Week 12 Practise Assignment

1) Let $f(x, y)$ be defined in a neighbourhood of (a, b) and $D(x, y) = f_{xx}f_{yy} - f_{xy}^2$. Suppose (a, b) is a critical point of f. ***1 point***
Which of the following options is(are) true?

- [x] If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a point of local minimum.
- [ ] If $D(a, b) < 0$, then (a, b) is a point of local maximum.
- [x] If $D(a, b) = 0$, then (a, b) may be a point of minimum or maximum or neither.
- [x] If $D(a, b) < 0$, then (a, b) is a saddle point.

2) Choose the correct statements about $f(x, y) = x^4 + y^4$, where $D(x, y) = f_{xx}f_{yy} - f_{xy}^2$. ***1 point***

- [x] $(0, 0)$ is the only critical point.
- [ ] $D(0, 0) > 0$ and $f_{xx}(0, 0) < 0$ and so $(0, 0)$ is a point of local maximum.
- [x] $D(0, 0) = 0$.
- [x] $(0, 0)$ is a point of local minimum.

3) Let $f(x, y) = (9x^2 - 1)(1 + 4y)$ be defined on the box $-2 \leq x \leq 3$ and $-1 \leq y \leq 4$. Then which of the following options is (are) true? ***1 point***

- ○ $(0, 1)$ is a point of local minimum.
- ◉ $(3, 4)$ is a point of absolute maximum.
- ○ -105 is the absolute minimum.
- ○ $(-2, -1)$ is a point of absolute minimum.

4) Suppose $z = f(x) + yg(x)$ for some polynomial functions f and g. Which of the following options is true? *1 point*

- (selected) $z_{yy} = 0$
- z_{yy} is a non-constant function of x.
- z_{yy} is a non-constant function of y.
- $z_{yy} = 1$

5) Define a function *1 point*

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if}(x,y) \neq (0,0) \\ 0 & \text{if}(x,y) = (0,0) \end{cases}$$

Choose the set of correct options.

- [x] $f_{xy}(0,0) = -1$
- [x] $f_{yx}(0,0) = 1$
- [ ] $f_{xy}(0,0) = f_{yx}(0,0)$
- [x] $f_x(0,0) = f_y(0,0)$

6) Consider a function $f(x,y,z) = x^3y + yz$. If A is the the Hessian matrix of $f(x,y,z)$ at (1,1,0), then which of the following options is(are) true? *1 point*

- [ ] $det(A) = 0$
- [ ] $Rank(A) = 2$
- [ ] $Nullity(A) = 1$
- [x] $det(A) = -6$
- [x] $Rank(A) = 3$

7) Let $f : \mathbb{R}^6 \to \mathbb{R}$ be a polynomial function. Which of the following options is true? *1 point*

- (selected) Hessian matrix of the function is a symmetric matrix of order 6.
- All possible mixed partial derivatives of second order have the same value at a point of $\mathbb{R}^6$
- Hessian matrix of the function is a symmetric matrix of order 5.
- Hessian matrix of the function is a symmetric matrix of order 4.

Let S be a rectangular box. Let x, y and z be the length of the sides of the box and volume of the box $V = xyz$. If d is the length of the diagonal of the rectangular box, then $d = \sqrt{x^2 + y^2 + z^2}$.
Use this information to answer the questions 8 and 9.

8) If the diagonal is of length 2 units and the volume of the box is the maximum possible, then which of the following options is (are) true? ***1 point***

- [x] S is a cube.
- [ ] All sides are of different length.
- [x] $x = \frac{2}{\sqrt{3}}$
- [ ] $y = \sqrt{\frac{1}{3}}$

9) If the length of the diagonal of the box is 3 units, then which of the followings is the maximum possible volume of the box? ***1 point***

- ( ) $\sqrt{3}$ cube units.
- (•) $3\sqrt{3}$ cube units.
- ( ) 1 cube units.
- ( ) 3 cube units

10) Consider a function $f(x, y)$.Let v_1, v_2, v_3 be the critical points of the function f and H_k be the Hessian matrix of the function at the point v_k.Then which of the following options is(are) true? ***1 point***

- [ ] If $H_1 = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$, then v_1 is a saddle point.
- [x] If $H_2 = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$, then v_2 is a saddle point.
- [x] If $H_3 = \begin{bmatrix} 3 & 1 \\ -3 & 1 \end{bmatrix}$, then v_3 is a point of local extremum.
- [x] If $H_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, then v_1 is a point of local minimum.

11) Consider the following function ***1 point***

$f(x, y, z) = x^2y^2 + xz^2$

Which of the following options is (are) true?

- [x] $(0, y, 0),$ where $y \in \mathbb{R}$ are critical points.
- [ ] $(0, 0, z),$ where $z \in \mathbb{R}$ are critical points.
- [ ] $(1, y, 0)$ where $y \in \mathbb{R}$ are critical points.
- [ ] $(0, y, 1),$ where $y \in \mathbb{R}$ are critical points.
- [x] $(x, 0, 0),$ where $x \in \mathbb{R}$ are critical points.

12) Consider a function $f(x,y,z)$. Let v_1, v_2, v_3 be the critical points of the function f and H_k be the Hessian matrix of the function at the point v_k.Then which of the following options is(are) true? ***1 point***

- [x] If $H_1 = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$, then v_1 is a saddle point.
- [ ] If $H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then v_2 is a saddle point.
- [ ] If $H_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then v_3 is a point of local maximum.
- [x] If $H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then v_1 is a point of local minimum.
- [x] If $H_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, then v_2 is a point of local maximum.
- [ ] If $H_3 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then v_3 is a point of local minimum.