Optimization

Illustration: Par, Inc:

- Operations involved in manufacturing a golf bag:
  - Cutting and dyeing the material.
  - Sewing.
  - Finishing (inserting umbrella holder, club separators, etc.).
  - Inspection and packaging.

	Production Time (hours)	
Department	**Standard Bag**	**Deluxe Bag**
Cutting and Dyeing	$\frac{7}{10}$	1
Sewing	$\frac{1}{2}$	$\frac{5}{6}$
Finishing	1	$\frac{2}{3}$
Inspection and Packaging	$\frac{1}{10}$	$\frac{1}{4}$

Total time available

Department	Number of hours
Cutting and Dyeing	630
Sewing	600
Finishing	708
Inspection and Packaging	135

Required profit contribution:

- Standard bag: $10/unit.
- Deluxe bag: $9/unit.

Problem formulation

- Describe the objective.
- Describe each constraint.
- Define the decision variables.
- Write the objective in terms of the decision variables.
- Write the constraints in terms of the decision variables.

Constraint	Description
1	Number of hours of cutting and dyeing time used must be less than or equal to the number of hours of cutting and dyeing time available.
2	Number of hours of sewing time used must be less than or equal to the number of hours of sewing time available.
3	Number of hours of finishing time used must be less than or equal to the number of hours of finishing time available.
4	Number of hours of inspection and packaging time used must be less than or equal to the number of hours of inspection and packaging time available.

Maximization problem

Decision variables

- S = number of standard bags
- D = number of deluxe bags

Objective

Maximize 10**S** + 9**D**

Constraints

$$\begin{pmatrix}\text{Hours of cutting and}\\ \text{dyeing time used}\end{pmatrix} \le \begin{pmatrix}\text{Hours of cutting and}\\ \text{dyeing time available}\end{pmatrix} \quad \cdots\cdots\blacktriangleright \quad \frac{7}{10}S + 1D \le 630$$

$$\begin{pmatrix}\text{Hours of sewing}\\ \text{time used}\end{pmatrix} \le \begin{pmatrix}\text{Hours of sewing}\\ \text{time available}\end{pmatrix} \quad \cdots\cdots\blacktriangleright \quad \frac{1}{2}S + \frac{5}{6}D \le 600$$

$$\begin{pmatrix}\text{Hours of finishing}\\ \text{time used}\end{pmatrix} \le \begin{pmatrix}\text{Hours of finishing}\\ \text{time available}\end{pmatrix} \quad \cdots\cdots\blacktriangleright \quad 1S + \frac{2}{3}D \le 708$$

$$\begin{pmatrix}\text{Hours of inspection and}\\ \text{packaging time used}\end{pmatrix} \le \begin{pmatrix}\text{Hours of inspection and}\\ \text{packaging time available}\end{pmatrix} \quad \cdots\cdots\blacktriangleright \quad \frac{1}{10}S + \frac{1}{4}D \le 135$$

Non-negativity constraints $\cdots\cdots\blacktriangleright$ $S \ge 0$ and $D \ge 0$ or $S, D \ge 0$

Mathematical formulation

Maximize $10\mathbf{S} + 9\mathbf{D}$ *subject to*:

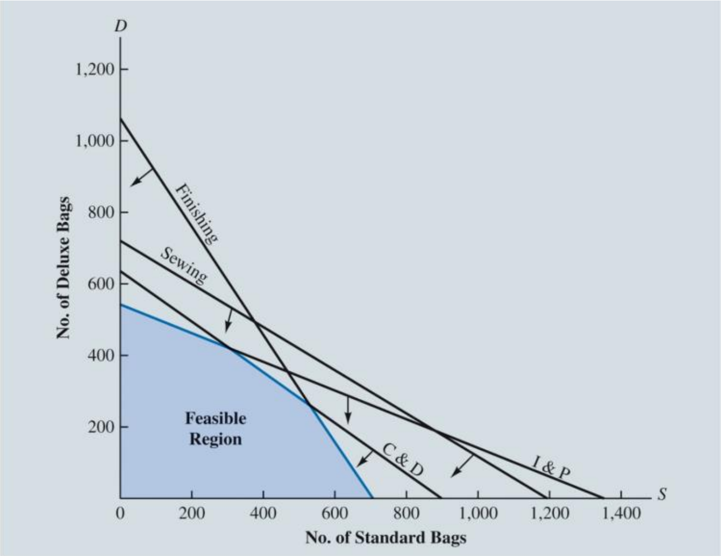
$$\begin{aligned}
&\frac{7}{10}S + 1D \leq 630 \quad \text{Cutting and dyeing} \\
&\frac{1}{2}S + \frac{5}{6}D \leq 600 \quad \text{Sewing} \\
&1S + \frac{2}{3}D \leq 708 \quad \text{Finishing} \\
&\frac{1}{10}S + \frac{1}{4}D \leq 135 \quad \text{Inspection and packaging} \\
&S, D \geq 0
\end{aligned}$$

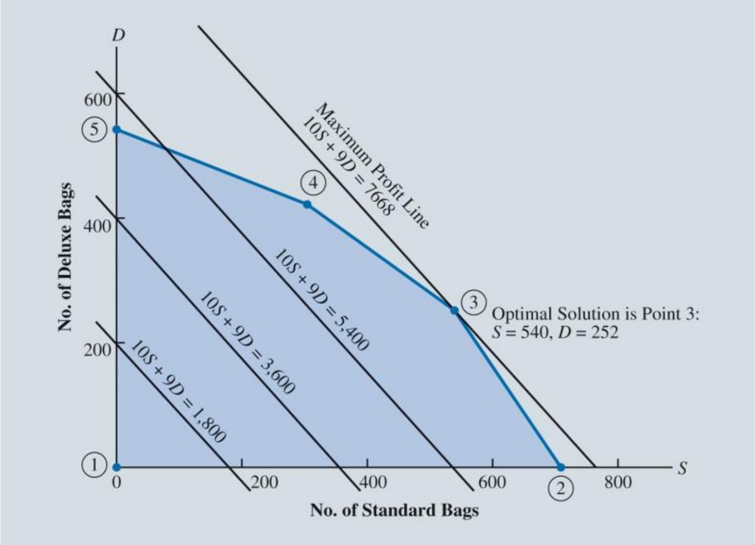
- This is a **linear programming model** (or **linear program**) because the objective function and all constraint functions are linear functions of the decision variables.

Optimal solution

- To find the optimal solution to the problem modeled as a **linear program**:
  - The optimal solution must have the highest objective function value.
  - The optimal solution must be a **feasible solution**—a setting of the decision variables that satisfies all of the constraints of the problem.
  - Search over the **feasible region**—a set of all possible solutions.
  - If constraints are inequalities, the constraint cuts the space in two:
    - The line and the area on one side of the line is the space the satisfies that constraint.
    - These subregions are called half spaces.
    - The intersection of the half spaces make up the feasible region.

Solving the Par, Inc. problem





- Based on the geometry of Figure, to solve a linear optimization problem we only have to search the **extreme points** of the feasible region to find the optimal solution.
- Extreme points are found where constraints intersect on the boundary of the feasible region.

Solving the Par, Inc. problem

Point	S	D	Profit = $10S + 9D$
1	0	0	$10(0) + 9(0) = 0$
2	708	0	$10(708) + 9(0) = 7,080$
3	540	252	$10(540) + 9(252) = 7,080$
4	300	420	$10(300) + 9(420) = 6,780$
5	0	540	$10(0) + 9(540) = 4,860$

- The approach of investigating only extreme points works well and generalizes for larger problems.
- The simplex algorithm, developed by George Dantzig, is quite effective at investigating extreme points in an intelligent way to find the optimal solution to even very large linear programs.

Excel Solver

	A	B	C	D
1	**Par, Inc.**			
2	**Parameters**			
3		**Production Time (Hours)**		**Time Available**
4	**Operation**	**Standard**	**Deluxe**	**Hours**
5	Cutting and Dyeing	=7/10	1	630
6	Sewing	=5/10	=5/6	600
7	Finishing	1	=2/3	708
8	Inspection and Packaging	=1/10	=1/4	135
9	Profit Per Bag	10	9	
10				
11	**Model**			
12				
13		**Standard**	**Deluxe**	
14	Bags Produced	1	1	
15				
16	**Total Profit**	=SUMPRODUCT(B9:C9,$B$14:$C$14)		
17				
18	**Operation**	**Hours Used**	**Hours Available**	
19	Cutting and Dyeing	=SUMPRODUCT(B5:C5,$B$14:$C$14)	=D5	
20	Sewing	=SUMPRODUCT(B6:C6,$B$14:$C$14)	=D6	
21	Finishing	=SUMPRODUCT(B7:C7,$B$14:$C$14)	=D7	
22	Inspection and Packaging	=SUMPRODUCT(B8:C8,$B$14:$C$14)	=D8	

	A	B	C	D
1	**Par, Inc.**			
2	**Parameters**			
3		**Production Time (Hours)**		**Time Available**
4	**Operation**	**Standard**	**Deluxe**	**Hours**
5	Cutting and Dyeing	0.7	1	630
6	Sewing	0.5	0.83333	600
7	Finishing	1	0.66667	708
8	Inspection and Packaging	0.1	0.25	135
9	Profit Per Bag	10	9.00	
10				
11	**Model**			
12				
13		**Standard**	**Deluxe**	
14	Bags Produced	1.00	1.00	
15				
16	**Total Profit**	$19.00		
17				
18	**Operation**	**Hours Used**	**Hours Available**	
19	Cutting and Dyeing	1.7	630	
20	Sewing	1.33333	600	
21	Finishing	1.66667	708	
22	Inspection and Packaging	0.35	135	

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Par, Inc.**												
2	**Parameters**												
3		**Production Time (Hours)**		**Time Available**									
4	**Operation**	**Standard**	**Deluxe**	**Hours**									
5	Cutting and Dyeing	0.7	1	630									
6	Sewing	0.5	0.83333	600									
7	Finishing	1	0.66667	708									
8	Inspection and Packaging	0.1	0.25	135									
9	Profit Per Bag	10	9.00										
10													
11	**Model**												
12													
13		**Standard**	**Deluxe**										
14	Bags Produced	540.00	252.00										
15													
16	**Total Profit**	$7,668.00											
17													
18	**Operation**	**Hours Used**	**Hours Available**										
19	Cutting and Dyeing	630	630										
20	Sewing	480.00000	600										
21	Finishing	708.00000	708										
22	Inspection and Packaging	117	135										
23													
24													
25													
26													
27													
28													
29													
30