



IIT Madras
ONLINE DEGREE

Mathematics for Data Science 2
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Week-6 Tutorial 07

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Another example $W = \left\{ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \mid \begin{array}{l} a_{11} + a_{12} + a_{13} = 0 \\ a_{21} + a_{22} + a_{23} = 0 \\ a_{31} + a_{32} + a_{33} = 0 \end{array} \right\} \subseteq M_{3 \times 3}(\mathbb{R})$

$a_{13} = -a_{11} - a_{12}, a_{23} = -a_{21} - a_{22}, a_{33} = -a_{31} - a_{32}$

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + a_{12} \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + a_{21} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} + a_{22} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$+ a_{31} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} + a_{32} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

In other way $\left\{ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \right\}$

Now let us see another example. We have given a set W , which is collection of 3×3 matrices, having some property, which is entries in first row, all sum up to 0, that is $a_{11} + a_{12} + a_{13} = 0$. Similarly, entry in second row all sum up to 0, and entry in third row all sum up to 0.

Now we have seen previous video that this is a subspace of all 3×3 matrices, and we know that subspace is a vector space. So, this is actually a vector space, independently, we can say this is a vector space over \mathbb{R} . Now, we want to find its basis.

So first, let us see its condition. The condition is that the entries in first row all sum up to 0. So, we can write this first condition as $a_{13} = -a_{12} - a_{11}$. Similarly, $a_{23} = -a_{22} - a_{21}, a_{33} = -a_{32} - a_{31}$.

Now let us take an element from W , let us say matrix A , in which we are writing entries as shown. So let us say this is a general matrix from W . And if we see clearly, entries a_{13}, a_{23}, a_{33} can be generated using the other two entries in each row based on the conditions discussed above. So, it means we can write this general matrix A as sum of these matrices, which are we can see that entry at (1,1) is 1 and entry at (1,3) is -1, similarly for other rows.

So we can see it here that this general matrix can be generated using these matrices. These six matrices, so let us collect these matrices in this set. And as we discussed, this collection of these matrices generate any element of W . Now, we will show that these matrices are linearly independent.

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$$\begin{aligned}
 & x_1 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \\
 & + x_6 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} = 0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
 \Rightarrow & \begin{bmatrix} x_1 & x_2 & -x_1-x_2 \\ x_3 & x_4 & -x_3-x_4 \\ x_5 & x_6 & -x_5-x_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow x_1=x_2=\dots=x_6=0
 \end{aligned}$$

And we know that how we will show those vectors are linearly independent. So, take the linear combination of these six matrices and equal it to 0. So, this linear combination, we can write it

as $\begin{bmatrix} x_1 & x_2 & -x_1-x_2 \\ x_3 & x_4 & -x_3-x_4 \\ x_5 & x_6 & -x_5-x_6 \end{bmatrix}$ and this is equal to 0 matrix. And if we compare this we will get that all the elements to be 0.

All are 0 that means, these vectors, these matrices are linearly independent. And as we discussed earlier that these matrices also generate whole vector space that is a spanning set of these matrices are whole vector space. And these matrices are linearly independent, so collection of these matrices is the basis of W . Thank you.