LOCALIZATION IN SENSOR NETWORKS WITH FADING AND MOBILITY

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***Abstract*— In sensor networks the device localization is an interesting topic due to its relationship with routing and energy consumption. We propose a scheme to perform localization, based on the estimation of the power received by only two beacons placed in known positions. By starting from the received powers, eventually averaged on a given window to counteract interference and fading, the actual distance between the sensor and the beacons is derived and the position obtained by means of triangulation. The paper shows the effectiveness of this approach in different environments, by including the possible disturbance due to fading channels and sensor mobility.**

I. Introduction

The interest in sensor networks has been strongly increased in the last years, due to their attractive applications, ranging from on-field physical and biological measurement to information advertising on large market center depending on user positions.

Different applications are driven by common problems, i.e., the single unit should be a very low power consumption device to assure long battery life. The message transmission could be multi hop by using intermediate sensors with respect to collecting infrastructure; eventually, elaboration may be distributed between sensors, optimizing the whole network battery life. Other problems are that the single unit cost must be very low and that the position unit has to be identified.

Regarding the last point, let us stress that the sensor position could be the measure itself the sensor is performing in a mobile environment (e.g. by using Global Position System [1]) or it could be useful for signaling purpose, such as for the routing path identification inside the sensor network covered space.

Many works in literature propose schemes and methods for localization in sensor networks by estimating the received power for many purposes [2] [3]. Often, triangulation is applied in order to self-estimate the position; in some cases, some special nodes whose position is known, called beacons, are used. Furthermore, some papers investigate the possibility to place the beacons, as in [4], in order to permit a more accurate estimation.

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Other works, such as [5], deal with the problems occurring when disturbs are considered and the associate localization is compromised. Unfortunately, these works present results obtained in simple scenarios, where the disturbs and noise are modeled as uniform random variables that are to be added to power estimation [4], or no noise at all is considered [6].

In this paper, we present a simple scheme by which localization is possible with low energy and computation complexity costs. In the following, it will be considered a two-dimensional rectangular environment, not necessarily indoor and not necessarily delimited by walls. In our proposal, we use fixed beacons placed in two corners on the same side of the rectangular space, whose signals are used by sensors to compute their relative position. The main idea is that the sensors estimate the power levels received by the beacon and estimate their position by means of a triangulation method.

We have focused on two problems that make the localization difficult. The former is Rayleigh fading: the variation of the received power level as a function of time compromises the accuracy of the estimation based on power. In stationary conditions, a large number of power estimations, averaged on a given window, can solve the problem of fading. The latter is the sensor mobility: it avoids to use a large number of power estimations, since the received power value is changing when the node is moving. Fading and motion account two opposite behaviors from the averaging point of view, giving an interesting trade-off to be investigated. The challenge is to find the optimal window length to minimize the mean error of the estimated position.

The paper is structured as in the following: in the second section, the proposed technique is defined in an analytical way; furthermore, some preliminary considerations are also discussed. In the third section, simulation results are shown; some considerations about the impact of Rayleigh fading and motion are discussed. Rayleigh fading samples are obtained by Jakes' simulator [7]; a simple mobility model takes into account the motion, even though the sensors may be considered fixed. In the fourth section, conclusions and work in progress are discussed.

II. Geometric Definition of the Proposal

We consider to have a two-dimensional rectangular area in which beacon B_1 is on the origin (position $x = 0$, $y = 0$) and the second beacon B_2 is at position $x = X_a$, $y = 0$, where X_a and Y_a are the widths of the area. The beacons are placed on the same side of the rectangular area to allow the triangulation without ambiguity that can be present when the beacons are on two opposite corners. The beacons transmit continuously on two different frequencies, tuned by two simple filters on each sensor.

Let the received power be:

$$P_{r_j} = k d_j^{-\alpha}$$

where k is a constant which takes into account carrier frequency and transmitted power (equal for each beacons), d_j is the distance between the sensor and the j-th beacon, $j = 1, 2$, and α is the attenuation exponent.

The sensor estimates the received power levels and stores this information to have the possibility of performing one average with a deep of w samples to increase the accuracy of the estimation. Each node performs an averaging process for the signal incoming from each beacon. Two different averaging windows should be used to consider possible different attenuation exponents, even though in the following we take the two windows of the same length.

The averaged received power from the j-th beacon at the i-th sample is:

$$\overline{P}_{r_j}(i) = \frac{1}{w} \sum_{h=0}^{w-1} P_{r_j}(i-h)$$

where $P_{r_j}(t)$ is the t-th power sample.

Let us suppose that k and α are known to each sensor, so the distances at time instant i may be estimated by starting from the averaged received power:

$$d_j(i) = \left(\frac{\overline{P}_{r_j}(i)}{k} \right)^{-1/\alpha}$$

Consider the following two circumferences: the first centered on B_1 with radius d_1 and the second centered on B_2 with radius d_2. They have at most one cross point in the considered area. So, the position $(x(i), y(i))$ can be found as the solution of the following system:

$$\begin{cases} x^2(i) + y^2(i) = d_1^2(i) \\ (x(i) - X_a)^2 + y^2(i) = d_2^2(i) \\ 0 \le x(i) \le X_a \\ 0 \le y(i) \le Y_a \end{cases}$$

where the last two conditions are set to avoid solutions out of the area. The system solution is:

$$x(i) = \frac{d_1^2(i) - d_2^2(i) + X_a^2}{2X_a}$$
$$y(i) = \sqrt{d_1^2(i) - x^2(i)}$$

The errors on power estimation due to fading and motion give an error on sensor location, which can be evaluated by means of the following expression:

$$\epsilon(i) = \sqrt{[x(i) - \tilde{x}(i)]^2 + [y(i) - \tilde{y}(i)]^2}$$

where the couples $(x(i), y(i))$ and $(\tilde{x}(i), \tilde{y}(i))$ are the estimated and actual sensor positions, respectively.

The errors on $d_1(i)$ and $d_2(i)$ lead to have intersections out of the considered area or not intersection at all. Let us classify these exceptions in three groups; for each group it is necessary to give a further definition to estimate the actual position, since the previous system does not have any solution.

1) **Intersection out of the area**. The couple $(x(i), y(i))$ is chosen as the most near point on rectangle sides to the intersection point on the region $y \ge 0$. With more detail, by starting from the solution of the above system (first two equations) there is not matching with the given constraints (last two equations); the constraints may be forced by using the positions:

$$x(i) = \begin{cases} 0 & x(i) \le 0 \\ X_a & x(i) \ge X_a \\ x(i) & \text{otherwise} \end{cases}$$

$$y(i) = \begin{cases} Y_a & y(i) \ge Y_a \\ y(i) & \text{otherwise} \end{cases}$$

2) **One circumference includes the other** (no intersection). This situation occurs when $d_j < d_n - X_a$ and $d_n > d_j$ for $n \ne j$. The resulting sensor position is chosen overlapped to the position of beacon B_j, i.e.,

$$x(i) = \begin{cases} 0 & d_1 > d_2 \\ X_a & d_2 > d_1 \end{cases}$$
$$y(i) = 0$$

3) **The circumferences are disjoint** (no intersection), i.e., when $d_n + d_j < X_a$ for $n \ne j$. In this case the sensor position is selected on axis x, in the middle of the two circumferences:

$$x(i) = \frac{X_a + d_1(i) - d_2(i)}{2}$$
$$y(i) = 0$$

Consider first all nodes to be stationary; when fading is absent, the sensor position estimation is just perfect by using a unitary window ($w = 1$), since no amplitude variation is present and the noise is assumed to be vanishing with respect to the received power. Conversely, if channel fading is considered, the power samples have to be multiplied to a factor r^2, where r is a random variable accounting for the fading amplitude, here modelled with a Rayleigh pdf:

$$f_r(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

where σ^2 is the variance of the two complex quadrature components forming the real Rayleigh process; σ^2 is assumed equal to $1/2$, leading to: $E[r^2] = 2\sigma^2 = 1$, as to say, the Rayleigh process has no gain nor attenuation in terms of power.

The received power estimation results to be:

$$\overline{P}_{r_j}(i) = \frac{1}{w} \sum_{h=0}^{w-1} P_{r_j}(i-h) r^2(i-h)$$

The challenging question is to determine the window size w to have a certain bound of error in estimation with a certain probability. Note that the Rayleigh process to be considered is correlated, and the degree of correlation impacts the window size strongly.

Now, consider the possibility to have mobile sensors. Mobility increases the complexity of self-localization mechanism when also fading is considered; in fact, fading requires a large window size to mitigate its effect while mobility requires a short window to avoid the insertion of spurious position errors due motion, and this error increases as much as the position speed or window length increases. However, we assume a low mobility degree, since the sensors usually do not move with high rates and/or speed.

III. Numerical Results

Even though the system description is quite simple, the number of constraints and the correlations between random variables make the system hard to be investigated analytically, so the results presented here are obtained by means of simulations.

At the beginning of the simulation, we consider to have a certain number N of nodes uniformly distributed on the simulation rectangle, that has dimensions $X_a = Y_a = 100$ m; each simulation considers 10000 fading samples per a single value of w.

The correlated fading samples are generated by means of a Jakes' simulator [7] and independent realizations are considered for each beacon. From an intuitive point of view, the more fading is uncorrelated, the more the averaged estimation is accurate with a lower number of samples. The Jakes' simulator is driven by a single parameter, $d = f_d * T_U$, called the normalized maximum Doppler frequency, where T_U is the time step between two successive samples computation and f_d is the maximum Doppler frequency.

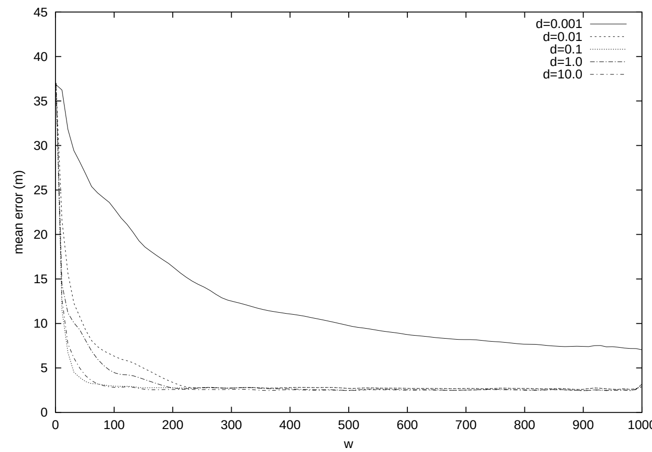

Fig. 1. Mean error as a function of window size (no mobility).

As far as the mobility model is concerning, we have integrated into our simulator the well-known Random Way-Point [8]: a node randomly chooses a destination point on the two-dimensional map; it walks to the target point with a constant speed, uniformly selected from the set $[0, V_{\max}]$, where $V_{\max}$ is the maximum speed of the mobile nodes. Once the target is reached, the sensor-node stops for a fixed time called Pause Time, then the mechanism is repeated. Since the target point is randomly chosen, the walk time is unpredictable.

In Figure 1 the mean error (i.e., the average of ϵ on many realizations) as a function of window size w is reported with $N = 20$ sensors on the simulation area. The nodes are considered stationary and the position errors are averaged on many different position estimations and on each node. When fading is not so correlated ($d = 10.0$), a short window size, e.g. $w = 50$, is enough to get the minima of the performance. The window size has to be increased when the normalized maximum Doppler frequency d decreases. For $d = 0.001$ a very huge window is necessary in order to have acceptable position errors. Note that the mean error never reaches the value of 0 and the best obtainable error is around few meters. In particular, an error lower bound of 2.5m may be identified for $w = 50$ for not correlated fading and this is the maximum resolution we have verified from this system in presence of Rayleigh fading. Furthermore, this spatial resolution is sufficient for a large set of applications.

The variance of the position error is reported in Figure 2 by taking into account the ϵ variation along the simulation and between the nodes. This variance gives a measure on the position error measure reliability. For some selected window size values, the mean error is about few meters and the relative

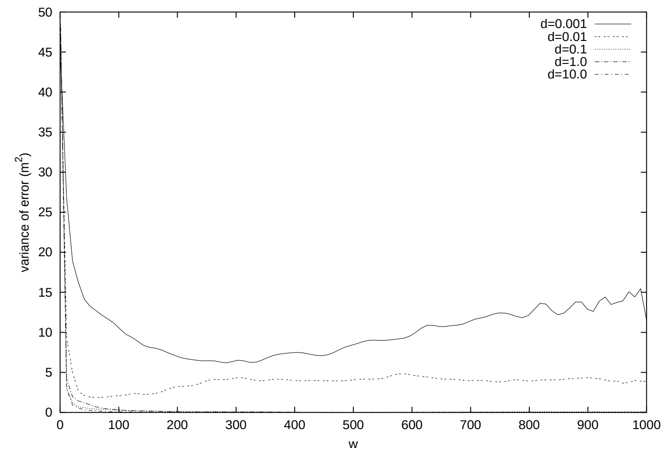

Fig. 2. Variance of error as a function of window size (no mobility).

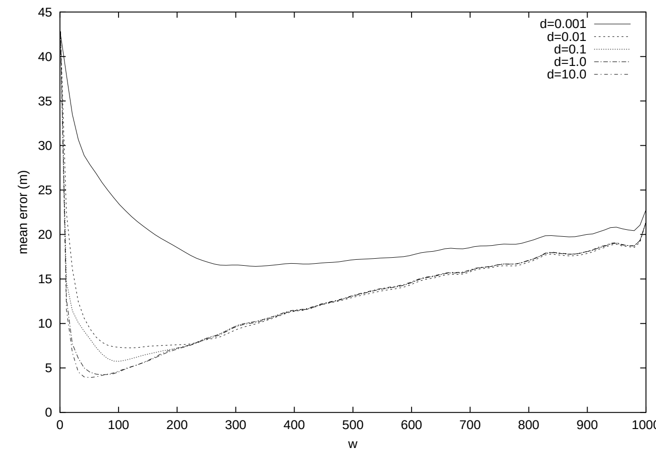

Fig. 4. Mean error as a function of window size ($V_{\max} = 0.3$ m $/T_U$).

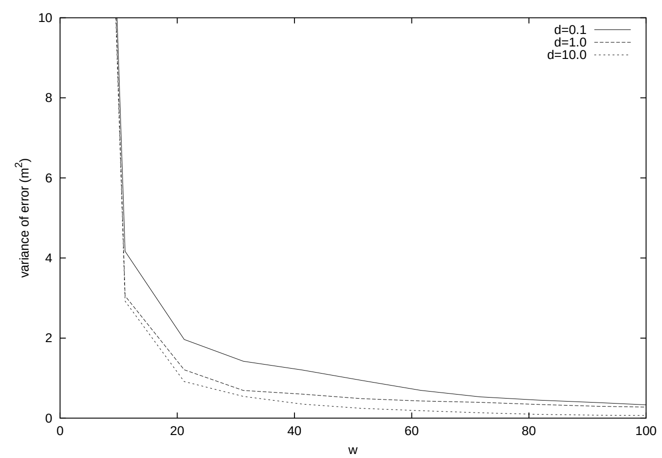

Fig. 3. Variance of error as a function of window size with zoom (no mobility).

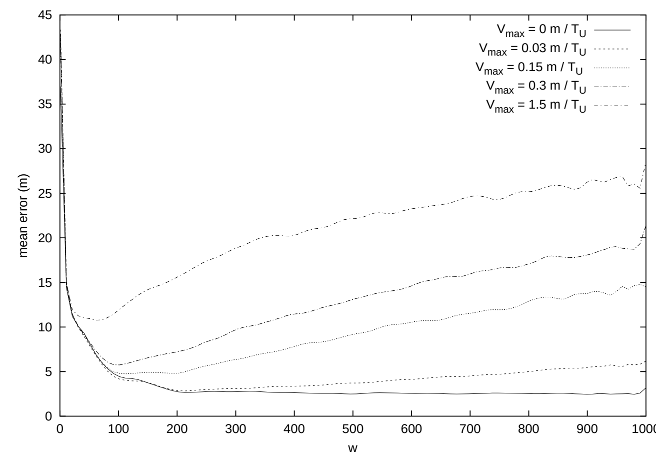

Fig. 5. Mean error as a function of window size ($d = 1.0$).

variance may be very close to zero; this occurs with not correlated fading. The variance oscillating behavior is due to the shortness of simulation time.

In Figure 3 a zoom of Figure 2 is plotted for $d = 0.1, 1.0, 10.0$. Note that for $w > 50$ the variance is very close to zero, giving to the mean value an high degree of reliability.

Until now, with stationary sensors, the system shows the best performance in fast fading environment. The effect of the mobility is introduced in Figure 4. All nodes are considered to move with $V_{\max} = 0.3\text{m}/T_U$ and a Pause Time of $30T_U$. Note that the best performance is yet obtained in fast fading environment, i.e., for high d values, as it is for the stationary case. Furthermore, with respect to the stationary case, the presence of a trade-off, underlined also in the introduction, is present. For low w values, the error increases due to fading, and this effect is mitigated from the average window as much as fading is fast, while for high w values, the error increases due to the mobility that make the old sample wrong with respect to the actual position. Note that the mobility case has a minimum error in the same region of the stationary one, around $w = 50$, even though the value of the position error doubles.

The impact of mobility speed is shown in Figure 5 for different value of $V_{\max}$ with $d = 1.0$. The error increases with the mobility speed, reaching value where the position estimation is difficult to be used. Fortunately, a sensor network is usually characterized to be stationary or with a low speed, so this result does not impair the system use.

One error cause is due to the three cases without intersection that we have overrun by fixing a conventional position; we refer to these types of cases as “bad events”. As the mean error reaches higher values with correlated fading, the same occurs for bad events.

Let us label the bad events as in the previous section, i.e., by means of three cases: 1) when intersection occurs out of the simulation area; 2) when one circumference includes the

d / w	$w = 10$	$w = 50$	$w = 100$	$w = 200$
$d = 0.001$	0.071	0.051	0.036	0.032
$d = 0.01$	0.053	0.024	0.013	0.011
$d = 0.1$	0.039	0.006	0.005	0.007
$d = 1.0$	0.041	0.014	0.010	0.009
$d = 10.0$	0.048	0.026	0.023	0.014

TABLE I

NUMBER OF BAD EVENTS OF TYPE 1 ON TOTAL ESTIMATIONS.

d / w	$w = 10$	$w = 50$	$w = 100$	$w = 200$
$d = 0.001$	0.136	0.031	0.003	< 0.001
$d = 0.01$	0.009	< 0.001	< 0.001	< 0.001
$d = 0.1$	0.003	< 0.001	< 0.001	< 0.001
$d = 1.0$	0.004	< 0.001	< 0.001	0.0
$d = 10.0$	0.008	< 0.001	< 0.001	< 0.001

TABLE II

NUMBER OF BAD EVENTS OF TYPE 2 ON TOTAL ESTIMATIONS.

d / w	$w = 10$	$w = 50$	$w = 100$	$w = 200$
$d = 0.001$	0.244	0.160	0.10	0.049
$d = 0.01$	0.117	0.030	0.006	0.003
$d = 0.1$	0.066	0.010	0.006	0.004
$d = 1.0$	0.067	0.013	0.004	0.003
$d = 10.0$	0.106	0.041	0.025	0.009

TABLE III

NUMBER OF BAD EVENTS OF TYPE 3 ON TOTAL ESTIMATIONS.

other; and 3) when circumferences are disjoint. In the following these three cases are investigated by means of tables, where the number of occurrences is reported normalized to the number of realizations.

The case 1) is reported in Table I. The bad events occurrence decreases as the window length increases and the fading increases in correlation. Similar considerations may be drawn for the case 2) and 3) reported in the tables II and III, respectively. Note that these errors occurrence is generically low, so their impact is vanishing and this is true as much as d is large and w is low.

IV. CONCLUSIONS AND FUTURE WORKS

In this paper, we proposed a simple localization system that can be applied in sensor networks, where energy consumption and computation complexity are critical. The system is based on triangulation of signal strength received from two beacons only and it is based on a buffer used as a sliding window. Our attention is focused on effects that can compromise the estimation of position. In particular, we focused on fading and mobility and we investigate the behavior by means of simulations.

The results prove that a small size of window is enough for an accurate estimation, at least for fading with low level of correlation. The numerical results also prove the existence of a window size trade-off when both fading and mobility are considered. Furthermore, some cases not resolvable from the system equation are identified and forced to a conventional value, by showing in the numerical results the vanishing impact of these decisions.

Possible further directions regards with: a more analytical investigation, by attempting to make hypothesis that leaves the system equation to be computed in a closed form; a more complex environment, by assuming more beacons; a more complex fading environment, such as Rice or Nakagami fading.

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