

## Sum of GCD(k,n)

I want to find this

$$\sum_{k=1}^n \gcd(k, n)$$

but I don't know how to solve. Does anybody can help me to finding this problem.

Thanks.

(elementary-number-theory) (summation) (divisibility)

edited Feb 14 '15 at 7:18



[Martin Sleziak](#)

41.5k 5 104 232

asked Apr 22 '12 at 15:58



[Elmi Ahmadov](#)

168 2 7

1 Do you know how many integers  $k$  in the range  $1 \leq k \leq n$  are relatively prime to  $n$  so that  $\gcd(k, n) = 1$ ? (Hint: Read about *Euler's totient function*.) – [Dilip Sarwate](#) Apr 22 '12 at 16:03

$n \leq 200000$ . Ok I will read Euler's totient function. – [Elmi Ahmadov](#) Apr 22 '12 at 16:32

### 1 Answer

This is Pillai's arithmetical function as in [OEIS A018804](#)

Formulae given there include

$$\sum_{d|n} d \phi(n/d)$$

and

$$\sum_{d|n} d \tau(d) \mu(n/d)$$

where  $\phi(n)$  is [Euler's totient function](#),  $\tau(n)$  is the [number of divisors](#) and  $\mu(n)$  is the [Möbius function](#).

answered Apr 22 '12 at 16:20



[Henry](#)

84k 3 55 128