IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 5 Tutorial 2

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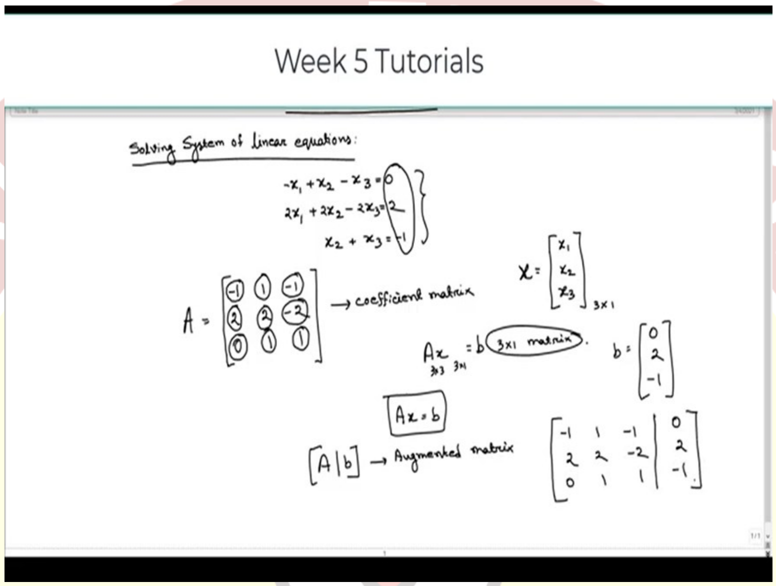

In this video, we are going to solve system of linear equation using Gauss elimination. Basically, we will take some examples and see how we can use in our row operations to solve the system of linear equations.

So, let us consider this system of linear equation first. So, which is written as $-x_1 + x_2 - x_3 = 0$. Let us consider the second equation to be $2x_1 + 2x_2 - 2x_3 = 2$ and the third equation to be $-x_1 + x_2 + x_3 = -1$. So, these three equations make this system of linear equation, so we can write the corresponding matrix of this system of equation as $\begin{bmatrix} -1 & 1 & -1 \\ 2 & 2 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

So, if you see the first equation, the coefficient of x_1 is - 1 which I write in this first place. Coefficient of x_2 is + 1, coefficient of x_3 is - 1. Similarly, coefficient of x_1 in the second equation is 2, coefficient of x_2 is also 2 and coefficient of x_3 is - 2. There is no x_1 in the third equation. So, the coefficient is 0. The coefficient of x_2 is 1 and coefficient of x_3 is also 1.

So, this is basically the coefficient matrix. We generally denote it by A. The variable we can write in this matrix, x such that this is a column matrix, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. So, if you multiply A with x, you will get a, so A is a 3×3 matrix and this is a 3 cross 1 matrix. So, $A\,x$ will be a 3 X 1 matrix and that is basically these terms, constant terms, these 3×1 matrix, we generally denote it by b. So, b is $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$.

So, this gives us this form Ax equal to b where A is our coefficient matrix and b is basically the right hand side of the equations. So, if we take A and there is a line in between and then we write b, this is known as the augmented matrix. So, using this augmented matrix, we can solve this system of linear equation.

So, let us see how we can do it. So, let us write the augmented matrix first for this system of equation. So, augmented matrix will be - 1, 1, - 1; 2, 2, - 2; 0, 1, 1 and then b is our $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ and we put as line in between them to separate these two. So, this is our augmented matrix in this system of linear equation.

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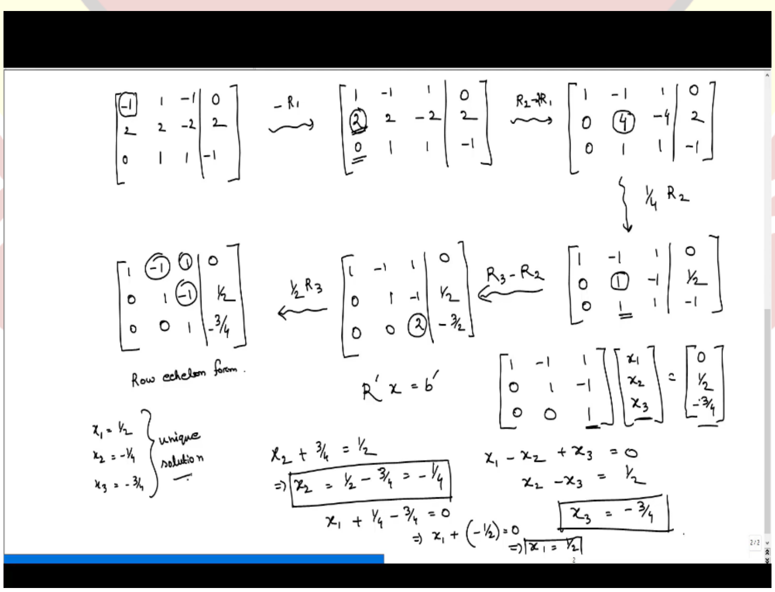

So, let us start with this augmented matrix. So, our first element, first non-zero element in first row is - 1. We have to make it 1. So, basically we will try to make it in reduced echelon form so, at first, we have to make the first pivot element to be 1. It is - 1, so basically we will do - of R_1 which will gives us 1, - 1, 1, 0 and the remaining rows will be same.

Now, we have to make this element to be 0. Basically, we have to make all the elements in the first column except the first pivot element to be 0. The third element, third row, the element in third row is already 0, so we have to make only the element in the second row to be 0. So, what we have to do? We have to multiply 2 with the first row and subtract it from the second. So, R_2 - 2 R_1. So, let us see what it will be give us.

The first row will remain unchanged, the second row will be 0, then 2 - of - 2, that is 4 and here it will be - 4, here it will be 2 and the third row will remain as it is. Now, the first non-zero element in the second row is 4, so this is our pivot element, we have to make it 1. So, what we will do? We will take $\frac{1}{4}$ of R_2. So, it will give us 1 - 1, 1, 0, first row will remain unchanged. So, this will give us 1 - 1 and here it will it be half. And the third row will remain unchanged.

Now, we have to make the element below this pivot element to be 0. So, we have to make this one to be 0. So, basically we have to just subtract R_2 from R_3. So, we will get the first row will be same because we are not doing any operation on that. The second is also same. And the third one will be 0 and this will be 1 of - 1, that is 2 and - $\frac{-1}{2}$, - 1, that will be $\frac{-3}{2}$.

Now, the first non-zero element in the third row is 2. So, we have to make it 1. So, we will do $\frac{1}{2}$of R3, it will give us 1, - 1, 1, 0, 0, 1, - 1, 1, 0, 0, 1, - 1 half, 0, 0, 1 and $\frac{-3}{4}$. So, this is already, now this is in row echelon form. So, using this row echelon form itself we can solve this equation. Or else, we can go to the reduced echelon form to solve this equation. For that, we have to make all these element to be 0.

But before going there, from here, let us try to solve this equation. So, what is this? Here, this first portion, this first 3 by 3 portion is basically our new coefficient matrix after row echelon form, after this set of row operations. So, this is our new matrix R, $Ax = b$ where R' is $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, this is my R', our x will remain same and our new b' , so this will be b' prime, will be this.

So, the new equations are $x_1 - x_2 + x_3$ is 0, $x_2 - x_3$ is $\frac{1}{2}$ and x_3 is $\frac{-3}{4}$. So, from the last one, what we are getting? x3 is equal to $\frac{-3}{4}$. So, we are basically solve the value for x3. Now, if we put x3 in the second equation, we will get $x_2 - x_3$ which is $\frac{-3}{4}$, so it will be + $\frac{3}{4}$ equal to half which gives us x2 equals to half $\frac{1}{4} - \frac{-3}{4}$ which is $\frac{-1}{4}$.

And now, we have got x_2 and x_3 and if we put this in the first equation which is $x_1 - x_2$ means 1 by 4 + x3 means $\frac{-3}{4}$, that is 0. So, x_1 will be, so we get x_1 is half. So, we have solved the equation and we get x1 to be half, x_2 to be $\frac{-1}{4}$ and x_3 to be $\frac{-3}{4}$. So, this is the solution and you can see that this is a unique solution. So, this system of linear equation has unique solution and the solution is given by this.

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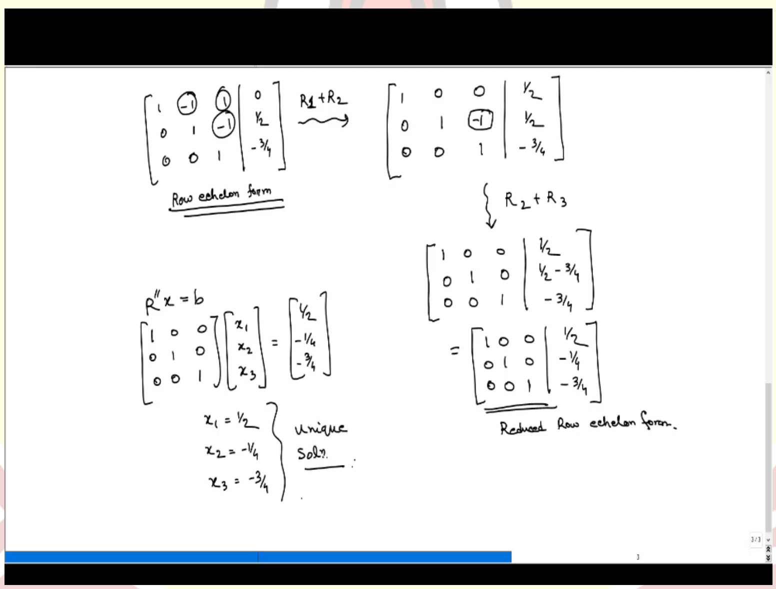

Now, in this example, we have already seen that this is in row echelon form. So, let us try to reduce this further to reduce row echelon form. So, basically, we have to make these 3 element 0. So, for making the element in the first row in second column to be 0, we have to basically add the second row to the first one. So, R_2, $R_1 + R_2$, this will give us $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$, here it will be half because we are adding the second row.

So, this will be unchanged, the second row will remain unchanged and the third row will also remain unchanged. So, we have already made this first row in $[1 \quad 0 \quad 0]$. So, only one element is remaining now, this - 1. So, what we have to do, we have to add the third row to the second row, so $R_2 + R_3$. So, it will give us $\begin{bmatrix}1 & 0 & 0 & \frac{1}{2}\end{bmatrix}$.

So, first row will remain unchanged, 0, 1, 0 and $\frac{1}{2}-\frac{3}{4}$ and 0, 0, 1 $-\frac{3}{4}$, the third row will remain unchanged so which is nothing but this matrix, $\frac{1}{2}$, 0, 1, 0 $\frac{1}{2}-\frac{3}{4}$ is $-\frac{1}{4}$, 0, 0, $1-\frac{3}{4}$. So, you can see that this is in row echelon form or reduced row echelon form. So, our new matrix R'' x is b, where R'' is the identity matrix, x is same, our variables and b is $\frac{1}{2}-\frac{3}{4}$ and $-\frac{3}{4}$. So, you can see that x_1 is $\frac{1}{2}$, x_2 is $-\frac{1}{4}$ and x_3 is $-\frac{3}{4}$.

So, from reduced row echelon form, we can directly solve this equation. Or else, what we have done in the previous page, from the row echelon form, we solve the equation by putting the values form the last variable, x_3. So, here these operations are doing the same thing in matrix form rather than putting the values of the variable from the last. So, this is the solution of the system equation we have taken and this is the unique solution for this system.

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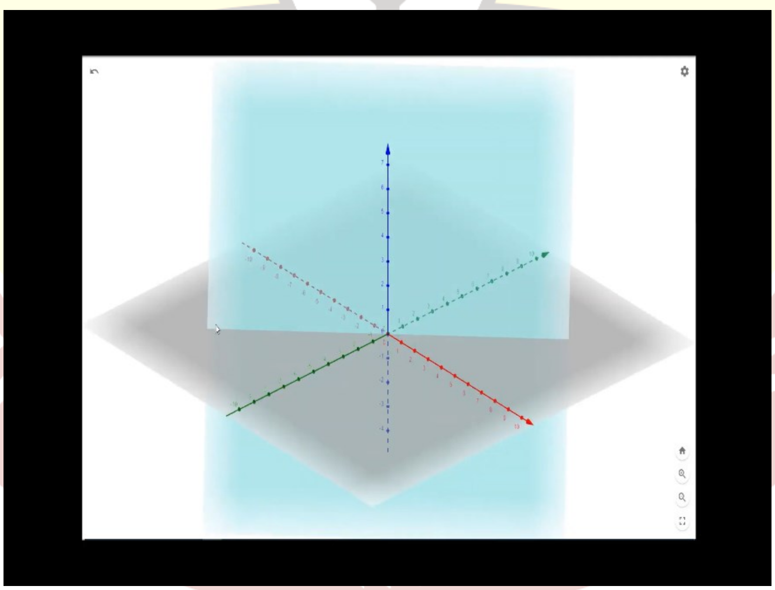

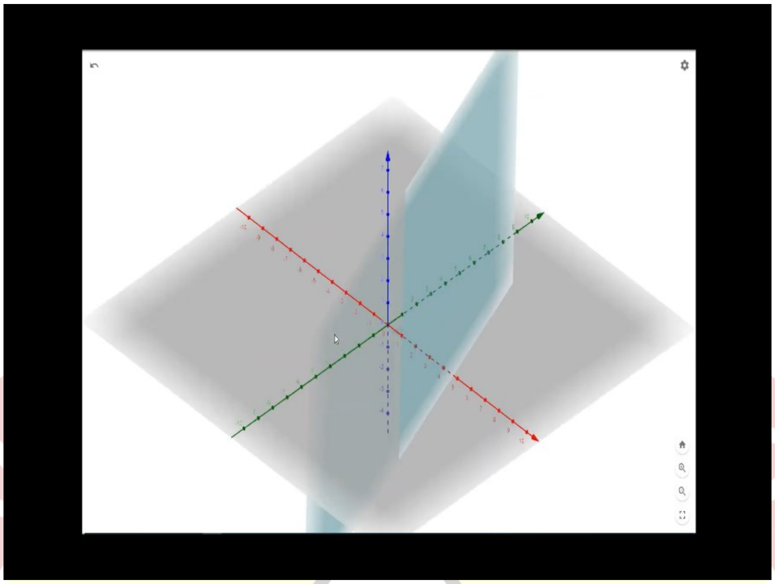

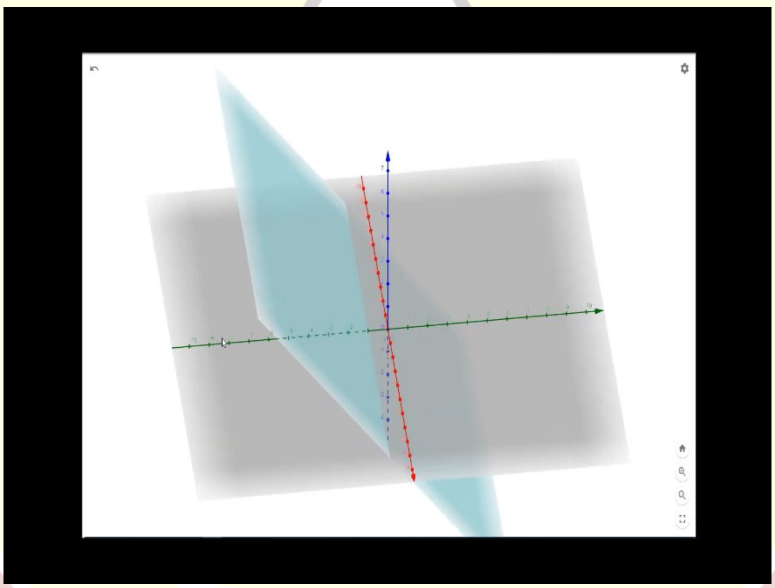

Now, let us see the geometric representation as we have seen in the earlier week. So, the first equation here was $-x_1 + x_2 - x_3 = 0$. So, in x, y, z plane, so suppose, our x_1 is basically the variable x, the x_2 is variable y and the x_3 is variable z, so we can represent these as a plane in this x, y, z coordinate system.

So, the first equation will denote this plane, it will pass through origin, as you can see and this is the plane $-x_1 + x_2 - x_3 = 0$ rather, $-x + y - z=0$. So, the second plane will look like this, second plane was $2x_1 + 2x_2 - 2x_3 = 2$. In this coordinate system, it is $2x + 2y - 2z= 2$.

Basically, we are changing the variable $x_1, x_2,$ to x, y, z to identify this in this coordinate system because in general, in coordinate system, we take the variable to be x, y and z. So, this is the second plane. And the third plane will look like this, that was $y + z$ equal to - 1. So, in the equation it was $x_2 + x_3$ equal to - 1. So, this is the third plane.

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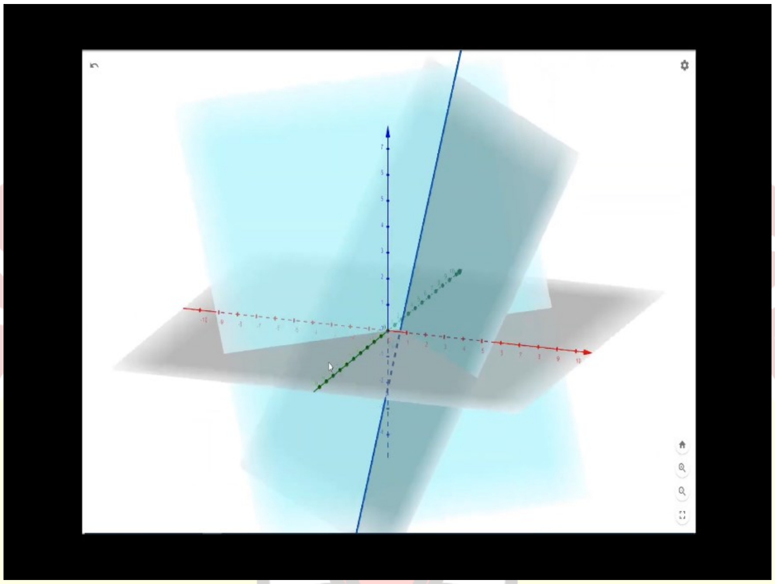

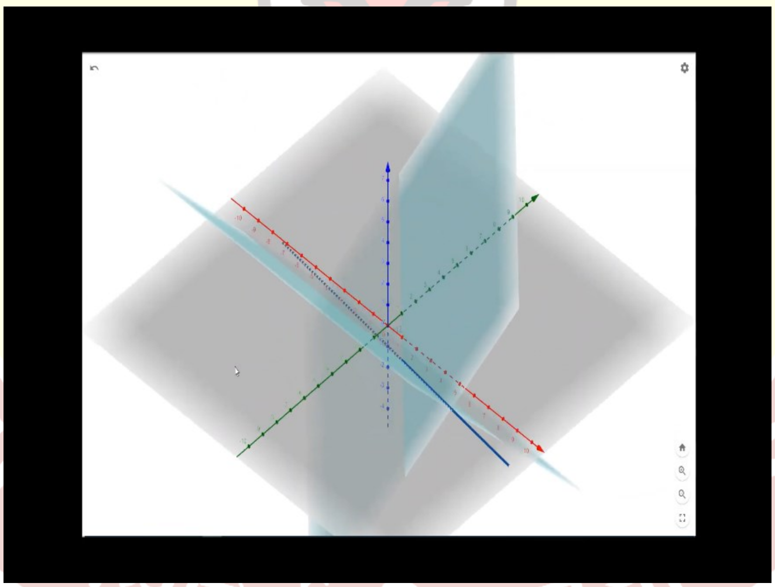

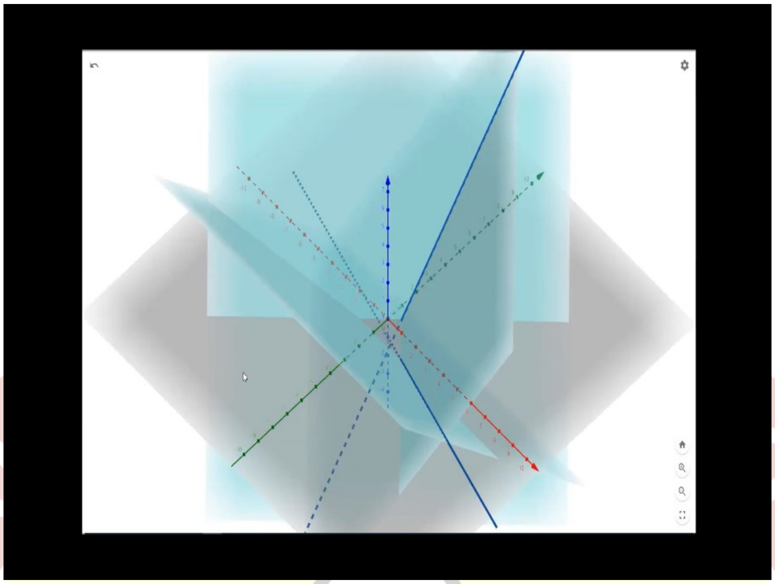

Now, let us see the first and the second equation where they intersect with each other. So, this is the first plane and this is the second plane. So, in this straight line, they will intersect with each other. So, this is the solution of the first two equation. Now, if we see the second and third equation, you will see, so this is the third equation, so this is the solution of second and third equation which is given by this straight line. So, in this straight line, these two plane intersect with each other.

So, let us consider the three equation at a time and see where they are intersecting. So, there are two line, where one line denotes the solution of first two equation and the second line denotes the solution of second and third equation. And you can see that one point, these two points are intersecting and that is basically the solution of these three equation where these three plane are intersecting with each other.

So, this point is basically the solution of these three plane. So, the intersection of these three plane is there and the solution of the three equation, we have considered. So, as these three plane are intersecting at a single point, the solution is unique. Thank you.