INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

# IIT Madras

ONLINE DEGREE

**Mathematics for Data Science 2**
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**Week 8 Tutorial 01**

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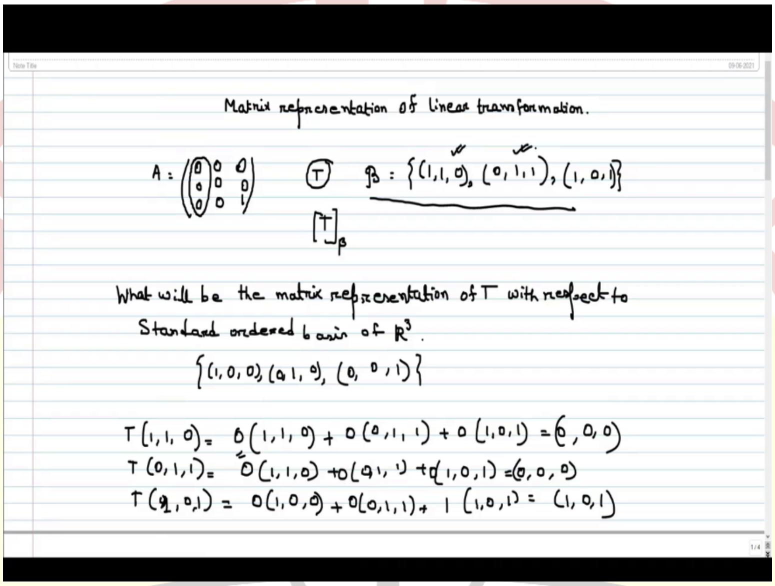


Hello everyone. Welcome to the tutorial video. So, in this video let us consider a matrix representation of linear transformation. So, suppose I am considering a matrix to be like this $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and I am saying that this is the matrix representation of T for some linear transformation T and my ordered basis which I have taken, let us consider it as β which is nothing but (1, 1, 0); (0, 1, 1), and (1, 0, 1).

So, with respect to these ordered basis for some linear transformation T I am getting my matrix representation of this linear transformation to be this A matrix. So, I can write it as T with respect to β. So, β for both domain and coronary. I am considering this order basis in both the sets. Now my question is what is the, what is the matrix representation of T? The matrix representation of T with respect to standard ordered basis of $\mathbb{R}^3$.

So, here what we are doing everything is in $\mathbb{R}^3$. So, standard ordered basis of $\mathbb{R}^3$ is nothing but (1, 0, 0); (0, 1, 0) and (0, 0, 1). So, we have to find the matrix representation of T with respect to standard ordered basis and so what we do? We will start with this given representation. So, here observe that T (1, 1, 0) is nothing but 0 × 1, 1, 0 + 0 × 0, 1, 1 + 0 × 1, 0, 1.

So, with respect to these ordered basis β; with this β, if I express this first vector T (1, 1, 0), then I am getting the coefficient for these vectors as 0, 0 and 0 and that is what come in this column. So, as this first column is nothing but 0, 0, 0, so the image of this first vector 1, 1, 0 should be 0. That is what the matrix representation is. So, similarly the second column is also 0, so the image of 0, 1, 1 is again the same thing 0 × 1, 1, 0 + 0 × 0, 1, 1 and 0 × 1, 0, 1; again it will be 0.

And what about the third one? The third column is 0, 0, 1, so if I take this last vector 1, 0, 1, so for the first element it will be 0 because 0 is in the first position of the first row of the third column. So, and the second is also 0, but the third will be 1. That is why we get 0, 0, 1 in the third column. So, it will give us 1, 0, 1. So, this is 0 mean 0 , 0 , 0 as we are considering in R3. So, these are the images of the given vectors in β. So, what we get here, just we will write just in the next page.

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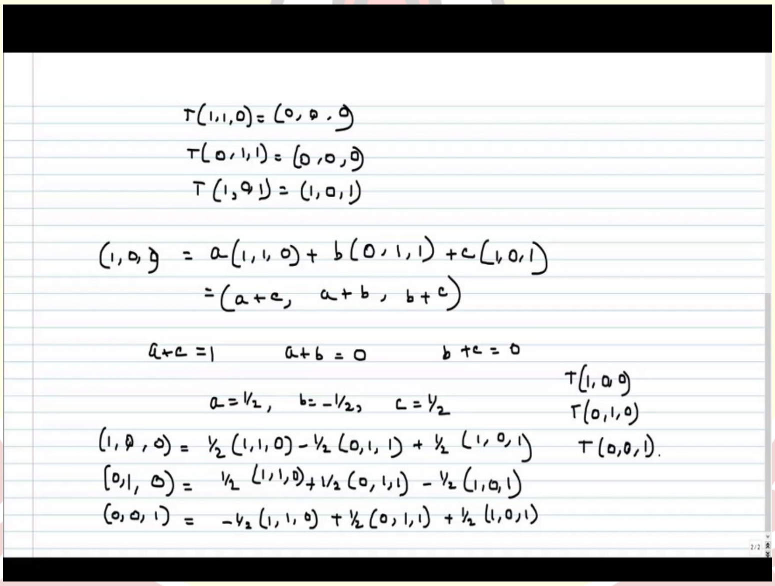


So, let us write what we have got here. T of 1, 1, 0 that is 0. T of 0, 1, 1 that is again 0 and T of 1, 0, 1 this is 1, 0, 1, the same write there. So, this we have got. Now what we have to do? We have to express the standard ordered basis with respect to these ordered basis. So, basically what we have to do? We have to express 1, 0, 0 the first vector in the standard ordered basis in terms of this vectors of β.

So, let me write express 1, 0, 0 as a linear combination of these 3 vectors. So, this is the, like this expression. So, it is $a + c, a + b$ and $b + c$. So, $a + c$ will be 1, $a + b$ will be 0 and $b + c$ will be 0 again. So, if we solve these 3 equations, then we get our a to be 1/2, b to be -

1/2 and c to be 1/2. If you solve these 3 equations, you will get this as a solution, this is a unique solution.

So, 1, 0, 0 is nothing but 1/2 of this first vector 1 , 1 , 0; − 1/2 of the second vector 0 , 1 , 1 + 1/2 of this third vector that is 1 , 0 , 1. So, you can express (1, 0, 0) in terms of these vectors in β. Similarly, we can do for others. So, others I am just writing here. 0, 1, 0 it will be 1/2 of 1 , 1 , 0 + 1/2 of 0 , 1 , 1 - 1/2 of 1 , 0 , 1.

And for the last one, 0 , 0 , 1 is nothing but - $\frac{1}{2}$ of 1 , 1 , 0 + 1/2 0 , 1 , 1 + 1/2 1 , 0 , 1. So, we expressed all the 3 vectors in standard ordered basis in terms of the basis which was given as β. So, we got this expression. Now what we have to do? We have to apply T here. So, we have to find the image of T( 1 , 0 , 0). Similarly, we have to find T of 0 , 1 , 0 and T of 0 , 0 , 1.

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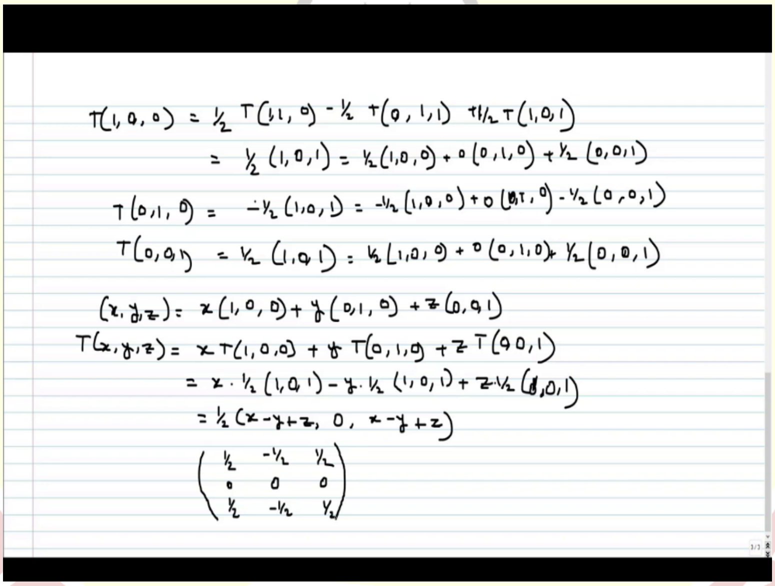


Now, if we apply T on 1 , 0 , 0 in this expression, so T is a linear transformation, so we can take this scalar out and it will be T 1/2 × 1 , 1 , 0 - 1/2 × T of 0 , 1 , 1 and 1/2 × T of 1 , 0 , 1. So, for the first two vectors T of 1 , 1 , 0 that is 0. T of 0 , 1 , 1 that is again 0. So, it is nothing but 1/2 × T of 1 , 0 , 1 which is the same T of 1 , 0 , 1. So, we get this vector.

Similarly, for the other two we can calculate that T of 0 , 1 , 0 is again - 1/2 of 1 , 0 , 1 and T of 0 , 0 , 1 is 1/2 of 1 , 0 , 1. So, this we got. Now any vector x, y, z can be written as x × 1 , 0 , 0 + y × 0 , 1 , 0 + z × 0 , 0 , 1. So, this we can do. So, T of x, y, z is nothing but x of T of 1 , 0 , 0.

Again we are using the properties of linear transformation and we are getting this T of 0 , 1 , 0 + z × T of 0 , 0 , 1. So, if we write this, if we put the values of T of 1 , 0 , 0 which we have calculated, so we get x × 1/2 × 1 , 0 , 1 - y × 1/2 × y , 0 , 1 + z × 1/2 0 ,, sorry, 1 , 0 , 1. So, if we calculate this together and put all the things together, we get 1/2 × x - y + z, 0, x - y + z.

So, this is the expression for T of x, y, z which were intended to find. Now we want to find the matrix representation of T with respect to standard ordered basis. So, for standard ordered basis T of 1 , 0 , 0 we already have got the image. So, we can just express it in terms of standard ordered basis and we will get this 1/2 × 1 , 0 , 0 + 0 × 0 , 1 , 0 + 1/2 × 0 , 0 , 1.

Similarly, for the second one, it will be just - 1/2 in these two places and 0 in the middle one. So, this is 0 , 0 , 1 and similarly for the third one again it will be 1/2 × 1 , 0 , 0 + 0 × 0 , 1 , 0 + 1/2 × 0 , 0 , 1. So, the matrix representation we got here with respect to standard ordered basis will be 1/2, 0, 1/2, - 1/2, 0, - 1/2, 1/2, 0, 1/2. So, this will be the matrix representation with respect to standard ordered basis of the given matrix which we have started.

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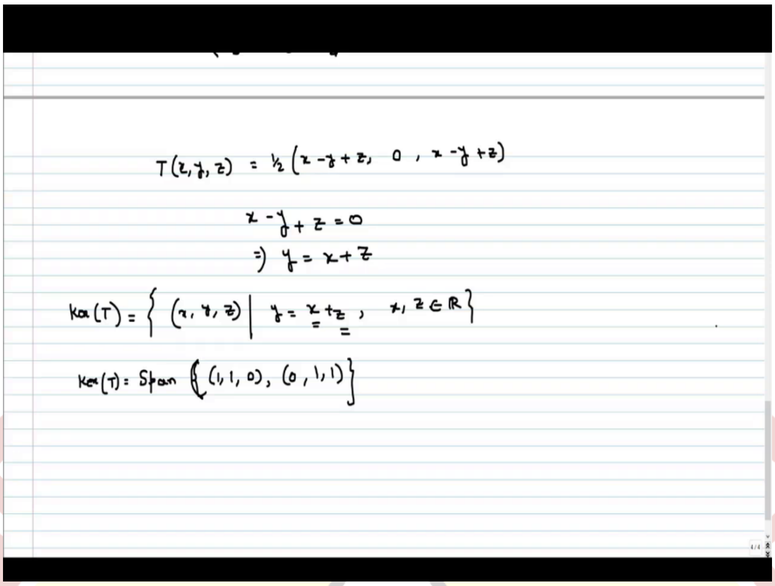


Now, for the same linear transformation which is T x, , y, , z is going to 1/2 of x - y + z , 0 , x - y + z. For this linear transformation if we want to visualize the kernel, so what we have to do? Basically, this image should I mean, kernel will about all those vectors x , y , z so that the image will be 0. So, if the image will be 0, we get x - y + z equal to 0 and that will give us y equal to x + z.

So, all these vectors x , y , z such that y equal to x + z and everything is from real x, z is from real, then this vector space is our kernel of T. So, this is our kernel of T. So, what will be the basis of this kernel? So, there are two independent variable x and z so we can get kernel as the span of these two vectors. So, if we put x, if we 1 in place of x we will get 1 , 1 , 0 and if we put 1 in place of z and 0 in place of x, we will get 0 , 1 , 1. So, these two stand out, these two vectors will be our kernel of T. So, this is our kernel of T.

So, you can also calculate this by using row operations on this matrix what we have got here. if we can do the row operation, you will again see that the one vector will be dependent variable and two are independent variables, so kernel will be of dimension 2 and it will be scanned by these two vectors. And so now if we can go back earlier from where we have started, you can see that for the first two vectors of these basis β, this 1 , 1 , 0 and 0 , 1 , 1 for these two vectors, the image is nothing but 0.

So, that we have written here, these two vectors, for these vectors the image are going to be 0 and for the last one the image is the same vector. So, for span of these two has vectors 1 , 1 , 0 and 0 , 1 , 1 will form a, these two vectors will form a basis of kernel of T. So, that we have seen in this example. Now let us try to visualize these thing in Algebra.

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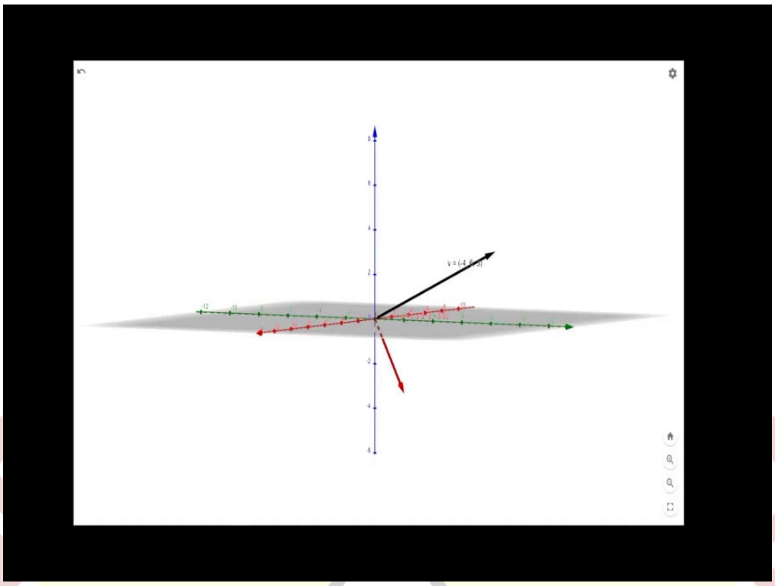

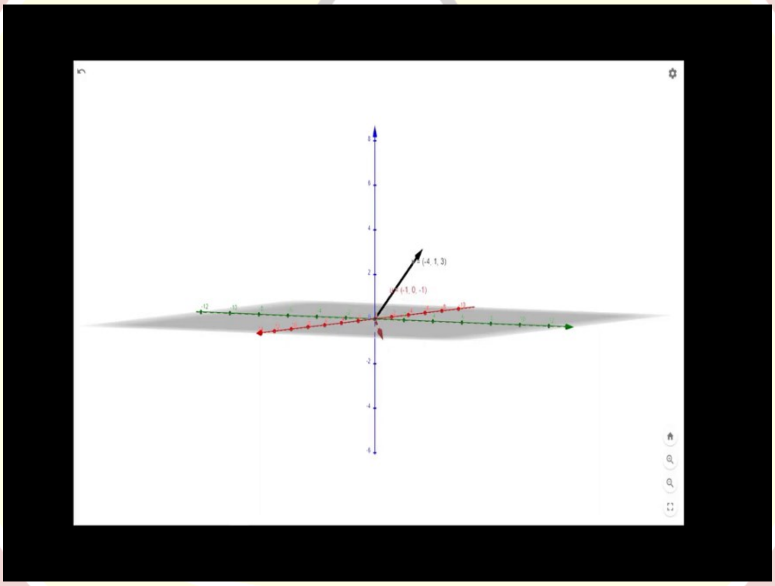

So, now if in three dimension plane in R3 if we choose the vector - 4 , 6 , 3 which we have seeing in the figure, in the diagram, then the image of this will be this new vector which we are seeing that is - 3.5 , 0 , - 3.5. So, now if we change the vectors there, we will see the image will also change here.

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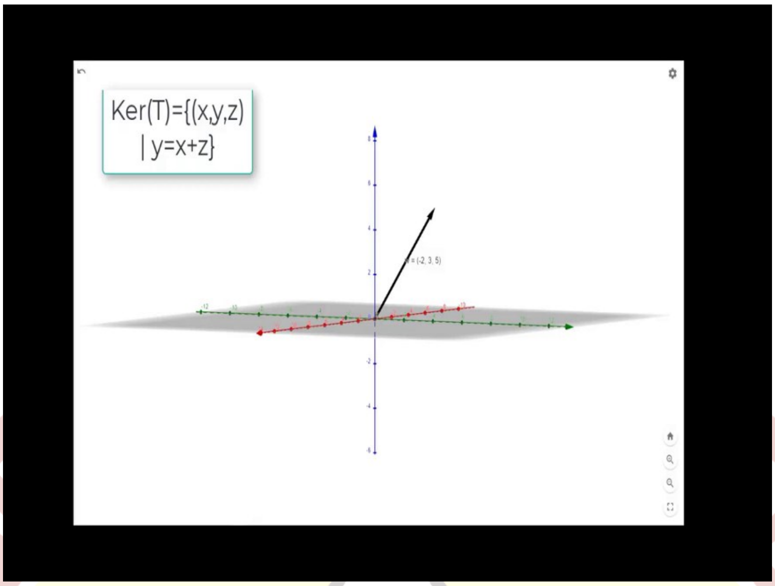


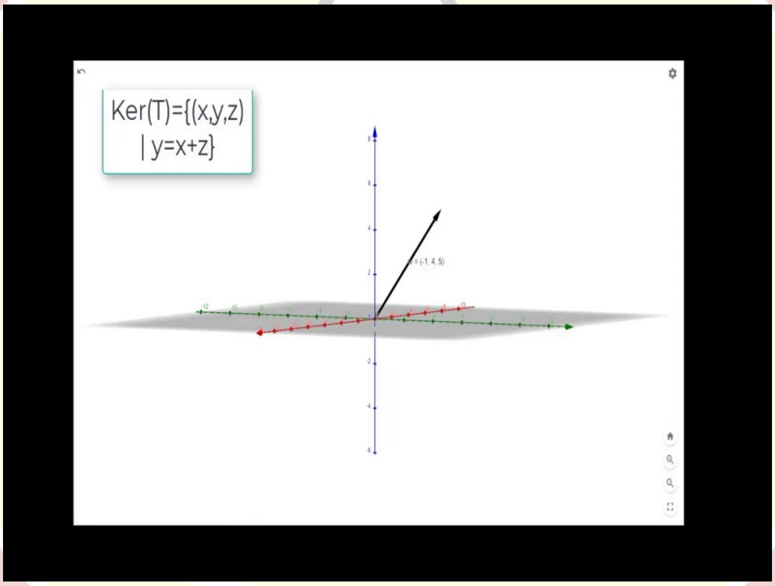


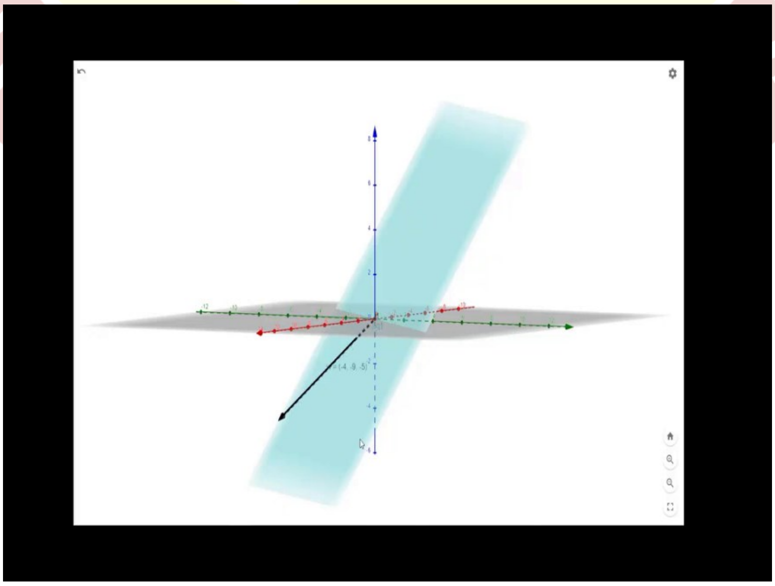

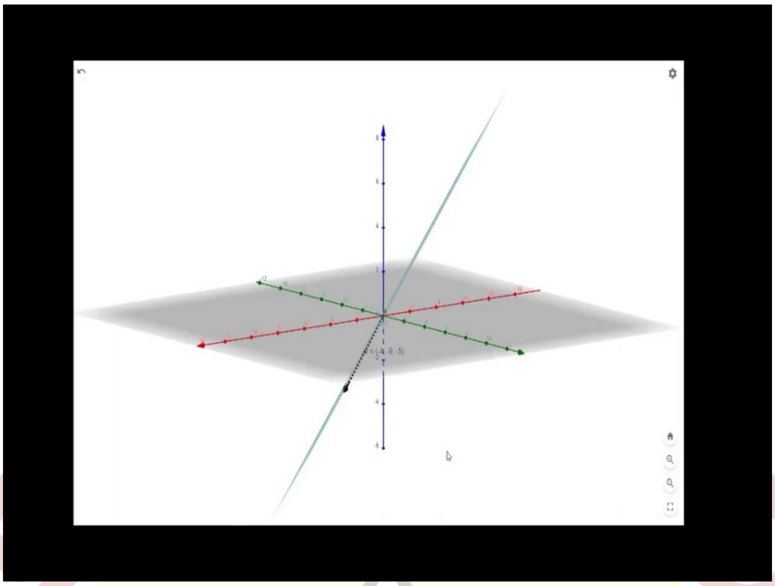

Now, if we only consider the vectors of this form such that x , y , z such that y equal to x + z which we have seen that is the kernel of the transformation. So, how the vectors will look like, so for that thing the vectors will look like this and in that case if we choose all the vectors in this form as we can see, so - 2 , 3 , 5. So, that means 5 + - 2 that will give us 3.

So, the second coordinate is basically the sum of the first coordinate and the third coordinate. If we consider these vectors only, then the image will always be 0. So, if we vary this, all these form of vectors, all these kind of vectors which are in this kernel, we will see that the image will be nothing but 0.

So, all these vectors which are of this form will lie in the plane as you can see here, it will lie in the plane as you can see this all these vectors are on the same plane and the plane is nothing but x - y + z equal to 0 that is y equal to x + z. So, all these vectors will lie in the plane and the image of these vectors will be 0, so this plane is basically the kernel of this linear transformation.

So, this is a two dimension vector space passing through origin and so this is the geometrical representation of the kernel. Thank you.