Week-1
Mathematics for Data Science - 2
Activity Question

Note:

- Here the set of natural numbers $\mathbb{N}$ does not include zero.
- A sequence $\{a_n\}$ is said to be an increasing sequence if $a_n \leq a_{n+1}$, for all $n \in \mathbb{N}$
- A sequence $\{a_n\}$ is said to be a decreasing sequence if $a_n \geq a_{n+1}$, for all $n \in \mathbb{N}$

1 Lecture 1:

1.1 Level 1:

1. Which of the following may represent a function $f : \mathbb{R} \to \mathbb{R}$?

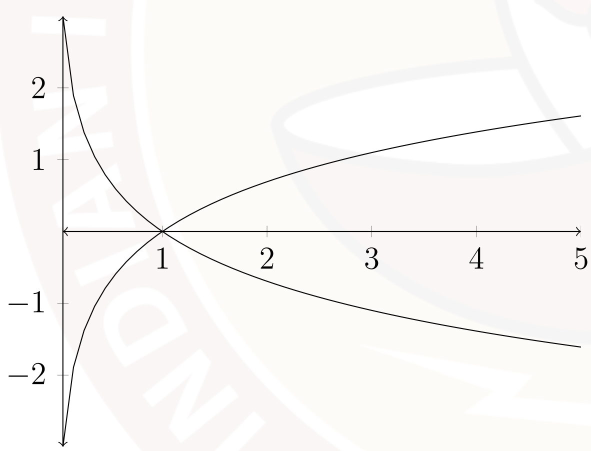


Figure:(a)

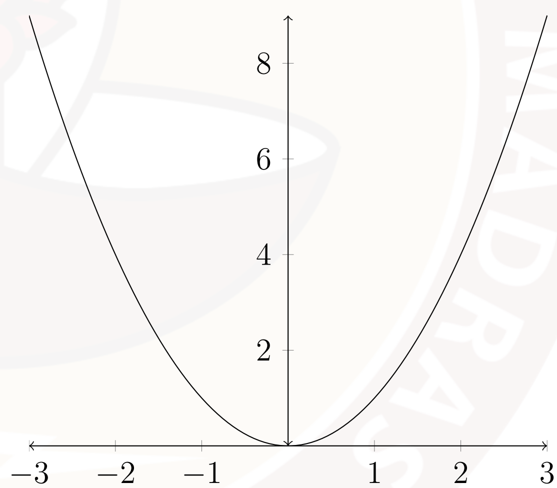


Figure: (b)

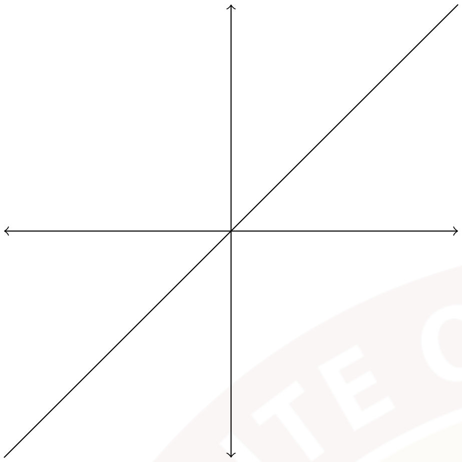


Figure: (c)

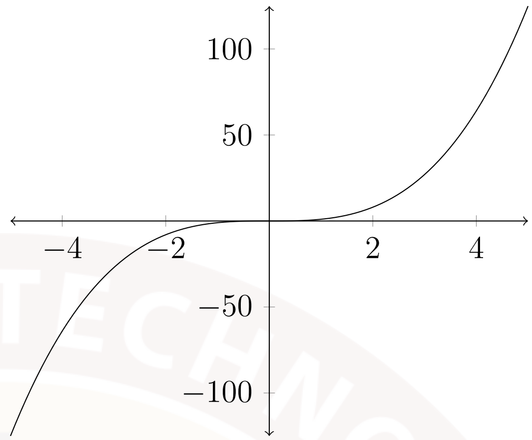


Figure: (d)

- ◯ Option 1: Figure (a)
- ◯ **Option 2:** Figure (b)
- ◯ **Option 3:** Figure (c)
- ◯ **Option 4:** Figure (d)

2. Which of the following represents a linear function?
   - ◯ **Option 1:** $f(x) = x + 5$
   - ◯ Option 2: $f(x) = 2x^2 + 5x + 10$
   - ◯ Option 3: $f(x) = \ln(x^2 + 5)$
   - ◯ Option 5: $f(x) = e^x$

3. Which of the following represents a quadratic function?
   - ◯ Option 1: $f(x) = 7x + 10$
   - ◯ **Option 2:** $f(x) = 3x^2 + 9x + 199$
   - ◯ Option 3: $f(x) = \ln(2x^2 + 10)$
   - ◯ Option 4: $f(x) = e^2 x$

4. Domain of the function $f(x) = \dfrac{x-1}{x^2 - 5x + 6}$ is
   - ◯ **Option 1:** $\mathbb{R} \setminus \{2, 3\}$
   - ◯ Option 2: $\mathbb{R} \setminus \{5, 6\}$
   - ◯ Option 3: $\mathbb{R} \setminus \{1, 5, 6\}$
   - ◯ Option 4: $\mathbb{R} \setminus \{1, 2, 3\}$
   - ◯ Option 5: $\mathbb{R} \setminus [2, 3]$
   - ◯ Option 6: $\mathbb{R} \setminus \{[2, 3] \cup [1, 2]\}$

1.2 Level 2:

5. Which of the following option(s) is(are) true?

   ◯ **Option 1:** All quadratic functions are polynomial functions.

   ◯ Option 2: All polynomial functions are quadratic functions.

   ◯ **Option 3:** All linear functions are polynomial functions.

   ◯ Option 4: All quadratic functions are linear functions.

6. Assume the following Figure M2W1AQ 1 represents the graph of a function $f : \mathbb{R} \to \mathbb{R}$.

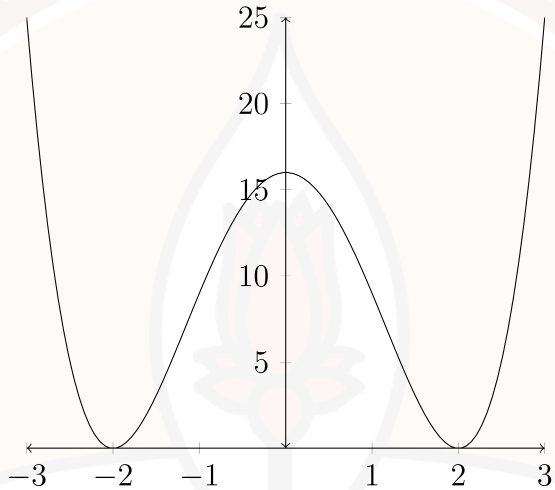


Figure M2W1AQ 1

Which of the following option(s) is(are) true?

   ◯ Option 1: In the interval $[-\infty, -2]$, the function is increasing.

   ◯ Option 2: In the interval $[-2, 2]$, the function is decreasing.

   ◯ **Option 3:** In the interval $[-2, 0]$, the function is increasing.

   ◯ **Option 4:** In the interval $[0, 2]$, the function is decreasing.

   ◯ Option 5: In the interval $[0, \infty]$, the function is increasing.

   ◯ **Option 6:** In the interval $[2, 3]$, the function is increasing.

7. In the following Figure M2W1AQ 2, $C_1, C_2,$ and C_3 represent curves.

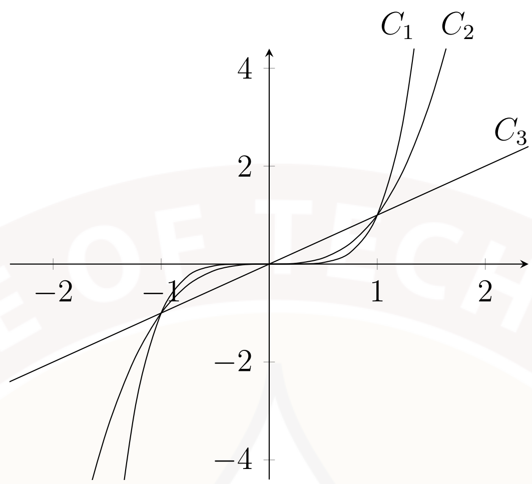


Figure M2W1AQ 2

Which of the following option(s) is(are) true?

- ◯ **Option 1:** Among these three curves, C_1 is the fastest growth curve.
- ◯ Option 2: Among these three curves, C_2 is the fastest growth curve.
- ◯ Option 3: Among these three curves, C_3 is the fastest growth curve.
- ◯ **Option 4:** C_1 is a faster growing curve than C_2.
- ◯ Option 5: C_3 is a faster growing curve than C_2.
- ◯ **Option 6:** C_2 is a faster growing curve than C_3.

8. Which of the following option(s) is(are) true?

- ◯ Option 1: Any linear function is always an increasing function.
- ◯ Option 2: Any quadratic function is always an increasing function.
- ◯ **Option 3:** e^x is always an increasing function.
- ◯ **Option 4:** $\ln x$ is always an increasing function.

2 Lecture 2:

2.1 Level 1:

Consider the following right angle triangle ACB in the Figure M2W1AQ 3.

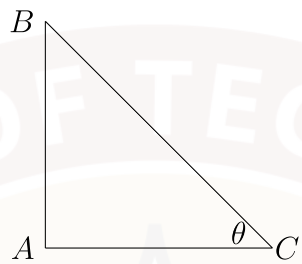


Figure M2W1AQ 3

1. Which of the following option(s) is(are) true?
   - ◯ Option 1: $\sin\theta = \frac{AB}{AC}$
   - ◯ Option 2: $\cos\theta = \frac{AB}{AC}$
   - ◯ **Option 3:** $\tan\theta = \frac{AB}{AC}$
   - ◯ Option 4: $\sin\theta = \frac{AC}{BC}$
   - ◯ **Option 5:** $\cos\theta = \frac{AC}{BC}$
   - ◯ **Option 6:** $\text{cosec}\ \theta = \frac{BC}{AB}$
   - ◯ **Option 7:** $\sec\theta = \frac{BC}{AC}$
   - ◯ Option 8: $\cot\theta = \frac{AB}{AC}$
2. If $f(x) = x^2 + 2x + 3$ and $g(x) = x + 1$, where $x \neq -1$, then which of the following is (are) true?
   - ◯ **Option 1:** $\frac{f}{g}(2) = \frac{11}{3}$
   - ◯ Option 2: $(f + g)(1) = 7$
   - ◯ Option 3: $(2f)(3) = 18$
   - ◯ **Option 4:** $(fg)(0) = 3$
3. Suppose $\sin(x + y) = \sin x \cos y + \cos x \sin y$, where $0 < x, y < \frac{\pi}{2}$, then which of the following is(are) true?
   - ◯ **Option 1:** $\sin 2x = 2 \sin x \cos x$
   - ◯ Option 2: $\sin 2x = 2 \sin x \sin x$
   - ◯ Option 3: $\sin 2x = \sin 2x \cos x$
   - ◯ Option 4: $\sin 2x = \sin x \cos 2x$

4. Suppose $\cos(x+y) = \cos x \cos y - \sin x \sin y$, where $0 < x, y < \frac{\pi}{2}$,, then which of the following is(are) true?

   - ◯ **Option 1:** $\cos 2x = \cos^2 x - \sin^2 x$
   - ◯ Option 2: $\cos 2x = \sin^2 x - \cos^2 x$
   - ◯ Option 3: $\cos 2x = \cos^2 2x - \sin^2 2x$
   - ◯ Option 4: $\cos 2x = \sin^2 2x - \cos^2 2x$

2.2 Level 2:

Figure M2W1AQ 4 shows the graph of $\sin\theta$ and Figure M2W1AQ 5 shows the graph of $\cos\theta$. Use the information to answer the questions 5,6 and 7

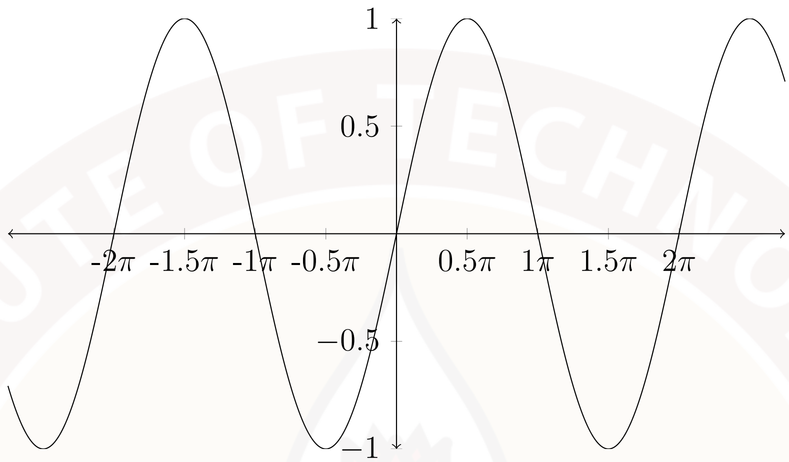


Figure M2W1AQ 4

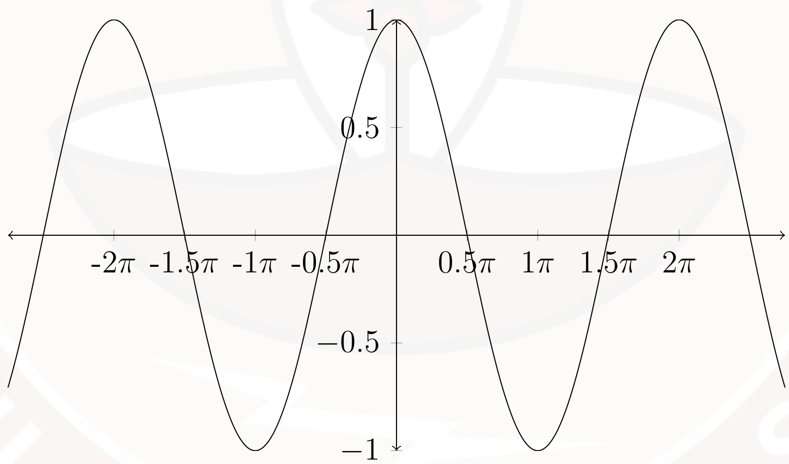


Figure M2W1AQ 5

5. Let $0 < \theta < \frac{\pi}{2}$, which of the following option(s) is(are) true?

   [**Hint:** Shift the curves in the figures by $\frac{\pi}{2}$ units.]

   - ◯ **Option 1:** $\sin(\frac{\pi}{2} + \theta) = \cos\theta$
   - ◯ Option 2: $\sin(\frac{\pi}{2} + \theta) = -\cos\theta$
   - ◯ Option 3: $\sin(\frac{\pi}{2} - \theta) = -\cos\theta$

- ◯ **Option 4:** $\sin(\frac{\pi}{2} - \theta) = \cos\theta$

6. Let $0 < \theta < \frac{\pi}{2}$, which of the following option(s) is(are) true?

   [**Hint:** Shift the curves in the figures by $\frac{\pi}{2}$ units.]

   - ◯ Option 1: $\cos(\theta + \frac{\pi}{2}) = \sin\theta$
   - ◯ **Option 2:** $\cos(\theta + \frac{\pi}{2}) = -\sin\theta$
   - ◯ **Option 3:** $\cos(\frac{\pi}{2} - \theta) = \sin\theta$
   - ◯ Option 4: $\cos(\frac{\pi}{2} - \theta) = -\sin\theta$

7. Let $0 < \theta < \frac{\pi}{2}$, which of the following option(s) is(are) true?

   [**Hint:** Shift the curves in the figures by π units.]

   - ◯ **Option 1:** $\sin(\pi + \theta) = -\sin\theta$
   - ◯ Option 2: $\sin(\pi + \theta) = \sin\theta$
   - ◯ **Option 3:** $\sin(\pi - \theta) = \sin\theta$
   - ◯ Option 4: $\sin(\pi - \theta) = -\sin\theta$
   - ◯ Option 5: $\sin(\pi + \theta) = \cos\theta$
   - ◯ Option 6: $\sin(\pi + \theta) = \cos\theta$

8. Let $f(x) = \frac{x}{x+a}$, where $x > 0$ and $a > 0$. If $f(f(x)) = \frac{x}{3x+4}$, then find the value of a [Ans: 2]

3 Lecture 3:

3.1 Level 1:

1. Consider a function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 + 1$. Which of the following options represents the graph of the function?

   ◯ **Option 1:** $\{(x, x^2 + 1) \mid x \in \mathbb{R}\}$

   ◯ Option 2: $\{(x, x^2) \mid x \in \mathbb{R}\}$

   ◯ Option 3: $\{(x, 1) \mid x \in \mathbb{R}\}$

   ◯ Option 4: $\{(1, x^2 + 1) \mid x \in \mathbb{R}\}$

2. Let C be a curve in the following Figure M2W1AQ 6.

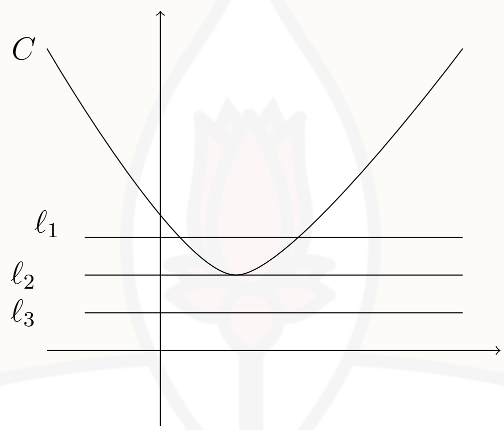


Figure M2W1AQ 6

Which of the following lines is the tangent at some point to the curve C?

   ◯ Option 1: ℓ_1

   ◯ **Option 2:** ℓ_2

   ◯ Option 3: ℓ_3

   ◯ Option 4: None of the above

3. Let C be a curve in the following Figure M2W1AQ 7.

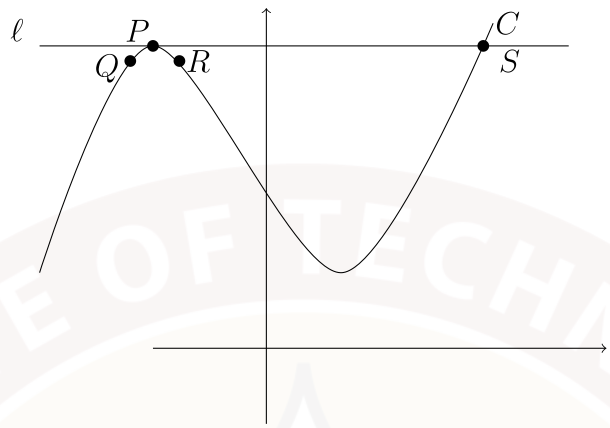


Figure M2W1AQ 7

Let ℓ be a straight line which touches the curve C at a point according to the Figure M2W1AQ 7. Then ℓ is the tangent to the curve at point

◯ **Option 1:** P

◯ Option 2: Q

◯ Option 3: R

◯ Option 4: S

4. Let C be a curve and three straight lines $\ell_1,\ \ell_2$ and ℓ_3 pass through the point P in the following Figure M2W1AQ 8.

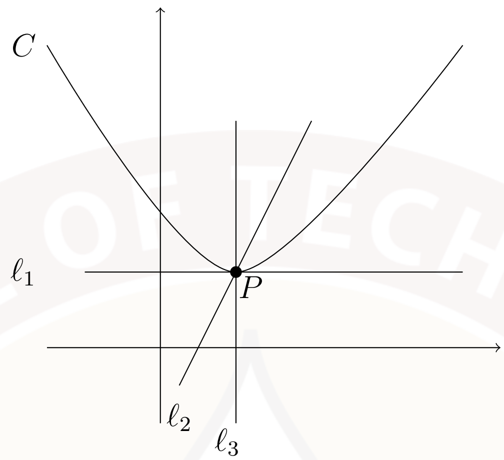


Figure M2W1AQ 8

Which of the following straight lines is the tangent to the curve C?

- ◯ **Option 1:** ℓ_1
- ◯ Option 2: ℓ_2
- ◯ Option 3: ℓ_3
- ◯ Option 4: None of the above

5. Which of the following options represents the graph of a quadratic function?
   - ◯ Option 1: $\{(x, x^3 + 2x + 1) \mid x \in \mathbb{R}\}$
   - ◯ **Option 2:** $\{(x, x^2 + 4x + 5) \mid x \in \mathbb{R}\}$
   - ◯ Option 3: $\{(x, x^4 + 5x^2 + 3) \mid x \in \mathbb{R}\}$
   - ◯ Option 4: $\{(x, 9x) \mid x \in \mathbb{R}\}$

3.2 Level 2:

6. Let $f : A \to B$ be a function, where A and B are subsets of $\mathbb{R}$ and cardinalities of the sets A and B are 4 and 1, respectively. What is the cardinality of $\Gamma(f)$, the graph of f? [Ans: 4]

Let C represent a curve in the Figure M2W1AQ 9. Answer the questions 7 and 8 using the curve C.

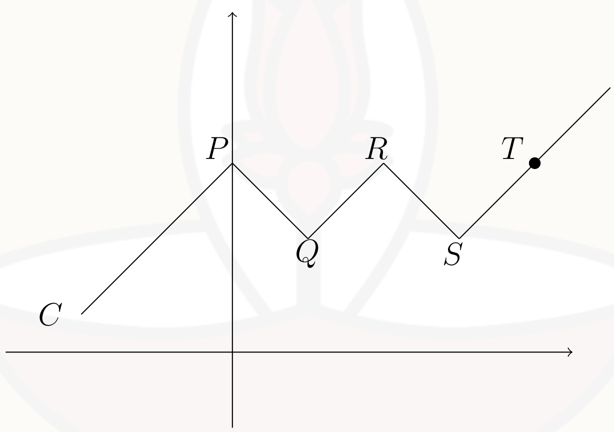


Figure M2W1AQ 9

7. Which of the following option(s) is(are) true?
   - ◯ Option 1: At point P, there is a tangent.
   - ◯ **Option 2:** At point Q, there is no tangent.
   - ◯ Option 3: At point P, there are an infinite number of tangents.
   - ◯ Option 4: None of the above.

8. Let A be the set points where the curve C has no tangents. Find a lower bound for the cardinality of the set A. [Ans: 4]

4 Lecture 4:

4.1 Level 1:

1. Consider the following functions, where $\mathbb{R}$ is the set of real numbers, $\mathbb{N}$ is the set of natural numbers, $\mathbb{Z}$ is the set of integer numbers, $\mathbb{R}^+$ is the set of positive real numbers, and $\mathbb{R}^-$ is the set of negative real numbers.
   $f_1 : \mathbb{R} \to \mathbb{R}$
   $f_2 : \mathbb{N} \to \mathbb{R}$
   $f_3 : \mathbb{R}^- \to \mathbb{R}$
   $f_4 : \mathbb{R}^+ \to \mathbb{R}$
   $f_5 : \mathbb{N} \to \mathbb{Z}$
   Which of the following represents a sequence?
   - ○ Option 1: f_1
   - ○ **Option 2:** f_2
   - ○ Option 3: f_3
   - ○ Option 4: f_4
   - ○ **Option 5:** f_5

2. Let $\{a_n\}$ be a sequence defined as $a_n = \dfrac{3n^2 + 5n + 2}{4n^2 + 2n + 1}$. The limit of the sequence $\{a_n\}$ is
   - ○ Option 1: $\frac{3}{2}$
   - ○ **Option 2:** $\frac{3}{4}$
   - ○ Option 3: 2
   - ○ Option 4: 0
   - ○ Option 5 :$+\infty$

3. Consider a sequence $\{n^2\}$. Which of the following options are subsequences of the given sequence?
   - ○ **Option 1:** $\{(n+5)^2\}$
   - ○ **Option 2:** $\{(2n+1)^2\}$
   - ○ Option 3: $\{(2n+1)^3\}$
   - ○ Option 4: $\{n^3\}$
   - ○ **Option 5:** $\{n^2\}$
   - ○ **Option 6:** $\{(2n)^2\}$
   - ○ Option 7: $\{(\frac{1}{2}n+1)^2\}$
   - ○ **Option 8:** $\{(an+1)^2\}$, where $a \in \mathbb{Z} \setminus \{-1\}$

4.2 Level 2:

4. Let $\{a_n\}$ be a sequence converging to the limit 1. Then which of the following option(s) about the mentioned subsequences is(are) true?

   - ◯ **Option 1:** $\lim\limits_{n\to\infty} a_{3n} = 1$
   - ◯ Option 2: $\lim\limits_{n\to\infty} a_{2n} = 2$
   - ◯ Option 3: $\lim\limits_{n\to\infty} a_{3n+1} = 2$
   - ◯ **Option 4:** $\lim\limits_{n\to\infty} a_{6n} = 1$

5. Consider the sequence $\{a_n\}$, defined as $a_n = \sqrt{2n+1} - \sqrt{2n}$. Which of the following option(s) is(are) true?

   - ◯ **Option 1:** $\{a_n\}$ is convergent.
   - ◯ **Option 2:** Limit of $\{a_n\}$ is 0.
   - ◯ Option 3: Limit of $\{a_n\}$ is 1.
   - ◯ Option 4: Limit of $\{a_n\}$ is $+\infty$.

6. Let $\{a_n\}$ be a sequence defined by $a_n = n^{\frac{1}{n}}$, has limit 1. Then which of the following option(s) is(are) true?

   - ◯ **Option 1:** $\lim\limits_{n\to\infty} \ln(n^{\frac{1}{n}}) = 0$
   - ◯ Option 2: $\lim\limits_{n\to\infty} \ln(1 + n^{\frac{1}{n}}) = 0$
   - ◯ **Option 3:** $\lim\limits_{n\to\infty} \dfrac{\ln 2 - \frac{1}{n}}{\ln(1 + n^{\frac{1}{n}})} = 1$
   - ◯ **Option 4:** $\lim\limits_{n\to\infty} (4n^{\frac{3}{n}} - 1) = 3$

7. If $\{a_n\}$ and $\{b_n\}$ be two sequences such that $|a_n| \le |b_n|$ for all $n \ge m$, where $m \in \mathbb{N}$ and $\lim\limits_{n\to\infty} b_n = 0$, then find the limit of a_n. [Ans: 0]

8. Which of the following option(s) is(are) true?

   - ◯ Option 1: $\left\{\dfrac{3n-5}{4n+2}\right\}$ is a decreasing sequence.
   - ◯ **Option 2:** $\left\{-\dfrac{1}{6n-5}\right\}$ is an increasing sequence.
   - ◯ Option 3: $\left\{\sqrt{n+1} - \sqrt{n}\right\}$ is an increasing sequence.
   - ◯ **Option 4:** $\left\{\sqrt{2n}\right\}$ is an increasing sequence.

5 Lecture 5:

5.1 Level 1:

1. Define a function $f(x) = \dfrac{x}{x^2 - 5x + 6}$. Then which of the following option(s) is(are) true?

   ◯ Option 1: $\lim_{x \to 0} f(x)$ does not exist.

   ◯ **Option 2:** $\lim_{x \to 1} f(x) = \frac{1}{2}$.

   ◯ Option 3: $\lim_{x \to 2} f(x)$ exists.

   ◯ Option 4: $\lim_{x \to 3} f(x)$ exists.

2. Define a function

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

   Which of the following option(s) is(are) true?

   ◯ Option 1: $\lim_{x \to 2} f(x) = -1$

   ◯ Option 2: $\lim_{x \to 0} f(x) = 1$

   ◯ **Option 3:** $\lim_{x \to -0.5} f(x) = -1$

   ◯ **Option 4:** Left limit at 0 i.e., $\lim_{x \to 0^-} f(x) = -1$

   ◯ Option 5: Right limit at 0 i.e., $\lim_{x \to 0^+} f(x) = -1$

   ◯ Option 6: Right limit at 0 i.e., $\lim_{x \to 0^+} f(x)$ does not exist.

   ◯ **Option 7:** Limit of the function at 0 does not exist.

   ◯ Option 8: Limit of the function does not exist at any real number.

3. Define a function

$$f(x) = \begin{cases} x & \text{if } x \leq 2 \\ 5 & \text{if } x > 2 \end{cases}$$

   Which of the following option(s) is(are) true?

   ◯ Option 1: Limit exists at $x = 2$.

   ◯ **Option 2:** Right limit exists at $x = 2$.

   ◯ **Option 3:** Left limit at $x = 2$ is 2.

   ◯ Option 4: Right limit at $x = 2$ is -1.

4. Consider the following graph of a function in the Figure M2W1AQ 10, where bullet point represents the point included in the line segment and circle represents the point does not included in the line segment.

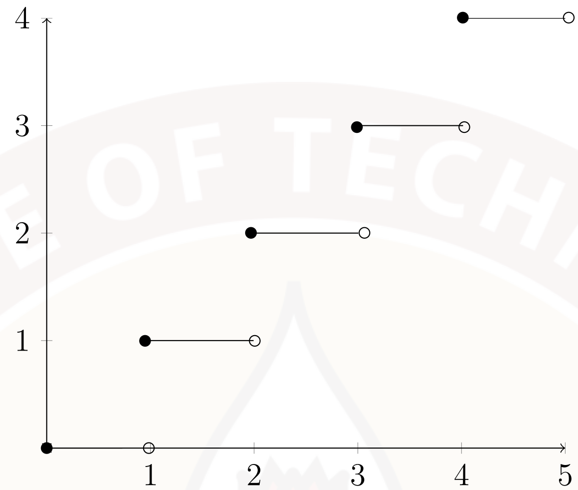


Figure M2W1AQ 10

Which of the following is(are) true?

- ◯ **Option 1:** Left limit exists at $x = 1$.
- ◯ **Option 2:** Left limit at $x = 3$ is 2.
- ◯ **Option 3:** Right limit at $x = 2$ is 2.
- ◯ Option 4: limit exists at $x = 3$.
- ◯ **Option 5:** Limit exists at every point $x \in (2, 3)$.

5.2 Level 2:

5. Consider the following graph of a function in the Figure M2W1AQ 11, where bullet point represents the point included in the curve and circle represents the point does not included in the line segment. Answer the following 2 questions

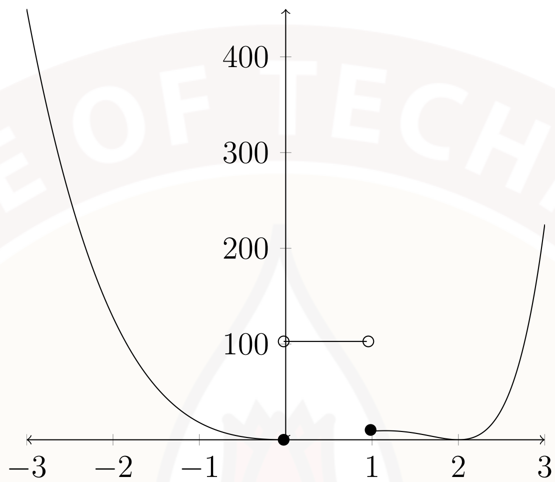


Figure M2W1AQ 11

Which of the following is(are) true?

- ◯ **Option 1:** Left limit at $x = 0$ is 0.
- ◯ Option 2: Limit exists at $x = 1$.
- ◯ **Option 3:** Limit exists at $x = 2$.
- ◯ Option 4: Limit exists at $x = 0$.
- ◯ **Option 5:** Limit exists at $x = \frac{1}{2}$ which is 100.
- ◯ Option 6: Limit exists at $x = \frac{1}{2}$ which is 0.

6. Which of the following is(are) true?

- ◯ Option 1: Function is increasing in the interval (1, 3).
- ◯ **Option 2:** Function is decreasing in the interval (1, 2).
- ◯ **Option 3:** Function is decreasing in the interval (-3, 0).
- ◯ Option 4: Function is decreasing in the interval (2, 3).

7. Which of the following option(s) is(are) true?

- ◯ **Option 1:** $\lim_{x \to -1} \dfrac{x^2 - 6x - 7}{x^2 + 3x + 2} = -8$
- ◯ Option 2:$\lim_{x \to 0} \dfrac{x^2 - 6x - 7}{x^2 + 3x + 2} = -8$

◯ **Option 3:** $\lim_{x \to 2}(x^3 + 4x^2 - 6x - 7) = 5$

◯ Option 4: $\lim_{x \to 3} \dfrac{x^2 - 6x + 9}{x - 3} = 1$

8. Which of the following option(s) is(are) true?

◯ **Option 1:** $\lim_{x \to \infty} \dfrac{1}{x} = 0$

◯ Option 2: $\lim_{x \to \infty} \dfrac{x^2}{1 + x} = 1$

◯ **Option 3:** $\lim_{x \to -\infty} \dfrac{1 + x}{x^2} = 0$

◯ **Option 4:** $\lim_{x \to \infty} \dfrac{1 + x + x^2}{5x^2 + 1} = \dfrac{1}{5}$

◯ Option 5: $\lim_{x \to \infty} \dfrac{x^{2021} + x^{2020} + \ldots + x + 1}{x^{2021} + 2021x^{2020} + \ldots + 2021} = 2021$

Week-2
Mathematics for Data Science - 2
Activity Questions

Continuity at a point:

A function f is said to be continuous at a point $x = a$ in its domain, if

1) f(a) exists; i.e value of $f(x)$ at $x = a$ exists.

2) $\lim_{x \to a} f(x)$ exists; i.e both left and right limits exist and are equal.

3) $\lim_{x \to a} f(x) = f(a)$.

Equivalently,

we can say that f is **continuous** at $x = a$, if $\{x_n\} \to a$ implies that $\{f(x_n)\} \to f(a)$.

- A function $f : X \to Y$ is bounded if we can find two real numbers m and M such that $m < f(x) < M$ for all $x \in X$, where X and Y are two subsets of $\mathbb{R}$.
- The greatest integer function $\lfloor \rfloor : \mathbb{R} \to \mathbb{Z}$ of a real number x denotes the greatest integer less than or equal to x.
- The smallest integer function $\lceil \rceil : \mathbb{R} \to \mathbb{Z}$ of a real number x denotes the least integer greater than or equal to x.

6 Lecture 1:

6.1 Level 1:

1. If $\lim_{x \to a} f(x) = L$, which of the following statements must be true?

   ◯ Option 1: f is defined at a.

   ◯ **Option 2**: $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

   ◯ Option 3: $f(a) = L$

   ◯ Option 4: f is continuous at a

2. Let f be a function and the Figure M2W2AQ1 represent the graph of function f. The solid points denote the value of the function at the points, and the values denoted by the hollow points are not taken by the functions.

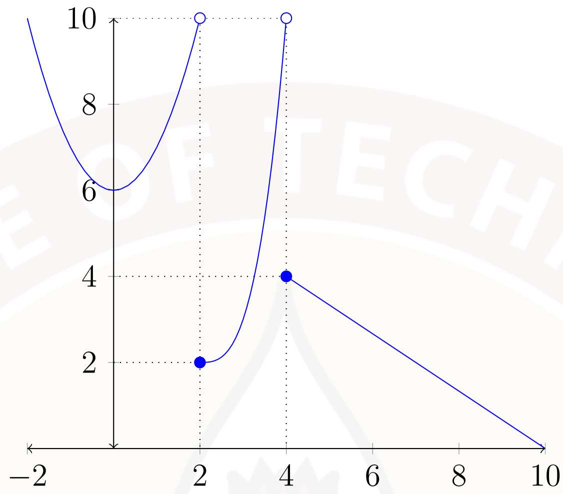


Figure: M2W2AQ1

Choose the set of correct options.

- ◯ **Option 1:** $\lim_{x \to 2^-} f(x) = 10$
- ◯ **Option 2:** $\lim_{x \to 2^+} f(x) = 2$
- ◯ Option 3: $\lim_{x \to 4^-} f(x) = 4$
- ◯ **Option 4:** $\lim_{x \to 4^+} f(x) = 4$
- ◯ Option 5: f is continuous at $x = 2$.
- ◯ **Option 6:** f is continuous at $x = 6$

3. Consider the following function:

$$f(x) = \begin{cases} \frac{x}{(x+1)(x+2)} & \text{if } x \geq 1 \\ \frac{1}{6} & \text{if } x < 1, \end{cases}$$

What is the number of points of discontinuity of f?

- ◯ Option 1: Two
- ◯ Option 2: Three
- ◯ **Option 3:** Zero
- ◯ Option 4: One

4. Define a function f as follows:

$$f(x) = \begin{cases} ax & \text{if } x \geq 1 \\ bx^2 & \text{if } x < 1, \end{cases}$$

where a and b are two arbitrary real numbers.

- ◯ Option 1: f is always continuous.
- ◯ Option 2: f is continuous if and only if $a = b = 0$.
- ◯ **Option 3:** f is continuous if $a = b = 0$.
- ◯ **Option 3:** f is continuous if $a = b$.

6.2 Level 2:

5. Which of the following option(s) is(are) true?

- ◯ **Option 1:** There exists a continuous function f from $[1, 5]$ to $[1, 7]$ such that $f(3) = \pi$.
- ◯ **Option 2:**: There exists a continuous function f from $[1, 10]$ to $[2, 8]$ such that $f(1) = 3$ and $f(10) = 5$.
- ◯ **Option 3:** There exists a non-constant continuous function f from $\mathbb{R}$ to $\mathbb{R}$ which is bounded.
- ◯ **Option 4**: There exists an unbounded continuous function from $(2, 3]$ to $\mathbb{R}$.

6. Consider a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(cx) = cf(x)$ for all $c, x \in \mathbb{R}$. Which of the following option(s) is(are) correct?

- ◯ **Option 1:** $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.
- ◯ Option 2: f is not continuous in $\mathbb{R}$.
- ◯ **Option 3:** f is continuous in $\mathbb{R}$.
- ◯ Option 4: $\lim\limits_{x \to a} f(x)$ exists for all $a \in R$, but f is not continuous in $\mathbb{R}$.

7. Define a function f as follows:

$$f(x) = \begin{cases} \frac{x}{tan^{-1}2x} & \text{if } x > 0 \\ b & \text{if } x = 0 \\ \frac{sin(ax)}{x} & \text{if } x < 0, \end{cases}$$

Which of the following options is true if f is continuous.

- ◯ Option 1: $a = b = 2$
- ◯ Option 2: $a = 2$, $b = \frac{1}{2}$

- ◯ Option 3: $a = \frac{1}{2}$, $b = 2$
- ◯ **Option 4:** $a = b = \frac{1}{2}$

8. Define a function f as follows:

$$f(x) = \begin{cases} \dfrac{1}{e^{\frac{1}{x}} + 1} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$$

Which of the following option(s) is(are) true?

- ◯ **Option 1**: $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$
- ◯ Option 2: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$
- ◯ **Option 3:** f is a bounded function on $\mathbb{R}$.
- ◯ Option 4: f is continuous at $x = 0$.

7 Lecture 2

7.1 Level 1

1. Choose the set of correct options.

   ◯ **Option 1:** If $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists, then f is differentiable at a.

   ◯ Option 2: A function f may be differentiable at a point a, even if it is not continuous at a.

   ◯ **Option 3:** If a function is differentiable at a point a, then it must be continuous at a.

   ◯ **Option 4:** There can exist some continuous functions which are not differentiable at some points in the domain.

2. Which of the following options showing step wise solution to check whether a function is differentiable or not are true?

   ◯ Option 1: Checking whether a constant function $f(x) = c$ is differentiable at any real number a or not: $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{c+h-c}{h} = \lim_{h\to 0} \frac{h}{h} = 1$.

   ◯ **Option 2:** Checking whether $f(x) = x - c$ is differentiable at a for some real number a, or not: $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h-c)-(a-c)}{h} = \lim_{h\to 0} \frac{h}{h} = 1$.

   ◯ Option 3: Checking whether $f(x) = x^2$ is differentiable at any real number a or not: $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^2-a^2}{h} = \lim_{h\to 0} \frac{2ah+h^2}{h} = 0$

   ◯ **Option 4:** Checking whether $f(x) = e^x$ is differentiable at any real number a or not: $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{e^{a+h}-e^a}{h} = \lim_{h\to 0} \frac{e^a(e^h-1)}{h} = e^a \lim_{h\to 0} \frac{e^h-1}{h} = e^a.1 = e^a$.

3. Consider the function defined as follows:

$$f(x) = \begin{cases} \lfloor x \rfloor & \text{if } x \geq 0 \\ \lceil x \rceil & \text{if } x < 0, \end{cases}$$

   Choose the set of correct options:

   ◯ **Option 1:** f is differentiable at $x = 0$.

   ◯ Option 2: f is differentiable at $x = 1$.

   ◯ Option 3: f is differentiable at $x = -1$.

   ◯ **Option 4:** f is differentiable at $x = 1.5$.

4. Consider the graphs given below:

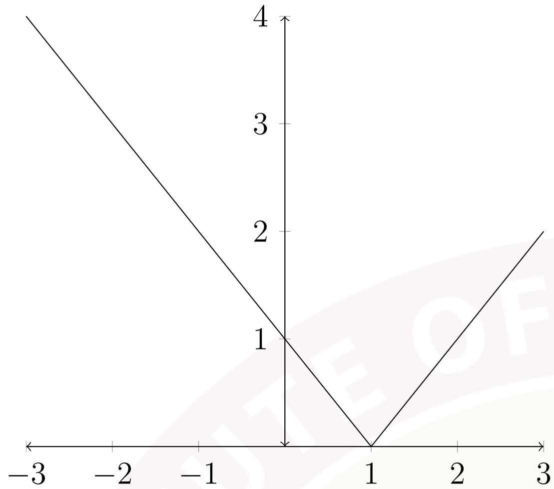


Figure: f_1

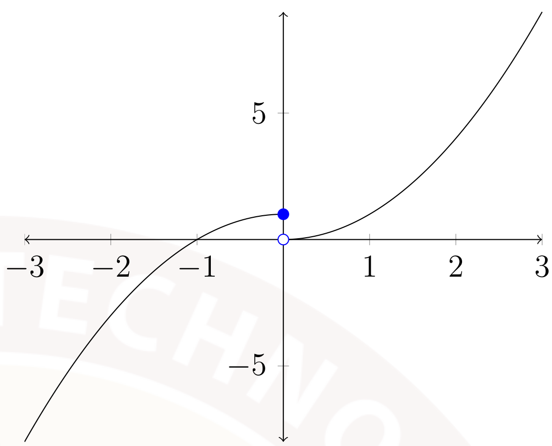


Figure: f_2

Choose the set of correct options.

- ◯ Option 1: f_1 is continuous and differentiable at each real number.
- ◯ **Option 2:** f_1 is not differentiable at 1.
- ◯ **Option 3:** f_2 is not continuous at 0.
- ◯ **Option 4:** f_2 is differentiable in the interval $[1, 2]$.

5. The following curve shown in Figure M2W2AQ2 represents the function $f : \mathbb{R} \to \mathbb{R}$, such that $f(x) = x^{\frac{1}{3}}$.

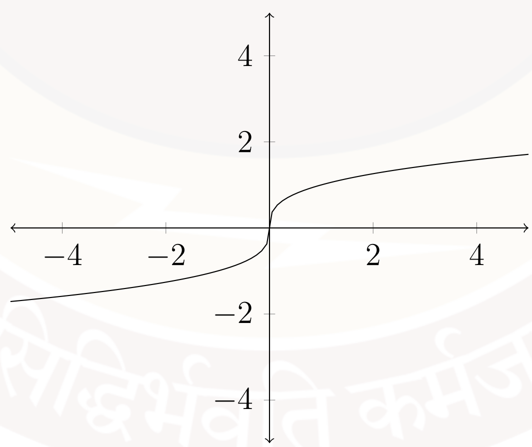


Figure: M2W2AQ2

Which of the following options is(are) correct?

- ◯ **Option 1:** f is continuous on $\mathbb{R}$.

○ Option 2: f is differentiable everywhere on $\mathbb{R}$.

○ Option 3: f is no where differentiable on $\mathbb{R}$.

○ **Option 4:** f is not differentiable at 0.

7.2 Level 2

6. Choose the set of correct options.

    ○ **Option 1:** There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that f is not differentiable exactly on the set of natural numbers.

    ○ Option 2: Inverse of a differntiable function is differentiable.

    ○ **Option 3:** Let $f(x) = x^2$ for x rational and $f(x) = 0$ for x irrational. Then f is differntiable at $x = 0$.

    ○ Option 4: none of the above

7. Consider the function $f(x) = |\sin x|$. Then f is

    ○ **Option 1:** periodic with period π.

    ○ Option 2: everywhere continuous and differentiable.

    ○ **Option 3:** everywhere continuous and not differentiable at $n\pi$, where $n \in \mathbb{Z}$.

    ○ Option 4: neither continuous nor differentiable at $n\pi$, where $n \in \mathbb{Z}$.

8. Consider Figure M2W2AQ3, which represents some function f, to choose the correct option from the following:

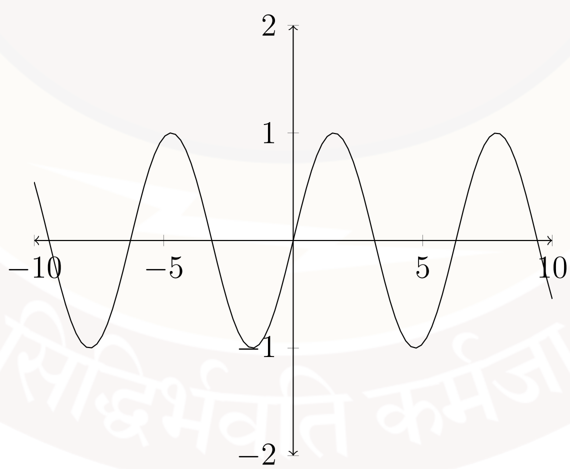


Figure: M2W2AQ3

    ○ **Option 1:** There are some points in the interval $[-5, 5]$ at which the derivative of f is 0.

- ◯ Option 2: There are some points in the interval $[-5, 5]$ at which f is not differentiable.
- ◯ Option 3: f is not differentiable at 0.
- ◯ **Option 4:** There are at least 2 points where the derivative of f are non-zero but the same.

8 Lecture 3

8.1 Level 1

1. If $f(x) = e^{x^2 \cos x}$, then the derivative of f is
   - ◯ Option 1: $e^x(2x\cos x - x^2 \sin x)$
   - ◯ Option 2: $e^x(x^2 \sin x + 2x\cos x)$
   - ◯ Option 3: $e^{x^2 \cos x}(2x\cos x + x^2 \sin x)$
   - ◯ **Option 4:** $e^{x^2 \cos x}(2x\cos x - x^2 \sin x)$

2. $\lim\limits_{x \to 0^+} \dfrac{\sin x}{\sqrt{x}}$
   - ◯ Option 1: -1
   - ◯ Option 2: Does not exist
   - ◯ **Option 3:** 0
   - ◯ Option 4: 1

3. Consider a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x|x|$. Which of the following option(s) is(are) true?
   - ◯ Option 1: f is not diffrentiable at any point of $\mathbb{R}$.
   - ◯ **Option 2:** $f'(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0, \end{cases}$.
   - ◯ Option 3: $f'(x) = \begin{cases} -2x & \text{if } x \geq 0 \\ 2x & \text{if } x < 0, \end{cases}$
   - ◯ Option 4: $f'(x) = 2x$
   - ◯ Option 5: $f'(x) = \begin{cases} 2x & \text{if } x > 0 \\ -2x & \text{if } x < 0, \end{cases}$, and f is not differentiable at 0.

4. If $f(x) = \sqrt{9 - x^2}$, then find out the value of $\lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x - 1}$
   - ◯ Option 1: $\dfrac{1}{\sqrt{8}}$
   - ◯ **Option 2:** $-\dfrac{1}{\sqrt{8}}$
   - ◯ Option 3: $\sqrt{8}$
   - ◯ Option 4: Does not exist

8.2 Level 2

5. Consider a function $f(x) = \log_x 5$, where $x > 0$, and $x \neq 1$. Derivative of f is

   ◯ Option 1: $\dfrac{-\log_x 5 \log_x e}{x^2}$

   ◯ **Option 2:** $\dfrac{-\log_x 5 \log_x e}{x}$

   ◯ Option 3: $\dfrac{\log_x 5 \log_x e}{x}$

   ◯ Option 4: $\dfrac{\log_x 5 \log_x e}{x^2}$

6. In which of the following, one can apply L'Hospital's rule to evaluate the limits?

   ◯ Option 1: $\lim\limits_{x \to \infty} \dfrac{x}{x + \sin x}$

   ◯ **Option 2**: $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{1 - \cos(2x)}$

   ◯ **Option 3:** $\lim\limits_{x \to \infty} \dfrac{7 + \ln x}{x^3 + 6}$

   ◯ Option 4: $\lim\limits_{x \to 0^+} \dfrac{1}{x}$

7. if $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(9) = 6$, $f'(0) = 4$, then $f'(9)$ is

   ◯ Option 1: 10

   ◯ **Option 2:** 24

   ◯ Option 3: 6

   ◯ Option 4: 4

8. Let f and g be two distinct functions from $\mathbb{R}$ to $\mathbb{R}$. Which of the following statements are true?

   ◯ Option 1: If fg is differentiable, then both f and g are differentiable.

   ◯ Option 2: Assume that $g(x) \neq 0$ for all $x \in \mathbb{R}$. If $\dfrac{f}{g}$ is differentiable, then both f and g are differentiable.

   ◯ **Option 3:** If f is an even differentiable function, then f' is an odd function.

   ◯ **Option 4:** If f is an odd differentiable function then, f' is an even function.

9 Lecture 4

9.1 Level 1

1. Suppose m_1, m_2, and m_3 denote the slopes of the tangents of the curve represented by the function $f(x) = x^3 + 3x$, at the points $(-1, f(1))$, $(0, f(0))$, and $(1, f(1))$, respectively. Which of the following options is correct?

   ◯ Option 1: $m_1 < m_2 < m_3$
   ◯ Option 2: $m_1 = m_3 < m_2$
   ◯ **Option 3:** $m_1 = m_3 > m_2$
   ◯ Option 4: $m_1 > m_2 > m_3$

2. What will be the equation of the tangent at the vertex of a parabola given by the equation $(y - k) = a(x - h)^2$?

   ◯ Option 1: $x = h$
   ◯ **Option 2:** $y = k$
   ◯ Option 3: $\dfrac{x}{h} + \dfrac{y}{k} = 1$
   ◯ Option 4: Tangent does not exist at the vertex.

3. What will be the equation of the tangent at the point (a, a^3) on the parabola given by the equation $y = ax^2$?

   ◯ **Option 1:** $y = a^2(2x - a)$
   ◯ Option 2: $y = 2ax + a^3$
   ◯ Option 3: $y = 2ax$
   ◯ Option 4: $y = 2a^2x$

4. Consider the function $f : \mathbb{R} \to \mathbb{R}$, such that $f(x) = mx + c$. Which of the following expression represents the linear approximation $L_f(x)$ at $(0, f(0))$?

   ◯ Option 1: $L_f(x) = c$
   ◯ Option 2: $L_f(x) = mx$
   ◯ **Option 3:** $L_f(x) = mx + c$
   ◯ Option 4: $L_f(x) = 0$

5. Consider the function $f : \mathbb{R} \to \mathbb{R}$, such that $f(x) = e^x sinx$. Which of the following expression represents the linear approximation $L_f(x)$ at $x = \dfrac{\pi}{2}$?

   ◯ **Option 1:** $L_f(x) = e^{\frac{\pi}{2}}x + e^{\frac{\pi}{2}}(1 - \frac{\pi}{2})$.
   ◯ Option 2: $L_f(x) = -e^{\frac{\pi}{2}}x + e^{\frac{\pi}{2}}(1 + \frac{\pi}{2})$.
   ◯ Option 3: $L_f(x) = -e^{\frac{\pi}{2}}x + \frac{\pi}{2}e^{\frac{\pi}{2}}$.
   ◯ Option 4: $L_f(x) = e^{\frac{\pi}{2}}x - \frac{\pi}{2}e^{\frac{\pi}{2}}$

9.2 Level 2

6. Let f be a differentiable function at $x = 1$. The tangent line to the curve represented by the function f at the point $(1, 0)$ passes through the point $(5, 8)$. What will be the value of $f'(1)$? [Answer: 2]

7. Suppose the tangent of the curve represented by a function f at the point $(1, f(1))$ is given by the equation $y = 3x + 2$. What is the value of $f(1)$? [Answer: 5]

8. Use the linear approximation of $f(x) = \sqrt{x}$ at $x = 4$, to find the approximate value of $\sqrt{4.4}$ from the given options.
   - ◯ Option 1: 3
   - ◯ **Option 2:** 2.1
   - ◯ Option 3: 2
   - ◯ Option 4: 2.2

Week-3
Mathematics for Data Science - 2
Activity Question

10 Lecture 1 (Part 1):

10.1 Level 1:

1. Let $f(x)$ be a function and P be a turning point of the function $f(x)$. Then which of the following option(s) is (are) true for the function $f(x)$ at the point P?

   ◯ **Option 1:** The function $f(x)$ may change from increasing to decreasing at the point P

   ◯ **Option 2:** The function $f(x)$ may change from decreasing to increasing at the point P

   ◯ Option 3: The function $f(x)$ remains the same.

   ◯ Option 4: None of the above.

2. Let $f(x)$ be a function and the graph of the function is given in Figure M2W3AQ1.

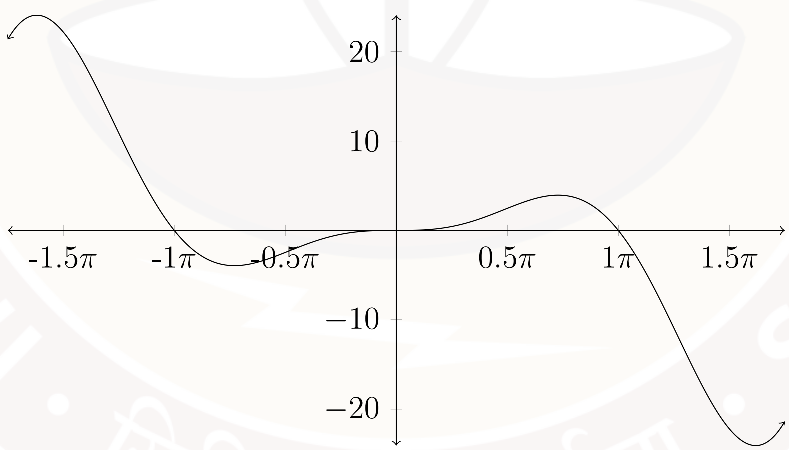


Figure: M2W3AQ1

In the given domain as shown in the figure, which of the following option is true, ?

   ◯ **Option 1:** The number of local maxima for the function $f(x)$ is 2.

   ◯ Option 2: The number of local minima for the function $f(x)$ is 3.

   ◯ Option 3: The function $f(x)$ has a local minimum at the origin.

◯ Option 4: The function $f(x)$ has a local maximum at the origin.

3. Let $f(x)$ be a function. If the function $f(x)$ has a local minimum at the point $x = 2$ and a local maximum at the point $x = 5$, then which of the following option(s) is(are) true?

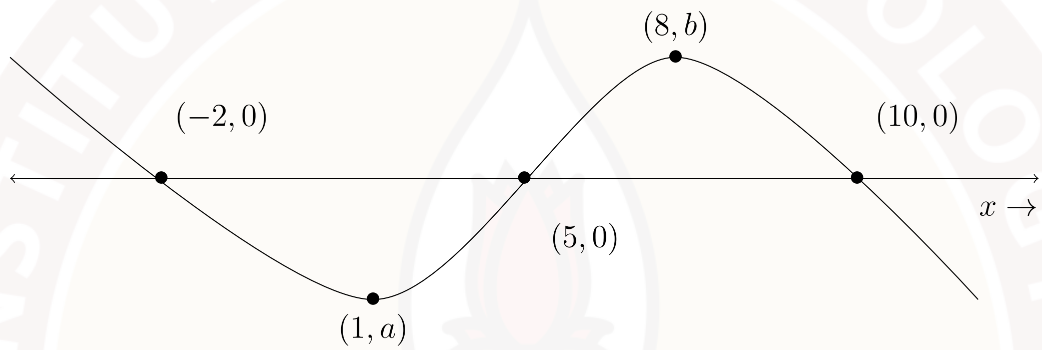
◯ Option 1: Slope of the tangent at the point $x = 2$ is 2.

◯ **Option 2:** Slope of the tangent at the point $x = 5$ is 0.

◯ **Option 3:** Slope of the tangent at the point $x = 2$ is 0.

◯ Option 4: Slope of the tangent at the point $x = 5$ is 5.

4. A function $f(x)$ represented by the curve is shown in Figure M2W3AQ2, where $a, b \in \mathbb{R}$.



M2W3AQ2

Choose the set of correct options.

◯ **Option 1**: If $x \in (2, 6)$, then $f(x)$ is an increasing function.

◯ **Option 2**: If $x \in (8, 10)$, then $f(x)$ is a decreasing function.

◯ Option 3: If $x \in (2, 6)$, then $f'(x) \leq 0$.

◯ Option 4: If $x \in (-2, 5)$, then $f'(x) \leq 0$.

◯ **Option 5**: The equation $f'(x) = 0$ has at least two solutions.

10.2 Level 2:

5. Let $f(x)$ be a differentiable function defined as $f(x) = \frac{x^4}{2} - \frac{13x^3}{3} + 11x^2 - 8x$. Which of the following option(s) is (are) true?

   ◯ **Option 1:** The number of critical points are 3.

   ◯ Option 2: The number of critical points are 4.

   ◯ **Option 3:** The point $x = 4$ is a critical point.

   ◯ Option 4: The point $x = 1$ is a critical point.

   ◯ **Option 5:** The point $x = 2$ is a critical point.

6. Let $f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x,\ x \in [0, \pi]$. Assume $f(x)$ does not have a saddle point in the interval. Which of the following may represent the graph of the function?

   [**Hint:** Differentiate the function $f(x)$ and then use the formula $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$ on $\cos x$ and $\cos 3x$ and then solve the equation $f'(x) = 0$ for critical points. Note: if $\cos x = 0$, then $x = \frac{\pi}{2}, \frac{3\pi}{2}$ and if $\cos x = -\frac{1}{2}$, then $x = \frac{2\pi}{3}$]

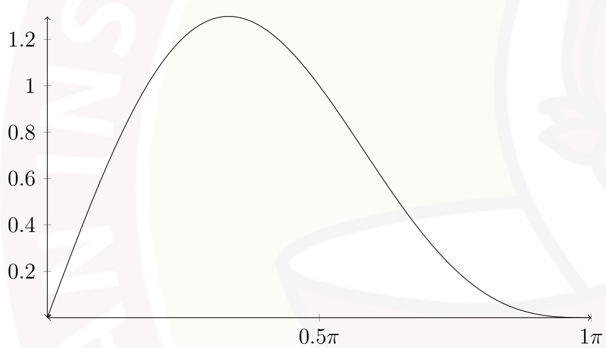


Figure: Curve 1

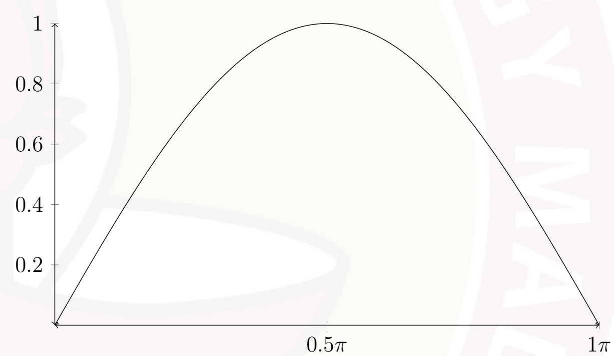


Figure: Curve 2

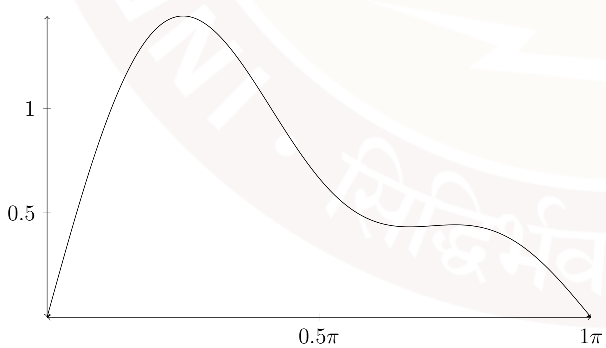


Figure: Curve 3

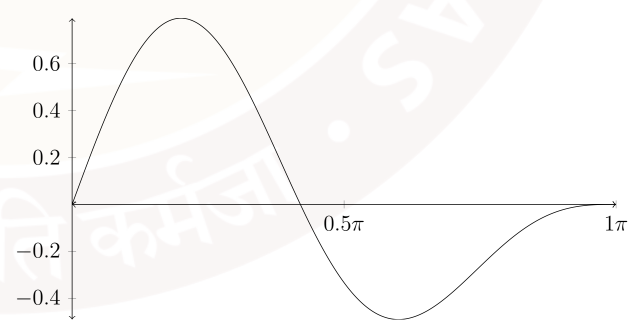


Figure: Curve 4

   ◯ Option 1: Curve 1

- ◯ Option 2: Curve 2
- ◯ **Option 3:** Curve 3
- ◯ Option 4: Curve 4

7. What would be the number of distinct solutions for the equation $f'(x) = 0$, if $f(x) = \frac{1}{3}x^3 + x^2 + x$?
   - ◯ Option 1: 3
   - ◯ Option 2: 2
   - ◯ **Option 3**: 1
   - ◯ Option 4: 0

8. Match the graphs of the functions $f(x)$ in column A with the graphs of their derivative functions $f'(x)$ in column B in Table M2W3AQ1.

   [**Hint:** Observe that if a is a point on the $X-$ axis at which a function $f(x)$ has a local maximum or a local minimum, then a is a root of $f'(x) = 0$.]

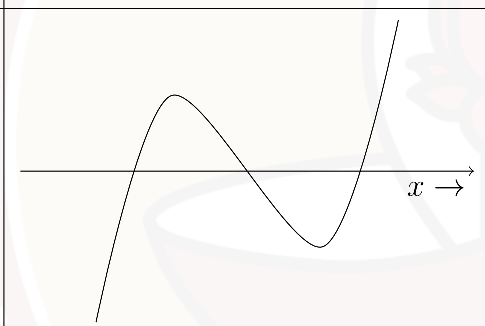
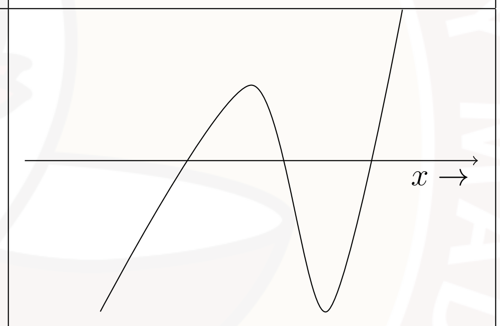
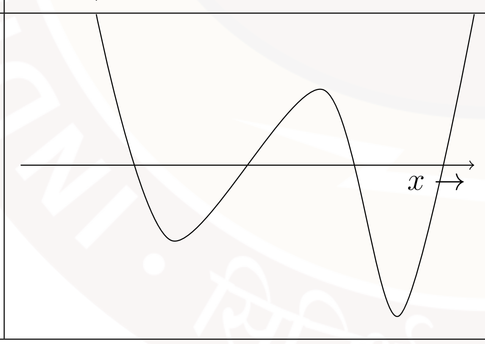
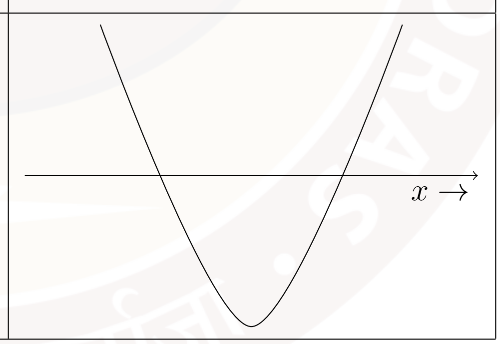
	Column A		Column B
a)		i)	
b)		ii)	

Table M2W3AQ1

   - ◯ Option 1: a $\rightarrow$ ii, b $\rightarrow$ i
   - ◯ **Option 2:** a $\rightarrow$ i, b $\rightarrow$ ii
   - ◯ Option 3: None of the above.

11 Lecture 1 (Part 2):

11.1 Level 1:

1. Let A be the set of local minima and B be the set of local maxima for the function $f(x) = \frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x$. Choose the correct option.

   ◯ Option 1: $A = \{1, 2\}, B = \{3\}$

   ◯ Option 2: $A = \{3\}, B = \{1, 2\}$

   ◯ Option 3: $A = \{2, 3\}, B = \{1\}$

   ◯ **Option 4**: $A = \{1, 3\}, B = \{2\}$

2. The minimum value of the polynomial $P(x) = (x - \alpha)(x - \beta)$ occurs at

   ◯ Option 1: $x = \dfrac{\alpha - \beta}{2}$

   ◯ Option 2: $x = \alpha - \beta$

   ◯ **Option 3:** $x = \dfrac{\alpha + \beta}{2}$

   ◯ Option 4: $x = \alpha + \beta$

3. If $x + y = 50$, then find the maximum value of $2xy$. [Ans: 1250]

4. Consider a rectangle of length l and width w such that the perimeter of the rectangle is 100 units. The maximum possible area of the rectangle is

   ◯ Option 1: 1250 square units

   ◯ **Option 2:** 625 square units

   ◯ Option 3: 1000 square units

   ◯ Option 4: 500 square units

11.2 Level 2:

5. Consider the function defined as follows:

$$f(x) = \begin{cases} -x^2 + 2x + 3 & \text{if } 0 \le x \le 50 \\ x^3 + 3 & \text{if } -50 \le x < 0. \end{cases}$$

   Which of the following options are correct?

   ◯ **Option 1:** 1 is a local maximum.

   ◯ **Option 2:** -50 is the global minimum.

   ◯ Option 3: 0 is the global maximum.

◯ Option 4: 50 is the global minimum.

6. Consider the function $f(x) = \dfrac{x}{n} + \dfrac{n}{x}$ from $(0, \infty)$ to $\mathbb{R}$, where n is a positive integer. Which of the following option(s) is(are) true?

   ◯ Option 1: n is a local maximum.

   ◯ Option 2: 1 is a local maximum.

   ◯ Option 3: 1 is the global maximum.

   ◯ **Option 4:** n is a local minimum.

   ◯ **Option 5:** n is the global minimum.

7. Let $f(x) = \sin x + \cos x + 5$. Which of the following is the maximum value of the function in the interval $[0, \pi]$?

   ◯ Option 1: $\sqrt{2}$.

   ◯ Option 2: 5.

   ◯ **Option 3:** $\sqrt{2} + 5$.

   ◯ Option 4: 2.

8. Let $f(x) = \frac{x^5}{5} - \frac{3x^4}{2} + \frac{11x^3}{3} - 3x^2$. Which of the following option(s) is(are) true about the function $f(x)$?

   ◯ Option 1: The function $f(x)$ is increasing in the interval (0, 1).

   ◯ **Option 2:** The function $f(x)$ is decreasing in the interval $(2, 3)$.

   ◯ **Option 3:** The function $f(x)$ is increasing in the interval (1, 2).

   ◯ **Option 4:** The function $f(x)$ is increasing in the interval $(3, \infty)$

   ◯ Option 5: The function $f(x)$ is decreasing in the interval [1, 3]

12 Lecture 2:

12.1 Level 1:

1. Let A be a square with each side of length 6 units. Which of the following option(s) is (are) true?

   ◯ **Option 1:** The maximum number of non- overlapping squares that can be drawn inside A such that each square has length 2 units is 9.

   ◯ Option 2: The maximum number of non- overlapping squares that can be drawn inside A such that each square has length 1 unit is 35.

   ◯ **Option 3:** The maximum number of non- overlapping rectangles that can be drawn inside A such that each square has length 2 units and breadth 3 units is 6.

   ◯ Option 4: The maximum number of non- overlapping rectangles that can be drawn inside A such that each square has length 2 units and breadth 3 units is 5.

2. Consider the parallelogram as shown in Figure M2W3AQ3. If $\theta = 60^\circ$ and perimeter of the parallelogram is 10 units then which of the following option(s) is(are) true?

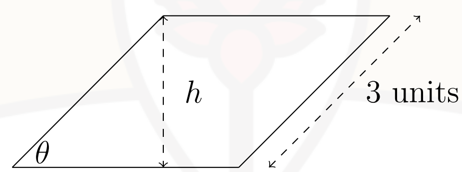


Figure M2W3AQ3

   ◯ **Option 1:** Height $h = \frac{3\sqrt{3}}{2}$ units.

   ◯ Option 2: Height $h = \frac{\sqrt{3}}{2}$ units.

   ◯ **Option 3:** Area of the parallelogram is $3\sqrt{3}$ square units.

   ◯ Option 4: Area of the parallelogram is $\sqrt{3}$ square units.

3. Consider the regular hexagon in Figure M2W3AQ4 and triangles which are made inside the hexagon. If the length of each side of the hexagon is 4 units, then find the area of the hexagon.

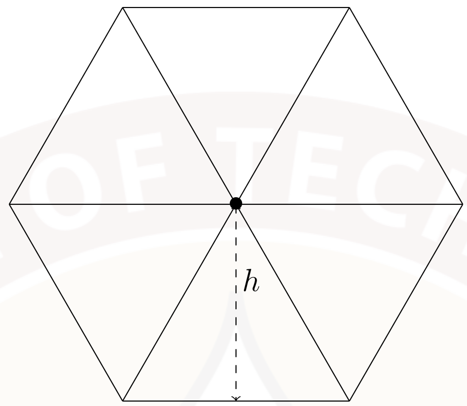


Figure M2W3AQ4

- ◯ Option 1: 24 square units.
- ◯ **Option 2:** $24\sqrt{3}$ square units.
- ◯ Option 3: 36 square units.
- ◯ Option 4: $36\sqrt{3}$ square units.

4. Let C_1 and C_2 be two circles where the radius are 2 units and 3 units, respectively. Which of the following option(s) is (are) true?

- ◯ **Option 1:** Area of $C_1 = 4\pi$ square units.
- ◯ Option 2: Area of $C_2 = 4\pi$ square units.
- ◯ Option 3: Area of $C_1 = 9\pi$ square units.
- ◯ **Option 4:** Area of $C_2 = 9\pi$ square units.

12.2 Level 2:

Consider the following two trapeziums $ACDB$ and $PQRS$ in Figure M2W3AQ5 and Figure M2W3AQ6 Respectively. In trapezium $ACDB$, $AB = 4$ units, $CD = 2$ units and height $h_1 = 2$ units. In trapezium $PQRS, \theta_1 = 60°, \theta_2 = 45°, PT = TU = 3$ units. Answer questions 5 and 6 based on the above information.

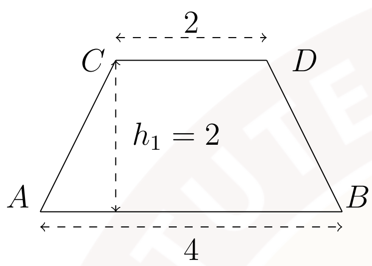


Figure M2W3AQ5

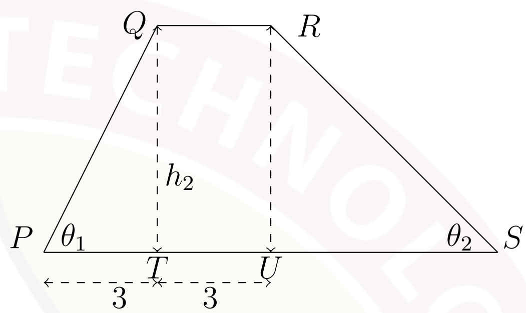


Figure M2W3AQ6

5. Find the area of the trapezium $ACDB$. [Ans: 6 square units]

   [Hint: Area of a trapezium= $\frac{1}{2} \times h \times (a+b)$, where a and b are the lengths of the parallel of sides, and h is the height, i.e., the distance between the parallel sides.]

6. Which of the option(s) is(are) true?

   ○ Option 1: $h_2 = 3$ units

   ○ Option 2: $h_2 = \sqrt{3}$ units

   ○ **Option 3:** $US = 3\sqrt{3}$ units

   ○ **Option 4:** Area of $PQRS = \frac{27(\sqrt{3}+1)}{2}$ square units

   ○ Option 5: Area of $PQRS = \frac{27}{2}$ square units

7. Let A be a triangle where all sides of the triangle are equal. If the length of a side of the triangle is 6 units, then area of the triangle is

   ◯ Option 1: $\sqrt{3}$ square units.

   ◯ Option 2: $3\sqrt{3}$ square units.

   ◯ **Option 3:** $9\sqrt{3}$ square units.

   ◯ Option 4: 9 square units.

8. Suppose there are infinitely many circles $C_1, C_2, \ldots, C_n, \ldots$. The sequence $\{r_n\}$ of the radii of these circles is defined as $r_n = \frac{2n-1}{2n+2}$ units. Now as $n \to \infty$, the radius of the circles is increasing so areas of the circles are also increasing. (for example if $n = 1$, then the radius of the circle C_1 is $\frac{1}{4}$ units so are area of circle(C_1) $= \frac{\pi}{4^2}$ square units, and if $n = 2$, then the radius of the circle C_2 is $\frac{3}{6} = \frac{1}{2} > \frac{1}{4}$ units and area of circle(C_2) $= \frac{\pi}{2^2} > \frac{\pi}{4^2}$ square units and so on). Which of the following option(s) is(are) true about the circles $C_1, C_2, \ldots, C_n, \ldots$.

   [**Hint:** Find the limit of the sequence $\frac{2n-1}{2n+1}$ and since the sequence is increasing so the radius of any circle is less than the limit of the sequence.]

   ◯ **Option 1:** Area of the biggest circle $\leq \pi$.

   ◯ Option 2: Area of the biggest circle $> \pi$.

   ◯ **Option 3:** Area of the smallest circle is $\frac{\pi}{16}$ square units.

   ◯ Option 4: Area of the smallest circle is π square units.

13 Lecture 3:

13.1 Level 1:

1. Consider the closed interval $[3, 9]$. Let $P = \{x_0, x_1, \ldots, x_n\}$ be an ordered set, where $n \in \mathbb{N}$. Which of the following option(s) is (are) true regarding partitions of the interval $[3, 9]$?

   ○ **Option 1:** If $x_0 = 3$ and $n = 1(i.e., x_1 = 9)$, then P is a partition of the interval $[3, 9]$.

   ○ **Option 2:** If $x_0 = 3$ and $x_n = 9$ and $3 < x_i < 9$ with $x_i < x_{i+1}$ for $i = 1, 2, \ldots, n-1$, then P is a partition of the interval $[3, 9]$.

   ○ Option 3: If $x_0 = 3$ and $x_n = 9$ and $3 < x_i < 9$ for $i = 2, 3 \ldots, n-1$, and $x_1 = 2$, then P is a partition of the interval $[3, 9]$.

   ○ Option 4: If S is the collection of all partitions of the interval $[3, 9]$, then cardinality of the set S is finite.

2. Consider an interval [0, 10] and the following partitions
   $P_1 : 0 = x_0 < x_1 < \ldots < x_{10} = 10$, where $x_i = x_{i-1} + 1$, $i = 1, 2, \ldots, 10$.
   $P_2 : 0 = x_0 < x_1 < \ldots < x_5 = 10$, where $x_i = x_{i-1} + 2$, $i = 1, 2, \ldots, 5$.
   $P_3 : 0 = x_0 < x_1 < x_2 < x_3 = 10$, where $x_1 = 3, x_2 = 5, x_3 = 10$.
   If $\triangle x_i = x_i - x_{i-1}$ and for a partition P_k, define $||P_k|| = \underset{i}{max}\{\triangle x_i\}$, then which of the following option(s) is (are) true?

   ○ **Option 1:** For P_1, $\triangle x_i = 1$ for all i

   ○ **Option 2:** For P_2, $\triangle x_i = 2$ for all i

   ○ Option 3: For P_3, $\triangle x_1 = 5$

   ○ **Option 4:** For P_3, $\triangle x_3 = 5$

   ○ Option 5: $||P_1|| = 2$

   ○ Option 6: $||P_2|| = 1$

   ○ **Option 7:** $||P_3|| = 5$

3. Let $f(x)$ be a function defined on a domain D which contains the interval $[a, b]$. Let P be a partition of the interval $[a, b]$ defined as $a = x_0 < x_1 < \ldots < x_n = b$, $x_i^* \in [x_{i-1}, x_i]$ and $\triangle x_i = x_i - x_{i-1}$. Which of the following option represents the Riemann sum?

   ○ **Option 1:** $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$

   ○ Option 2: $\sum_{i=1}^{n} \triangle x_i$

   ○ Option 3: $\sum_{i=1}^{n} f(x_i^*)$

   ○ Option 4: None of the above.

4. Which of the following curves shown in the following figures shows negative area on the X axis in the interval $[0, 1]$?

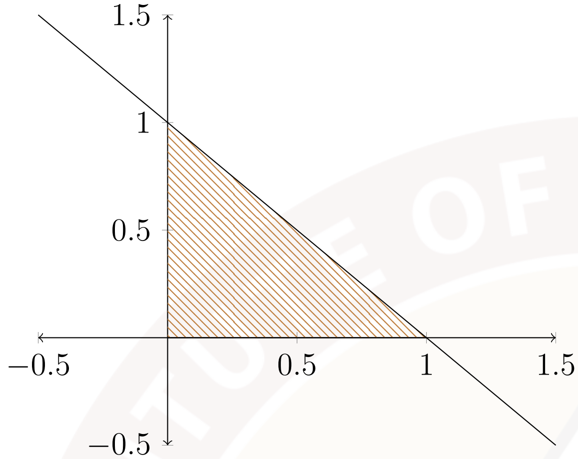


Figure: Curve 1

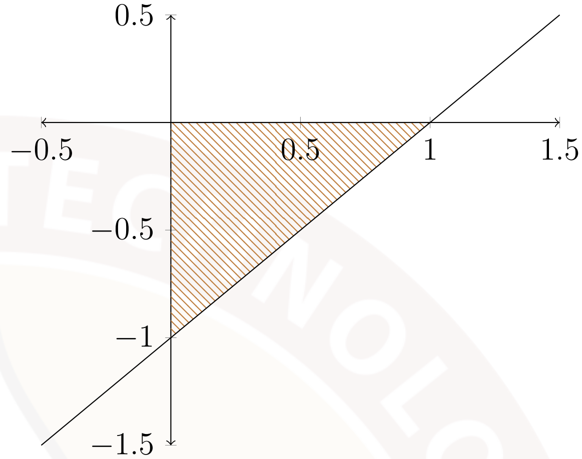


Figure: Curve 2

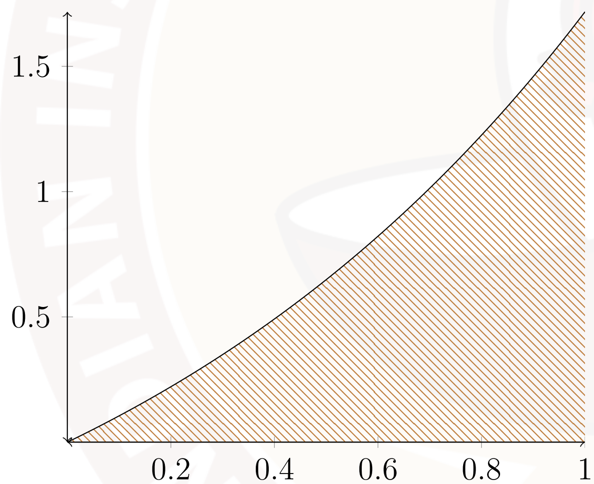


Figure: Curve 3

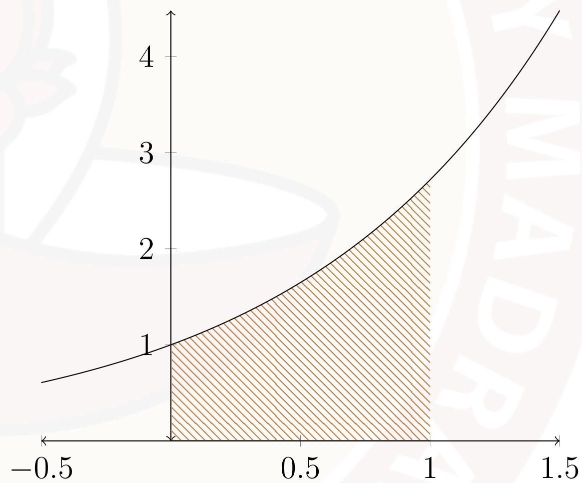


Figure: Curve 4

◯ Option 1: Curve 1

◯ **Option 2:** Curve 2

◯ Option 3: Curve 3

◯ Option 4: Curve 4

13.2 Level 2:

5. Consider an interval [3, 7] and the following partitions
P_1: $3 = x_0 < x_1 < \ldots < x_n = 7$, where $x_i = 4 + \frac{i \times 3}{n}, i = 1, 2, \ldots, n$
P_2: $3 = x_0 < x_1 < \ldots < x_n = 7$, where $x_i = 3 + \frac{i \times 4}{n}, i = 1, 2, \ldots, n$
P_3 : $3 = x_0 < x_1 < \ldots < x_n = 7$, where $x_i = 5 + \frac{i \times 2}{n}, i = 1, 2, \ldots, n$
Which of the following partitions has norm ($||P_k|| = \max_i\{\triangle x_i\}$) tends to 0 as $n \to \infty$(i.e., $||P_k|| \to 0$ as $n \to \infty$.)?
[**Hint:** First calculate $\triangle x_1$, then compare with other $\triangle x_i, i = 2, 3, \ldots, n$]
   - ◯ Option 1: P_1
   - ◯ **Option 2:** P_2
   - ◯ Option 3: P_3
   - ◯ Option 4: None of the above.

6. Let $f(x) = 3x + 1$ then find the value of the integral $\int_0^2 f(x)\,dx$ using limit of Riemann sums as $n \to \infty$, for the given partition $P = \{0 = x_0, x_1 = \frac{2}{n}, \ldots, x_i = \frac{2 \times i}{n}, \ldots, x_n = 2\}, i = 1, 2, \ldots, n$ and $x_i^* \in [x_{i-1}, x_i]$, where $x_i^* = \frac{2 \times i}{n}$. [Ans : 8]

Let $f(x) = x$ be a function defined on the domain D which contains the interval $[0, 5]$. Let P be a partition of the interval $[0, 5]$ defined as $0 = x_0 < x_1 < \ldots < x_n = 5$ where $x_i = \frac{i \times 5}{n}$ i.e., $x_0 = 0 < \frac{5}{n} < \frac{2 \times 5}{n} < \frac{3 \times 5}{n} < \ldots < \frac{5(n-1)}{n} < \frac{5n}{n} = 5 = x_n, i = 1, 2, \ldots, n$ and $x_i^* \in [x_{i-1}, x_i], \triangle x_i = x_i - x_{i-1}$. Answer the questions 7 and 8 based on the above information.

7. Which of the following option(s) is(are) true?
   - ◯ **Option 1:** If $x_i^* = x_i$, then the Riemann sum $\sum_{i=1}^n f(x_i^*)\triangle x_i$ equals $\frac{25(n+1)}{2n}$.
   - ◯ Option 2: If $x_i^* = x_i$, then the Riemann sum $\sum_{i=1}^n f(x_i^*)\triangle x_i$ equals $\frac{25(n+1)}{n}$.
   - ◯ **Option 3:** If $x_i^* = x_{i-1}$, then the Riemann sum $\sum_{i=1}^n f(x_i^*)\triangle x_i$ equals $\frac{25(n-1)}{2n}$.
   - ◯ Option 4: If $x_i^* = x_{i-1}$, then the Riemann sum $\sum_{i=1}^n f(x_i^*)\triangle x_i$ equals $\frac{25(n-1)}{n}$.
   - ◯ Option 5: If $x_i^* = x_{i-1}$, then the Riemann sum $\sum_{i=1}^n f(x_i^*)\triangle x_i$ equals $\frac{25}{2}$.

8. If $x_i^* = \frac{x_i + x_{i-1}}{2}$, then find the Riemann sum $\sum_{i=1}^n f(x_i^*)\triangle x_i$. [Ans 12.5]
[**Hint:** Observe that n will get canceled from $\sum_{i=1}^n f(x_i^*)\triangle x_i$.]

14 Lecture 4:

14.1 Level 1:

1. Suppose f is a continuous function on the domain D which includes interval $[4, 9]$ and F is the anti-derivative of f such that $F(4) = 3$ and $F(9) = 6$. Then which of the following option(s) is (are) true?

   ◯ **Option 1:**$F'(x) = f(x)$

   ◯ **Option 2:** $F(x) - F(4) = \int_4^x f(x)\,dx$, where $x \in (4, 9)$.

   ◯ Option 3: $\int_4^9 f(x)\,dx = 9$

   ◯ **Option 4:** $\int_4^9 f(x)\,dx = 3$

2. Let $f(x) = x^3 - x^2 + x$. Let $F(x)$ be the anti-derivative of $f(x)$ such that $F(2) = 6$. Then $F(x)$ equals

   ◯ Option 1: $\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} + 8$

   ◯ Option 2: $\frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} + \frac{8}{3}$

   ◯ Option 3: $\frac{x^3}{3} - \frac{x^2}{2} + x + \frac{8}{3}$

   ◯ **Option 4:** $\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} + \frac{8}{3}$

3. Let $F(x)$ be the anti-derivative of a function $f(x)$ and $F(x)$ is defined as $F(x) = \sin 2x + \tan 5x + 5^{9x} + \ln 3x + 5$, then $f(x) =$

   ◯ **Option 1:** $2\cos 2x + 5\sec^2 5x + 9(5^{9x})\ln 5 + \frac{1}{x}$.

   ◯ Option 2: $\cos 2x + \sec^2 5x + \ln 5.5^{9x} + \frac{1}{x}$.

   ◯ Option 3: $2\cos 2x + 5\sec^2 5x + 9\ln 5.5^{9x} + x$.

   ◯ Option 4: $2\cos 2x + 5\sec^2 5x + 9\ln 5.5^{9x} + \frac{1}{x} + 5$.

4. Which of the following option(s) is(are) true?

   ◯ Option 1: $\int_2^3 x^2\,dx = \frac{3}{2}$

   ◯ **Option 2:** $\int_1^2 \frac{1}{x}\,dx = \ln 2$

   ◯ **Option 3:** $\int_0^{\frac{\pi}{3}} \tan x \sec x\,dx = 1$

   ◯ **Option 4:** $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = \frac{\pi}{2}$

   ◯ Option 5: $\int_{2020}^{2021} 1\,dx = 2021$

   ◯ **Option 6:** $\int_1^2 x^{-5}\,dx = \frac{15}{64}$

   ◯ **Option 7:** $\int_0^1 \sqrt{x}\,dx = \frac{2}{3}$

   ◯ **Option 8:** $\int_0^1 x^{\frac{3}{2}}\,dx = \frac{2}{5}$

14.2 Level 2:

5. Match the functions $f(x)$ in column A with their anti-derivatives in column B in Table M2W3AQ2.

	Function (Column A)		Anti-derivative (Column B)
a)	$f(x) = -\sin 5x + \cos 2x$	i)	$F(x) = \frac{e^{2x}}{2} + \frac{2^{3x}}{3\ln 2} + \frac{\ln 2x}{2} + 7$
b)	$f(x) = x^4 + \frac{2}{x^3}$	ii)	$F(x) = \frac{\cos 5x}{5} + \frac{\sin 2x}{2} + 2$
c)	$f(x) = e^{2x} + 2^{3x} + \frac{1}{2x}$	iii)	$F(x) = x\ln 3x - x + \sin^{-1}\frac{x}{4} + \tan^{-1}\frac{x}{5} - 1$
d)	$f(x) = \ln 3x + \frac{1}{\sqrt{16-x^2}} + \frac{5}{25+x^2}$	iv)	$F(x) = \frac{x^5}{5} - \frac{1}{x^2} + 10$

Table : M2W3AQ2

Choose the correct option.

- ◯ Option 1: a → ii, b → iv, c → iii, d → i.
- ◯ Option 2: a → ii, b → iii, c → i, d → iv.
- ◯ **Option 3:** a → ii, b → iv, c → i, d → iii.
- ◯ Option 4: a → iv, b → iii, c → i, d → ii.

Let $f(x) = 2x^2$ be a function defined on the domain D which contains the interval $[0, 4]$. Let P be a partition of the interval $[0, 4]$ defined as $0 = x_0 < x_1 < \ldots < x_n = 4$ where $x_i = \frac{i\times 4}{n}$ i.e., $x_0 = 0 < \frac{4}{n} < \frac{2\times 4}{n} < \frac{3\times 4}{n} < \ldots < \frac{4(n-1)}{n} < \frac{4n}{n} = 4 = x_n, i = 1, 2, \ldots, n$ and $x_i^* \in [x_{i-1}, x_i]$, $\triangle x_i = x_i - x_{i-1}$. Answer the questions 6 and 7 based on the above information.

6. Which of the following option(s) is(are) true?
   - ◯ **Option 1:** $\triangle x_i = \frac{4}{n}$.
   - ◯ Option 2: $||P|| = \frac{1}{n}$.
   - ◯ **Option 3:** If $x_i^* = x_{i-1}$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $\frac{2\times 4^3}{n^3}[1^2 + 2^2 + \ldots + (n-1)^2]$.
   - ◯ Option 4: If $x_i^* = x_{i-1}$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $\frac{2\times 4^3}{n^3}$.
   - ◯ Option 5: If $x_i^* = x_{i-1}$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $1^2 + 2^2 + \ldots + (n-1)^2$.

7. Which of the following option(s) is (are) true?
   - ◯ Option 1: If $x_i^* = x_i$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $\frac{2\times 4^3}{n^3}[1^2 + 2^2 + \ldots + (n-1)^2]$.
   - ◯ **Option 2:** If $x_i^* = x_i$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $\frac{2\times 4^3}{n^3}[1^2 + 2^2 + \ldots + (n-1)^2 + n^2]$.
   - ◯ Option 3: If $x_i^* = \frac{x_i + x_{i-1}}{2}$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $1^2 + 2^2 + \ldots + (n-1)^2 + n^2$.
   - ◯ **Option 4:** If $x_i^* = \frac{x_i + x_{i-1}}{2}$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $\sum_{i=1}^{n} \frac{32}{n^3}(4i^2 - 4i + 1)$.
   - ◯ Option 5: If $x_i^* = \frac{x_i + x_{i-1}}{2}$, then the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\triangle x_i$ equals the expression $\frac{32}{n^3}$.

8. Which of the following option(s) is(are) true?
   - ◯ **Option 1:** $\int_1^{\infty} e^{-x}\, dx = \frac{1}{e}$
   - ◯ Option 2:$\int_0^{\infty} \frac{1}{4+x^2}\, dx = \pi$
   - ◯ **Option 3:** $\int_1^{\infty} \frac{1}{x}\, dx$ does not exist.
   - ◯ Option 4: $\int_1^{\infty} x^{-3}\, dx = -\frac{1}{2}$

15 Lecture 5:

15.1 Level 1:

1. Which of the following option(s) is (are) true for some constant c?
   - ◯ Option 1: $\int x^3 \ln x \, dx = x^4[\ln x - 1] + c$
   - ◯ Option 2: $\int x^3 \ln x \, dx = \frac{x^4}{4}[\ln x + \frac{1}{4}] + c$
   - ◯ Option 3: $\int x^2 2^{3x} \, dx = \frac{6x \ (2^{3x})}{\ln 2} - \frac{18 \ (2^{3x})}{(\ln 2)^2} + c$
   - ◯ **Option 4:** $\int 3cosec \ 2x + 2\cot 3x - x^4 \, dx = \frac{3\ln|cosec \ 2x - \cot 2x|}{2} + \frac{2\ln|\sin 3x|}{3} - \frac{x^5}{5} + c$
   - ◯ **Option 5:** $\int 5\tan 3x - \sec 4x \, dx = \frac{5\ln|\sec 3x|}{3} - \frac{\ln|\sec 4x + \tan 4x|}{4} + c$
2. Use basic properties of definite integrals to choose which of the following option(s) is (are) true?
   - ◯ Option 1: $\int_{-\pi}^{\pi} x^2 \cos 3x \, dx = \int_0^{\pi} x^2 \cos 3x \, dx$
   - ◯ **Option 2:** $\int_2^3 \cot 5x + \ln(\sin 3x) \, dx = -\int_3^2 \cot 5x + \ln(\sin 3x) \, dx$
   - ◯ Option 3:$\int_0^3 x^{100} \, dx = \int_0^1 x^{100} \, dx - \int_1^2 x^{100} \, dx - \int_2^3 x^{100} \, dx$
   - ◯ **Option 4:** $\int_0^3 x^{100} \, dx = \int_0^1 x^{100} \, dx + \int_1^2 x^{100} \, dx + \int_2^3 x^{100} \, dx$
3. If $f(x) = x^2$ then find the area under the curve represented by $f(x)$, which lie above to X- axis in the interval [-1,2]. [Ans: 3]
4. Find the value of given definite integral $\int_{-2021}^{2021} (x^{2021}. \cos 2021x + \sin 2021x) \, dx$. [Ans 0]

15.2 Level 2:

5. Match the definite integral expressions in column A with their values in column B in Table M2W3AQ3.

	Definite integral expression (Column A)		Definite integral value (Column B)
a)	$\int_1^2 3^{3x}\,dx$	i)	$\pi - 2$
b)	$\int_1^2 \frac{\ln 3x}{3x}\,dx$	ii)	$\frac{234}{\ln 3}$
c)	$\int_0^{\frac{\pi}{2}} 2x\cos x$	iii)	2
d)	$\int_0^{\frac{\pi}{2}} 2x\sin x$	iv)	$\frac{1}{6}[(\ln 6)^2 - (\ln 3)^2]$

Table : M2W3AQ3

Choose the correct option.

- ◯ Option 1: a → ii, b → iv, c → iii, d → i.
- ◯ **Option 2:** a → ii, b → iv, c → i, d → iii.
- ◯ Option 3: a → iv, b → ii, c → iii, d → i.
- ◯ Option 4: a → iv, b → ii, c → i, d → iii.

6. If $f_1(x) = \lfloor x \rfloor + 1$ and $f_2(x) = \lceil x \rceil + 2$, then which of the following option(s) is(are) true?

- ◯ **Option 1:** $\int_0^4 f_1(x)\,dx = 10$
- ◯ Option 2: $\int_0^4 f_1(x)\,dx = 6$

◯ Option 3: $\int_0^4 f_2(x)\,dx = 10$

◯ **Option 4:** $\int_0^4 f_2(x)\,dx = 18$

◯ Option 5: $\int_0^4 f_1(x)\,dx = \int_0^4 f_2(x)\,dx$

◯ **Option 6:** $\int_0^{100} f_2(x)\,dx = 5250$

◯ Option 7: $\int_0^{100} f_2(x)\,dx = 5050$

7. Which of the following option(s) is(are) true?

   [**Hint:** Use the function $\frac{1}{1-\frac{x}{2}}$ and $\frac{1}{1+x^2}$]

   ◯ Option 1: $\int_0^1 \frac{1}{1+x^3}\,dx \le \frac{\pi}{4}$

   ◯ **Option 2:** $\int_0^1 \frac{1}{1+x^3}\,dx \ge \frac{\pi}{4}$

   ◯ Option 3: $\int_0^1 \frac{1}{1+x^3}\,dx \ge 2\ln 2$

   ◯ **Option 4:** $\int_0^1 \frac{1}{1+x^3}\,dx \le 2\ln 2$

8. Let $f_1(x) = 3x^2$ and $f_2(x) = 4 - x^2$ represent two curves (see Figure M2W3AQ7 for reference). If A is the area which is enclosed by the curves $f_1(x)$ and $f_2(x)$, then find the value of $3A$. [Ans: 16]

   [**Hint:** Find the area under both curve$f(x) = 3x^2$ and $f(x) = 4 - x^2$, and between the intersection point of the curves and above to X- axis.]

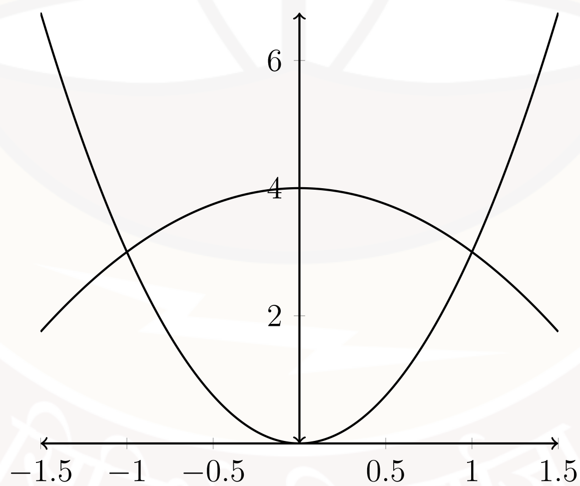


Figure: M2W3AQ7

Math-2 : Activity Questions
Week-4

Instructions:

- $(i, j)^{th}$ entry in each given matrix is a real number and scalars are also real numbers.
- If order of a matrix M is not specified, then you may assume that M is a square matrix of order 2 or 3.

16 Lecture 1.1

16.1 Level 1

1. Choose the set of correct options using Figure M2W4AQ1.

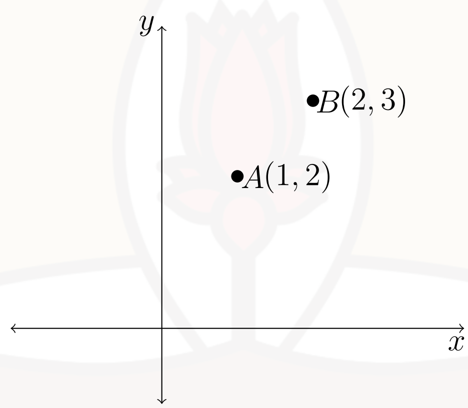


Figure M2W4AQ1

[Hint: Recall that, vector addition and scalar multiplication are done coordinatewise.]

◯ **Option 1:** $2A$ is the vector $(2, 4)$.

◯ **Option 2:** $3B$ is the vector $(6, 9)$.

◯ **Option 3:** $A + B$ is the vector $(3, 5)$.

◯ **Option 4:** $A - B$ is the vector $(-1, -1)$.

2. Let $V_1 = (1, 1)$, $V_2 = (1, 0)$, and $V_3 = (0, 1)$ be three vectors. Find out the correct set of options.

◯ **Option 1:** $(2, 3) = 2V_1 + 0V_2 + V_3$

◯ **Option 2:** $(2, 3) = 0V_1 + 2V_2 + 3V_3$

◯ Option 3: $(2, 3) = 2V_1 + V_2 + 0V_3$

- ◯ Option 4: $(2,3) = 0V_1 + 3V_2 + 2V_3$

The marks obtained by Karthika, Romy and Farzana in Quiz 1, Quiz 2 and End sem (with the maximum marks for each exam being 100) are shown in Table M2W4AQ1.

	Quiz 1	Quiz 2	End sem
Karthika	51	50	61
Romy	33	41	45
Farzana	38	21	35

Table: M2W4AQ1

Use the above information answer questions 3, 4, and 5:

3. Choose the following set of correct options.
   - ◯ **Option 1:** Marks obtained by Romy in Quiz 1, Quiz 2 and End sem represent a row vector.
   - ◯ **Option 2:** Quiz 2 marks of Karthika, Romy and Farzana represent a column vector.
   - ◯ Option 3: Number of components in column vector representing Quiz 2 marks are 9.
   - ◯ **Option 4:** Number of components in row vector representing Romy's marks are 3.

4. In order to improve her marks, Farzana undertook project work and succeeded in increasing her marks. Her marks became doubled for each exam. Choose the correct options.
   - ◯ **Option 1:** To obtain the marks obtained by Farzana after completion of the project, scalar multiplication has to be done by 2 to the row vector representing Farzana's marks.
   - ◯ Option 2: To obtain the marks obtained by Farzana after completion of the project, scalar multiplication has to be done by 1 to the row vector representing Farzana's marks.
   - ◯ **Option 3:** After completion of the project the row vector representing Farzana's marks is (76, 42, 70)
   - ◯ Option 4: After completion of the project the row vector representing Farzana's marks is (76, 21, 35).
   - ◯ Option 5: After completion of the project the row vector representing Farzana's marks is (66, 82, 90)

5. Following Farzana's improved marks due to her project (i.e her marks become doubled for each exam), all students were given bonus marks in Quiz 2, which is given by the column vector $\begin{pmatrix}10\\12\\15\end{pmatrix}$. What will be the column vector representing the final marks obtained in Quiz 2 by Karthika, Romy and Farzana?

   ◯ **Option 1:** $\begin{pmatrix}60\\53\\57\end{pmatrix}$

   ◯ Option 2: $\begin{pmatrix}60\\53\\36\end{pmatrix}$.

   ◯ Option 3: $\begin{pmatrix}61\\45\\53\end{pmatrix}$

   ◯ Option 4: $\begin{pmatrix}71\\57\\85\end{pmatrix}$

16.2 Level 2

6. Let A and B be two vectors. Which of the following statements is (are) true?

   ◯ **Option 1:** $3A + 5B = 3(A + B) + [(A + B) - (A - B)]$

   ◯ Option 2: $3A + 5B = 5(A + B) - [(A + B) - (A - B)]$

   ◯ Option 3: $3A + 5B = 3(A + B) + [(A + B) + (A - B)]$

   ◯ **Option 4:** $3A + 5B = 5(A + B) - [(A + B) + (A - B)]$

7. Let $V_1 = (1, 0, 0)$, $V_2 = (0, 1, 0)$ and $V_3 = (0, 0, 1)$ be three vectors and $a, b,$ and c be three real numbers (scalars). Which of the following is (are) true?

   ◯ **Option 1:** $(a, b, c) = aV_1 + bV_2 + cV_3$

   ◯ Option 2: $(a, b, c) = abV_1 + bcV_2 + caV_3$

   ◯ Option 3: $(a, 0, c) = aV_1 + cV_2 + 0V_3$

   ◯ **Option 4:** $(a, 0, c) = aV_1 + 0V_2 + cV_3$

8. Consider vectors $A(-1, 2)$ and $B(2, -2)$ in $\mathbb{R}^2$ as shown in Figure M2W4AQ2.

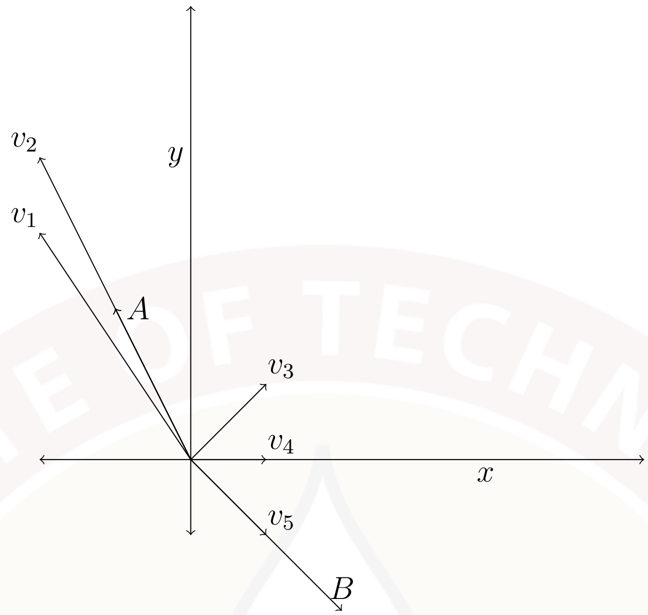


Figure M2W4AQ2

Choose the set of correct options.
[Hint: Recall the geometric representation of vectors, scalar multiplication and vectors addition.]

- ◯ Option 1: v_1 represents a scalar multiple of A.
- ◯ **Option 2:** v_2 represents a scalar multiple of A.
- ◯ **Option 3:** v_5 represents a scalar multiple of B.
- ◯ Option 4: v_1 represents a scalar multiple of B.
- ◯ **Option 5:** v_4 represents a scalar multiple of $A + B$.
- ◯ Option 6: v_3 represents a scalar multiple $A + B$.

17 Lecture 1.2

17.1 Level 1:

1. Suppose $A = \begin{bmatrix} 2 & 4 & 5 & 1 \\ 11 & -2 & 9 & -6 \\ -3 & 4 & 7 & 7 \end{bmatrix}$. Which of the following is true about the matrix A?

   [Hint: The (i, j)-th entry is the entry which is at the i-th row and j-th column.]

   - ◯ Option 1: It is a 4×3 matrix.
   - ◯ **Option 2:** It is a 3×4 matrix.
   - ◯ Option 3: (2,3)-th entry of the matrix A is 4.
   - ◯ **Option 4:** (2,3)-th entry of the matrix A is 9.

2. Which of the following statements is(are) TRUE?

   [Hint: Recall, the definitions of scalar matrix, diagonal matrix, and identity matrix.]

   - ◯ Option 1: Any diagonal matrix is a scalar matrix.
   - ◯ Option 2: Scalar matrices may not be square matrices.
   - ◯ **Option 3:** Scalar matrices must be square matrices.
   - ◯ Option 4: Any scalar matrix is an identity matrix.

3. Given below is the system of linear equations:
   $7x_1 + 10x_2 + 12x_3 = 36$
   $8x_1 + 4x_2 - 9x_3 = 11$
   $4x_1 - x_2 + 3x_3 = 10$

   If the matrix representation of the system of equations is $Ax = b$, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then

   - ◯ **Option 1:** $A = \begin{bmatrix} 7 & 10 & 12 \\ 8 & 4 & -9 \\ 4 & -1 & 3 \end{bmatrix}, b = \begin{bmatrix} 36 \\ 11 \\ 10 \end{bmatrix}$
   - ◯ Option 2: $A = \begin{bmatrix} 7 & 10 & 12 & 36 \\ 8 & 4 & 9 & 11 \\ 4 & 1 & 3 & 10 \end{bmatrix}, b = \begin{bmatrix} 36 \\ 11 \\ 10 \end{bmatrix}$
   - ◯ Option 3: $A = \begin{bmatrix} 7 & 10 & 12 & 36 \\ 8 & 4 & -9 & 11 \\ 4 & -1 & 3 & 10 \end{bmatrix}$
   - ◯ Option 4: $A = \begin{bmatrix} 7 & 10 & 36 & 12 \\ 8 & 4 & 11 & -9 \\ 4 & -1 & 10 & 3 \end{bmatrix}$

4. Which of the following statements is(are) TRUE?

◯ Option 1: Addition of two matrices is possible only if the number of columns in the first matrix is same as the number of rows in the second matrix.

◯ **Option 2:** Addition of two matrices is possible only if the orders of both the matrices are the same.

◯ Option 3: Defining AB is possible if the number of rows in the matrix A is same as the number of columns in the matrix B.

◯ **Option 4:** Defining AB is possible if the number of columns in the matrix A is same as the number of rows in the matrix B.

5. Suppose $P = \begin{bmatrix} 3 & -1 & 7 \\ 4 & 0 & 1 \\ 2 & -5 & 2 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 4 & -9 \end{bmatrix}$, $R = \begin{bmatrix} 0 & -3 & 10 \end{bmatrix}$, $D = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$

[Hint: If A is a matrix of order $m \times n$ and B is a matrix of order $n \times p$, then the order of AB is $m \times p$.]

◯ **Option 1:** The matrix PD is of order 3×1.

◯ Option 2: The matrix PD is of the order 1×3.

◯ Option 3: The matrix QD is of order 3×3.

◯ **Option 4:** The matrix QD is of order 1×1.

◯ **Option 5:** The matrix DQ is of order 3×3.

◯ Option 6: The matrix DQ is of order 1×1.

◯ Option 7: The product QD is not defined.

◯ **Option 8:** The product QR is not defined.

◯ **Option 9:** The addition $P + Q$ is not defined.

◯ **Option 10:** The addition $P + D$ is not defined.

17.2 Level 2:

6. Choose the set of correct options.

[Hint: Assume $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and use the given conditions to find matrix A, for first two options.]

◯ Option 1: If A is a square matrix of order 2 and $A^2 = I$, then $A = I$ or $A = -I$ where I is the identity matrix of order 2.

◯ Option 2: If A is a square matrix of order 2 and $A^2 = 0$, then $A = 0$.

◯ **Option 3:** If A and B are square matrices of order 3 and $A + B = 0$, then $B = -A$.

- ◯ **Option 4:** If A is a scalar matrix of order 3, B is a non-zero square matrix of order 3 and $AB = 0$, then $A = 0$.

7. If A is a square matrix of order 2 whose first column is denoted by C_1 and second column is denoted by C_2 (the left most column is taken as the first one) and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ then choose the set of correct options.
[Hint: Recall the definition of matrix multiplication in terms of rows and columns.]
    - ◯ **Option 1:** The first column of AB is $b_{11}C_1 + b_{21}C_2$.
    - ◯ **Option 2:** The second column of AB is $b_{12}C_1 + b_{22}C_2$.
    - ◯ Option 3: The first column of AB is $b_{21}C_1 + b_{11}C_2$.
    - ◯ Option 4: The second column of AB is $b_{22}C_1 + b_{21}C_2$.

8. Choose the set of correct options.
[Hint: Assume $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and use the given condition to find matrix A for last two options.]
    - ◯ **Option 1:** There exist some real matrices A and B, such that $AB = BA$.
    - ◯ Option 2: There do not exist any real matrices A and B, such that $AB = BA$.
    - ◯ Option 3: There does not exist any real matrix A, such that $A^2 = A$.
    - ◯ **Option 4:** There exists some real 2×2 matrix A such that $A^2 + A + I = 0$

18 Lecture 1.3

18.1 Level 1

Consider a system of linear equations (System 1):

$$\begin{aligned} -2x_1 + 3x_2 + x_3 &= 1 \\ -x_1 + x_3 &= 0 \\ 2x_2 &= 5 \end{aligned} \tag{1}$$

Answer questions 1 and 2 based on the above data.

1. If the matrix representation of system (1) is $Ax = b$, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then

   ◯ **Option 1:** $A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$

   ◯ Option 2: $b = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$

   ◯ Option 3: $A = \begin{bmatrix} -2 & -1 & 0 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$, and $b = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

   ◯ **Option 4:** $A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$, and $b = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

2. System (1) has

   [Hint: Solve for x_1, x_2, and x_3.]

   ◯ **Option 1:** a unique solution.

   ◯ Option 2: no solution.

   ◯ Option 3: infinitely many solutions.

   ◯ Option 4: None of the above.

3. Choose the set of correct options.

   [Hint: Think of A as a 2×2 or 3×3 matrix, and b accordingly.]

   ◯ **Option 1:** Every system of linear equations has either a unique solution, no solution or infinitely many solutions.

   ◯ Option 2: If each equation of a system of linear equations is multiplied by a non-zero constant c, then the solution of the new system of equations is c times the solution of the old system of equations.

◯ **Option 3:** If $Ax = b$ is a system of linear equations which has a solution, then the system of linear equations $cAx = b$, where $c \neq 0$, will also have a solution.

◯ **Option 4:** If $Ax = b$ is a system of linear equations which has a solution, then $\frac{1}{c}Ax = b$, where $c \neq 0$, will also have a solution.

4. The Plane 1 and Plane 2 in Figure M2W1AQ3, correspond to two different linear equations, which form a system of linear equations.

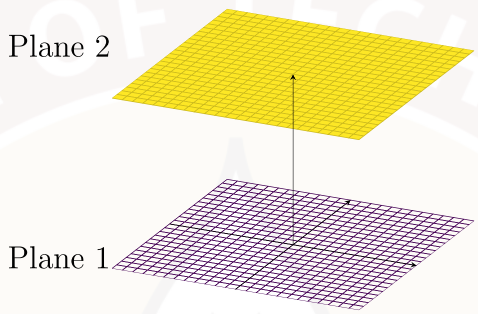


Figure M2W1AQ3

The above system of linear equations has

[Hint: If a system of linear equations has a solution, then the point corresponding to the solution must lie on each plane corresponding to each linear equation of the given system.]

◯ Option 1: a unique solution.

◯ **Option 2:** no solution.

◯ Option 3: infinitely many solutions.

◯ Option 4: None of the above.

5. Consider the geometric representations (Figures $(a), (b),$ and (c)) of three systems of linear equations.

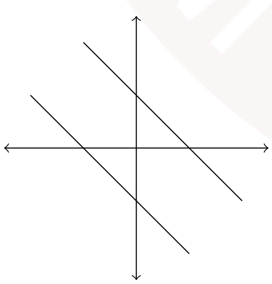


Figure (a)

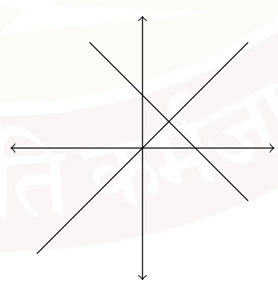


Figure (b)

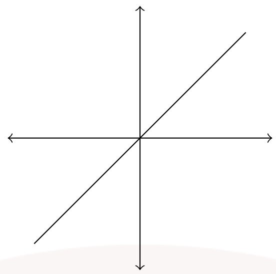


Figure (c)

Choose the set of correct options.

- ◯ **Option 1:** Figure (a) represents a system of linear equations which has no solution.
- ◯ Option 2: Figure (a) represents a system of linear equations which has infinitely many solutions.
- ◯ **Option 3:** Figure (b) represents a system of linear equations which has a unique solution.
- ◯ Option 4: Figure (b) represents a system of linear equations which has infinitely many solutions.
- ◯ **Option 5:** Figure (c) represents a system of linear equations which has infinitely many solutions.
- ◯ Option 6: Figure (c) represents a system of linear equations which has no solution.

18.2 Level 2

6. Consider a system of equations:

$$\begin{aligned} 2x_1 + 3x_2 &= 6 \\ -2x_1 + kx_2 &= d \\ 4x_1 + 6x_2 &= 12 \end{aligned}$$

Choose the set of correct options.

- ◯ **Option 1:** $Ax = b$ represents the above system, where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} 2 & 3 \\ -2 & k \\ 4 & 6 \end{bmatrix}$, and $b = \begin{bmatrix} 6 \\ d \\ 12 \end{bmatrix}$
- ◯ **Option 2:** The system has no solution if $k = -3$, $d = 0$.

◯ **Option 3:** The system has a unique solution if $k = 3$, $d = 0$.

◯ Option 4: The system has infinitely many solutions if $k = -3$, $d = 6$.

◯ **Option 5:** The system has infinitely many solutions if $k = -3$, $d = -6$.

7. Let x_1 and x_2 be solutions of the system of linear equations $Ax = b$. Which of the following options are correct?

   ◯ Option 1: $x_1 + x_2$ is a solution of the system of linear equations $Ax = b$.

   ◯ **Option 2:** $x_1 + x_2$ is a solution of the system of linear equations $Ax = 2b$.

   ◯ Option 3: $x_1 - x_2$ is a solution of the system of linear equations $Ax = b$.

   ◯ **Option 4:** $x_1 - x_2$ is a solution of the system of linear equations $Ax = 0$.

8. Let v be a solution of the systems of linear equations $A_1x = b$ and $A_2x = b$. Which of the following options are correct ?

   ◯ Option 1: v is a solution of the system of linear equations $(A_1 + A_2)x = b$.

   ◯ **Option 2:** v is a solution of the system of linear equations $(A_1 + A_2)x = 2b$.

   ◯ **Option 3:** v is a solution of the system of linear equations $(A_1 - A_2)x = 0$.

   ◯ Option 4: v is a solution of the system of linear equations $(A_1 - A_2)x = b$.

19 Lecture 1.4

19.1 Level 1

1. Let A be a 3×3 matrix with non-zero determinant. If $det(2A) = k\ det(A)$, then what will be the value of k?

   [Hint: If a scalar (c) is multiplied with one row of a matrix A, then the determinant of the new matrix will be c times the determinant of A.]

   ◯ Option 1: 2

   ◯ Option 2: 4

   ◯ **Option 3:** 8

   ◯ Option 4: 16

2. If $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -3 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 3 & 4 & 2 \end{bmatrix}$, then choose the set of correct options.

   ◯ Option 1: $det(A) = -9$ and $det(B) = -3$.

   ◯ Option 2: $det(A) = 9$ and $det(B) = 3$.

   ◯ **Option 3:** $det(A) = -9$ and $det(B) = 3$.

   ◯ Option 4: $det(AB) = -27$ and $det(BA) = 27$.

   ◯ **Option 5:** $det(AB) = det(BA) = -27$.

3. Let A be a 2×2 matrix, which is given as $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. Define the following matrices :

$$B = \begin{bmatrix} a_{11} - a_{21} & a_{12} - a_{22} \\ a_{21} & a_{22} \end{bmatrix},\ C = \begin{bmatrix} a_{11} - a_{12} & a_{12} \\ a_{21} - a_{22} & a_{22} \end{bmatrix},$$

$$D = \begin{bmatrix} a_{11} + a_{21} & a_{12} - a_{22} \\ a_{21} & a_{22} \end{bmatrix},\ E = \begin{bmatrix} a_{11} - a_{21} & a_{12} + a_{22} \\ a_{21} & a_{22} \end{bmatrix}$$

   Which of the matrices among $B, C, D,$ and E have the same determinant as that of the matrix A, for any real numbers $a_{11}, a_{12}, a_{21}, a_{22}$?

   ◯ Option 1: B and D

   ◯ Option 2: B and E

   ◯ **Option 3:** B and C

   ◯ Option 4: D and E

   ◯ Option 5: C and E

4. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ ta_{11} - sa_{31} & ta_{12} - sa_{32} & ta_{13} - sa_{33} \\ ra_{31} & ra_{32} & ra_{33} \end{bmatrix}$ be a matrix and $r, s, t \neq 0$. Find $det(A)$ [Ans: 0]

19.2 Level 2

Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle. The area of the triangle ABC is given by $\frac{1}{2}|det(D)|$ square units, where $D = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$

Use this information to answer questions 5 and 6.

5. If $P(-1, 1), Q(1, 1)$ and $R(1, 0)$ are the vertices of a triangle, then find the area (in square units) of the triangle PQR. [Ans: 1]

6. Let $P(x-1, 2x), Q(0, x-2)$ and $R(-x+1, 1)$ be the vertices of a triangle. If the area of the triangle is 5 square units and $x > 0$, then choose the set of correct options
   - ◯ **Option 1:** length of side $PQ = \sqrt{29}$ units.
   - ◯ Option 2: length of side $PR = \sqrt{40}$ units.
   - ◯ **Option 3:** length of side $QR = 2$ units.
   - ◯ Option 4: length of side $QR = 1$ units.

7. Find out the correct value of $det(A)$, where $A = \begin{bmatrix} 1 & 2010 & 2020 \times 2030 \\ 1 & 2020 & 2030 \times 2010 \\ 1 & 2030 & 2010 \times 2020 \end{bmatrix}$

   [Hint: Observe that the given matrix is of the form: $\begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$]
   - ◯ Option 1: 0
   - ◯ Option 2: 200
   - ◯ Option 3: -2000
   - ◯ **Option 4:** 2000

8. Let A be a square matrix of order 3 and B be a matrix that is obtained by adding 2 times the first row of A to the third row of A and adding 3 times the second row of A to the first row of A. What is the value of $det(6A^2B^{-1})$?
   - ◯ Option 1: 6 $det(A)$
   - ◯ Option 2: 6 $det(A)det(B)$
   - ◯ **Option 3:** 6^3 $det(A)$
   - ◯ Option 4: 6^3 $det(A)det(B)$

20 Lecture 1.5

20.1 Level 1

1. Choose the set of correct options

   ◯ **Option 1:** If A is a real 3×3 matrix, then $det(A) = det(A^T)$.

   ◯ Option 2: If $A = [a_{ij}]$ is a real 4×4 matrix, then the order of the sub matrix obtained by deleting the i-th row and j-th column of A is 4×4.

   ◯ **Option 3:** If $A = [a_{ij}]$ is a real 4×4 matrix, then the order of the sub matrix obtained by deleting the i-th row and j-th column of A is 3×3.

   ◯ **Option 4:** If A is a real 3×3 matrix, then the orders of all possible square sub matrices of A are $1 \times 1, 2 \times 2,$ and 3×3.

2. Let $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ be a 3×3 matrix. Which of the following is(are) correct?

   [Hint: Determinant can be calculated by expanding with respect to different rows or columns.]

   ◯ **Option A:**
   $$det(A) = 3 \times det\left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\right) - 2 \times det\left(\begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix}\right) + 2 \times det\left(\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\right)$$

   ◯ **Option B:**
   $$det(A) = 3 \times det\left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\right) + 2 \times det\left(\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}\right) + 2 \times det\left(\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\right)$$

   ◯ **Option C:**
   $$det(A) = -2 \times det\left(\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}\right) + 3 \times det\left(\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\right) - 2 \times det\left(\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\right)$$

   ◯ Option D:
   $$det(A) = 2 \times det\left(\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}\right) - 3 \times det\left(\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\right) + 2 \times det\left(\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\right)$$

3. If all the elements of a 3×3 real matrix A are the same, then which of the following is (are) correct?

   ◯ **Option 1:** Determinant of matrix A is 0.

   ◯ Option 2: Determinant of matrix A cannot be determined from the given information.

   ◯ Option 3: Determinant of matrix A will be the sum of the elements of a row.

   ◯ **Option 4:** Determinant of matrix $A+A^T$ is 0, where A^T denotes the transpose of A.

◯ Option 5: Determinant of matrix $A+A^T$ cannot be determined from the given information, where A^T denotes the transpose of A.

4. Choose the set of correct options

◯ **Option 1:** If two rows are the same in a 3×3 real matrix, then the determinant of that matrix is zero.

◯ **Option 2:** If two columns are the same in a 3×3 real matrix, then the determinant of that matrix is zero.

◯ Option 3: If one row is a non-zero multiple of another row in a 3×3 real matrix, then the determinant of that matrix is not zero.

◯ **Option 4:** If one column is a non-zero multiple of another column in a 3×3 real matrix, then the determinant of that matrix is zero.

5. Let $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ be a matrix of order 4×4. Find $det(A)$. [Ans: -1]

20.2 Level 2

6. Let $A, B,$ and C be 3×3 real matrices. Which of the following options is (are) correct?

◯ **Option 1:** $det(ABC) = det(A)\ det(B)\ det(C)$

◯ **Option 2:** $det(A^3) = det(A)^3$

◯ Option 3: $det(A + B + C) = det(A) + det(B) + det(C)$

◯ **Option 4:** $det(AB^T) = det(A)\ det(B)$, where B^T denotes the transpose of B.

7. Let $A = [\delta_{ij}]$ be a matrix of order 3×3 such that

$$\delta_{ij} = \begin{cases} 0 & \text{if } i > j \\ j & \text{if } i \leq j \end{cases}$$

Choose the set of correct options

[Hint: Write down matrix A first.]

◯ **Option 1:** A is an upper triangular matrix.

◯ Option 2: A is a lower triangular matrix.

◯ **Option 3:** $det(A) = 6$

◯ Option 4: $det(A) = 3$

8. Let $A = [\delta_{ij}]$ be a matrix of order 3×3 such that

$$\delta_{ij} = \begin{cases} 0 & \text{if } i < j \\ 2 & \text{if } i \geq j \end{cases}$$

Choose the set of correct options

- ◯ Option 1: A is an upper triangular matrix.
- ◯ **Option 2:** A is a lower triangular matrix.
- ◯ Option 3: $det(A) = 2$
- ◯ **Option 4:** $det(A) = 8$

Maths-2 : Activity Questions
Week-5

21 Lecture 5.1

21.1 Level 1

1. Consider a square matrix $A = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$. If

   - $P = -1M_{11} - 2M_{12} + 0M_{13}$
   - $Q = -2M_{21} + 0M_{22} + 1M_{23}$
   - $R = -1M_{31} - 0M_{32} + 1M_{33}$

   where M_{ij} is the minor with respect to the (i, j)-th entry, then choose the set of correct of options.

   ◯ **Option 1:** $det(A) = -2$.
   ◯ Option 2: $P \neq Q$.
   ◯ Option 3: $P \neq Q \neq R$.
   ◯ **Option 4:** $P = Q = R$.
   ◯ Option 5: $Q \neq R$

2. Suppose for a real 3×3 matrix A, there exists a real 3×3 matrix P such that $D = PAP^{-1}$ is a real 3×3 diagonal matrix. Choose the correct set of options.

   ◯ Option 1: $det\ (A)$ must be equal to $det\ (P)$.
   ◯ **Option 2:** $det\ (A)$ must be equal to $det\ (D)$.
   ◯ Option 3: The matrix A must be equal to D.
   ◯ **Option 4:** If D is the identity matrix of order 3, then A must be the identity matrix of order 3.

3. Choose the set of correct options for square matrices of order 3.

   ◯ **Option 1:** If $AB = 0$, then one of the matrices between A and B must have determinant 0.
   ◯ Option 2: If $AB = 0$, then both the matrices A and B must have determinant 0.
   ◯ Option 3: If $AB = 3I$, where I denotes the identity matrix of order 3, then $detA = \frac{3}{det(B)}$.

- Option 4: If $AB = 3I$, where I denotes the identity matrix of order 3, then $detA = \frac{9}{det(B)}$.
- **Option 5:** If $AB = 3I$, where I denotes the identity matrix of order 3, then $detA = \frac{27}{det(B)}$.
- **Option 6:** If $AB = 3I$, where I denotes the identity matrix of order 3, then both A and B must have non-zero determinant.

4. Consider a matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$. If $a+b+c$ is divisible by 6 then choose the set of correct options.
   - Option 1: $det(A)$ is divisible by 5.
   - **Option 2:** $det(A)$ is divisible by 6.
   - **Option 3:** $det(A)$ is divisible by 2.
   - **Option 4:** $det(A)$ is divisible by 3.

21.2 Level 2

5. Let $A = [\delta_{ij}]$ be a square matrix of order 3, where $\delta_{ij} = i \times j$. Find $det(A)$. [Ans: 0]

6. Consider the following square matrices:
   $A = \begin{bmatrix} 2013 & 2014 & 2015 \\ 2016 & 2017 & 2022 \\ 2019 & 2020 & 2021 \end{bmatrix}, B = \begin{bmatrix} 2016 & 2017 & 2022 \\ 2013 & 2014 & 2015 \\ 2019 & 2020 & 2021 \end{bmatrix}$ and $C = \begin{bmatrix} 4032 & 4034 & 4044 \\ 2013 & 2014 & 2015 \\ 2019 & 2020 & 2021 \end{bmatrix}$
   Choose the set of correct options.
   - Option 1: $det(B) = det(A)$.
   - **Option 2:** $det(A) = -det(B)$.
   - Option 3: $det(C) \neq -2det(B)$.
   - **Option 4:** $det(C) = -2det(A)$.

Let A be a real 3×3 matrix as given below:

$$A = \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix}$$

where C_i represents the i-th column of the matrix A. Now let us consider the following set of matrices.

- $A_1 = \begin{bmatrix} C_1 & C_2 + 5C_3 & C_3 \end{bmatrix}$
- $A_2 = \begin{bmatrix} C_1 + C_2 + C_3 & C_2 & C_3 \end{bmatrix}$
- $A_3 = \begin{bmatrix} C_1 & C_2 + 5C_3 & 0 \end{bmatrix}$
- $A_4 = \begin{bmatrix} C_1 + C_2 & C_2 + C_3 & C_3 + C_1 \end{bmatrix}$

Based on the above information answer questions 7 and 8.

7. Which of the matrices have the same determinant as that of A?

   ◯ **Option 1:** A_1

   ◯ **Option 2:** A_2

   ◯ Option 3: A_3

   ◯ Option 4: A_4

8. Choose the set of correct options.

   ◯ Option 1: $det(A_3)$ cannot be determined from the given information.

   ◯ **Option 2:** $det(A_3) = 0$

   ◯ **Option 3:** $det(A_4) = 2\ det(A)$

   ◯ Option 4: $det(A_2) = 3\ det(A)$

22 Lecture 5.2

22.1 Level 1

1. Consider a system of linear equations as follows:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1\\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2\\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
\end{aligned}
$$

Let the matrix representation of the above system be $Ax = b$, where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

Let A_{x_i} be the matrix obtained by replacing the i-th column of A $\left(\text{i.e., } \begin{bmatrix} a_{1i} \\ a_{2i} \\ a_{3i} \end{bmatrix}\right)$ by b, for $i = 1, 2, 3$. If $det(A_{x_i}) = 0$ for $i = 1, 2, 3$, then which of the following is (are) true?

- ○ Option 1: $x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ must be a solution.
- ○ **Option 2:** $x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is a solution if $det(A) \neq 0$.
- ○ **Option 3:** If $det(A) = 0$, then we cannot conclude about the solution using Cramer's rule.
- ○ **Option 4:** If $det(A) \neq 0$, then $b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Consider a system of linear equations

$$\begin{aligned} x_1 + x_3 &= 1 \\ -x_1 + x_2 - x_3 &= 1 \\ -x_2 + x_3 &= 1 \end{aligned}$$

Let matrix representation of the above system be $Ax = b$, where $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Let A_{x_i} be the matrix obtained by replacing the i-th column of A (i.e., $\begin{bmatrix} a_{1i} \\ a_{2i} \\ a_{3i} \end{bmatrix}$) by b, for $i = 1, 2, 3$.

Use the above information to answer questions 2 and 3.

2. Choose the set of correct options

   - ◯ Option 1: $A_{x_1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$
   - ◯ **Option 2:**$A_{x_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$
   - ◯ Option 3: $A_{x_2} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$
   - ◯ **Option 4:** $A_{x_3} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$

3. Choose the set of correct options.

   - ◯ **Option 1:** $x_1 = -2$.
   - ◯ Option 2: $x_2 = -2$.
   - ◯ **Option 3:** $x_3 = 3$.
   - ◯ Option 4: None of the above.

22.2 Level 2

4. Consider the system of linear equations $Ax = b$, where $A = \begin{bmatrix} 1 & a & 0 \\ a & 1 & 2 \\ 1 & 0 & a \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ 0 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, and the solution for x is partially known.

   What is the value of a^2, if $a \neq 0, \sqrt{3}, -\sqrt{3}$ is given?

   [Hint: Observe that the second row of the vector x is given as 0, which implies that x_2 is known, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$] [Answer: 2]

23 Lecture 5.3

23.1 Level 1

1. Match the matrices in Column A with their inverses in Column B.

	Column A		Column B
a)	$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 3 \end{bmatrix}$	i)	Inverse does not exist
b)	$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$	ii)	$B_2 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{3} \end{bmatrix}$
c)	$A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$	iii)	$B_3 = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

Table: M2W5AQ1

- ◯ Option 1: a → iii, b → ii, c → i
- ◯ Option 2: a → ii, b → i, c → iii
- ◯ **Option 3:** a → ii, b → iii, c → i
- ◯ Option 4: a → i, b → ii, c → iii

Consider a system of linear equations

$$\begin{aligned} 2x_1 - x_2 &= 3 \\ x_1 - x_3 &= 3 \\ x_2 - x_3 &= 2 \end{aligned}$$

Let the matrix representation of the above system be $Ax = b$, where $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$. Use the above information to answer questions 2 and 3.

2. Choose the set of correct options.

   ◯ Option 1: $A^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

   ◯ **Option 2:** $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -2 & 1 \end{bmatrix}$

   ◯ **Option 3:** Adjoint of the matrix A is $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -2 & 1 \end{bmatrix}$

   ◯ **Option 4:** $det(A) = 1$.

3. Choose the set of correct options.

   ◯ Option 1: $x_1 = -2$.

   ◯ **Option 2:** $x_2 = 1$.

   ◯ **Option 3:** $x_3 = -1$

   ◯ Option 4: None of the above.

4. Choose the set of correct options.

   ◯ Option 1: A system of linear equations $Ax = b$ is called a homogeneous system of linear equations if $b \neq 0$.

   ◯ **Option 2:** A system of linear equations $Ax = b$ is called a non-homogeneous system of linear equations if $b \neq 0$.

   ◯ **Option 3:** If v is a solution of the system of linear equations $Ax = b$, then $\frac{1}{c}v$ is a solution of system of linear equations $cAx = b$, where $c \neq 0$.

   ◯ **Option 4:** Let $Ax = b$ be a system of linear equations. If A is invertible, then $adj(A)x = b$ also has a solution.

5. Which of the following is the cofactor matrix of $\begin{bmatrix} 3 & 2 & 3 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$?

   ◯ Option 1: $\begin{bmatrix} 1 & 1 & 1 \\ -1 & -3 & -1 \\ -2 & 0 & 2 \end{bmatrix}$

   ◯ Option 2: $\begin{bmatrix} -1 & 1 & 2 \\ 1 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix}$

   ◯ **Option 3:** $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}$

◯ Option 4: $\begin{bmatrix} 1 & -1 & -1 \\ -1 & 3 & -1 \\ -2 & 0 & 2 \end{bmatrix}$

23.2 Level 2

6. Which of the following square matrices of order 3 are the same as their adjoint matrices?

   a) Identity matrix.

   b) Zero matrix.

   c) Any scalar matrix.

   d) Any diagonal matrix.

   ◯ **Option 1:** Only the matrices in a and b.

   ◯ Option 2: Only the matrices in a, b and c.

   ◯ Option 3: Only the matrices in b.

   ◯ Option 4: Only the matrices in d.

7. Choose the set of correct options.

   [Hint: For the first option, find out the adjoint matrix of an arbitrary square upper triangular matrix of order 2, i.e., take the matrix $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ and find out its adjoint matrix.]

   ◯ **Option 1:** The adjoint of a 2×2 real upper triangular matrix is an upper triangular matrix.

   ◯ **Option 2:** There exists a square matrix of order 3, such that $A = A^{-1} = adj(A)$.

   ◯ Option 3: If $A = A^{-1}$, then $det(A)$ must be 1.

   ◯ Option 4: If $A = A^{-1}$, then A must be an identity matrix.

   ◯ **Option 5:** If $A = A^{-1}$, then $A^2 = I$.

   ◯ **Option 6:** If $A^{-1} = adj(A)$, then $det(A)$ must be 1.

8. If A and B are squares matrices of order 2, then choose the set of correct options.

   [Hint: Find out the adjoint matrix of an arbitrary square matrix of order 2, i.e., take the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and find out its adjoint matrix.]

   ◯ Option 1: $adj(AB) = adj(A)adj(B)$.

   ◯ **Option 2:** $adj(AB) = adj(B)adj(A)$.

   ◯ **Option 3:** $adj(A + B) = adj(A) + adj(B)$.

   ◯ **Option 4:** $adj(A^T) = adj(A)^T$.

   ◯ **Option 5:** $adj(A^{-1}) = adj(A)^{-1}$.

24 Lecture 5.4

24.1 Level 1

1. Let $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ be a matrix (where the first row denotes the top most row, and the ordering of the rows is in the order top to bottom). Among the given set of options, identify the correct statements.

   ◯ Option 1: The first non-zero element in the first row is 3.

   ◯ **Option 2:** The first non-zero element in the second row is 1.

   ◯ Option 3: There is a non-zero element in the third row.

   ◯ **Option 4:** Since there is a row with all elements as zero, $det(A) = 0$.

   Let $I_{3\times3}$ denote the identity matrix of order 3. Answer questions 2 and 3 about the set S defined as :

$$S = \left\{ A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \right.$$

$$E = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, F = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\left. I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \ K = I_{3\times3}, \ L = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \right\}.$$

2. If S_1 is the subset of S consisting of all the matrices in S that are in row echelon form, then choose the correct option from the following.

   ◯ Option 1: $S_1 = S$

   ◯ Option 2: $S_1 = \{A, B, C, D, F, G, I\}$

   ◯ Option 3: $S_1 = \{B, C, D, E, F, G, H, I, J, K, L\}$

   ◯ Option 4: $S_1 = \{C, D, E, F, G, H, I, J, K\}$

   ◯ Option 5: $S_1 = \{C, D, E, F, G, H, J, K, L\}$

   ◯ **Option 6:** $S_1 = \{C, D, F, G, H, J, K, L\}$

3. If S_2 is the subset of S consisting of all the matrices in S that are in reduced row echelon form, then choose the correct option from the following.

- ◯ Option 1: $S_2 = \{A, C, D, F, G, L\}$
- ◯ Option 2: $S_2 = \{B, C, D, E, F, G, H, J, K\}$
- ◯ Option 3: $S_2 = \{C, D, G, H, J, K, L\}$
- ◯ **Option 4:** $S_2 = \{C, D, H, J, K, L\}$
- ◯ Option 5: $S_2 = \{C, D, H, I, J, K, L\}$

4. Consider the following system of linear equations:

$$\begin{aligned} 0x_1 + x_2 + 0x_3 + 0x_4 &= 1 \\ 0x_1 + 0x_2 + 0x_3 + 0x_4 &= 1 \\ x_1 + x_2 + 0x_3 + 0x_4 &= 1 \\ 0x_1 + 0x_2 + x_3 + x_4 &= 1. \end{aligned}$$

Choose the the set of correct options.

- ◯ Option 1: The system of linear of equations has a solution.
- ◯ **Option 2:** The system of linear equations has no solution.
- ◯ **Option 3:** $det(A) = 0$, where A is the coefficient matrix of the given system of linear equations.
- ◯ Option 4: None of the above.

5. Consider a system of linear equations:

$$\begin{aligned} 0x_1 + x_2 + 0x_3 + 0x_4 &= 1 \\ 0x_1 + 0x_2 + x_3 + 0x_4 &= 1 \end{aligned}$$

Choose the set of correct options.

[Hint: Recall the definitions of independent and dependent variable with respect to reduced row echelon form.]

- ◯ Option 1: x_1 and x_2 are dependent variables.
- ◯ **Option 2:** x_2 is a dependent variable.
- ◯ Option 3: x_3 and x_4 are independent variables.
- ◯ **Option 4:** x_4 is an independent variable.

24.2 Level 2

6. Suppose a system of linear equations consists of only one equation and four variables as follows:

$$x_1 + x_2 + x_3 + x_4 = a$$

where a is a constant. Among the below options identify the correct statement.

[Hint:"Think about" how many variables can be expressed in terms of the other variables in the given system of linear equations?]

- ◯ Option 1: There are two independent and two dependent variables.
- ◯ **Option 2:** There are three independent and one dependent variable.
- ◯ Option 3: There are one independent and three dependent variables.
- ◯ Option 4: All the four variables are dependent.
- ◯ Option 5: All the four variables are independent.

7. Let $[A|b]$ denote the augmented matrix of the system of linear equations

$$2x_1 + x_2 = 3$$

$$x_1 + 3x_2 = 4$$

where, $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, and $b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

Let the matrix

$$\left[\begin{array}{cc|c} 1 & 0 & \text{a} \\ 0 & 1 & \text{b} \end{array}\right]$$

denote the reduced row echelon form of the augmented matrix corresponding to the system linear equations above. Which of the following option(s) is (are) correct?

- ◯ Option 1: The values of a and b cannot be determined from the given information.
- ◯ Option 2: $a = b$ but their exact values cannot be determined from the given information.
- ◯ **Option 3:** $a = b = 1$
- ◯ Option 4: $a = 2$ and $b = 3$
- ◯ Option 5: The solutions for x_1 and x_2 are not unique.
- ◯ **Option 6:** $x_1 = x_2 = 1$, and the system of linear equations has a unique solution.
- ◯ Option 7: $x_1 = x_2 = 1$ is the solution. However, it is not possible to determine whether the system of equations has a unique solution or not from the given information.

8. Let $Ax = b$ be a matrix representation of a system of linear equations, where A is a 4×4 matrix, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$.

Let the reduced row echelon form of A be $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Which of the following options are correct?

[Hint: Recall the definitions of independent and dependent variable with respect to reduced row echelon form.]

- ◯ Option 1: x_1 is dependent on x_2.
- ◯ **Option 2:** x_3 is dependent on x_4.
- ◯ **Option 3:** x_2 is an independent variable.
- ◯ Option 4: The solution of the system of linear equations (if it exists) is unique.
- ◯ **Option 5:** There exist infinitely many solutions for the given system of linear equations.

25 Lecture 5.5

25.1 Level 1

1. Match the matrices in Column A with their row operation steps (in the exact sequence given) in Column B, and their corresponding reduced row Echelon forms in Column C of Table M2W5AQ2.

	Matrices (Column A)		Steps for row operation (Column B)		Reduced row Echelon form (Column C)
i)	$\begin{bmatrix}1 & 3 & 2 & 0\\0 & 2 & 1 & 0\\2 & 4 & 4 & 3\end{bmatrix}$	a)	$[\ \ ]$ $\rightsquigarrow$ $R_3 - 2R_1$ $[\ \ ]$ $\rightsquigarrow$ $\frac{1}{2}R_2$ $[\ \ ]$ $\rightsquigarrow$ $R_3 + 2R_2$ $[\ \ ]$ $\rightsquigarrow$ $R_2 - \frac{1}{2}R_3$ $[\ \ ]$ $\rightsquigarrow$ $R_1 - 2R_3$ $[\ \ ]$ $\rightsquigarrow$ $R_1 - 3R_2$ $[\ \ ]$	1)	$\begin{bmatrix}1 & 0 & 0 & \frac{4}{5}\\0 & 1 & 0 & \frac{1}{5}\\0 & 0 & 1 & -\frac{2}{5}\end{bmatrix}$
ii)	$\begin{bmatrix}1 & 0 & 2 & 0\\0 & 2 & 1 & 0\\2 & 3 & 3 & 1\end{bmatrix}$	b)	$[\ \ ]$ $\rightsquigarrow$ $R_3 - 2R_1$ $[\ \ ]$ $\rightsquigarrow$ $\frac{1}{2}R_2$ $[\ \ ]$ $\rightsquigarrow$ $R_3 - 3R_2$ $[\ \ ]$ $\rightsquigarrow$ $-\frac{2}{5}R_3$ $[\ \ ]$ $\rightsquigarrow$ $R_2 - \frac{1}{2}R_3$ $[\ \ ]$ $\rightsquigarrow$ $R_1 - 2R_3$ $[\ \ ]$	2)	$\begin{bmatrix}1 & 0 & 0 & -\frac{3}{2}\\0 & 1 & 0 & -\frac{3}{2}\\0 & 0 & 1 & 3\end{bmatrix}$

Table: M2W5AQ2

Find the correct option.

◯ Option 1: Applying (b) on (i) gives (2) and applying (a) on (ii) gives (1).

◯ Option 2: Applying (a) on (i) gives (1) and applying (b) on (ii) gives (2).

◯ **Option 3:** Applying (a) on (i) gives (2) and applying (b) on (ii) gives (1).

2. Choose the correct set of options.

   ◯ Option 1: If two matrices A and B have the same reduced row echelon form, then A must be equal to B.

   ◯ Option 2: If two matrices A and B have the same row echelon form, then A must be equal to B.

   ◯ **Option 3:** The reduced row echelon form of a diagonal matrix must be the identity matrix.

   ◯ **Option 4:** The reduced row echelon form of a scalar matrix must be the identity matrix.

   The three different types of elementary row operations that can be performed on a matrix are:

   - **Type 1:** Interchanging two rows.
   - **Type 2:** Multiplying a row by some constant.
   - **Type 3:** Adding a scalar multiple of a row to another row.

   Answer questions 3 and 4 based on the above information.

3. Consider the four matrices given below:
   $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & -3 & 3 \end{bmatrix}$.
   Choose the set of correct options.

   ◯ **Option 1:** Matrix B is obtained from matrix A by an elementary row operation of Type 1.

   ◯ Option 2: Matrix C is obtained from matrix A by an elementary row operation of Type 1.

   ◯ **Option 3:** Matrix D is obtained from matrix A by an elementary row operation of Type 3.

   ◯ **Option 4:** Matrix A is obtained from matrix C by an elementary row operation of Type 2.

4. Let A and B be square matrices of order 3. Consider the three equations below.

   - **Equation 1:** $det(A) = -det(B)$

   - **Equation 2:** $det(A) = -c\ det(B), c \in \mathbb{R}$

- **Equation 3:** $det(A) = det(B)$

Choose the set of correct options.

◯ **Option 1:** If matrix B is obtained from matrix A by an elementary row operation of type 1, then equation 1 is satisfied.

◯ **Option 2:** If matrix B is obtained from matrix A by an elementary row operation of type 1 followed by an elementary operation of type 2, then equation 2 is satisfied for some c.

◯ Option 3: If matrix B is obtained from A by an elementary row operation of type 2, then equation 3 is satisfied.

◯ **Option 4:** If matrix B is obtained from A by an elementary row operation of type 3, then equation 3 is satisfied.

5. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ be a square matrix of order 3. Which of the statements below are true for matrix A?

◯ **Option 1:** A can be transformed via elementary row operations into the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ which is in row echelon form.

◯ Option 2: The reduced row echelon form of matrix A is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

◯ **Option 3:** The reduced row echelon form of matrix A is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

◯ **Option 4:** A can be transformed via elementary row operations into the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ which is in row echelon form.

◯ **Option 5:** A can be transformed via elementary row operations into the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ which is in row echelon form.

25.2 Level 2

6. Let the reduced row echelon form of a matrix A be

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

If the first and third columns of A are $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ respectively, then the second column of the matrix A is,

[Hint: Start with an arbitrary column $\begin{bmatrix} a \\ b \end{bmatrix}$ as the second column of A.]

- ◯ Option 1: $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- ◯ Option 2: $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- ◯ Option 3: $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$
- ◯ **Option 4:** $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

7. Let the different types of elementary row operations be defined as follows:

   - **Type 1:** Interchanging two rows.
   - **Type 2:** Multiplying a row by some constant.
   - **Type 3:** Adding a scalar multiple of a row to another row.

   Which of the following statements are true?

   - ◯ **Option 1:** If matrix A is obtained from matrix B by a finite number of elementary row operations, then matrix B can also be obtained from matrix A by a finite number of elementary row operations.
   - ◯ Option 2: The reduced row echelon form of a matrix cannot be the identity matrix.
   - ◯ **Option 3:** An upper triangular matrix, with value of all the diagonal elements equal to 1, is in row echelon form.
   - ◯ **Option 4:** Identity matrix is in reduced row echelon form.
   - ◯ Option 5: The reduced row echelon form of a scalar matrix (other than identity matrix) can be obtained by applying only elementary row operations of type 1.
   - ◯ **Option 6:** The reduced row echelon form of a diagonal matrix (other than identity matrix) can be obtained by applying only elementary row operations of type 2.

8. Let A be a 3×3 real matrix whose sum of entries of each column is 5 and sum of first two elements of each column is 3. Which of the following statements is (are) true? [Hint: Row operation: adding one row to other row.]

   - ◯ **Option 1:** The determinant of matrix A is a multiple of 5.

- ◯ **Option 2:** The determinant of matrix A is a multiple of 3.
- ◯ **Option 3:** The determinant of matrix A is a multiple of 15.
- ◯ **Option 4:** The determinant of matrix A is a multiple of 2.
- ◯ **Option 5:** The determinant of matrix A is a multiple of 8.

26 Lecture 5.6

26.1 Level 1

1. Let $Ax = b$ be a matrix representation of a system of linear equations. Let $[R|c]$ be the reduced row echelon of the augmented matrix $[A|b]$ corresponding to the system. Choose the set of correct options.

   [Hint: Recall reduced row echelon form.]

   - ◯ **Option 1:** If the system $Rx = c$ has infinitely many solutions, then the system $Ax = b$ has infinite solutions.
   - ◯ Option 2: If the system $Rx = c$ has no solutions, then the system $Ax = b$ has a unique solution.
   - ◯ Option 3: If the system $Rx = c$ has a unique solution, then the system $Ax = b$ has no solution.
   - ◯ **Option 4:** If the system $Rx = c$ has a unique solution, then the system $Ax = b$ has a unique solution.

2. Consider a system of linear equations

$$\begin{aligned} 2x_1 + x_2 &= 1 \\ -x_1 + x_3 + x_4 &= -1 \\ x_1 + x_2 - x_3 + x_4 &= 2 \\ -x_1 + x_3 + x_4 &= 1. \end{aligned}$$

   Which of the following represents the augmented matrix of the system?

   - ◯ Option 1: $\left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 \\ -1 & 0 & 1 & 1 & 0 \end{array}\right]$
   - ◯ **Option 2:** $\left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & 2 \\ -1 & 0 & 1 & 1 & 1 \end{array}\right]$
   - ◯ Option 3: $\left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 2 \\ -1 & 0 & 1 & 1 & -1 \end{array}\right]$
   - ◯ Option 4: $\left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 & -1 \\ 1 & 1 & -1 & 1 & 2 \\ -1 & 1 & 1 & 0 & -1 \end{array}\right]$

3. Let $Ax = b$ be a matrix representation of a system of linear equations and $b = 0$. Choose the set of correct options.

   [Hint: Recall the definition of the trivial solution of a system of linear equations and applicability of Gauss Elimination Method.]

   - ◯ Option 1: If A is an invertible matrix then the system has no solution.
   - ◯ **Option 2:** If A is an invertible matrix then the system has a unique solution.
   - ◯ **Option 3:** If A is an invertible matrix then the trivial solution is the only solution for the system.
   - ◯ **Option 4 :** If $det(A) = 0$, then the system has infinitely many solutions.

4. Let $Ax = b$ be a matrix representation of a system of linear equations and $b \neq 0$. Choose the set of correct options.

   - ◯ Option 1: If A is an invertible matrix, then the system has no solution.
   - ◯ **Option 2:** If A is an invertible matrix, then the system has a unique solution.
   - ◯ Option 3: If A is an invertible matrix, then the trivial solution is a solution for the system.
   - ◯ **Option 4 :** If $det(A) = 0$, then either the system has no solution or the system has infinitely many solutions.

5. Consider the following systems of linear equations:

System I:
$$\begin{aligned} x - y &= 3 \\ -y + 2z &= 1 \\ x + y + z &= 0 \end{aligned}$$

System II:
$$\begin{aligned} x - y &= 3 \\ 2y + z &= 1 \\ 6y + 3z &= 0 \end{aligned}$$

System III:
$$\begin{aligned} x - y &= 3 \\ 2y + z &= 1 \\ 6y + 3z &= 3 \end{aligned}$$

   Choose the correct option.

   - ◯ Option 1: All the three systems have a unique solution.

- ◯ Option 2: System I has a unique solution, whereas, System II and System III have no solution.
- ◯ Option 3: System I has a unique solution, whereas, System II and System III have infinitely many solutions.
- ◯ **Option 4:** System I has a unique solution, System II has no solution, and System III has infinitely many solutions.
- ◯ Option 5: System I has no solution, System II and System III have infinitely many solutions.

26.2 Level 2

6. Suppose P is a 3×3 real matrix as follows:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The vector X_n is defined by the recurrence relation $PX_{n-1} = X_n$. If $X_2 = \begin{bmatrix} 7 \\ 8 \\ 3 \end{bmatrix}$, what is X_0?

- ◯ Option 1: $\begin{bmatrix} 7 \\ 8 \\ 3 \end{bmatrix}$
- ◯ **Option 2:** $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- ◯ Option 3: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
- ◯ Option 4: $\begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$

7. Consider the system of linear equations given below:

$$x - y + z = 2$$

$$x + y - z = 3$$

$$-x + y + z = 4.$$

The system of linear equations has

[Hint: Use the relation between a system of linear equations and determinant of corresponding coefficient matrix.]

- ◯ Option 1: no solution.
- ◯ Option 2: infinitely many solutions.
- ◯ **Option 3:** a unique solution.
- ◯ Option 4: finitely many solutions.

8. Consider the system of linear equations:

$$\begin{aligned} -x_1 + x_2 + 2x_3 &= 1 \\ 2x_1 + x_2 - 2x_3 &= -1 \\ 3x_2 + cx_3 &= d \end{aligned}$$

Choose the set of correct options.

[Hint: Recall the Gauss elimination method.]

- ◯ **Option 1:** If $c = 2$ and $d = 1$, then the system has infinitely many solutions.
- ◯ Option 2: If $c = 1$ and $d = 1$, then the system has infinitely many solutions.
- ◯ Option 3: If $c = 1$ and $d = 2$, then the system has no solution.
- ◯ **Option 4:** If $c = 3$ and $d = 2$, then the system has a unique solution.

Maths 2 : Activity Questions
Week-6

27 Lecture 6.1

27.1 Level 1

1. Consider the following expression:
$v_1 = (1,0), v_2 = (0,0), v_3 = (0,1), v_5 = 1, v_6 = 5, v_7 = (1,0,1), v_8 = (0,0,1), v_9 = (0,0,0), v_{10} = (0,1,0)$
Choose the set of correct options.

   - ◯ Option 1: v_1, v_7 represent vectors in $\mathbb{R}^2$.
   - ◯ Option 2: v_2, v_5 represent vectors in $\mathbb{R}^1$.
   - ◯ Option 3: v_2, v_5, v_9 represent vectors in $\mathbb{R}^3$.
   - ◯ **Option 4:** v_1, v_2, v_3 represent vectors in $\mathbb{R}^2$.
   - ◯ **Option 5:** v_5, v_6 represent vectors in $\mathbb{R}^1$.
   - ◯ Option 6: v_1, v_6 represent vectors in $\mathbb{R}^1$.
   - ◯ **Option 7:** v_8, v_9, v_{10} represent vectors in $\mathbb{R}^3$.

2. Let v, v_1, v_2 represent vectors in a vector space V over $\mathbb{R}$. Which of the following expressions make sense and represent vectors in V?

   - ◯ **Option 1:** $cv \in V$, where $c \in \mathbb{R}$.
   - ◯ **Option 2:** $\frac{v}{2}$.
   - ◯ Option 3: v^2.
   - ◯ **Option 4:** $v_1 - v_2$.
   - ◯ Option 5: $v_1^2 - v_2^2$.

3. Consider the set $V = \{(z-x, -y) \mid x, y, z \in \mathbb{R}\} \subseteq \mathbb{R}^2$ with the usual addition and scalar multiplication as in $\mathbb{R}^2$ and associated statements given below.

   - **P:** V is closed under addition i.e $(v_1, v_2 \in V \implies v_1 + v_2 \in V)$.
   - **Q:** V is closed under scalar multiplication i.e $(c \in \mathbb{R}, v \in V \implies cv \in V)$.
   - **R:** V has a zero element with respect to addition. i.e., there exists some element 0 such that $v + 0 = v$, for all $v \in V$.
   - **S:** V is not a vector space.
   - **T:** $(a+b)v = av + bv$ where $a, b \in \mathbb{R}$ and $v \in V$.

Choose the set of correct options.

- ◯ Option 1: Only P is true.
- ◯ Option 2: Only S is true.
- ◯ **Option 3:** Both P and Q are true.
- ◯ **Option 4:** Both R and T are true.
- ◯ **Option 5:** All statements are true except S.

4. Consider a set $V = \{(1, x) \mid x \in \mathbb{R}\} \subseteq \mathbb{R}^2$ with the usual addition and scalar multiplication as in $\mathbb{R}^2$ and associated statements given below.

- **P:** V is closed under addition.
- **Q:** V has no zero element with respect to addition. i.e., there does not exists any element 0 such that $v + 0 = v$, for all $v \in V$.
- **R:** $(a + b)v \neq av + bv$ where $a, b \in \mathbb{R}$ and $v \in V$.

Choose the correct option.

- ◯ Option 1: Only P is true.
- ◯ **Option 2:** Only Q is true.
- ◯ Option 3: Both P and R are true.
- ◯ Option 4: Both Q and R are true.

5. Which of the following sets with the given addition and scalar multiplication (scalars are real numbers in every case) form vector spaces?

$V_1 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$
Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + 1, z_2 + 1);$
$(x_1, y_1, z_1), (x_2, y_2, z_2) \in V_1$
Scalar multiplication: $c(x, y, z) = (x, y, cz);\ (x, y, z) \in V_1,\ c \in \mathbb{R}$

$V_2 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$
Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1, y_1, z_1);$
$(x_1, y_1, z_1), (x_2, y_2, z_2) \in V_2$
Scalar multiplication: $c(x, y, z) = (x, cy, z);\ (x, y, z) \in V_2,\ c \in \mathbb{R}$

$V_3 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$
Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, 0, 0);$
$(x_1, y_1, z_1), (x_2, y_2, z_2) \in V_3$
Scalar multiplication: $c(x, y, z) = (cx, y, z);\ (x, y, z) \in V_3,\ c \in \mathbb{R}$

- ◯ Option 1: Only V_3 is vector space.
- ◯ Option 2: Both V_1 and V_2 are vector spaces.
- ◯ Option 3: All are vector spaces.
- ◯ **Option 4:** None of the above is a vector space.

27.2 Level 2

6. Consider the following sets:

   - $V_1 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a symmetric matrix, i.e., $A = A^T\}$
   - $V_2 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a scalar matrix$\}$
   - $V_3 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a diagonal matrix$\}$
   - $V_4 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a upper triangular matrix$\}$
   - $V_5 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a lower triangular matrix$\}$

   Choose the set of correct options.

   - ◯ Option 1: Only V_1 is a subspace of $M_{2\times 2}(\mathbb{R})$.
   - ◯ Option 2: Only V_4 is a subspace of $M_{2\times 2}(\mathbb{R})$.
   - ◯ **Option 3:** Both V_2 and V_3 are subspaces of $M_{2\times 2}(\mathbb{R})$
   - ◯ **Option 4:** All are subspaces of $M_{2\times 2}(\mathbb{R})$

7. Choose the set of correct options.

   - ◯ **Option 1:** A vector in $\mathbb{R}^3$ can be thought of as matrix of order 1×3 or 3×1.
   - ◯ Option 2: Intersection of any two subspaces of a vector space V is not a subspace.
   - ◯ Option 3: Intersection of any two subsets of a vector space V is a subspace.
   - ◯ Option 4: The empty subset of a vector space V is a subspace of V.
   - ◯ **Option 5:** $V = \{(x, 0, 0) \mid x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.

8. Consider a set $V = \{(x, y) \mid x, y \in \mathbb{R}\} \subseteq \mathbb{R}^2$ with the usual addition as in $\mathbb{R}^2$ and scalar multiplication is defined as

   $$c(x,y) = \begin{cases} (0,0) & c = 0 \\ (\frac{cx}{2}, \frac{y}{c}) & c \neq 0 \end{cases} \qquad (x,y) \in V,\ c \in \mathbb{R}$$

   Consider the statements given below.

   - **P:** V is closed under addition.

- **Q:** V has zero element with respect to addition. i.e., there exists some element 0 such that $v + 0 = v$, for all $v \in V$.
- **R:** $1.v = v$ where $1 \in \mathbb{R}$ and $v \in V$.
- **S:** $a(v_1 + v_2) = av_1 + av_2$ where $v_1, v_2 \in V$ and $a \in \mathbb{R}$.
- **T:** $(a + b)v = av + bv$ where $a, b \in \mathbb{R}$ and $v \in V$.

Choose the set of correct options.

- ◯ Option 1: Only P is true.
- ◯ Option 2: Only Q is true.
- ◯ **Option 3:** Both P and Q are true.
- ◯ **Option 4:** P,Q and S are true.
- ◯ **Option 5:** Both R and T are not true.

28 Lecture 6.2

28.1 Level 1

1. If $(1, 2, 3) + v = (3, 2, 1)$ in the vector space $\mathbb{R}^3$ with ususal addition and scalar multiplication, then which of the following vectors will be $-v$?

   ◯ Option 1: $(2, 0, 2)$

   ◯ Option 2: $(-2, 0, -2)$

   ◯ **Option 3:** $(-2, 0, 2)$

   ◯ Option 4: $(2, 0, -2)$

2. The cancellation law for vector spaces states that for any vectors x, y, z in a vector space

   ◯ **Option 1:** $x + y = z + y \implies x = z.$

   ◯ Option 2: $x + y = z + y \implies x = y.$

   ◯ Option 3: $x + y = z + y \implies y = z.$

   ◯ Option 4: None of the above.

3. Which of the following options is/are true for a non zero vector space V?

   ◯ **Option 1:** $x + y = z + y \implies x = z \ \forall x, y, z \in V.$

   ◯ **Option 2:** $ax = bx, \ \forall\, x \in V \implies a = b,$ where $a, b \in \mathbb{R}.$

   ◯ **Option 3:** $ax = ay, \ \forall\, a \in \mathbb{R} \implies x = y,$ where $x, y \in V.$

   ◯ Option 4: None of the above.

4. Which of the following options is/are true for a non zero vector space V?

   ◯ **Option 1:** $av = 0, \ \forall\, v \in V \implies a = 0.$

   ◯ **Option 2:** $av = 0, \ \forall\, a \in \mathbb{R} \implies v = 0.$

   ◯ Option 3: There exists $c \in \mathbb{R}$ and $v, w \in V$ such that, $c(v - w) \neq cv - cw.$

   ◯ **Option 4:** $c((v + w) - (v - w) - 2w) = 0 \ \forall c \in \mathbb{R},$ and $v, w \in V$

28.2 Level 2

Let V be a plane plane parallel to the XY-plane. Any plane parallel to XY-plane is given by $z = c$. We define addition of two vectors $v_1 = (x_1, y_1, c)$ and $v_2 = (x_2, y_2, c)$ on V as follows: First project v_1 and v_2 on the XY-plane (we will get the vectors $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ by projection on the XY-plane) and then calculate the addition of the vectors we obtained by the projection on the XY-plane (we will obtain $(x_1 + x_2, y_1 + y_2, 0)$). Then project the obtained vector back to the plane V (we will obtain the vector $(x_1 + x_2, y_1 + y_2, c)$).
Let V be a plane which is parallel to the XY-plane, defined by $z = 2$. Let $v_1 = (1, 2, 2)$ and $v_2 = (0, 3, 2)$ be in V. Answer questions 5 and 6.

5. Which of the following pairs of vectors will be the projections of v_1 and v_2 on the XY-plane?
   - ◯ Option 1: $(1, 0, 2)$ and $(0, 0, 2)$
   - ◯ **Option 2:** $(1, 2, 0)$ and $(0, 3, 0)$
   - ◯ Option 3: $(0, 2, 2)$ and $(0, 3, 2)$
   - ◯ Option 4: $(1, 0, 0)$ and $(0, 0, 0)$

6. Which of the following vectors will be $v_1 + v_2$ as per the addition defined above?
   - ◯ Option 1: $(1, 0, 4)$
   - ◯ Option 2: $(1, 5, 0)$
   - ◯ **Option 3:** $(1, 5, 2)$
   - ◯ Option 4: $(1, 5, 4)$

Let V be a plane plane parallel to the XY-plane. Any plane parallel to the XY-plane is given by $z = c$. We define scalar multiplication of a vector $v = (x, y, c)$ on V as follows: First project v on the XY-plane first (we will get the vector $(x, y, 0)$ by projection on the XY-plane) and then calculate the scalar (α) multiple of the vector we obtained by the projection on the XY-plane (we will obtain $(\alpha x, \alpha y, 0)$). Then project the obtained vector back to the plane V (we will obtain the vector $(\alpha x, \alpha y, c)$).
Let V be a plane which is parallel to the XY-plane, defined by $z = 2$. Let $v = (1, 2, 2)$ be in V. Answer questions 7 and 8.

7. Which of the following vectors will be the projection of v on XY-plane?
   - ◯ Option 1: $(1, 0, 2)$
   - ◯ Option 2: $(1, 0, 0)$
   - ◯ **Option 3:** $(1, 2, 0)$
   - ◯ Option 4: $(0, 2, 2)$

8. Which of the following vectors will be $4v$ as per the scalar multiplication defined above?
   - ◯ Option 1: $(4, 2, 8)$
   - ◯ Option 2: $(4, 8, 8)$
   - ◯ **Option 3:** $(4, 8, 2)$
   - ◯ Option 4: $(4, 2, 2)$

29 Lecture 6.3

29.1 Level 1

1. Which of the following vectors can be written as the linear combination of vectors $(2,3)$ and $(1,2)$ in $\mathbb{R}^2$ with usual addition and scalar multiplication?

   ◯ **Option 1:** $(1,0)$

   ◯ **Option 2:** $(0,1)$

   ◯ **Option 3:** $(0,0)$

   ◯ **Option 4:** Any vector in $\mathbb{R}^2$.

2. Suppose the set $\{(3,7,4), v\}$ is a linearly dependent set in the vector space $\mathbb{R}^3$ with usual addition and scalar multiplication. Which of the following vectors are possible as a candidate for v?

   ◯ **Option 1:** $(0,0,0)$

   ◯ **Option 2:** $(5, \frac{35}{3}, \frac{20}{3})$

   ◯ Option 3: $(1,0,0)$

   ◯ Option 4: $(6,14,-8)$

3. Suppose the set $\{(2,3,0),(0,1,0), v\}$ is a linearly dependent set in the vector space $\mathbb{R}^3$ with usual addition and scalar multiplication. Which of the following vectors are possible as a candidate for v?

   ◯ Option 1: $(2,3,1)$

   ◯ **Option 2:** $(1,2,0)$

   ◯ **Option 3:** $(1,3,0)$

   ◯ Option 4: $(0,1,1)$

   ◯ **Option 5:** $(\pi, e, 0)$

4. Let V be a vector space and v_1, v_2, v_3 and $v_4 \in V$. If v_1 is a linear combination of $v_i, i = 2,3,4$ *i.e.* $v_1 = av_2 + bv_3 + cv_4$ then

   ◯ Option 1: v_2 is a linear combination of $v_i, i = 1,3,4$.

   ◯ Option 2: v_3 is a linear combination of $v_i, i = 1,4$.

   ◯ **Option 3:** v_2 is a linear combination of v_2.

   ◯ Option 4: None of the above.

5. Consider three vectors $v_1 = (1,0,0), v_2 = (1,1,0), v_3 = (1,1,1)$ in the vector space $\mathbb{R}^3$, with usual addition and scalar multiplication. Which of the following sets is (are) true?

   ◯ Option 1: $\{v_1, v_2, v_3\}$ is a linearly dependent set.

- ◯ Option 2: $\{v_1 + v_2, v_1 + v_3, v_3\}$ is a linearly dependent set.
- ◯ **Option 3:** $\{v_2 - v_1, v_1, v_2\}$ is a linearly dependent set.
- ◯ **Option 4:** $\{v_1, v_2 - v_1, v_3 - v_2, v_3\}$ is a linearly dependent set.

29.2 Level 2

6. Let S be a subset of $\mathbb{R}^3$ which is linearly dependent. Which of the following options are true?
   - ◯ **Option 1:** $S \cup \{(1, 0, 0)\}$ must be linearly dependent.
   - ◯ **Option 2:** $S \cup \{v\}$ must be linearly dependent for any $v \in \mathbb{R}^3$.
   - ◯ Option 3: $S \setminus \{v\}$ must be linearly dependent for any $v \in \mathbb{R}^3$.
   - ◯ **Option 4:** There may exist some $v \in S$, such that $S \setminus \{v\}$ is still linearly dependent.

$M_{3\times3}(\mathbb{R})$ denotes the vector space consisting of real square matrices of order 3 with usual matrix addition and scalar multiplication. Consider the following matrices:

$$M_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, M_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, M_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}, M_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

Use the above information to answer questions 7 and 8.

7. Which of the following options are correct?
   - ◯ Option 1: $M_1 - M_2 = 0$, where 0 denotes the zero matrix of order 3.
   - ◯ Option 2: $2M_1 + M_2 = M_3$
   - ◯ **Option 3:** $2M_1 - M_2 - M_3 = 0$, where 0 denotes the zero matrix of order 3.
   - ◯ **Option 4:** $2M_2 - M_1 - M_4 = 0$, where 0 denotes the zero matrix of order 3.
   - ◯ Option 5: The tuple (a, b, c) is unique for which $aM_1 + bM_3 + cM_4 = 0$ holds, where 0 denotes the zero matrix of order 3.
   - ◯ **Option 6:** The pair (a, b) is unique for which $aM_1 + bM_2 = 0$ holds, where 0 denotes the zero matrix of order 3.

8. Which of the following options is(are) true?
   - ◯ Option 1: $\{M_1, M_2\}$ is a linearly dependent set.
   - ◯ **Option 2:** $\{M_1, M_2, M_3\}$ is a linearly dependent set.
   - ◯ **Option 3:** $\{M_1, M_2, M_4\}$ is a linearly dependent set.
   - ◯ **Option 4:** $\{M_1, M_3, M_4\}$ is a linearly dependent set.

30 Lecture 6.4

30.1 Level 1

1. Consider the following statements:

   - **Statement 1:** $0(0,0) + 0(1,1) = (0,0)$ implies that the set $\{(0,0),(1,1)\}$ is a linearly independent subset of $\mathbb{R}^2$.
   - **Statement 2:** $2(1,0) + 2(0,1) = (2,2)$ implies that the set $\{(1,0),(0,1),(2,2)\}$ is a linearly dependent subset of $\mathbb{R}^2$.

   Which of the following options is correct?

   ◯ Option 1: Both the statements are true.

   ◯ Option 2: Statement 1 is true, but Statement 2 is false.

   ◯ **Option 3:** Statement 2 is true, but Statement 1 is false.

   ◯ Option 4: Both the statements are false.

2. If $\alpha(1,1,0) + \beta(0,1,1) + \gamma(1,0,1) = v$ for some $\alpha, \beta, \gamma \in \mathbb{R}$ and $v \in \mathbb{R}^3$, then choose the set of correct options.

   ◯ Option 1: If $v = (0,0,0)$, then one of the possible solutions of $\alpha, \beta,$ and γ is $\alpha = \frac{1}{2} = \gamma$, and $\beta = -\frac{1}{2}$.

   ◯ **Option 2:** If $v = (0,0,0)$, then one of the possible solutions of $\alpha, \beta,$ and γ is $\alpha = \beta = \gamma = 0$.

   ◯ **Option 3:** If $v = (0,0,0)$, then the possible solution of $\alpha, \beta,$ and γ is unique.

   ◯ **Option 4:** If $v = (1,0,0)$, then one of the possible solutions of $\alpha, \beta,$ and γ is $\alpha = \frac{1}{2} = \gamma$, and $\beta = -\frac{1}{2}$.

   ◯ **Option 5:** The set $\{(1,1,0),(0,1,1),(1,0,1)\}$ is linearly independent.

3. Consider a set of vectors $S = \{(1,2),(-1,2),(3,1),(-3,1)\}$ in $\mathbb{R}^2$. Choose the set of correct options.

   ◯ Option 1: $\{(1,2),(-1,2)\}$ is a linearly dependent set.

   ◯ **Option 2:** $\{(-1,2),(-3,1)\}$ is a linearly independent set.

   ◯ **Option 3:** $\{(-3,1),(-1,2),(3,1)\}$ is a linearly dependent set.

   ◯ Option 4: $\{(3,1),(-3,1)\}$ is a linearly dependent set.

   ◯ **Option 5:** $\{(3,1),(1,2),(-3,1)\}$ is a linearly dependent set.

4. Let V be a vector space and non zero vectors $v_1, v_2, v_3,$ and $v_4 \in V$. If v_1 is a linear combination of v_i, for $i = 2,3,4$, then which of the following is (are) correct?

   ◯ **Option 1:** The set $\{v_1, v_2, v_3, v_4\}$ cannot be linearly independent.

- ◯ Option 2: The set $\{v_2, v_3, v_4\}$ must be be linearly dependent.
- ◯ Option 3: The set $\{v_2, v_3, v_4\}$ must be be linearly independent.
- ◯ **Option 4:** The set $\{v_1\}$ must be be linearly independent.

5. Choose the correct set of options.
   - ◯ Option 1: Union of two distinct linearly independent sets must be linearly independent.
   - ◯ Option 2: Union of two distinct linearly independent sets must be linearly dependent.
   - ◯ **Option 3:** Non-empty intersection of two linearly independent sets must be linearly independent.
   - ◯ Option 4: Non-empty intersection of two linearly independent sets must be linearly dependent.

30.2 Level 2

6. Let S be a linearly independent subset of $\mathbb{R}^3$. Choose the correct set of options.
   - ◯ Option 1: $S \cup \{(1, 0, 0)\}$ must be linearly independent.
   - ◯ Option 2: $S \cup \{v\}$ must be linearly independent for any $v \in \mathbb{R}^3$.
   - ◯ **Option 3:** $S \setminus \{v\}$ must be linearly independent for any $v \in \mathbb{R}^3$.
   - ◯ **Option 4:** There may exist some $v \in \mathbb{R}^3$, such that $S \cup \{v\}$ is still linearly independent.

7. Consider a set of vectors $S = \{(1, 1, -1), (-1, 1, 1), (0, \frac{1}{2}, 0), (0, 1, -2), (1, 0, -2)\}$. Choose the set of correct options.
   - ◯ Option 1: The singleton set $\{(0, 1, -2)\}$ is linearly dependent.
   - ◯ **Option 2:** If $\alpha, \beta \in S$ and α, β are distinct then $\{\alpha, \beta\}$ is a linearly independent set of vectors.
   - ◯ **Option 3:** The set {(-1, 1, 1), (0, $\frac{1}{2}$, 0),(0, 1, -2)} is a linearly independent set of vectors.
   - ◯ Option 4: The set S is a linearly independent set of vectors.
   - ◯ Option 5: The set $\{\alpha, \beta, \gamma\}$ is a linearly dependent set of vectors for any $\alpha, \beta, \gamma \in S$, where all the three are distinct vectors.
   - ◯ Option 6: The set $\{\alpha, \beta, \gamma, \delta\}$ is a linearly independent set of vectors for any $\alpha, \beta, \gamma, \delta \in S$, where all the four are distinct vectors.

8. Let S be the solution set of a system of homogeneous linear equations with 3 variables and 3 equations, whose matrix representation is as follows:

$$Ax = 0$$

where A is the 3×3 coefficient matrix and x denotes the column vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Choose the set of correct options.

- ◯ **Option 1:** If v_1 and v_2 are in S, then any linear combination of v_1 and v_2 will also be in S.
- ◯ **Option 2:** The set S will be a subspace of $\mathbb{R}^3$, with respect to usual addition and scalar multiplication as in $\mathbb{R}^3$.
- ◯ **Option 3:** The set $\{v_1, v_2, v_1 - v_2\}$ is a linearly dependent subset in S.
- ◯ **Option 4:** The set $\{v_1, v_2\}$ is a linearly independent subset in S if v_1 is not a scalar multiple of v_2.

31 Lecture 6.5

31.1 Level 1

1. Choose the set of correct options.

   ◯ **Option 1:** The set $\{(1,2,3),(4,5,6),(7,8,9)\}$ is linearly dependent.

   ◯ Option 2: The set $\{(1,2,3),(4,5,6),(5,7,9)\}$ is linearly independent.

   ◯ **Option 3:** The set $\{(1,2,3),(0,5,6),(0,0,9)\}$ is linearly independent.

   ◯ Option 4: The set $\{(1,2,3),(0,0,6),(0,0,9)\}$ is linearly independent.

2. If $ad - bc = 0$ then which of the following options are true?

   ◯ **Option 1:** The set $\{(a,b),(c,d)\}$ is linearly dependent in $\mathbb{R}^2$.

   ◯ **Option 2:** The set $\{(a,c),(b,d)\}$ is linearly dependent in $\mathbb{R}^2$.

   ◯ **Option 3:** The set $\{(1,0,0),(0,a,b),(0,c,d)\}$ is linearly dependent in $\mathbb{R}^3$.

   ◯ **Option 4:** The set $\{(1,0,0),(0,a,c),(0,b,d)\}$ is linearly dependent in $\mathbb{R}^2$.

3. For $c \in \mathbb{Z}$, consider the set of three vectors $S = \{(1,c,0),(c,0,c),(2c,3c,4c)\}$ in $\mathbb{R}^3$ with usual addition and scalar multiplication. Find the value of c such that the given set is linearly dependent. [Answer: 0]

4. If the set $\{(a_1,b_1,c_1),(a_2,b_2,c_2),(a_3,b_3,c_3)\}$ is given to be linearly independent, then the following system of linear equations

$$\begin{aligned} a_1x + b_1y + c_1z &= 1 \\ a_2x + b_2y + c_2z &= 2 \\ a_3x + b_2y + c_3z &= 3 \end{aligned}$$

   has

   ◯ **Option 1:** a unique solution.

   ◯ Option 2: no solution.

   ◯ Option 3: infinitely many solutions.

   ◯ Option 4: Cannot be concluded about any one of the above options from the above information.

5. If the set $\{(a_1,b_1,c_1),(a_2,b_2,c_2),(a_3,b_3,c_3)\}$ is given to be linearly dependent, then the following system of linear equations

$$\begin{aligned} a_1x + b_1y + c_1z &= 1 \\ a_2x + b_2y + c_2z &= 2 \\ a_3x + b_2y + c_3z &= 3 \end{aligned}$$

   has

◯ Option 1: either a unique solution or no solution.

◯ **Option 2:** either no solution or infinitely many solutions.

◯ Option 3: either infinitely many solutions or a unique solution.

◯ Option 4: a unique solution.

31.2 Level 2

6. Let the set $\{(a,b),(c,d)\}$ be a linearly independent subset of $\mathbb{R}^2$. Choose the set of correct options.

   ◯ **Option 1:** $\{(a,b,0),(c,d,0)\}$ must be a linearly independent subset of $\mathbb{R}^3$.

   ◯ **Option 2:** $\{(a,b,0),(c,d,0),(0,0,1)\}$ must be a linearly independent subset of $\mathbb{R}^3$.

   ◯ Option 3: $\{(a,b,0),(c,d,0),(1,0,0)\}$ must be a linearly independent subset of $\mathbb{R}^3$.

   ◯ Option 4: $\{(a,b,0),(c,d,0),(0,1,0)\}$ must be a linearly independent subset of $\mathbb{R}^3$.

7. Each column of an $n \times n$ matrix A can be thought of as vectors in $\mathbb{R}^n$. If a set S consists of all the column vectors of A, then which of the following is(are) true?

   ◯ Option 1: S is always linearly independent.

   ◯ **Option 2:** S is linearly independent if $det(A) \neq 0$.

   ◯ Option 3: S is linearly independent if $det(A) = 0$.

   ◯ Option 4: If $det(A) = 1$, then S is linearly dependent.

8. Suppose it is known that the three vectors $v_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $v_2 = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$, $v_3 = \begin{bmatrix} g \\ h \\ i \end{bmatrix}$ are linearly dependent in $\mathbb{R}^3$. Which of the following is correct?

   ◯ **Option 1:** Some linear combination of a, b, c must be 0.

   ◯ **Option 2:** Some linear combination of a, d, g must be 0.

   ◯ **Option 3:** Some linear combination of d, e, f must be 0.

   ◯ **Option 4:** Some linear combination of c, f, i must be 0.

32 Lecture 6.6

32.1 Level 1

1. Choose the set of correct options.

   ◯ **Option 1:** If S is a spanning set of the vector space V, then $S \cup \{v\}$ must be a spanning set of V, for all $v \in V$.

   ◯ **Option 2:** Span of an empty set is the zero vector space.

   ◯ Option 3: If S is a spanning set of the vector space V, then $S \setminus \{v\}$ must be a spanning set of V, for all $v \in S$.

   ◯ Option 4: If S is a spanning set of the vector space V, then $S \cup \{v\}$ may not be a spanning set of V, for all $v \in V$.

2. Consider the following set of matrices: $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$. What is $Span(S)$?

   ◯ Option 1: The vector space consisting of only lower triangular square matrices of order 2.

   ◯ **Option 2:** The vector space consisting of only upper triangular square matrices of order 2.

   ◯ Option 3: The vector space consisting of all the square matrices of order 2.

   ◯ Option 4: The vector space consisting of only scalar matrices of order 2.

3. Consider the following set of matrices: $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$. What is $Span(S)$?

   ◯ Option 1: The vector space consisting of only the Identity matrix of order 2.

   ◯ **Option 2:** The vector space consisting of only diagonal matrices of order 2.

   ◯ Option 3: The vector space consisting of all the square matrices of order 2.

   ◯ Option 4: The vector space consisting of only scalar matrices of order 2.

4. Consider the following set of vectors $S = \{(1, 1, -1), (-1, 1, 1), (0, \frac{1}{2}, 0), (0, 1, -2), (1, 0, -2)\}$. Choose the set of correct options.

   ◯ **Option 1:** The set { (1, 1, -1), (-1, 1, 1), (0, $\frac{1}{2}$,0) } is not a basis of $\mathbb{R}^3$.

   ◯ Option 2: The set {(0, $\frac{1}{2}$,0), (0, 1, -2), (1, 0 ,-2),(1, 0, -2)} is a basis of $\mathbb{R}^3$.

   ◯ **Option 3:** The set {(-1, 1, 1), (0, $\frac{1}{2}$,0), (0, 1, -2)} is a basis of $\mathbb{R}^3$.

   ◯ Option 4: None of the above.

5. Consider a set of vectors $S = \{(1, 2), (-1, 2), (3, 1), (-3, 1)\}$ in $\mathbb{R}^2$. Choose the set of correct options.

◯ Option 1: The set { (1, 2) ,(-1, 2), (3, 1)} is a basis of $\mathbb{R}^2$.

◯ **Option 2:** The set { (-1, 2), (3, 1) } is a basis of $\mathbb{R}^2$.

◯ **Option 3:** The set { (3, 1), (-3, 1) } is a basis of $\mathbb{R}^2$.

◯ Option 4: None of the above.

32.2 Level 2

6. Choose the set of correct options.

◯ Option 1: If a subset S of $\mathbb{R}^2$ contains only two elements then S is a basis of $\mathbb{R}^2$.

◯ Option 2: If a subset S of $\mathbb{R}^3$ contains only two elements then S is a basis of $\mathbb{R}^3$.

◯ **Option 3:** If a subset S of $\mathbb{R}$ contains only one non zero element then S is a basis of $\mathbb{R}$.

◯ Option 4: If S_1 and S_2 are two bases of $\mathbb{R}^3$ then $S_1 = S_2$.

7. Let V be the subspace of $\mathbb{R}^3$ defined as follows:

$$V = \{(x, y, z) \mid x = y - z, \text{ and } x, y, z \in \mathbb{R}\}$$

Choose the set of correct options from the following.

◯ **Option 1:** $\{(1, 1, 0), (1, 0, -1)\}$ is a linearly independent set of V.

◯ Option 2: $\{(1, 1, 0), (1, 0, -1), (0, 1, 1)\}$ is a linearly independent set of V.

◯ **Option 3:** $\{(0, 1, 1), (1, 0, -1)\}$ is a spanning set of V.

◯ Option 4: $\{(1, 1, 0)\}$ is a spanning set of V.

8. Let V and W be vector spaces which are defined as follows:

$V = \{(x, y) \mid y = mx, \text{ where } m \neq 0 \text{ and } x, y, m \in \mathbb{R}\}$ with usual addition and scalar multiplication as in $\mathbb{R}^2$.
$W = \{(x, y) \mid x = 0\}$ with usual addition and scalar multiplication as in $\mathbb{R}^2$.

Choose the correct set of options.

◯ Option 1: The set $\{(1, m), (\frac{1}{m}, 1)\}$ is a linearly independent set in V.

◯ **Option 2:** The set $\{(1, m), (\frac{1}{m}, 1)\}$ is a spanning set for V.

◯ **Option 3:** The set $\{(1, m)\}$ is a linearly independent set in V.

◯ **Option 4:** The set $\{(\frac{1}{m}, 1)\}$ is a linearly independent set in V.

◯ Option 5: The set $\{(0, 1), (0, 2)\}$ is a linearly independent set in W.

◯ **Option 6:** The set $\{(0, 1)\}$ is a linearly independent set in W.

◯ **Option 7:** The set $\{(0, 5)\}$ is a spanning set for W.

33 Lecture 6.7

33.1 Level 1

1. Match the vector spaces (with the usual scalar multiplication and vector addition as in $M_{3\times 3}(\mathbb{R})$) in column A with their bases in column B in Table : M2W6AQ1.

	Vector space (Column A)		Basis (Column B)
a)	$V = \left\{ \begin{bmatrix} x & y & 0 \\ 0 & y & 0 \\ 0 & x & x \end{bmatrix} \mid x+y=0, \text{ and } x, y \in \mathbb{R} \right\}$	i)	$\left\{ \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \right\}$
b)	$V = \{A \mid A \in M_{3\times 3}(\mathbb{R}),\ A$ is a diagonal matrix, and the sum of the diagonal entries is zero. $\}$	ii)	$\left\{ \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \right\}$
c)	$V = \left\{ \begin{bmatrix} 0 & y & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{bmatrix} \mid x+y=0, \text{ and } x, y \in \mathbb{R} \right\}$	iii)	$\left\{ \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$

Table : M2W6AQ1

Choose the correct option.

- ◯ Option 1: a → ii, b → i, c → iii.
- ◯ **Option 2:** a → ii, b → iii, c → i.
- ◯ Option 3: a → i, b → ii, c → iii.

◯ Option 4: a → iii, b → ii, c → i.

2. If S_1 is a maximal linear independent set and S_2 is a minimal spanning set of a vector space V, then which of the following option(s) is (are) true?

   ◯ **Option 1:** For any $v \in S_1$, $S_1 \setminus \{v\}$ is a linearly independent.

   ◯ Option 2: For any $v \in S_2$, $S_2 \setminus \{v\}$ is a spanning set of V.

   ◯ **Option 3:** For any $v \in V \setminus S_1$, $S_1 \cup \{v\}$ is a linearly dependent.

   ◯ **Option 4:** For any $v \in V$, $S_2 \cup \{v\}$ is a spanning set of V.

3. Consider the following subset of $\mathbb{R}^2$ with usual addition and scalar multiplication as in $\mathbb{R}^2$.

   - $V_1 = \{(x, 0) \mid x \in \mathbb{R}\}$
   - $V_2 = \{(0, y) \mid y \in \mathbb{R}\}$

   Which of the following options are correct?

   ◯ **Option 1:** Both V_1 and V_2 are subspaces of $\mathbb{R}^2$.

   ◯ **Option 2:** $V_1 \cap V_2$ is a subspace of $\mathbb{R}^2$.

   ◯ Option 3: $V_1 \cup V_2$ is a subspace of $\mathbb{R}^2$.

   ◯ Option 4: V_1 is a subspace of $\mathbb{R}^2$, but V_2 is not.

   ◯ Option 5: V_2 is a subspace of $\mathbb{R}^2$, but V_1 is not.

   ◯ **Option 6:** $\{(1, 0)\}$ is a basis of V_1.

   ◯ **Option 7:** $\{(0, 1)\}$ is a basis of V_2.

4. Consider the following subset of $\mathbb{R}^2$ with usual addition and scalar multiplication as in $\mathbb{R}^2$.

   - $V_1 = \{(x, 0) \mid x \in \mathbb{R}\}$
   - $V_2 = \{(2x, 0) \mid x \in \mathbb{R}\}$

   Which of the following options are correct?

   ◯ **Option 1:** Both V_1 and V_2 are subspaces of $\mathbb{R}^2$.

   ◯ **Option 2:** $V_1 \cap V_2$ is a subspace of $\mathbb{R}^2$.

   ◯ **Option 3:** $V_1 \cup V_2$ is a subspace of $\mathbb{R}^2$.

   ◯ Option 4: V_1 is a subspace of $\mathbb{R}^2$, but V_2 is not.

   ◯ Option 5: V_2 is a subspace of $\mathbb{R}^2$, but V_1 is not.

   ◯ **Option 6:** $\{(1, 0)\}$ is a basis of V_1.

   ◯ **Option 7:** $\{(1, 0)\}$ is a basis of V_2.

5. Let V be a vector space which is defined as follows:

$$V = \{(x, y, z, w) \mid x + z = y + w\} \subseteq \mathbb{R}^4$$

   with usual addition and scalar multiplication. Which of the following set forms a basis of V?

- ◯ Option 1: $\{(1,0,0,0),(0,1,0,0),(0,0,0,1)\}$.
- ◯ Option 2: $\{(1,1,0,0),(0,1,-1,0),(0,-1,0,1)\}$.
- ◯ Option 3: $\{(1,-1,0,0),(0,1,1,0),(0,-1,0,1)\}$.
- ◯ **Option 4:** $\{(1,1,0,0),(0,1,1,0),(0,-1,0,1)\}$.

33.2 Level 2

6. Which of the following sets form a basis of the vector space of 2×2 lower triangular real matrices with usual matrix addition and scalar multiplication? (More than one option may be correct)
   - ◯ **Option 1:** $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$
   - ◯ **Option 2:** $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
   - ◯ Option 3: $\left\{ \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$
   - ◯ **Option 4:** $\left\{ \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$
7. If $\{v_1, v_2, v_3\}$ forms a basis of $\mathbb{R}^3$, then which of the following are true?
   - ◯ **Option 1:** $\{v_1, v_2, v_1 + v_3\}$ forms a basis of $\mathbb{R}^3$.
   - ◯ **Option 2:** $\{v_1, v_1 + v_2, v_1 + v_3\}$ forms a basis of $\mathbb{R}^3$.
   - ◯ **Option 3:** $\{v_1, v_1 + v_2, v_1 - v_3\}$ forms a basis of $\mathbb{R}^3$.
   - ◯ **Option 4:** $\{v_1, v_1 - v_2, v_1 - v_3\}$ forms a basis of $\mathbb{R}^3$.
8. Which of the following options is(are) true?
   - ◯ **Option 1:** Any minimal spanning set of a vector space V must be a basis of V.
   - ◯ Option 2: Any maximal spanning set of a vector space V must be a basis of V.
   - ◯ Option 3: Any minimal linear independent set of vector space V must be a basis of V.
   - ◯ **Option 4:** Any maximal linear independent set of vector space V must be a basis of V.
   - ◯ Option 5: The basis of a vector space is unique.
   - ◯ **Option 6:** The number of elements in a basis of $\mathbb{R}^3$ is 3.
   - ◯ Option 7: The number of elements in a basis of $M_{3\times 3}(\mathbb{R})$ is 3.
   - ◯ **Option 8:** There are infinite number of bases of $\mathbb{R}^3$.
   - ◯ **Option 9:** Any subset of a minimal spanning set of V cannot be a spanning set.

Maths 2 : Activity Questions
Week-7

34 Lecture 7.1

34.1 Level 1

1. The dimension of a vector space is the :
   - ◯ Option 1: Cardinality of any spanning set.
   - ◯ **Option 2:** Cardinality of a basis.
   - ◯ **Option 3:** Cardinality of minimal spanning set.
   - ◯ **Option 4:** Cardinality of maximal linearly independent set.

2. Match the sets of vectors in column A with their properties of linear dependence or independence in column B and the dimension of the vector spaces in column C spanned by the sets.

	Set of vectors (Column A)		Linear dependence or independence (Column B)		Dimension of the vector space spanned by the set (Column C)
a)	$\{(1,0,0),(0,1,0),(1,1,1),(1,0,1)\}$	i)	Linearly independent	1)	1
b)	$\{(1,0,-1),(0,-1,0),(-1,1,-1)\}$	ii)	Linearly dependent	2)	2
c)	$\{(2,3,6),(\frac{1}{3},\frac{1}{2},1),(4,6,12)\}$	iii)	Linearly dependent	3)	3
d)	$\{(1,0,-\frac{1}{6}),(0,1,-\frac{1}{3}),(3,2,-\frac{7}{6})\}$	iv)	Linearly dependent	4)	3

Table : M2W4AQ2

Choose the correct option.

◯ Option 1: a → ii → 3, b → iii → 2, c → i → 4, d → iv → 1
◯ Option 2: a → ii → 3, b → i → 4, c → iii → 2, d → iv → 1
◯ **Option 3:** a → ii → 4, b → i → 3 , c → iv → 1, d → iii → 2
◯ Option 4: a → ii → 4, b → i → 3 , c → iv → 2, d → iii → 1

3. Choose the set of correct options.

◯ **Option 1:** The dimension of the vector space $M_{1\times 2}(\mathbb{R})$ is 2.
◯ Option 2: The dimension of the vector space $M_{2\times 1}(\mathbb{R})$ is 1.
◯ Option 3: The dimension of the vector space $M_{3\times 3}(\mathbb{R})$ is 3.
◯ **Option 4:** A basis of $M_{2\times 2}(\mathbb{R})$ is the set $\left\{\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\right\}$.

4. Consider the following sets:

- $V_1 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a symmetric matrix, i.e., $A = A^T\}$
- $V_2 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a scalar matrix$\}$
- $V_3 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a diagonal matrix$\}$
- $V_4 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is an upper triangular matrix$\}$
- $V_5 = \{A \mid A \in M_{2\times 2}(\mathbb{R})$ and A is a lower triangular matrix$\}$

All $V_i, i = 1, 2, 3, 4, 5$ are subspaces of the vector space $M_{2\times 2}(\mathbb{R})$.
Choose the set of correct options.

◯ **Option 1:** The dimension of V_1 is 3.
◯ Option 2: The dimension of V_2 is 3.
◯ Option 3: The dimension of V_3 is 1.
◯ **Option 4:** The dimension of V_4 is 3.
◯ **Option 5:** The dimension of V_5 is 3.

5. Consider the following two matrices M_1 and M_2.

$$M_1 = \begin{bmatrix}1 & 4 & 3 & 2\\ 2 & 3 & 1 & 4\\ 3 & 4 & 2 & 1\end{bmatrix}, M_2 = \begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}$$

Choose the correct set of options.

◯ Option 1: The row rank of both the matrices, M_1 and M_2, is 2.
◯ **Option 2:** The row rank of both the matrices, M_1 and M_2, is 3.
◯ Option 3: The column rank of M_1 is 4, but the column rank of M_2 is 3.
◯ Option 4: The column rank of M_2 is 4, but the column rank of M_1 is 3.
◯ Option 5: The column rank of both the matrices, M_1 and M_2, is 4.
◯ **Option 6:** The column rank of both the matrices, M_1 and M_2, is 3.

34.2 Level 2

6. Find the dimension of the vector space

$$V = \{A \mid \text{ sum of entries in each row is 0, and } A \in M_{3\times 2}(\mathbb{R})\}.$$

[Answer: 3]

7. Find the dimension of the vector space

$$V = \{(x, y, z, w) \mid x + y = z + w, x + w = y + z, \text{ and } x, y, z, w \in \mathbb{R}\}.$$

[Answer: 2]

8. Choose the set of correct options.

   ◯ Option 1: If $S \subset \mathbb{R}^2$ contains three vectors, then vectors in S must be linearly independent.

   ◯ **Option 2:** If $S \subset \mathbb{R}^3$ contains three vectors, then vectors in S may or may not be linearly independent.

   ◯ **Option 3:** If $S \subset \mathbb{R}^4$ contains five vectors, then vectors in S must be linearly dependent.

   ◯ Option 4: If $S \subset \mathbb{R}$ contains one vector, then the vector in S must be linearly dependent.

35 Lecture 7.2

35.1 Level 1

1. Find the rank of the following matrix:

$$\begin{bmatrix} 0 & 1 & 3 & 1 \\ 1 & 0 & 0 & 1 \\ 3 & 2 & 0 & 3 \end{bmatrix}$$

[Answer: 3]

Consider the following set of vectors S in $\mathbb{R}^3$ to answer the questions 2, 3 and 4.
$S = \{(1, 2, 0), (0, 3, 1), (3, 3, -1), (3, 0, -2)\}$.

2. Let V be the vector space spanned by the vectors $(1, 2, 0)$ and $(0, 3, 1)$. Which one of the following is correct?

   ◯ **Option 1:** $V = \{(a, 2a + 3b, b) \mid a, b \in \mathbb{R}\}$
   ◯ Option 2: $V = \{(0, 2a - 3b, b) \mid a, b \in \mathbb{R}\}$
   ◯ Option 3: $V = \{(-a, 2a + 3b, 0) \mid a, b \in \mathbb{R}\}$
   ◯ Option 4: $V = \{(0, 2a + 3b, 0) \mid a, b \in \mathbb{R}\}$

3. If the vectors in S are written as the columns of a matrix A, then what will be the rank of A?

   ◯ Option 1: 4
   ◯ Option 2: 3
   ◯ **Option 3:** 2
   ◯ Option 4: 1

4. What will be the dimension of the vector space spanned by S?

   ◯ Option 1: 4
   ◯ Option 2: 3
   ◯ **Option 3:** 2
   ◯ Option 4: 1

5. Choose the set of correct options from the following.

   ◯ **Option 1:** Row rank and column rank of a matrix is always the same.
   ◯ **Option 2:** The rank of a zero matrix is always 0.
   ◯ **Option 3:** The rank of a matrix, all of whose entries are the same non-zero real number, must be 1.
   ◯ Option 4: Rank of the $n \times n$ identity matrix is 1 for any $n \in \mathbb{N}$.

35.2 Level 2

6. Consider the following upper triangular matrix to choose the correct options.

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

where $a, b, c, d, e, f \in \mathbb{R}$.

- ◯ **Option 1:** If $f = 0$, then the rank of the matrix must be less than or equal to 2.
- ◯ Option 2: If $f = 0$, then the rank of the matrix must be exactly 2.
- ◯ **Option 3:** If a, b, c, d, e, f are all non-zero then the rank of the matrix must be 3.
- ◯ **Option 4:** If a, d, f are non-zero then the rank of the matrix must be 3.

7. If rank of the matrix $\begin{bmatrix} 2 & -3 & 4 \\ 0 & 1 & -2 \\ 1 & -3 & a \end{bmatrix}$ is 2 then find the value of a. [Answer: 5]

8. Find the rank of the matrix A, where $A = [a_{ij}]$ is of order 3×3 and $a_{i,j} = \min\{i, j\}$, $i, j = 1, 2, 3$. [Answer: 3]

   [Hint: Write down the matrix and apply row operations to derive it's reduced row echelon form.]

36 Lecture 7.3

36.1 Level 1

Consider the system of linear equations $Ax = 0$, where $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ and

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Use the above information to answer questions 1, 2 and 3.

1. Choose the correct statement from the following options.
   - ◯ Option 1: x_1, x_2 and x_3 all are dependent variables.
   - ◯ Option 2: x_2 is the only dependent variable.
   - ◯ **Option 2:** x_1 and x_3 are the dependent variables and x_2 is the only independent variable.
   - ◯ Option 4: x_1, x_2 and x_3 all are independent variables.

2. Which one of the following vector spaces represents the null space of A?
   - ◯ Option 1: $\{(t_1, 0, t_2) \mid t_1, t_2 \in \mathbb{R}\}$
   - ◯ **Option 2:** $\{(0, t, 0) \mid t \in \mathbb{R}\}$
   - ◯ Option 3: $\{(t_1, t_2, 0) \mid t_1, t_2 \in \mathbb{R}\}$
   - ◯ Option 4: $\{(0, 0, t) \mid t \in \mathbb{R}\}$

3. What will be the nullity of A? [Answer: 1]

4. Let S denote the set of solutions of the homogeneous system of linear equations $Ax = 0$. Which of the following statements is (are) true?
   - ◯ **Option 1:** If x_1 and x_2 are in S, then $x_1 - x_2$ is in S.
   - ◯ **Option 2:** If x is in S, then cx is in S, for any $c \in \mathbb{R}$.
   - ◯ **Option 3:** If x_1 and x_2 are in S, then $\alpha x_1 + \beta x_2$ is in S, for any $\alpha, \beta \in \mathbb{R}$.
   - ◯ **Option 4:** If x is in S, then $A^n x = 0$ for any $n \in \mathbb{N} \setminus \{0\}$.

5. Find out the value of a for which the matrix $\begin{bmatrix} 1 & 2 \\ 3 & a \end{bmatrix}$ has nullity 1. [Answer: 6]

36.2 Level 2

6. Consider the coefficient matrix A of the following system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_4 &= 0 \\ x_2 + x_3 &= 0 \\ x_1 - x_3 + x_4 &= 0 \end{aligned}$$

Which one of the following vector spaces represents the null space of A?

◯ Option 1: $\{(t_1 + t_2, t_1, t_1, t_2) \mid t_1, t_2 \in \mathbb{R}\}$

◯ **Option 2:** $\{(t_1 - t_2, -t_1, t_1, t_2) \mid t_1, t_2 \in \mathbb{R}\}$

◯ Option 1: $\{(t_1 + t_2, -t_1, t_1, t_2) \mid t_1, t_2 \in \mathbb{R}\}$

◯ Option 1: $\{(t_1 - t_2, -t_1, t_1, -t_2) \mid t_1, t_2 \in \mathbb{R}\}$

7. Find the nullity of the matrix A, where $A = [a_{ij}]$ is of order 3×3 and $a_{i,j} = \min\{i, j\}$, $i, j = 1, 2, 3$. [Answer: 0]

8. Choose the correct set of options from the following.

◯ Option 1: The nullity of a non-zero scalar matrix of order 3 must be 3.

◯ **Option 2:** The nullity of a non-zero scalar matrix of order 3 must be 0.

◯ Option 3: The nullity of a non-zero diagonal matrix of order 3 must be 3.

◯ **Option 4:** The nullity of a non-zero diagonal matrix of order 3 can be at most 2.

37 Lecture 7.4

37.1 Level 1

1. If A is an $m \times n$ matrix, then which of the following statements is true?
   - ◯ Option 1: $rank(A) + nullity(A) = m$
   - ◯ **Option 2:** $rank(A) + nullity(A) = n$
   - ◯ Option 3: $rank(A) + nullity(A) = mn$
   - ◯ Option 4: $rank(A) + nullity(A) = max\{m, n\}$
   - ◯ Option 5: $rank(A) + nullity(A) = min\{m, n\}$

2. Which of the following is true for a homogeneous system of linear equations $Ax = 0$?
   - ◯ Option 1: It may have no solution.
   - ◯ **Option 2:** It can have infinitely many solutions.
   - ◯ **Option 3:** It can have a unique solution.
   - ◯ **Option 4:** Whenever it has a non-trivial(non zero) solution, it must have infinitely many solutions.

3. Which of the following options are correct for a square matrix A of order $n \times n$, where n is any natural number?
   - ◯ **Option 1:** If the determinant is non-zero, then the nullity of A must be 0.
   - ◯ Option 2: If the determinant is non-zero, then the nullity of A may be non-zero.
   - ◯ **Option 3:** If the nullity of A is non-zero, then the determinant of A must be 0.
   - ◯ Option 4: If the nullity of A is non-zero, then the determinant of A may be non-zero.

4. Choose the set of correct statements.
   - ◯ **Option 1:** If nullity of a 3×3 matrix is c for some natural number c, $0 \le c \le 3$, then the nullity of $-A$ will also be c.
   - ◯ Option 2: $nullity(A + B) = nullity(A) + nullity(B)$.
   - ◯ **Option 3:** Nullity of the zero matrix of order $n \times n$, is n.
   - ◯ Option 4: Nullity of the zero matrix of order $n \times n$, is 0.
   - ◯ **Option 5:** There exist square matrices A and B of order $n \times n$, such that nullity of both A and B is 0, but the nullity of $A + B$ is n.

5. If A is a 3×4 matrix, then which of the following options are true?
   - ◯ **Option 1:** $rank(A)$ must be less than or equal to 3.

- ◯ **Option 2:** $nullity(A)$ must be greater than or equal to 1.
- ◯ **Option 3:** If A has 2 columns which are non-zero and not multiples of each other, while the remaining columns are linear combinations of these 2 columns, then $nullity(A) = 2$.
- ◯ Option 4: If A has 2 columns which are non-zero and not multiples of each other, while the remaining columns are linear combinations of these 2 columns, then $nullity(A) = 1$.

37.2 Level 2

6. Let $Ax = 0$ be a homogeneous system of linear equations which has infinitely many solutions, where A is an $m \times n$ matrix (where, $m > 1, n > 1$). Which of the following statements are possible?
   - ◯ **Option 1:** $rank(A) = m$ and $m < n$.
   - ◯ Option 2: $rank(A) = m$ and $m > n$.
   - ◯ Option 3: $rank(A) = m$ and $m = n$.
   - ◯ **Option 4:** $nullity(A) = n$.
   - ◯ **Option 5:** $nullity(A) \neq 0$.

Consider the coefficient matrix A of the following system of linear equations to answer questions 7 and 8:

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 0 \\ x_1 + x_3 &= 0 \end{aligned}$$

7. Which one of the following vector spaces represents the null space of A appropriately?
   - ◯ **Option 1:** $\{(-t, t, t) \mid t \in \mathbb{R}\}$.
   - ◯ Option 2: $\{(t_1, t_2, \frac{t_2 - t_1}{2}) \mid t_1, t_2 \in \mathbb{R}\}$.
   - ◯ Option 3: $\{(t, -t, t) \mid t \in \mathbb{R}\}$.
   - ◯ Option 4: $\{(t_1, t_2, \frac{t_1 + t_2}{2}) \mid t_1, t_2 \in \mathbb{R}\}$.
8. What will be the rank of A and nullity of A?
   - ◯ Option 1: $rank(A) = 3, nullity(A) = 2$
   - ◯ **Option 2:** $rank(A) = 2, nullity(A) = 1$
   - ◯ Option 3: $rank(A) = 1, nullity(A) = 2$
   - ◯ Option 4: $rank(A) = 2, nullity(A) = 0$

38 Lecture 7.5

A book shop is organizing an year end sale. Price of any Bengali, Hindi, Tamil, and Urdu book is fixed as ₹200, ₹180, ₹230, and ₹250, respectively. Let $T(x, y, z, w)$ denote the total price of x number of Bengali books, y number of Hindi books, z number of Tamil books, and w number of Urdu books. Table M2W7AQ1 shows the numbers of books of different languages purchased by some customers.

	Bengali	Hindi	Tamil	Urdu
Samprita	3	0	0	2
Srinivas	0	1	2	1
Anna	0	1	0	3
Tiyasha	2	2	0	1
Hasan	2	2	1	1

M2W7AQ1

Answer Questions 1 to 8 from the given data.

38.1 Level 1

1. What will be the correct expression for $T(x, y, z, w)$?
   - ◯ Option 1: $T(x, y, z, w) = (200 + 180 + 230 + 250)(x + y + z + w)$
   - ◯ Option 2: $T(x, y, z, w) = x + y + z + w$
   - ◯ Option 3: $T(x, y, z, w) = 200x + 230y + 180z + 250w$
   - ◯ **Option 4:** $T(x, y, z, w) = 200x + 180y + 230z + 250w$
2. Which of the following expressions represents the total price of the books purchased by Samprita?
   - ◯ **Option 1:** $T(3, 0, 0, 2)$
   - ◯ Option 2: $T(3, 0, 0, 2, 2)$
   - ◯ Option 3: $T(3, 2)$
   - ◯ Option 4: $T(5)$
3. What will be total price (in ₹) of the books purchased by Tiyasha? [Answer: 1010]
4. What will be total price (in ₹) of the books purchased by Hasan? [Answer: 1240]

38.2 Level 2

5. Which of the following expressions represent the total price of the books purchased by Srinivas?

   ◯ Option 1: $2T(0,1,1,0) + T(0,0,0,1)$

   ◯ Option 2: $T(0,1,0,0) + T(0,0,1,1)$

   ◯ **Option 3:** $T(0,1,0,0) + 2T(0,0,1,0) + T(0,0,0,1)$

   ◯ **Option 4:** $T(0,1,1,0) + T(0,0,1,1)$

6. Which of the following expressions represents the total price of the books purchased by Anna?

   ◯ Option 1: $T(0,1,0,0) + T(0,0,0,1)$

   ◯ Option 2: $T(0,1,0,0) + T(0,0,1,0)$

   ◯ **Option 3:** $T(0,1,0,0) + 3T(0,0,0,1)$

   ◯ Option 4: $T(0,1,0,0) + 3T(0,0,1,0)$

7. Which of the following expressions represent the total price of the books purchased by Srinivas and Anna together?

   ◯ **Option 1:** $T(0,2,2,4)$

   ◯ Option 2: $T(2,2,4)$

   ◯ Option 3: $T(0,2,2,1)$

   ◯ **Option 4:** $T(0,1,2,1) + T(0,1,0,3)$

8. Which of the following expressions represent the difference between the total price of the books purchased by Samprita and Tiyasha?

   ◯ Option 1: $|T(5,2,0,3)|$

   ◯ **Option 2:** $|T(1,-2,0,1)|$

   ◯ **Option 3:** $|T(3,0,0,2) - T(2,2,0,1)|$

   ◯ **Option 4:** $|T(3,0,0,2) + T(-2,-2,0,-1)|$

39 Lecture 7.6

Let $T(x, y, z) = mx + ny + pz$ denote the total amount of money a multiplex made from a specific movie, in one day by selling x number of tickets for the morning show, y number of tickets for the evening show, and z number of tickets for the night show, where m, n and p are the prices of each ticket of that movie in the morning, evening, and night show, respectively.

In the multiplex, 2 Malayalam movies, 1 Hindi movie, 1 Bengali movie, and 1 English movie are running parallelly in one week. Suppose there are 3 shows (Morning, Evening, Night) per day for each of the Malayalam movies and the English movie, whereas 2 shows (Evening, Night) per day for the Hindi and the Bengali movie. Ticket price (irrespective of morning, evening, and night shows) for each Malayalam movie is ₹200, Hindi movie is ₹250, Bengali movie is ₹250, and English movie is ₹300. The number of tickets sold in a particular day is given in Table M2W7AQ2:

Shows	Morning	Evening	Night
Malayalam Movie 1	m_1	m_2	m_3
Malayalam Movie 2	m'_1	m'_2	m'_3
Hindi Movie	0	h_2	h_3
Bengali Movie	0	b_2	b_3
English Movie	e_1	e_2	e_3

Table: M2W7AQ2

Answer questions 1 to 8 using the given data above.

39.1 Level 1

1. Which expression accurately expresses the total amount of money the multiplex made in the specific day mentioned above by selling the tickets of the morning shows only?

   ◯ Option 1: $200m_1 + 200m_2 + 200m_3$

   ◯ Option 2: $200m_1 + 250m'_1 + 300e_1$

   ◯ Option 3: $200m_1 + 200m'_1 + 250e_1$

   ◯ **Option 4:** $200m_1 + 200m'_1 + 300e_1$

2. Which expression accurately expresses the total amount of money the multiplex made in the specific day mentioned above by selling the tickets of the Bengali movie only?

   ◯ Option 1: $200(b_2 + b_3)$

   ◯ **Option 2:** $250(b_2 + b_3)$

- ◯ Option 3: $250(h_2 + h_3)$
- ◯ Option 4: $250b_2 + 200b_3$

3. Which expression accurately expresses the total amount of money the multiplex made in the specific day mentioned above by selling the tickets of the night shows only?
   - ◯ Option 1: $200(m_2 + m'_2) + 250(h_2 + b_2) + 300e_2$
   - ◯ Option 2: $200(m_3 + m'_3 + h_3 + b_3) + 300e_3$
   - ◯ **Option 3:** $200(m_3 + m'_3) + 250(h_3 + b_3) + 300e_3$
   - ◯ Option 4: $200(m_3m'_3) + 250(h_3b_3) + 300e_3$

4. Which expression accurately expresses the total amount of money the multiplex made in the specific day mentioned above by selling the tickets of the Bengali and English movie only?
   - ◯ Option 1: $250(b_2 + b_3)$
   - ◯ Option 2: $300e_1 + 250(b_2 + e_2) + 300(b_3 + e_3)$
   - ◯ **Option 3:** $250(b_2 + b_3) + 300(e_1 + e_2 + e_3)$
   - ◯ Option 4: $b_2 + b_3 + e_1 + e_2 + e_3$

39.2 Level 2

5. What was the total amount of money the multiplex made in the specific day mentioned above by selling the tickets of the Malayalam movies together?
   - ◯ **Option 1:** $T(m_1 + m'_1, m_2 + m'_2, m_3 + m'_3)$
   - ◯ Option 2: $200T(m_1 + m'_1, m_2 + m'_2, m_3 + m'_3)$
   - ◯ Option 3: $T(m_1m'_1, m_2m'_2, m_3m'_3)$
   - ◯ Option 4: $200T(m_1m'_1, m_2m'_2, m_3m'_3)$

6. Which of the following options are correct?
   - ◯ **Option 1:** $T(x, y, z) = T(x, 0, 0) + T(0, y, 0) + T(0, 0, z)$
   - ◯ **Option 2:** $T(x, y, z) = T(x, y, 0) + T(0, 0, z)$
   - ◯ Option 3: $T(x, y, z) = T(x, 0, 0) + T(y, 0, 0) + T(z, 0, 0)$
   - ◯ **Option 4:** $T(x, 0, 0) + T(y, 0, 0) + T(z, 0, 0) = T(x + y + z, 0, 0)$

7. Which of the following equalities correctly represent the total amount of money the multiplex made in the specific day mentioned above by selling the tickets of the Bengali and Hindi movie together?
   - ◯ **Option 1:** $T(0, b_2 + h_2, b_3 + h_3) = T(0, (h_2 + b_2), (h_3 + b_3))$

- ◯ Option 2: $T(0, b_2 + h_2, b_3 + h_3) = T(0, 0, 0) + T(0, h_2 + b_2, 0) + T(0, h_3 + b_3, 0)$
- ◯ **Option 3:** $T(0, b_2 + h_2, b_3 + h_3) = T(0, (h_2 + b_2), 0) + T(0, 0, (h_3 + b_3))$
- ◯ **Option 4:** $T(0, b_2 + h_2, b_3 + h_3) = T(0, b_2, b_3) + T(0, h_2, h_3)$

8. Which of the following equality correctly represents the total amount of money the multiplex made in the specific day mentioned above by selling the tickets of the English movie?

- ◯ Option 1: $T(e_1, e_2, e_3) = 200(e_1 + e_2 + e_3)$
- ◯ Option 2: $T(e_1, e_2, e_3) = 250(e_1 + e_2 + e_3)$
- ◯ Option 3: $T(e_1, e_2, e_3) = 200e_1 + 250e_2 + 300e_3$
- ◯ **Option 4:** $T(e_1, e_2, e_3) = 300(e_1 + e_2 + e_3)$

40 Lecture 7.7

40.1 Level 1

1. Choose the correct set of options:

   ◯ **Option 1:** T is a one to one linear transformation if and only if there does not exist any $v \neq 0 \in V$ so that $T(v) = 0$.

   ◯ Option 2: If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a surjective linear transformation, then it cannot be injective.

   ◯ **Option 3:** If $T : \mathbb{R} \to \mathbb{R}$ is a linear transformation which is not injective, then T must be 0, i,e., $T(v) = 0$ for all $v \in \mathbb{R}$.

   ◯ **Option 4:** If there exists some non-zero vector $v \in V$, such that $T(v) = 0$, for a linear transformation $T : V \to W$, then T cannot be an isomorphism.

2. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation, such that $T(x, y) = (x, 0)$, then which of the following options is true?

   ◯ Option 1: T is both one to one and onto.

   ◯ Option 2: T is one to one, but not onto.

   ◯ Option 3: T is onto, but not one to one.

   ◯ **Option 4:** T is neither one to one nor onto.

3. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation, such that $T(x, y) = (x + y, x - y)$, then which of the following options is true?

   ◯ **Option 1:** T is both one to one and onto.

   ◯ Option 2: T is one to one, but not onto.

   ◯ Option 3: T is onto, but not one to one.

   ◯ Option 4: T is neither one to one nor onto.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, such that $T(2, 1) = (3, 2)$ and $T(1, 3) = (0, 0)$. Which of the following options is true?

   ◯ Option 1: $T(x, y) = \frac{1}{5}(3x, 2y)$

   ◯ Option 2: $T(x, y) = (x + 1, y + 1)$

   ◯ **Option 3:** $T(x, y) = \frac{1}{5}(9x - 3y, 6x - 2y)$

   ◯ Option 4: T cannot be determined from the given information.

5. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(v) = Av$, where $v = \begin{bmatrix} x \\ y \end{bmatrix}$, and $A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$. Which of the following options is true?

- ◯ **Option 1:** T is an isomorphism.
- ◯ Option 2: T is one to one, but not onto.
- ◯ Option 3: T is onto, but not one to one.
- ◯ Option 4: T is neither one to one nor onto.

40.2 Level 2

6. Let $S : V_1 \to V_2$ and $T : V_2 \to V_3$ be two linear transformations. Let us define $T \circ S : V_1 \to V_3$ by $T \circ S(v) = T(S(v))$. Choose the correct set of options.
   - ◯ Option 1: If $T \circ S$ is injective, then T must be injective.
   - ◯ **Option 2:** If $T \circ S$ is injective, then S must be injective.
   - ◯ **Option 3:** If $T \circ S$ is surjective, then T must be surjective.
   - ◯ Option 4: If $T \circ S$ is surjective, then S must be surjective.

7. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation defined by $T(x, y) = (2x + 3y, 5x - y, x + 6y)$. Which of the following options is true?
   - ◯ Option 1: T is both one to one and onto.
   - ◯ **Option 2:** T is one to one, but not onto.
   - ◯ Option 3: T is onto, but not one to one.
   - ◯ Option 4: T is neither one to one nor onto.

8. If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined as follows,

$$T : \mathbb{R}^3 \to \mathbb{R}^3$$
$$T(x, y, z) = (a_1x + b_1y + c_1z, a_2x + b_2y + c_2z, a_3x + b_3y + c_3z).$$

This can be represented as $T(v) = Av$, where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, and $v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Which of the following options are true?

   - ◯ **Option 1:** T is injective if and only if the system of linear equations $Av = 0$ has a unique solution.
   - ◯ **Option 2:** T is injective if and only if $rank(A) = 3$.
   - ◯ **Option 3:** If the system of linear equations $Av = b$ for any $b \in \mathbb{R}^3$, has a solution, then T must be surjective.
   - ◯ **Option 4:** If T is not surjective, then there exists some $b \in \mathbb{R}^3$, such that the system of linear equations $Av = b$ has no solution.

Maths 2 : Activity Questions
Week-8

41 Lecture 8.1

41.1 Level 1

1. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation, such that $T(x, y) = (x, 0)$. Which of the following matrices corresponds to T with respect to the standard ordered basis of $\mathbb{R}^2$, i.e., $\{(1, 0), (0, 1)\}$, for both the domain and co-domain?

   ◯ Option 1: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   ◯ Option 2: $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

   ◯ **Option 3:** $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

   ◯ Option 4: $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

2. If the matrix corresponding to a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, with respect to standard ordered basis of $\mathbb{R}^2$, i.e., $\{(1, 0), (0, 1)\}$, for both the domain and co-domain, is $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. Which of the following is the appropriate definition of T?

   ◯ Option 1: $T(x, y) = (2x + y, 3x + 4y)$

   ◯ Option 2: $T(x, y) = (2x + y, 3x - 4y)$

   ◯ Option 3: $T(x, y) = (x + 4y, 2x + 3y)$

   ◯ **Option 4:** $T(x, y) = (2x + 3y, x + 4y)$

Let $W = \{(x, y, z) \mid x = 2y + z\}$ be a subspace of $\mathbb{R}^3$. Let $\beta = \{(2, 1, 0), (1, 0, 1)\}$ be a basis of W. Let $T : W \to \mathbb{R}^2$ be a linear transformation such that $T(2, 1, 0) = (1, 0)$ and $T(1, 0, 1) = (0, 1)$. Answer the questions 3, 4 and 5 using the given information.

3. Which of the following is the appropriate definition of T?

   ◯ Option 1: $T(x, y, z) = (x, y)$

   ◯ Option 2: $T(x, y, z) = (x, z)$

   ◯ **Option 3:** $T(x, y, z) = (y, z)$

   ◯ Option 4: $T(x, y, z) = (x - y, z)$

4. Choose the correct options.
   - ◯ Option 1: T is one to one but not onto.
   - ◯ Option 2: T is onto but not one to one.
   - ◯ Option 3: T is neither one to one nor onto.
   - ◯ **Option 4:** T is an isomorphism.
5. What will be the matrix representation of T with respect to the basis β for W and $\gamma = \{(1,1),(1,-1)\}$ for $\mathbb{R}^2$?
   - ◯ **Option 1:** $\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
   - ◯ Option 2: $\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
   - ◯ Option 3: $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
   - ◯ Option 4: $\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$

41.2 Level 2

6. Choose the set of correct options.
   - ◯ **Option 1:** If the Identity matrix of order 2 is the matrix representation of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, with respect to the standard ordered basis for both the domain and co-domain, then it is also the matrix representation of T with respect to any other basis β for both the domain and co-domain.
   - ◯ **Option 2:** Let T be an isomorphism between two vector spaces, and A be the matrix representation of T with respect to some basis. Then the nullity of A is 0.
   - ◯ **Option 3:** If $A = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$ is a matrix representation of $T : \mathbb{R}^2 \to \mathbb{R}^2$ with respect to the standard ordered basis of $\mathbb{R}^2$, for both the domain and co-domain, then the matrix representation of $T \circ T$ with respect to the same bases will also be A.
   - ◯ Option 4: If $A = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$ is a matrix representation of $T : \mathbb{R}^2 \to \mathbb{R}^2$ with respect to the standard ordered basis of $\mathbb{R}^2$, for both the domain and co-domain, then the matrix representation of $T \circ T$ will be the identity matrix.

Let $\beta = \{(1,0),(0,1)\}$ and $\gamma = \{(1,1),(1,-1)\}$ be two bases of $\mathbb{R}^2$. If $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is the matrix representation of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with respect to β for both the domain and the co-domain. Answer questions 7 and 8 using the information given above.

7. What is the matrix representation of T with respect to γ for the domain and β for the co-domain?

   ◯ Option 1: $\begin{bmatrix} a+b & c+d \\ a-b & c-d \end{bmatrix}$

   ◯ Option 2: $\begin{bmatrix} a & -b \\ c & -d \end{bmatrix}$

   ◯ **Option 3:** $\begin{bmatrix} a+b & a-b \\ c+d & c-d \end{bmatrix}$

   ◯ Option 4: $\begin{bmatrix} a+b & -a-b \\ c+d & -c-d \end{bmatrix}$

8. What is the matrix representation of T with respect to γ for both the domain and the co-domain?

   ◯ Option 1: $\begin{bmatrix} a+b+c+d & a-b+c-d \\ a+b-c-d & a-b-c+d \end{bmatrix}$

   ◯ **Option 2:** $\begin{bmatrix} \frac{a+b+c+d}{2} & \frac{a-b+c-d}{2} \\ \frac{a+b-c-d}{2} & \frac{a-b-c+d}{2} \end{bmatrix}$

   ◯ Option 3: $\begin{bmatrix} \frac{a+b+c+d}{2} & \frac{a+b-c-d}{2} \\ \frac{a-b+c-d}{2} & \frac{a-b-c+d}{2} \end{bmatrix}$

   ◯ Option 4: $\begin{bmatrix} a+b+c+d & a+b-c-d \\ a-b+c-d & a-b-c+d \end{bmatrix}$

42 Lecture 8.2

42.1 Level 1

1. Choose the set of correct options.

   ◯ **Option 1:** If v_1 and v_2 are in the kernel of a linear transformation $T : V \to W$, where V and W are two vector spaces, then $v_1 + v_2$ will also be in the kernel of T.

   ◯ **Option 2:** Kernel of a linear transformation $T : V \to W$ is a vector subspace of V, where V and W are two vector spaces.

   ◯ **Option 3:** If w_1 and w_2 are in the image of a linear transformation $T : V \to W$, where V and W are two vector spaces, then $w_1 + w_2$ will also be in the image of T.

   ◯ **Option 4:** Image of a linear transformation $T : V \to W$ is a vector subspace of W, where V and W are two vector spaces.

Consider the following linear transformation:

$$T : \mathbb{R}^3 \to \mathbb{R}^2$$
$$T(x, y, z) = (2x + 3z, 4y + z)$$

Answer questions 2,3,4 and 5, using the information given above.

2. Which of the following matrices corresponds to the given linear transformation T with respect to the standard ordered basis for $\mathbb{R}^3$ and the standard ordered basis for $\mathbb{R}^2$?

   ◯ Option 1: $\begin{bmatrix} 2 & 3 & 0 \\ 4 & 1 & 0 \end{bmatrix}$

   ◯ Option 2: $\begin{bmatrix} 2 & 0 \\ 0 & 4 \\ 3 & 1 \end{bmatrix}$

   ◯ **Option 3:** $\begin{bmatrix} 2 & 0 & 3 \\ 0 & 4 & 1 \end{bmatrix}$

   ◯ Option 4: $\begin{bmatrix} 2 & 4 \\ 3 & 1 \\ 0 & 0 \end{bmatrix}$

3. Which of the following represents a basis of the kernel of T?

   ◯ Option 1: $\{(-\frac{3}{2}, 0, 1), (0, -\frac{1}{4}, 1)\}$.

   ◯ **Option 2:** $\{(-\frac{3}{2}, -\frac{1}{4}, 1)\}$.

   ◯ Option 3: $\{(-\frac{3}{2}, -\frac{1}{4}, 2)\}$.

◯ Option 4: $\{(2, 0, 3), (0, 4, 1)\}$.

4. What will be the dimension of the subspace $Im(T)$? [Answer: 2]

5. Choose the correct option.

◯ Option 1: T is an isomorphism.

◯ Option 2: T is one to one but not onto.

◯ **Option 3:** T is onto but not one to one.

◯ Option 4: T is neither one to one, nor onto.

42.2 Level 2

6. Which option represents the kernel and image of the following linear transformation?

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$
$$T(x, y) = (x, 0)$$

◯ Option 1: $ker(T) = Span\{(1, 0)\}, Im(T) = Span\{(1, 0)\}$.

◯ Option 2: $ker(T) = Span\{(1, 0)\}, Im(T) = Span\{(0, 1)\}$.

◯ **Option 3:** $ker(T) = Span\{(0, 1)\}, Im(T) = Span\{(1, 0)\}$.

◯ Option 4: $ker(T) = Span\{(0, 1)\}, Im(T) = Span\{(0, 1)\}$.

7. Which of the following linear transformations is an isomorphism between the vector spaces $V = \{(x, y, z) \mid x = y - z, \text{ and } x, y, z \in \mathbb{R}\} \subset \mathbb{R}^3$ and $W = \{(x, y, z) \mid x = y, \text{ and } x, y, z \in \mathbb{R}\} \subset \mathbb{R}^3$?

◯ Option 1: $T : V \to W$, such that $T(x, y, z) = (y, y, 0)$.

◯ **Option 2:** $T : V \to W$, such that $T(x, y, z) = (y, y, z)$.

◯ Option 3: $T : V \to W$, such that $T(x, y, z) = (x, y, z)$.

◯ Option 4: There does not exist any isomorphism between V and W.

8. Which of the following linear transformations is an isomorphism between the vector spaces $V = \{(x, y, z) \mid x = y - z = 0, \text{ and } x, y, z \in \mathbb{R}\} \subset \mathbb{R}^3$ and $W = \{(x, y, z) \mid x = y = 0, \text{ and } x, y, z \in \mathbb{R}\} \subset \mathbb{R}^3$?

◯ **Option 1:** $T : V \to W$, such that $T(x, y, z) = (x, 0, y)$.

◯ Option 2: $T : V \to W$, such that $T(x, y, z) = (0, y, z)$.

◯ Option 3: $T : V \to W$, such that $T(x, y, z) = (0, 0, x)$.

◯ Option 4: There does not exist any isomorphism between V and W.

43 Lecture 8.3

43.1 Level 1

1. Choose the set of correct options.

   ◯ **Option 1:** Nullity and rank of the identity transformation on a vector space of dimension n are 0 and n respectively.

   ◯ Option 2: Nullity and rank of the identity transformation on a vector space of dimension n are 1 and $n-1$ respectively.

   ◯ Option 3: Nullity and rank of the identity transformation on a vector space of dimension n are n and 0 respectively.

   ◯ Option 4: Nullity and rank of an isomorphism between two vector spaces V and W (both of dimension n) are n and 0 respectively.

   ◯ **Option 5:** Nullity and rank of an isomorphism between two vector spaces V and W (both of dimension n) are 0 and n respectively.

   ◯ **Option 6:** There cannot exist an isomorphism between two vector spaces whose dimensions are not the same.

Consider the following linear transformation:

$$T : \mathbb{R}^3 \to \mathbb{R}^3$$
$$T(x, y, z) = (x - z, 2x + 3y + z, 3y + 3z)$$

Answer questions 2,3,4, and 5, using the information given above.

2. Which of the matrices corresponds to the given linear transformation T with respect to the standard ordered basis of $\mathbb{R}^3$ for both the domain and the co-domain?

   ◯ Option 1: $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 3 \\ -1 & 1 & 3 \end{bmatrix}$

   ◯ Option 2: $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 3 \\ 0 & 1 & 0 \end{bmatrix}$

   ◯ Option 3: $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 3 & 3 & 0 \end{bmatrix}$

   ◯ **Option 4:** $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 1 \\ 0 & 3 & 3 \end{bmatrix}$

3. What will be the kernel of T?

- ◯ Option 1: $Span(S)$, where $S = \{(x, 0, z), (0, y, z) \mid x, y, z \in \mathbb{R}\}$.
- ◯ Option 2: $Span(S)$, where $S = \{(z, 0, z), (0, -z, z) \mid z \in \mathbb{R}\}$.
- ◯ **Option 3:** $Span(S)$, where $S = \{(z, -z, z) \mid z \in \mathbb{R}\}$.
- ◯ Option 4: $Span(S)$, where $S = \{(x, 0, z), (0, -x, z) \mid x, z \in \mathbb{R}\}$.

4. What will be the $rank(T)$? [Answer: 2]

5. Choose the correct option.
   - ◯ Option 1: T is an isomorphism.
   - ◯ Option 2: T is one to one but not onto.
   - ◯ Option 3: T is onto but not one to one.
   - ◯ **Option 4:** T is neither one to one, nor onto.

43.2 Level 2

6. Choose the set of correct options.
   - ◯ **Option 1:** Any injective linear transformation between any two vector spaces which have the same dimensions, must be an isomorphism.
   - ◯ **Option 2:** Any surjective linear transformation between any two vector spaces which have the same dimensions, must be an isomorphism.
   - ◯ **Option 3:** There does not exist any surjective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$.
   - ◯ Option 4: There does not exist any injective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$.

7. Let T be an injective linear transformation as follows :

$$T : \mathbb{R}^3 \to \mathbb{R}^3 \text{ such that, } T(1, 0, 0) = (0, 1, 1) \text{ and } T(0, 1, 0) = (1, 0, 1)$$

   Which of the following can be a possible definition of T?
   - ◯ Option 1: $T(x, y, z) = (y, x, x + y)$
   - ◯ Option 2: $T(x, y, z) = (y + z, x + 2z, x + y + 3z)$
   - ◯ **Option 3:** $T(x, y, z) = (y + z, x + 2z, x + y + 4z)$
   - ◯ Option 4: $T(x, y, z) = (y, x + y, x + y + z)$

8. Consider the linear transformation
$M_{2\times2}(\mathbb{R}) \to M_{2\times2}(\mathbb{R})$ such that $T(A) = PA$, where $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and a, b, c, d are in $\mathbb{R}$.
Let $\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ be an ordered basis of $M_{2\times2}(\mathbb{R})$. Which of the following matrices represents the matrix corresponding to the linear transformation T with respect to β for both domain and co-domain?

◯ Option 1: $\begin{bmatrix} a & b & 0 & 0 \\ 0 & 0 & c & d \\ a & b & 0 & 0 \\ 0 & 0 & c & d \end{bmatrix}$

◯ Option 2: $\begin{bmatrix} a & 0 & b & 0 \\ c & 0 & d & 0 \\ c & a & 0 & b \\ 0 & c & 0 & d \end{bmatrix}$

◯ Option 3: $\begin{bmatrix} a & 0 & 0 & a \\ c & 0 & 0 & c \\ b & 0 & 0 & b \\ d & 0 & 0 & d \end{bmatrix}$

◯ **Option 4:** $\begin{bmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ c & 0 & d & 0 \\ 0 & c & 0 & d \end{bmatrix}$

44 Lecture 8.4

44.1 Level 1

1. Choose the set of correct options.

   ◯ **Option 1:** If a 3×3 matrix A is similar to the identity matrix of order 3, then A must be the identity matrix of order 3.

   ◯ Option 2: If a 3×3 matrix A is similar to a diagonal matrix of order 3, then A must be a diagonal matrix of order 3.

   ◯ **Option 3:** If A and B are similar matrices, then they are also equivalent matrices.

   ◯ Option 4: If A and B are equivalent matrices, then they are also similar matrices.

Consider the linear transformation given below:

$$T : \mathbb{R}^3 \to \mathbb{R}^2$$

$$T(x, y, z) = (x - y, y + z)$$

Consider two ordered bases $\beta_1 = \{(1,0,0),(0,1,0),(0,0,1)\}$, and $\beta_2 = \{(1,0,1),(0,1,1),(1,1,0)\}$ of $\mathbb{R}^3$ and consider the standard ordered basis $\gamma = \{(1,0),(0,1)\}$ of $\mathbb{R}^2$. Let A and B be the matrices corresponding to the linear transformation T with respect to the bases β_1 and β_2 for the domain, respectively and γ for the co-domain. Let Q and P be matrices such that $B = QAP$.
Answer the questions 2,3, and 4 using the above information.

2. Which of the following matrices represents A?

   ◯ Option 1: $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

   ◯ **Option 2:** $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

   ◯ Option 3: $\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$

   ◯ Option 4: $\begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

3. Which of the following matrices can be P?

   ◯ Option 1: There does not exist any matrix P which satisfies the property $B = QAP$.

- ◯ Option 2: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$
- ◯ Option 3: $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$
- ◯ **Option 4:** $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

4. Which of the following statements is true for Q?
   - ◯ Option 1: Q can be the identity matrix of order 3.
   - ◯ **Option 2:** Q can be the identity matrix of order 2.
   - ◯ Option 3: Q is unique.
   - ◯ **Option 4:** Q can be the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

5. Choose the set of correct options.
   - ◯ Option 1: If two square matrices of the same order have the same determinants, then they must be similar to each other.
   - ◯ **Option 2:** Similar matrices have the same rank.
   - ◯ **Option 3:** If A and B are similar matrices, and $Ax = b$ has a unique solution, then $Bx = b$ also has a unique solution.
   - ◯ **Option 4:** If A and B are similar matrices, and $Ax = b$ does not have a unique solution, then $Bx = b$ also does not have a unique solution.

44.2 Level 2

6. Let E be the set of 2×2 matrices which are similar to a scalar matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. Choose the set of correct statements.
   - ◯ Option 1: E is an infinite set.
   - ◯ **Option 2:** Cardinality of E is finite.
   - ◯ **Option 3:** E is singleton set.
   - ◯ Option 4: E consists of all the scalar matrices of order n.

7. Which of the following option(s) is(are) true?
   - ◯ **Option 1:** If A and B are similar matrices then A^{-1} and B^{-1} are similar matrices.

◯ **Option 2:** If A and B are similar matrices then A^2 and B^2 are similar matrices.

◯ Option 3: If A^2 and B^2 are similar matrices then A and B are similar matrices.

◯ **Option 4:** If A and B are similar matrices then A^T and B^T are similar matrices.

8. Let us consider the following matrices.

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Choose the set of correct options.

◯ **Option 1:** A and B have the same rank, and same determinant, but they are not similar matrices.

◯ Option 2: A and C have the same rank, and same determinant, but they are not similar matrices.

◯ **Option 3:** B and C have the same rank, and same determinant, but they are not similar matrices.

◯ Option 4: None of the options are true.

45 Lecture 8.5

45.1 Level 1

1. Let L be an affine subspace of $\mathbb{R}^3$ defined as $L = \{(x, y, z) \mid x + y + 2z = 6\}$, then which of the following subspaces of $\mathbb{R}^3$ corresponds to the affine subspace L?
   - ◯ Option 1: $\{(x, y, z) \mid x + y + z = 5\}$
   - ◯ **Option 2:** $\{(x, y, z) \mid x + y + 2z = 0\}$
   - ◯ Option 3: $\{(x, y, z) \mid x + 2y + z = 0\}$
   - ◯ Option 4: $\{(x, y, z) \mid 2x + y + z = 0\}$
2. Which of the following subsets of $\mathbb{R}^2$ represent an affine subspace of $\mathbb{R}^2$?
   - ◯ **Option 1:** $\{(x, y) \mid x^2 + y^2 = 0\}$
   - ◯ Option 2: $\{(x, y) \mid x^2 + y^2 = 4\}$
   - ◯ Option 3: $\{(x, y) \mid x^5 + 1 = y\}$
   - ◯ **Option 4:** $\{(x, y) \mid 3x + y = 1\}$
   - ◯ **Option 5:** $\{(x, y) \mid x = 1\}$
   - ◯ Option 6: $\{(x, y) \mid y = 5x^2\}$
   - ◯ Option 7: $\{(x, y) \mid y = x^3 + 2\}$
   - ◯ **Option 8:** $\{(1,2)\}$
3. Which of following subsets of $\mathbb{R}^3$ represent an affine subspace of $\mathbb{R}^3$?
   - ◯ **Option 1:** $\{(x, y, z) \mid x^2 + y^2 + z^2 = 0\}$
   - ◯ Option 2: $\{(x, y, z) \mid x^2 + z^2 + y = 1\}$
   - ◯ **Option 3:** $\{(x, y, z) \mid x + y + z = 2\}$
   - ◯ **Option 4:** $\{(x, y, z) \mid 2x + y = 3z\}$
   - ◯ Option 5: $\{(x, y, z) \mid xy = z\}$
   - ◯ **Option 6 :** $\{(x, y, z) \mid y = x + z\}$
4. Let U be a subspace of the vector space $\mathbb{R}^3$ and a basis of U is given by $\{(0, 1, -3), (1, 0, -1)\}$. Then which of the following subsets of $\mathbb{R}^3$ is an affine subspace of $\mathbb{R}^3$ such that the corresponding vector subspace is U?
   - ◯ **Option 1:** $L = \{(x, y, z) \mid x + 3y + z = 3\}$
   - ◯ Option 2: $L = \{(x, y, z) \mid x + 3y + 2z = 0\}$
   - ◯ **Option 3:** $L = \{(x, y, z) \mid x + 3y + z = 0\}$
   - ◯ **Option 4:** $L = \{(x, y, z) \mid x + 3y + z = 8\}$
5. Which of the following mappings is an affine mapping from $\mathbb{R}^2$ to an affine subspace of $\mathbb{R}^3$?
   - ◯ Option 1: $T(x, y) = (x + 1, y, xy)$
   - ◯ **Option 2:** $T(x, y) = (x, y, x + y + 5)$

◯ **Option 3:** $T(x, y) = (2x - y + 2, 3y + 3, x + y - 1)$

◯ Option 4: $T(x, y) = (3x, y^2, x)$

◯ **Option 5:** $T(x, y) = (x, x + y, 0)$

45.2 Level 2

6. Consider a system of linear equations

$$\begin{aligned} -x + y - z &= 1 \\ x - y + z &= -1 \\ x + z &= 0 \end{aligned}$$

Which of the following is an affine subspace L of $\mathbb{R}^3$ such that if $v \in L$, then v is a solution of the system of linear equations?

- ◯ Option 1: $\{(t, -1, t) \mid t \in \mathbb{R}\}$
- ◯ **Option 2:** $\{(-t, 1, t) \mid t \in \mathbb{R}\}$
- ◯ Option 3: $\{(-t, 1, -t) \mid t \in \mathbb{R}\}$
- ◯ Option 4: $\{(-t, -1, t) \mid t \in \mathbb{R}\}$

7. Consider the following affine mappings from $\mathbb{R}^3$ to an affine subspace of $\mathbb{R}^3$
$f_1(x, y, z) = (x + 1, x + 2y - 2, x + y + 2z)$
$f_2(x, y, z) = (x + 5, x + 2y - 3, x + y + 2z + 8)$
$f_3(x, y, z) = (2x + 4, y + z - 1, x + y + 5)$.
Which of the following option(s) is(are) true?

- ◯ **Option 1:** A linear transformation corresponding to f_1 is $T(x, y, z) = (x, x + 2y, x + y + 2z)$
- ◯ Option 2: The linear transformations corresponding to f_1 and f_2 are not the same.
- ◯ Option 3: A linear transformation corresponding to f_3 is $T(x, y, z) = (x, y + z, x + y)$
- ◯ **Option 4:** A linear transformation corresponding to f_3 is $T(x, y, z) = (2x, y + z, x + y)$

8. Let L and L' be affine subspaces of $\mathbb{R}^3$, where $L = U$ and $L' = (2,0,1) + U'$, for some vector subspaces U and U' of $\mathbb{R}^3$. Let a basis for U be given by $\{(2,0,1),(1,1,0),(0,1,0)\}$ and a basis for U' be given by $\{(1,0,1),(0,1,1)\}$. Suppose there is a linear transformation $T : U \to U'$ such that $(0,1,0) \in ker(T)$, $T(2,0,1) = (0,1,1)$ and $T(1,1,0) = (1,0,1)$. An affine mapping $f : L \to L'$ is obtained by defining $f(u) = (2,0,1) + T(u)$, for all $u \in U$. Which of the following options are true?

   - ◯ Option 1: $L = \{(x,y,z) \mid x - y - 2z = 0\}$
   - ◯ **Option 2:** $L = \mathbb{R}^3$
   - ◯ **Option 3:** $L' = \{(x,y,z) \mid x + y - z = 1\}$
   - ◯ Option 4: $L' = \mathbb{R}^3$
   - ◯ **Option 5:** $f(x,y,z) = (x - 2z + 2, z, x - z + 1)$
   - ◯ Option 6: $f(x,y,z) = (x - 2z + 2, \frac{x}{2}, x - z + 1)$
   - ◯ Option 7: $f(x,y,z) = (x - 2z, z, x - z)$

Maths 2 : Activity Questions
Week-9

46 Lecture 9.1

46.1 Level 1

1. Given that $\|a\| = 3$ and $\|b\| = 5$ and $a \cdot b = 7.5$, for two vectors a and b in $\mathbb{R}^n$. Find the angle (in degrees) between the two vectors a, b. Note: $\cos 60° = 1/2, \cos 30° = \sqrt{3}/2$.
   - ◯ Option 1: 30°
   - ◯ **Option 2:** 60°
   - ◯ Option 3: 120°
   - ◯ Option 4: 150°

2. Consider two vectors $a = (3, 4, -1), b = (2, -1, 2)$ in $\mathbb{R}^3$. Choose the correct options.
   - ◯ **Option 1:** Length/Norm of a is $\sqrt{26}$ and that of b is 3.
   - ◯ Option 2: Length/Norm of a is 5 and that of b is 3.
   - ◯ Option 3: Length/Norm of a is $\sqrt{24}$ and that of b is $\sqrt{7}$.
   - ◯ **Option 4:** Angle between the two vectors is 90 degrees.
   - ◯ Option 5: Angle between the two vectors is approximately 0 degrees.

3. If the dot product of two vectors is zero, then choose the correct option.
   - ◯ **Option 1:** The two vectors are perpendicular to each other.
   - ◯ Option 2: The two vectors are parallel to each other.
   - ◯ Option 3: The two vectors are exactly the same.
   - ◯ Option 4: The length of the two vectors must be the same.

46.2 Level 2

4. If $a = (6, -1, 3), b = (4, c, 2)$ where $a, b \in \mathbb{R}^3$ and a and b are perpendicular to each other, then find the value of c? (Answer: 30)

5. Consider two vectors $a = (1, 2), b = (2, 2)$ and θ is the angle between them. The sum of the two vectors is given by $c = a + b$. Choose the correct options.
   - ◯ **Option 1:** Length/Norm of c is 5.
   - ◯ Option 2: Length/Norm of c is 25.

- ◯ Option 3: Length/Norm of c is 4.
- ◯ **Option 4:** $\cos\theta = \frac{3}{\sqrt{10}}$.
- ◯ Option 5: $\cos\theta = \frac{3}{\sqrt{5}}$.

6. Consider 3 vectors a, b, c in $\mathbb{R}^3$ and a scalar λ in $\mathbb{R}$. Choose the set of correct options.

   Note: (.) represents the dot product.

   - ◯ **Option 1:** $\lambda(a \cdot b) = (\lambda a) \cdot b$
   - ◯ **Option 2:** $\lambda(a \cdot b) = (\lambda b) \cdot a$
   - ◯ Option 3: $\lambda(a \cdot b) = (\lambda a) \cdot (\lambda b)$
   - ◯ **Option 4:** $(a + c) \cdot b = a \cdot b + c \cdot b$
   - ◯ **Option 5:** $(a + c) \cdot b = a \cdot c + c \cdot b$
   - ◯ **Option 6:** $a \cdot a = 0, b \cdot b = 0$ if and only if a, b are null vectors.

7. Consider two vectors $a = (1, 3, 5, 7, 9)$ and $b = (2, 4, 6, 8, 10)$. Choose the set of correct options.

   - ◯ **Option 1:** The length of b is more than a.
   - ◯ Option 2: The length of a is more than b.
   - ◯ **Option 3:** The length of $(a - b)$ is $\sqrt{5}$.
   - ◯ Option 4: The length of $(a - b)$ is $\sqrt{50}$.

47 Lecture 9.2

47.1 Level 1

1. Consider a function $f : V \times V \to \mathbb{R}$ where $V \subseteq \mathbb{R}^2$ defined by $f(v, w) = 2v_1w_1 + 5v_2w_2$, where $v = (v_1, v_2), w = (w_1, w_2)$. Choose the set of correct options.

   ◯ **Option 1:** f satisfies the symmetry condition of the inner product.

   ◯ **Option 2:** f satisfies the bilinearity condition of the inner product.

   ◯ **Option 3:** f satisfies the positivity condition of the inner product.

   ◯ **Option 4:** f is an inner product.

   ◯ Option 5: f is not an inner product.

2. Consider a function $f : V \times V \to \mathbb{R}$ where $V \subseteq \mathbb{R}^2$ defined by $f(v, w) = v_1w_1 - v_1w_2 - v_2w_1 + 4v_2w_2$, where $v = (v_1, v_2), w = (w_1, w_2)$. Choose the set of correct options.

   ◯ **Option 1:** f satisfies the symmetry condition of the inner product.

   ◯ **Option 2:** f satisfies the bilinearity condition of the inner product.

   ◯ **Option 3:** f satisfies the positivity condition of the inner product.

   ◯ **Option 4:** f is an inner product.

   ◯ Option 5: f is not an inner product.

3. Consider two vectors $a = (0.4, 1.3, -2.2), b = (2, 3, -5)$ in $\mathbb{R}^3$. Choose the set of correct options.

   ◯ **Option 1:** The two vectors satisfy the triangle inequality given by $\|a + b\| \leq \|a\| + \|b\|$.

   ◯ Option 2: The two vectors do not satisfy the triangle inequality.

   ◯ **Option 3:** The two vectors satisfy the Cauchy-Schwarz inequality given by $| < a, b > | \leq \|a\| \, \|b\|$.

   ◯ Option 4: The two vectors do not satisfy the Cauchy-Schwarz inequality.

47.2 Level 2

4. Consider a function $f : V \times V \to \mathbb{R}$ where $V \subseteq \mathbb{R}^2$ defined by $f(v, w) = v_1^2w_1^2 + v_1w_2^2 + v_2^2w_1$, where $v = (v_1, v_2), w = (w_1, w_2)$. Choose the set of correct options.

   ◯ **Option 1:** f satisfies the symmetry condition of the inner product.

   ◯ Option 2: f satisfies the positivity condition of the inner product.

   ◯ Option 3: f is an inner product.

   ◯ **Option 4:** f is not an inner product.

5. Consider a vector $a = (a_1, a_2, a_3)$ in $\mathbb{R}^3$. Which of these is (are) possible candidates for a norm?

   - ◯ **Option 1:** $\sqrt{a_1^2 + b_1^2 + c_1^2}$
   - ◯ Option 2: $a_1 - b_1$
   - ◯ Option 3: $a_1 + b_1 + c_1$
   - ◯ Option 4: $max(a_1, b_1, c_1)$
   - ◯ Option 5: $min(a_1, b_1, c_1)$

48 Lecture 9.3

48.1 Level 1

1. Consider two vectors $a = (2, 0, 3, 0, 8), b = (3, 2, -2, 4, 0)$ in $\mathbb{R}^5$. Choose the set of correct options.

   ◯ **Option 1:** a and b are orthogonal.

   ◯ Option 2: a and b are not orthogonal.

   ◯ Option 3: $(a - b) \cdot a = 0$.

   ◯ **Option 4:** $(a - b) \cdot a = 77$.

2. Choose the set of correct statements.

   ◯ Option 1: In an orthogonal set, the norms of all the vectors are equal.

   ◯ **Option 2:** In an orthogonal set, the vectors are linearly independent.

   ◯ Option 3: In an orthogonal set, the vectors are linearly dependent.

   ◯ **Option 4:** If the columns of an $n \times n$ coefficient matrix A comprises the individual vectors of an orthogonal set in $\mathbb{R}^n$, then there must be a unique solution to the system $AX = b$, where X, b are $n \times 1$ vectors.

   ◯ Option 5: If the columns of an $n \times n$ coefficient matrix A comprises the individual vectors of an orthogonal set in $\mathbb{R}^n$, then there are no solutions to the system $AX = b$, where X, b are $n \times 1$ vectors.

   ◯ Option 6: The determinant of a square matrix formed by a set of orthogonal vectors in $\mathbb{R}^n$ is zero.

   ◯ **Option 7:** A set of n vectors can never form an orthogonal basis in $\mathbb{R}^{n-1}$.

3. Which of the following is an orthogonal basis of the given vector spaces with respect to the standard inner product (dot product)?

   ◯ **Option 1:** $\{(1, 0), (0, 1)\}$ is an orthogonal basis of $\mathbb{R}^2$.

   ◯ **Option 2:** $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is an orthogonal basis of $\mathbb{R}^3$.

   ◯ Option 3: $\{(3, 4), (4, -3), (2, -3)\}$ is an orthogonal basis of $\mathbb{R}^2$.

   ◯ Option 4: $\{(2, 1, -1), (-1, 1, -1), (3, -3, 3)\}$ is an orthogonal basis of $\mathbb{R}^3$.

48.2 Level 2

4. Find a vector in $\mathbb{R}^4$ that is orthogonal to the subspace spanned by $(1, 1, 0, 0)$ and $(0, 1, 1, 0)$ with respect to the dot product as the inner product.

   ◯ **Option 1:** $(1, -1, 1, 0)$

○ Option 2: $(2, 3, 4, 5)$

○ Option 3: $(1, -1, -1, 1)$

○ Option 4: $(1, 1, 1, 0)$

5. Which of the following are orthogonal to the vector $(1, 2)$ in $\mathbb{R}^2$ with respect to the inner product $< v, w >= v_1w_1 - v_1w_2 - v_2w_1 + 4v_2w_2$?

○ Option 1: $(-7, 1)$

○ Option 2: $(6, -1)$

○ **Option 3:** $(7, 1)$

○ Option 4: $(-9, 1)$

6. Consider the system of linear equations:

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 1 \\ 2x_1 + x_2 &= 5 \\ 3x_1 - 6x_2 - 5x_3 &= 9. \end{aligned}$$

Choose the correct option.

○ **Option 1:** The system has a unique solution.

○ Option 2: The system has no solution.

○ Option 3: The system can have infinitely many solutions.

○ Option 4: None of the above.

7. Let A be the coefficient matrix of the system of linear equations:

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 1 \\ 2x_1 + x_2 &= 5 \\ 3x_1 - 6x_2 - 5x_3 &= 9. \end{aligned}$$

Which one of the following is true about the matrix AA^T?

○ Option 1: A scalar matrix.

○ Option 2: The identity matrix.

○ **Option 3:** A diagonal matrix.

○ Option 4: A lower triangular matrix.

○ Option 5: An upper triangular matrix.

○ Option 6: None of the above.

49 Lecture 9.4

49.1 Level 1

1. Which of these sets form an orthonormal basis of $\mathbb{R}^3$ with respect to the dot product as the inner product in $\mathbb{R}^3$?
   - ◯ Option 1: $\{(1,0,0),(0,1,0)\}$
   - ◯ Option 2: $\{(2,0,0),(0,2,0),(0,0,2)\}$
   - ◯ **Option 3:** $\{(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}),(\frac{-3}{\sqrt{11}},\frac{1}{\sqrt{11}},\frac{1}{\sqrt{11}}),(\frac{1}{\sqrt{66}},\frac{7}{\sqrt{66}},\frac{-4}{\sqrt{66}})\}$
   - ◯ Option 4: $\{(2,1,-1),(-1,-1,1),(1,2,-2)\}$
2. Consider two orthogonal vectors a, b in $\mathbb{R}^2$. If $a+b$ and $a-b$ are orthogonal, then choose the correct option.
   - ◯ Option 1: $\|a\| = \|b\| = 1$
   - ◯ **Option 2:** $\|a\| = \|b\|$
   - ◯ Option 3: $\|a\| = 2\|b\|$
   - ◯ Option 4: $2\|a\| = \|b\|$
3. Consider $a = (1,1), b = (1,-1)$. Let $V = Span\{a,b\}$. Choose the correct options by considering the standard inner product (dot product).
   - ◯ **Option 1:** Vectors a, b form an orthogonal basis for V.
   - ◯ Option 2: Vectors a, b form an orthonormal basis for V.
   - ◯ **Option 3:** There exist scalar multiples of a, b which form an orthonormal basis
   - ◯ Option 4: There do not exist scalar multiples of a, b which form an orthonormal basis

49.2 Level 2

4. Choose the set of correct statements.
   - ◯ **Option 1:** The determinant of a matrix formed by 3 orthonormal vectors in $\mathbb{R}^3$ is ± 1.
   - ◯ Option 2: The determinant of a matrix formed by 3 orthonormal vectors in $\mathbb{R}^3$ is 0.
   - ◯ **Option 3:** The determinant of a matrix formed by 2 orthonormal vectors in $\mathbb{R}^2$ is ± 1.
   - ◯ Option 4: The determinant of a matrix formed by 2 orthonormal vectors in $\mathbb{R}^2$ is 0.

5. Consider a system of linear equations:

$$2x_1 + 2x_2 + 7x_3 = b_1$$
$$2x_1 + x_2 - 10x_3 = b_2$$
$$3x_1 - 2x_2 + 2x_3 = b_3.$$

Let A be the coefficient matrix of the given system of linear equations. Let a matrix B contain the column vectors of A, which are normalized by their respective norms, as its columns (i.e. first column vector of A normalized by its norm is the first column of B). Which of the following statements are true ?

- ◯ **Option 1:** The determinant of BB^T is 1.
- ◯ **Option 2:** BB^T is an identity matrix.
- ◯ **Option 3:** BB^T is a scalar matrix.
- ◯ **Option 4:** BB^T is a diagonal matrix.

6. Choose the correct option(s).

- ◯ **Option 1:** The vectors in an orthonormal set are linearly independent.
- ◯ Option 2: A set of linearly dependent vectors can be orthonormal.
- ◯ Option 3: A set of linearly independent vectors is always orthonormal.
- ◯ Option 4: A set of linearly independent vectors is always orthogonal but not orthonormal.

Consider the inner product $\langle u, v\rangle = 3u_1v_1 + 2u_2v_2$ where $u = (u_1, u_2), v = (v_1, v_2)$ are vectors in $\mathbb{R}^2$. Answer questions 7 and 8 based on this information.

7. Which of the following is an orthonormal basis with respect to the inner product defined above?

- ◯ Option 1: $\{(2, 3), (-4, 4)\}$
- ◯ **Option 2:** $\{\frac{1}{\sqrt{30}}(2, 3), \frac{1}{\sqrt{80}}(-4, 4)\}$
- ◯ Option 3: $\{\frac{1}{\sqrt{13}}(2, 3), \frac{1}{\sqrt{32}}(-4, 4)\}$
- ◯ Option 4: $\{(2, 3), (-3, 2)\}$
- ◯ Option 5: $\{\frac{1}{\sqrt{13}}(2, 3), \frac{1}{\sqrt{13}}(-3, 2)\}$

8. Use the orthonormal basis $\{u, v\}$ obtained in question 7 with respect to the defined inner product. Express the vector $(4, 0)$ as a linear combination of the basis vectors u and v, as $(4, 0) = c_1u + c_2v$. Which of the following gives the coefficients of the linear combination?

- ◯ **Option 1:** $c_1 = \frac{24}{\sqrt{30}}, c_2 = \frac{-48}{\sqrt{80}}$
- ◯ Option 2: $c_1 = \frac{24}{\sqrt{13}}, c_2 = \frac{-48}{\sqrt{32}}$

◯ Option 3: $c_1 = \frac{8}{\sqrt{13}}, c_2 = \frac{-16}{\sqrt{32}}$

◯ Option 4: $c_1 = \frac{24}{\sqrt{30}}, c_2 = \frac{48}{\sqrt{80}}$

50 Lecture 9.5

50.1 Level 1

1. Consider two vectors $a = (1, -2, 2), b = (4, 0, -3)$. Using the standard dot product as the inner product, the projection of a in the direction of b is given by

   ◯ **Option 1:** $(\frac{-8}{25}, 0, \frac{6}{25})$

   ◯ Option 2: $(\frac{-8}{25}, \frac{6}{25})$

   ◯ Option 3: $(\frac{8}{25}, 0, \frac{-6}{25})$

   ◯ Option 4: $(\frac{8}{25}, \frac{-6}{25})$

2. Consider an orthonormal basis $\{\frac{1}{\sqrt{5}}(1, 2), \frac{1}{\sqrt{5}}(-2, 1)\}$ of $\mathbb{R}^2$. If $(3, 5) = c_1v_1 + c_2v_2$ where $c_1, c_2 \in \mathbb{R}$ and $v_1 = \frac{1}{\sqrt{5}}(1, 2), v_2 = \frac{1}{\sqrt{5}}(-2, 1)$, then choose the correct option.

   ◯ Option 1: $c_1 = \frac{13}{\sqrt{5}}, c_2 = \frac{1}{\sqrt{5}}$

   ◯ Option 2: $c_1 = \frac{-13}{\sqrt{5}}, c_2 = \frac{1}{\sqrt{5}}$

   ◯ **Option 3:** $c_1 = \frac{13}{\sqrt{5}}, c_2 = \frac{-1}{\sqrt{5}}$

   ◯ Option 4: $c_1 = \frac{-13}{\sqrt{5}}, c_2 = \frac{-1}{\sqrt{5}}$

3. Consider an orthonormal basis $\{(1, 0, 0), (0, 1, 0)\}$ for a subspace W in $\mathbb{R}^3$. If $x = (1, 2, 3)$ is a vector in $\mathbb{R}^3$, then which of the following represents a vector in W whose distance from x is the least? Consider dot product as the standard inner product.

   ◯ Option 1: $(2, 4, 0)$

   ◯ Option 2: $(3, 4, 0)$

   ◯ Option 3: $(4, 5, 0)$

   ◯ **Option 4:** $(1, 2, 0)$

50.2 Level 2

Consider the inner product $\langle a, b\rangle = a_1b_1 - a_1b_2 - a_2b_1 + 4a_2b_2$ where $a = (a_1, a_2), b = (b_1, b_2)$ are vectors in $\mathbb{R}^2$. Use this information to answer questions 4 and 5.

4. Let $x = (1, 2)$. Find the projection of x in the direction of $(3, 4)$ using the inner product defined above.

   ◯ **Option 1:** $\frac{25}{49}(3, 4)$

   ◯ Option 2: $\frac{11}{25}(3, 4)$

   ◯ Option 3: $\frac{25}{49}(1, 2)$

   ◯ Option 4: $\frac{11}{25}(1, 2)$

5. Let $x = (1, 2)$, find the projection of x in a direction perpendicular to $(3, 4)$.

   ◯ Option 1: $\frac{25}{49}(-4, 3)$

   ◯ Option 2: $\frac{11}{25}(4, -3)$

   ◯ **Option 3:** $(-\frac{26}{49}, -\frac{2}{49})$

   ◯ Option 4: $(\frac{26}{49}, -\frac{2}{49})$

6. Consider an orthogonal basis $\{(1, 2, 1), (-2, 0, 2)\}$ of a subspace W, of the inner product space $\mathbb{R}^3$ with respect to the dot product. If $y = (1, 2, 3) \in \mathbb{R}^3$, then find $Proj_W(y)$.

   ◯ **Option 1:** $(\frac{1}{3}, \frac{8}{3}, \frac{7}{3})$

   ◯ Option 2: $(\frac{18}{\sqrt{6}} - \frac{32}{\sqrt{8}}, \frac{36}{\sqrt{8}}, \frac{18}{\sqrt{6}} + \frac{32}{\sqrt{8}})$

   ◯ Option 3: $(\frac{1}{3}, -\frac{8}{3}, \frac{7}{3})$

   ◯ Option 4: $(\frac{18}{\sqrt{6}} - \frac{32}{\sqrt{8}}, \frac{36}{\sqrt{8}}, -\frac{18}{\sqrt{6}} + \frac{32}{\sqrt{8}})$

51 Lecture 9.6

51.1 Level 1

Consider an ordered basis $\gamma = \{b_1, b_2, b_3\}$ of the inner product space $\mathbb{R}^3$ with respect to the dot product, where $b_1 = (1,2,3), b_2 = (1,1,1)$ and $b_3 = (0,2,1)$. Let $\beta = \{a_1, a_2, a_3\}$ be the orthonormal basis which is obtained using the Gram-Schmidt process from the basis γ. Use this information to answer the questions 1 and 2.

1. The vector a_2 is
   - ◯ Option 1: $(\frac{4}{\sqrt{21}}, \frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}})$
   - ◯ **Option 2:** $(\frac{4}{\sqrt{21}}, \frac{1}{\sqrt{21}}, \frac{-2}{\sqrt{21}})$
   - ◯ Option 3: $(\frac{-4}{\sqrt{21}}, \frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}})$
   - ◯ Option 4: $(\frac{4}{\sqrt{21}}, \frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}})$
2. The vector a_3 is
   - ◯ **Option 1:** $\frac{1}{\sqrt{6}}(-1, 2, -1)$
   - ◯ Option 2: $\frac{1}{\sqrt{6}}(-1, -2, -1)$
   - ◯ Option 3: $\frac{1}{\sqrt{6}}(1, 2, -1)$
   - ◯ Option 4: $\frac{1}{\sqrt{6}}(1, 2, 1)$

51.2 Level 2

3. Let $v_1 = (1,0,1,1)$ and $v_2 = (0,1,1,1)$ be the vectors from the inner product space $\mathbb{R}^4$ with respect to the dot product. If $v_3 = v_2 + av_1$ where $a \in \mathbb{R}$ and v_1, v_3 are orthogonal, then
   - ◯ **Option 1:** $a = -2/3$
   - ◯ Option 2: $a = 2/3$
   - ◯ Option 3: $a = 1/3$
   - ◯ Option 4: $a = -1/3$
4. Let W be a subspace of the inner product space $\mathbb{R}^4$ with respect to dot product and $\{v_1, v_2, v_3\}$ be an ordered basis of W, where $v_1 = (1,1,1,1), v_2 = (1,1,1,0), v_3 = (1,1,0,0)$. Which of these is the orthonormal basis of W obtained using the Gram-Schmidt process?
   - ◯ **Option 1:** $\{\frac{1}{2}(1,1,1,1), \frac{1}{2\sqrt{3}}(1,1,1,-3), \frac{1}{\sqrt{6}}(1/3, 1/3, -2/3, 0)\}$
   - ◯ Option 2: $\{\frac{1}{2}(1,1,1,1), \frac{1}{2\sqrt{3}}(1,1,1,-3), \frac{1}{\sqrt{6}}(1/3, 1/3, 2/3, 0)\}$

- ◯ Option 3: $\{\frac{1}{2}(1,1,1,1), \frac{1}{2\sqrt{3}}(1,1,1,-3), \frac{1}{\sqrt{6}}(3,1,-2,0)\}$
- ◯ Option 4: $\{\frac{1}{2}(1,1,1,1), \frac{1}{2\sqrt{3}}(1,1,1,-3), \frac{1}{\sqrt{6}}(3,1,2,-2)\}$

5. Let $a = (\frac{2}{3}, \frac{2}{3}, \frac{1}{3})$ be a vector from the inner product space $\mathbb{R}^3$ with respect to dot product and $W = \{(x,y,z) \in \mathbb{R}^3 \mid \langle (x,y,z), (\frac{2}{3}, \frac{2}{3}, \frac{1}{3}) \rangle = 0\}$ be a subspace of $\mathbb{R}^3$. Then which of the following is (are) a basis of W?
   - ◯ Option 1: $\{(1,0,2),(0,1,2)\}$
   - ◯ **Option 2:** $\{(1,0,-2),(0,1,-2)\}$
   - ◯ **Option 3:** $\{(-1,0,2),(0,-1,2)\}$
   - ◯ Option 4: $\{(1,0,2),(0,1,-2)\}$

52 Lecture 9.7

52.1 Level 1

1. Choose the correct options.

   ◯ Option 1: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, where $\mathbb{R}^2$ is the inner product space with respect to the dot product. Then $Tu \cdot Tv = u \cdot v$.

   ◯ **Option 2:** Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be an orthogonal linear transformation, where $\mathbb{R}^2$ is the inner product space with respect to the dot product. Then $Tu \cdot Tv = u \cdot v$.

   ◯ Option 3: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, where $\mathbb{R}^2$ is the inner product space with respect to the inner product given by $< a, b >= 2a_1b_1 + 5a_2b_2$. Then $\langle Tu, Tv\rangle = \langle u, v\rangle$.

   ◯ **Option 4:** Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be an orthogonal linear transformation, where $\mathbb{R}^2$ is the inner product space with respect to the inner product given by $< a, b >= 2a_1b_1 + 5a_2b_2$. Then $\langle Tu, Tv\rangle = \langle u, v\rangle$.

2. If A is the matrix representation of an orthogonal transformation $T : \mathbb{R}^n \to \mathbb{R}^n$, then choose the set of correct statements.

   ◯ **Option 1:** $AA^T = I$

   ◯ Option 2: $A = A^T$

   ◯ **Option 3**: AA^T is an upper triangular matrix

   ◯ **Option 4:** AA^T is a lower triangular matrix

   ◯ **Option 5:** $A^T = A^{-1}$

   ◯ Option 6: $A^T = -A^{-1}$

   ◯ Option 7: A^{-1} does not exist

   ◯ **Option 8:** A^{-1} is also an orthogonal matrix

3. If A is the matrix representation of an orthogonal transformation $T : \mathbb{R}^n \to \mathbb{R}^n$, then choose the set of correct statements.

   ◯ Option 1: The column vectors of A are orthogonal but not orthonormal.

   ◯ **Option 2:** The column vectors of A are orthonormal.

   ◯ Option 3: The row vectors of A are orthogonal but not orthonormal.

   ◯ **option 4:** The row vectors of A are orthonormal.

52.2 Level 2

4. Consider the orthogonal set $B = \{(1,1,1), (-1,1,0)\}$ in $\mathbb{R}^3$. Find a third vector in $\mathbb{R}^3$ which is orthogonal to both the vectors in B.

   ◯ **Option 1:** $(1, 1, -2)$

   ◯ Option 2: $(1, -1, 2)$

   ◯ Option 3: $(-1, 1, -2)$

   ◯ Option 4: $(-1, -1, -2)$

5. Consider the orthogonal set $B = \{(1,1,1), (-1,1,0)\}$ and vector obtained in question 4 which is orthogonal to the set B. Form the columns of an orthogonal matrix U by scaling these vectors appropriately. Choose the correct option.

   ◯ Option 1: $U = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$

   ◯ **Option 2:** $U = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \end{bmatrix}$

   ◯ Option 3: $U = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$

   ◯ Option 4: $U = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \end{bmatrix}$

6. Consider a vector $v = (3, 0, 4)$ and U from Question 5. Choose the correct option using the dot product as the standard inner product.

   ◯ **Option 1:** $\|v\| = \|Uv\|$

   ◯ Option 2: $\|v\| \neq \|Uv\|$

   ◯ Option 3: $2\|v\| = \|Uv\|$

   ◯ Option 4: $\|v\| = 2\|Uv\|$

7. Let the matrix representation of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ be $U = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$.

   Let $B = \begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$. Choose the set of correct statements.

◯ **Option 1:** T is an orthogonal transformation.

◯ Option 2: T is not an orthogonal transformation.

◯ **Option 3:** B is a real symmetric matrix.

◯ **Option 4:** $U^{-1}BU$ is a diagonal matrix.

◯ Option 5: $U^{-1}BU$ is a scalar matrix.

◯ Option 6: $U^{-1}BU$ is an upper triangular matrix.

◯ Option 7: $U^{-1}BU$ is a lower triangular matrix.

Maths-II
Activity Questions
Week-10

53 Lecture 10.1

Functions of several variables:
Level-1:

1. Which of the following are correct with respect to $f(x, y) = x^2 - 2y$, where x, y are real?
   (a) Domain of f is $\mathbb{R}^2$
   (b) Domain of f is $\mathbb{R}$
   (c) Range of f is $\mathbb{R}$
   (d) Range of f is $\mathbb{R}^2$

2. Let $D \subset \mathbb{R}^n$ and $f : D \longrightarrow \mathbb{R}^m$, $g : D \longrightarrow \mathbb{R}^m$ be multivariable functions on D. Which of the following statements is(are) TRUE?
   (a) The product function fg is defined on D by $fg(\underset{\sim}{x}) = f(\underset{\sim}{x}) \times g(\underset{\sim}{x}), \underset{\sim}{x} \in D$
   (b) The sum function $f + g$ is defined on D by $(f + g)(\underset{\sim}{x}) = f(\underset{\sim}{x}) + g(\underset{\sim}{x}), \underset{\sim}{x} \in D$
   (c) Let $c \in \mathbb{R}$. The function cf is defined on D by $(cf)(\underset{\sim}{x}) = \frac{1}{c} \times f(\underset{\sim}{x}), \underset{\sim}{x} \in D$
   (d) If $m = 1$ and $g(\underset{\sim}{x}) \neq 0, \underset{\sim}{x} \in D$, then the function f/g is defined on D by $(f/g)(\underset{\sim}{x}) = f(\underset{\sim}{x})/g(\underset{\sim}{x}), \underset{\sim}{x} \in D$

3. Suppose $f : D \longrightarrow \mathbb{R}$ is a function defined on a domain $D \subset \mathbb{R}$. Which of the following can not be f?
   (a) $f(x) = \frac{x^2}{x+1}$
   (b) $f(x) = log(\sqrt{x^4 + 3})$
   (c) $f(x) = x^3 + x^2 - x$
   (d) $f(x) = 4 \pm \sqrt{x - 10}$

4. Which of the following statements is(are) TRUE?
   (a) The output of a scalar valued multivariable function is always a real number.
   (b) The output of a vector valued multivariable function can be a real number.

(c) The output of a single variable vector valued function is always a vector.

(d) The output of a vector valued multivariable function can not be a real number.

5. Consider the function $S(t) = (\cos t, \sin t, 2t)$, where t is real. Choose the set of correct options.

   (a) S is a single-variable vector-valued function

   (b) S is a multi-variable vector-valued function

   (c) The domain of S is $\mathbb{R}$ and the co-domain of S is $\mathbb{R}^3$

   (d) The domain of S is $\mathbb{R}^3$ and the co-domain of S is $\mathbb{R}^3$

6. Which of the following functions is(are) a vector valued multivariable function?

   (a) $f(x,y) = \frac{1}{3\pi}e^{x^2+y^2}$

   (b) $f(x,y) = (x^3, y^2)$

   (c) $f(x,y) = (x+y, x-y, 2xy)$

   (d) $f(x,y,z) = x^2 + xyz - z^2$

7. Suppose $g : D \longrightarrow \mathbb{R}^3$ is a function defined on a domain $D \subset \mathbb{R}^2$. Which of the following functions may be g?

   (a) $g(x,y) = 4x + 3y - 12xy$

   (b) $g(x,y) = (x^4+1, y^4+1, xy)$

   (c) $g(x,y,z) = (e^x - e^z, e^z - e^y)$

   (d) $g(x,y) = (sin(x+y), cos(x-y))$

Level-2:

8. Which of the following are correct with respect to $f(x,y) = \sqrt{1 - \frac{x^2}{9} - \frac{y^2}{16}}$, where x, y are real? (MSQ Ans: a,c)

   (a) Domain of f is $D = \{(x,y) | \frac{x^2}{9} + \frac{y^2}{16} \leq 1\}$

   (b) Domain of f is $D = \{(x,y) | \frac{x^2}{9} + \frac{y^2}{16} \geq 1\}$

   (c) Range of f is $[0,1]$

   (d) Range of f is $[0,1)$

   (e) Range of f is $[1,\infty)$

9. A function $F : \mathbb{R}^2 \to \mathbb{R}^3$ has the form $F(x,y) = (P(x,y), Q(x,y), R(x,y))$. Choose the correct option(s).

(a) F is a multi-variable vector-valued function

(b) F is a single-variable vector-valued function

(c) P, Q, R are single-variable vector-valued functions

(d) P, Q, R are multi-variable vector-valued functions

(e) P, Q, R are multi-variable scalar-valued functions

10. Consider $f(x, y) = \sqrt{1 - \frac{x^2}{9} - \frac{y^2}{4}}$. Note: An ellipse is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a, b are the dimensions of the ellipse.

(a) Ellipses of increasing size represent the curves $\{(x, y)|f(x, y) = c\}$, where c increases from 0 towards 1

(b) Ellipses of decreasing size represent the curves $\{(x, y)|f(x, y) = c\}$, where c increases from 0 towards 1

(c) Ellipses of constant size represent the curves $\{(x, y)|f(x, y) = c\}$, where c increases from 0 towards 1

(d) The curves $\{(x, y)|f(x, y) = c\}$, where $c \in [0, 1)$, cannot be represented by an ellipse

(e) $f(x, y) = 1$ is a point

11. Consider a point source $S = (1, 2, 3)$ radiating energy. The intensity I at a given point $P = (x, y, z)$ in space is inversely related to the square of the distance d between S and P : $I(x, y, z) = \frac{k}{d^2}$, where k is a real positive constant. Choose the correct option(s).

(a) Intensity I is constant on $\{(x, y, z)|(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 1\ \}$

(b) Intensity I is constant on $\{(x, y, z)|(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 2\}$

(c) Intensity I is constant on $\{(x, y, z)|x^2 + y^2 + z^2 = 1\ \}$

(d) Intensity I is constant on $\{(x, y, z)|x^2 + y^2 + z^2 = 2\ \}$

54 Lecture 10.2

Partial Derivatives:
Level-1:

1. What is the rate of change of $f(x,y,z) = 5x^2 - 6xy + 3z^2 - 2yz$ at (1,2,-1) with respect to z? [Answer: -10]

2. Which of the following is equivalent to the rate of change of a function $f(x,y) = x^3 + y^3 - x^2y^2$ at (1,1) with respect to y?

   (a) $\lim\limits_{h\to 0}\frac{f((1,1)+h(1,0))-f(1,1)}{h}$
   (b) $\lim\limits_{h\to 0}\frac{h^3-2h^2+h}{h}$
   (c) $\lim\limits_{h\to 0}\frac{f((1,1)+h(0,1))-f(1,1)}{h}$
   (d) 1

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = x^3 + 3xy$. Identify the correct option with respect to the partial derivatives of f.

   (a) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$
   (b) $\frac{\partial f}{\partial x}(1,1) = \frac{\partial f}{\partial y}(1,1)$
   (c) $\frac{\partial f}{\partial x}(1,0) = \frac{\partial f}{\partial y}(1,0)$
   (d) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} + 3$

4. Let $f(x,y) = \frac{xy}{x^2+y}$. Identify the correct options with respect to the partial derivatives of f.

   (a) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are not defined at $(0,0)$
   (b) $\frac{\partial f}{\partial x}(1,1) = 0, \frac{\partial f}{\partial y}(1,1) = 1/4$
   (c) $\frac{\partial f}{\partial x}(1,1) = 0, \frac{\partial f}{\partial y}(1,1) = 0$
   (d) $\frac{\partial f}{\partial x}(1,0) = 0, \frac{\partial f}{\partial y}(1,0) = 1$

   Level-2:

5. Suppose $f(x,y,z) = xy + yz + zx - 3xyz$ is a multivariable function defined on domain $D \subset \mathbb{R}^3$. Which of the following is(are) correct?

   (a) f is a scalar valued multivariable function.
   (b) The rate of change of the function f at (0,1,1) with respect to x is 1.

(c) The rate of change of the function f at (2,2,2) with respect to x is same as the rate of change of the function f at (2,2,2) with respect to z.

(d) The rate of change of the function f at (1,3,5) with respect to y is -9.

6. Suppose $f : D \longrightarrow \mathbb{R}$ is a multivariable function defined on domain $D \subset \mathbb{R}^2$ and also satisfying the following conditions:

   (i) $\frac{\partial f}{\partial x} = 6x + 4xy + 3y$

   (ii) $\frac{\partial f}{\partial y} = 2x^2 + 3x - 3y$

   Which of the following functions can be f?

   (a) $f(x, y) = 3x^2 + 2x^2y - 3xy - \frac{3}{2}y^2$

   (b) $f(x, y) = 3x^2 + 2x^2y + 3xy - \frac{3}{2}y^2$

   (c) $f(x, y) = 3x^2 - 3xy - 2x^2y + 3y^2$

   (d) $f(x, y) = 3x^2 + 3y^2 + 3xy$

7. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (P(x, y), Q(x, y))$, where $P(x, y) = 3x^2y, Q(x, y) = 5x + y^3$. Consider a matrix $A = \begin{bmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{bmatrix}$. Find the determinant of A.

   (a) $18xy^3 - 15x^2$

   (b) $18x^3y - 15x^2$

   (c) $15x^3y - 18x^2$

   (d) $15xy^3 - 18x^2$

8. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (P(x, y), Q(x, y))$, where $P(x, y) = 6x + y^2, Q(x, y) = 3x^2 + 2y$. Consider a matrix $A = \begin{bmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{bmatrix}$. If $det(A) = 0$, then identify the correct option.

   (a) $x = 0, y = 0$

   (b) $xy = 1$

   (c) $x + y = 1$

   (d) $x = 1, y = -1$

55 Lecture 10.3

Directional Derivatives:
Level-1:

1. Which of the following statements is(are) true?

   (a) The directional derivative of $f(x, y)$ at a point (a, b) in the direction of a unit vector u is a non negative real number.

   (b) The directional derivative of any function is always a scalar.

   (c) The rate of change of $f(x_1, x_2, ..., x_n)$ in the direction of a unit vector $u = (a_1, a_2, ..., a_n)$ is called the directional derivative of f in the direction of u.

   (d) The directional derivative of any function is a vector because it is dependent on the direction of a unit vector.

2. The directional derivative of $f(x, y) = x + xy - 2y^2$ at the point (2,3) in the direction of the vector $(2\sqrt{2}, 2\sqrt{2})$ is

   (a) $3\sqrt{2}$

   (b) $2\sqrt{3}$

   (c) $-3\sqrt{2}$

   (d) $-2\sqrt{3}$

3. The directional derivative of $f(x, y, z) = xe^y + ye^z + ze^x$ at point (1,0,1) in the direction of a vector (2,-2,1) is

   (a) e

   (b) $3e$

   (c) 1

   (d) $\frac{e}{3}$

4. Suppose f is a multivariable function defined on domain $\mathbb{R}^2$. Which of the following is equal to the rate of change of f at point (x_1, y_1) in the direction of a vector $u = (u_1, u_2)$?

   (a) $\lim\limits_{h \to 0} \frac{f((x_1,y_1)+h(u_1,u_2))+f(x_1,y_1)}{h}$

   (b) $\lim\limits_{h \to 0} \frac{f((x_1,y_1)+h(u_1,u_2))-f(x_1,y_1)}{h}$

   (c) $\lim\limits_{h \to 0} \frac{f((x_1,y_1)+h(\frac{u_1}{\sqrt{{u_1}^2+{u_2}^2}}, \frac{u_2}{\sqrt{{u_1}^2+{u_2}^2}}))-f(x_1,y_1)}{h}$

   (d) $\lim\limits_{h \to 0} \frac{f((x_1,y_1)+h(\frac{u_1}{\sqrt{{u_1}^2+{u_2}^2}}, \frac{u_2}{\sqrt{{u_1}^2+{u_2}^2}}))+f(x_1,y_1)}{h}$

Level-2:

5. Consider the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as:

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^4+y^2} & x, y \neq 0 \\ 0 & x = y = 0 \end{cases}$$

Find the directional derivative of f at (0,0) in the direction of the vector $u = (\frac{\sqrt{3}}{2}, \frac{1}{2})$.

(a) $\frac{3}{2}$
(b) $\frac{3}{8}$
(c) 3
(d) $\sqrt{3}$

6. Suppose d is the directional derivative of a scalar valued multivariable function $f(x,y) = x^2 + y^2$ at a point $P(1,2)$ in the direction of the line PQ, where point Q is at (3,5). The approximate value of d is

(a) 4.2
(b) 4.8
(c) 4.4
(d) 5

7. Suppose $f(x,y,z) = ax^2y + yz^2 - z^2x$, where $a \in \mathbb{R}$, is a scalar valued multivariable function defined on domain $D \subset \mathbb{R}^3$. For what value of a, does the directional derivative of the given function at the point (1,1,1) in the direction of the vector $u = (1, 2, -2)$ equal 3?
(Answer: 2)

8. The temperature at a point (x, y, z) is given by

$$T(x,y,z) = 10e^{-x^2+y^2-4z^2}$$

where T is measured in $^{\circ}C$ and x, y, z are measured in meters. What will be the rate of change of temperature at (2,-1,0) in the direction of a vector $u = (1, -1, 1)$?

(a) $\frac{20e^{-3}}{\sqrt{3}}$
(b) $\frac{-20e^{-3}}{\sqrt{3}}$
(c) $\frac{20e^{-3}}{3}$
(d) $\frac{-20e^{-3}}{3}$

56 Lecture 10.4

Limits:
Level-1:

1. Find the value of the following limit:

$$\lim_{(x,y)\to(2,3)} \frac{x^2+y^2-y}{y^2-x^2}$$

(Answer: 2)

2. Suppose f and g are multivariable functions defined on domain $\mathbb{R}^2$ with the following limits:

$$\lim_{(x,y)\to(0,0)} f(x,y) = l \text{ and } \lim_{(x,y)\to(0,0)} g(x,y) = k$$

where $l, k \in \mathbb{R}$. Which of the following options is(are) correct?

(a) $\lim\limits_{(x,y)\to(0,0)} (f(x,y)+g(x,y)) = l+k$
(b) $\lim\limits_{(x,y)\to(0,0)} (f(x,y).g(x,y)) = lk$
(c) $\lim\limits_{(x,y)\to(0,0)} (5f(x,y)) \neq 5l$
(d) $\lim\limits_{(x,y)\to(0,0)} (\frac{f(x,y)}{g(x,y)}) = \frac{l}{k}$, when $k \neq 0$

3. Consider the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as:

$$f(x,y) = \begin{cases} \frac{4x^2y^2}{x^2+y^2} & x,y \neq 0 \\ 0 & x=y=0 \end{cases}$$

What is the value of $\lim\limits_{(x,y)\to(0,0)} (f(x,y))$?

(a) 4
(b) 0
(c) $\frac{1}{4}$
(d) Limit does not exist at (0,0).

• Consider the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as:

$$f(x,y) = \begin{cases} \frac{3xy}{x^2+y^2} & x,y \neq 0 \\ 0 & x=y=0 \end{cases}$$

Use the given information to answer the questions 4-6:

4. If f approaches to L as (x,y) approaches to the origin along Y-axis, then find the value of L? [Answer: 0]

Level-2:

5. If f approaches to L as (x,y) approaches to the origin along a straight line $y = mx$, $m \in \mathbb{R}$, then find the value of L?

   (a) 0

   (b) $\frac{m}{1+m^2}$

   (c) $\frac{3m}{1+m^2}$

   (d) $\frac{3}{1+m^2}$

6. Which of the following options is true?

   (a) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$

   (b) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 1$

   (c) Limit of the given function at (0,0) exists but is not equal to 0.

   (d) Limit of the given function at (0,0) does not exist.

7. Which of the following functions have limit value equal to 5 at the point (1,1,-1)?

   (a) $f(x,y,z) = \frac{x+3yz^2-zx}{xy+yz+xz^2}$

   (b) $f(x,y,z) = \frac{3(x+y-z)+7x^2y+yz}{xyz+zy+xz}$

   (c) $f(x,y,z) = x^3 + y^3 + z^3 - 2xy - 2yz - 4zx$

   (d) $f(x,y,z) = \frac{x^2e^{\frac{x}{2}}+y^2e^{\frac{y}{2}}+z^2e^{\frac{-z}{2}}}{e^{\frac{x+y+z}{2}}}$

8. Consider the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as:

$$f(x,y) = \begin{cases} \frac{x^k y}{x^{2n}+y^{2n}} & x, y \neq 0 \\ 0 & x = y = 0 \end{cases}$$

   where $k, n \in \mathbb{N} \setminus \{0\}$. Which of the following statements is(are) true about f?

   (a) If $k = 2n - 1$, then the limit at (0,0) exists and is equal to 0.

   (b) If $k < 2n - 1$, then the limit at (0,0) does not exist.

   (c) If $k > 2n$, then the limit at (0,0) always exists and is equal to 0.

   (d) If $k > 2n$, then the limit at (0,0) does not exist.

57 Lecture 10.5

Continuity: (Level 1)

1. Let $f(x, y, z) = (xy, x + y + z^2)$, where $x, y, z \in \mathbb{R}$. Choose the set of correct options.
   (a) The domain of f is $\mathbb{R}^2$ and the co-domain of f is $\mathbb{R}^2$
   (b) The domain of f is $\mathbb{R}^3$ and the co-domain of f is $\mathbb{R}^2$
   (c) f is continuous everywhere in its domain
   (d) f is not continuous anywhere in its domain

2. Let $f(x, y) = \begin{cases} \frac{\cos y \sin x}{x} & x \neq 0 \\ \cos y & x = 0 \end{cases}$. Choose the correct options.
   (a) f is not defined at $(0, 0)$
   (b) f is defined at $(0, 0)$
   (c) f is continuous at $(0, 0)$
   (d) f is not continuous at $(0, 0)$

3. Let $h(u, v) = \sin(uv), g(x) = x^3, f(y) = \cos y$. Let $p(x, y) = h(g(x), f(y))$ Choose the correct option(s).
   (a) p is a scalar-valued function
   (b) p is a vector-valued function
   (c) p is continuous everywhere since $g.f$ is continuous everywhere in its domain
   (d) p is continuous everywhere, g is also continuous everywhere while f is not continuous everywhere in its domain

4. Let $p(t) = (ln(1 - t), \frac{1}{t}, 3t)$. Choose the correct option(s). (MSQ Ans: a)
   (a) p is continuous for all real t less than 1 but not equal to zero
   (b) p is continuous on $\mathbb{R}$
   (c) p is continuous in $\{t \in \mathbb{R} | t < 1\}$
   (d) p is continuous in $\{t \in \mathbb{R} | t \neq 1, t \neq 0\}$

5. Let $f(x, y) = ln(x^2 + y^2 - 1)$. Identify where f is continuous. Note: Equation of a circle with radius r and center (a, b) is given by $(x - a)^2 + (y - b)^2 = r^2$
   (a) Everywhere in $\mathbb{R}^2$
   (b) Everywhere in $\mathbb{R}^2 \setminus \{(1, 0), (0, 1)\}$
   (c) Everywhere in $\mathbb{R}^2$ outside the circle with radius 1 and center $(0, 0)$
   (d) None of the above

Level2

6. Let $g(u,v,w) = \frac{uv}{w}$ and $f(x,y,z) = g(P(x,y,z), Q(x,y,z), R(x,y,z))$ where $P(x,y,z) = e^{x^2+y}, Q(x,y,z) = \sqrt{y^2+z^2+3}, R(x,y,z) = \sin(xyz)+5$. Choose the correct option(s).

   (a) f is a vector-valued function

   (b) f is a scalar-valued function

   (c) f is continuous everywhere in its domain

   (d) f is continuous everywhere in its domain. P is also continuous everywhere in its domain, while Q is not continuous everywhere in its domain

7. Let $f : \mathbb{R}^2 \to \mathbb{R}, f(x,y) = \begin{cases} \frac{x^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$. Choose the correct option(s).

   (a) $\lim_{(x,y)\to(0,0)} f$ does not exist

   (b) $\lim_{(x,y)\to(0,0)} f$ exists

   (c) f is continuous at $(0,0)$

   (d) f is not continuous at $(0,0)$

8. Let $f : \mathbb{R}^2 \to \mathbb{R}, f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$. Choose the correct option(s).

   (a) $\lim_{(x,y)\to(0,0)} f$ does not exist

   (b) $\lim_{(x,y)\to(0,0)} f$ exists

   (c) f is continuous at $(0,0)$

   (d) f is not continuous at $(0,0)$

9. Let $f(r) = \frac{1}{<r,r>-1}$, where $r = (x,y,z)$ and the inner product $<,>$ is the dot product. Choose the correct option.

   (a) f is continuous throughout $\mathbb{R}^3$

   (b) f is continuous throughout $\mathbb{R}^3 \setminus \{(0,0,0)\}$

   (c) f is discontinuous on the unit sphere $\{r \in \mathbb{R}^3 : \|r\| = 1\}$

   (d) f is continuous throughout $\mathbb{R}^3 \setminus \{(1,0,0)\}$

   (e) None of the above

58 Lecture 10.6

Gradients

Level1

1. Consider a function $f : \mathbb{R}^3 \to \mathbb{R}$. Choose the correct statement with respect to ∇f.
   (a) ∇f has co-domain $\mathbb{R}^2$
   (b) ∇f has co-domain $\mathbb{R}$
   (c) ∇f has co-domain $\mathbb{R}^3$
   (d) None of the above

2. Let $f : \mathbb{R}^3 \to \mathbb{R}$ and suppose its gradient ∇f is continuous. Which of the following statements is true for the directional derivative of f in the direction of $a \in \mathbb{R}^3$? Consider dot product to be the inner product $<,>$.
   (a) $D_a f = \frac{1}{\|a\|} < a, \nabla f >$
   (b) $D_a f =< a, \nabla f >$
   (c) $D_a f = \frac{1}{2\|a\|} < a, \nabla f >$
   (d) $D_a f = \frac{1}{\|a\|^2} < a, \nabla f >$

3. Let $f(x, y) = e^x \sin y$. Find ∇f at the point $(ln(2), \pi/4)$.
   (a) $(\sqrt{2}, \sqrt{2})$
   (b) $(\frac{1}{2}, \frac{1}{2})$
   (c) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
   (d) $(\frac{e}{\sqrt{2}}, \frac{e}{\sqrt{2}})$

Level2

4. Let $p = -\frac{k}{(\|r\|)^3} r$, where $r = (x, y, z) \in \mathbb{R}^3 \setminus (0,0,0), k \in \mathbb{R}$. Amongst the options below, choose those for which the function f satisfies $\nabla f = p$ for some $k \in \mathbb{R}$. Consider dot product as the inner product for finding the norm.

   (a) $f(x, y, z) = \frac{1}{\|r\|}$
   (b) $f(x, y, z) = \frac{1}{2\|r\|}$
   (c) $f(x, y, z) = \frac{1}{\|r\|+2}$
   (d) $f(x, y, z) = \|r\|$

5. Let $f(x, y, z) = 4(x^2 + y^2) - z^2$. Choose the correct option(s).

   (a) ∇f at $(1, 0, 2)$ is $(8, 0, -4)$
   (b) ∇f is continuous everywhere in $\mathbb{R}^3$
   (c) ∇f is not continuous everywhere in $\mathbb{R}^3$
   (d) ∇f at $(1, 0, 2)$ is $(-8, 0, -4)$

6. Consider a function defined as $f(x, y) = \begin{cases} \frac{y^3}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$. Choose the correct option(s).

   (a) $\frac{\partial f}{\partial x} = \frac{-2xy^3}{(x^2+y^2)^2}, \frac{\partial f}{\partial y} = \frac{3x^2y^2+y^4}{(x^2+y^2)^2}$ for $(x, y) \neq (0, 0)$
   (b) $\frac{\partial f}{\partial x} = \frac{2xy^3}{(x^2+y^2)^2}, \frac{\partial f}{\partial y} = \frac{3x^2y^2+y^4}{(x^2+y^2)^2}$ for $(x, y) \neq (0, 0)$
   (c) $\frac{\partial f}{\partial x} = \frac{2x^3y}{(x^2+y^2)^2}, \frac{\partial f}{\partial y} = \frac{3x^2y^2+y^4}{(x^2+y^2)^2}$ for $(x, y) \neq (0, 0)$
   (d) $\frac{\partial f}{\partial x} = \frac{2xy^3}{(x^2+y^2)^2}, \frac{\partial f}{\partial y} = \frac{3x^2y^2-y^4}{(x^2+y^2)^2}$ for $(x, y) \neq (0, 0)$

7. Consider the function definition given in Q6 above. Choose the correct option(s).

   (a) $\frac{\partial f}{\partial x}$ is continuous at $(0, 0)$
   (b) $\frac{\partial f}{\partial x}$ is not continuous at $(0, 0)$
   (c) $\frac{\partial f}{\partial y}$ is not continuous at $(0, 0)$
   (d) ∇f is discontinuous at the origin $(0, 0)$
   (e) Directional derivative of f in the direction of a vector $a \in \mathbb{R}^2$ at $(0, 0)$ can be obtained using the gradient of f

8. Consider a function defined as $f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\frac{1}{\sqrt{x^2+y^2}}\right) & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$.

   Choose the correct option(s).

   (a) $\frac{\partial f}{\partial x}(0,0) = 0, \frac{\partial f}{\partial y}(0,0) = 0$

   (b) $\frac{\partial f}{\partial x} = 0$

   (c) $\frac{\partial f}{\partial y} = 0$

   (d) None of the above

9. Consider the function definition given in Q8 above. Choose the correct option(s) with respect to the partial derivative of f. Note $sign(x) = \pm 1$ depending on the sign of x.

   (a) $\frac{\partial f}{\partial x}(x,0) = 2x\sin(\frac{1}{|x|}) - sign(x)\cos(\frac{1}{|x|})$ for $x \neq 0$

   (b) $\frac{\partial f}{\partial x}(x,0) = 2x\sin(\frac{1}{|x|}) - sign(x)\cos(\frac{1}{|x|})$

   (c) $\frac{\partial f}{\partial x}(x,0) = 2x\sin(\frac{1}{|x|}) - \cos(\frac{1}{|x|})$ for $x \neq 0$

   (d) $\frac{\partial f}{\partial y}(0,y) = 2y\sin(\frac{1}{|y|}) - sign(y)\cos(\frac{1}{|y|})$ for $y \neq 0$

   (e) $\frac{\partial f}{\partial y}(0,y) = 2y\sin(\frac{1}{|y|}) - sign(y)\cos(\frac{1}{|y|})$

   (f) $\frac{\partial f}{\partial y}(0,y) = 2y\sin(\frac{1}{|y|}) - \cos(\frac{1}{|y|})$ for $y \neq 0$

10. Consider the function definition given in Q8 above. Choose the correct option.

   (a) ∇f is continuous everywhere

   (b) ∇f is discontinuous at the origin $(0,0)$

   (c) $\frac{\partial f}{\partial x}$ is continuous at the origin but $\frac{\partial f}{\partial y}$ is discontinuous at the origin

   (d) $\frac{\partial f}{\partial y}$ is continuous at the origin but $\frac{\partial f}{\partial x}$ is discontinuous at the origin

   (e) Directional derivative of f in the direction of a vector $a \in \mathbb{R}^2$ at $(0,0)$ can be obtained using the gradient of f

Week 11

AQ 11.1 : The directional of steepest ascent/descent

1) Consider a function $f:\mathbb{R}^2 \to \mathbb{R}$ defined as $f(x,y) = x^2 + 2xy^2 + 2x^2y + y^2$. Which of the following options is true?

1 point

- (selected) $\nabla f = (2x + 2y^2 + 4xy, 4xy + 2x^2 + 2y)$.
- $\nabla f = (2x + 2y + 4xy, 4xy + 2x^2 + 2y)$.
- $\nabla f = (2x + 2xy^2 + 2xy, 4xy + 2x^2 + 2y)$.
- $\nabla f = (2x + 2y^2 + 4xy, 2xy + 2x^2y + 2y)$

Yes, the answer is correct.

Score: 1

Accepted Answers:

$\nabla f = (2x + 2y^2 + 4xy, 4xy + 2x^2 + 2y)$.

2) Consider the following function defined as

1 point

$$f_1(x,y) = \begin{cases} \frac{x}{y} & \text{if} y \neq 0 \\ 1 & \text{if y} = 0. \end{cases}$$

$$f_2(x,y) = xy + x^2y^2$$

Which of the following option(s) is (are) true?

- f_1 has continuous gradient at the origin.
- (selected) f_2 has continuous gradient at the origin.
- $f_1(x,y)$ and $f_2(x,y)$ have continuous gradients at the origin.
- None of the above

Yes, the answer is correct.

Score: 1

Accepted Answers:

f_2 has continuous gradient at the origin.

3) Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f(x,y) = e^{(x^2+xy+y^3)}$.

1 point

Which of the following option(s) is (are) true?

- [ ] $\nabla f = \left(e^{(x^2+xy+y^3)}, e^{(x^2+xy+y^3)}\right)$
- [ ] $\nabla f = (2x+y, 3y^2+x)$
- [ ] $\nabla f = \left(2xe^{(x^2+xy+y^3)}, 3y^2e^{(x^2+xy+y^3)}\right)$
- [x] $f(x,y)$ increases most rapidly in the direction of $\frac{1}{5}(3,4)$ at the point (1,1).
- [x] $f(x,y)$ decreases most rapidly in the direction of $\frac{1}{\sqrt{17}}(-1,-4)$ at the point (1,-1).

Yes, the answer is correct.
Score: 1

Accepted Answers:

$f(x,y)$ increases most rapidly in the direction of $\frac{1}{5}(3,4)$ at the point (1,1).
$f(x,y)$ decreases most rapidly in the direction of $\frac{1}{\sqrt{17}}(-1,-4)$ at the point (1,-1).

Level 2:

With a particular frame of reference (in $\mathbb{R}^2$), a corner of a hot rectangular iron plate with some finite length and breadth is placed at the origin $(0,0)$. The temperature of the plate at a point (x,y) is given by the function:

$$T(x,y) = xy^2 + x^2y$$

Use this information to answer the following questions

4) At the point (1,3), in which direction is the rate of change in temperature the minimum?

1 point

- (•) $\frac{1}{\sqrt{274}}$(-15, -7)
- ( ) $\frac{1}{\sqrt{82}}$(9, 1)
- ( ) $\frac{1}{\sqrt{82}}$(-9, -1)
- ( ) $\frac{1}{\sqrt{274}}$(15, 7)

Yes, the answer is correct.
Score: 1

Accepted Answers:

$\frac{1}{\sqrt{274}}$(-15, -7)

5) At the point (1,3), in which direction is the rate of change in temperature of the iron plate the maximum? ***1 point***

- ( ) $\frac{1}{\sqrt{274}}$(-15, -7)
- ( ) $\frac{1}{\sqrt{82}}$(9, 1)
- ( ) $\frac{1}{\sqrt{82}}$(-9, -1)
- (•) $\frac{1}{\sqrt{274}}$(15, 7)

Yes, the answer is correct.

Score: 1

Accepted Answers:

$\frac{1}{\sqrt{274}}$(15, 7)

6) From a point (1,3), in which direction the temperature of the iron plate is maintained at the same value? ***1 point***

- [x] $\frac{1}{\sqrt{274}}$(-7, 15)
- [ ] $\frac{1}{\sqrt{82}}$(1, -9)
- [ ] $\frac{1}{\sqrt{82}}$(-1, 9)
- [x] $\frac{1}{\sqrt{274}}$(7, -15)

Yes, the answer is correct.

Score: 1

Accepted Answers:

$\frac{1}{\sqrt{274}}$(-7, 15)

$\frac{1}{\sqrt{274}}$(7, -15)

7) Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f(x, y, z) = 2e^{2x} \sin yz$. At the point $(0, \frac{\pi}{2}, 1)$, which are the directions that have the directional derivative to be 0 ? **1 point**

- [x] $\frac{1}{\sqrt{2}}$ (0,1,1)
- [x] $\frac{1}{\sqrt{2}}$ (0,1,-1)
- [ ] $\frac{1}{\sqrt{3}}$ (1,1,1)
- [x] $\frac{1}{2021\sqrt{2}}$ (0,2021,-2021)

Yes, the answer is correct.
Score: 1

Accepted Answers:

$\frac{1}{\sqrt{2}}$ (0,1,1)
$\frac{1}{\sqrt{2}}$ (0,1,-1)
$\frac{1}{2021\sqrt{2}}$ (0,2021,-2021)

8) Consider a function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x, y, z) = \alpha x + \beta xy^2 + \gamma yz^2$. If the maximum value of the directional derivative of $f(x, y, z)$ at (1, 1, 1) is in the direction parallel to $X-$ axis and has magnitude 5, then f can be **1 point**

- ○ $f(x, y, z) = 5x + x^2$
- ○ $f(x, y, z) = 2x + xy^2$
- ◉ $f(x, y, z) = 5x$
- ○ $f(x, y, z) = xy^2 + 2yz^2$
- ○ $f(x, y, z) = 2x$

Yes, the answer is correct.
Score: 1

Accepted Answers:

$f(x, y, z) = 5x$

AQ 11.2 : Tangents for scaler-valued multivariable functions

1) Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as: **1 point**

$$f(x,y) = \begin{cases} x^2 \sin\frac{1}{x} + y & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Which of the following option is true?

○ f_y is continuous at the origin.

○ f_x is continuous at the origin.

○ Both f_x and f_y are continuous at the origin.

◉ None of the above.

Yes, the answer is correct.
Score: 1

Accepted Answers:

None of the above.

2) A function $f : \mathbb{R}^2 \to \mathbb{R}$ satisfies that $f_x(x,y) = c$ and $f_y(x,y) = k$ of all points of $\mathbb{R}^2$, where c and k are some constants, then which of the following option(s) is (are) true? **1 point**

☑ The tangent line exists at any point of $\mathbb{R}^2$ in any direction.

☐ The tangent lines at the origin do not lie on a plane.

☐ There exist points in $\mathbb{R}^2$ for which tangent points in all directions do not exist.

☑ $f(x,y)$ can be $f(x,y) = cx + ky + \alpha$, where α is any real number.

Yes, the answer is correct.
Score: 1

Accepted Answers:

The tangent line exists at any point of R^2 in any direction.
$f(x,y)$ can be $f(x,y) = cx + ky + \alpha$, where α is any real number.

3) Consider $f : \mathbb{R}^2 \to \mathbb{R}$ and suppose partial derivatives of $f(x,y)$ with respect to x and y are continuous at a point (a,b). Then which of the following option(s) is (are) true? **1 point**

☑ $f_x(a,b) = \lim\limits_{(x,y)\to(a,b)} f_x(x,y)$

☑ $f_y(a,b) = \lim\limits_{(x,y)\to(a,b)} f_y(x,y)$

☐ $f_x(a,b) \neq \lim\limits_{(x,y)\to(a,b)} f_x(x,y)$.

☐ $f_y(a,b) \neq \lim\limits_{(x,y)\to(a,b)} f_y(x,y)$

Yes, the answer is correct.
Score: 1

4) Consider a function $f(x, y) = x^2 + y^2$. Which of following is the tangent at the point (1,2) in the direction of (1, 1)? **1 point**

◉ $x(t) = 1 + \frac{t}{\sqrt{2}}, y(t) = 2 + \frac{t}{\sqrt{2}}, z(t) = 5 + \frac{6t}{\sqrt{2}}$

○ $x = 1 + t, y(t) = 2 + t, z(t) = 5 + 6t$

○ $x = 1, y(t) = 2 + t, z(t) = 5 + \frac{6t}{\sqrt{2}}$

○ $x(t) = 1 + \frac{t}{\sqrt{2}}, y(t) = 2 + \frac{t}{\sqrt{2}}, z(t) = 4 + \frac{6t}{\sqrt{2}}$

Yes, the answer is correct.
Score: 1

Accepted Answers:

Level 2:

5) Consider a function **1 point**

$$f(x, y) = \begin{cases} \frac{x}{y} & \text{if } y \neq 0 \\ 0 & \text{if } y = 0. \end{cases}.$$

Which of the following is the tangent line passing through the point (1, -1) and parallel to the line $x(t) = t, y(t) = 1 + 2t, z(t) = -\frac{3}{\sqrt{5}}t$?

○ $x(t) = 1 + \frac{t}{\sqrt{5}}, y(t) = -1 + \frac{2t}{\sqrt{5}}, z(t) = -\frac{3}{\sqrt{5}}t$

○ $x(t) = 1 + t, y(t) = 1 + 2t, z(t) = -1 - \frac{3}{\sqrt{5}}t$

◉ $x(t) = 1 + \frac{t}{\sqrt{5}}, y(t) = -1 + \frac{2t}{\sqrt{5}}, z(t) = -1 - \frac{3}{\sqrt{5}}t$

○ $x(t) = -1 + t, y(t) = -1 + 2t, z(t) = -1 - \frac{3}{\sqrt{5}}t$

Yes, the answer is correct.
Score: 1

Accepted Answers:

6) Consider a function $f(x, y, z) = 2x^2 + xy + z^2$. Which of following is the tangent at the point (1,1,1) in the direction of (1, 0, 0) ***1 point***

○ $(1, 1, 1, 5 + 5t)$

○ $(1 + t, 1, 1, 1 + t)$

○ $(1, 1 + t, 1 + t, 5 + 5t)$

◉ $(1 + t, 1, 1, 4 + 5t)$

Yes, the answer is correct.
Score: 1

Accepted Answers:

7) Consider the following functions f defined as: ***1 point***

$$f(x, y, z) = xyz.$$

Then the tangent line at the point (1,0,1) in the direction of (1,1,1) is parallel to the line

○ $x(t) = 2t, y(t) = 2 + 2t, z(t) = 3 + 2t, u(t) = \frac{2}{\sqrt{3}}t$

○ $x(t) = 2t, y(t) = 2 + 1t, z(t) = 3 + 2t, u(t) = \frac{2}{\sqrt{3}}t$

○ $x(t) = t, y(t) = 2 + t, z(t) = 3 + t, u(t) = \frac{2}{\sqrt{3}}t$

◉ None of the above.

Yes, the answer is correct.
Score: 1

AQ 11.3 : Finding the tangent hyper (plane)

1) Consider the following functions. ***1 point***

$f_1(x,y) = 5\sqrt{x^2+y^2}$

$f_2(x,y,z) = x^2z^2 + yz + x^2y^2z^2$

$f_3(x,y) = \begin{cases} y\sin\frac{1}{y} + xy & if\ y \neq 0 \\ 0 & if\ y = 0 \end{cases}$

Which of the following options is true?

- ◯ Tangent plane exists at the origin for f_1.
- ◉ Tangent plane exists at the origin for f_2.
- ◯ Tangent plane exists at the origin for f_3.
- ◯ None of the above.

Yes, the answer is correct.

Score: 1

Accepted Answers:

Tangent plane exists at the origin for f_2.

2) Which of the following equations represents the tangent plane to the function $f(x, y) = 3x^2 + xy - 2y^2$ at the point (1,-1) ***1 point***

○ $5x + 5y = 0.$

◉ $5x + 5y - z = 0.$

○ $x + y - z = 0.$

○ $5x + 5y + z = 0.$

Yes, the answer is correct.
Score: 1

Accepted Answers:

$5x + 5y - z = 0.$

---

3) Consider a function $f(x, y) = \frac{1}{2}(xy - \frac{x}{y})$. Let T be the tangent plane at point (1,2). Which of the following points lie on the tangent plane T? ***1 point***

☑ (0,0,$-\frac{5}{4}$)

☐ (1,0,$\frac{1}{4}$)

☑ (2, -4, $-\frac{9}{4}$)

☐ (1, 2, -2)

Yes, the answer is correct.
Score: 1

Accepted Answers:

4) Consider a function $f(x, y) = xy$. Let T_1 be the tangent plane at point (1,1), T_2 be the tangent plane at point (1,-1). Then which of the following option is true? *1 point*

- ○ T_1, T_2 intersect in only one point.
- ◉ T_1, T_2 intersect in infinitely many points.
- ○ T_1, T_2 do not intersect.
- ○ None of the above

Yes, the answer is correct.

Score: 1

Accepted Answers:

T_1, T_2 intersect in infinitely many points.

5) Let f be a function and T be the tangent plane at a point. Then which of the following option(s) is (are) true. *1 point*

- [x] T is an affine subspace of the vector space $\mathbb{R}^3$ with respect to usual addition and scalar multiplication.
- [ ] T is never an affine subspace of the vector space $\mathbb{R}^3$ with respect to usual addition and scalar multiplication.
- [x] T can be a subspace of the vector space $\mathbb{R}^3$ with respect to usual addition and scalar multiplication.
- [ ] None of the above.

Yes, the answer is correct.

Score: 1

Accepted Answers:

T is an affine subspace of the vector space R^3 with respect to usual addition and scalar multiplication.

T can be a subspace of the vector space R^3 with respect to usual addition and scalar multiplication.

6) Consider a function $f(x, y, z) = e^{x+y} \sin 2z$. Which of the followings is the tangent (hyper) plane at the point $(1, 0, \frac{\pi}{4})$? ***1 point***

- (selected) $u = ex + ey$
- $u = ex + ey + z$
- $u = ex + ey + ez$
- None of the above.

Yes, the answer is correct.
Score: 1

Accepted Answers:

$u = ex + ey$

7) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a function and L_f be the linear approximation of f at point (a, b). Which of the following options is true? ***1 point***

- (selected) L_f is a linear function.
- L_f is a quadratic function.
- L_f is a logarithmic function.
- None of the above.

Yes, the answer is correct.
Score: 1

Accepted Answers:

L_f is a linear function.

8) Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f(x,y) = xy\tan(x)\sin^{-1}(y + \frac{\sqrt{3}}{2}) + 1$. Which of the following is the linear approximation of f at the point $(\frac{\pi}{4}, 0)$

1 point

○ $L_f(x,y) = x + \frac{\pi^2}{12}y$

◉ $L_f(x,y) = 1 + \frac{\pi^2}{12}y$

○ $L_f(x,y) = 1 + y$

○ $L_f(x,y) = \frac{\pi^2}{12}y$

Yes, the answer is correct.

Score: 1

Accepted Answers:

$L_f(x,y) = 1 + \frac{\pi^2}{12}y$

9) Consider the three functions $f_1 = x^2 + y^2$, $f_2 = xy$ and $f_3 = x + y$. Let T_1, T_2 and T_3 be the tangent planes at points $(-\frac{1}{2}, \frac{1}{2})$ of f_1, (-1, 1,) of f_2 and (0,0) of f_3 respectively. If (a, b, c) is the intersection point of the T_1, T_2 and T_3, then find the value of $a + 2b + c$.

1

Yes, the answer is correct.

Score: 1

Accepted Answers:

(Type: Numeric) 1

1 point

10) Which of the following is the linear approximation of the function $f(x,y) = e^x \sin y + xe^x$ at the point (0, 0)

1 point

○ $L_f(0,0) = 2x + y$

○ $L_f(0,0) = x$

○ $L_f(0,0) = 2x + 2y$

◉ $L_f(0,0) = x + y$

Yes, the answer is correct.

Score: 1

Accepted Answers:

$L_f(0,0) = x + y$

11) Production of number of breads depends upon the quantity of raw material which includes egg, wheat and sugar . x denotes the number of eggs, y denotes the quantity of wheat (in kg) and z denotes the quantity of sugar (in kg) and defined as $f(x, y, z) = xyz$. Quality of the bread is measured by the value of linear approximation $(L_f(x, y, z))$ with respect to 1 egg, 1 kg wheat, and 1 kg sugar. If value of the linear approximation is greater than 10, then quality of product is not good and if less than 10 then quality of the product is good. ***1 point***

Use this information to answer question.

Which of the following option(s) is (are) true?

- [x] If 2 eggs, 5 units wheat, 3 units sugar is used to produce breads, then the quality of produced breads
- [x] If 5 eggs, 10 units wheat, 3 units sugar is used to produce breads, then the quality of produced breads is not good.
- [ ] If 7 eggs, 8 units wheat, 9 units sugar is used to produce breads, then the quality of produced breads is good.
- [ ] If 5 eggs, 3 units wheat, 3 units sugar is used to produce breads, then the quality of produced breads is not good.

Yes, the answer is correct.

Score: 1

Accepted Answers:

If 2 eggs, 5 units wheat, 3 units sugar is used to produce breads, then the quality of produced breads

If 5 eggs, 10 units wheat, 3 units sugar is used to produce breads, then the quality of produced breads is not good.

12) Consider a functions $f : \mathbb{R}^3 \to \mathbb{R}$ defined as: $f(x, y, z) = x^2 + y^2 + z^2$. If $L_f(x, y, z)$ is the linear approximation of $f(x, y, z)$ at point (1, 1, 0), then find the value of $\nabla L_f(x, y, z) \cdot (3, 2, 1)$.

10

Yes, the answer is correct.

Score: 1

Accepted Answers:

(Type: Numeric) 10

1 point

13) Consider a function $f(x, y, z) = ax + by + cz$, where $a, b, c \in \mathbb{R}$. If $L_f(x, y, z)$ is the linear approximation of $f(x, y, z)$ at the origin, then which of the following options is true? ***1 point***

- ○ $L_f(x, y, z) = 2f(x, y, z)$
- ○ $L_f(x, y, z) = -f(x, y, z)$
- ◉ $L_f(x, y, z) = f(x, y, z)$
- ○ $L_f(x, y, z) = 0$

Yes, the answer is correct.

Score: 1

Accepted Answers:

$L_f(x, y, z) = f(x, y, z)$

AQ 11.4 Critical points for multivariable functions

1) Let $f(x, y)$ be defined in a neighbourhood of (a, b). When is (a, b) a critical point of f? **_1 point_**

☐ $f(a, b) = 0$.

☐ $f_x(a, b) = 0$.

☑ $f_x(a, b) = 0 = f_y(a, b)$.

☑ f_x does not exist.

Yes, the answer is correct.

Score: 1

Accepted Answers:

$f_x(a, b) = 0 = f_y(a, b)$.

f_x does not exist.

2) Consider $f(x, y) = \sqrt{5x^2 + 3y + 2}$. **_1 point_**

☑ Any point on the y-axis in the domain of definition of f satisfies $f_x = 0$.

☐ Any point on the line $x + y = 1$ in the domain of definition of f satisfies $f_y = 0$.

☑ There is no (x, y) such that $f_y = 0$.

☑ Solutions of $5x^2 + 3y + 2 = 0$ are critical points of f.

Yes, the answer is correct.

Score: 1

Accepted Answers:

Any point on the y-axis in the domain of definition of f satisfies $f_x = 0$.

There is no (x, y) such that $f_y = 0$.

3) A critical point

1 point

☐ is a point of maximum.

☑ may be a saddle point.

☑ may be a point of local minimum.

☑ can be obtained by equating f_x and f_y to 0.

Yes, the answer is correct.

Score: 1

Accepted Answers:

may be a saddle point.
may be a point of local minimum.
can be obtained by equating f_x and f_y to 0.

4) Let $f(x, y) = x^2 y$. Then

1 point

☑ $(0, 0)$ is a critical point.

☑ $(0, y)$ for $y \in \mathbb{R}$ are critical points.

☐ $f_x > 0$ at the critical points.

☑ $f_y = 0$ at the critical points.

Yes, the answer is correct.

Score: 1

Accepted Answers:

(0, 0) is a critical point.

5) For $f(x, y) = x^3 + 2xy - 2x - 4y$, which of the following is/are true? **1 point**

☑ $(2, -5)$ is a critical point.

☐ $(2, 5)$ is a critical point.

☑ $f_y = 0$ is a straight line parallel to the y-axis.

☑ The system of equations $f_x = 0$, $f_y = 0$ has a unique solution.

Yes, the answer is correct.

Score: 1

Accepted Answers:

$(2, -5)$ is a critical point.

$f_y = 0$ is a straight line parallel to the y-axis.

The system of equations $f_x = 0, f_y = 0$ has a unique solution.

6) Choose the correct statement(s) about the function $f(x, y) = xy^2 + 2x$. **1 point**

☐ $f_x = 0$ only for 2 values of x.

☑ $f_y = 0$ along both x and y axes.

☑ There are no critical points of f.

☐ The point $(0, 0)$ is a point of local minimum.

Yes, the answer is correct.

Score: 1

Accepted Answers:

$f_y = 0$ along both x and y axes.

There are no critical points of f.

7) For $f(x,y) = \frac{x^3}{3} + y^2 + 2xy - 6x - 3y + 4$ **1 point**

- [ ] $f_y = 0$ is a straight line passing through the origin.
- [x] $(-1, \frac{5}{2})$ and $(3, -\frac{3}{2})$ are critical points.
- [x] $f_x(-1, \frac{5}{2}) = 0$.
- [ ] $f_y(3, -\frac{3}{2}) = 2$.

Yes, the answer is correct.

Score: 1

Accepted Answers:

$(-1, \frac{5}{2})$ and $(3, -\frac{3}{2})$ are critical points.

$f_x(-1, \frac{5}{2}) = 0$.

8) Choose the correct statement(s) about $f(x,y) = x^4 + y^4$. **1 point**

- [ ] There are no critical points.
- [x] There is exactly one critical point.
- [x] $(0,0)$ is a point of global minimum.
- [x] $(0,0)$ is a point of local minimum.

Yes, the answer is correct.

Score: 1

Accepted Answers:

9) Let $f(x,y) = x^2 - 2xy + 4y^2 - 4x - 2y + 2$. Then **1 point**

- [x] $(3,1)$ is a critical point.
- [ ] $(2,1)$ is a critical point.
- [x] Value of f at $(2,1)$ is -4.
- [ ] Value of f at $(3,1)$ is -9.

Yes, the answer is correct.
Score: 1

Accepted Answers:

(3, 1) is a critical point.
Value of f at (2, 1) is -4.

10) Let $f(x,y) = x^2 - 2xy + 4y^2 - 4x - 2y + 2$ be defined on the box $0 \leq x \leq 2$ and $0 \leq y \leq 1$. Then which of the following options is (are) true? **1 point**

- [x] Along the line joining $(0,0)$ and $(2,0)$, the minimum is attained at $(2,0)$.
- [x] $(0,1)$ is a point of extremum along the line joining $(0,1)$ and $(2,1)$.
- [ ] $f(3, -\sqrt{5}, 1) = 1$.
- [ ] None of the above.

Yes, the answer is correct.
Score: 1

Accepted Answers:

Along the line joining (0, 0) and (2, 0), the minimum is attained at (2, 0).
(0, 1) is a point of extremum along the line joining (0, 1) and (2, 1).

11) Let $f(x, y) = sin\, x\, cos\, y$. Then

1 point

☑ $((2k+1)\frac{\pi}{2}, k\pi)$ for $k \in \mathbb{N}$ are critical points of f.

☐ $((2k+1)\frac{\pi}{2}, (2k+1)\frac{\pi}{2})$ for $k \in \mathbb{N}$ are critical points of f.

☐ $(k\pi, k\pi)$ for $k \in \mathbb{N}$ are critical points of f.

☑ $(k\pi, (2k+1)\frac{\pi}{2})$ for $k \in \mathbb{N}$ are critical points of f.

Yes, the answer is correct.
Score: 1

Accepted Answers:

$((2k+1)\frac{\pi}{2}, k\pi)$ for $k \in \mathbb{N}$ are critical points of f.

$(k\pi, (2k+1)\frac{\pi}{2})$ for $k \in \mathbb{N}$ are critical points of f.

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12) Let $f(x, y) = e^{-x^2+y-\frac{y^3}{3}}$. Then

1 point

☑ $f_x(1, 0) = -\frac{2}{e}$.

☐ $f_x(0, 0) = 1$.

☑ $f_y(0, -1) = 0$.

☐ $f_y(1, 0) = e$.

Yes, the answer is correct.
Score: 1

Accepted Answers:

13) Let $f(x,y) = e^{-x^2+y-\frac{y^3}{3}}$. Then

1 point

- [x] $(0,1)$ is a critical point.
- [x] $(0,-1)$ is a critical point.
- [ ] $(1,1)$ is a critical point.
- [ ] $(1,-1)$ is a critical point.

Yes, the answer is correct.
Score: 1

Accepted Answers:

$(0,1)$ is a critical point.
$(0,-1)$ is a critical point.

14) Find three positive numbers whose sum is 12 such that the sum of their squares is minimum. The mathematical expression that best defines this problem is

1 point

- (●) minimize $x^2+y^2+z^2$ such that $x+y+z=12, x,y,z \geq 0$.
- ( ) minimize $x+y+z$ such that $x^2+y^2+z^2=12, x,y,z \geq 0$.
- ( ) maximize $x^2+y^2+z^2$ such that $x+y+z=12, x,y,z \geq 0$.
- ( ) maximize $x+y+z$ such that $x^2+y^2+z^2=12, x,y,z \geq 0$.

Yes, the answer is correct.
Score: 1

Accepted Answers: