This was what I first drew on a sheet of paper when I read the text, and I saw that there were three possible comb can have, and that if n is odd the solution is 0 because there is no way to fill up the entire rectangle then (one piece by a domino).

So I thought the solution was simply 3ⁿ, if n was even, and 0, if n was odd. turns out, I was wrong.

I found a relatively simple solution here:

```
#include <iostream>
using namespace std;
int main()
     int arr[31];
     arr[0]=1;
     arr[1]=0;
     arr[2]=3;
     arr[3]=0;
     for(int i = 4; i < 31; i++) {</pre>
         arr[i] = arr[i-2] * 4 - arr[i-4]; //this is the only line i don't get
     int n;
     while(1) {
         cin >> n;
         if(n == -1) {
              break;
         cout << arr[n] << endl;</pre>
     }
     return 0;
}
Why does this work?!
       algorithm
                 math
                         permutation
 edited Jan 21 '15 at 23:50
                                                                                    asked May 5 '13 at 20:02
                                                                                          Abrf Kled
       iCodez
       91.4k
              20
```

3 Answers

Let T(n) be the number of ways one can tile a $3\times n$ board with 2×1 tiles. Also, let P(n) be the number of ways one can tile a $3\times n$ board with one corner removed with 2×1 tiles. Assume n sufficiently large (>= 4).

Then consider how you can start the tiling from the left (or right, doesn't matter).

You can place the tile covering the top left corner in two ways, vertical or horizontal. If you place it vertical, the tile covering the bottom left corner must be placed horizontally, giving a configuration

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and then you have no choice but to place another tile horizontally at the top, leaving you

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with a 3×(n-3) board minus a corner,

$$P(n-1) = T(n-2) + P(n-3)$$

Adding up,

$$T(n) = T(n-2) + 2*(T(n-2) + P(n-3))$$

= 3*T(n-2) + 2*P(n-3) (2)

But, using (1) with n-2 in place of n, we see that

$$T(n-2) = T(n-4) + 2*P(n-3)$$

or

$$2*P(n-3) = T(n-2) - T(n-4)$$

Inserting that into (2) yields the recurrence

$$T(n) = 4*T(n-2) - T(n-4)$$

q.e.d.

answered May 5 '13 at 20:33



Daniel Fischer

153k 14 243 377

Nice proof! More information available at oeis.org/A001835 - Peter de Rivaz May 5 '13 at 20:40

- @Daniel Could you explain the base case corresponding to n = 0. ATul Singh Aug 3 '14 at 10:25
- @ATulSingh For n = 0, we have a board with no cells at all. There is precisely one way to tile it: place no tiles on it [a 3×0 board has $3\cdot0$ = 0 cells, so you need $0/(2\cdot1)$ = 0/2 = 0 tiles]. Daniel Fischer Aug 12 '14 at 14:41

Sharing my image tut. Hope it helps.

answered Dec 16 '15 at 8:24



M3TILE

answered Dec 23 '15 at 9:37

