



IIT Madras
ONLINE DEGREE

Mathematics for Data Science 2
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Basis
 $V = M_{2 \times 2}(\mathbb{R})$

Hello friends, in this video we will discuss about basis. So, first of all what is basis? Basis is your set, which is collection of linearly independent vector from the vector space and spanning set of these vectors is the given whole vector space.

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$V = M_{2 \times 2}(\mathbb{R})$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + a_{12} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + a_{21} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + a_{22} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

\Rightarrow

$$\alpha \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \delta \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \alpha = 0, \beta = 0, \gamma = 0, \delta = 0$

$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

Now, consider a vector space V , which is a set of all 2×2 matrices over R . So, we know that this is a vector space over R with as usual matrix addition and scalar multiplication with a matrix. So, this set forms a vector space over R . Now let A be a 2×2 matrix, we are writing as $a_{11}, a_{12}, a_{21}, a_{22}$, so this matrix represents a central element in $M_{2 \times 2}$ set of all 2×2 matrices.

Now, this matrix we can write it as $a_{10} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + a_{12} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + a_{21} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + a_{22} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. So, this matrix is actually linear combination of these matrices. And we are saying any matrix will be in this form, and this matrix can be generated by these four matrices. Now, let us check if these

matrices are linearly independent? So, let us say $\alpha \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \delta \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, which we can write it is equal to 0. So, this is the way to checking linearly independent.

Here, if we compare these imply that $\alpha = 0, \beta = 0, \gamma = 0, \delta = 0$, these matrices are linearly independent, and this matrix can be generated by these four matrices.

And we have seen that these matrices are linearly independent. It means linear combination of these matrices, generates whole 2×2 , and these are linearly independent. So this is form a basis of set of all 2×2 matrices.