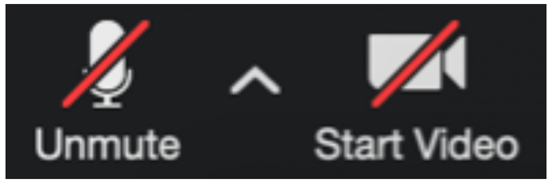
AEROSP 584 - Navigation and Guidance: From Perception to Control



Lectures start at 10:30am EST

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Lecture 14b

Slides by Ankit Goel

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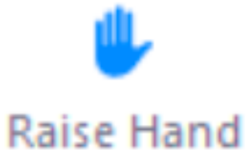
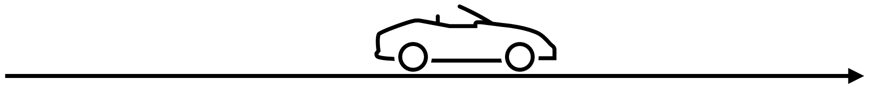
To ask questions:



or

Measurements Vs Model Vs Filter



$\dot{x} = f(x, u)$

- $\dot{x} = f(x, u)$ describes the evolution of the state x

Measurements

- If we can measure $x(t)$, no need to rely on the model
- Accuracy depends on the sensor

Model

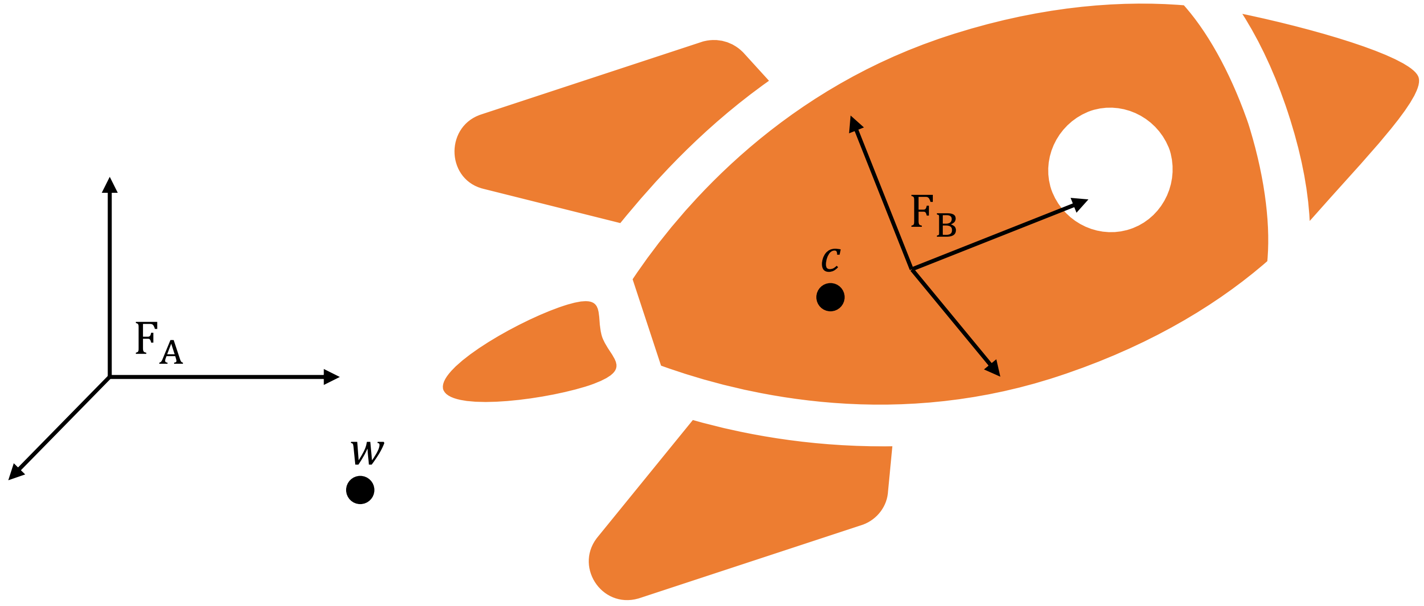
- If we know $x(0)$ and $u(t)$, we can integrate the model $\dot{x} = f(x, u)$ to get $x(t)$
- Accuracy depends on $x(0), u(t)$, and $f(x, u)$

Filter

- If we measure a part of $x(t)$ and the input $u(t)$, we can use the model in the filtering framework that provides an estimate $\hat{x}(t)$ of the state $x(t)$
- Accuracy depends on $y(t)$, and $f(x, u)$

Navigation

- Navigation – Process of finding the state of the vehicle
  - State includes position, velocity, attitude, angular velocity



- $\vec{r}_{c/w}\big|_A$
- $\vec{v}_{c/w/A}\big|_A$
- (Φ, Θ, Ψ)
- $\omega_{B/A}$

Measurement Based Navigation

- Measure using sensors
  - Position $\vec{r}_{c/w}\big|_{\mathrm{A}}$ – GPS
  - Velocity $\vec{v}_{c/w/A}\big|_{\mathrm{A}}$ – Differentiate position signal
  - Attitude (Φ, Θ, Ψ) – Track at least 4 points on the vehicle (MOCAP)
  - Angular velocity – Use the equation $\omega_{\mathrm{B/A}} = S(\Phi, \Theta)[\dot{\Phi} \quad \dot{\Theta} \quad \dot{\Psi}]^{\mathrm{T}}$

- What are the problems?
  - GPS/Motion Capture technology too expensive and not very reliable
  - Don’t want to differentiate noisy signal

Inertial Navigation

- Use inertial sensors

$$a(t) = \vec{a}_{c/w/\mathrm{A}}(t)\Big|_{\mathrm{B}} + \vec{g}\,\Big|_{\mathrm{B}}, \qquad \omega(t) = \vec{\omega}_{\mathrm{B/A}}(t)\Big|_{\mathrm{B}}$$

  - Acceleration and the angular velocity are easier to measure
- Use Kinematic model to obtain the state
  - Double integrator $\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\vec{r}_{c/w}\big|_A\right) = \vec{a}_{c/w/\mathrm{A}}\big|_A$
  - Poisson's equation $\dot{\mathcal{O}}_{\mathrm{B/A}} = -\vec{\omega}_{\mathrm{B/A}}\big|_{\mathrm{B}}^{\times}\mathcal{O}_{\mathrm{B/A}} = -\omega(t)^{\times}\mathcal{O}_{\mathrm{B/A}}$
- What are the problems?
  - Equations are coupled

$$\vec{a}_{c/w/\mathrm{A}}\Big|_A = \mathcal{O}_{\mathrm{A/B}}\,\vec{a}_{c/w/\mathrm{A}}\Big|_{\mathrm{B}} = \mathcal{O}_{\mathrm{A/B}}\left(a(t) - \vec{g}\,\Big|_{\mathrm{B}}\right) = \mathcal{O}_{\mathrm{A/B}}a(t) - \vec{g}\,\Big|_{\mathrm{A}}$$

  - Both systems are unstable $\Rightarrow$ Initial condition effect does not disappear
  - Measurements $a(t)$ and $\omega(t)$ are noisy

Inertial Navigation

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Double Integrator

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\vec{r}_{c/w}\Big|_A\right) = \vec{a}_{c/w/\mathrm{A}}\Big|_A$$

- $\vec{r}_{c/w}\Big|_A = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = r$

- $\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \ddot{r} = \mathcal{O}_{\mathrm{A/B}}a(t) - \vec{g}\,|_{\mathrm{A}}$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} r \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}\left(\mathcal{O}_{\mathrm{A/B}}a(t) - \vec{g}\,\Big|_{\mathrm{A}}\right)$$

Poisson's Equation

$$\dot{\mathcal{O}}_{\mathrm{B/A}} = -\omega(t)^{\times}\mathcal{O}_{\mathrm{B/A}}$$

- $\mathcal{O}_{\mathrm{B/A}} = Y$

$$\dot{Y} = \Omega(t)Y$$

$$\dot{X} = AX + B\left(Y^{\mathrm{T}}a(t) - g_{\mathrm{A}}\right)$$

$$\dot{Y} = \Omega(t)Y$$

Inertial Navigation

- $\dot{X} = AX + B\left(Y^{\mathrm{T}} a(t) - g_{\mathrm{A}}\right)$
- $\dot{Y} = \Omega(t) Y$
  - Typically, no analytical solution exists
  - Convert to DT systems and propagate
- $X_{k+1} = A_{\mathrm{d}} X_k + B_{\mathrm{d}}\left(Y_k^{\mathrm{T}} a_k - g_{\mathrm{A}}\right)$
- $Y_{k+1} = \Omega_{\mathrm{d},k} Y_k$

$$A_{\mathrm{d}} = e^{AT}, \qquad B_{\mathrm{d}} \triangleq \left(\int_0^T e^{A(\mathrm{T}-\tau)} \mathrm{d}\tau\right) B, \qquad \Omega_{\mathrm{d},k} = e^{-\|\omega_k\| T \hat{n}_k^{\times}}, \qquad \hat{n}_k = \frac{\omega_k}{\|\omega_k\|}$$

Inertial Navigation

- $A_\mathrm{d} = e^{AT}$,
  - $A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$
  - $A_\mathrm{d} = e^{AT} = \sum_{i=0}^{\infty} \frac{(AT)^i}{i!} = I + AT = \begin{bmatrix} I & TI \\ 0 & I \end{bmatrix}$

- $B_\mathrm{d} \triangleq \left(\int_0^T e^{A(T-\tau)} \mathrm{d}\tau \right) B$
  - $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$
  - $B_\mathrm{d} \triangleq \left(\int_0^T e^{A(T-\tau)} \mathrm{d}\tau \right) B = \int_0^T \begin{bmatrix} I & (T-\tau)I \\ 0 & I \end{bmatrix} \mathrm{d}\tau\, B = \begin{bmatrix} TI & \frac{T^2}{2} I \\ 0 & TI \end{bmatrix} \begin{bmatrix} 0 \\ I \end{bmatrix} = \begin{bmatrix} \frac{T^2}{2} I \\ TI \end{bmatrix}$

Inertial Navigation – Simple Example

$$\mathrm{F_A} \xrightarrow[2]{\Theta} \mathrm{F_B}$$

- $\omega(t) = \vec{\omega}_{\mathrm{B/A}}\big|_\mathrm{B} = \begin{bmatrix} 0 \\ \omega_y \\ 0 \end{bmatrix}$
- $\mathcal{O}_{\mathrm{B/A}} = \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{bmatrix}$
- $\Theta(t) = \int_0^t \omega_y(t)\mathrm{d}t + \Theta(0)$
- $\omega_y(t) = \alpha \Rightarrow \Theta(t) = \alpha t + \theta(0)$
- $\vec{a}_{c/w/\mathrm{A}}\big|_\mathrm{B} = \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix} \Rightarrow \vec{a}_{c/w/\mathrm{A}}\big|_\mathrm{A} = \mathcal{O}_{\mathrm{A/B}}\vec{a}_{c/w/\mathrm{A}}\big|_\mathrm{B} = \begin{bmatrix} \cos\Theta \\ 0 \\ -\sin\Theta \end{bmatrix}\beta$

$$r(t) = \iint_0^t \begin{bmatrix} \cos(\alpha t + \Theta_0) \\ 0 \\ -\sin(\alpha t + \Theta_0) \end{bmatrix}\beta\mathrm{d}t = \begin{bmatrix} -\cos(\alpha t + \Theta_0) + \cos\Theta_0 \\ 0 \\ \sin(\alpha t + \Theta_0) - \sin\Theta_0 \end{bmatrix}\frac{\beta}{\alpha^2} + t\left(\dot{r}(0) - \frac{\beta}{\alpha}\begin{bmatrix} \sin\Theta_0 \\ 0 \\ \cos\Theta_0 \end{bmatrix}\right) + r(0)$$

Inertial Navigation – Simple Example

$$\mathrm{F_A} \xrightarrow[2]{\Theta} \mathrm{F_B}$$

- $\omega(t) = \vec{\omega}_{\mathrm{B/A}}\big|_{\mathrm{B}}$
- $a(t) = \left(\vec{a}_{c/w/\mathrm{A}} + \vec{g}\,\right)\big|_{\mathrm{B}}$

- $Y_{k+1} = e^{-\omega_{y,k} T e_2^{\times}} Y_k$

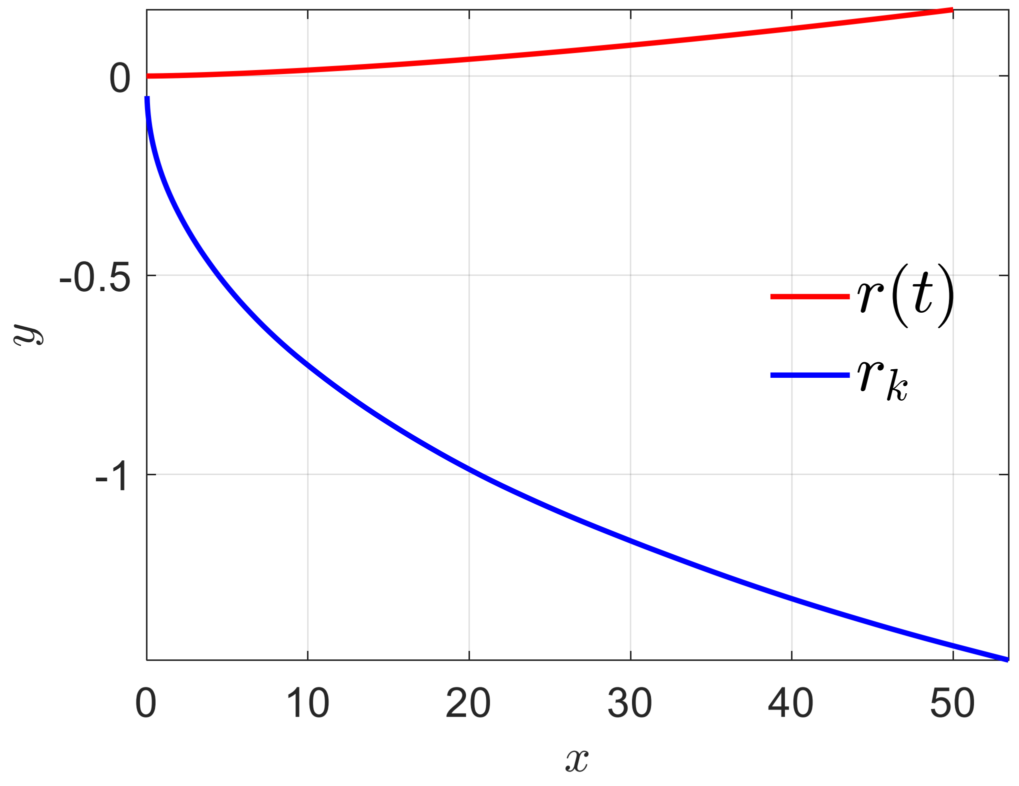
- $X_{k+1} = A_\mathrm{d} X_k + B_\mathrm{d}\left(Y_k^\mathrm{T} a_k - g_\mathrm{A}\right)$

Inertial Navigation – Simple Example

- $\alpha = -10^{-3}$ rad/sec
- $\beta = 1\ \mathrm{m}^2$/sec

- $T = 10^{-1}$ sec
- $\hat{X}_0 = 10^{-1}\mathrm{randn}(6{,}1)$
- Gyro error $\sim \mathcal{N}(0{,}10^{-5}I_3)$
- Acc error $\sim \mathcal{N}(0{,}10^{-2}I_3)$

- Errors due to
  - Discretization
  - Inaccurate initial conditions
  - Noise

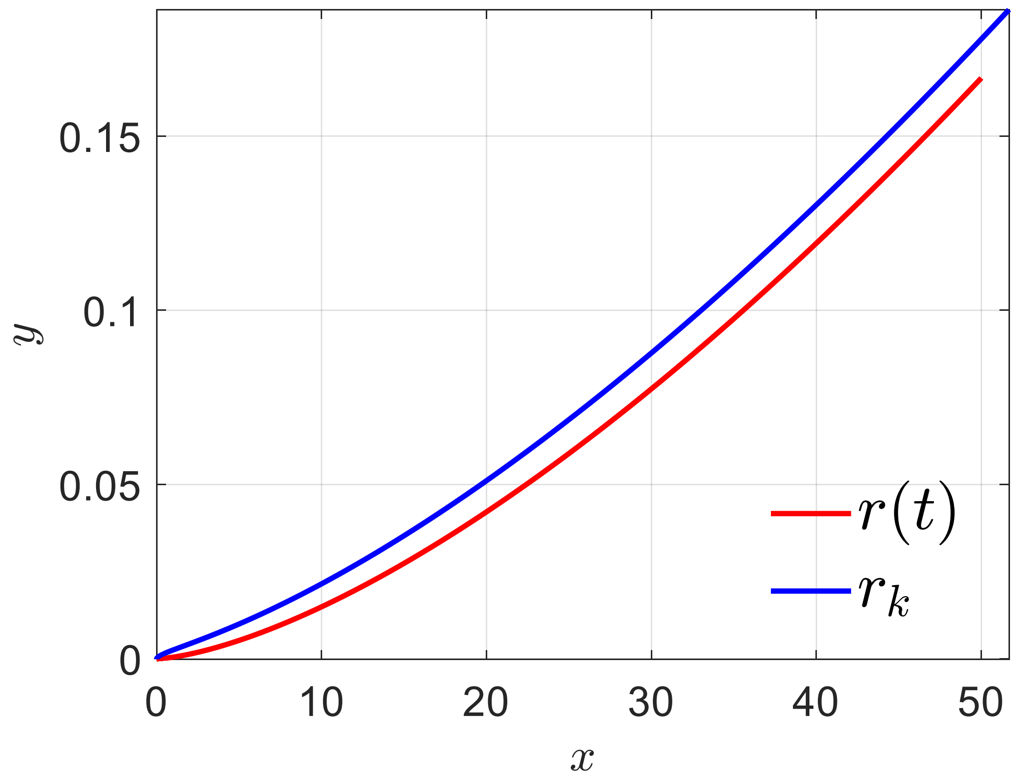


Inertial Navigation – Simple Example

- $\alpha = -10^{-3}$ rad/sec
- $\beta = 1\ \mathrm{m}^2$/sec

- $T = 10^{-3}$sec
- $\hat{X}_0 = 10^{-3}\mathrm{randn}(6,1)$
- Gyro error $\sim \mathcal{N}(0,10^{-7}I_3)$
- Acc error $\sim \mathcal{N}(0,10^{-3}I_3)$

- Errors due to
  - Discretization
  - Inaccurate initial conditions
  - Noise



Can We Use KF?

$$Y_{k+1} = \Omega_{\mathrm{d},k} Y_k$$

$$X_{k+1} = A_\mathrm{d} X_k + B_\mathrm{d}\left(Y_k^\mathrm{T} a_k - g_\mathrm{A}\right)$$

$$x_{k+1} = A_k x_k$$
$$y_k = \omega_k + D_2 w_k$$

$$x_{k+1} = A x_k + B g_\mathrm{A} + B_k u_k + B_k w_k$$
$$y_k = a_k + D_2 w_k$$

$$A_k = e^{-\|\omega_k + D_2 w_k\|T \hat{n}_k^{\times}}$$

$$\hat{x}_{k+1|k} = A_k \hat{x}_{k|k}$$
$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_k\left(y_{k+1} - \quad\right)$$

$$\hat{x}_{k+1|k} = A_k \hat{x}_{k|k} + B g_\mathrm{A} + B_k y_k$$
$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_k\left(y_{k+1} - \quad\right)$$

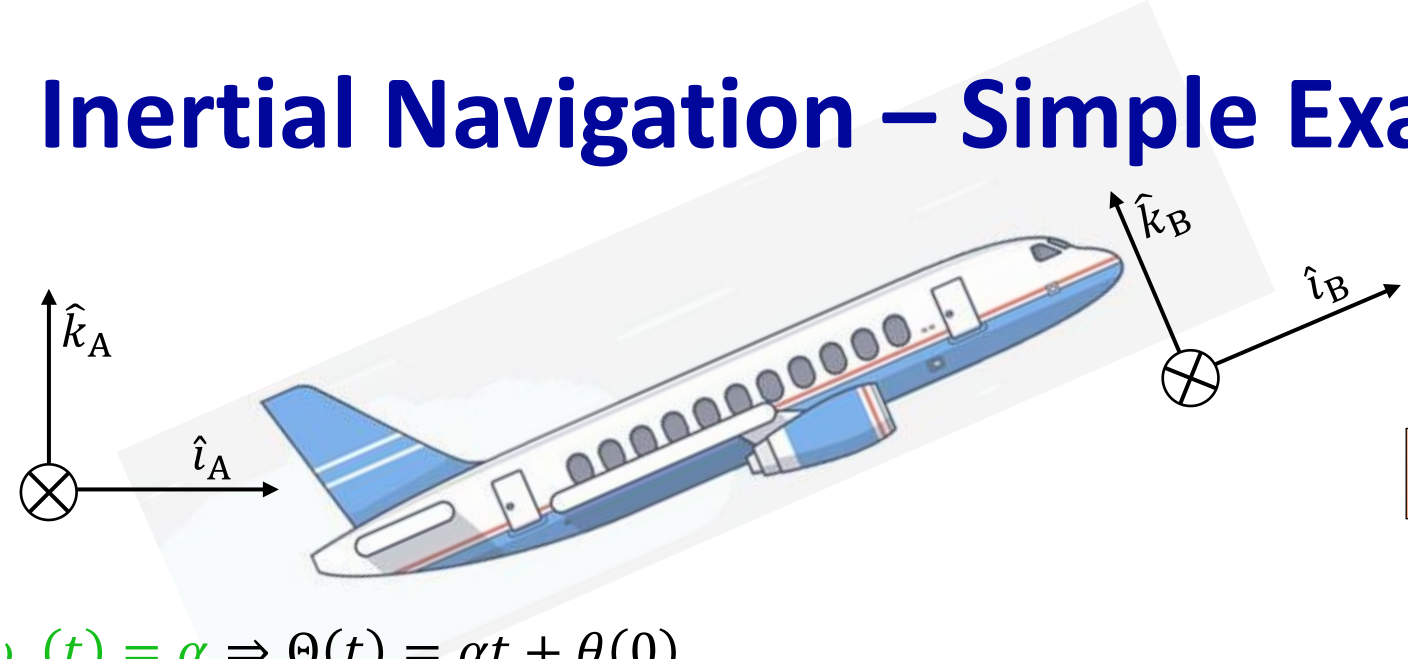
GPS-based Navigation

- GPS provides position measurements by using position fixing method
    - GPS accuracy $\sim$ 5 meters
    - Sampling period is not constant
    - $y_k = [I \quad 0]X_k + D_2 w_k$

- We use the "dynamics" to update the state in between measurement

- When GPS measurement is available, use KF to **correct** the state

Inertial Navigation – Simple Example



- $\omega_y(t) = \alpha \Rightarrow \Theta(t) = \alpha t + \theta(0)$
- $\vec{a}_{c/w/\mathrm{A}}\big|_\mathrm{B} = \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix} \Rightarrow \vec{a}_{c/w/\mathrm{A}}\big|_\mathrm{A} = \mathcal{O}_{\mathrm{A/B}}\vec{a}_{c/w/\mathrm{A}}\big|_\mathrm{B} = \begin{bmatrix} \cos\Theta \\ 0 \\ -\sin\Theta \end{bmatrix}\beta$

$$Y_{k+1} = e^{-\omega_{y,k} T e_2^\times} Y_k$$

$$X_{k+1} = A_\mathrm{d} X_k + B_\mathrm{d}\left(Y_k^\mathrm{T} a_k - g_\mathrm{A}\right)$$
$$y_k = r + D_2 w_k = [I \quad 0] X_k + D_2 w_k$$

$$\hat{x}_{k+1|k} = A_k \hat{x}_{k|k} + B g_\mathrm{A} + B_k y_k$$

GPS Measurement available?

No:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k}$$

Yes:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_k\left(y_{k+1} - C\hat{x}_{k+1|k}\right)$$

GPS-based Navigation

- $\alpha = -10^{-3}$ rad/sec
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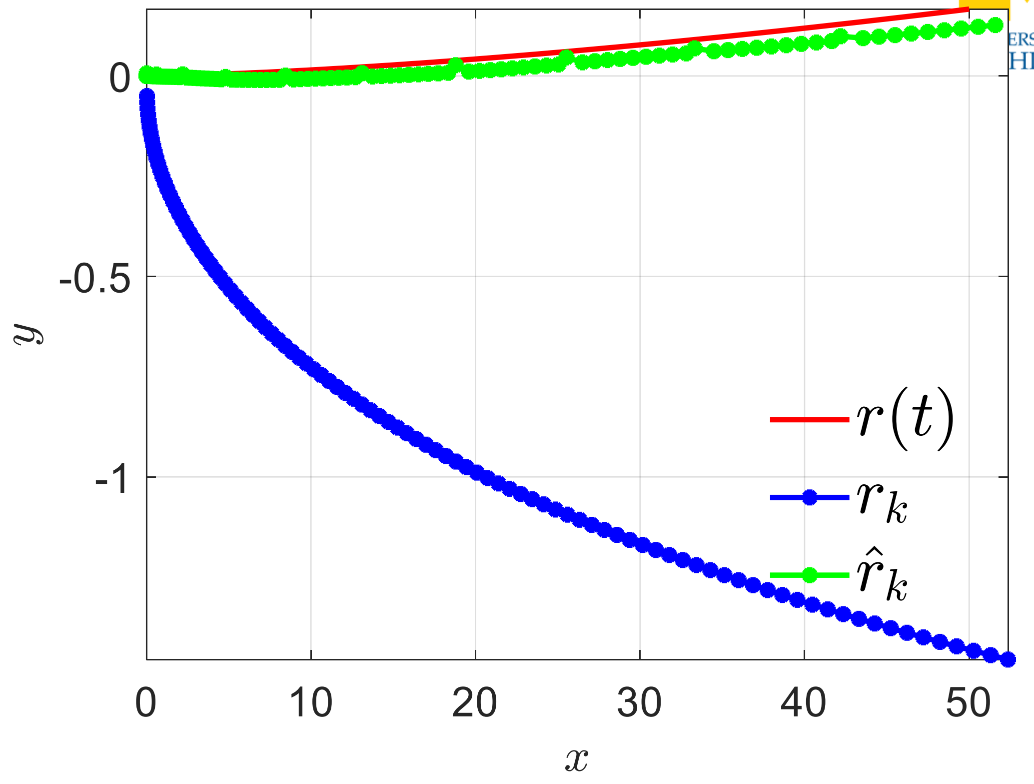

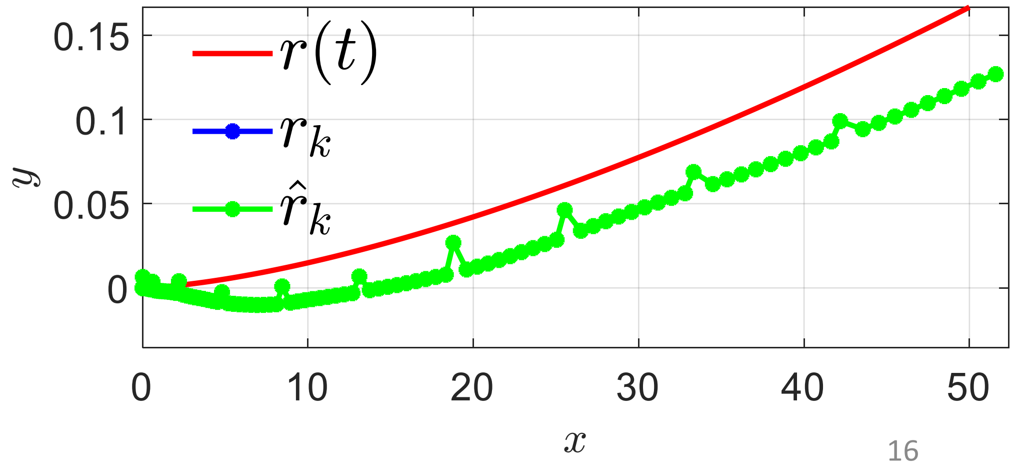

Summary

- Kalman Filter provides a mechanism to use the measurements to improve the accuracy of the state estimate
  - KF provides estimates of unmeasured state
  - KF improves the accuracy of the measured state

- Kalman Filter is NOT useful in Inertial Navigation
  - Inertial Navigation entirely depends on good sensors and available computational resources

- Kalman Filter IS applicable in GPS-based Navigation