IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 10 - Tutorial 05

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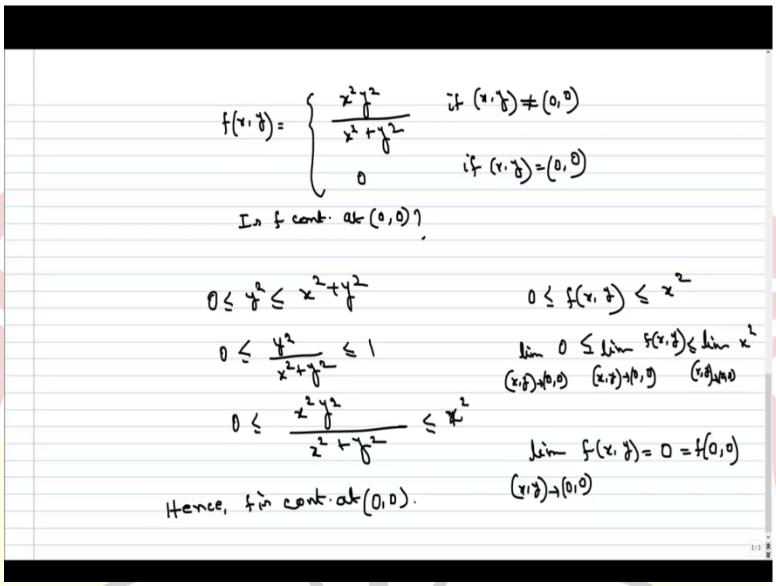
y 

Hello everyone, so in this video we will check the continuity of this function at origin. So again, this function is defined in two parts. So, in the origin it is 0, and otherwise it is $\frac{x^2y^2}{x^2+y^2}$. So, observe that this, we can write $0 \leq y^2 \leq y^2 + x^2$, this inequality is always true, because both x^2, y^2 are positive. And, obviously, $x^2 + y^2 \geq y^2$, because x^2 can be 0 or more than that, it cannot be negative, and y^2 is greater than or equal to 0. So, this inequality holds.

Now, I can write it as $0 \leq \frac{y^2}{y^2+x^2} \leq 1$. Now, if I multiply it with x^2, now x^2 is positive, so we will get $\frac{x^2y^2}{x^2+y^2}$ because this x^2 is basically greater than equal to 0, so it is non negative. So, I can write it, write this inequality.

So, my function, given function f(x, y), when (x,y) ≠ (0,0), then I am getting , now, we can use the theorem here $0 \leq f(x,y) \leq x^2$. So, I am taking (x, y) tending to origin. So, this is constant function 0. And this is the limit which we want to calculate, $\lim_{(x,y)\to 0} 0 \leq \lim_{(x,y)\to(0,0)} f(x,y) \leq \lim_{(x,y)\to 0} x^2$.

Now, this is only a function of x^2, and we as are tending to the origin, this is obviously going to 0. So, by theorem, we can conclude that limit of f(x,y) is 0 as (x,y) tends to origin, and which matches with the value at the origin f(0,0). So, this function, this limit exists and it matches with the value of the function at that point. Hence, we can conclude that f is continuous at origin. Thank you.