AEROSP 584
Homework #4

Due Friday, November 18, by 11:59 PM.

Instructions:

1. Please check out and follow the homework preparation and uploading guidelines posted in the Section “Assignment Policies and Specification” of the syllabus, especially with regard to how to submit if you are working on a team (optional for Homework #4).

   Extra credit will occasionally be given for especially neat work.

2. Attach all your Matlab and Simulink code in a single zip file, along with a single PDF file with your answers to the homework problems; please include in the PDF a copy-paste of your code.

   In this homework, you will be using MATLAB’s ode45 and fminunc. Examples have been posted in the course Canvas page.

3. Please read and follow the honor code guidelines in the Section “Additional Policies” of the syllabus.

Problem 1 (30 points). Position fixing using the Kalman Filter Let L_1, L_2, L_3 be three beacons and let P be your position. Let $\mathrm{F_A}$ be a frame and w be a point such that $\vec{r}_{P/w}\big|_{\mathrm{A}} = \begin{bmatrix}0.7212 & 2.4080 & 0\end{bmatrix}^{\mathrm{T}}$ and, for all $i \in \{1,2,3\}$,

$$\vec{r}_{L_i/w}\big|_{\mathrm{A}} = \begin{bmatrix} x_i \\ y_i \\ 0 \end{bmatrix} \text{ m}, \tag{3}$$

where $x_i, y_i \in \mathbb{R}$. Furthermore, for all $i \in \{1,2,3\}$, let the 2D distance from the beacons L_i to P be r_i, such that $r_i = \sqrt{(0.7212 - x_i)^2 + (2.4080 - y_i)^2}$. Suppose that $(x_1, y_1) = (0,0)$ m, $(x_2, y_2) = (5,5)$ m, and $(x_3, y_3) = (2.5, 0)$ m, Thus, it follows that $r_1 = 2.5$ m, $r_2 = 5$ m, $r_3 = 3$ m approximately. Furthermore, suppose that, for all $i \in \{1,2,3\}$, noisy 2D distance measurements from beacon L_i are available every one second. That is, for all $k > 0$,

$$Y_k = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} + D w_k, \tag{4}$$

where $D = \mathrm{diag}(0.1, 0.1, 0.1)$ and $w_k \sim \mathcal{N}(0, I_3)$.

We will use the Kalman Filter to obtain an estimate of $\vec{r}_{P/w}\big|_{\mathrm{A}}$, which we will refer to as $\vec{r}_{\hat{P}/w}\big|_{\mathrm{A}}$. In order to do so, define, for all $k \geq 0$, $\hat{X}_k = \begin{bmatrix}\hat{x}_k & \hat{y}_k\end{bmatrix}^{\mathrm{T}}$, such that $\vec{r}_{\hat{P}/w,k}\big|_{\mathrm{A}} = \begin{bmatrix}\hat{x}_k & \hat{y}_k & 0\end{bmatrix}^{\mathrm{T}}$. For all $k \geq 0$, the dynamics model that will be used in the Kalman Filter is given by $\hat{X}_{k+1} = \hat{X}_k$. Furthermore, for all $k \geq 0$, the measurement model is given by

$$g(\hat{X}_k) = \begin{bmatrix} \sqrt{(\hat{x}_k - x_1)^2 + (\hat{y}_k - y_1)^2} \\ \sqrt{(\hat{x}_k - x_2)^2 + (\hat{y}_k - y_2)^2} \\ \sqrt{(\hat{x}_k - x_3)^2 + (\hat{y}_k - y_3)^2} \end{bmatrix}. \tag{5}$$

a) Write the Kalman filter equations to obtain an estimate of x_k.

b) Let $Q = 0$ and $R = 0.1$*eye(3), and suppose that $\hat{X}_0 = \begin{bmatrix}4 & 4\end{bmatrix}^{\mathrm{T}}$ m. For all $p_0 \in \{0.01, 0.1, 1, 10\}$ such that $P_{0|0} = p_0$*eye(2), obtain the estimates $\hat{X}_k$ for all $k \in \{0, 1, \ldots, 50\}$. In one figure, plot the 2D trajectory $\hat{X}_k$ for all $k \in \{0, 1, \ldots, 50\}$, for each value of p_0 and place a red point on the position of P. In another figure, use semilogy to plot the frobenius norm of $P_{k|k}$ versus k for all $k \in \{0, 1, \ldots, 50\}$.

Problem 2 (60 points). Let $g = 9.80665$ m/s^2 be the acceleration due to gravity, and let $\phi = \pi/6$ rad. Suppose that a 3-axis accelerometer and a 3-axis rate gyro are attached to a quadcopter following an inclined, circular trajectory. Let $\mathrm{F_A}$ be an inertial frame, let $\mathrm{F_B}$ be a frame fixed to the quadcopter, and suppose that the axes of both the rate gyro and the accelerometer are aligned with $\mathrm{F_B}$. Let c be the center of mass of the quadcopter, let w be a point with zero inertial acceleration, and let $\vec{r}_{c/w}(t)$ and $\mathcal{O}_{\mathrm{B/A}}(t)$ be the position vector of the quadcopter center of mass and the orientation matrix of $\mathrm{F_B}$ relative to $\mathrm{F_A}$ at time t, respectively. Furthermore, suppose that $g_A = \begin{bmatrix} 0 & 0 & -g \end{bmatrix}$ m/s^2 and, for all $k \in \{0, 1, \ldots, 2000\}$, the measurements from the sensors are given by

$$\omega_k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + D_1 w_{1,k} \ \ \text{rad}, \tag{6}$$

$$a_k = \begin{bmatrix} -1 - g \sin\phi \sin kT \\ -g \sin\phi \cos kT \\ -g \cos\phi \end{bmatrix} + D_2 w_{2,k} \ \ \text{m/s}^2, \tag{7}$$

where $T = 0.01$ s is the sampling rate of these sensors, $D_1 = D_2 = \mathrm{diag}(0.1, 0.1, 0.1)$, $w_{1,k} \sim \mathcal{N}(0, I_3)$, and $w_{2,k} \sim \mathcal{N}(0, I_3)$. Furthermore, suppose that

$$\vec{r}_{c/w}\big|_{\mathrm{A}}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{m}, \tag{8}$$

$$\overset{\mathrm{A}\bullet}{\vec{r}}_{c/w}\big|_{\mathrm{A}}(0) = \begin{bmatrix} 0 \\ \cos\phi \\ \sin\phi \end{bmatrix} \text{m/s}, \quad \mathcal{O}_{\mathrm{B/A}}(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}. \tag{9}$$

a) For all $k \in \{0, 1, \ldots, 2000\}$, obtain the estimates $\vec{r}_{c,w}\big|_{\mathrm{A}}(kT)$ via a Kalman filter, using only the rate-gyro and accelerometer measurements. Compare it against the trajectory in the rcwA.mat file by plotting both this reference trajectory and the estimated trajectory in a 3D plot.

b) For all $k \in \{0, 1, \ldots, 2000\}$, let $y_{\mathrm{meas},k} \in \mathbb{R}^3$ be the position vectors from the rcwA.mat file and suppose that these are sampled every T_{MOCAP} seconds to obtain noisy position measurements $y_k \in \mathbb{R}^3$, such that, for all $k \in \{0, 1, \ldots, 2000\}$ such that $\mathrm{mod}(kT, T_{\mathrm{MOCAP}}) = 0$,

$$y_k = y_{\mathrm{meas},k} + D_3 w_{3,k}, \tag{10}$$

where $D_3 = \mathrm{diag}(0.005, 0.005, 0.005)$, and $w_{3,k} \sim \mathcal{N}(0, I_3)$. Use the position measurements in your Kalman filter with $R = 0.001 I_3, Q = 10 I_6$, and $P_{0|0} = 10 I_6$. Do this for $T_{\mathrm{MOCAP}} = 1$ and $T_{\mathrm{MOCAP}} = 0.1$ s. In one figure, use a 3D plot to plot the trajectory in the rcwA.mat file and the estimated trajectories obtained for $T_{\mathrm{MOCAP}} = 1$ and $T_{\mathrm{MOCAP}} = 0.1$ s. In another figure, plot the 3 components of the trajectory in the rcwA.mat file and the estimated trajectories obtained for $T_{\mathrm{MOCAP}} = 1$ and $T_{\mathrm{MOCAP}} = 0.1$ s versus time in a 3-by-1 figure grid using the subplot function.

NOTE: You will find the rcwA.mat file in the folder "Matlab Material" posted on Canvas.