Introduction

Business Analytics → Better business decision

In most problems, the decision maker is faced with **several decision alternatives** and an **uncertain or risk-filled pattern** of *future events*.

Risk analysis → **Favorable & unfavorable scenarios**

Decision analysis → **Decision alternatives & possible future events**

- Decision trees & payoff tables
- Sensitivity analysis
- Use of Bayes' theorem

Problem formulation

- Create verbal statement of the problem.
- Identify the **decision alternatives**:
  - The uncertain future events, referred to as **chance events**.
  - The **outcomes** associated with each combination of decision alternative and chance event outcome.

Pittsburgh Development Corporation (PDC):

- PDC commissioned preliminary architectural drawings for three different projects:
  - One with 30 condominiums.
  - One with 60 condominiums.
  - One with 90 condominiums.
- The financial success of the project depends on:
  - Selection of the size of the condominium complex.
  - The chance event concerning the demand for the condominiums.

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Decision alternatives

- The financial success of the project depends on:
  - The size of the condominium complex.
  - The chance event concerning the demand for the condominiums.

Chance

- In decision analysis, the possible outcomes for a chance event are the **states of nature**.
- The states of nature are mutually exclusive (no more than one can occur) and collectively exhaustive (at least one must occur).
- Thus, one and only one of the possible states of nature will occur.

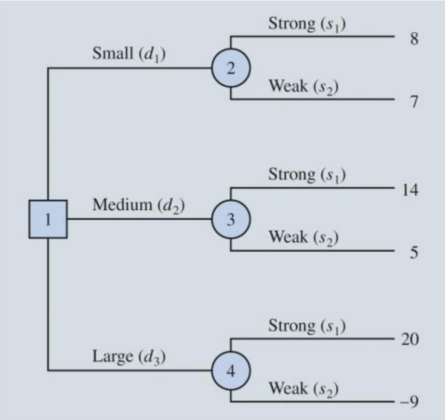
Payoff table

- **Payoff** is the outcome resulting from a specific combination of a decision alternative and a state of nature.
- **Payoff table** is a table showing payoffs for all combinations of decision alternatives and states of nature

	State of Nature	
Decision Alternative	**Strong Demand, s_1**	**Weak Demand, s_2**
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

Decision tree

- A **decision tree** provides a graphical representation of the decision-making process.
- Shows the natural or logical progression that will occur over time.



- Four nodes, numbered 1–4.
- **Nodes** → decisions and chance events.
  - Squares → **decision nodes**
  - Circles → **chance nodes**.
- The **branches** connect the nodes; those leaving the decision node correspond to the decision alternatives.
- The branches leaving each chance node correspond to the states of nature.
- The outcomes (payoffs) are shown at the end of the states-of-nature branches.

Decision analysis without probabilities

Optimistic approach

Conservative approach

Minimax regret approach

Optimistic approach

- The **optimistic approach** evaluates each decision alternative in terms of the best payoff that can occur.
- The decision alternative that is recommended is the one that provides the best possible payoff.
- For minimization problems, this approach leads to choosing the alternative with the smallest payoff.

	State of Nature	
Decision Alternative	**Strong Demand, s_1**	**Weak Demand, s_2**
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	−9

Decision Alternative	Maximum Payoff	
Small complex, d_1	8	
Medium complex, d_2	14	
Large complex, d_3	20 ←	Maximum of the maximum payoff values

Conservative approach

- The **conservative approach** evaluates each decision alternative in terms of the worst payoff that can occur.
- The decision alternative recommended is the one that provides the best of the worst possible payoffs.
- For problems involving minimization (for example, when the output measure is cost), this approach identifies the alternative that will minimize the maximum payoff.

Decision Alternative	Minimum Payoff ($ Millions)	
Small complex, d_1	7	← Maximum of the minimum payoff values
Medium complex, d_2	5	
Large complex, d_3	−9	

Minmax regret approach

- **Regret** is the difference between the payoff associated with a particular decision alternative and the payoff associated with the decision that would yield the most desirable payoff for a given state of nature.
- Regret is often referred to as **opportunity loss**.
- Under the **minimax regret approach**, one would choose the decision alternative that minimizes the maximum state of regret that could occur over all possible states of nature.

$$R_{ij} = |V_j^* - V_{ij}| \qquad (15.1)$$

where

R_{ij} = the regret associated with decision alternative d_i and state of nature s_j

V_j^* = the payoff value corresponding to the best decision for the state of nature s_j

V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_j

20 - 8

Decision Alternative	State of Nature Strong Demand, s_1	Weak Demand, s_2
Small complex, d_1	12	0
Medium complex, d_2	6	2
Large complex, d_3	0	16

Decision Alternative	Maximum Regret ($ millions)	
Small complex, d_1	12	
Medium complex, d_2	6	Minimum of the maximum regret
Large complex, d_3	16	

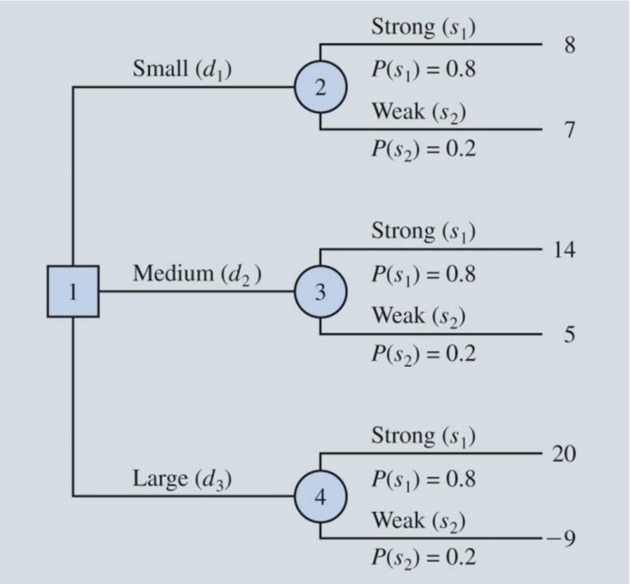
Decision analysis with probabilities

Expected value approach

- In decision-making situations where probability assessments for the states of nature are available, we can use the **expected value approach** to identify the best decision alternative.
- The **expected value (EV)** of a decision alternative is the sum of weighted payoffs for the decision alternative.

EXPECTED VALUE OF DECISION ALTERNATIVE d_i

$$\mathrm{EV}(d_i) = \sum_{j=1}^{N} P(s_j)V_{ij}$$

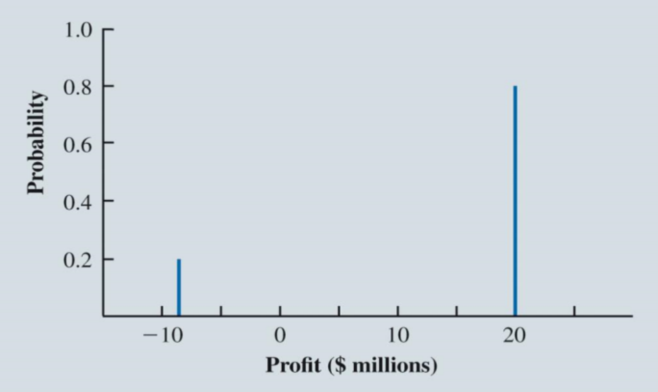


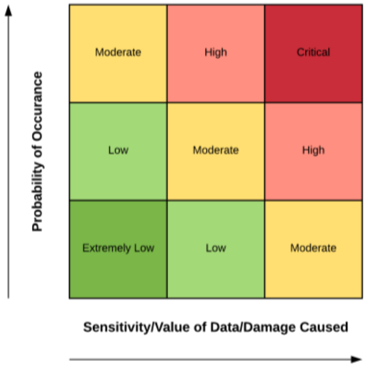
0.8(8) + 0.2(7) = 7.8
0.8(14) + 0.2(5) = 12.2
0.8(20) + 0.2(-9) = 14.2

Decision analysis with probabilities

Risk analysis

- Decision alternative and a state of nature combine to generate the payoff associated with a decision.
- **Risk profile** for a decision alternative shows the possible payoffs along with their associated probabilities.





https://www.lucidchart.com/techblog/2019/09/04/startup-security-conduct-risk-analysis/

Decision analysis with probabilities

Sensitivity Analysis

- **Sensitivity analysis** determines how changes in the probabilities for the states of nature or changes in the payoffs affect the recommended decision alternative.
- In many cases, the probabilities for the states of nature and the payoffs are based on subjective assessments.
- Sensitivity analysis helps the decision maker understand which of these inputs are critical to the choice of the best decision alternative.
- If a small change in the value of one of the inputs causes a change in the recommended decision alternative, the solution to the decision analysis problem is sensitive to that particular input.

0.7(8) + 0.3(7) = 7.7
0.7(14) + 0.3(5) = 11.3
0.7(20) + 0.3(-9) = 11.3

0.8(8) + 0.2(7) = 7.8
0.8(14) + 0.2(5) = 12.2
0.8(20) + 0.2(-9) = 14.2

0.9(8) + 0.1(7) = 7.9
0.9(14) + 0.1(5) = 13.1
0.9(20) + 0.1(-9) = 17.1

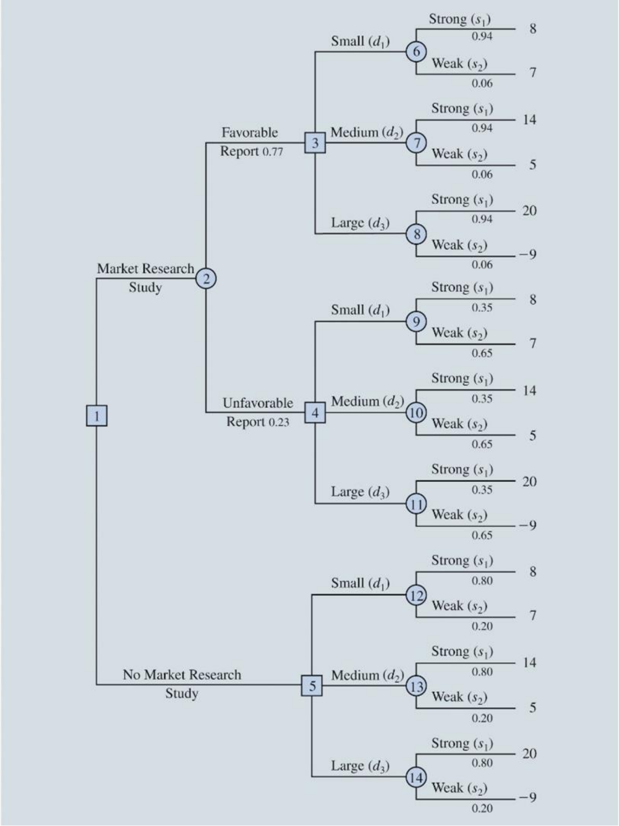
Decision analysis with Sample information

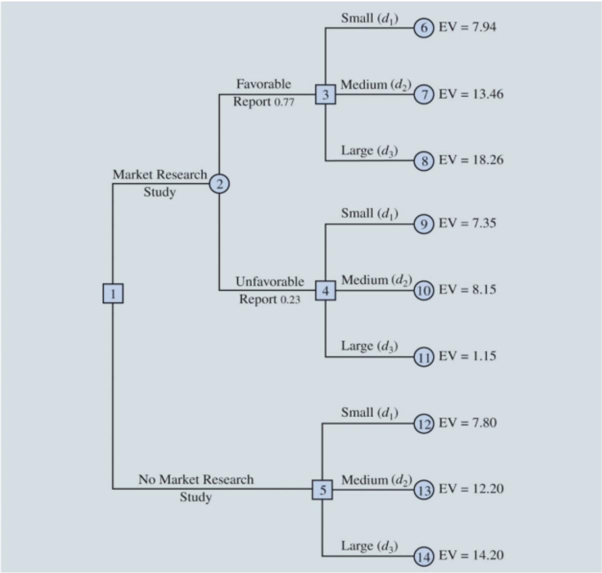
- Decision makers have the ability to collect additional information about the states of nature.
- Additional information is obtained through experiments designed to provide **sample information** about the states of nature.
- The preliminary or **prior probability** assessments for the states of nature that are the best probability values available prior to obtaining additional information.
- **Posterior probabilities** are revised probabilities after obtaining additional information.

➢ **Favorable report:** A substantial number of the individuals contacted express interest in purchasing a PDC condominium.

➢ **Unfavorable report:** Very few of the individuals contacted express interest in purchasing a PDC condominium.

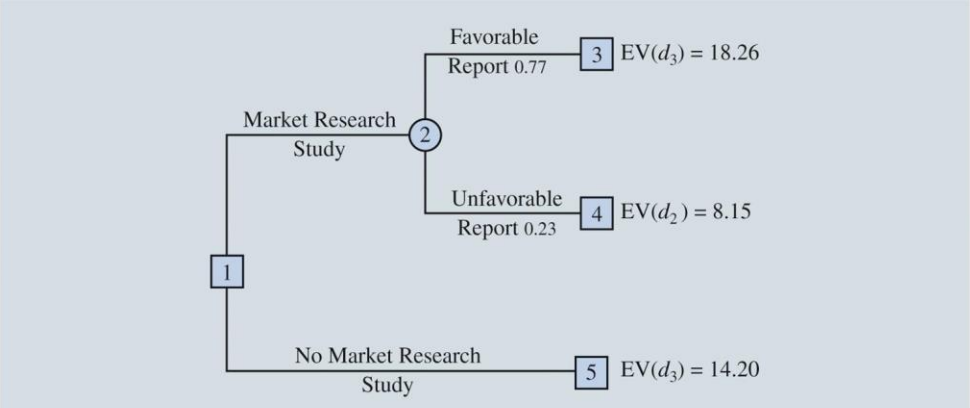
Decision analysis with Sample information

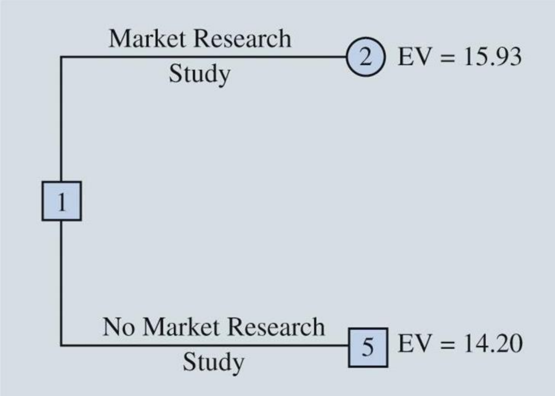




A **decision strategy** is a sequence of decisions and chance outcomes in which the decisions chosen depend on the yet-to-be-determined outcomes of chance events.

Decision analysis with Sample information





EXPECTED VALUE OF SAMPLE INFORMATION (EVSI)

$$\text{EVSI} = |\text{EVwSI} - \text{EVwoSI}| \tag{15.3}$$

where

EVSI = expected value of sample information

EVwSI = expected value *with* sample information about the states of nature

EVwoSI = expected value *without* sample information about the states of nature

$\boxed{15.93 - 14.20 = 1.73}$

Expected value of sample information

Decision analysis with Perfect information

EXPECTED VALUE OF PERFECT INFORMATION (EVPI)

$$\text{EVPI} = |\text{EVwPI} - \text{EVwoPI}| \tag{15.4}$$

where

EVPI = expected value of perfect information

EVwPI = expected value *with* perfect information about the states of nature

EVwoPI = expected value *without* perfect information about the states of nature

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Expected value with perfect information (EVwPI)

$$\boxed{0.8(20) + 0.2(7) = 17.4}$$

Expected value without perfect information (EVwoPI) = 14.2 (decision alternative, d_3)

Expected value of perfect information (EVPI) = 17.4 - 14.2 = 3.2 million

Computing with Bayes' theorem

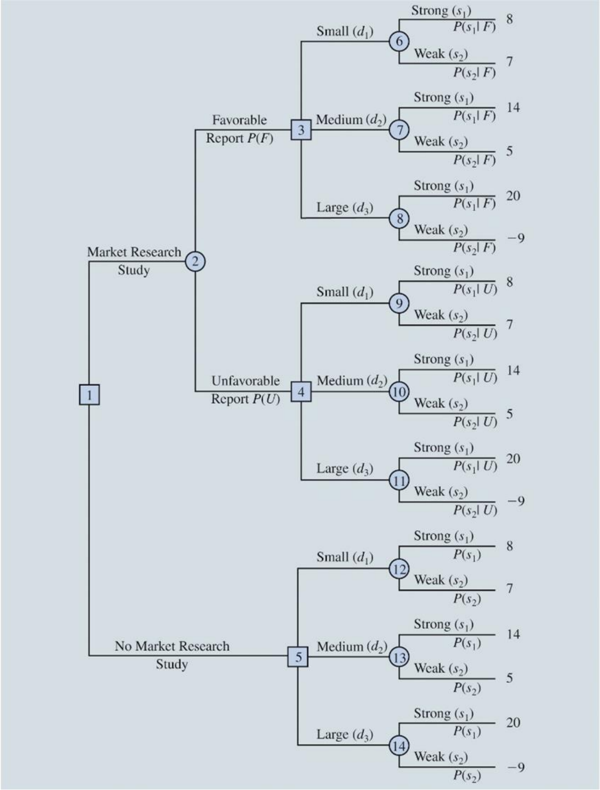
Bayes' theorem can be used to compute **branch probabilities** for **decision trees.**

$F =$ favorable market research report

$U =$ unfavorable market research report

$S_1 =$ strong demand (state of nature 1)

$S_2 =$ weak demand (state of nature 2)



Computing with Bayes' theorem

BAYES' THEOREM

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \cdots + P(A_n)P(B \mid A_n)} \quad \textbf{(15.5)}$$

	Market Research	
State of Nature	**Favorable, *F***	**Unfavorable, *U***
Strong demand, s_1	$P(F \mid s_1) = 0.90$	$P(U \mid s_1) = 0.10$
Weak demand, s_2	$P(F \mid s_2) = 0.25$	$P(U \mid s_2) = 0.75$

$$P(s_1 \mid U) = \frac{P(U \mid s_1)P(s_1)}{P(U \mid s_1)P(s_1) + P(U \mid s_2)P(s_2)}$$

$$= \frac{0.10 \times 0.80}{(0.10 \times 0.80) + (0.20 \times 0.75)} = 0.35$$