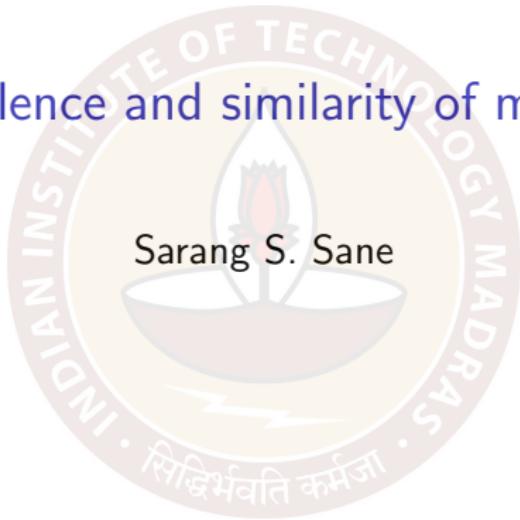


Equivalence and similarity of matrices



Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

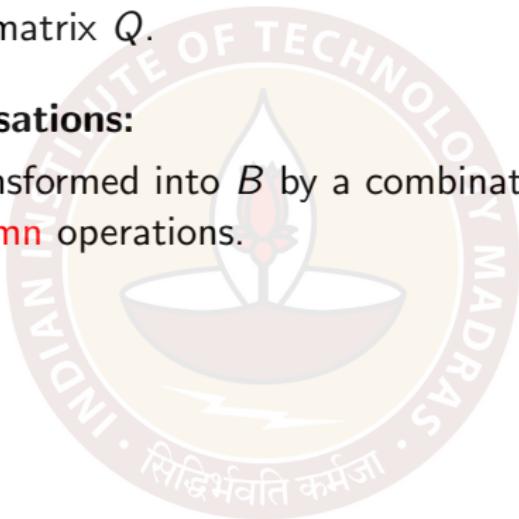


Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

Other characterisations:

- 1) A can be transformed into B by a combination of **elementary row and column** operations.

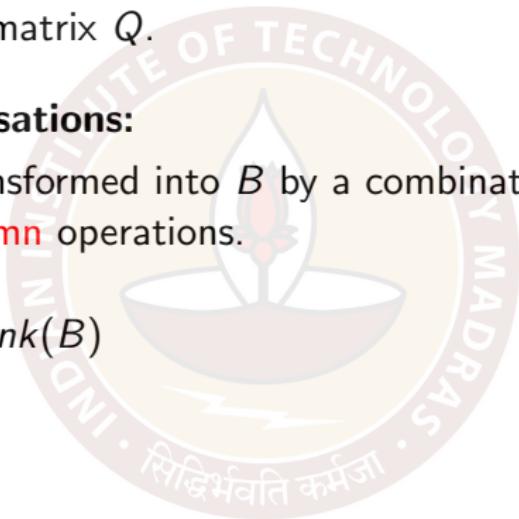


Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

Other characterisations:

- 1) A can be transformed into B by a combination of **elementary row and column operations**.
- 2) $\text{rank}(A) = \text{rank}(B)$



Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

Other characterisations:

- 1) A can be transformed into B by a combination of **elementary row and column operations**.
- 2) $\text{rank}(A) = \text{rank}(B)$

Equivalence of matrices is an **equivalence relation**

Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

Other characterisations:

- 1) A can be transformed into B by a combination of **elementary row and column operations**.
- 2) $\text{rank}(A) = \text{rank}(B)$

Equivalence of matrices is an **equivalence relation** i.e.

- ▶ A is equivalent to itself

Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

Other characterisations:

- 1) A can be transformed into B by a combination of **elementary row and column operations**.
- 2) $\text{rank}(A) = \text{rank}(B)$

Equivalence of matrices is an **equivalence relation** i.e.

- ▶ A is equivalent to itself
- ▶ A is equivalent to B implies B is equivalent to A .

Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

Other characterisations:

- 1) A can be transformed into B by a combination of **elementary row and column operations**.
- 2) $\text{rank}(A) = \text{rank}(B)$

$$QAP = \begin{bmatrix} I_{n \times n} & 0 \\ 0 & 0 \end{bmatrix} = Q'BP'$$

Equivalence of matrices is an **equivalence relation** i.e.

- ▶ A is equivalent to itself $A = I_{m \times m} A I_{n \times n}$ $B = QAP \Rightarrow A = Q'BP^{-1}$
- ▶ A is equivalent to B implies B is equivalent to A . $\Rightarrow A = Q'BP^{-1}$
- ▶ A is equivalent to B and B to C implies A is equivalent to C .
 $B = QAP, C = Q'BP' \Rightarrow C = \underbrace{Q'}_{\sim} \underbrace{QAP}_{\sim} \underbrace{P'}_{\sim}$

Equivalence of matrices

Let A and B be two matrices of order $m \times n$. We say A is **equivalent** to B if $B = QAP$ for some invertible $n \times n$ matrix P and for some invertible $m \times m$ matrix Q .

Other characterisations:

- 1) A can be transformed into B by a combination of **elementary row and column operations**.
- 2) $\text{rank}(A) = \text{rank}(B)$

Equivalence of matrices is an **equivalence relation** i.e.

- ▶ A is equivalent to itself
- ▶ A is equivalent to B implies B is equivalent to A .
- ▶ A is equivalent to B and B to C implies A is equivalent to C .

Example

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$, defined as :
$$f(x, y, z) = (x + y, y + z).$$



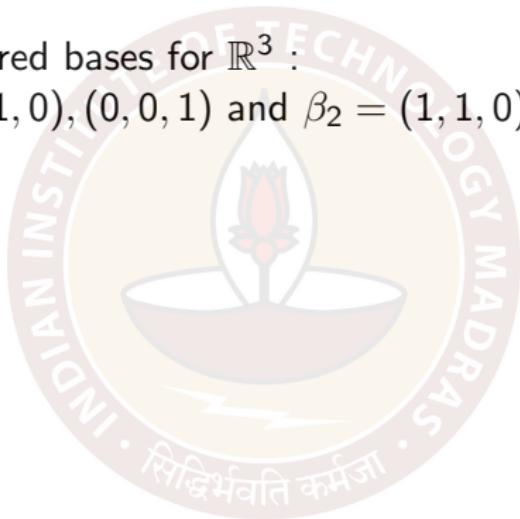
Example

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$, defined as :

$$f(x, y, z) = (x + y, y + z).$$

Consider two ordered bases for \mathbb{R}^3 :

$$\beta_1 = (1, 0, 0), (0, 1, 0), (0, 0, 1) \text{ and } \beta_2 = (1, 1, 0), (0, 1, 1), (0, 0, 1).$$



Example

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$, defined as :

$$f(x, y, z) = (x + y, y + z).$$

Consider two ordered bases for \mathbb{R}^3 :

$$\beta_1 = (1, 0, 0), (0, 1, 0), (0, 0, 1) \text{ and } \beta_2 = (1, 1, 0), (0, 1, 1), (0, 0, 1).$$

Similarly, consider two ordered bases for \mathbb{R}^2 :

$$\gamma_1 = (1, 0), (0, 1) \text{ and } \gamma_2 = (1, 0), (1, 1).$$

Example

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$, defined as :

$$f(x, y, z) = (x + y, y + z).$$

Consider two ordered bases for \mathbb{R}^3 :

$$\beta_1 = (1, 0, 0), (0, 1, 0), (0, 0, 1) \text{ and } \beta_2 = (1, 1, 0), (0, 1, 1), (0, 0, 1).$$

Similarly, consider two ordered bases for \mathbb{R}^2 :

$$\gamma_1 = (1, 0), (0, 1) \text{ and } \gamma_2 = (1, 0), (1, 1).$$

$$f(1, 0, 0) = (1, 0),$$

$$f(0, 1, 0) = (1, 1) = 1(1, 0) + 1(0, 1),$$

$$f(0, 0, 1) = (0, 1).$$

Example

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$, defined as :

$$f(x, y, z) = (x + y, y + z).$$

Consider two ordered bases for \mathbb{R}^3 :

$$\beta_1 = (1, 0, 0), (0, 1, 0), (0, 0, 1) \text{ and } \beta_2 = (1, 1, 0), (0, 1, 1), (0, 0, 1).$$

Similarly, consider two ordered bases for \mathbb{R}^2 :

$$\gamma_1 = (1, 0), (0, 1) \text{ and } \gamma_2 = (1, 0), (1, 1).$$

$$f(1, 0, 0) = (1, 0), \quad \leftarrow$$

$$f(0, 1, 0) = (1, 1) = 1(1, 0) + 1(0, 1), \quad \leftarrow$$

$$f(0, 0, 1) = (0, 1). \quad \leftarrow$$

Hence the matrix corresponding to f with respect to the bases β_1

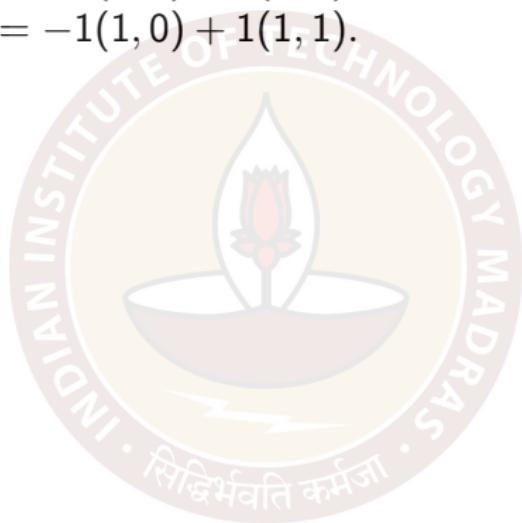
and γ_1 is $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Example (contd.)

$$f(1, 1, 0) = (2, 1) = 1(1, 0) + 1(1, 1),$$

$$f(0, 1, 1) = (1, 2) = -1(1, 0) + 2(1, 1),$$

$$f(0, 0, 1) = (0, 1) = -1(1, 0) + 1(1, 1).$$



Example (contd.)

$$f(1, 1, 0) = (2, 1) = 1(1, 0) + 1(1, 1),$$

$$f(0, 1, 1) = (1, 2) = -1(1, 0) + 2(1, 1),$$

$$f(0, 0, 1) = (0, 1) = -1(1, 0) + 1(1, 1).$$

Hence the matrix corresponding to f with respect to the bases β_2 and γ_2 is $B = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$.

Example (contd.)

$$f(1, 1, 0) = (2, 1) = 1(1, 0) + 1(1, 1),$$

$$f(0, 1, 1) = (1, 2) = -1(1, 0) + 2(1, 1),$$

$$f(0, 0, 1) = (0, 1) = -1(1, 0) + 1(1, 1).$$

Hence the matrix corresponding to f with respect to the bases β_2

and γ_2 is $B = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$.

Choose $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

Example (contd.)

$$(1,1,0) = 1e_1 + 1e_2 + 0e_3$$

$$f(1,1,0) = (2,1) = 1(1,0) + 1(1,1),$$

$$f(0,1,1) = (1,2) = -1(1,0) + 2(1,1),$$

$$f(0,0,1) = (0,1) = -1(1,0) + 1(1,1).$$

$$\begin{aligned}(1,0) &= 1(1,0) + 0(1,1) \\ (0,1) &= -1(1,0) + 1(1,1)\end{aligned}$$

Hence the matrix corresponding to f with respect to the bases β_2 and γ_2 is $B = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

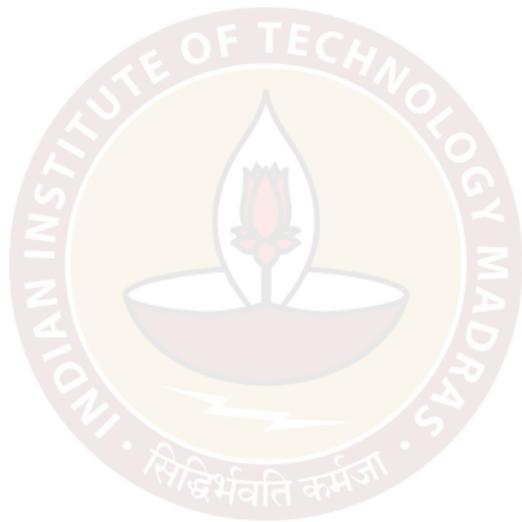
Choose $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$. Then

$$QAP = \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}_{\text{ }} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}_{\text{ }} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}_{\text{ }} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \checkmark = B \checkmark$$

Hence A and B are equivalent to each other.

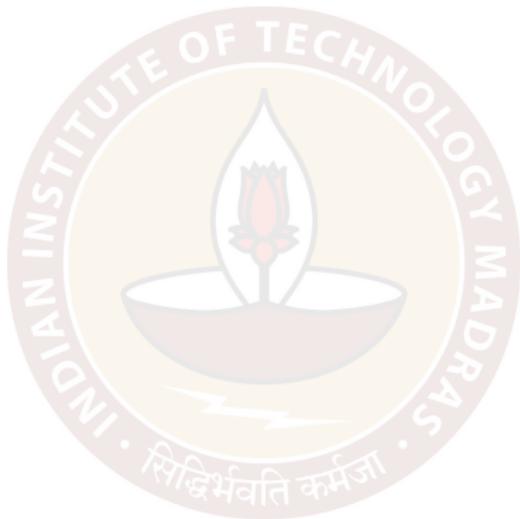
Linear transformations and equivalence of matrices

Consider a linear transformation $T : V \rightarrow W$,



Linear transformations and equivalence of matrices

Consider a linear transformation $T : V \rightarrow W$, two ordered bases β_1 and β_2 for V ,



Linear transformations and equivalence of matrices

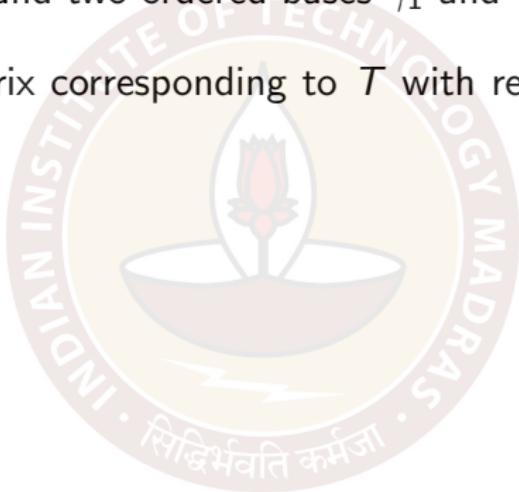
Consider a linear transformation $T : V \rightarrow W$, two ordered bases β_1 and β_2 for V , and two ordered bases γ_1 and γ_2 for W .



Linear transformations and equivalence of matrices

Consider a linear transformation $T : V \rightarrow W$, two ordered bases β_1 and β_2 for V , and two ordered bases γ_1 and γ_2 for W .

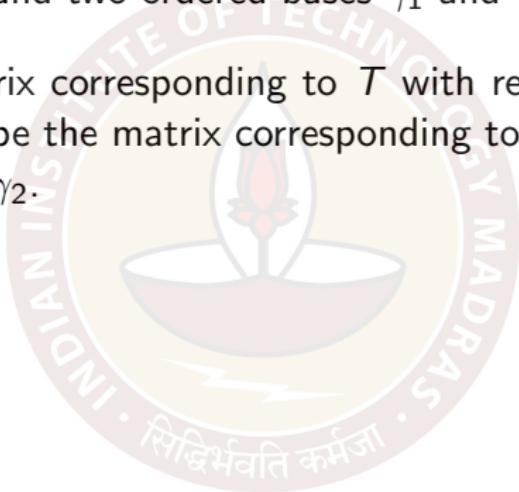
Let A be the matrix corresponding to T with respect to the bases β_1 and γ_1



Linear transformations and equivalence of matrices

Consider a linear transformation $T : V \rightarrow W$, two ordered bases β_1 and β_2 for V , and two ordered bases γ_1 and γ_2 for W .

Let A be the matrix corresponding to T with respect to the bases β_1 and γ_1 and B be the matrix corresponding to T with respect to the bases β_2 and γ_2 .



Linear transformations and equivalence of matrices

Consider a linear transformation $T : V \rightarrow W$, two ordered bases β_1 and β_2 for V , and two ordered bases γ_1 and γ_2 for W .

Let A be the matrix corresponding to T with respect to the bases β_1 and γ_1 and B be the matrix corresponding to T with respect to the bases β_2 and γ_2 .

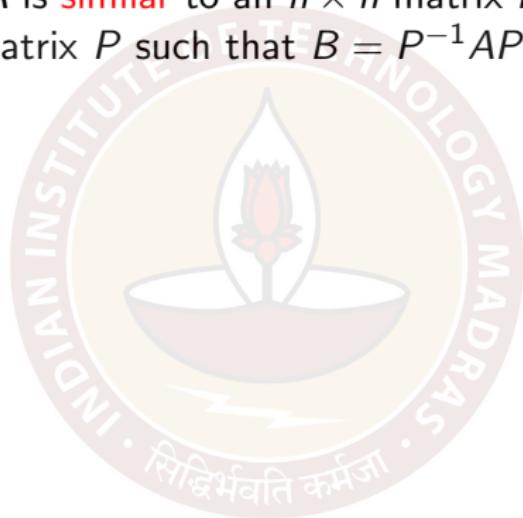
Then A is equivalent to B !

$P \rightarrow$ express the ordered basis β_2 in terms of β_1 .
 $Q \rightarrow$ express the ordered basis γ_2 in terms of γ_1 .

Then $B = QAP$.

Similar matrices

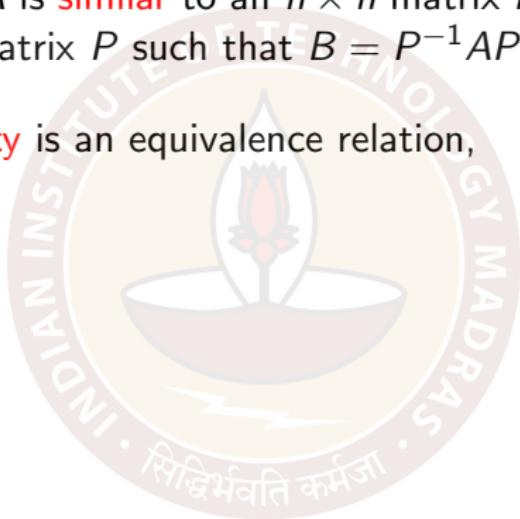
An $n \times n$ matrix A is **similar** to an $n \times n$ matrix B if there exists an $n \times n$ invertible matrix P such that $B = P^{-1}AP$.



Similar matrices

An $n \times n$ matrix A is **similar** to an $n \times n$ matrix B if there exists an $n \times n$ invertible matrix P such that $B = P^{-1}AP$.

Note that **similarity** is an equivalence relation,

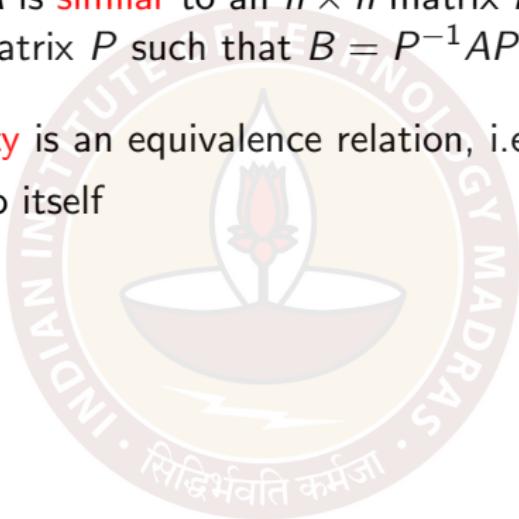


Similar matrices

An $n \times n$ matrix A is **similar** to an $n \times n$ matrix B if there exists an $n \times n$ invertible matrix P such that $B = P^{-1}AP$.

Note that **similarity** is an equivalence relation, i.e. :

- ▶ A is similar to itself



Similar matrices

An $n \times n$ matrix A is **similar** to an $n \times n$ matrix B if there exists an $n \times n$ invertible matrix P such that $B = P^{-1}AP$.

Note that **similarity** is an equivalence relation, i.e. :

- ▶ A is similar to itself
- ▶ A is similar to B implies B is similar to A .

Similar matrices

An $n \times n$ matrix A is **similar** to an $n \times n$ matrix B if there exists an $n \times n$ invertible matrix P such that $B = P^{-1}AP$.

Note that **similarity** is an equivalence relation, i.e. :

- ▶ A is similar to itself $P = I$: $A = I^{-1}AI = A$.
- ▶ A is similar to B implies B is similar to A . ✓
- ▶ A is similar to B and B to C implies A is similar to C . ✓

$$B = P^{-1}AP \Rightarrow PBP^{-1} = A \Rightarrow A = (P^{-1})^{-1}B(P^{-1})$$

$$\begin{aligned} B &= P^{-1}AP, \quad C = Q^{-1}BQ \\ \Rightarrow C &= Q^{-1}(P^{-1}AP)Q = Q^{-1}P^{-1}A(PQ) \\ &= (PQ)^{-1}A(PQ) \end{aligned}$$

Similar matrices

An $n \times n$ matrix A is **similar** to an $n \times n$ matrix B if there exists an $n \times n$ invertible matrix P such that $B = P^{-1}AP$.

Note that **similarity** is an equivalence relation, i.e. :

- ▶ A is similar to itself
- ▶ A is similar to B implies B is similar to A .
- ▶ A is similar to B and B to C implies A is similar to C .

Important properties of similar matrices

Suppose A and B are similar matrices. Then the following properties hold :



Important properties of similar matrices

Suppose A and B are similar matrices. Then the following properties hold :

- ▶ A and B are equivalent.



Important properties of similar matrices

Suppose A and B are similar matrices. Then the following properties hold :

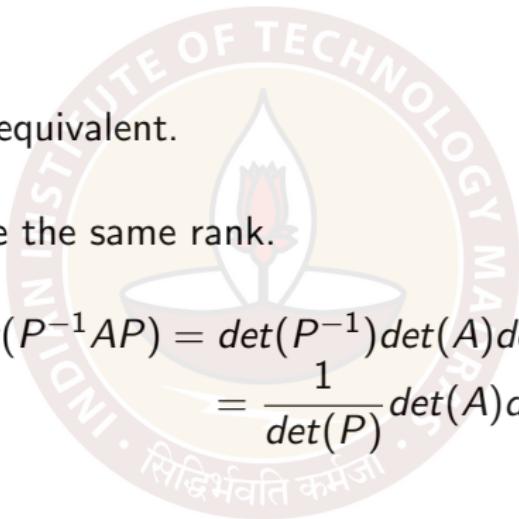
- ▶ A and B are equivalent.
- ▶ A and B have the same rank.



Important properties of similar matrices

Suppose A and B are similar matrices. Then the following properties hold :

- ▶ A and B are equivalent.
- ▶ A and B have the same rank.
- ▶ $\det(B) = \det(P^{-1}AP) = \det(P^{-1})\det(A)\det(P)$
 $= \frac{1}{\det(P)}\det(A)\det(P) = \det(A).$



Important properties of similar matrices

Suppose A and B are similar matrices. Then the following properties hold :

- ▶ A and B are equivalent.
- ▶ A and B have the same rank.
- ▶ $\det(B) = \det(P^{-1}AP) = \det(P^{-1})\det(A)\det(P)$
 $= \frac{1}{\det(P)}\det(A)\det(P) = \det(A).$
- ▶ Several other invariants of A and B are the same such as the characteristic polynomial, minimal polynomial and eigen values (with multiplicity).

An example of similar matrices

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ where $f(x, y, z) = (-x + y + z, x - y + z, x + y - z)$.



An example of similar matrices

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ where $f(x, y, z) = (-x + y + z, x - y + z, x + y - z)$.

Let $\beta = \gamma$ both be the standard ordered basis of \mathbb{R}^3 .



An example of similar matrices

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ where $f(x, y, z) = (-x + y + z, x - y + z, x + y - z)$.

Let $\beta = \gamma$ both be the standard ordered basis of \mathbb{R}^3 .

Then we get :

$$f(1, 0, 0) = (-1, 1, 1) = -1(1, 0, 0) + 1(0, 1, 0) + 1(0, 0, 1)$$

$$f(0, 1, 0) = (1, -1, 1) = 1(1, 0, 0) - 1(0, 1, 0) + 1(0, 0, 1)$$

$$f(0, 0, 1) = (1, 1, -1) = 1(1, 0, 0) + 1(0, 1, 0) - 1(0, 0, 1)$$

An example of similar matrices

Consider the linear transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ where $f(x, y, z) = (-x + y + z, x - y + z, x + y - z)$.

Let $\beta = \gamma$ both be the standard ordered basis of \mathbb{R}^3 .

Then we get :

$$f(1, 0, 0) = (-1, 1, 1) = -1(1, 0, 0) + 1(0, 1, 0) + 1(0, 0, 1) \quad \leftarrow$$

$$f(0, 1, 0) = (1, -1, 1) = 1(1, 0, 0) - 1(0, 1, 0) + 1(0, 0, 1) \quad \leftarrow$$

$$f(0, 0, 1) = (1, 1, -1) = 1(1, 0, 0) + 1(0, 1, 0) - 1(0, 0, 1) \quad \leftarrow$$

Hence the matrix of f corresponding to the standard ordered basis

is
$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$
.

Example (contd.)

Consider another ordered basis $\beta' = (1, 1, 1), (-1, 1, 0), (-1, 0, 1)$.



Example (contd.)

Consider another ordered basis $\beta' = (1, 1, 1), (-1, 1, 0), (-1, 0, 1)$.

Then we have the following:

$$f(1, 1, 1) = (1, 1, 1) = 1(1, 1, 1) + 0(-1, 1, 0) + 0(-1, 0, 1)$$

$$f(-1, 1, 0) = (2, -2, 0) = 0(1, 1, 1) - 2(-1, 1, 0) + 0(-1, 0, 1)$$

$$f(-1, 0, 1) = (2, 0, -2) = 0(1, 1, 1) + 0(-1, 1, 0) - 2(-1, 0, 1)$$

Example (contd.)

Consider another ordered basis $\beta' = (1, 1, 1), (-1, 1, 0), (-1, 0, 1)$.

Then we have the following:

$$f(1, 1, 1) = (1, 1, 1) = 1(1, 1, 1) + 0(-1, 1, 0) + 0(-1, 0, 1)$$

$$f(-1, 1, 0) = (2, -2, 0) = 0(1, 1, 1) - 2(-1, 1, 0) + 0(-1, 0, 1)$$

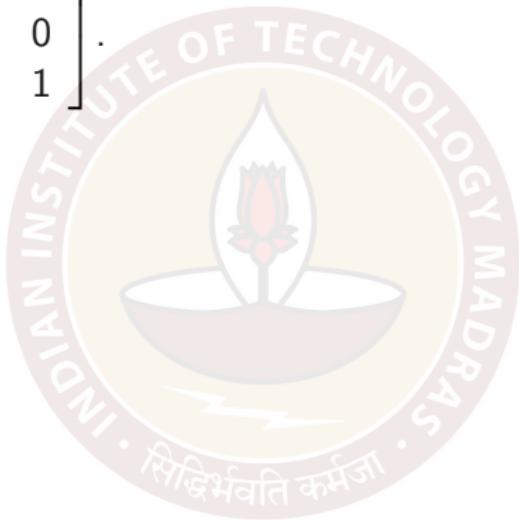
$$f(-1, 0, 1) = (2, 0, -2) = 0(1, 1, 1) + 0(-1, 1, 0) - 2(-1, 0, 1)$$

Hence the matrix of f corresponding to the ordered basis β' is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

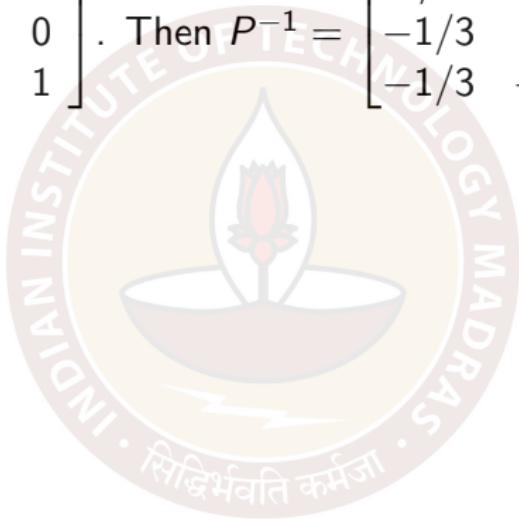
Example (contd.)

$$\text{Let } P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$



Example (contd.)

$$\text{Let } P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \text{ Then } P^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}.$$



Example (contd.)

$$\text{Let } P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \text{ Then } P^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}.$$

$$\text{Then } P^{-1}AP = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Example (contd.)

$$\text{Let } P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \text{ Then } P^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}.$$

$$\begin{aligned} \text{Then } P^{-1}AP &= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -4/3 & 2/3 \\ 2/3 & 2/3 & -4/3 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \end{aligned}$$

Example (contd.)

$$\text{Let } P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \text{ Then } P^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}.$$

$$\begin{aligned} \text{Then } P^{-1}AP &= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -4/3 & 2/3 \\ 2/3 & 2/3 & -4/3 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \end{aligned}$$

Hence A and B are similar matrices.

Another example

Consider the linear transformation seen earlier :



Another example

Consider the linear transformation seen earlier :

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$
$$f(x, y) = (2x, y)$$



Another example

Consider the linear transformation seen earlier :

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$
$$f(x, y) = (2x, y)$$

Consider the ordered basis $(1, 0), (1, 1)$ for \mathbb{R}^2 . Then we have the following:

$$f(1, 0) = (2, 0) = 2(1, 0) + 0(1, 1)$$

$$f(1, 1) = (2, 1) = 1(1, 0) + 1(1, 1)$$

Another example

Consider the linear transformation seen earlier :

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$
$$f(x, y) = (2x, y)$$

Consider the ordered basis $(1, 0), (1, 1)$ for \mathbb{R}^2 . Then we have the following:

$$f(1, 0) = (2, 0) = 2(1, 0) + 0(1, 1) \quad \leftarrow$$

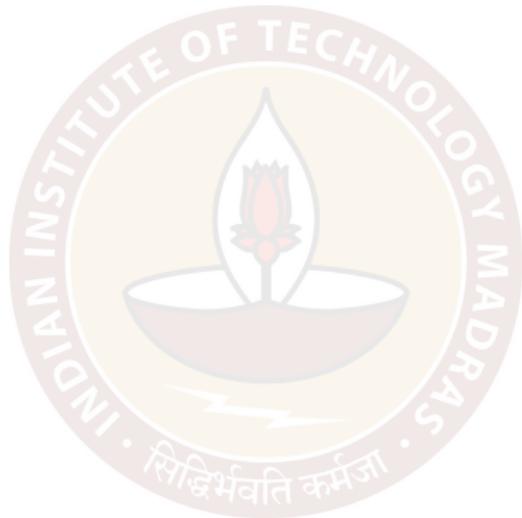
$$f(1, 1) = (2, 1) = 1(1, 0) + 1(1, 1) \quad \leftarrow$$

Hence the matrix of f corresponding to this ordered basis is :

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

Another example (contd.)

Consider the standard ordered basis $(1, 0), (0, 1)$ for \mathbb{R}^2 .



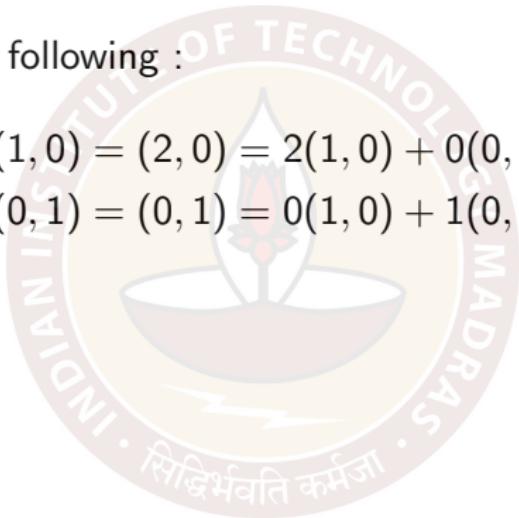
Another example (contd.)

Consider the standard ordered basis $(1, 0), (0, 1)$ for \mathbb{R}^2 .

Then we have the following :

$$f(1, 0) = (2, 0) = 2(1, 0) + 0(0, 1)$$

$$f(0, 1) = (0, 1) = 0(1, 0) + 1(0, 1)$$



Another example (contd.)

Consider the standard ordered basis $(1, 0), (0, 1)$ for \mathbb{R}^2 .

Then we have the following :

$$f(1, 0) = (2, 0) = 2(1, 0) + 0(0, 1)$$

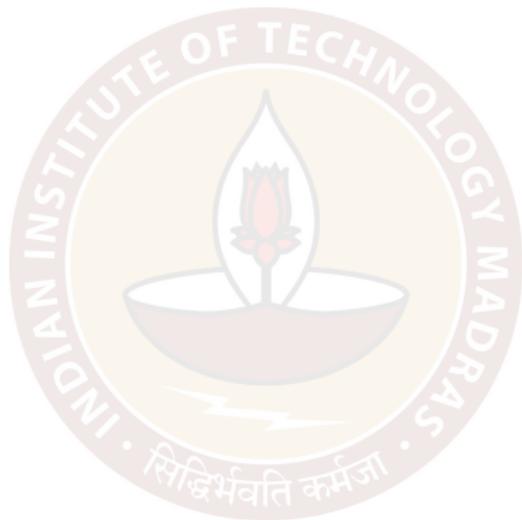
$$f(0, 1) = (0, 1) = 0(1, 0) + 1(0, 1)$$

Hence the matrix of f corresponding to this ordered basis is :

$$B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

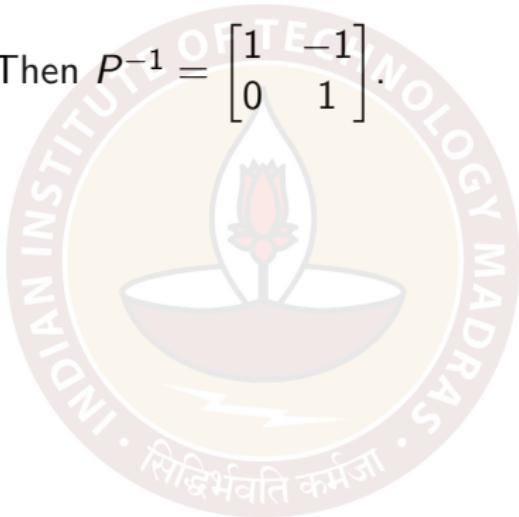
Another example (contd.)

Let $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.



Another example (contd.)

Let $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Then $P^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.



Another example (contd.)

Let $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Then $P^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

$$\begin{aligned} \text{Then } P^{-1}AP &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = B. \end{aligned}$$

Hence the matrices $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ are similar.

Linear transformations and similarity of matrices

Consider a linear transformation $T : V \rightarrow V$ and two ordered bases β and γ for V .



Linear transformations and similarity of matrices

Consider a linear transformation $T : V \rightarrow V$ and two ordered bases β and γ for V .

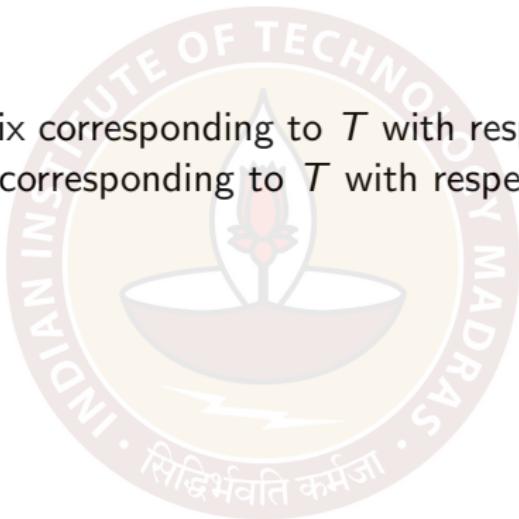
Let A be the matrix corresponding to T with respect to the basis β and



Linear transformations and similarity of matrices

Consider a linear transformation $T : V \rightarrow V$ and two ordered bases β and γ for V .

Let A be the matrix corresponding to T with respect to the basis β and B the matrix corresponding to T with respect to the basis γ .



Linear transformations and similarity of matrices

Consider a linear transformation $T : V \rightarrow V$ and two ordered bases β and γ for V .

Let A be the matrix corresponding to T with respect to the basis β and B the matrix corresponding to T with respect to the basis γ .

Then A is similar to B !

$$B = P^{-1} A P$$

$P \rightarrow$ Express γ in terms of β .
 $P^{-1} \rightarrow$ Express β "सिद्धिर्भवति कर्मजा" γ .

Linear transformations and similarity of matrices

Consider a linear transformation $T : V \rightarrow V$ and two ordered bases β and γ for V .

Let A be the matrix corresponding to T with respect to the basis β and B the matrix corresponding to T with respect to the basis γ .

Then A is similar to B !

Why do we care about similarity?

Linear transformations and similarity of matrices

Consider a linear transformation $T : V \rightarrow V$ and two ordered bases β and γ for V .

Let A be the matrix corresponding to T with respect to the basis β and B the matrix corresponding to T with respect to the basis γ .

Then A is similar to B !

Why do we care about similarity? Because under some basis, we hope that the corresponding matrix is a diagonal matrix which gives an easy geometric understanding of the linear transformation.

Thank you

