IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 5 Tutorial 3

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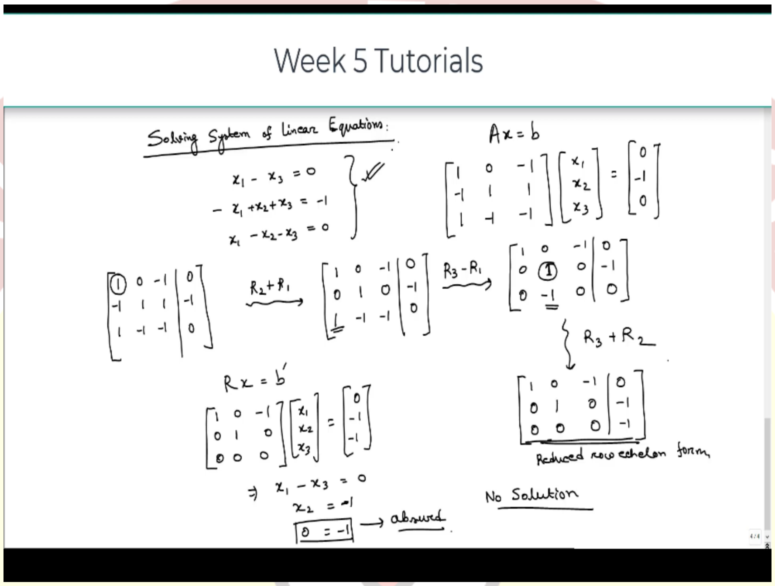

Hello. Let us consider another system of equations and try to solve that one. So, here we are taking this system of linear equation, $x_1 - x_3 = 0, -x_1 + x_2 + x_3 = -1, x_1 - x_2 - x_3 = 0$. The matrix representation of this system of equations will be like this, Ax= b, where A is the coefficient matrix, which is $\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ and x = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, this column vector and our b = $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$.

So, our augmented matrix will be $\begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$. So, this is our augmented matrix. So, the first pivot element is 1, which is the first column and first row. So, at first, we make the element in the second row and first column to be 0. So, we will just add this,$R_1 + R_2$. So it will give us, the first row will remain unchanged, the second row will become $[\,0 \quad 1 \quad 0 \; -1\,]$ and the third row will remain unchanged.

Now, we have to make x_{31} to be 0. So, what we have to do? We have to do$R_3 - R_1$. So, the first two rows will be same and the third row will become $[\,0 \quad 1 \quad 0 \; -1\,]$. So, this is the third row.

Now, the second row, the pivot element is this one, which is in the second column and this is already 1, so we do not have to do anything to make it pivot.

And so, we have to make the element below of that to be 0, so we have to do $R_2 + R_3$, so what we will get? We will get the first two rows will be same as we are not doing any row operation there and the third row will become $[\,0 \quad 0 \quad 0 \; -1\,]$. So, we get a matrix which is in reduced row echelon form, so we will write it as Rx= b′, so where R= $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, x = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and b′ = $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$.

So, if we do that, we will get $x_1 - x_3 = 0,\ x_2 = -1$ and for the third one we are getting $0 = -1$, which is absurd, this is not possible. So, this system of equation has no solution because from the last row, we are getting $0 = -1$, which is not possible. So, our system equation from where we have started with has no solution. So, let us see the geometric representation of this system of linear equations.

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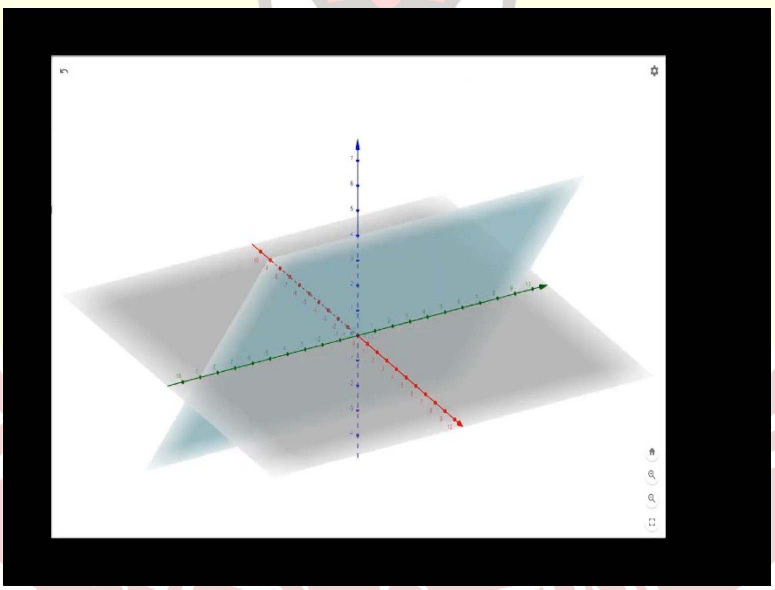

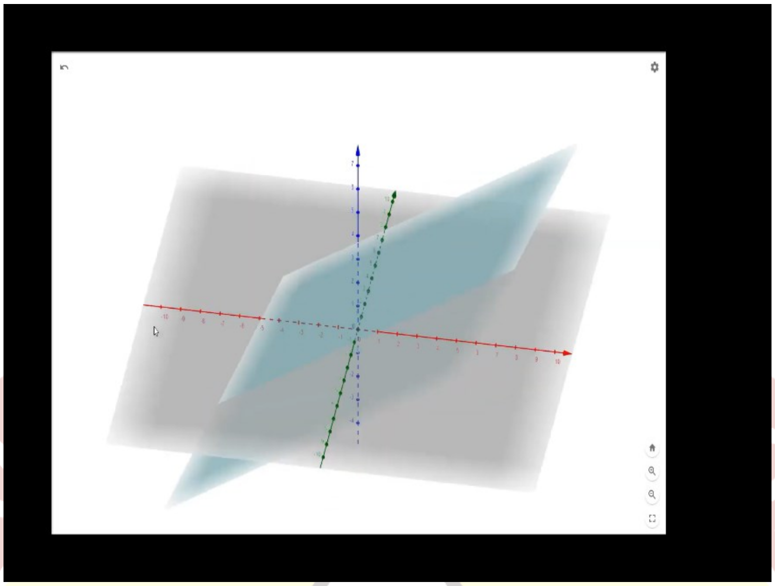

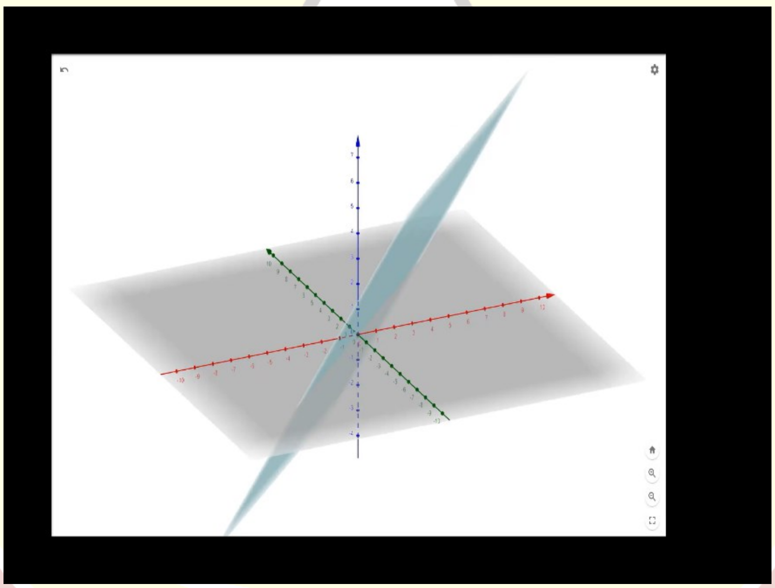

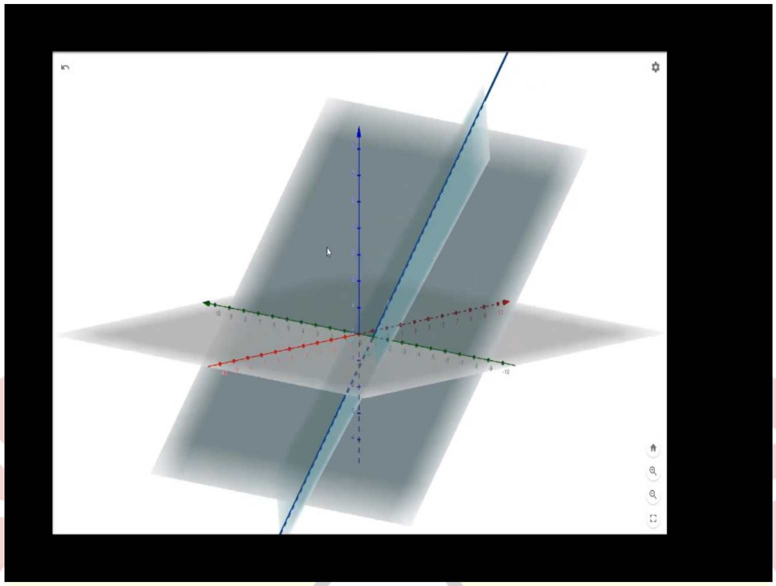

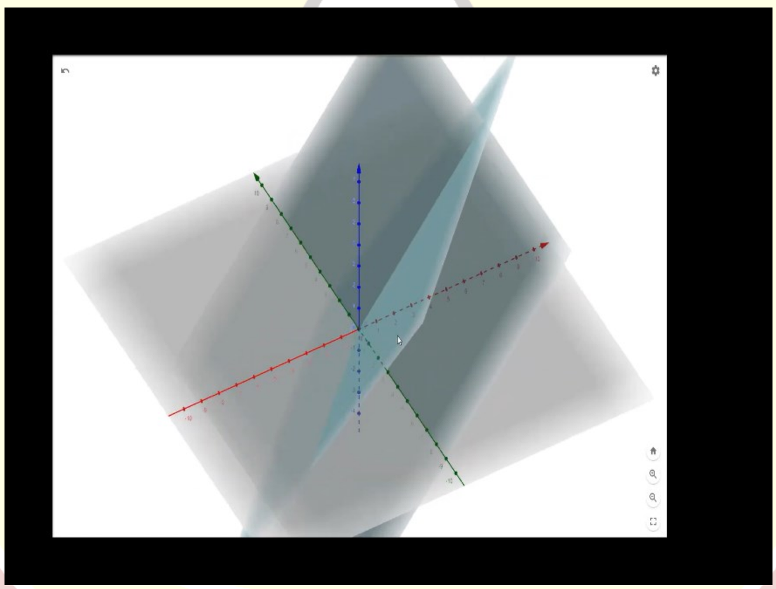

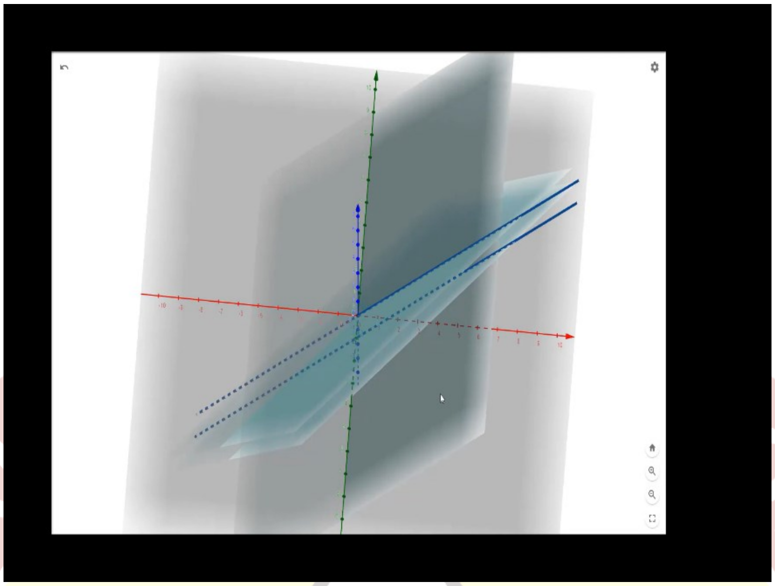

So, this first plane look like this which is given by $x_1 - x_3 = 0$, so it will look like this. So, this is the representation of the first equation. The representation of the second equation is this plane which is $-x_1 + x_2 + x_3 = -1$. So, this is the second plane. The third equation is $x_1 - x_2 - x_3 = 0$, so it will pass through origin and this is the third plane.

So, if we consider the first and the second plane, we will see that they will intersect at a straight line which is given by this straight line. Now, if we see the first and the third equation, they will again intersect in another straight line, so this will be the straight line where they are intersecting.

So, the first and third equation have infinite number of solution and the first and second equation also have infinite number of solution but when you considering three planes together, we can see the solutions which are two straight lines, they are basically parallel to each other and they are not meeting at anywhere because they are parallel.

So, these three planes are not meeting at any point, I mean there is not any point where all these three planes are intersecting each other. So, these three equations have no solution. Thank you.