Determinants (Part 2)

Sarang S. Sane

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  Expanding with respect to the $1st$ row :

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$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}$$

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$$A = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 8 & 7 \\ 0 & 0 & 9 \end{bmatrix}$$

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An example

Lower Triangular

$A_{ij} = 0$ if $i<j$

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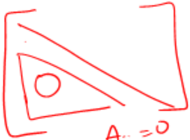
$A_{n \times n} =$



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U.T.

This is an upper triangular matrix. For such matrices, the determinant is the product of the diagonal elements.

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Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then $A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

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$$\text{Notation} : \ A^T \qquad \text{Definition} : \ (A^T)_{ij} = A_{ji}$$

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$$\begin{aligned} det(A^T) &= a_{11} \times det \begin{bmatrix} a_{22} & a_{32} \\ a_{23} & a_{33} \end{bmatrix} - a_{21} \times det \begin{bmatrix} a_{12} & a_{32} \\ a_{13} & a_{33} \end{bmatrix} + a_{31} \times det \begin{bmatrix} a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix} \\ &= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{21}(a_{12}a_{33} - a_{32}a_{13}) + a_{31}(a_{12}a_{23} - a_{22}a_{13}) \\ &= a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - a_{21}a_{12}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{31}a_{22}a_{13} \\ &= det(A) \end{aligned}$$

Minors and Cofactors

If A is an $n \times n$ square matrix with $n \leq 4$. Then the minor of the entry in the i-th row and j-th column is the determinant of the submatrix formed by deleting the i-th row and j-th column.

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Example : $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $M_{11} = det \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$.

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Above example : $C_{11} = (-1)^{1+1} M_{11} = M_{11}$

$C_{23} = (-1)^{2+3} M_{23} = -M_{23}$

Determinant in terms of minors and cofactors

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Observe that : For $A_{3\times 3}$

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This formula holds for $A_{2\times 2}$.

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This formula holds for $A_{2\times 2}$. We use it to generalize the determinant beyond $n = 3$. Generalization to $A_{4\times 4}$:

Determinant in terms of minors and cofactors

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Definition

$det(A) = \Sigma_{j=1}^{4}(-1)^{1+j}a_{1j}M_{1j} = \Sigma_{j=1}^{4}a_{1j}C_{1j}$

$$= a_{11}\times det\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44}\end{bmatrix} - a_{12}\times det\begin{bmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44}\end{bmatrix} + a_{13}\times det\begin{bmatrix} \end{bmatrix} - a_{14}\times det\begin{bmatrix} \end{bmatrix}$$

Inductive definition of the determinant

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$$det(I_n) = det(I_{n-1}) = det(I_{n-2}) = \cdots det(I_3) = det(I_2) = 1.$$

Definition

$det(A) = \Sigma_{j=1}^{n}(-1)^{1+j}a_{1j}M_{1j} = \Sigma_{i=1}^{n}a_{1j}C_{1j}$

$$det(A) = a_{11}\times det\begin{bmatrix} a_{22} & a_{23} & \cdots & a_{2n} \\ a_{32} & a_{33} & \cdots & a_{3n} \\ & & \cdots & a_{nn}\end{bmatrix} - a_{12}\times det\begin{bmatrix} a_{21} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ & & & a_{nn}\end{bmatrix} + a_{13}\times det[\ \ ] - a_{14}\times det[\ \ ]$$

$$det(I_{n\times n}) = det\left(\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1\end{bmatrix}\right) = 1\times det\left(\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1\end{bmatrix}\right) - 0\times det(\ \ ) + 0\times det(\ \ ) \cdots$$

$$= 1\times det(I_{n-1})$$

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expansion along the i^{th} row

$$= \Sigma_{i=1}^{n}(-1)^{i+j}a_{ij}M_{ij} \quad \text{for a fixed } j$$

expansion along the j^{th} column

$$det(A_{3\times3}) = (-1)^{2+1} a_{21} \times M_{21} + (-1)^{2+2} a_{22} \times M_{22} + (-1)^{2+3} a_{23} \times M_{23}$$

$$= -a_{21} \times det\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} + a_{22} \times det\begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} - a_{23}\, det\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix}$$

$$= (-1)^{1+2} a_{12} \times det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + (-1)^{2+2} a_{22} \times det\begin{bmatrix} & \\ & \end{bmatrix} + (-1)^{2+3} a_{32} \times det\begin{bmatrix} & \\ & \end{bmatrix}$$

Important properties and identities

Property 1 : Determinant of a product is product of the determinants.

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$det(A^n) = det(A)^n$.

$det(A^{-1}) = \frac{1}{det(A)} = det(A)^{-1}$.

$det(P^{-1}AP) = det(A)$.

$det(AB) = det(BA)$

$det(A^TA) = det(A)^2$.

$det(A^T)$

$\|$ expand along 1^{st} column + induction

$det(A)$

Important properties and identities

Property 2 : Switching two rows or columns changes the sign.

$$A = \begin{array}{c} i^{th} \rightarrow \\ \\ j^{th} \rightarrow \end{array} \begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \\ & & & \\ a_{j1} & a_{j2} & \cdots & a_{jn} \end{bmatrix} \qquad \tilde{A} = \begin{bmatrix} a_{j1} & a_{j2} & \cdots & a_{jn} \\ & & & \\ a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix} \begin{array}{c} \leftarrow i^{th} \text{ row} \\ \\ \leftarrow j^{th} \text{ row} \end{array}$$

$$\det(\tilde{A}) = -\det(A)$$

Pf Expand along i^{th} row & use induction

Property 3 : Adding multiples of a row to another row leaves the determinant unchanged.

A

$$\tilde{A} = \begin{array}{c} \\ \left[\begin{array}{ccccc} a_{i1}+ta_{j1} & a_{i2}+ta_{j2} & \cdots & & a_{in}+ta_{jn} \\ & & & & \\ a_{j1} & a_{j2} & \cdots & \cdots & a_{jn} \\ & & & & \end{array}\right] \end{array} \begin{array}{l} \leftarrow i^{th} \text{ row} \\ \\ \\ \\ \end{array}$$

k^{th} column

$$\tilde{\tilde{A}} = \left[\begin{array}{ccc} a_{1k}+ta_{1\ell} & & \\ a_{2k}+ta_{2\ell} & & \\ \vdots & & \\ a_{nk}+ta_{n\ell} & & \end{array}\right]$$

$\det(\tilde{A}) = \det(A)$

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Important properties and identities

Property 3 : Adding multiples of a row to another row leaves the determinant unchanged.

Property 3' : Adding multiples of a column to another column leaves the determinant unchanged.

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A

$$\tilde{A} = \begin{bmatrix} & & & \\ ta_{i1} & ta_{i2} & \cdots & ta_{in} \\ & & & \end{bmatrix}$$

$$\det(\tilde{A}) = \sum_{j=1}^{n} (-1)^{i+j} t a_{ij} \tilde{M}_{ij} = t\left(\sum_{j=1}^{n} (-1)^{i+j} a_{ij} \tilde{M}_{ij}\right) = t\left(\sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij}\right) = t \det(A).$$

Warning: $\det(tA_{n\times n}) = t^n \det(A)$

Property 4' : Scalar multiplication of a column by a constant t multiplies the determinant by t.

Useful computational tips

1) The determinant of a matrix with a row or column of zeros is 0.

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2) The determinant of a matrix in which one row (or column) is a linear combination of other rows (resp. columns) is 0.
3) Scalar multiplication of a row by a constant t multiplies the determinant by t.
4) While computing the determinant, you can choose to compute it using expansion along a suitable row or column.

Thank you