DEPARTMENT OF MATHEMATICS

Sample Questions **ONE MARK / TWO MARK**- Graph Theory [V Sem ISE & VII Sem Global]

Unit-1[Introduction to Graph theory]

1. Draw the line graph of the Konigsberg graph.
2. Determine the values of m and n such that graph Km,nis Eulerian.
3. What is the maximum number of edges a simple graph with 400 - vertices can have?
4. Write a graph which is regular, complete bipartite, complete and hypercube.
5. Is it possible to draw a graph that has a trail of length seven but no path of length seven?

   If so, draw such an example.
6. If Gis a simple graph with 15 edges and G has 13 edges then G has 13 edges then G

   has_________ vertices.
7. What is the maximum number of edges a simple graph with 13- vertices can have?
8. The complete bipartite graphKm,nis regular if and only if __________.
9. Is there exists a graph with 12 vertices and 28 edges, if the degree of each vertex is either 3 or 4?
10. With an example prove (or) disprove whether the complement of every regular graph is regular.
11. Decompose the following graph into copies of K1,3.
12. An undirected graph possesses Eulerian path if and only if it has _____________ number

    ofodddegree vertices.
13. Find a closed walk of length 12 in the following graph.
14. Model a simple graph on acquaintance relation among six people contain three mutual

    acquaintances and three mutual strangers.
15. Let G be the graph with vertex set {1,………,15} in whichi and j are adjacent if and

    only if their gcd exceeds1. Count the component of G.

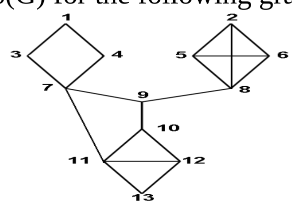
16. Provide a construction or a proof, whether the simple graph exists for the sequence {1,1,3,3,3,4,6,7}.

17. An undirected graph possesses Eulerian circuit if and only if it has _______number of odd degree vertices.

18. Find how many cycles of length 6 present in the Peterson graph?

19. Construct an example to show that d (u, v) > 2, then d (u) + d (v) ≤ (n + 1) – d (u, v), where u and v be any two vertices of a n-vertex simple graph, where d (u) – degree of the vertex u and d (u, v) is the distance betweenany two vertices u and v.

20. Determining K(G), λ(G) and δ(G) for the following graph given below



Unit-2[**Trees & Fundamental Circuits**]

1. With an example prove (or) disprove the number of components is the number of vertices minus the number of edges in a forest.

2. Let T be a tree with average degree of vertices is **a**. In terms of **a**, determine n (T).[vertices of T]

3. Let F1= (V1, E1) be a forest of seven trees, where E1 = 40. What is V1?

4. What is the eccentricity of each vertex in K5?

5. Compute the diameter and radius of K2,3.

6. Is it possible to have a tree with more than 5-vertices such that every vertex except the root and leaves has exactly the same number of ancestors and descendants? Explain by providing proof or construction.

7. Can the graph below be decomposed into edge-disjoint spanning trees? If yes write any two such trees.

8. If the following is the list of edges of all spanning trees of a graph G, determine G.

9. Define with a suitable example (i) Vertex connectivity (ii) Edge connectivity

10. Construct a graph G with edge connectivity of G = 4, vertex connectivity of G = 3 and degree of every vertex of G ≥ 3.

11. Construct a 3-regular planar graph of diameter 3 with 12-vertices.

12. Sketch 6 different binary trees on 6-vertices having 6 pendant vertices each.

13. Let $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees. If $|E_1|=19$ and $|V_2|=3|V_1|$, find $|V_1|,|V_2|$ and $|E_2|$.

14. Is it possible to have a tree with more than 5-vertices such that every vertex except the root and leaves has exactly the same number of ancestors and descendants? Explain by providing proof or construction.

15. How many internal vertices does a complete 5-any tree with 817 leaves have?

Unit-3[Planar Graphs]

1. Construct a 3-regular planar graph of diameter 3 with 12-vertices.

2. Write the dual of the wheel graph in 6-vertices.

3. The minimum number of edges deleted from the Peterson graph to obtain a planar sub graph is ___________.

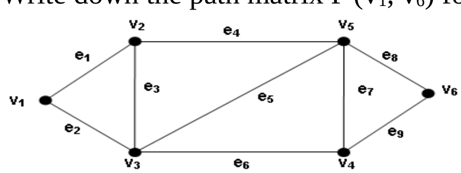
4. Is the compliment of 3-dimentional cube Q_3 is planar? Justify your answer with a neat sketch.

5. The minimum number of vertices & edges deleted from the Peterson graph to obtain a disconnected graph is _______&_______.

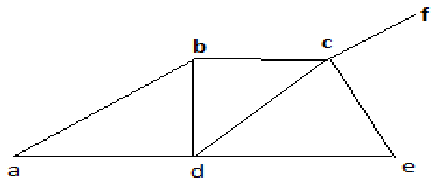
6. Define the term block.
7. What is the chromatic polynomial of a tree with 20-vertices?
8. The chromatic number of a cycle with 16-vertices is equal to ___________.
9. State decomposition theorem.
10. If 5- colors are used, how many ways can the vertices of K_7 are properly colored?
11. How many ways a null graph on 12-vertices can be properly colored with at most 4-colors?

Unit-4[Matrix Representation of Graph]

1. Write down the path matrix P (v_1, v_6) for the graph given below.



2. Write the incidence matrix of the following graph.



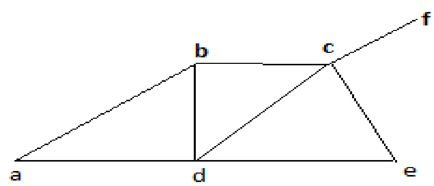
3. The rank of the incidence matrix A(G) of order 11 x 15 is _______________

4. Identical columns in a incidence matrix produce ___________ in a graph

5. Define adjacency matrix of the digraph

<u>Sample Questions **Essay type**- Graph Theory [V Sem ISE & VII Sem Global]</u>

Unit-1[Introduction to Graph theory]

1. A simple graph with n-vertices and k-components can have at most (n-k)(n-k+1)/2 edges.
2. Define the following terms with one example each i) Bipartite graph ii) Peterson graph
3. Prove that the number of odd degree vertices in a graph is always even.
4. For a graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degrees of vertices, show that $\delta \leq 2m/n \leq \Delta$.
5. With one example each define: i) Cycle ii) Eulerian circuit iii) Wheel graph ii) Regular graph
6. In the following graph, find (i) **b– b** circuit of length 6. (ii) **a – a** cycle of maximum length.
7. 

8. Show that in a complete graph of n vertices the degree of everyvertex is (n-1) and that the total number of edges is n(n-1)/2.
9. Let G be athe simple graph with the vertex set {1,…,11} defined by i□j if and only if i , j have acommon factor bigger than 1. Determine the blocks of G (i□j adjacency).
10. Let G be a connected graph with at least three vertices. Form G' from G by adding an edge withend points x, y whenever d(x, y) = 2. Prove that G' is 2-connected.
11. Define digraph. In every diagraph D, prove that $\sum d^-(v_i) = \sum d^+(v_i)$.
12. Let G be a graph with p-vertices, q-edges and regularity k.Prove that 2q = kp.
13. Prove that every tree has either one or two centers.
14. Give an example of a graph which contains

(i) Eulerian path but not Eulerian circuit(ii) Hamiltonian path but not Hamiltonian circuit

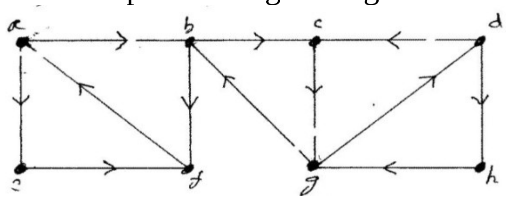
15. Define Euler digraph and Hamiltonian digraph with one example each.

16. For the digraph shown below, find the following:

(i) The directed walk of length 8

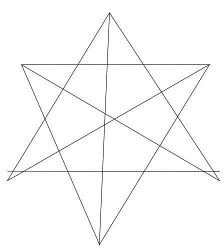
(ii)The directed Trail of longest length

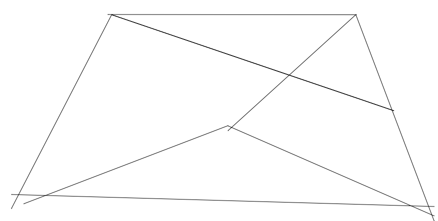
(iii) The directed path of longest length



17. Define Hamilton cycle. How many edge - disjoint Hamilton cycles exist in the complete graph with seven vertices? Also, draw the graph to show these Hamilton cycles

18. Define isomorphism of graphs. Show that no two of the following three graphs as shown in the figure are isomorphic.





19. Define the following with one example each.
i) Component of a graph. ii) Complete bipartite graph iii) complete graph.

20. Draw graphs representing problems of (a) two houses and three utilities (b) Four houses and four utilities, say water, gas, electricity & telephone.

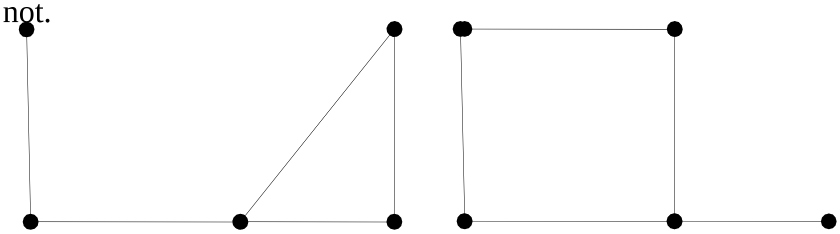
21. With suitable examples define the following:
a) Pendent & isolated vertex b) Regular graph c) Adjacent edges & vertices

22. Define the following terminology with a suitable example of a graph:
a) Sub graph and proper sub graph. b) Edge-disjoint and vertex-disjoint sub graph.
c) Union, Intersection and Ring sum of a graph. d) Complement of a sub graph.

23. Define the following terminology with a suitable example of a graph:
i) Finite and infinite graphs. ii) Labeled and unlabeled graphs. iii) Wheel graph.

24. With suitable examples define the following: i) Isolated graph ii) Connected & disconnected graph.

25. Describe the Konigsberg bridge problem. Does the solution exist according to Euler's theorem? For the graph to be a Euler graph what property should it possess?
26. What are the common properties of non-planar graph K_5 and $K_{3,3}$.
27. Determine (V) for the following graphs:
    i) G has 9 edges and all vertices have degree 3.
    ii) G is regular with 15 edges.
28. Define Euler circuit. Discuss Konigsberg bridge problem.
29. Define Isomorphism of graphs. Verify whether the following graphs are Isomorphic (or) not.

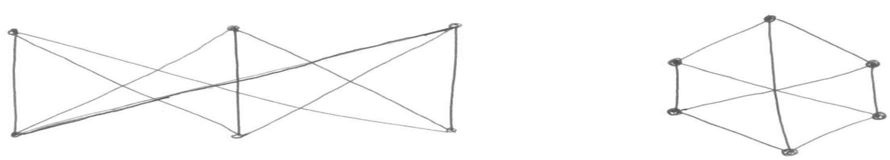


30. Prove the following:
    (i) A path with n vertices is of length (n-1).

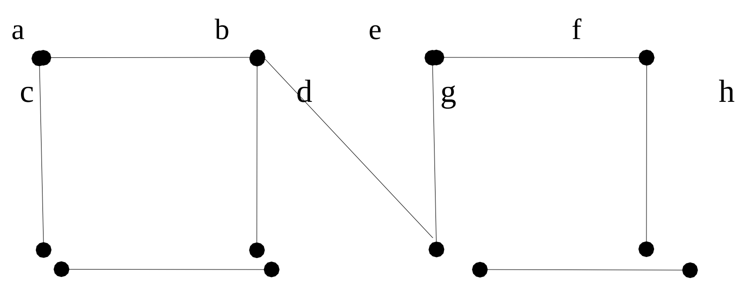
    (ii) If a circuit has n vertices, it has n edges.

    (iii)The degree of every vertex in a circuit is two.

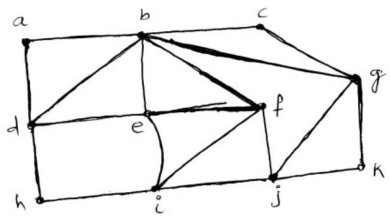
31. Show that in a (p, q) graph G is the number of vertices of odd degree is even.
32. Define isomorphism of graphs. Show that the following graphs are isomorphic.



33.
34. Let G = (V, E) be the undirected graph in the figure given below. How many paths are there in G from a to h? How many of these paths have a length 5?



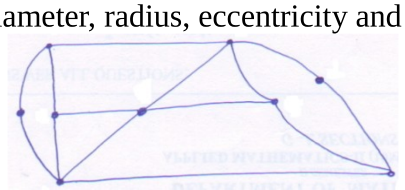
35. Find the Euler circuit for the graph shown in the figure



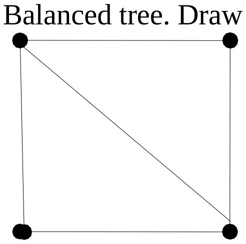
36. Show that the sum of the degrees of vertices of a undirected graph G is twice the number of edges.
37. Prove that every Euler digraph is strongly connected, but the converse is not necessarily true.

Unit-2[Trees & Fundamental Circuits]

1. Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and threevertices of degree 4. Find the number of pendant vertices in T.
2. Sketch a 15-vertex binary tree with one havingmaximum height & one having minimum height.
3. Define (i) Cut-set (ii) Fundamental cut set (iii) Fundamental circuit with one example each.
4. Define a tree, rooted tree, binary tree and also discuss some properties of a tree (at least two)
5. Show that a connected graph is a tree if and only if it is a minimally connected.
6. Prove that tree with n – vertices has (n-1) edges.
7. Prove that a tree with two or more vertices contains at least two pendant vertices.
8. Find the diameter, radius, eccentricity and center of the following graph.

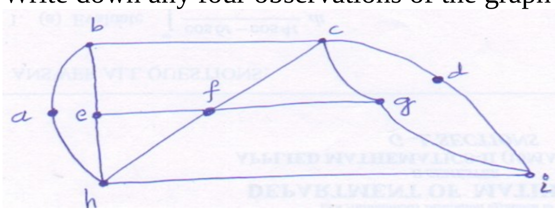


9. With suitable examples define the following:
   a. Edge connectivity & Vertex connectivity of a graph.
   b. Cut set & Fundamental cut set.
   c. Branch & Chord with respect to spanning tree.
10. Define i) Tree ii) Rooted tree. Prove that in every tree T = (V, E), $|V| = |E| + 1$.
11. Define: (i) Spanning tree (ii) Balanced tree. Draw all the spanning trees of the graph shown below.



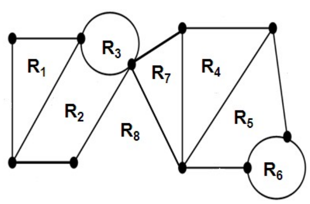
Unit-3[Planar Graphs]

1. A connected planar graph G with n-vertices and m-edges has exactly m-n+2 regions. If G is a connected simple planar graph with n ($\geq$ 3) vertices, m (>2) edges and r regions, then show that (i) m $\leq$ 3n-6 Further, if G is a triangle-free, then (ii) m $\leq$ 2n-4.
2. Define dual of a planar graph. Find the dual graph for the following planar graph shown in figure. Write down any four observations of the graph given below and its dual.

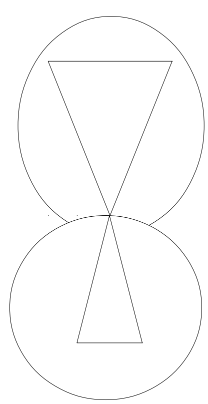


3. Let λ be the number of colors available to properly color the vertices of the graph K 2, 3. Then find
   (i) How many proper colorings of the graph have vertices {a , b} colored the same?
   (ii) How many proper colorings of the graph have vertices {a , b} colored with different colors?
   (iii) Find the chromatic polynomial for the graph $K_{2,n}$

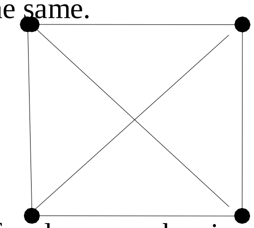
4. Find the dual graph for the following planar graph shown in figure. Write down any four observations of the graph given below and its dual.



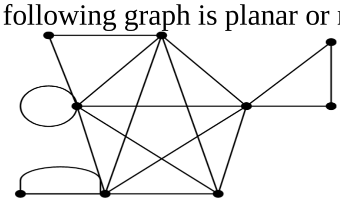
5. What isthe maximum number of edges possible in a simple connected planar graph with eightvertices? What is the minimum number of vertices necessary for a simple connected graph with11 edges to be a planar?
6. Prove that every tree with two or more vertices is 2-chromatic. Is the converse true?
7. Show that the graphs $K_{2,2}$ and $K_{2,3}$ are planar graphs.
8. Find the dual graph for the following planar graph shown in figure. Write down any four observations of the graph given below and its dual.



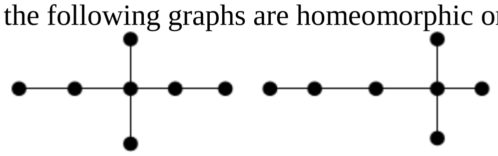
9. Define chromatic number of a graph. Find the chromatic polynomial for the graph shown below and also find the chromatic number for the same.



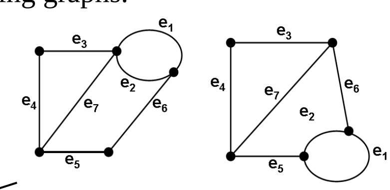
10. Explain the steps involved in detection of a planar graph using elementary reduction.
11. Determine whether the following graph is planar or not using elementary reduction.



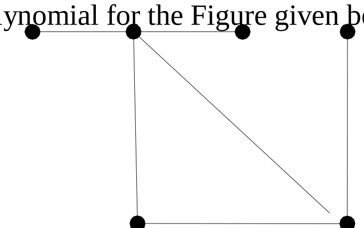
12. Verify that the following graphs are homeomorphic or not?



13. With suitable example verify the following:
14. The chromatic number of a graph equals the maximum of the chromatic numbers of its components.
15. If G is a connected graph, then $\chi(G) \leq 1 + a(G)$, where a (G) is the average of the vertex degrees.
16. Find the chromatic number of the following graphs.



17. Define the following terms with suitable example.
    (i)Homeomorphic graphs (ii) Chromatic polynomial of a planar graph
    (iii)Proper coloring of a graph (iv) Block cut point graph (v) k-connected graph
18. Find the chromatic polynomial for the Figure given below:

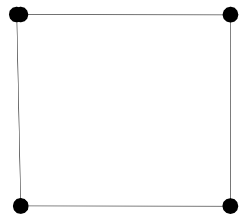


    If 4 colors are used, in how many ways can the graph be properly colored?
19. Define: (a) Matching (b) Maximal matching (c) Vertex covering (d) Minimal edge covering
20. Define: i) Edge covering ii) Chromatic number iii) Complete matching
21. Three boys b_1, b_2, b_3 and four girls g_1,g_2, g_3, g_4 are such that
    1. b_1 is a cousin of g_1, g_3, g_4
    2. b_2 is a cousin of g_2 and g_4

3. b_3 is a cousin of g_2 and g_3

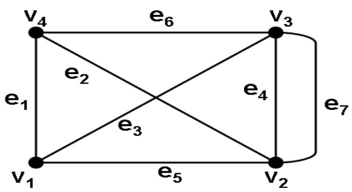
Can every one of the boys marry a girl who is one of his cousins? If so find the possible sets.

22. Define chromatic number. Find the chromatic polynomial for the cycle of length 4 as shown in the figure below. Hence find the chromatic number.

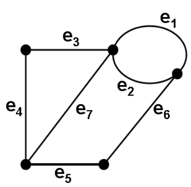


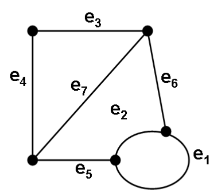
Unit-4[Matrix Representation of Graph]

1. Find a circuit matrix for the following graph. Write any four observations.



2. Find the incidence matrix of the following graphs.

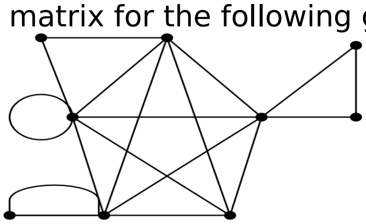




3. Draw the graph for which the following incidence matrix.

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

4. Write the incidence matrix for the following graph



5. Define the following with a suitable illustration:

   i) Path matrix ii) Incidence matrix iii) Cut-set matrix

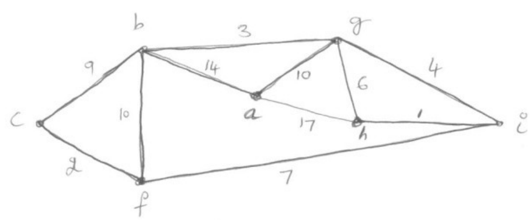
6. Define: (i) Incidence matrix of a graph. (ii) Isomorphic digraph.

7. List any three observations about the adjacency matrix.Draw the graph representing thefollowing matrix.

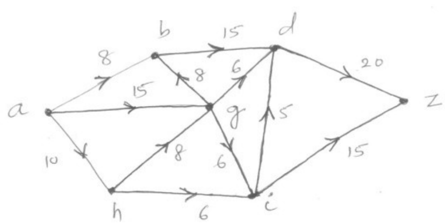
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Unit-5[Applications of graph -Algorithms]

1. State Kruskal's algorithm and using this algorithm find a minimal spanning tree for the weighted graph shown below:



2. Apply Dijkstra algorithm to the weighted graph G = (V, E) shown in the figure below and determine the shortest distance from vertex**'a'** to the vertex **' z'** in the graph.



3. Use Prim's algorithm to generate an optimal tree for the graph, shown in the figure.