IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 9 Tutorial 3

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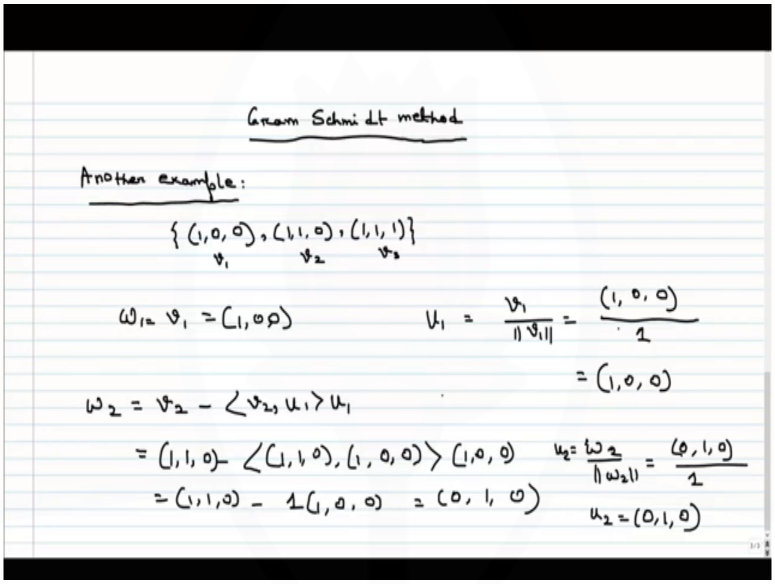

Hello everyone, so in this video we will take another example. So, here the example is written, we have three vectors, v_1, v_2, v_3 and you can see that these are linearly independent vector of $\mathbb{R}^3$, so this forms the basis. Now we will try to convert this basis into an orthogonal basis, rather I can say it is into an orthonormal basis using Gram Schmidt method.

So, what we do first, by the algorithm what we see, the w_1, the first vector will remain unchanged, so as we have to make it orthonormal, so we will find the norm of the first vector and divide it with them. So, our new vector $u_1 = \frac{v_1}{\|v_1\|} = \frac{(1,0,0)}{1} = (1,0,0)$. Now we have to calculate the second one w_2.

Now, $w_2 = v_2 - \langle v_2, u_1 \rangle u_1 = (1,1,0) - \langle (1,1,0),(1,0,0) \rangle (1,0,0) = (0,1,0)$. So, it will give us a new vector which is orthogonal to u1 and then we have to normalize it. Basically then we have to divide it with the norm.

Now, $u_2 = \frac{w_2}{\|w_2\|} = \frac{(0,1,0)}{1} = (0,1,0)$. So, you can check that u_1, u_2 are orthogonal to each other and both u_1, u_2 are orthonormal also, because their norm is 1. So, let us calculate the third one.

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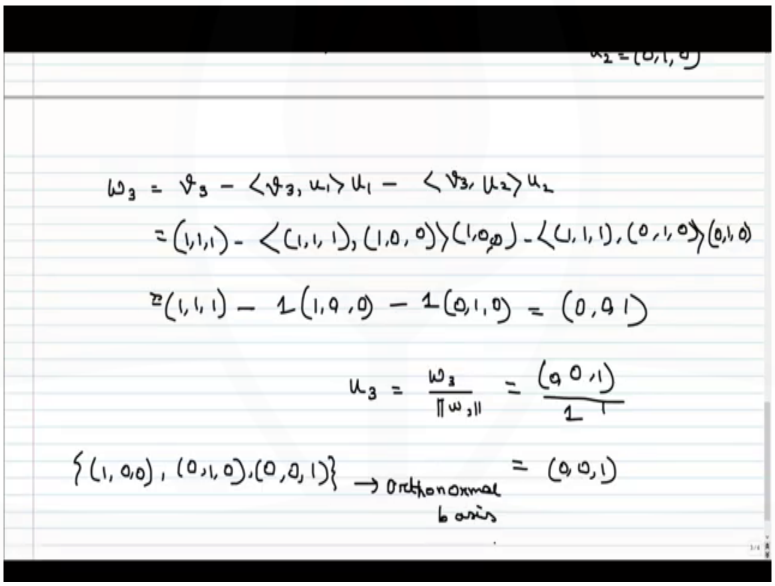

So, for the third one, similarly we have to find w_3 $w_3 = v_3 - \langle v_3, u_1 \rangle u_1 - \langle v_3, u_2 \rangle u_2 = (1,1,1) - \langle (1,1,1),(1,0,0) \rangle (1,0,0) - \langle (1,1,1),(0,1,0) \rangle (0,1,0) = (0,0,1)$. And now we have to normalize it. So, after normalizing we will get the new vector

$$u_3 = \frac{w_3}{\|w_3\|} = \frac{(0,0,1)}{1} = (0,0,1)$$.

So, our new set of vector here is $\{(1,0,0),(0,1,0),(0,0,1)\}$. So, these are the standard basis which we have got by orthogonalization of that v_1 basis. So, this is the standard basis and if you see, if you calculate the inner product of any two you will get 0 and if you calculate the norm of each vector, the norm of each vector is 1.

So, it is an orthonormal basis corresponding to the basis which we have started with and we do this Gram Schmidt algorithm and get these set of vectors which form an orthonormal basis. Thank you.