LNCT UNIVERSITY, BHOPAL

Enrollment No. ......

CS - 102
B.TECH (CS/AIML) I SEMESTER
EXAMINATION [DEC-2024]
ADVANCED CALCULUS

Maximum Marks: 70   Time Allowed: 3Hours

Note:- Attempt all questions. Internal choices are given.

(SECTION –A)

1. **Short Answer Type Questions (Attempt Any Five) [5x6=30]**

   i.   Verify Lagrange's Mean value Theorem for the function
        $f(x) = x^3 + x^2 - 6x$ in $[-1, 4]$

   ii. If $\log \sec x = \frac{1}{2}.x^2 + A.x^4 + B.x^6 + \cdots \ldots$, find A and B.

   iii. Prove that $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$.

   iv. Evaluate $\int_0^1 \int_0^{x^2} e^{y/x} \, dy \, dx$.

   v.   Solve the Differential Equation $\frac{dy}{dx} + y = 1$.

   vi. Define Fuzzy subset of a set.

   vii. Reduce the Matrix A to its normal form. Hence find the Rank;

   $$where \; A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}.$$

(SECTION –B)

2. **Long Answer Type Questions (Attempt Any Four) [4x10=40]**

   i.   Expand $\log(1 + e^x)$ in ascending power of x as far as the term containing x^4.

   ii. Prove that; $\Gamma n \, \Gamma 1 - n = \frac{\pi}{\sin n\pi}$ when $0 < n < 1$.

   iii. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dz \, dy \, dx$.

   iv. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$.

   v.   Verify Cayley-Hamilton theorem of the Matrix $A =$

   $$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \; and \; hence \; find \; A^{-1}.$$

   vi. Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.