Week-4
Mathematics for Data Science - 2
Vectors and Matrices
Practice Assignment Solution

1 Multiple Choice Questions (MCQ)

1. If $A = \begin{bmatrix} 2x & 2z & y+x \\ -x+z & 2x & -y+z \\ x+y-2z & y-z-2x & 2y-2z \end{bmatrix}$, then $det(A)$ is

Hint:

- Replace column 1 with (column 1 - column 3)
  - ◯ Option 1: $(x-y)(y-z)(z-x)$
  - ◯ Option 2: $(x-y)(y+z)(z-x)$
  - ◯ **Option 3:** $(x-y)(y-z)(z+x)$
  - ◯ Option 4: $(x+y)(y-z)(z-x)$

Note:- Because of $det(A) = det(A^T)$, both the row operation and column operations have the same effects on the determinant of matrix A

Solution:

Given $A = \begin{bmatrix} 2x & 2z & y+x \\ -x+z & 2x & -y+z \\ x+y-2z & y-z-2x & 2y-2z \end{bmatrix}$

Use the given hint, replacing column 1 with Column 1 - Column 3

$$det(A) = \begin{vmatrix} x-y & 2z & y+x \\ -x+y & 2x & -y+z \\ x-y & y-z-2x & 2y-2z \end{vmatrix}$$

$$= \begin{vmatrix} x-y & 2z & y+x \\ -(x-y) & 2x & -y+z \\ x-y & y-z-2x & 2y-2z \end{vmatrix}$$

Replacing, Row 3 + Row 2 in Row 3

$$\det(A) = \begin{vmatrix} x-y & 2z & y+x \\ -(x-y) & 2x & -y+z \\ 0 & y-z & y-z \end{vmatrix}$$

Replacing, Row 2 + Row 1 in Row 2

$$\det(A) = \begin{vmatrix} x-y & 2z & y+x \\ 0 & 2(x+z) & x+z \\ 0 & y-z & y-z \end{vmatrix}$$

Observe, Row 2 is multiple of $(x+z)$ & Row 3 is multiple of $(y-z)$

$$\text{So, } \det(A) = (x+z)(y-z) \begin{vmatrix} x-y & 2z & y+x \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

To get determinant we can expand along the first column.

$$\text{So, } \det(A) = (x+z)(y-z)\Big[(x-y)(2-1) - 0\cdot(2z-(y+x)) + 0\,(2z-2(y+x))\Big]$$

$$\det(A) = (x+z)(y-z)(x-y)$$

Hence, third option is correct.

2. Match the systems of linear equations in Column A with their number of solutions in Column B and their geometric representation in Column C.

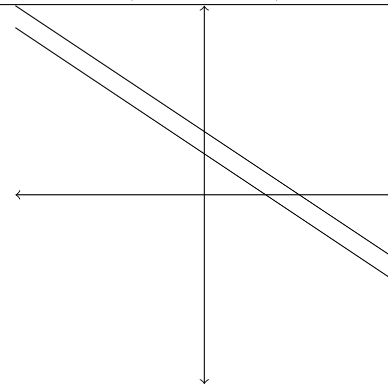
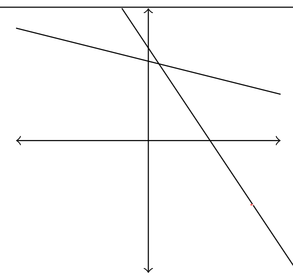
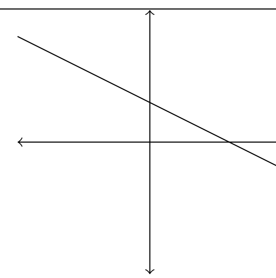
	System of linear equations (Column A)		Number of solutions (Column B)		Geometric representations (Column C)
i)	$3x + 2y = 7$, $x + 4y = 12$	a)	Infinite solutions	1)	
ii)	$2x + 3y = 5$, $8x + 12y = 13$	b)	No solution	2)	
iii)	$x + 2y = 3$, $4x + 8y = 12$	c)	Unique solution	3)	

Table: W4PT1

- ◯ **Option 1:** i) → c) → 2); ii) → b) → 1); iii) → a) → 3)
- ◯ Option 2: i) → c) → 1); ii) → b) → 3); iii) → a) → 2)
- ◯ Option 3: i) → b) → 3); ii) → c) → 2); iii) → a) → 1)
- ◯ Option 4: i) → a) → 3); ii) → b) → 1); iii) → c) → 2)

Solution:

System (i)

$$3x + 2y = 7$$
$$x + 4y = 12$$

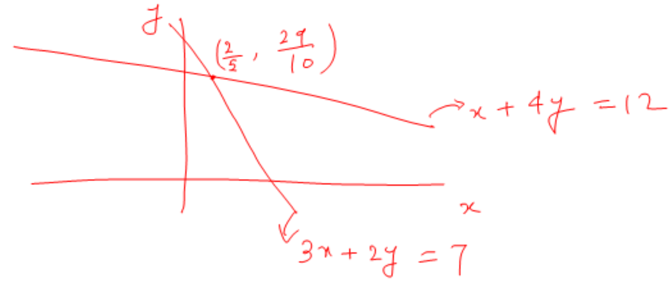
Multiplying 3 to the both sides of the second equation and substracting from first equation, will give $y = \frac{29}{10}$.

(both sides of)
Similarly multiplying 2 to the first equation and substracting second equation from first equation, we'll give $x = \frac{2}{5}$

So, the system (i) has unique solution.

Geometrically, both equations in system (i) represent lines in X-Y co-ordinate plane and both lines intersect at a unique point and that is $\left(\frac{2}{5}, \frac{29}{10}\right)$

Below figure shows lines in X-Y co-ordinate plane.



So, (i) → (c) → (2)

System (ii)

$$2x + 3y = 5$$
$$8x + 12y = 13$$

Multiplying 4 to the both sides of the first equation, to get

$$8x + 12y = 20$$

& let the 2nd equation be as it is $8x + 12y = 13$

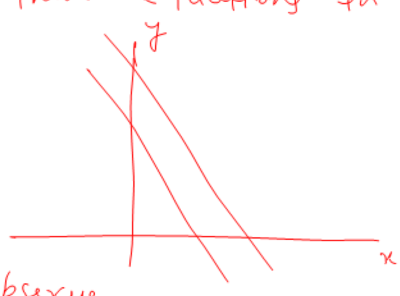
here, left sides of both the equations are the same.

So, $20 = 13$.

But this is not possible.

Hence, we can conclude that there is no solution for the system of linear equations (ii)

geometrically, these equations in the system (ii) represents parallel lines in X-Y plane as shown in figure.



Observe that, there is no point of intersection of $8x + 12y = 13$ and $2x + 3y = 5$

So, there is no solution for system (ii)

So (ii) → (b) → (1)

System (iii)

$$x + 2y = 3$$
$$4x + 8y = 12$$

Multiply 4 to the both sides of the first equation to get, $4x + 8y = 12$

which is exactly the second equation.

Now $x + 2y = 3 \Rightarrow x = 3 - 2y$

let $y = t$, for some real number $t \in \mathbb{R}$,

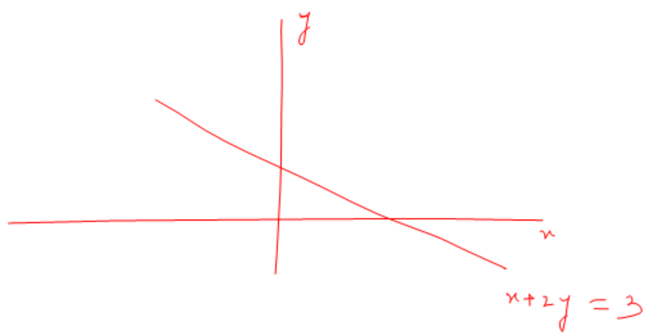
$\Rightarrow x = 3 - 2t$

Now, if we change value of 't', we will get different value of x

i.e there are infinitely many solutions.

Geometrically, Observe, second equation is multiple of first equation.

So, the figure below shows first line $x + 2y = 3$. This will be the same as $4x + 8y = 12$ i.e second line in x-y plane.



So, (iii) → (a) → (3)

Hence, option 1 is the correct option.

2 Multiple Select Questions (MSQ):

3. Choose the set of correct options

   ◯ **Option 1:** If both A and B are 2×2 real matrices and $det(AB) = 0$, then $det(A) = 0$ or $det(B) = 0$.

   ◯ **Option 2:** If A is a 3×3 real matrix with non-zero determinant and k is some real number, then $det(k\ A) = k^3 \times det(A)$.

   ◯ **Option 3:** If $A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$, then $A^{10} = 2^{10} \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$ (where A^n is the matrix $A \times A \times \ldots \times A$, n-times).

   ◯ **Option 4:** The number of scalar additions to be done to compute the matrix AB, where A is a 3×2 matrix and B is a 2×3 matrix, is 9.

Solution: Option 1:

Given $det(AB) = 0$

we know that, $det(AB) = det(A) \cdot det(B) = 0$

Observe that $det(A) = 0$ or $det(B) = 0$

So this option1 is true.

Option 2:

Given $A_{3\times3}$ matrices

let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

so, $kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{32} \end{bmatrix}$, Now $det(kA) = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{vmatrix}$

Using property - If multiplying a real number 'k' with a row in a matrix then determinant of the new matrix is k times determinant of the earlier matrix.

Since, all three rows are multiple of k.

So, $\det(kA) = k \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{vmatrix} = k \cdot k \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{vmatrix}$

$$= k \cdot k \cdot k \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k^3 \det(A)$$

Hence, $\det(kA) = k^3 \det(A)$

So, option 2 is true.

Option 3:

Given $A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Let $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\quad B^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$B^3 = B^2 \cdot B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$B^4 = B^3 \cdot B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

$\vdots \qquad \vdots \qquad \vdots$

$$B^9 = B^8 \cdot B = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}$$

$$B^{10} = B^9 \cdot B = \begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$

Now, $A^{10} = (2 \cdot B)^{10}$

$= 2^{10} \cdot B^{10}$

$A^{10} = 2^{10} \cdot \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$

So, option 3 is also true.

<u>Option 4</u>:

Given A is a 3×2 matrix & B is 2×3 matrix

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ & $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} & a_{11} b_{13} + a_{12} b_{23} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} & a_{21} b_{13} + a_{22} b_{23} \\ a_{31} b_{11} + a_{32} b_{21} & a_{31} b_{12} + a_{32} b_{22} & a_{31} b_{13} + a_{32} b_{23} \end{bmatrix}$$

Observe that, AB is order of 3×3 matrix.

To obtain each row of AB, 3 scalar addition are performed.

So, total 9 scalar additions are performed in the matrix multiplication AB.

Hence, option 4 is also true.

4. Choose the set of correct options

- ○ **Option 1:** A triangular 3×3 matrix has non-zero determinant if and only if all the diagonal entries are non-zero.
- ○ Option 2: If A and B are 3×3 matrices then $det(A+B) = det(A) + det(B)$.
- ○ Option 3: If A and B are 3×3 matrices then $adj(A+B) = adj(A) + adj(B)$.
- ○ **Option 4:** If A is a 3×3 matrix and B is a matrix obtained from A by multiplying each column of A by its column number, then $det(B) = 6det(A)$.
- ○ **Option 5:** If the sum of the first and the third row vectors of a 3×3 matrix A is equal to the second row vector of A, then $det(A) = 0$.

Solution: Option 1: Let A be an upper triangular matrix as follows

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Observe det (A) = multiple of all diagonal entries.

i.e. $\det(A) = a_{11} a_{22} a_{33}$

Clearly, $\det(A) = 0$ if and only if one of the a_{11}, a_{22} or a_{33} will be zero which are the diagonal entries.

In other word, $\det(A) \neq 0$ if and only if none of a_{11}, a_{22} or a_{33} will be zero i.e. diagonal entries are non zero.

Similarly, the above statements are true for lower triangular matrices also.

Hence, option 1 is true.

Option 2: let $A = B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Clearly, $\det(A) = \det(B) = 1 \Rightarrow \det(A) + \det(B) = 1 + 1 = 2$

$$A+B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Observe $\det(A+B) = 2^3 \cdot \det(I)$, where I is 3×3 identity matrix.

$\Rightarrow \det(A+B) = 8 \neq 2 = \det(A)+\det(B)$ (we know that $\det(I) = 1$)

i.e $\det(A+B) \neq \det(A) + \det(B)$

Hence option 2 is not true.

Option 3:

let $A = B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Observe, $\operatorname{adj}(A) = \operatorname{adj}(B) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \operatorname{adj}(A)+\operatorname{adj}(B) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Also, $A+B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

observe, $\operatorname{adj}(A+B) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Clearly, $\operatorname{adj}(A+B) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \operatorname{adj}(A) + \operatorname{adj}(B)$

Hence $\operatorname{adj}(A+B) \neq \operatorname{adj}(A) + \operatorname{adj}(B)$

Option 4:

let $A = \begin{array}{c} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{Column 1} & \text{Column 2} & \text{Column 3} \end{array} \end{array}$

From the given condition, $B = \begin{bmatrix} a_{11} & 2\cdot a_{12} & 3\cdot a_{13} \\ a_{21} & 2\cdot a_{22} & 3\cdot a_{23} \\ a_{31} & 2\cdot a_{32} & 3\cdot a_{33} \end{bmatrix}$

Observe, $\det(B) = 1\cdot 2\cdot 3 \det(A) = 6\cdot\det(A)$

Hence option 4 is true.

Option 5 : let A be a matrix of order 3×3 with given properties

i.e let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and $a_{11} + a_{31} = a_{21}$
$a_{12} + a_{32} = a_{22}$
$a_{13} + a_{33} = a_{23}$

Use property - Adding a row with multiple of some real number to another row, which does not effect to determinant of the matrix.

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}+a_{31} & a_{12}+a_{32} & a_{13}+a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \left(\begin{array}{c}\text{Row} \\ \text{operation} \\ R_3+R_1\end{array}\right)$$

$$= \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$ (Because two row of the matrix are the same)

$\Rightarrow \det(A) = 0$

Hence, option 5 is true.

5. Mahesh bought 2 kg potato and c kg dal from a shop, and paid ₹200 to the shopkeeper. Gaurav bought 4 kg potato and 4 kg dal, and paid ₹d to the shopkeeper. If x_1 represents the price of 1 kg potato and x_2 represents the price of 1 kg dal, then choose the set of correct options.

- ◯ Option 1: The matrix representation to find x_1 and x_2 is

$$\begin{bmatrix} 2 & 4 \\ 4 & c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ d \end{bmatrix}$$

- ◯ **Option 2:** The matrix representation to find x_1 and x_2 is

$$\begin{bmatrix} 2 & c \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ d \end{bmatrix}$$

- ◯ **Option 3:** If Mahesh tries to find the price of 1 kg potato and 1 kg dal using appropriate matrix representation by taking $c = 2$ and $d = 400$, then the price of 1 kg potato that he thus arrives at, will not be unique.
- ◯ Option 4: If Mahesh tries to find the price of 1 kg potato and 1 kg dal using appropriate matrix representation by taking $c = 2$ and $d = 400$, then the price of 1 kg potato that he thus arrives at, will be unique.
- ◯ Option 5: If Mahesh tries to find the price of 1 kg potato and 1 kg dal using the appropriate matrix representation by taking $c = 2$ and $d \neq 400$, then he will be able to find the price (as a numerical value) of 1 kg potato.
- ◯ **Option 6:** If Mahesh tries to find the price of 1 kg potato and 1 kg dal using the appropriate matrix representation by taking $c = 2$ and $d \neq 400$, then he will fail to find the price (as a numerical value) of 1 kg potato.

Solution: Given, x_1 represents the price of 1 kg potato.
x_2 represents the price of 1 kg dal

$\Rightarrow$ price of 2 kg potato $= 2x_1$

price of c kg dal $= cx_2$

price of 4 kg potato $= 4x_1$

price of 4 kg dal $= 4x_2$

Now, Gaurav bought 4 kg potato & 4 kg dal, & paid 'd' rupees.

i.e $4x_1 + 4x_2 = d$

again, Mahesh bought 2 kg potato & c kg dal from a shop & paid 200 rupees.

i.e $2x_1 + cx_2 = 200$

The system of linear equations will be

$$2x_1 + cx_2 = 200$$
$$4x_1 + 4x_2 = d$$

Now, Matrix form of the above system is $Ax = b$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} 2 & c \\ 4 & 4 \end{bmatrix}$ & $b = \begin{bmatrix} 200 \\ d \end{bmatrix}$

Hence, Option 2 is true, but option 1 is not true.

Option 3: If $c = 2$ & $d = 400$

then, the system becomes

$$2x_1 + 2x_2 = 200$$
$$4x_1 + 4x_2 = 400 \Rightarrow 2x_1 + 2x_2 = 200$$

Observe, both the equations are the same.

(Recall question 2)

this system of linear equations have infinitely many solution.

Hence, By solving the system, the price of 1 kg potato that Mahesh arrives at, will not be unique.

Hence, Option 3 is true.

Option 4: Observe from option 3, we conclude that the price of 1 kg potato that Mahesh thus arrives at, will not be unique.

Hence, option 4 is not true.

Option 5: Observe from option 3, $c = 2$ & $d = a$ where $a \in \mathbb{R} \setminus \{400\}$

The system is

$$2x_1 + 2x_2 = 200 \Rightarrow 4x_1 + 4x_2 = 400$$

$$4x_1 + 4x_2 = a$$

from the above equations $a = 400$

But it is given that $a \neq 400$

Hence, for $c = 2$ & $d \neq 400$ the the above system has no solution.

Hence, Mahesh will not be able to find the price of 1 kg potato

Hence, option 5 is not true & option 6 is true.

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6. The marks obtained by Safina, Ram and Pratiksha in Quiz 1, Quiz 2 and End sem (with the maximum marks for each exam being 100) are shown in Table W1PT2.

	Quiz 1	Quiz 2	End sem
Safina	89	95	88
Ram	92	81	98
Pratiksha	85	93	98

Table: W4PT2

The weightage of marks in final grade(in percent) of Quiz 1, Quiz 2, and End sem is shown in Table W1PT3.

	In percent (%)
Quiz 1	20
Quiz 2	20
End sem	60

Table: W4PT3

Choose the set of correct options.

- ◯ Option 1: Final grades (in 100) of Safina, Ram and Pratiksha can be represented by the matrix:

$$\begin{bmatrix} 93.4 \\ 94.4 \\ 89.6 \end{bmatrix}$$

- ◯ **Option 2:** Final grades (in 100) of Safina, Ram and Pratiksha can be represented by the matrix :

$$\begin{bmatrix} 89.6 \\ 93.4 \\ 94.4 \end{bmatrix}$$

- ◯ **Option 3:** The order of the matrix which represents final grades(in 100) of Safina, Ram and Pratiksha is 3×1

- ◯ **Option 4:** If bonus marks given to Safina, Ram and Pratiksha are represented by the following matrix $\begin{bmatrix} 1.4 \\ 2.6 \\ 0 \end{bmatrix}$ then the overall final grades (in 100) can be represented by the matrix:

$$\begin{bmatrix} 91 \\ 96 \\ 94.4 \end{bmatrix}$$

Solution:

Matrix form of marks obtained by Safina, Ram & Pratiksha in Quiz 1, Quiz 2 & End sem is $\begin{bmatrix} 89 & 95 & 88 \\ 92 & 81 & 98 \\ 85 & 93 & 98 \end{bmatrix}$

& matrix of the weightage of marks is $\begin{bmatrix} 20/100 \\ 20/100 \\ 60/100 \end{bmatrix}$

So, final grades of Safina, Ram & Pratiksha will be

$$\begin{bmatrix} 89 & 95 & 88 \\ 92 & 81 & 98 \\ 85 & 93 & 98 \end{bmatrix} \begin{bmatrix} 20/100 \\ 20/100 \\ 60/100 \end{bmatrix} = \begin{bmatrix} \frac{89\times20}{100} + \frac{95\times20}{100} + \frac{88\times60}{100} \\ \frac{92\times20}{100} + \frac{81\times20}{100} + \frac{98\times60}{100} \\ \frac{85\times20}{100} + \frac{93\times20}{100} + \frac{98\times60}{100} \end{bmatrix}$$

$$= \begin{bmatrix} 89.6 \\ 93.4 \\ 94.4 \end{bmatrix}$$

Hence, the second option is true.

Observe that, the matrix which obtained in option 2 have 3 rows & 1 column

Hence the order of matrix is 3×1

So, option 3 is true.

Now, the matrix representing the bonous marks is $\begin{bmatrix} 1.4 \\ 2.6 \\ 0 \end{bmatrix}$

So overall final grades (in 100) can be obtained by addition i.e, $\begin{bmatrix} 89.6 \\ 93.4 \\ 94.4 \end{bmatrix} + \begin{bmatrix} 1.4 \\ 2.6 \\ 0 \end{bmatrix} = \begin{bmatrix} 91 \\ 96 \\ 94.4 \end{bmatrix}$

Hence, the fourth option is true.

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3 Numerical Answer Type (NAT):

7. Let A be a 3×3 matrix with non-zero determinant and B be a matrix obtained by adding 5 times of first row of A to the third row of A and adding 10 times of second row of A to the first row of A. What is the value of $det(3AB^{-1})$? [Ans: 27]

Solution:

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Hence, $B = \begin{bmatrix} a_{11}+10a_{21} & a_{12}+10a_{22} & a_{13}+10a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31}+5a_{11} & a_{32}+5a_{12} & a_{33}+5a_{13} \end{bmatrix}$

Observe, $det(A) = det(B)$

We know $det(B^{-1}) = \dfrac{1}{det(B)} = \dfrac{1}{det(A)}$

Now, $3A = \begin{bmatrix} 3a_{11} & 3a_{12} & 3a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3a_{31} & 3a_{32} & 3a_{33} \end{bmatrix}$

So $\det(3A) = 3^3 \det(A)$

We know that

$$\det(3AB^{-1}) = \det(3A) \cdot \det(B^{-1})$$

$$= 3^3 \cdot \det(A) \cdot \frac{1}{\det(A)}$$

$$= 3^3$$

$$\det(3AB^{-1}) = 27$$

Hence $\det(3AB^{-1}) = 27$.

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4 Comprehension Type Question:

A shopkeeper sells three types of clothes- shirts, jeans, and T- shirts- in three different sizes: small, medium, and large. In a week, he sold 1 small, 1 medium and 2 large sized shirts; 2 small, c medium and 6 large sized jeans, and 1 small, 3 medium and $c-5$ large sized T-shirts (where c is an integer). The price of shirts, jeans, and T-shirts remain same for different sizes (i.e., small, medium, and large sized shirts have same price; similarly, small, medium, large sized jeans have same price; and small, medium, large sized T-shirts have same price). The shopkeeper earned ₹7 , ₹27 and ₹43 (in thousand) in that week, for small, medium, and large sized clothes respectively.
Answer the following questions using the given data.

8. If s, j, t represents the price of 1 shirt, 1 jeans and 1 T-shirt respectively and we want to find s, j, t by solving a system of linear equations represented by the matrix form $Ax = b$, where $x = (s, j, t)^T$, then which of the following options is correct? (MCQ)

○ Option 1: $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & c & 6 \\ 1 & 3 & c-5 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix}$

○ **Option 2:** $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & c & 6 \\ 1 & 3 & c-5 \end{bmatrix}^T$ and $b = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix}$

○ Option 3: $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & c & 6 \\ 1 & 3 & c-5 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix}^T$

○ Option 4: $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & c & 6 \\ 1 & 3 & c-5 \end{bmatrix}^T$ and $b = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix}^T$

Solution:

Given, the prices of one shirt, one jeans & one T-shirt is s, j, and t respectively. Also price of shirts in all different sized are the same, similarly for jeans and T-shirts

Since, the shopkeeper sold 1 small shirt, 2 small jeans & 1 small T-shirt and earned 7 rupees (in thousand) in a week, we get the equation:

$$s + 2j + t = 7$$

Similarly, the shopkeeper earned 27 rupees (in thousand) in that week for medium sized clothes after selling 1 medium shirt, c medium jeans & 3 medium T-shirts so we get the equation:

$$s + cj + 3t = 27$$

Similarly for large sized clothes,

$$2s + 6j + (c-5)t = 43$$

So, the system of linear equation is

$$s + 2j + t = 7$$
$$s + cj + 3t = 27$$
$$2s + 6j + (c-5)t = 43$$

Now, matrix representation of the system of linear equations is $Ax = b$.

where $x = \begin{bmatrix} s \\ j \\ t \end{bmatrix}$, $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & c & 3 \\ 2 & 6 & c-5 \end{bmatrix}$ & $b = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix}$

Observe that, in second option $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & c & 6 \\ 1 & 3 & c-5 \end{bmatrix}^T$

& $b = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix}$

Hence, the second option is true.

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9. If $det(A) = 122$, then how many medium sized jeans were sold in the week? (NAT)
Ans : 16

Solution: Given $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & c & 3 \\ 2 & 6 & c-5 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 1 & 2 & 1 \\ 1 & c & 3 \\ 2 & 6 & c-5 \end{vmatrix}$$

$$= 1\left(c(c-5) - 18\right) - 2\left(c-5-6\right) + 1\left(6-2c\right)$$

$$= c^2 - 5c - 18 - 2c + 22 + 6 - 2c$$

$$\det(A) = c^2 - 9c + 10$$

Also Given $\det(A) = 122$

$$\Rightarrow \quad c^2 - 9c + 10 = 122$$

$$\Rightarrow \quad c^2 - 9c - 112 = 0$$

$$\Rightarrow \quad (c-16)(c+7) = 0$$

$$\Rightarrow \quad c = 16 \quad \text{or} \quad c = -7$$

But c is the number of medium sized jeans so it cannot be negative.

Hence $c = 16$.

Hence, 16 medium sized pens sold in that week.

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10. If A is the matrix as above (in question 8) and $det(A) = 122$, then What is the price(in thousand) of a shirt, a jeans, and a T-shirt in the matrix form $x = (s, j, t)^T$? (MCQ)

   ◯ Option 1: $x = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

   ◯ Option 2: $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

   ◯ **Option 3:** $x = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

   ◯ Option 4: $x = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$

Solution:- Given $\det(A) = 122$.

from Question 9 we got $c = 16$

Hence, $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 16 & 3 \\ 2 & 6 & 11 \end{bmatrix}$

Matrix representation of system of equations is $Ax = b$

where $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 16 & 3 \\ 2 & 6 & 11 \end{bmatrix}$, $x = \begin{bmatrix} s \\ j \\ t \end{bmatrix}$, $b = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix}$

So, to get the price (in thousand) of a shirt, a jeans, and a T-shirt, we have to choose such x which satisfies the equation $Ax = b$

Observe for $x = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$$Ax = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 16 & 3 \\ 2 & 6 & 11 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+2+3 \\ 2+16+9 \\ 4+6+33 \end{bmatrix} = \begin{bmatrix} 7 \\ 27 \\ 43 \end{bmatrix} = b$$

Hence, the third option is true.