Course: Machine Learning - Foundations
Week 2 - Practice Questions

---

1. (1 point)

   Answer: False
   $sin(x)/x$ is not defined at x=0

2. (2 points)

   Answer: $A^c = [10, 29] \cup [51, 100]$
   $B^c = [10, 49] \cup [91, 100]$
   $A \cup B = [30, 90]$
   $A \cap B = [50]$
   $A^c \cap B^c = [10, 29] \cup [91, 100]$

3. (1 point)

   Answer: D
   $\mathbf{x}^T\mathbf{y} = \mathbf{x}.\mathbf{y} = \sum_{i=1}^{d} x_i y_i$

4. (1 point)

   Answer: D
   Linear approximation at $x = a$ is given by
   $L(x) = f(a) + (x - a)f'(a)$
   $tanx - tan0 = (x - 0)sec^2 0 = x$

5. (1 point)

   Answer: 3
   $\frac{\partial f}{\partial x}$
   $= 3x^2$
   $= 3(1)$

6. (1 point)

   Answer: A
   $\lim_{x \to 1^-} f(x) = 9$
   $\lim_{x \to 1^+} f(x) = 7 \times 1 + 2 = 9$
   $f(1) = 9$

7. (1 point)

   Answer: D
   Linear approximation at $x = a$ is given by
   $L(x) = f(a) + (x - a)f'(a)$
   Here a=0 and $f(x) = e^x$
   $e^x \approx 1 + x$

8. (2 points)

Answer: B
$f(x) \approx f(v) + \nabla f(v)^T (x - v)$
$= 2 + [2 \quad 2] \begin{bmatrix} x-1 \\ y-1 \end{bmatrix}$
$= 2x + 2y - 2$

9. (1 point)

Answer: B
$[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]$
$= [2xy \quad x^2]$
$= [6 \quad 1]$

10. (1 point)

Answer: -0.816
$\nabla f = [2x \quad 3 \quad 2z]^T$
at (1,2,1)
$\nabla f = [2 \quad 3 \quad 2]^T$
$\parallel i - 2j + k \parallel = \sqrt{6}$
Directional derivative=$\frac{2\times1-3\times2+2\times1}{\sqrt{6}}$

11. (1 point)

Answer: A
$\nabla f = [2x \quad 3y^2 \quad 4z^3]^T$
at (1,1,1)
$\nabla f = [2 \quad 3 \quad 4]^T$
$\parallel [2 \quad 3 \quad 4]^T \parallel = \sqrt{29}$
direction of steepest ascent=$\nabla f / \parallel f \parallel$

12.

Answer: 0.577
$\nabla f = [1 \quad 1 \quad 1]^T$
at (-1,1,-1)
$\nabla f = [1 \quad 1 \quad 1]^T$
$\parallel i - j + k \parallel = \sqrt{3}$
Directional derivative=$\frac{1\times1-1\times1+1\times1}{\sqrt{3}}$

13. (1 point)

Answer: Equation of a line passing through points a and b is given by
$[x, y, z] = a + \alpha(b - a)$
or
$[x, y, z] = b + \alpha(a - b)$

14. (1 point)

Answer: As per Cauchy Schwartz inequality
$a.b \leq \| a \| \| b \|$
Also,
$a.b = \| a \| \| b \|$, when
$a = cb$ or $b = ca$
Here b=-ca
so, $a.b = - \| a \| \| b \|$