Rahul Vemuri

D16A 33

Tutorial no: 1

Q1. Determine the Discrete Fourier Transform (DFT) of four point sequence $x(n) = \{0, 1, 2, 3\}$

The 4-point DFT in the matrix form is given by

$$X_4 = [W_4] \cdot x(n)$$

Thus

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ 2j-2 \\ -2 \\ -2j-2 \end{bmatrix}$$

$$X_4 = \{6, 2j-2, -2, -2j-2\}$$

Q2. Compute the length-4 sequence from its DFT which is given by $X(K) = \{4, 1-j, -2, 1+j\}$

→ We know that the IDFT in matrix form is expressed as

$$IDFT = x(n) = x_N = \frac{1}{N} [W_N^*] X_N$$

Here, X_N is the given DFT matrix. Also '*' indicates complex conjugate. To obtain the complex conjugate we have to change the sign of j term. For example complex conjugate of $1-j1$ is $1+j1$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \quad \text{-----} \ \text{II}$$

$$\therefore \quad W_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \quad \text{-----} \ \text{III}$$

Given matrix of DFT is

$$X_N = X_4 = \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix} \quad \text{-----} \ \text{IV}$$

Substituting eqn III & IV $N=4$ in eqn 1 we shall have

$$x_N = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$x_N = \frac{1}{4} \begin{bmatrix} 4+1+-j-2+1+j \\ 4+j+j^2+2-j-j^2 \\ 4-1+j-2-1-j \\ 4-j+j^2+2+j+j^2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 \\ 4+2+1+1 \\ 4-4 \\ 4+2-2 \end{bmatrix}$$

$$x_N = \{1, 2, 0, 1\}$$

Q3 Determine the DFT of a sequence $x(n) = \{1,1,0,0\}$ & check the validity of your answer by calculating it's IDFT.

Let us compute 4-point DFT. We have already obtained the matrix for $[W_4]$ in previous example. It is reproduced here

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -j \\ 1 & j & -1 & -j \end{bmatrix}$$

The given sequence is $x(n) = \{1,1,0,0\}$. The DFT of this sequence has been computed in previous example. It is $x_N = x(k) = \{2, 1-j, 0, 1+j\}$
Now, let us check this answer by using the expression for IDFT.

The IDFT is given by

$$x(n) = \frac{1}{N} [W_N^*] \text{ by }.$$

$$x(n) = \frac{1}{N} [W_N^*] \cdot X_N$$

Here, $[W_N^*] = [W_N^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$

& $X_N = X_4 = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$

$\therefore$ We have.

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x_n = \{1, 1, 0, 0\}$$

Therefore calculated DFT is correct. Hence Proved.

Q4 Use four-point DFT & IDFT to determine the circular convolution of following sequences

$$x_1(n) = (1, 2, 3, 1) \qquad x_2(n) = (4, 3, 2, 2)$$

The four point DFT of $x_1(n)$ is $X_1(K)$ & it is given by

$$X_1(K) = [W_4]\, x_{1N}$$

$$\text{we have } [W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\therefore X_1(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+3+1 \\ 1-2j-3+j \\ 1-2+3-1 \\ 1+2j-3-j \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$\therefore X_1(K) = \{7, -2-j, 1, -2+j\}$$

$$X_2(K) = [W_4]\, x_{2N}$$

Also $X_2(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix}$ $\qquad X_2(K) = \{11, 2-j, 1, 2+j\}$

Now according to property of circular convolution we have

$$x_1(n) \, \textcircled{N} \, x_2(n) = X_1(K) \cdot X_2(K) = X_3(K)$$

or $X_3(K) = \{7, -2-j, 1, -2+j\} \cdot \{11, 2-j, 1, 2+j\}$

$$X_3(K) = \{77, -5, 1, -5\}$$

Let result of $x_1(n) \, \textcircled{N} \, x_2(n)$ be sequence $x_3(n)$. It is obtained by computing IDFT of $X_3(K)$. According to the definition of IDFT we have.

$$x_3(n) = \frac{1}{N}[W_N^*] \cdot X_N$$

Hence $x_3(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 76 \\ -5 \\ 1 \\ -5 \end{bmatrix}$

$$x_3(n) = \frac{1}{4} \begin{bmatrix} 77-5+1-5 \\ 77-5j-1+5j \\ 77+5+1+5 \\ 77+5j-1-5j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 68 \\ 76 \\ 88 \\ 76 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

Considering only real part, approximately sequence $x_3(n)$ can be written as under

$$x_3(n) = \{17, 19, 22, 19\}$$

Q5 Write applications of Digital Signal Processing

As a matter of fact, there are various application areas of DSP due to the availability of high resolution spectral analysis. It requires high speed processor to implement the fast fourier Transform (FFT). Some of these areas can be listed as under:

1) Speech Processing

2) Image Processing

3) Radar Signal Processing

4) Digital Communication

5) Spectral Analysis

6) Sonar Signal Processing

Few other applications of DSP can be listed.

1 Transmission Lines
2 Advanced optical fiber communication
3 Analysis of sound & vibration analysis
4 Implementation of speech recognition algorithm.
5 Very large scale Integration (VLSI) technology.
6 Telecommunication n/ws.
7 Microprocessor systems
8 satelite communications
9 Telephony communication.

1) **Speech Processing:**

Speech is a one-dimensional signal. DSP of speech is applied to a wide range of speech problems such as speech spectrum analysis channel vocoders etc. DSP is applied to speech coding, speech analysis & synthesis; speech recognition & speaker recognition.

2) **Image Processing**

Any two-dimensional pattern is called an image. DSP of images requires 2-D DSP tools such as DFT, FFT and Z-transforms.

3) **Radar Signal Processing.**

Radar stands for "Radio Detection & Ranging". Improvement is signal processing is possible by digital technology. Development of DSP has led to greater sophistication in radar tracking algos.

4) **Digital Communications.**

Application of DSP in digital communication specifically telecommunications comprises of digital transmission using PCM, digital switching using TDM echo control & digital tape recorders. DSP in telecomunication system are found to be cost effective due to availability.

Q6 Derive relationship between z-transform & DFT

Let $X(z)$ be the z-transform for a sequence $x(n)$ which is expressed as:

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}}$$

with a ROC which includes the unit circle. If $X(z)$ is sampled at the N equally spaced points on the unit circle

$$Z_K = e^{j2\pi K/N}, \quad K = 0, 1, 2, \ldots, N-1$$

then $$X(K) = X(z)\Big|_{\text{at } z = e^{j2\pi K/N}}$$

Now it may be noted that this is identical to fourier transform $X(e^{j\omega})$ evaluated at N equally spaced frequencies.

$$\omega_K = 2\pi K/N \quad K = 0, 1, \ldots N-1$$

If the sequence $x(n)$ has a finite duration of length N, then z-transform is given as

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$$

Now substituting the IDFT relationship for $x(n)$, we obtain

$$X(z) = 1/N \sum_{n=0}^{N-1} \left[1/N \sum_{K=0}^{N-1} X(K) e^{j2\pi nK/N} \right] \cdot z^{-n}$$

$$= 1/N \sum_{K=0}^{N-1} X(K) \sum_{n=0}^{N-1} (e^{j2\pi K/N} z^{-1})^{n}$$

or $$X(z) = 1/N \sum_{K=0}^{N-1} X(K) \cdot \frac{1-z^{-N}}{1-e^{j2\pi K/N} z^{-1}} = \frac{1-z^{-N}}{N} \sum_{K=0}^{} \frac{X(K)}{1-e^{j2\pi K/N} z^{-1}}$$

This equation is identical to that of frequency sampling form

$$X(e^{j\omega}) = \frac{1-e^{-j\omega N}}{N} \sum_{K=0}^{N-1} \frac{X(K)}{1-e^{j(\omega - 2\pi K/N)}}$$

Q7. Explain circular convolution using matrix method.

The graphical method which we have just discussed is quite tedious, especially when many samples are present. while the matrix method is more convenient. In the matrix method one sequence is repeated via circular shifting of sampes

we have $y(m) = x(n) \, N \, h(n) = h(n) \, N \, x(n)$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \\ y(N-2) \end{bmatrix} = \begin{bmatrix} h(0) & h(N-1) & h(N-2) \\ h(1) & h(0) & h(N-1) \\ h(2) & h(1) & h(0) \\ \vdots & & \\ h(N-2) & h(N-3) & \vdots \\ & h(N-2) & \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-2) \end{bmatrix}$$

Q8 Determine the following sequence $y(n) = x(n) \, (N) \, h(n)$

Where $x(n) = \{1, 2, 3, 1\}$ & $h(n) = \{4, 3, 2, 1\}$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4+4+6+3 \\ 3+8+6+2 \\ 2+6+12+2 \\ 2+4+9+4 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

$y(n) = \{17, 19, 22, 19\}$

Q9 Find the linear convolution using circular convolution of the following sequences $x(n) = (1, 2, 1)$, $h(n) = (1, 2)$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2j \\ 0 \\ j \end{bmatrix}$$

$X(K) = \{4, -2j, 0, j\}$

$$H(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1-2j \\ -1 \\ 1+2j \end{bmatrix}$$

$H(K) = \{3, 1-2j, -1, 1+2j\}$

$Y(K) = x(K) \cdot H(K)$

$Y(K) = \{4, -2j, 0, j\} \cdot \{3, 1-2j, -1, 1+2j\}$

$Y(3) = x(3) \cdot H(3) = j \times (1+2j) = j + 2j^2 = j-2$

$Y(K) = \{12, 2j-4, 0, j-2\}$