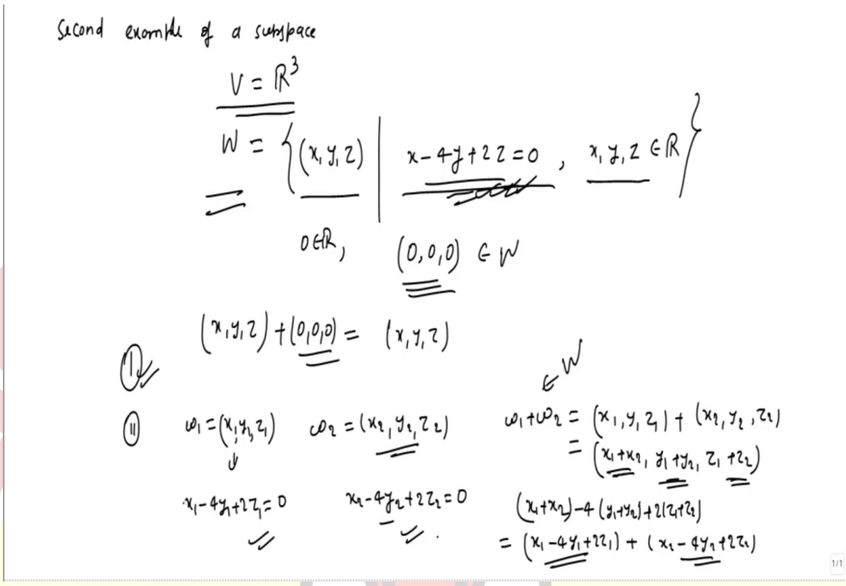
IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
Professor Sarang S. Sane
Department of Mathematics
Indian Institute of Technology Madras
Week-6 Tutorial 2

(Refer Slide Time: 00:14)



Now let us discuss another example of a subspace. So, suppose given vector spaces $\mathbb{R}^3$, we know that $\mathbb{R}^3$ is a form of vector space over $\mathbb{R}^1$ and we let W is a subset which having the coordinate x, y, z and with the condition $x - 4y + 2z = 0$ and these x, y, z are taken from R.

So, now, we need to check is this W follow those three criteria, which we discussed earlier to check that W is a subspace or not. So, we know that 0 belongs to $\mathbb{R}^1$. So, if you take 0, 0, 0 so this is an element of W, because this is follow the condition. Now, this is this element is 0 element of W, so let us take an element x, y, z and we add with 0 this 0 element, this element. So, we will get the same coordinate while addition will happen.

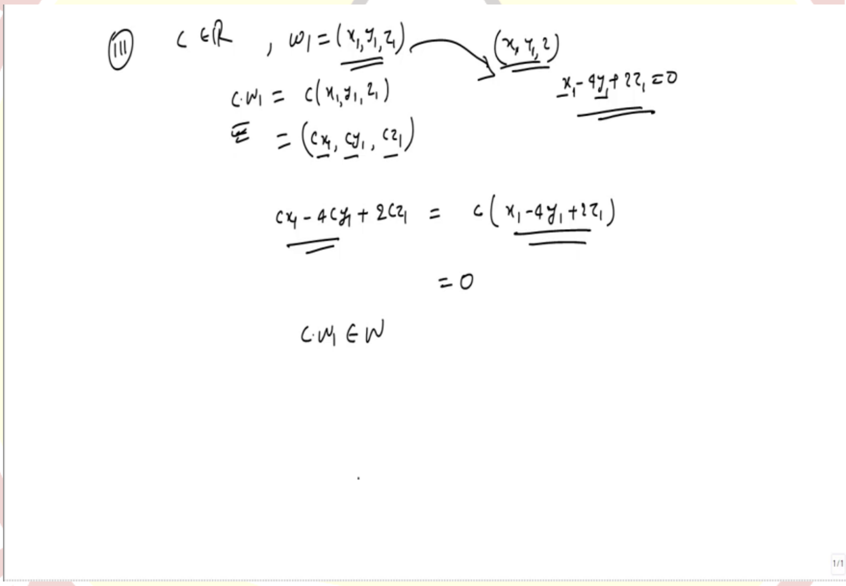
So, because as we discussed the addition and scalar, that operation which we which will be in $\mathbb{R}^3$ follow a W. So, it means coordinate y addition will happen so it will again x, y, z, so it means 0, 0, 0 is a 0 element of W. So, it means first test passed by W. Now, we need to check the second criteria, the second criteria is W should be closed under addition.

So, it means suppose W1 is an element of W, which is $x_1 y_1 z_1$ and W_2 is another element of W which is $x_2 y_2 z_2$. Now W1 is element of W it means it would follow the condition this it means $x_1 - 4y_1 + 2z_1 = 0$ and this will also follow the condition it means $x_2 - 4y_2 + 2z_2 = 0$.

Now, adding this W₁ + W₂ this will become $x_1, y_{1,} z_1 + x_2, y_{2,} z_2$ the coordinate as addition will happen. So, it will become the $x_1 + x_2, y_1 + y_2, z_1 + z_2$. Now, let us check is these three coordinates follows this condition or not. Then if this follow, if these recordings follow this condition, then we will say W is closed under addition. So, let us check $x_1 + x_2 - 4(y_1 + y_2) + 2(z_1 + z_2)$.

Now, if we arrange this term, because we can swap because these elements are from $\mathbb{R}^3$ and $\mathbb{R}^3$ having those things and we can arrange $x_1 - 4y_1 + 2z_1$ and the second one we can arrange as $x_2 - 4y_2 + 2z_2$ and this is 0 from here and second one is 0 from here it means these three coordinates make 0 with some scalar multiplication means follows this condition. So, it means $W_1 + W_2$ is element of W, it means W is closed under addition.

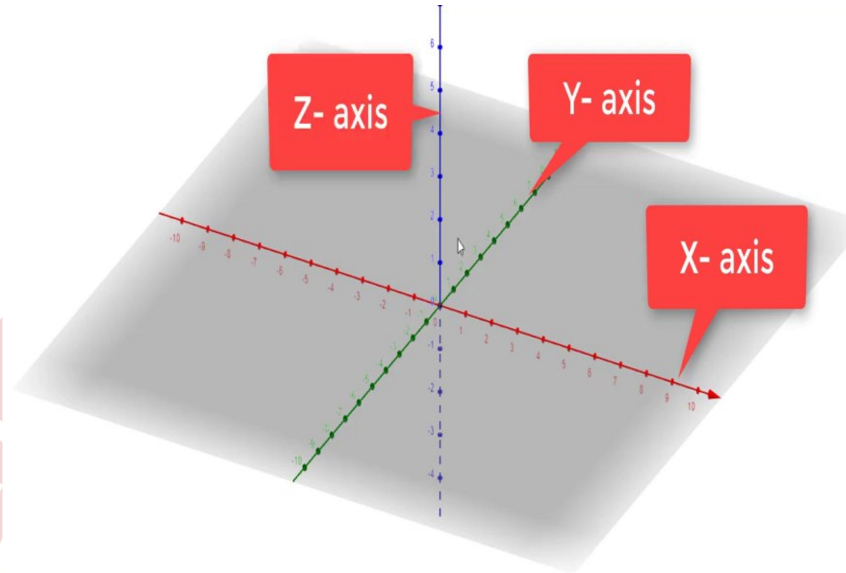
(Refer Slide Time: 04:35)

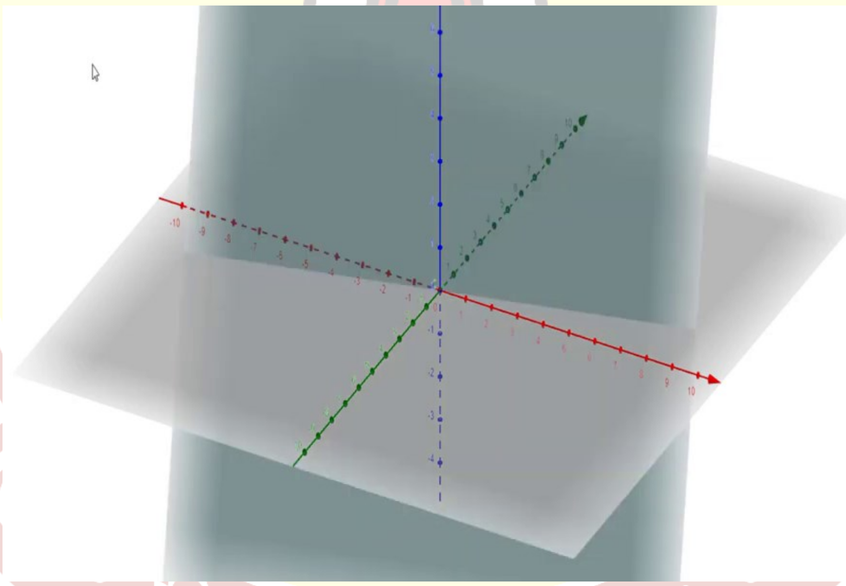


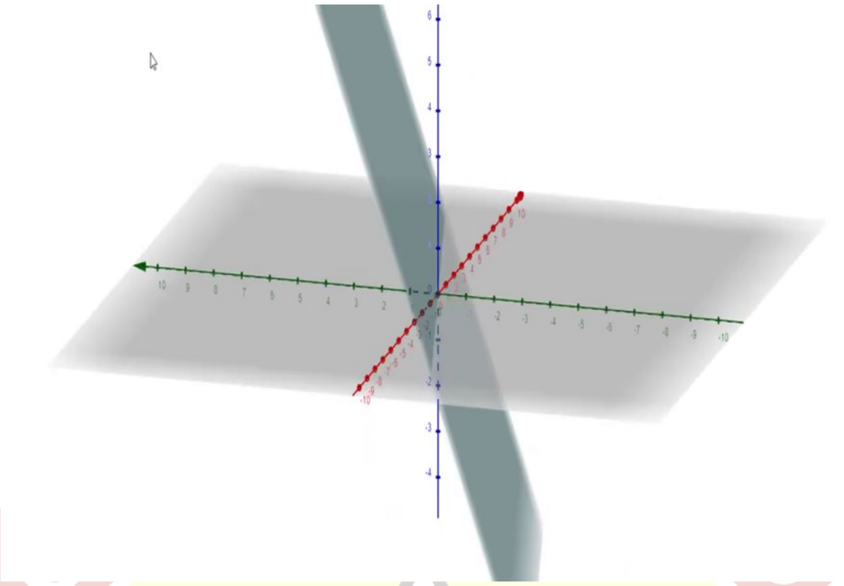
Now, we need to check the third criteria. The third criteria is, taking any event from real number and an element from W_1, from W which is let us say W_1, $x_1, y_{1,} z_1$ and this scalar multiplication $C \times W$. If this is belongs to W, then W will follow $\mathbb{R}^3$ criteria and W become a subspace. So, let us C time W_1 let us check, then it will become C times $x_1, y_{1,} z_1$ and the coordinate wise multiplication will happen because these elements are from $\mathbb{R}^3$ and $\mathbb{R}^3$ is a vector space. So, C × x_1, C × $y_{1,}$, C × z_1.

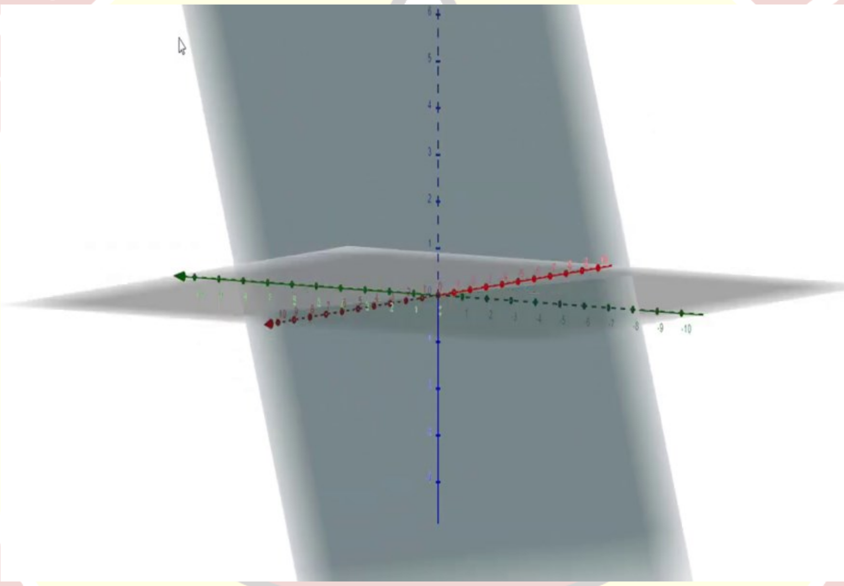
Now, C × x_1 - 4 × C$y_{1,}$ + 2 time, C z_1 which is actually you can write C we can take common. So, C× x_1 - 4 × C$y_{1,}$ + 2 × $z_{1,}$ time and actually W_1, $x_1, y_{1,} z_1$ is element of W so it will follow these, it means this is, this will happen with W_1, so it means this is 0, so it will become 0 it means this scalar multiplication is element of W. So it means, W follows all three criteria. So, W is a subspace of $\mathbb{R}^3$ over $\mathbb{R}^1$.

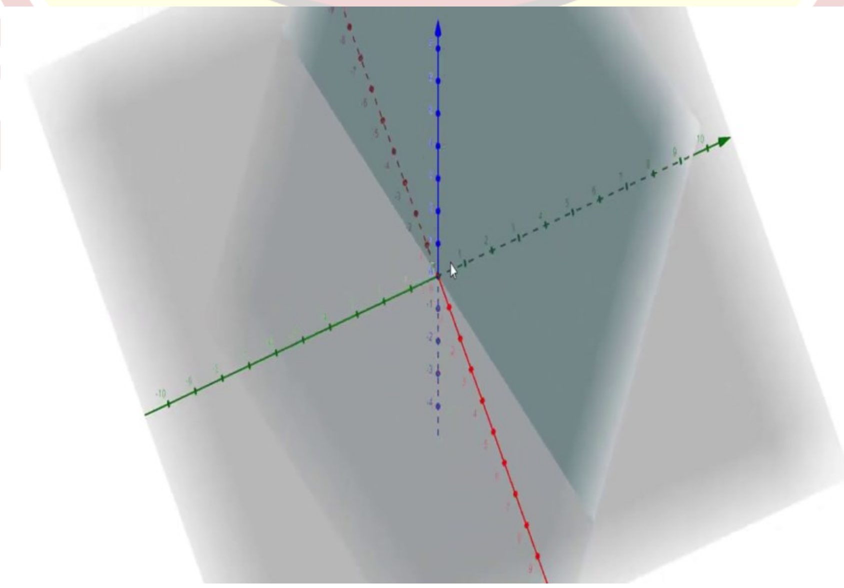
(Refer Slide Time: 06:35)











Now, let us see it geometrically. So, geometrically $\mathbb{R}^3$vector space $\mathbb{R}^3$ over $\mathbb{R}^1$ means, x axis y axis and z axis. So, this red line is actually x axis, this green line is y axis and this blue line is z axis. So, this is we can say this is a vector space whole $\mathbb{R}^3$.

Now our subspace is given x, y, z $c$oordinate with condition $x - 4y + 2z = 0, x, y\ z$ is from $\mathbb{R}^1$. So, actually, $x - 4y + 2z$ is actually a plane, which we can see it here. This is the actually, this in $\mathbb{R}^3$ this is the subspace, this is the W, which having condition $x - 4y + 2z$ =0. So actually, this is representing in $\mathbb{R}^3$ a plane, which passes through origin. So, this plane is actually passes through origin, we can see it here.