

Sigmoid

$$\text{sig}(\text{val}) \rightarrow [0-1]$$

$$\text{Sig}(x) = \frac{1}{1 + e^{-x}}$$

$$\begin{aligned} w^* &= \arg \max_{\omega} \sum_{i=1}^n y_i \omega^T x_i \\ &= \arg \max_{\omega} \sum_{i=1}^n \text{sig}(y_i \omega^T x_i) \\ &= \sum_{i=1}^n \frac{1}{1 + \exp(-y_i \omega^T x_i)} \end{aligned}$$

Optimal

$$w^* = \arg \max_{\omega} \sum_{i=1}^n \frac{1}{1 + \exp(-y_i \omega^T x_i)}$$

Model

$$bx \rightarrow [] \rightarrow f()$$

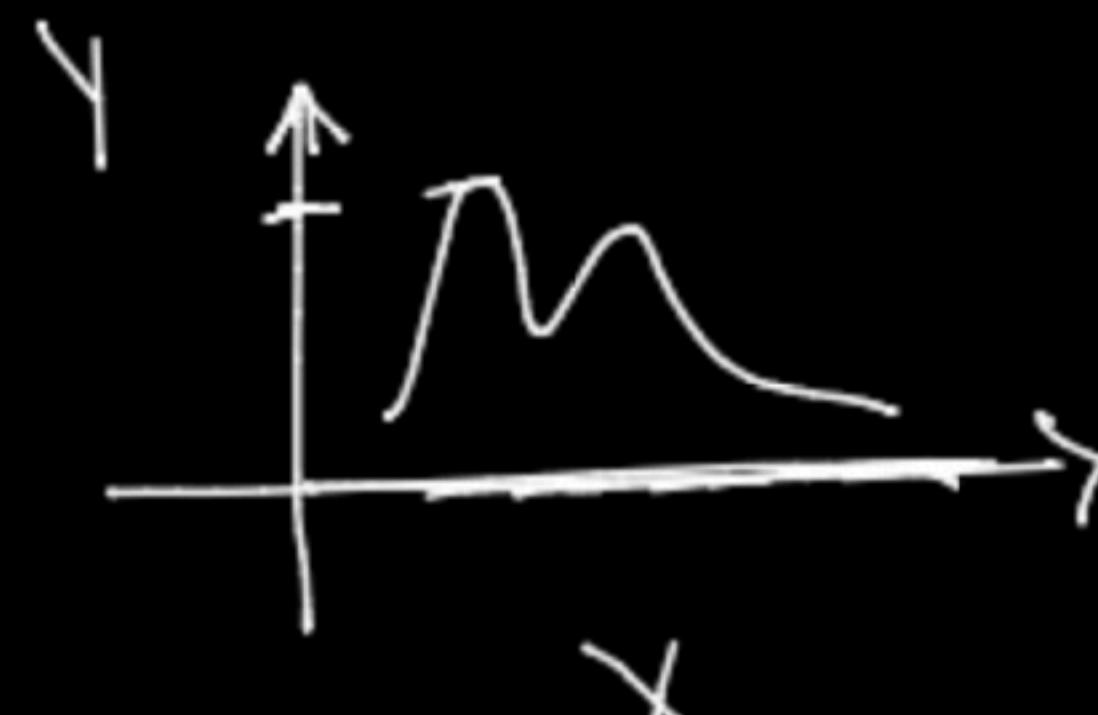
$$y = f(x)$$

MAX

$$f(x) \rightarrow$$

x^*

Optimisation



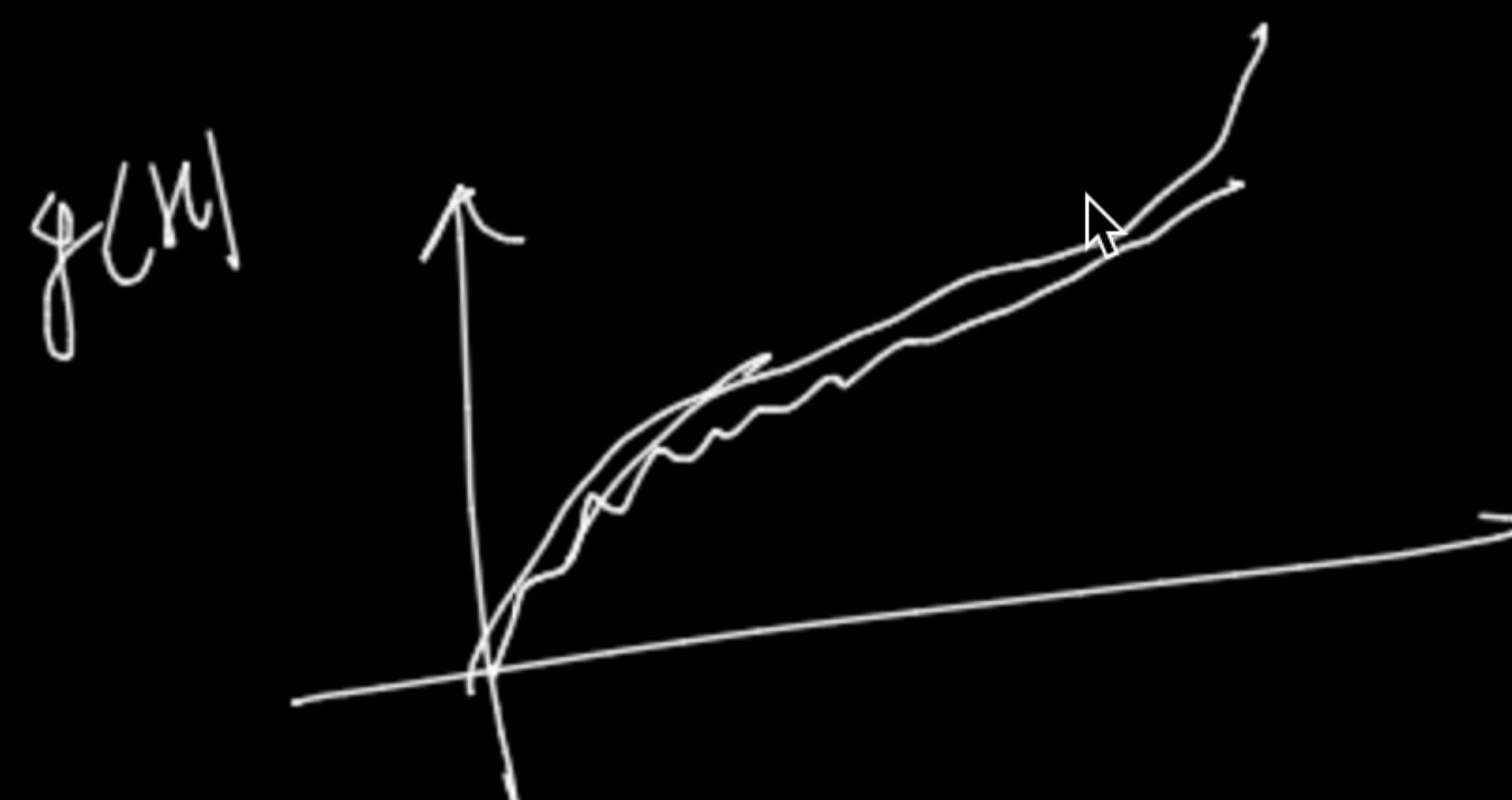
Monotonic f^h

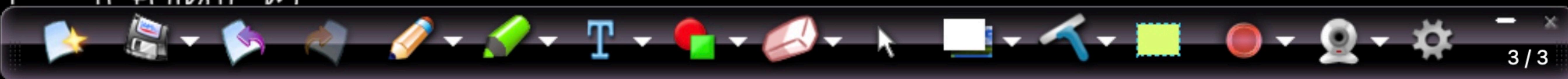
$g(x)$

$x \uparrow \quad g(x) \uparrow$

then $x_1 > x_2$ then $g(x_1) > g(x_2)$

↳ then $g(x)$ is said to be
monotonically increasing





①

$$x^* = \arg \min x^2 \text{ is monotonically}$$

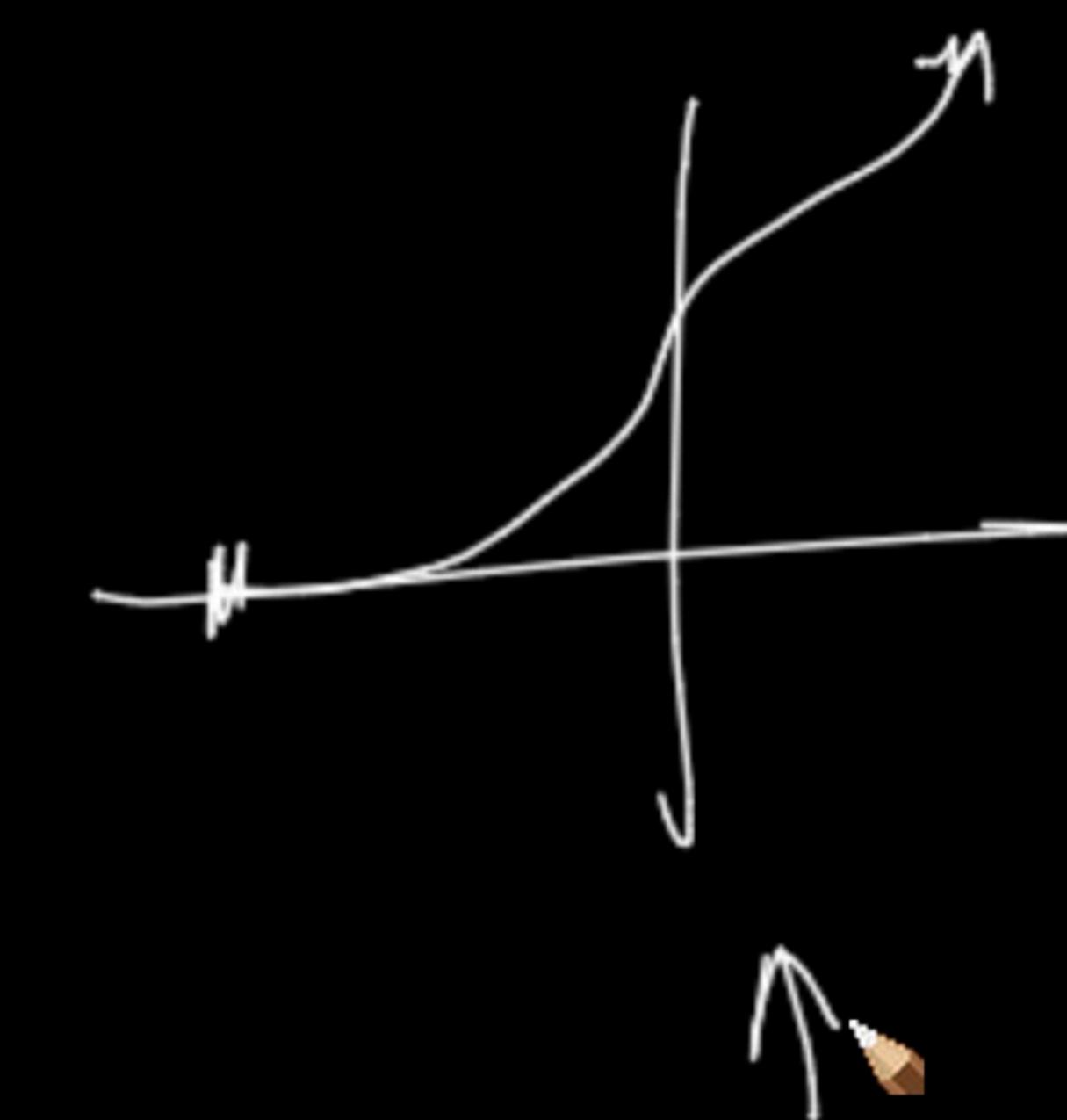


②

$$g(x) = \log(x)$$

$$x^* = \arg \min f(x)$$

$$\because f(x) = x^2$$



$$x = \arg \min (g(f(x)))$$

monotonically

$$= \arg \min (\log(x^2))$$

monotonically

$$w^* = \underset{\omega}{\operatorname{argmax}} \sum_{i=1}^n \frac{1}{1 + \exp(-y_i \omega^T x_i)}$$

$$\text{wt } g(x) = \log(x)$$

$$\log\left(\frac{w}{n}\right) \rightarrow \log w - \log n$$

$$w^* = \underset{\omega}{\operatorname{argmax}} \sum_{i=1}^n \log \left(\sigma(y_i \omega^T x_i) \right)$$

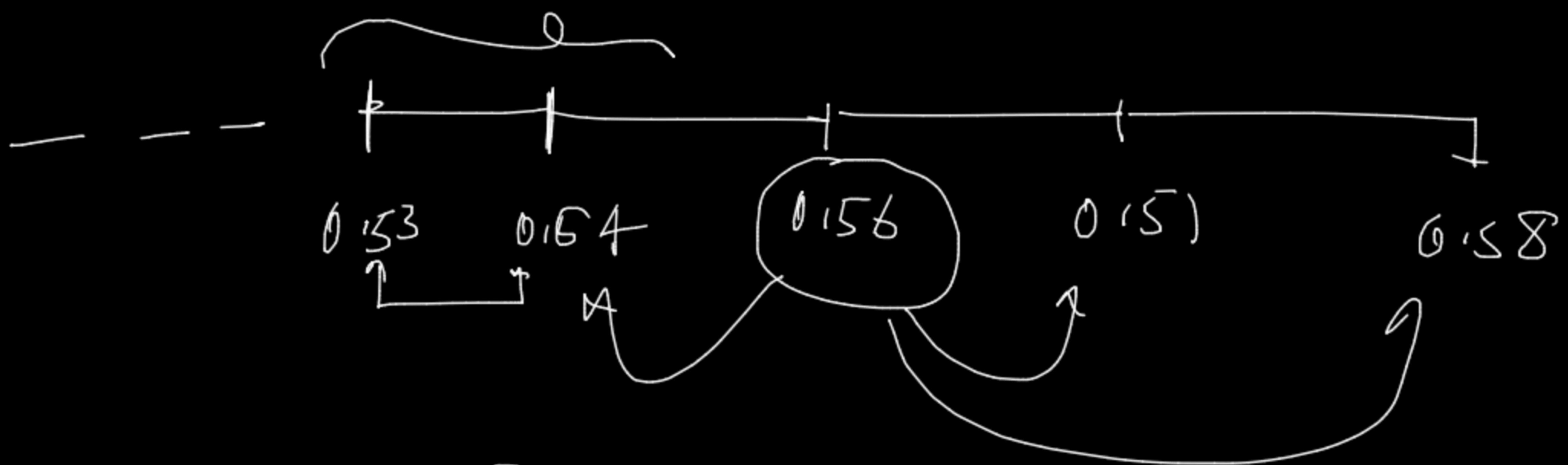
$$\begin{aligned} w^* &= \underset{\omega}{\operatorname{argmax}} \sum_{i=1}^n \log \left(\frac{1}{1 + \exp(-y_i \omega^T x_i)} \right) \\ &= \underset{\omega}{\operatorname{argmax}} \sum_{i=1}^n \log 1 - \log \left(1 + \exp(-y_i \omega^T x_i) \right) \end{aligned}$$

$$w^* = \underset{w}{\operatorname{arg\,max}} \sum_{i=1}^n -\log \left(1 + \exp(-y_i w^T x_i) \right)$$

$$\underset{w}{\operatorname{arg\,max}} -f(x) = \underset{w}{\operatorname{arg\,min}} f(x)$$

$$w^* = \underset{w}{\operatorname{arg\,min}} \sum_{i=1}^n \log \left(1 + \exp \left(-y_i w^T x_i \right) \right)$$

optimization problem for logistic regression



Loss

t_0

$\rightarrow \text{turns}$

$[-] - - - \square$

ω^*

LR

\rightarrow

θ_x



w^*

weight vector



$$\sigma(w^T x_i) = P(Y_q = +w)$$

(case - i)

If $w_i = +w$

$$\hookrightarrow x_{q_i} \uparrow \Rightarrow (w_i \times q_i) \uparrow$$

implies

$$\sum_{i=1}^d w_i x_{q_i} \uparrow$$

$$w \leftarrow w_1 \leftarrow w_3 \leftarrow w_4 \dots w_d$$

$x_i \in \mathbb{R}^d$

$$f_1 \ f_2 \ f_3 \dots f_d$$

$$(x_q) \xrightarrow{?} Y_q$$

[]

if $w^T x_q > 0$
 $\rightarrow +w$ bits

if $w^T x_q < 0$
 $\rightarrow -w$

$$\sigma(w^T x_{q_i}) \uparrow$$

$$\Rightarrow P(Y_q = +w) \uparrow$$

(use 11)

$$\text{if } \boxed{w_i = -w}$$

$$x_{q_i} \uparrow \Rightarrow w_i x_{q_i} \downarrow$$

$$\begin{aligned} & \Rightarrow \sum_{i \leftarrow 1}^d (w_i x_{q_i}) \downarrow \\ & \Rightarrow \sigma(w_i x_{q_i}) \downarrow \quad \text{ie} \\ & \quad P(Y_q = +w) \downarrow \\ & \quad P(Y_q = -w) \uparrow \end{aligned}$$