Partial derivatives

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Very important notation

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scalar-valued multivariable function.

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If $\underset{\sim}{a}$ is a point in $\mathbb{R}^n$, then an open ball of radius r around $\underset{\sim}{a}$ is the set defined as

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$e_1, e_2, \ldots, e_n$ is the standard ordered basis of $\mathbb{R}^n$.

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Let $f(x_1, x_2, \dots, x_n)$ be a function defined on a domain D in $\mathbb{R}^n$ containing a point $\underset{\sim}{a}$ and an open ball around it.

Then the rate of change of f at $\underset{\sim}{a}$ w.r.t. the variable x_i is

$$\lim_{h\to 0} \frac{f\left(\underset{\sim}{a} + he_i\right) - f\left(\underset{\sim}{a}\right)}{h}.$$

$g(h) = f(\underset{\sim}{a} + he_i)$

g is a fn. of 1-variable

$$\lim_{h\to 0} \frac{g(h) - g(0)}{h}$$

$e_i = (0, 0, \dots, 0, 1, 0, \dots, 0)$

ith place

$\underset{\sim}{a} = (a_1, a_2, \dots, a_n)$

$$\lim_{h\to 0} \frac{f((a_1, a_2, \dots, a_n) + h(0, \dots, 1, \dots, 0)) - f((a_1, \dots, a_n))}{h}$$

$$= \lim_{h\to 0} \frac{f((a_1, a_2, \dots, a_{i-1}, a_i + h, a_{i+1}, \dots, a_n)) - f(a_1, \dots, a_n)}{h}$$

Examples

The rate of change of $f(x, y) = x + y$ at $(0, 0)$ w.r.t. x

$$\lim_{h\to 0} \frac{f((0,0)+h(1,0)) - f(0,0)}{h} = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{h-0}{h} = \lim_{h\to 0} \frac{h}{h} = 1.$$

The rate of change of $f(x, y, z) = xy + yz + zx$ at $(1, 2, 3)$ w.r.t. y

$$\lim_{h\to 0} \frac{f((1,2,3)+h(0,1,0)) - f(1,2,3)}{h} = \lim_{h\to 0} \frac{f(1,2+h,3) - f(1,2,3)}{h}$$

$$= \lim_{h\to 0} \frac{1\times(2+h) + (2+h)\times 3 + 1\times\cancel{3} - 1\times\cancel{2} - \cancel{2\times 3} - 1\times\cancel{3}}{h} = \lim_{h\to 0} \frac{h+3h}{h} = 4.$$

The rate of change of $f(x, y) = sin(xy)$ at $(1, 0)$ w.r.t. x

$$\lim_{h\to 0} \frac{f((1,0)+h(1,0)) - f(1,0)}{h} = \lim_{h\to 0} \frac{f(1+h,0) - f(1,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0.$$

w.r.t. y :

$$\lim_{h\to 0} \frac{f((1,0)+h(0,1)) - f(1,0)}{h} = \lim_{h\to 0} \frac{\sin(h) - 0}{h} = \lim_{h\to 0} \frac{\sin(h)}{h} = 1.$$

Partial derivatives

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Let $f(x_1, x_2, \ldots, x_n)$ be a function defined on a domain D in $\mathbb{R}^n$. The partial derivative of f w.r.t. x_i is the function denoted by $f_{x_i}\left(\underset{\sim}{x}\right)$ or $\dfrac{\partial f}{\partial x_i}\left(\underset{\sim}{x}\right)$ and defined as

$$\frac{\partial f}{\partial x_i}\left(\underset{\sim}{x}\right) = \lim_{h \to 0} \frac{f\left(\underset{\sim}{x} + he_i\right) - f\left(\underset{\sim}{x}\right)}{h} \quad .$$

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Its domain consists of those points of D at which the limits exists.

Example

$f(x, y) = x + y$

$$\frac{\partial f}{\partial x}(x,y) = \lim_{h \to 0} \frac{f\left((x,y) + h(1,0)\right) - f(x,y)}{h} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h} = \lim_{h \to 0} \frac{x+h+y-(x+y)}{h} = 1.$$

$$\frac{\partial f}{\partial y}(x,y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h} = \lim_{h \to 0} \frac{x+y+h-(x+y)}{h} = 1.$$

Calculating partial derivatives

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To calculate the partial derivative w.r.t. x_i, think of f only as a function of x_i while treating all other variables as constants. Then calculate it as the derivative of a function of one variable.

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Examples :

- $f(x, y, z) = xy + yz + zx$

$$\frac{\partial f}{\partial x}(x,y,z) = y + 0 + z = y + z.$$

$$\frac{\partial f}{\partial y}(x,y,z) = x + z \qquad \frac{\partial f}{\partial z}(x,y,z) = x + y.$$

$x\,3 + 20 + 18x$

$3 + 0 + 18$

$\sin(3x)$

$\cos(3x) \times 3$

- $f(x, y, z) = sin(xy)$

$$\frac{\partial f}{\partial x}(x,y) = \cos(xy) \times y = y\cos(xy)$$

$$\frac{\partial f}{\partial y}(x,y) = x\cos(xy).$$

Another example :

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

$\boxed{(x,y) \neq (0,0)}$

What is $\frac{\partial f}{\partial x}(x,y)$ & $\frac{\partial f}{\partial y}(x,y)$

$\frac{3x}{x^2+4}$

$$\frac{\partial f}{\partial x}(x,y) = \frac{(x^2+y^2)y - xy(2x)}{(x^2+y^2)^2} = \frac{y^3 - x^2 y}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{x^3 - y^2 x}{(x^2+y^2)^2}$$

$(x,y) = (0,0)$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h \to 0} \frac{0 - 0}{h} = 0.$$

$$\frac{\partial f}{\partial y}(0,0) = 0.$$

Thank you