

Probability

⇒ conditional prob (wiki)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

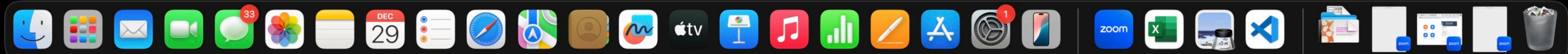
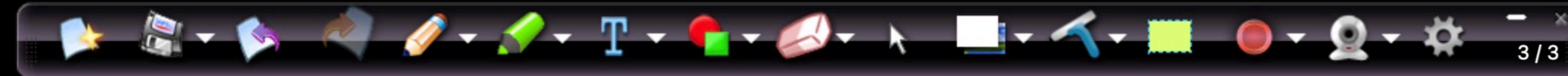
A

B

}  $P\left(\frac{A}{B}\right)$

already

$P(B)$  ↗  
occluded



Bayes theorem



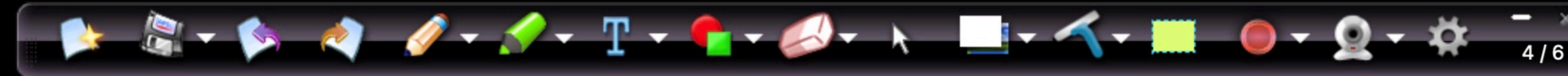
$$P(A|B) =$$

→  
posterior  
↓

$$\frac{P(B|A) \times P(A)}{P(B)}$$

↑ Evidence  
↓ Prior  
Likelihood

What is probability  
of getting A  
if B already  
happened



Data

$x_1$  text →

$x_2$  →  
→  
→  
→

Class

1

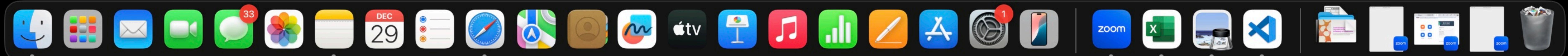
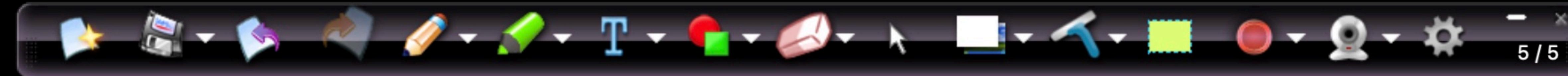
0

1 = spam

0 = not spam

$$P(Y=1 \mid \text{text}_q) = ?$$

$$P(Y=0 \mid \text{text}_q)$$



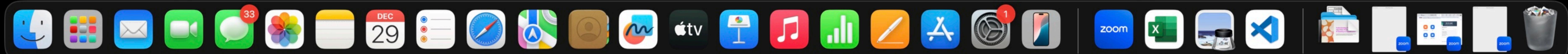
text → vector space

→  $\{w_1, w_2, w_3, \dots, w_d\}$

$$P(Y=1 | \{w_1, w_2, w_3, \dots, w_d\})$$

$$= P(Y=1) * P(w_1 | Y=1) * P(w_2 | Y=1) * \dots * P(w_d | Y=1)$$

$$= P(Y=1) * \prod_{i=1}^d P(w_i | Y=1)$$

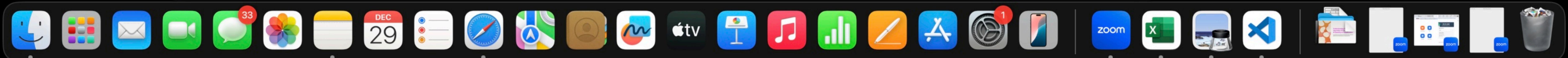
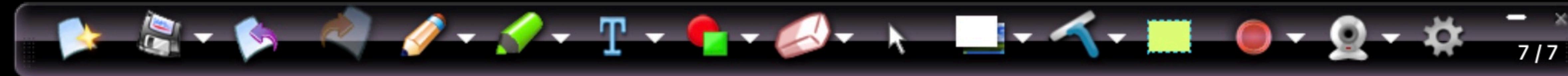


Very simple

$$P(Y_0 \mid \text{few}) = P(Y_0) \stackrel{u}{\sim} P(\omega_i \mid Y=0)$$

$\downarrow$

$$P(Y=0) = \frac{\# \text{ train pts with } Y=1}{\text{total } \# \text{ train pts}}$$



e4

Data

Label

0 → Not spam

1 → Spam

training

✓ w<sub>1</sub> w<sub>2</sub> w<sub>2</sub>  
✓ w<sub>1</sub> w<sub>2</sub> w<sub>1</sub> (0.5)  
w<sub>3</sub> w<sub>4</sub> w<sub>1</sub>  
w<sub>5</sub> w<sub>1</sub> w<sub>1</sub>  
w<sub>1</sub> w<sub>2</sub> w<sub>3</sub>, w<sub>4</sub>

0 }  
0 }  
0 }  
1 } ✓  
1 }

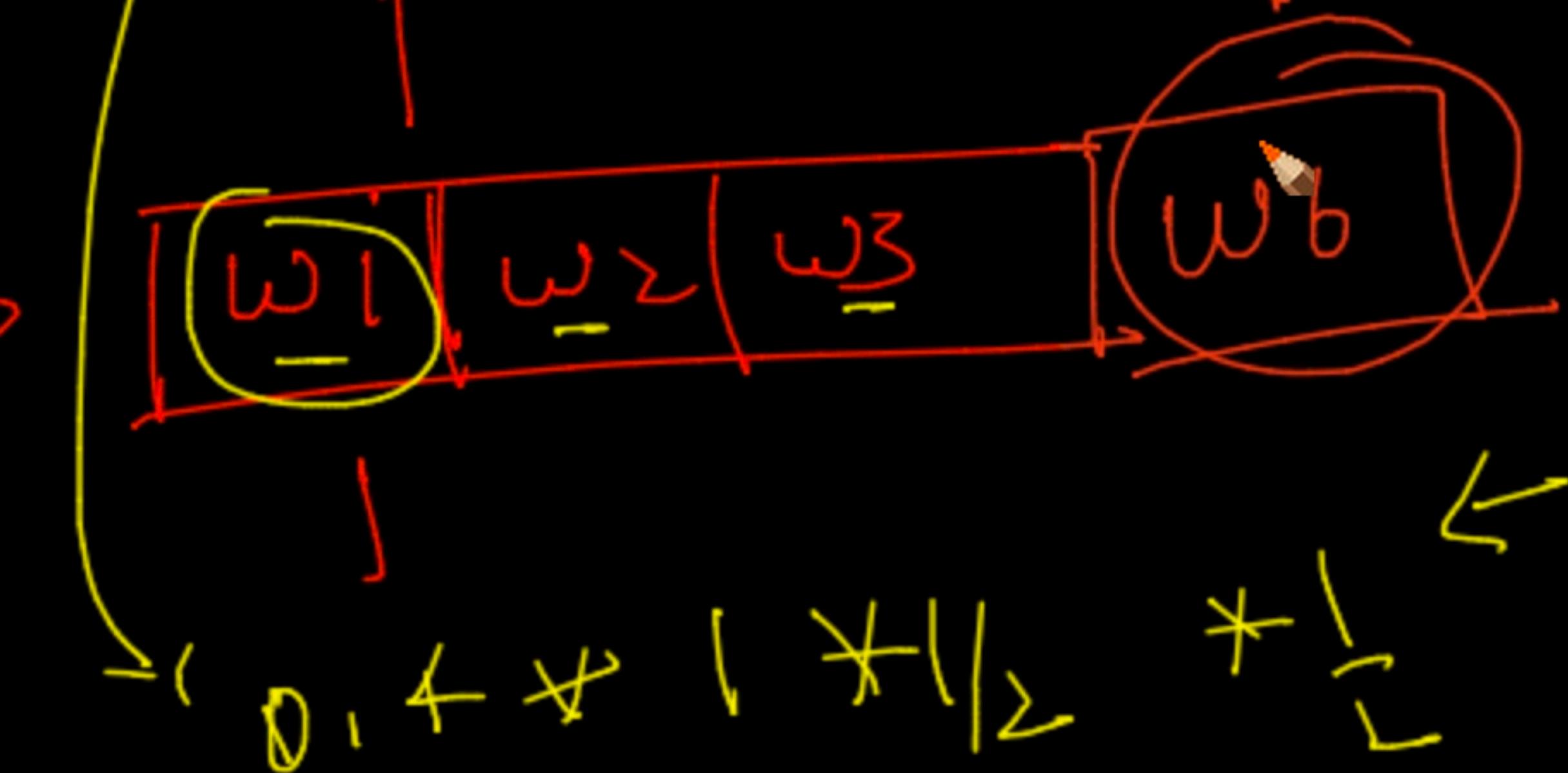
outcome of training

$P(Y=0) = \frac{3}{5} \Rightarrow 0.6$

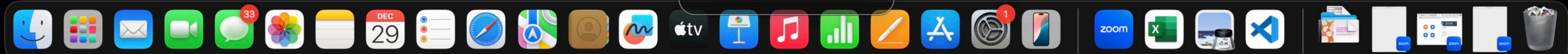
$P(Y=1) = \frac{2}{5} \Rightarrow 0.4$  ✓

Unique $\rightarrow$  testing $\rightarrow$  tentq

vector

 $\rightarrow$  $\pi$  $\rightarrow$ 

$$P(Y=1 | w_1, w_2, w_3) = P(Y=1) * P(w_1 | Y=1) * P(w_2 | Y=1) * P(w_3 | Y=1)$$



at time of training

⇒ at time of testing

↓,

$$P(Y=1)$$

$$P(Y=0)$$

textq ⇒

$$P(\omega_1 | Y=1)$$

$$P(\omega_0 | Y=0)$$

$$\{\omega_1, \omega_2, \omega_3, \omega^*\}$$

:

:

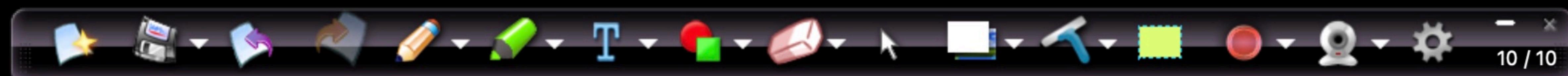
:

:

$$P(\omega_d | Y=1)$$

$$P(\omega_d | Y=0)$$

'New word'



$$P(Y=1 | \text{text}q) = P(Y=1) \times P(\omega_1 | Y=1) * P(P(\omega_2 | Y=1))$$

↗ ↘

$\rightarrow 0'$

$\downarrow$

$$* P(\omega_3 | Y=1) * P(\omega' | Y=1)$$

$\rightarrow 1$

$\downarrow$

$$P(Y=0 | \text{text}q) =$$

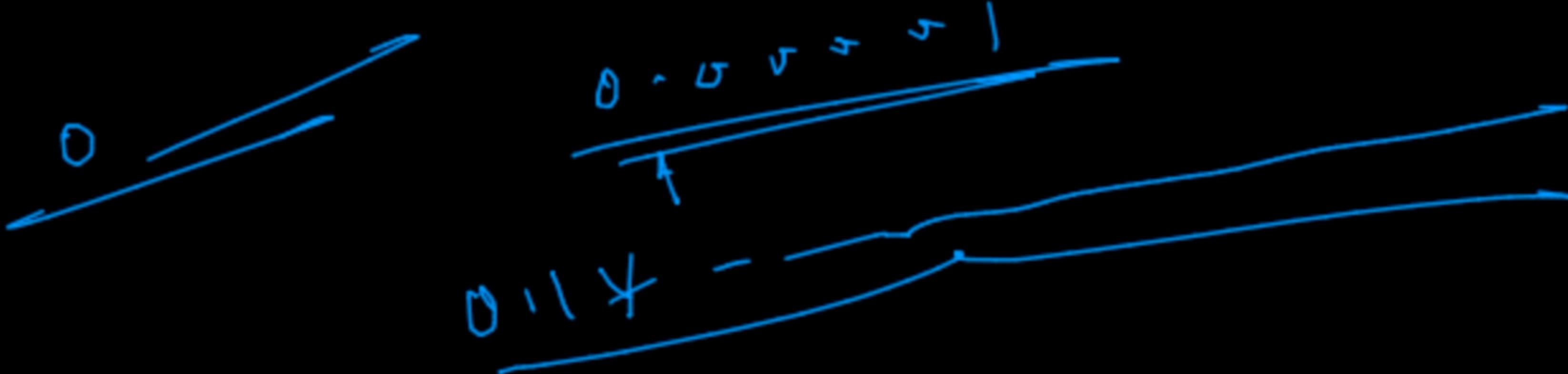
$\rightarrow$

$\downarrow 0'$

$\uparrow$

$$P(\omega' | Y=1) \Rightarrow \# \text{time } (\omega') \text{ occur in training \& } Y=1$$

$\# \text{ pts where } Y=1$



$$\frac{0}{m} \rightarrow 0$$

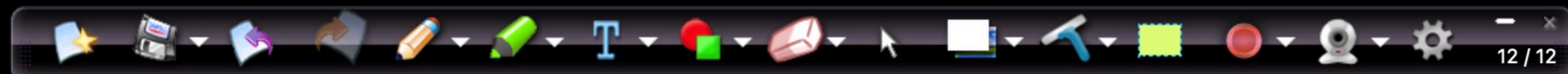
Updale smoothing

$$p(\omega^i | Y=1) = \frac{o + d}{n_i + Kd}$$

$\nearrow$

$K \Rightarrow \# \text{distinct value } \omega^i \text{ counter}$

$\swarrow K=2 \downarrow$



(case i)  $\rightarrow 1$

$$n_1 = 1^{50}$$

$$P(\omega^1 | Y=1) =$$

$$\frac{0 + d}{50 + 2d}$$

$$= \frac{d}{100 + 2d}$$

(case i)

$$d = 1$$

$$\frac{1}{100 + 2} > 0$$

(case ii)

( $d \uparrow$ )

$$d = 10^{-100}$$

$$\Rightarrow \frac{0 + 10^{-100}}{100 + 2 \times 10^{-100}}$$

50%

$$\frac{10^{-100}}{2 \times 10^{-100}} = \frac{1}{2} \approx 0.5$$

(good)

$1 \rightarrow 1/n$

$\{ \begin{matrix} w_1 \\ w_2 \end{matrix} \} \rightarrow -ve$

$w_1 \rightarrow \text{small}$

$\text{small}$

$\alpha$   
 $\beta$   
 $\gamma$

Hyper parameters



$f(\cdot)$

$$P(w_i | y=1) = \frac{\# \text{data points with } w_i \& y=1 + \alpha}{\# \text{data points } y=1 + \alpha K}$$



Naive Bayes  $\rightarrow$  sklearn