Week-5
Mathematics for Data Science - 2
Solutions of System of Linear Equations
Graded Assignment Solutions

1 Multiple Choice Questions (MCQ)

1. Consider the system of equations given below:

$$\frac{x^{2021}}{2021} + \frac{y^{2021}}{2021} - \frac{z^{2021}}{2021} = \pi$$

$$\frac{x^{2021}}{2021} + \frac{y^{2021}}{2021} - \frac{z^{2021}}{2021} = e$$

$$\frac{x^{2021}}{2021} - \frac{y^{2021}}{2021} + \frac{z^{2021}}{2021} = 1729.$$

The system has

- ◯ **Option 1:** no solution.
- ◯ Option 2: a unique solution.
- ◯ Option 3: infinitely many solutions.
- ◯ Option 4: finitely many solutions.

Solution:
Observe that left side of first two equations in the system of linear equations are the same.
This implies that $\pi = e$ which is not true.
Hence the system of linear equations has no solution.

2. Let the reduced row echelon form of a matrix A be

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{6} \\ 0 & 0 & 1 & \frac{1}{6} \end{bmatrix}.$$

If the first, second and fourth columns of A are $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ then the third column of matrix A is,

- ○ Option 1: $\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$
- ○ Option 2: $\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$
- ○ **Option 3:** $\begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$
- ○ Option 4: $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Solution:
Let us assume the elements of the third column of matrix A be x, y, z such that

$$A = \begin{bmatrix} 1 & 3 & x & -1 \\ 0 & 2 & y & 0 \\ -1 & 1 & z & 0 \end{bmatrix}$$

To transform this matrix into reduced row echelon form, we do the following row operations in the same sequence:
$R_3 + R_1, R_2/2, R_3 - 4R_2, R_1 - 3R_2, R_3/(-6), R_2 - R_3, R_1 + 3R_3$.

Now, the transformed matrix is $\begin{bmatrix} 1 & 0 & \frac{x-y-z}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{y+z+x}{6} & -\frac{1}{6} \\ 0 & 0 & \frac{-z-x+2y}{6} & \frac{1}{6} \end{bmatrix}$.

Compare this matrix with the reduced row echelon form of the matrix A.
We get,

$$\frac{x-y-z}{2} = 0, \frac{y+z+x}{6} = 0, \frac{-z-x+2y}{6} = 1.$$

Solving these three equations, we get, $x = 0, y = 2, z = -2$ which are the elements of the third column of the matrix A.

2 Multiple Select Questions (MSQ)

3. In a particular year, the profit (in lakhs of ₹) of Star Fish company is given by the polynomial $P(x) = ax^2 + bx + c$ where x denotes the number of months since the beginning of the year (i.e., $x = 1$ denotes January, $x = 2$ denotes February, and so on). In January and February the company made a loss of ₹45(in lakhs), and ₹19(in lakhs) respectively, and in March the company made profit of ₹3(in lakhs). Let the loss be represented by negative of profit.
Choose the correct set of options based on the given information.
   - ◯ Option 1: The maximum profit will be in the month of May.
   - ◯ **Option 2:** The maximum profit will be in the month of August.
   - ◯ **Option 3:** The maximum monthly profit amount is ₹53 lakh.
   - ◯ Option 4: The maximum monthly profit amount is ₹35 lakh.

Solution: Based on the given information of losses in January and February, and profit in March, we formulate the following set of equations:

$$\begin{aligned} a + b + c &= -45 \\ 4a + 2b + c &= -19 \\ 9a + 3b + c &= 3 \end{aligned}$$

This is a system of linear equations where a, b, c being the unknowns. The augmented matrix is written as:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -45 \\ 4 & 2 & 1 & -19 \\ 9 & 3 & 1 & 3 \end{array}\right]$$

The following sequence of row operations are performed to transform this augmented matrix into reduced row echelon form:

$$R_2 - 4R_1, R_3 - 9R_1, \frac{-1}{2}R_2, R_1 - R_2, R_3 + 6R_2, R_1 + \frac{1}{2}R_3, R_2 - \frac{3}{2}R_3.$$

The resultant augmented matrix is given by

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 32 \\ 0 & 0 & 1 & -75 \end{array}\right]$$

Thus, we find that $a = -2, b = 32, c = -75$. Now, we can represent the polynomial as $P(x) = -2x^2 + 32x - 75$. We have to find the point of global maximum and the corresponding maximum value of this function. Differentiating $P(x)$, we get $P'(x) = -4x + 32$. We get $x = 8$ as a critical point. Note that $P''(x) = -4$. Thus, the function has a maximum at the critical point. So, $P(8) = 53$ which is the maximum monthly profit and it happens in the month of August ($x = 8$).

4. If A be a 3×4 matrix and b be a 4×1 matrix, then choose the set of correct options.

   - ◯ **Option 1:** If $(A|b)$ be the a ugmented matrix and $(A'|b')$ be the matrix obtained from $(A|b)$ after a finite number of elementary row operations then the system $Ax = b$ and the system $A'x = b'$ have the same set of solutions.
   - ◯ Option 2: If $(A'|b')$ is the reduced row echelon form of $(A|b)$ then the system $A'x = b'$ has at least one solution.
   - ◯ **Option 3:** If $(A'|b')$ is the reduced row echelon form of $(A|b)$, then A' is also in reduced row Echelon form.
   - ◯ **Option 4:** If $(A'|b')$ is the reduced row echelon form of $(A|b)$ and there is no row such that the only non zero entry lies in the last column of $(A'|b')$ then the system $Ax = b$ has at least one solution.

Solution:

Option 1: We know from the Gauss elimination method that any number of elementary row operations on an augmented matrix $(A|b)$ does not alter the solutions of $Ax = b$. Hence this option is true.

Option 2: Note that if there is a row with all zeros in the row echelon form such that the corresponding row in the b vector is non-zero, then we have no solution for the system $Ax = b$.

For example, Let

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

be the reduced row echelon form of a augmented matrix of a system of linear equations. Observe that the system of linear equation has no solution.

Option 3: Observe that when we transform the augmented matrix into reduced row echelon form, the coefficient matrix A also gets transformed into reduced row echelon form.

Option 4: Observe that if $(A'|b')$ is the reduced row echelon form of $(A|b)$ and there is no row such that the only non zero entry lies in the last column of $(A'|b')$, then all variable can be dependent or there can be at least one variable which will be independent. In both the cases the system of linear equations has at least one solution.

To better understand this statement, consider the following example

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & b_1 \\ 0 & 1 & 0 & 0 & b_2 \\ 0 & 0 & 1 & 0 & b_3 \end{array}\right]$$

Since there is no row such that the only non-zero entry lies in the last column, x_4, the unknown variable, becomes an independent variable. Thus, there can be infinitely many solutions for the system $Ax = b$.

5. Choose the set of correct options

- ◯ **Option 1:** If the sum of all the elements of each row of a matrix A is 0, then A is not invertible.
- ◯ **Option 2:** If E is a matrix of order 3×3 obtained from the identity matrix by a finite number of elementary row operations then E is invertible.
- ◯ Option 3: Any system of linear equations has at least one solution.
- ◯ Option 4: If A is a matrix of order 3×3 and $det(A) = 3$ then $det(Adj(A)) = 3$.
- ◯ **Option 5:** If A is a matrix of order 3×3 and $det(A) = 3$ then $det(Adj(A)) = 9$.

Solution:

Note: we have shown a 3×3 matrix here only as an example. We cannot consider A to be a $m \times n$ matrix $(m \neq n)$ because inverse exists only for square matrices (where determinant is defined).

Option 1: Consider a matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

Given

$$\begin{aligned} a_{11} + a_{12} + a_{13} &= 0 \\ a_{21} + a_{22} + a_{23} &= 0 \\ a_{31} + a_{32} + a_{33} &= 0 \end{aligned}$$

$$\begin{aligned} \text{So, } det(A) = & \text{ determinant of } \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \\ = & \text{ determinant of } \begin{bmatrix} a_{11} & a_{12} & a_{11} + a_{12} + a_{13} \\ a_{21} & a_{22} & a_{21} + a_{22} + a_{23} \\ a_{31} & a_{32} & a_{31} + a_{32} + a_{33} \end{bmatrix} \\ = & \text{ determinant of } \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 0 \end{bmatrix} \end{aligned}$$

$\implies det(A) = 0$

Hence matrix A is not invertible. Similarly, we can show for a square matrix of any order.

Option 2: Observe that if E is a 3×3 matrix obtained from $I_{3\times3}$ using finite number of elementary row operations, then $det(E) = c.det(I_{3\times3})$, where c is any non zero real number. so $det(E) = c \neq 0$ (Since $\det(I_{3\times3} = 1$).

Hence, E is invertible.

Option 3: A system of linear equations can have a unique solution, infinitely many solutions or no solution.

For example, consider a system linear equation with only one equation $x + y + z = 1$.

Observe that this system of linear equation has infinite many solutions.

Options 4 and 5: Let A be a square matrix of order 3.
we know that $A^{-1} = \frac{adj(A)}{det(A)}$
$\implies (det(A))A^{-1} = adj(A)$
$\implies det((det(A))A^{-1}) = det(adj(A))$
$\implies det(adj(A)) = det(A)^n det(A^{-1})$ Since $det(k.A) = k^n det(A)$
$\implies det(adj(A)) = det(A)^{n-1}$ Since $det(A^{-1}) = \frac{1}{det(A)}$

So, $det(Adj(A)) = 3^{3-1} = 9$.

6. Ramya bought 1 comic book, 2 horror books, and 1 novel from a bookshop which cost her ₹1000. Romy bought 2 comic books, 5 horror books, and 1 novel which cost him ₹2000. Farjana bought 4 comic books, 5 horror books, and c novel from a shop which cost her ₹d. If $x_1, x_2,$ and x_3 represent the price of each comic book, horror book, and novel, respectively, then choose the set of correct options.

   ◯ **Option 1:** The matrix representation to find x_1, x_2 and x_3 is
   $$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 1 \\ 4 & 5 & c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2000 \\ d \end{bmatrix}$$

   ◯ **Option 2:** The matrix representation to find x_1, x_2 and x_3 is
   $$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 5 \\ 1 & 1 & c \end{bmatrix} = \begin{bmatrix} 1000 & 2000 & d \end{bmatrix}$$

   ◯ Option 3: The matrix representation to find x_1, x_2 and x_3 is
   $$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 5 \\ 1 & 1 & c \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 1000 & 2000 & d \end{bmatrix}$$

   ◯ Option 4: If Farjana tries to find $x_1, x_2,$ and x_3 using appropriate matrix representation by taking $c = 2$ and $d = 4000$, then the price of each comic book that she thus arrives at, will not be unique.

   ◯ **Option 5:** If $c = 7$ and $d = 4000$, then the price of each comic book cannot be determined from this data.

   ◯ **Option 6:** If $c = 7$ and $d = 3000$, then the shopkeeper has made a mistake.

   ◯ **Option 7:** If $c = 2$ and $d = 3000$, then the price of each comic book can be determined from the data.

Solution:
Given $x_1, x_2,$ and x_3 represent the price of each comic book, horror book, and novel, respectively.
So, prices of 1 comic book, 2 horror books, and 1 novel are $x_1, 2x_2$ and x_3 respectively. Similarly, we can get for others.
So, we write three equations for the total price of all the books purchased by Ramya, Romy and Farjana respectively as the following:

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1000 \\ 2x_1 + 5x_2 + x_2 &= 2000 \\ 4x_1 + 5x_2 + cx_3 &= d \end{aligned}$$

Option 1: The matrix representation of the system of linear equations $Ax = b$ is given by

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 1 \\ 4 & 5 & c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2000 \\ d \end{bmatrix}$$

Option 2 and 3: Observe that $(Ax)^T = b^T \implies x^T A^T = b^T$. Thus,

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 5 \\ 1 & 1 & c \end{bmatrix} = \begin{bmatrix} 1000 & 2000 & d \end{bmatrix}$$

Now, the augmented matrix of the above system of linear equations is

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1000 \\ 2 & 5 & 1 & 2000 \\ 4 & 5 & c & d \end{array}\right]$$

In order to find whether the system of equations has no solution, unique solution or infinitely many solutions, we transform the obtained augmented matrix to a row echelon form using the following sequence of row operations:

$$R_2 - 2R_1, R_3 - 4R_1, R_1 - 2R_2, R_3 + 3R_2.$$

The resultant augmented matrix is given by

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1000 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & c-7 & d-4000 \end{array}\right]$$

Option 4: Substitute $c = 2, d = 4000$, we get

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1000 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -5 & 0 \end{array}\right]$$

Observe that in this case the system has a unique solution.
Option 5: Now, substitute $c = 7, d = 4000$, we get

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1000 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Observe that in this case the system has infinitely many solutions.

Hence the price of each comic book cannot be determined from this data.
Option 6: Now, substitute $c = 7, d = 3000$, we get

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1000 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1000 \end{array}\right]$$

Observe that in this case the system has no solutions.
Hence the shopkeeper has made a mistake.
Option 7: Now, substitute $c = 2, d = 3000$, we get,

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1000 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -5 & -1000 \end{array}\right]$$

Thus, we get $x_3 = 200, x_1 + 3x_3 = 1000 \implies x_1 = 400$.
Hence the price of each comic book can be determined from the data.

3 Numerical Answer Type (NAT):

7. Three mobile shops- shop A, shop B and shop C, sell three brands of mobile phones: brand R, brand S and brand T. In a week, shop A sold 1 mobile phone of brand R, $3k$ mobile phones of brand S, and $3k+4$ mobile phones of brand T. Shop B sold 1 mobile phone of brand R, $k+4$ mobile phones of brand S, and $4k+2$ mobile phones of brand T. Shop C sold 1 mobile phone of brand R, $2k+2$ mobile phones of brand S, and $3k+4$ mobile phones of brand T (assume, $k \neq 2$). Assume that the price of a given model of a given brand is the same in all the shops. Shop A, shop B, and shop C earned ₹61, ₹65 and ₹66 (in thousands), respectively by selling these three brands of mobile phones. If the price of each mobile phone of brand S is ₹5 (in thousands), then what is the price of each mobile phone of brand T (in thousands)? [Note: Suppose the price comes out to be 20,000, then the answer should be 20]? [Answer: 6]

Solution:
Let x_R, x_S, x_T be the price of each mobile of brand R, S, T respectively.
So, In a week, shop A earned ₹$(x_R + 3kx_S + (3k+4)x_T)$ by selling 1 mobile phone of brand R, $3k$ mobile phones of brand S, and $3k+4$ mobile phones of brand T which is equal to ₹61 (in thousands). Similarly we can calculate amount in a week for the shop B and shop C.
Hence, The system of equations representing the total earnings of the shops A, B, C can be represented by the following:

$$\begin{aligned} x_R + 3kx_S + (3k+4)x_T &= 61 \\ x_R + (k+4)x_S + (4k+2)x_T &= 65 \\ x_R + (2k+2)x_S + (3k+4)x_T &= 66. \end{aligned}$$

So augmented matrix of the above system of linear equations is

$$\left[\begin{array}{ccc|c} 1 & 3k & 3k+4 & 61 \\ 1 & k+4 & 4k+2 & 65 \\ 1 & 2k+2 & 3k+4 & 66 \end{array}\right].$$

Now, using the following sequence of elementary row operations:

$$R_2 - R_1, R_3 - R_1, R_2/(4-2k), R_3/(2-k), R_3 - R_2, R_2 + R_3, 2R_3$$

Transformed the above augmented matrix into a row echelon form given below:

$$\left[\begin{array}{ccc|c} 1 & 3k & 3k+4 & 61 \\ 0 & 1 & 0 & \frac{5}{2-k} \\ 0 & 0 & 1 & \frac{6}{2-k} \end{array}\right]$$

Note that $k \neq 2$.
Based on the above, we can write $x_S = \frac{5}{2-k}, x_T = \frac{6}{2-k}$.
Since, the price of each mobile phone of brand S is ₹5 (in thousands) i.e., $x_S = 5$.
After substituting the value $x_S = 5$ in the equation $x_S = \frac{5}{2-k}$, we got $k = 1$ and so $x_T = 6$. (We can use cramer's rule also to solve this system of linear equations.)
Hence, the price of each mobile phone of brand T is ₹6 (in thousands).

4 Comprehension Type Question:

The network in Figure: M2W5GA1 shows a proposed plan for flow of traffic around a park. All the streets are assumed to be one-way and the arrows denote the direction of flow of traffic. The plan calls for a computerized traffic light at the South Street. Let $2x_1, 3x_2, 2x_3,$ and x_4 denote the average number (per hour) of vehicles expected to pass through the connecting streets (e.g., $2x_1$ denote the average number (per hour) of vehicles expected to pass through the street connecting the North Street and West Street as shown in Figure: M2W5GA1). 400, 1000, 900, and c denote the average number (per hour) of vehicles expected to pass through West, North, East, and South Streets respectively.

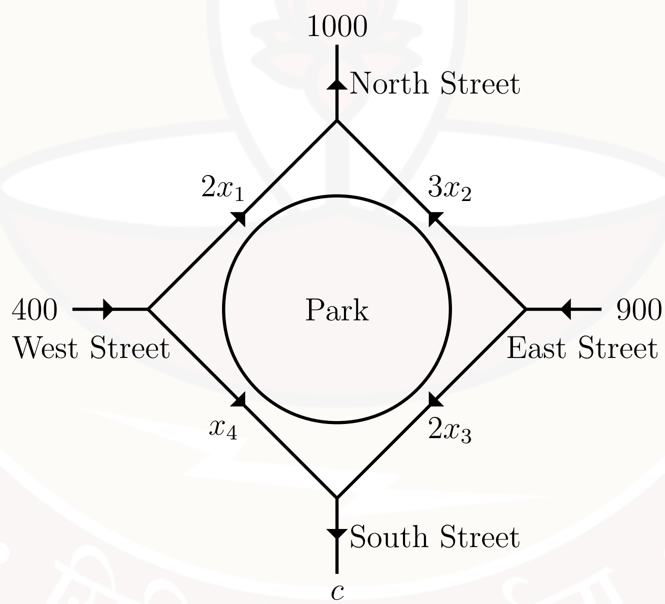

Figure: M2W5GA1

8. The system of equations corresponding to the flow of expected traffic according to the given data above, will be

◯ **Option 1:**

$$\begin{aligned} 2x_1 + 3x_2 &= 1000 \\ 3x_2 + 2x_3 &= 900 \\ 2x_3 + x_4 &= c \\ 2x_1 + x_4 &= 400 \end{aligned}$$

◯ Option 2:

$$\begin{aligned} 2x_1 + 3x_2 &= 900 \\ 3x_2 + 2x_3 &= 1000 \\ 2x_3 + x_4 &= 400 \\ 2x_1 + x_4 &= c \end{aligned}$$

◯ Option 3:

$$\begin{aligned} 2x_1 + 3x_2 &= 1000 \\ 3x_2 + 2x_3 &= c \\ 2x_3 + x_4 &= 900 \\ 2x_1 + x_4 &= 400 \end{aligned}$$

◯ Option 4:

$$\begin{aligned} 2x_1 + 3x_2 &= 400 \\ 3x_2 + 2x_3 &= 900 \\ 2x_3 + x_4 &= c \\ 2x_1 + x_4 &= 1000 \end{aligned}$$

Solution:
The average number of vehicles expected to pass through West street is 400. $2x_1$ and x_4 are the average number of vehicles expected to pass through the street connecting West and North and West and South streets respectively. Hence, $2x_1 + x_4 = 400$. Similarly, we arrive at the other three equations:
$2x_3 + x_4 = c$
$3x_2 + 2x_3 = 900$
$2x_1 + 3x_2 = 1000$.
Hence, option 1 is true.

9. How many vehicles per hour in average are expected to pass through the South Street? (NAT) [Answer: 300]

Solution:

According to the question we need to find the value of c.

The system of linear equation are:

$$2x_1 + 3x_2 = 1000 \quad ...(1)$$
$$3x_2 + 2x_3 = 900 \quad ...(2)$$
$$2x_3 + x_4 = c \quad ...(3)$$
$$2x_1 + x_4 = 400 \quad ...(4)$$

After subtracting of the both side of the equation from equation (4) to equation (3), and from equation (1) to equation (2) we get,

$2x_1 - 2x_3 = 400 - c$ and

$2x_1 - 2x_3 = 100$

Since left side of the both equations are the same.

Hence, $400 - c = 100 \implies c = 300$.

10. Match the names of the street in Column A with the maximum and minimum number of vehicles expected to pass through the street in average (per hour) in Column B and Column C, respectively; in Table M1W5GA1.

	Name of the connecting street		The maximum number of vehicles expected to pass through the street (per hour)		The minimum number of vehicles expected to pass through the street (per hour)
	Column A		Column B		Column C
a)	Connecting West and North street	i)	300	1)	0
b)	Connecting East and North street	ii)	300	2)	100
c)	Connecting East and South street	iii)	400	3)	600
d)	Connecting West and South street	iv)	900	4)	0

Table : M1W5GA1

(MCQ)

- ◯ Option 1: a → iv → 1; b → ii → 3; c → iii → 2; d → i → 4
- ◯ Option 2: a → ii → 1; b → iii → 3; c → iv → 2; d → i → 4
- ◯ Option 3: a → ii → 2; b → iv → 3; c → iii → 4; d → i → 1
- ◯ **Option 4:** a → iii → 2; b → iv → 3; c → i → 4; d → ii → 1

Solution:
As we get the system of linear equations

$$\begin{aligned} 2x_1 + 3x_2 &= 1000 \\ 3x_2 + 2x_3 &= 900 \\ 2x_3 + x_4 &= 300 \\ 2x_1 + x_4 &= 400 \end{aligned}$$

So we can write last two equations $2x_1 = 400 - x_4$ and $2x_3 = 300 - x_4$
Observe that the average number of vehicles expected to pass can not be negative, that means x_4 can not be grater than 300. i.e., $x_4 \leq 300$.
Therefore, the maximum number of vehicles expected to pass through West street to south street is 300. But the minimum number of vehicles expected to pass through West street to south street can be 0. So, $0 \leq x_4 \leq 300$.

Again, since the third equation in the system of linear equations is $2x_3 = 300 - x_4$ and the second equation in the system of linear equations is $3x_2 + 2x_3 = 900 \implies 3x_2 = 900 - 2x_3 = 900 - 300 + x_4 = 600 + x_4$

Therefore, $600 \leq 3x_2 \leq 900$, (as $0 \leq x_4 \leq 300$).

Now, since the average number of vehicles expected to pass through West street is 400 and the maximum number of vehicles expected to pass through West street to south street is 300 (as we obtained) so remaining 100 vehicles pass through West street to North street that means the minimum number of vehicles expected to pass through West street to North street is 100. i.e., $100 < 2x_1$

From the last equation of the system of linear equation $2x_1 = 400 - x_4 \implies 2x_1 \leq 400$ (as $0 \leq x_4 \leq 300$).

So, $100 \leq 2x_1 \leq 400$.

Now, since the third equation in the system of linear equations is $2x_3 = 300 - x_4$. So, $0 \leq 2x_3 \leq 300$ (as $0 \leq x_4 \leq 300$).