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Rahul R. Adsul.
D16B / ETRX
DSP

(1)

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Assignment - 1

Q.1. Determine the Discrete Fourier Transform (DFT) of four point sequence $x(n) = \{0, 1, 2, 3\}$.

→ The 4-point DFT in the matrix form is given by:

$$X_4 = [W_4] \cdot x(n).$$

Thus,

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -J & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & J & -1 & -J \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ 2j-2 \\ -2 \\ -2j-2 \end{bmatrix}$$

Simplifying, we shall get,

$$X_4 = \{6, 2j-2, -2, -2j-2\}$$

$\uparrow$

Q.2. Compute the length-4 sequence from its DFT which is given by $X(K) = \{4, 1-j, -2, 1+j\}$

→ We know that the IDFT in matrix form is expressed as:

$$IDFT = x(n) = x_N = \frac{1}{N}[W^*_N] \cdot X_N \quad --- \text{(I)}$$

Here, X_N is the given DFT matrix. Also, '*' indicates complex conjugate. To obtain the complex conjugate, we have to change the sign of j term. For example, complex conjugate of $1-j1$ is $1+j1$.

Now, we have already obtained the matrix W_4 in previous examples. It is reproduced here i.e.,

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$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \quad \text{---} \; \text{II}$$

Therefore,
$$[W_4^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \quad \text{---} \; \text{III}$$

Given matrix of DFT is,

$$X_N = X_4 = \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix} \quad \text{----} \; \text{IV}$$

Substituting equations III & IV & substituting $N=4$ in equation I, we shall have.

$$x_N = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

or
$$x_N = \frac{1}{4} \begin{bmatrix} 4+1-j-2+1+j \\ 4+j-j^2+2-j-j^2 \\ 4-1+j-2-1-j \\ 4-j+j^2+2+j+j^2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 \\ 4+2+1+1 \\ 4-4 \\ 4+2-2 \end{bmatrix}$$

$(\because j^2 = -1)$

simplifying, we get

or,
$$x_N = \frac{1}{4} \begin{bmatrix} 4 \\ 8 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

or, $x(n) = \{1, 2, 0, 1\}$ --- ANS.

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Q.3. Determine the DFT of a sequence $x(n) = \{1, 1, 0, 0\}$ & check the validity of your answer by calculating it's IDFT.

→ Sol^n:- Let us compute 4-point DFT. we have already obtained the matrix for $[W_4]$ in previous example. It is reproduced here.

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -j \\ 1 & +j & -1 & -j \end{bmatrix}$$

The given sequence is $x(n) = \{1, 1, 0, 0\}$.

The DFT of this sequence has been computed in previous example. It is $X_N = X(k) = \{2, 1-j, 0, 1+j\}$

Now, let us check this answer by using the expression for IDFT.

The IDFT is given by,

$$x(n) = \frac{1}{N}[W_N^*] \cdot X_N$$

Here, $$[W_N^*] = [W_4^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

& $$X_N = X_4 = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

Therefore, we have,

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2+1-j+0+1+j \\ 2+j+1+0-j+1 \\ 2-1+j+0-1-j \\ 2-j-1+0+j-1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

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This means that $x(n) = \{1, 1, 0, 0\}$

But, this is same as the given sequence. Therefore, calculated DFT is correct. ... Hence Proved.

Q.31. Use the four point DFT & IDFT to determine the circular convolution of following sequences.

$$x_1(n) = (\underset{\uparrow}{1}, 2, 3, 1), \quad x_2(n) = (\underset{\uparrow}{4}, 3, 2, 2)$$

→ The four point DFT of $x_1(n)$ is $X_1(k)$ & it is given by,

$$X_1(k) = [W_4]\, x_{1N}$$

we have, $[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

Therefore, $X_1(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+3+1 \\ 1-2j-3+j \\ 1-2+3-1 \\ 1+2j-3-j \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$

Therefore, $X_1(k) = \{7, -2-j, 1, -2+j\}$

similarly, $X_2(k) = [W_4]\, x_{2N}$

Also, $X_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix}$

Therefore, $X_2(k) = \begin{bmatrix} 4+3+2+2 \\ 4-3j-2+2j \\ 4-3+2-2 \\ 4+3j-2-2j \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$

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or. $X_2(k) = \{11, 2-j, 1, 2+j\}$

Now according to property of circular convolution, we have

$$x_1(n) \; (N) \; x_2(n) = X_1(k) \cdot X_2(k) = X_3(k)$$

or $X_3(k) = \{7, -2-j, 1, -2+j\} \cdot \{11, 2-j, 1, 2+j\}$

or $X_3(k) = \{77, -5, 1, -5\}$

Let the result of $x_1(n) \; (N) \; x_2(n)$ be sequence $x_3(n)$. It is obtained by computing IDFT of $X_3(k)$. According to the definition of IDFT, we have,

$$x_3(n) = \frac{1}{N}[W^*_N] \cdot X_N$$

Hence, $x_3(n) = \frac{1}{4}[W^*_4] \cdot X_{3N}$

or $$x_3(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

or $$x_3(n) = \frac{1}{4}\begin{bmatrix} 77-5+1-5 \\ 77-5j-1+5j \\ 77+5+1+5 \\ 77+5j-1-5j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 68 \\ 76 \\ 88 \\ 19 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

considering only real part, approximately sequence $x_3(n)$ can be written as under:

$$x_3(n) = \{17, 19, 22, 19\}$$

Q.5. Write applications of Digital Signal Processing.

→ As a matter of fact, there are various application areas of Digital Signal Processing (DSP) due to the availability of high resolution spectral analysis. It requires high speed processor to implement the Fast Fourier Transform (FFT). Some of these areas can be listed as under:

1. Speech Processing.
2. Image Processing
3. Radar Signal Processing.

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4. Digital communications
5. Spectral Analysis.
6. Sonar signal Processing.

Few other applications of Digital signal Processing (DSP) can be listed as under.

1. Transmission Lines
2. Advanced optical fiber communication.
3. Analysis of sound & vibration signals.
4. Implementation of speech recognition algorithms.
5. Very Large scale Integration (VLSI) technology.
6. Telecommunication networks.
7. Microprocessor systems.
8. Satellite communications.
9. Telephony transmission.
10. Aviation
11. Astronomy.
12. Industrial noise control.

Now, let us discuss few major applications in brief:

1. Speech Processing.

Speech is a one-dimensional signal. Digital processing of speech is applied to a wide range of speech problems such as speech spectrum analysis, channel vocoders (voice coders) etc. DSP is applied to speech coding, speech enhancement, speech analysis & synthesis, speech recognition & speaker recognition.

2. Image Processing.

Any two-dimensional pattern is called an image. Digital processing of images requires two-dimensional DSP tools such as Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT) algorithms & z-transforms. Processing of electrical signals extracted from image by digital technique

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include image formation & recording, image compression, image restoration, image reconstruction & image enhancement.

3. Radar Signal Processing.

Radar stands for "Radio Detection & Ranging". Improvement in signal processing is possible by digital technology. Development of DSP has led to greater sophistication of radar tracking algorithms. Radar systems consist of transmit-receive antenna, digital processing system & control unit.

4. Digital communications.

Application of DSP in digital communication specially telecommunications comprises of digital transmission using PCM, digital switching using Time Division Multiplexing (TDM), echo control & digital tape-recorders. DSP in telecommunication system are found to be cost effective due to availability of medium & large scale digital IC's. These IC's have desirable properties such as small size, low cost, low power, immunity to noise & reliability.

5. Spectral Analysis.

Frequency-domain analysis is easily & effectively possible in digital signal processing using Fast Fourier Transform (FFT) algorithms. These algorithms reduce computational complexity & also reduce the computational time.

6. Sonar Signal Processing.

Sonar stands for "Sound Nevigation & Ranging". Sonar is used to determine the range, velocity & direction of targets that are remote from the observer. Sonar uses sound waves at lower frequencies to detect objects under water. DSP can be used to process sonar signals, for the purpose of nevigation & ranging.

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Q.6. Derive relationship between z-transform & DFT.

→ Let X(z) be the z-transform for a sequence x(n) which is expressed as:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

with a ROC which includes the unit circle. If X(z) is sampled at the N equally spaced points on the unit circle.

$$z_k = e^{j2\pi k/N}, \quad k = 0, 1, 2, \ldots N-1.$$

then $X(k) = X(z)\big|_{at\ z = e^{j2\pi k/N}}, \quad k = 0, 1, \ldots N-1.$

or $$X(k) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi nk/N}$$

Now, it may be noted that this is identical to the Fourier transform $X(e^{j\omega})$ evaluated at the N equally spaced frequencies i.e.,

$$\omega_k = 2\pi k/N, \quad k = 0, 1, \ldots N-1.$$

If the sequence x(n) has a finite duration of length N, then the z-transform is given as:

$$X(z) = \sum_{n=0}^{N-1} x(n) \cdot z^{-n}$$

Now, substituting the IDFT relationship for x(n), we obtain,

$$X(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N} \right] \cdot z^{-n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} (e^{j2\pi k/N} \cdot z^{-1})^{n}$$

or $$X(z) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot \frac{1-z^{-N}}{1-e^{j2\pi k/N} z^{-1}} = \frac{1-z^{-N}}{N} \cdot \sum_{k=0}^{N-1} \frac{X(k)}{1-e^{j2\pi k/N} \cdot z^{-1}}$$

This equation is identical to that of frequency sampling form.

Now, when this is evaluated over an unit circle, then we write

$$X(e^{j\omega}) = \frac{1-e^{-j\omega N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1-e^{j(\omega-2\pi k/N)}}$$

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Q.7. Explain circular convolution using matrix method.

→ The graphical method which we have just discussed is quite tedious, especially when many samples are present. While the matrix method is more convenient. In the matrix method, one sequence is repeated via circular shifting of samples. It is represented as under.

We have $y(m) = x(n) \, (N) \, h(n) = h(n) \, (N) \, x(n)$.

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(N-2) \\ y(N-1) \end{bmatrix} = \begin{bmatrix} h(0) & h(N-1) & h(N-2) & \cdots & h(2) & h(1) \\ h(1) & h(0) & h(N-1) & \cdots & h(3) & h(2) \\ h(2) & h(1) & h(0) & \cdots & h(4) & h(3) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ h(N-2) & h(N-3) & h(N-4) & \cdots & h(0) & h(N-1) \\ h(N-1) & h(N-2) & h(N-3) & \cdots & h(1) & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-2) \\ x(N-1) \end{bmatrix}$$

Q.8. Determine the following sequence $y(n) = x(n) \, (N) \, h(n)$ where $x(n) = \{1, 2, 3, 1\}$ & $h(n) = \{4, 3, 2, 2\}$
↑ ↑

Sol^n → We have, $y(m) = x_2(n) \, (N) \, x_1(n) = h(n) \, (N) \, x(n)$.

Using matrix method, we shall have.

Here, $x(0) = 1$, $x(1) = 2$, $x(2) = 3$, $x(3) = 4$. &

$h(0) = 4$, $h(1) = 3$, $h(2) = 2$, $h(3) = 2$.

Here, $N = 4$.

In the matrix form, we have.

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} h(0) & h(3) & h(2) & h(1) \\ h(1) & h(0) & h(3) & h(2) \\ h(2) & h(1) & h(0) & h(3) \\ h(3) & h(2) & h(1) & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} (4\times1)+(2\times2)+(2\times3)+(3\times1) \\ (3\times1)+(4\times2)+(2\times3)+(2\times1) \\ (2\times1)+(3\times2)+(4\times3)+(2\times1) \\ (2\times1)+(2\times2)+(3\times3)+(4\times1) \end{bmatrix} = \begin{bmatrix} 4+4+6+3 \\ 3+8+6+2 \\ 2+6+12+2 \\ 2+4+9+4 \end{bmatrix}$$

hence, we have, $\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$

Therefore,

$y(n) = x(n) \; Ⓝ \; h(n) = \{17, 19, 22, 19\}$ -- Ans.

Q.9. Find the linear convolution using circular convolution of the following sequences: $x(n) = (1,2,1)$, $h(n) = (1,2)$.

Sol^n → We have $x(n) = (1,2,1)$, $h(n) = (1,2)$.

Here, length of $x(n) = L = 3$.

& length of $h(n) = M = 2$.

Therefore, $N = L+M-1 = 3+2-1 = 4$.

i.e., we have to calculate 4-point DFT. i.e., $N=4$.

Let us make length of $x(n)$ & $h(n)$ equal to 4 by adding zeros at end.

Hence, $x(n) = \{1,2,1,0\}$. --- (i)

& $h(n) = \{1,2,0,0\}$ --- (ii)

we have, $X(k) = W_N \cdot X_N$ --- (iii).

The matrix for twiddle factor W_4 is given by

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \quad \text{--- (iv)}$$

Input matrix is given by, $X_N = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ --- (v)

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Substituting eq^n (iv) & (v) in eq^n (iii), we get

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+2+1+0 \\ 1-2j-1+0 \\ 1-2j+1-0 \\ 1+2j-1-0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2j \\ 0 \\ j \end{bmatrix}$$

Therefore, $X(k) = \{4, -2j, 0, j\}$

Now, let us find $H(k)$: $\quad H(k) = W_N \cdot h_N$

$$H(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+2+0+0 \\ 1-2j+0+0 \\ 1-2+0+0 \\ 1+2j+0+0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1-2j \\ -1 \\ 1+2j \end{bmatrix}$$

Therefore, $H(k) = \{\underset{\uparrow}{3}, 1-2j, -1, 1+2j\}$

Now, $Y(k) = X(k) \cdot H(k)$

Therefore, we have, $Y(k) = \{4, -2j, 0, j\} \cdot \{3, 1-2j, -1, 1+2j\}$

$Y(0) = X(0) \cdot H(0) = 4 \times 3 = 12.$

$Y(1) = X(1) \cdot H(1) = (-2j) \times (1-2j) = -2j + 4j^2 = -2j - 4.$

$Y(2) = X(2) \cdot H(2) = 0 \times (-1) = 0.$

$Y(3) = X(3) \cdot H(3) = (j) \times (1+2j) = j + 2j^2 = j - 2.$

Therefore, $Y(k) = \{Y(0), Y(1), Y(2), Y(3)\}$

or $\quad Y(k) = \{12, -2j-4, 0, j-2\}$ --- Ans.

Q.10. Determine the 8-point DFT of the following sequence.

$x(n) = \{1/2, 1/2, 1/2, 1/2, 0, 0, 0, 0\}.$

Use in-place radix-2 decimation in time FFT algorithm.

Sol^n → This flow graph has been shown in fig.

Here, $s_1(n)$ represents output of stage 1 & $s_2(n)$ represents output of stage 2. The different value twiddle factor are

$W_8^0 = e^0 = 1.$

$W_8^1 = e^{-j\frac{\pi}{4}} = 0.707 - j0.707.$

$W_8^2 = e^{-j\frac{\pi}{2}} = -j$

$W_8^3 = -0.707 - j0.707.$

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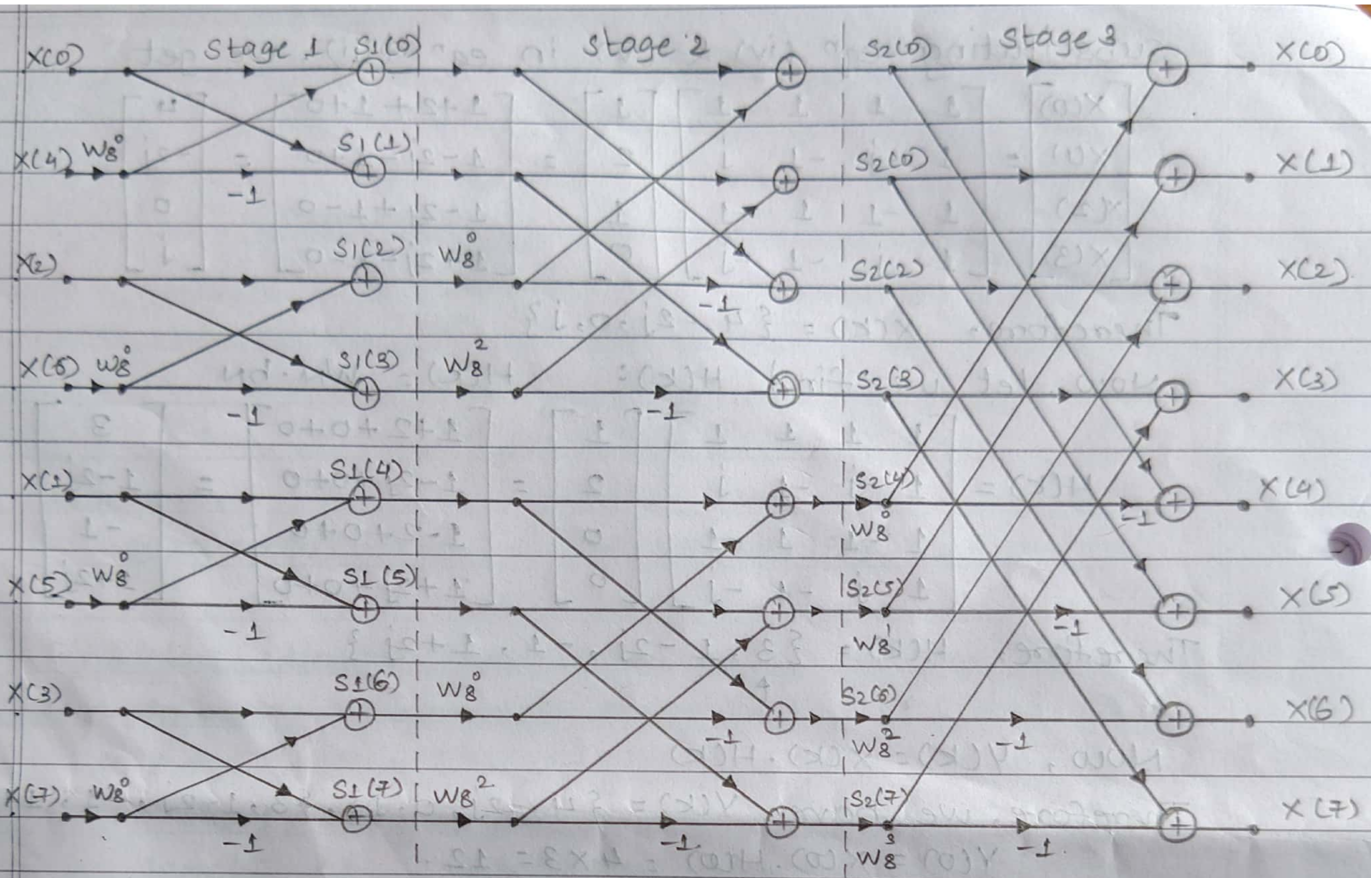

Output of stage 1:

$S_1(0) = x(0) + W_8^0 . x(4) = 1/2 + 1(0) = 1/2$

$S_1(1) = x(0) - W_8^0 . x(4) = 1/2 - 1(0) = 1/2$

$S_1(2) = x(2) + W_8^0 . x(6) = 1/2 + 1(0) = 1/2$

$S_1(3) = x(2) - W_8^0 . x(6) = 1/2 - 1(0) = 1/2$

$S_1(4) = x(1) + W_8^0 . x(5) = 1/2 + 1(0) = 1/2$

$S_1(5) = x(1) - W_8^0 . x(5) = 1/2 - 1(0) = 1/2.$

$S_1(6) = x(3) + W_8^0 . x(7) = 1/2 + 1(0) = 1/2$

$S_1(7) = x(3) - W_8^0 . x(7) = 1/2 - 1(0) = 1/2.$

Output of stage 2.

$S_2(0) = S_1(0) + W_8^0 \, S_1(2) = 1/2 + 1.(1/2) = 1.$

$S_2(1) = S_1(1) + W_8^2 \, S_1(3) = 1/2 - j\,1/2.$

$S_2(2) = S_1(0) - W_8^0 \, S_1(2) = 1/2 - 1/2 = 0$

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$S_2(3) = S_1(1) - W_8^2\, S_1(3) = 1/2 + j\,1/2.$

$S_2(4) = S_1(4) + W_8^0\, S_1(6) = 1/2 + 1/2 = 1.$

$S_2(5) = S_1(5) + W_8^2\, S_1(7) = 1/2 - j\,1/2.$

$S_2(6) = S_1(4) - W_8^0\, S_1(6) = 1/2 - 1/2 = 0$

$S_2(7) = S_1(5) - W_8^2\, S_1(7) = 1/2 + j\,1/2.$

Final output.

$X(0) = S_2(0) + W_8^0\, S_2(4) = 1 + 1 = 2.$

$X(1) = S_2(1) + W_8^1\, S_2(5) = (1/2 - j\,1/2) + (0.707 - j\,0.707)(1/2 - j\,1/2)$

$X(1) = 0.5 - j\,1.207.$

$X(2) = S_2(2) + W_8^2\, S_2(6) = 0 + (-j)(0) = 0.$

$X(3) = S_2(3) + W_8^3\, S_2(7) = (1/2 - j\,1/2) + (-0.707 - j\,0.707)(1/2 - j\,1/2)$

$X(3) = (1/2 - j\,1/2) + (0 - j\,0.707) = 0.5 - j\,0.207.$

$X(4) = S_2(0) - W_8^0\, S_2(4) = 1 - 1\cdot(1) = 0.$

$X(5) = S_2(1) - W_8^1\, S_2(5) = (1/2 - j\,1/2) - (0.707 - j\,0.707)(1/2 - j\,1/2)$

or $X(5) = (1/2 - j\,1/2) - (-0.707j) = 0.5 + j\,0.207.$

$X(6) = S_2(2) - W_8^2\, S_2(6) = 0 + j\cdot(0) = 0$

$X(7) = S_2(3) - W_8^3\, S_2(7) = (1/2 + j\,1/2) - (-0.707 - j\,0.707)(1/2 + j\,1/2)$

$X(7) = (1/2 + j\,1/2) + 0.707j = 0.5 + j\,1.21.$

Thus, we have

$X(k) = \{X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)\}$

or $X(k) = \{2, 0.5 - j\,1.207, 0, 0.5 - j\,0.207, 0, 0.5 + j\,0.207, 0, 0.5 + j\,1.21\}$

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