INDIAN INSTITUTE OF TECHNOLOGY MADRAS
सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
Professor. Sarang S. Sane
Department of Mathematics
Indian Institute of Technology Madras
Week-6 Tutorial

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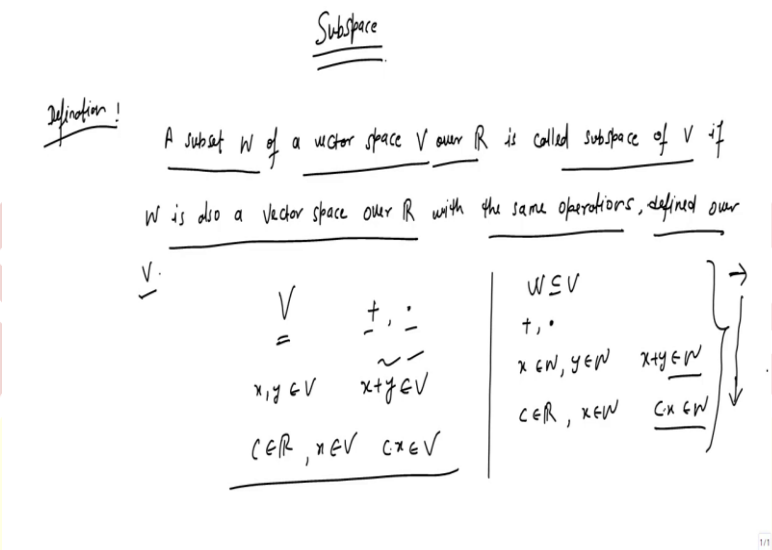

Hello guys, today we will discuss about the subspace, what is subspace. Suppose V is a vector space, then subspace is actually subset of vector space V with W having some property. So, let us go through by formal definition. Suppose W is a subset of a vector space V over the, V is vector space over ℝ, we call W is a subspace of V, if W is also a vector space over ℝwith and this vector space form over some operation and that operation will be same whatever the operation on V is there.

Now, we know that what is subset. So, suppose it is V is a set then some collection of element of V, if you take that then if you form a set of those elements then we will say that set is actually a subset of V and what is vector space? So, we know that suppose V is a set and we define two operation addition and scalar multiplication on V with this addition and scalar multiplication having closed properties, suppose x and y belongs to V then if you add them, add them and if it should V belongs to V and this scalar multiplication.

So, suppose C is a real number and x is an element of V and this x, C times V also should belongs to V. If suppose these properties followed in V and with as we have seen in video lecture, there are eight properties which we use to say oximes, if those properties followed by this operation, then we will say V is a vector space.

Now, suppose W is a subset of V and if we define the same operation addition and scalar multiplication with the property same like this, the x belongs to W and Y belongs to W and to making a subspace to, subspace of V it should be $x + y$ also belongs to W and suppose C is a real number and x belongs to W then Cx should also belongs to W. This is closed under addition and this is closed and this is called closed under scalar multiplication. With same properties, this eight properties which V follows, if W will also follow then we will say W is a subspace of V.

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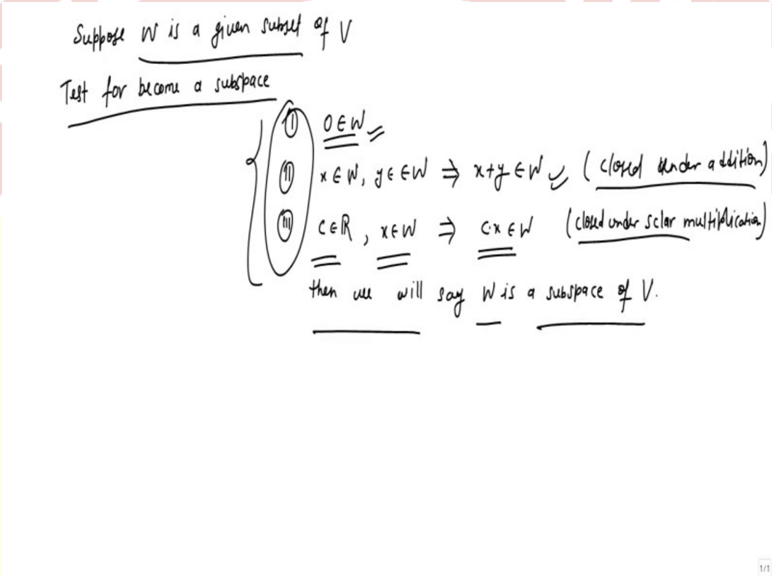

Now, suppose W is a given subset of V, then how we will check that W is also a subspace. W is a subset of, it will be given then how we will check that W is subspace of V then there is three criteria. If W will satisfy that then we will say W is subspace of V. Those three criteria are 0 should belong to W and the second criteria is if x suppose x an element belongs to W, y is an element of W then addition should also so belongs to W.

This is also called closed under addition and the third criteria is, suppose c is a real number, and x is an element of W then their multiplication which is called a scalar multiplication and if that one, that multiplication is also element of W, which is called a closed under scalar multiplication then these, if these criteria followed by W, then we will say W is a subspace of V.

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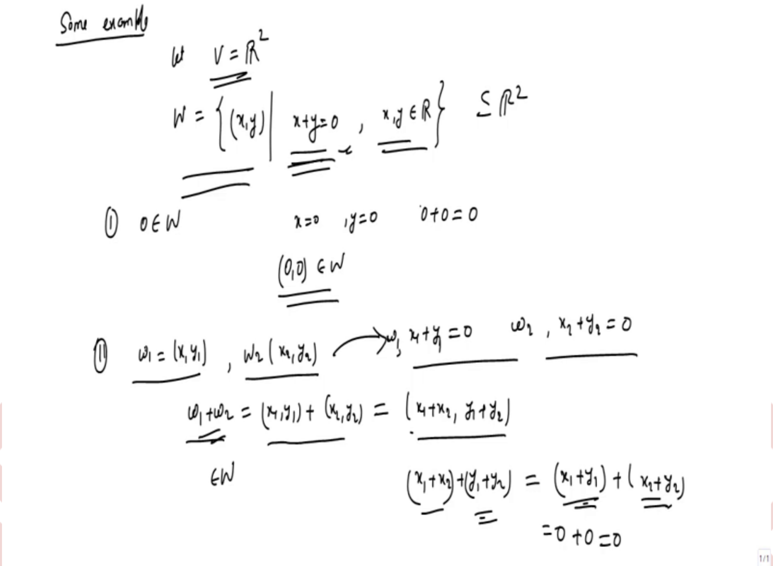

Now, let us do some example. Let the given vector space is $\mathbb{R}^2$ over R and we have given a subset W, W is x, y coordinate and this, if we add $x + y$ it will become 0 and x, y element of R, these two conditions. So, W is a subset given of $\mathbb{R}^2$. We need to check W is a subspace or not. So, let us our first criteria that 0 should belongs to W.

So, if we take $x = 0$ and $y = 0$, and if we add them 0 + 0 it becomes 0, it means, 0, 0 belongs to W that means 0 element of $\mathbb{R}^2$ which is also belongs to W. Now, we need to check the second criteria, the second criteria is let W_1 is one element is x_1, y_1 and the second W_2 second element in x_2, y_2.

Now, if we add them $W_1 + W_2$, so it will become $x_1, x_2 + y_1, y_2$ this = coordinate y addition will happen same operation, as we did in R2. So, this will become the $x_1 + x_2, y_1 + y_2$.

Now, w1 is element of W, W_2 is element or W, it means, this two will satisfy this condition means $x_1 + y_1$ = 0, and this is the for W_1 and for $W_2, x_2 + y_2 = 0$. So, these two conditions we have given because W_1, W_2 two are element of W. So, now, what we did in addition of $W_1 + W_2$ we got these coordinates $x_1 + x_2, y_1 + y_2$. So, let us check is this addition is belongs to W, so we need to add them $y_1 + y_2$ we need, we added them.

And we, as we said these are the element of $\mathbb{R}^2$. So, that swapping can we do easily so, we can write $x_1 + y_1 + x_2 + y_2$ and this is 0 and this is 0 as w1. This is 0 as w1 is that coordinate addition is becomes 0 as given condition and this is 0, so 0 + 0 becomes 0. It means, this addition belongs to W. So, second criteria is followed.

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(iii) $c \in \mathbb{R}$, $w_1 = (x_1, y_1) \in W$

$x_1 + y_1 = 0$

$c \cdot w_1 = (cx_1, cy_1)$

$cx_1 + cy_1 = c(x_1 + y_1) = 0$

W is a subspace of $\mathbb{R}^2$

Now, we need to check third criteria. The third criteria is a real number C and suppose W_1 = x_1, y_1 an element of W and so W_1 is element W it means $x_1 + y_1 = 0$. Now, let us do multiplication C into W_1 = it will become coordinate wise multiplication will happen, same operation as in $\mathbb{R}^2$. So, it will become Cx_1, Cy_1.

Now we need to check this coordinates make sum 0 or not, then we will say this is scalar multiplication is in W happened that is closed under scalar multiplication. So, let us do the sum. So, sum $x_1 + Cy_1$ this we can write $x_1 + y_1$. We are able to do this because as we can able to do in $\mathbb{R}^2$, so we can do this and we have given and this is 0 it means this is 0. It means C times W_1 is element of W. So, all three criteria done by W it means W is a subspace of $\mathbb{R}^2$.

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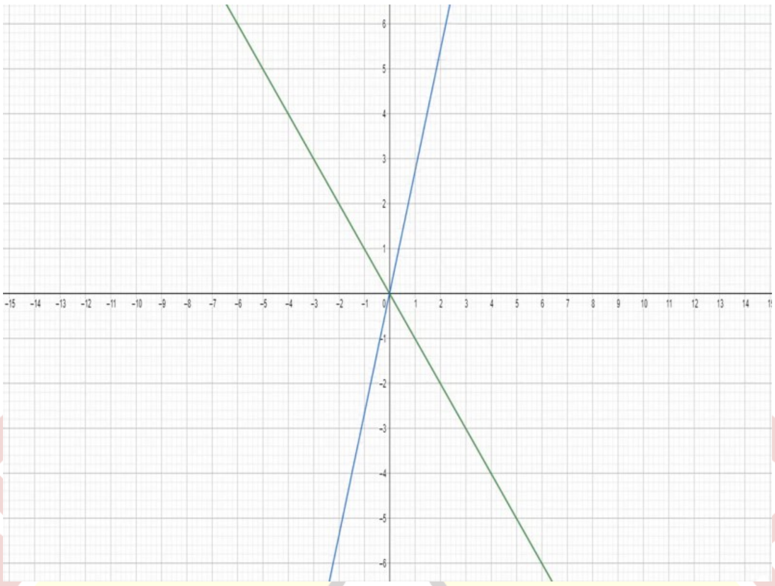

So, let us see it geometrically what do we mean, what we mean by a subspace. So, this is the vector space $\mathbb{R}^2$representing this plane is actually x, y, plane $\mathbb{R}^2$ means x, y plane. Now, we have given a subset W, which is coordinates x, y and having some condition $x + y = 0$. If you see it as in x, y plane so actually x this W represent a line, which pass through origin with having slope - 1.

So, actually W is this line which pass through origin and we if we see clearly the first condition 0 it passes through origin, so 0 element is here is present in W and if we do addition with any taking to, any two elements then it will be the same in the line and again and the third criteria is by taking any real number. If you multiply with this any element of W, then it will be again in the line. So, it means W is a subspace of $\mathbb{R}^2$.

Now, suppose, another W_1 is a subspace of $\mathbb{R}^2$ and which having condition the same coordinates with changing the slope actually. So, subspace of $\mathbb{R}^2$ will be the passing represent the line, which passes through origin. Why this is passed through origin because of the first criteria, 0 element would be in W. So, any subspace of $\mathbb{R}^2$ other than trivial subspaces, which are 0 subspace and $\mathbb{R}^2$ itself, represent a line which pass through origin.

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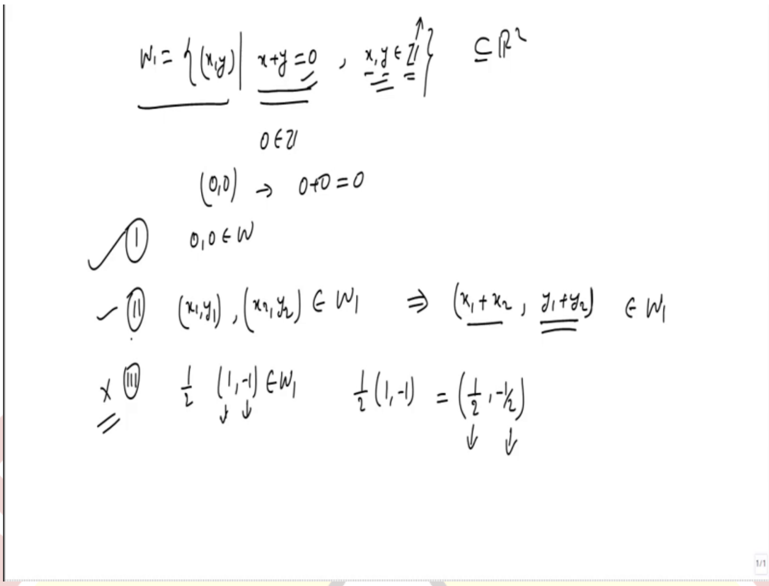

Now, we will see a subset of $\mathbb{R}^2$, which will not form a subspace, it means, we have to check those three criteria, which we discussed earlier and we will see that, which conditions will not followed by this W_1. So, let us check this, the first criteria that 0 element in W_1. So, first see clearly W_1 is actually this subset of $\mathbb{R}^2$ with the same condition, as we discussed earlier $x + y = 0$, but that x is that x and y belong, taken from this integer that means x, y are integer.

So, we know that 0 belongs to z, so we can take the coordinate 0, 0 and this follow the condition 0 + 0 which is 0. So, it means 0 element is in W. So, first criteria satisfied W_1, now we need to check second criteria. The second criteria is, you will take two element from W_1. Let us take x_1, y_1 and second element is the x_2, y_2. So, if we from W_1 if we add them this will become coordinate wide addition and addition of two integer is integer.

Also previously we have seen that addition of coordinates that is the $x_1 + x_2 + y_1 + y_2 = 0$, here, same thing will also happen. So, conditions of W_1 are satisfied, it means it follow the addition, this W_1 is closed under addition. So, it means this belongs to W_1. So second criteria followed by W_1.

Now, the third criteria is, any real number multiplied with an element, which is at a scalar multiplication and this is lacking W1. So, let us take any real number we can take. So, let us take any 1 by 2, this is a rational number and an element of W1 which is we can take 1, n - 1. So, this is from W_1 satisfying this condition and also these are the integer. So, this is the element of W_1.

Now, if we multiply them, so this is, this will be a coordinate wise multiplication, so this will become 1/2 – 1/ 2. Now, here is the missing part that this is not an integer, this is not an integer, it means W_1 and is not closed under scalar multiplication. So, here the third condition is lacking for W_1. So, the W_1 is a subset, but not a subspace.