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Lecture (2) @ Unit (2)

* Joint probability Mass function.

→ If X and Y are random variables.

$$P_X(x_i) = P_X(x = x_i).$$

$$P_{XY}(x,y) = P_{XY}(x = x_i,\ y = y_j) = P_{ij}$$

$$\boxed{P_{ij} = P_{XY}(x,y) = \sum_x \sum_y P_{xy}(x,y).}$$

CDF for D.R.V

Properties

1) $P_{xy}(x,y) \geq 0$

2) $\sum_{x_i} \sum_{y} P_{xy}(x,y) = 1.$

$$\left.\begin{aligned} \text{If } P_{XY}(x,y) &= P_{xy}(x = x_i) \\ &\text{or} \\ &= P_{xy}(y = y_1) \end{aligned}\right\} \text{Marginal PMF}$$

→ If X, Y are independent Random Variable

$$\Rightarrow \boxed{P_{XY}(x,y) = P_X(x) \cdot P_Y(y)}$$

for ~~DRV~~

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$$\left. f_{XY}(x,y) = f_X(x) \cdot f_Y(y) \right] \longrightarrow \text{For CRV}$$

→ Expected value of f^n of RV.

If X, Y are RV.

$g(X,Y) \longrightarrow f^n$ of RV.

$f_{XY}(x,y) \longrightarrow$ joint pdf.

$$E[g(X,Y)] = \overline{g(X,Y)} = \overline{g}(\cdot) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{XY}(x,y)\, dx\, dy$$

for CRV

For BRV.

$$E[g(X,Y)] = \sum_x \sum_y g(x,y) \cdot (PMF)$$

↳ $P_{XY}(x,y)$.

Q. Let $P_{XY}(x,y) = \begin{cases} cxy & x = 1,2,4 \\ & y = 1,3 \\ 0 & \text{o.w} \end{cases}$

Find out $E[g(X,Y)]$

(i) $g(x,y) = y/x$ (ii) $g(x,y) = xy$.

Soln $\sum_x \sum_y cxy \cdot y/x = \sum\sum cy^2$

$= c(1+1+1+9+9+9)$

$= 30c$.

For $x=1$, $x=1$
$y=1$, $y=3$
$x=2$, $x=2$ so on
$y=1$, $y=3$

(iii) $g(x,y) = xy$

$$\Rightarrow \sum_x \sum_y xy \cdot cxy = \sum\sum c x^2 y^2$$

$$= c(1 + 1\times 9 + 4\times 1 + 4\times 9 + 16\times 1 + 16\times 9)$$

$$= 210\,c.$$