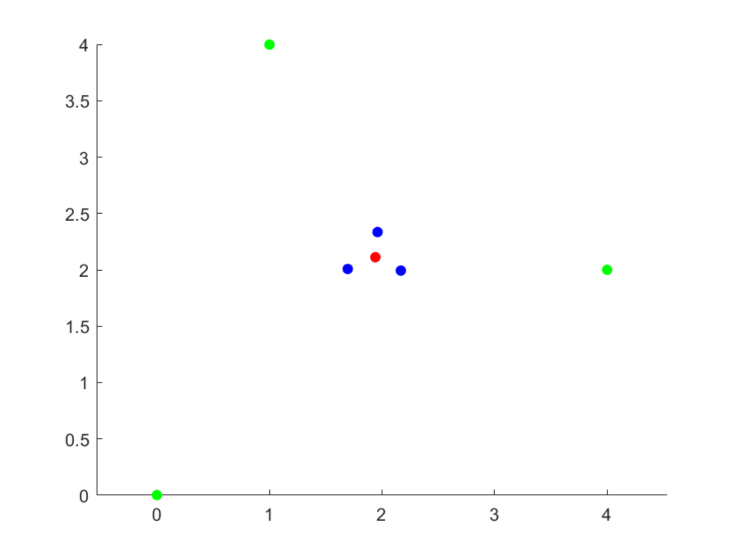


Optimal solution: Mean [`1.9410 ;2.1121]`

```
Sol1 : [1.6782,   2.0000]
Sol2:  [2.1547,   2.0000]
Sol3:  [1.9605,   2.3364]
```

Problem 2:
Optimal point : [`1.6358;6.0980]`

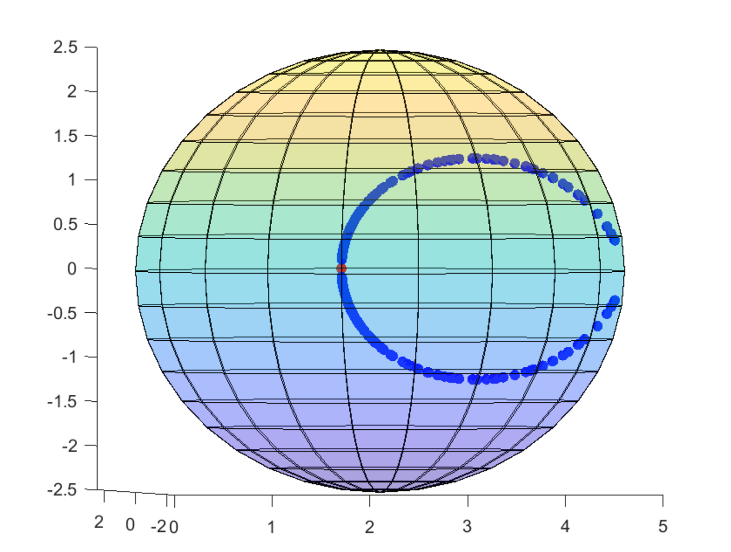




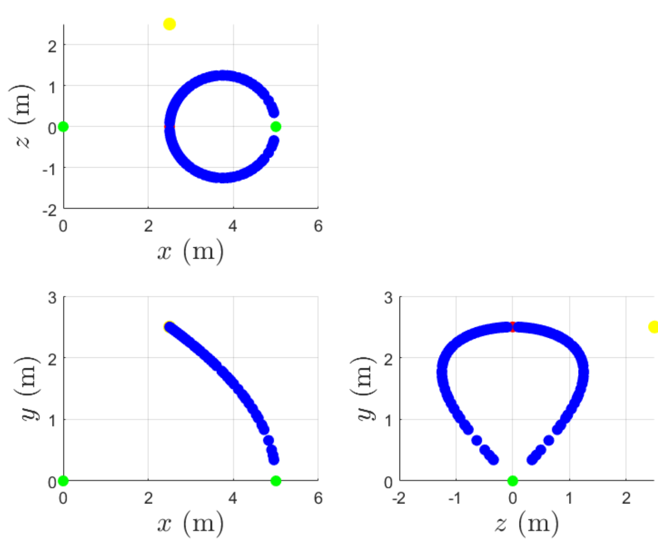
Optimal solution: Mean [`1.9410 ;2.1121]`

```
Sol1 : [1.6953,   2.0076]
Sol2:  [2.1673,   1.9932]
Sol3:  [1.9606,   2.3354]
```

Problem 3:

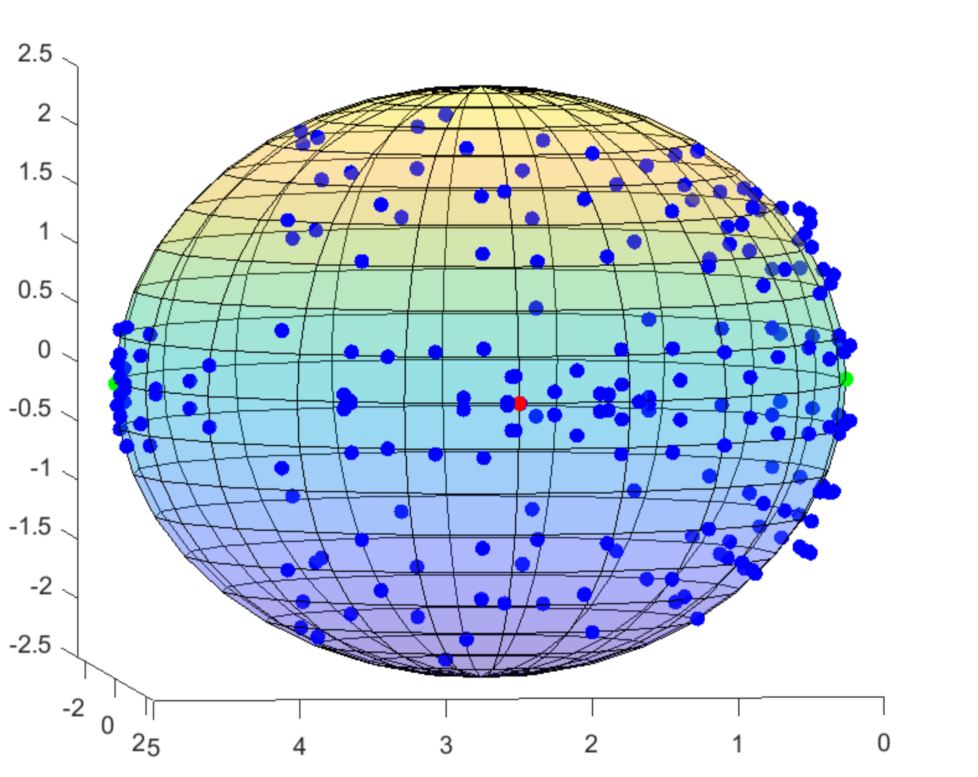


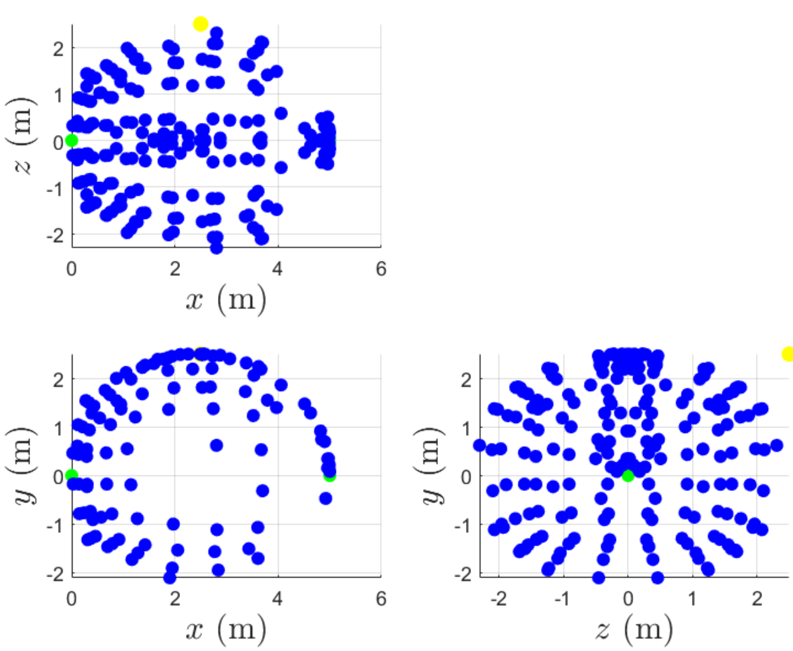
The Solution was cone from the lecture, when it meets the sphere so the visualization of it looks like a circle/ellipse.



Solution : [2.624, 2.436, -0.5443]

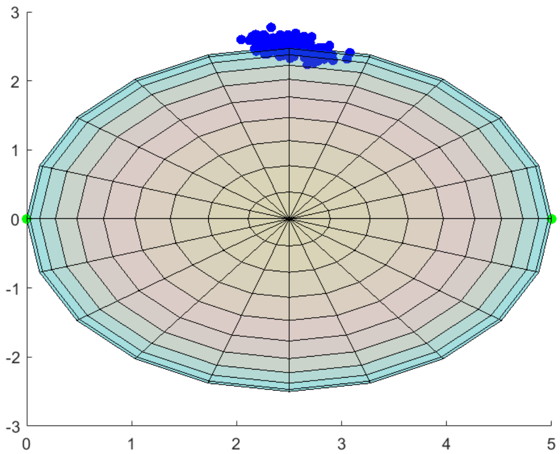
Problem 4:

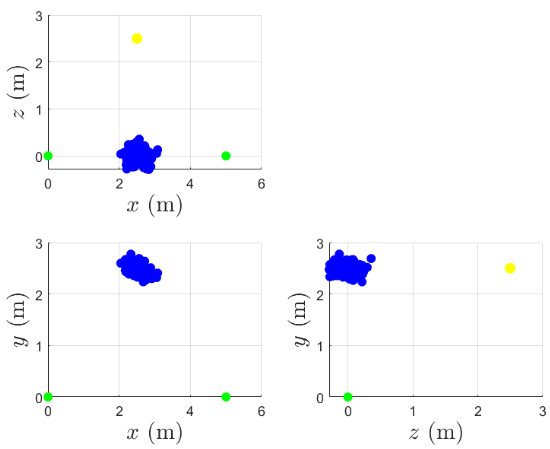


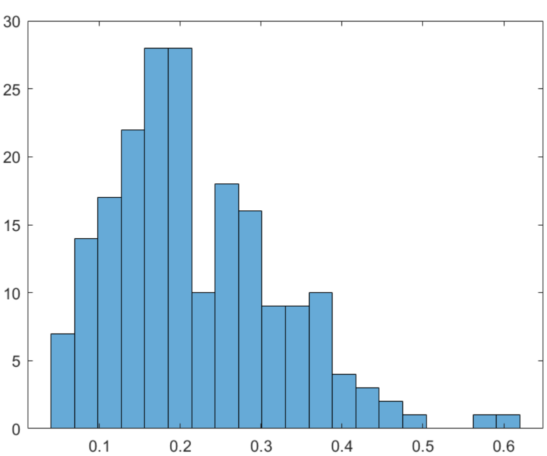


Optimal soln : [2.5,2.5,0]

Problem 5:







Mean of the distribution: 0.2185
Variance of the distribution: 0.0108

Codes
Question 1:

```
p1 = [0;0];
p2 = [4;2];
p3 = [1;4];
%% part c
theta1 = -165;
theta2 = 150;
T1 = tan(deg2rad(90-theta1));
T2 = tan(deg2rad(90-theta2));
P_sol = pos_fix(p1,p2,T1,T2);
fig = figure();
scatter(p1(1),p1(2),'g','filled', 'o');
hold on;
scatter(p2(1),p2(2),'g','filled', 'o');
scatter(P_sol(1),P_sol(2),'r','filled', 'o');
axis equal
hold off
%% part d
theta1 = -140;
theta2 = 90;
theta3 = -30;
T1 = tan(deg2rad(90-theta1));
T2 = tan(deg2rad(90-theta2));
T3 = tan(deg2rad(90-theta3));
p_sol1 = pos_fix(p1,p2,T1,T2);
p_sol2 = pos_fix(p2,p3,T2,T3);
p_sol3 = pos_fix(p3,p1,T3,T1);
p_mean = (p_sol3 + p_sol2 + p_sol1)/3;
fig = figure();
scatter(p1(1),p1(2),'g','filled', 'o');
hold on;
scatter(p2(1),p2(2),'g','filled', 'o');
scatter(p3(1),p3(2),'g','filled', 'o');
scatter(p_sol1(1),p_sol1(2),'b','filled', 'o');
scatter(p_sol2(1),p_sol2(2),'b','filled', 'o');
scatter(p_sol3(1),p_sol3(2),'b','filled', 'o');
scatter(p_mean(1),p_mean(2),'r','filled', 'o');
axis equal
hold off


function p = pos_fix(p1,p2, T1, T2)
A = [T1, -1; T2, -1];
B = [(T1*p1(1))-p1(2);(T2*p2(1))-p2(2)];
p = inv(A)*B;
end
```

Question 2

```
p1 = [0;0];
p2 = [4;2];
p3 = [1;4];
%% part a
theta1 = -165;
theta2 = 150;
P_sol = get_best(p1, p2, theta1, theta2);
fig = figure();
scatter(p1(1),p1(2),'g','filled', 'o');
hold on;
scatter(p2(1),p2(2),'g','filled', 'o');
scatter(P_sol(1),P_sol(2),'r','filled', 'o');
axis equal
hold off
% %% part b
%
theta1 = -140;
theta2 = 90;
theta3 = -30;
p_sol1 = get_best(p1,p2,theta1,theta2);
p_sol2 = get_best(p2,p3,theta2,theta3);
p_sol3 = get_best(p3,p1,theta3,theta1);
p_mean = (p_sol3 + p_sol2 + p_sol1)/3;
fig = figure();
scatter(p1(1),p1(2),'g','filled', 'o');
hold on;
scatter(p2(1),p2(2),'g','filled', 'o');
scatter(p3(1),p3(2),'g','filled', 'o');
scatter(p_sol1(1),p_sol1(2),'b','filled', 'o');
scatter(p_sol2(1),p_sol2(2),'b','filled', 'o');
scatter(p_sol3(1),p_sol3(2),'b','filled', 'o');
scatter(p_mean(1),p_mean(2),'r','filled', 'o');
axis equal
hold off

function cost = parta_cost(p, p1, p2, theta1, theta2)
psi = theta2;
psi1 = theta1;
% theta = 45;
check1 = psi/abs(psi);
check2 = psi1/abs(psi1);
if(check2==check1)
   theta = abs(theta2-theta1);
else
   theta = 360 - (abs(theta1)+abs(theta2));
end
% theta
```

```
pn = [0;1];
l1 = sqrt((p-p1)'*(p-p1));
l2 = sqrt((p-p2)'*(p-p2));
l = sqrt((p1-p2)'*(p1-p2));
part1 = (p-p1)'*(p2-p1);
part2 = l1*l2*cos(deg2rad(theta));
part3 = l1*l1;
E2 = part3 - part2 - part1;
part4 = (p2-p)'*pn;
part5 = l2*cos(deg2rad(psi));
E3 = part5 - part4;
part6 = (p1-p)'*pn;
part7 = l1*cos(deg2rad(psi1));
E1 = part6 - part7;
% cost = E1^2 + E2^2 + E3^2 ;
flag =0;
if(l1<=0.01 || l2<=0.01)
   flag = 1;
end
cost = 10*flag+ E2^2 + E3^2 ;
end

function optimal_sol = get_best(p1, p2, theta1, theta2)
optimal = Inf;
optimal_sol = [0;0];
for i = -2:2:10
   for j = -2:2:10
       p = [i;j];
       cost = @(p) parta_cost(p, p1, p2, theta1, theta2);
       options = optimoptions(@fminunc,'OptimalityTolerance',1e-2);
       [sol1,~] = fminunc(cost,p,options);
       cost_final = parta_cost(sol1, p1, p2, theta1, theta2);
       if(cost_final<optimal)
           optimal = cost_final;
           optimal_sol = sol1;
       end
   end
end
end
```

Problem 3:

```
p1 = [0;0;0];
p2 = [5;0;0];
theta = 90;
psi = 135;
[my_sol,good_sol] = get_best(p1, p2, theta, psi);
fig = figure();
[Xsp,Ysp,Zsp] = sphere;
hold on;
surf(2.5.*Xsp+ 2.5,2.5.*Ysp,2.5.*Zsp,'FaceAlpha',0.25);
for i = 1:length(my_sol)
   sol = cell2mat(my_sol(i));
   scatter3(sol(1),sol(2),sol(3),'b','filled', 'o');
end
scatter3(2.5,2.5,0,'r','filled', 'o');
hold off
center = [2.5;2.5;2.5];
figure(2)
subplot(2,2,1)
scatter(center(1), center(3),50,'yellow','fill')
hold on
scatter(2.5,0,'r','filled', 'o');
scatter(p1(1),p1(3),'g','filled', 'o');
scatter(p2(1),p2(3),'g','filled', 'o');
for i = 1:length(my_sol)
   sol = cell2mat(my_sol(i));
   scatter(sol(1),sol(3),'b','fill');
end
hold off
grid on
ylabel('$z$ (m)','interpreter','latex','fontsize',15)
xlabel('$x$ (m)','interpreter','latex','fontsize',15)
subplot(2,2,3)
scatter(center(1), center(2),50,'yellow','fill')
hold on
scatter(2.5,2.5,'red','filled', 'o');
scatter(p1(1),p1(2),'g','filled', 'o');
scatter(p2(1),p2(2),'g','filled', 'o');
for i = 1:length(my_sol)
   sol = cell2mat(my_sol(i));
   scatter(sol(1),sol(2),'b','fill');
end
hold off
grid on
ylabel('$y$ (m)','interpreter','latex','fontsize',15)
xlabel('$x$ (m)','interpreter','latex','fontsize',15)
subplot(2,2,4)
scatter(center(3), center(2),50,'yellow','fill')
```

```matlab
hold on
scatter(0,2.5,'red','filled', 'o');
scatter(p1(3),p1(2),'g','filled', 'o');
scatter(p2(3),p2(2),'g','filled', 'o');
for i = 1:length(my_sol)
   sol = cell2mat(my_sol(i));
   scatter(sol(3),sol(2),'b','fill');
end
hold off
ylabel('$y$ (m)','interpreter','latex','fontsize',15)
xlabel('$z$ (m)','interpreter','latex','fontsize',15)
grid on

function [optimal_sol,good_sol] = get_best(p1, p2, theta, psi)
optimal = Inf;
optimal_sol = [];
good_sol = [];
for i = -5:2:5
   for j = -5:2:5
       for k=-5:2:5
           p = [i;j;k];
           cost = @(p) parta_cost(p, p1, p2, theta, psi);
           options = optimoptions(@fminunc,'OptimalityTolerance',1e-12);
           [sol1,~] = fminunc(cost,p,options);
           optimal_sol = [optimal_sol;mat2cell(sol1,3,1)];
           cost_final = parta_cost(sol1, p1, p2, theta, psi);
           if(cost_final<optimal)
               optimal = cost_final;
               good_sol = sol1;
           end
       end
   end
end
end

function cost = parta_cost(p, p1, p2, theta, psi)
r_hat = [0;1;0];
l1 = sqrt((p-p1)'*(p-p1));
l2 = sqrt((p-p2)'*(p-p2));
l = sqrt((p1-p2)'*(p1-p2));
part1 = (p-p1)'*(p2-p1);
part2 = l1*l2*cos(deg2rad(theta));
part3 = l1*l1;
E2 = part3 - part2 - part1;
part4 = (p2-p)'*r_hat;
part5 = l2*cos(deg2rad(psi));
E3 = part5 - part4;
flag =0;
if(l1<=0.01 || l2<=0.01)
```

```
   flag = 1;
end
cost = 10*flag+ E2^2 + E3^2 ;
end
```

Question 4

```
p1 = [0;0;0];
p2 = [5;0;0];
theta = 90;
psi1 = 135;
psi2 = 90;
[my_sol,good_sol] = get_best(p1, p2, theta, psi1, psi2);
fig = figure();
[Xsp,Ysp,Zsp] = sphere;
hold on;
surf(2.5.*Xsp+ 2.5,2.5.*Ysp,2.5.*Zsp,'FaceAlpha',0.25);
scatter3(p1(1),p1(2),p1(3),'g','filled', 'o');
scatter3(p2(1),p2(2),p2(3),'g','filled', 'o');
for i = 1:length(my_sol)
   sol = cell2mat(my_sol(i));
   scatter3(sol(1),sol(2),sol(3),'b','filled', 'o');
end
scatter3(2.5,2.5,0,'r','filled', 'o');
hold off
center = [2.5;2.5;2.5];
figure()
subplot(2,2,1)
scatter(center(1), center(3),50,'yellow','fill')
hold on
scatter(2.5,0,'r','filled', 'o');
scatter(p1(1),p1(3),'g','filled', 'o');
scatter(p2(1),p2(3),'g','filled', 'o');
for i = 1:length(my_sol)
   sol = cell2mat(my_sol(i));
   scatter(sol(1),sol(3),'b','fill');
end
hold off
grid on
ylabel('$z$ (m)','interpreter','latex','fontsize',15)
xlabel('$x$ (m)','interpreter','latex','fontsize',15)
subplot(2,2,3)
scatter(center(1), center(2),50,'yellow','fill')
hold on
scatter(2.5,2.5,'red','filled', 'o');
scatter(p1(1),p1(2),'g','filled', 'o');
scatter(p2(1),p2(2),'g','filled', 'o');
for i = 1:length(my_sol)
```

```
    sol = cell2mat(my_sol(i));
    scatter(sol(1),sol(2),'b','fill');
end
hold off
grid on
ylabel('$y$ (m)','interpreter','latex','fontsize',15)
xlabel('$x$ (m)','interpreter','latex','fontsize',15)
subplot(2,2,4)
scatter(center(3), center(2),50,'yellow','fill')
hold on
scatter(0,2.5,'red','filled', 'o');
scatter(p1(3),p1(2),'g','filled', 'o');
scatter(p2(3),p2(2),'g','filled', 'o');
for i = 1:length(my_sol)
    sol = cell2mat(my_sol(i));
    scatter(sol(3),sol(2),'b','fill');
end
hold off
ylabel('$y$ (m)','interpreter','latex','fontsize',15)
xlabel('$z$ (m)','interpreter','latex','fontsize',15)
grid on

function [optimal_sol,good_sol] = get_best(p1, p2, theta, psi1, psi2)
optimal = Inf;
optimal_sol = [];
good_sol = [];
for i = -5:2:5
    for j = -5:2:5
        for k=-5:2:5
            p = [i;j;k];
            cost = @(p) parta_cost(p, p1, p2, theta, psi1, psi2);
            options = optimoptions(@fminunc,'OptimalityTolerance',1e-2);
            [sol1,~] = fminunc(cost,p,options);
            optimal_sol = [optimal_sol;mat2cell(sol1,3,1)];
            cost_final = parta_cost(sol1, p1, p2, theta, psi1, psi2);
            if(cost_final<optimal)
                optimal = cost_final;
                good_sol = sol1;
            end
        end
    end
end
end

function cost = parta_cost(p, p1, p2, theta, psi1, psi2)
r_hat1 = [0;1;0];
r_hat2 = [0;0;1];
l1 = sqrt((p-p1)'*(p-p1));
l2 = sqrt((p-p2)'*(p-p2));
```

```
l = sqrt((p1-p2)'*(p1-p2));
part1 = (p-p1)'*(p2-p1);
part2 = l1*l2*cos(deg2rad(theta));
part3 = l1*l1;
E2 = part3 - part2 - part1;
part4 = (p2-p)'*r_hat1;
part5 = l2*cos(deg2rad(psi1));
E3 = part5 - part4;
part6 = (p2-p)'*r_hat2;
part7 = l2*cos(deg2rad(psi2));
E1 = part6 - part7;
flag =0;
if(l1<=0.01 || l2<=0.01)
   flag = 1;
end
cost = 100*flag+ E2^2 + E3^2 + E1^2 ;
end
```

Question 5:

```
p1 = [0;0;0];
p2 = [5;0;0];
apt_sol = [2.5;2.5;0];
good_sols = [];
err = [];
for i = 1:200
   noise = 2*randn(1);
   theta = 90+noise;
   noise = 2*randn(1);
   psi1 = 135+noise;
   noise = 2*randn(1);
   psi2 = 90+noise;

   [~,good_sol] = get_best(p1, p2, theta, psi1, psi2);
   err = [err;norm(good_sol-apt_sol)];
   good_sols = [good_sols;mat2cell(good_sol,3,1)];
end
fig = figure();
[Xsp,Ysp,Zsp] = sphere;
hold on;
surf(2.5.*Xsp+ 2.5,2.5.*Ysp,2.5.*Zsp,'FaceAlpha',0.25);
scatter3(p1(1),p1(2),p1(3),'g','filled', 'o');
scatter3(p2(1),p2(2),p2(3),'g','filled', 'o');
for i = 1:length(good_sols)
   sol = cell2mat(good_sols(i));
   scatter3(sol(1),sol(2),sol(3),'b','filled', 'o');
end
scatter3(2.5,2.5,0,'r','filled', 'o');
```

```matlab
hold off
nbins = 20;
fig = figure();
h = histogram(err,nbins);
mean_data = mean(err);
variance = var(err);

function [optimal_sol,good_sol] = get_best(p1, p2, theta, psi1, psi2)
optimal = Inf;
optimal_sol = [];
good_sol = [];
for i = [-1,2,5]
   for j = [-1,2,5]
       for k= [-1,2,5]
           p = [i;j;k];
           cost = @(p) parta_cost(p, p1, p2, theta, psi1, psi2);
           options = optimoptions(@fminunc,'OptimalityTolerance',1e-12);
           [sol1,~] = fminunc(cost,p,options);
           cost_final = parta_cost(sol1, p1, p2, theta, psi1, psi2);
           if(cost_final<optimal)
               optimal = cost_final;
               good_sol = sol1;
           end
       end
   end
end
end

function cost = parta_cost(p, p1, p2, theta, psi1, psi2)
r_hat1 = [0;1;0];
r_hat2 = [0;0;1];
l1 = sqrt((p-p1)'*(p-p1));
l2 = sqrt((p-p2)'*(p-p2));
l = sqrt((p1-p2)'*(p1-p2));
part1 = (p-p1)'*(p2-p1);
part2 = l1*l2*cos(deg2rad(theta));
part3 = l1*l1;
E2 = part3 - part2 - part1;
part4 = (p2-p)'*r_hat1;
part5 = l2*cos(deg2rad(psi1));
E3 = part5 - part4;
part6 = (p2-p)'*r_hat2;
part7 = l2*cos(deg2rad(psi2));
E1 = part6 - part7;
flag =0;
if(l1<=0.01 || l2<=0.01)
   flag = 1;
end
cost = 10*flag+ E2^2 + E3^2 + E1^2 ; end
```