IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 8 Tutorial 02

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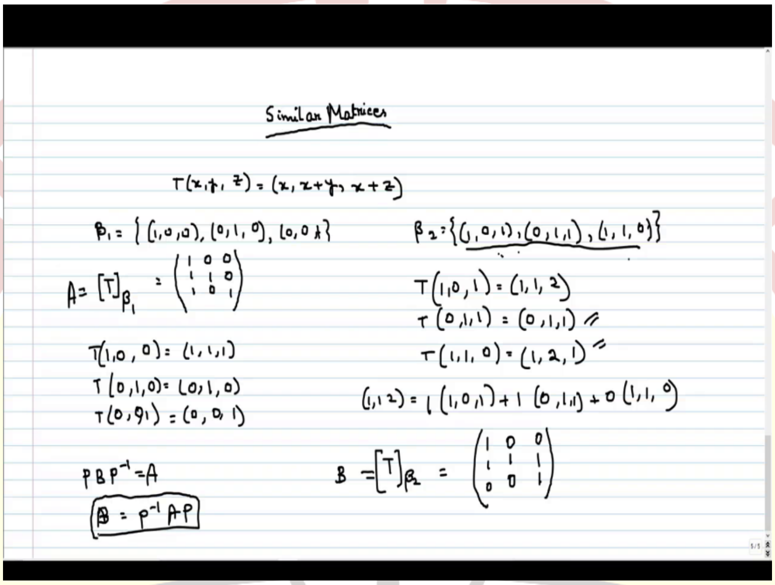

Hello everyone. In this lecture, in this tutorial video we will talk about similar matrices. So, let us take an example. So, we start with some linear transformation $T(x, y, z) = (x, x+y, x+z)$

So, this is a linear transformation, from $\mathbb{R}^3 \to \mathbb{R}^3$ and now we consider ordered basis, so this is β_1 this is the standard ordered basis we are considering {(1, 0, 0), (0, 1, 0), (0, 0, 1)}. So, this is the standard ordered basis.

Let us consider another ordered basis β_2 let us take $\beta_2 = \{(1,0,1),(0,1,1),(1,1,0)\}$ So, if we write the matrix representation with respect to both the ordered basis, so T with respect to β_1 and for the domain and codomain in both places if we consider the basis to be β_1 then this will give us the matrix. So, let us write it T(1, 0, 0) that is nothing but (1,1,1); T(0, 1, 0) that is the matrix, so (0, 1, 0) and T(0, 0, 1) this will give us (0, 0, 1).

So, the matrix we got here, so this matrix is nothing but first column will be (1, 1, 1); second column will be (0, 1, 0) and the third column will be (0, 0, 1). So, this is the matrix representation with respect to standard ordered basis. Now, similarly if we calculate T of these vectors in β_2 and we write, we then get T (1, 0, 1) is nothing but (1, 1, 2). T(0, 1, 1) is nothing but (0, 1, 1) and T(1, 1, 0) = (1, 2, 1).

Now (1, 1, 2) we can express in terms in a linear combination of vectors in β_2 So, we will get $1(1,0,1)+1(0,1,1)+0(1,1,0)$ Similarly, we can express the other two also, so the matrix we get is $[T]_{\beta_2}=\begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}$ So, I want all of you to check this representation. You can express all the images these vectors and these vectors as a linear combination of vectors in β_2 and then write the matrix representation.

So, we get $[T]_{\beta_1}$ and $[T]_{\beta_2}$ those are the matrix representation with respect to β_1 and β_2 respectively. Now, these two matrix are similar, but for similar matrix we know that there should exists some, so let us denote this matrix as A for removing the complication of notation and this matrix as B.

So, for as A and B are similar matrix, so there should exists some P such that $PBP^{-1}=A$ or even you can write sorry $B=P^{-1}AP$ So either of these, basically I just change the sides of P. So, we want to find this P. What will be this P?

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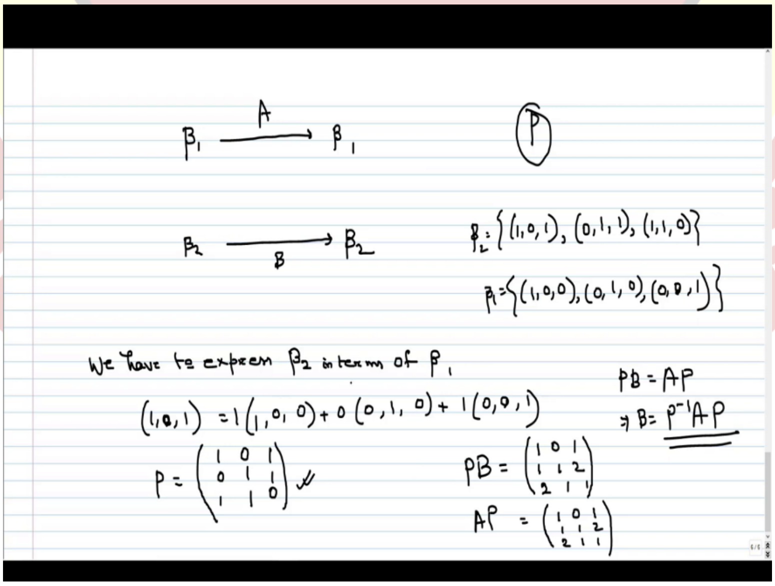

So, what we got till now, till now we got that if we consider β_1 for both domain and codomain we get the matrix A as the representation of linear transformation. Similarly, for β_2 we got the matrix B. So, now if we represent this β_2 in terms of β_1 then we get the change of basis matrix that will give us the matrix P. So, what we have to do? We have to do, we have to express the vectors of β_2 in terms of vectors of β_1

So, here beta 2 is nothing but $\{(1,0,1;),(0,1,1)(1,1,0)\}$ Now we have to express this in terms of β_1. β_1 is our standard ordered basis. So, this is $\beta_1=\{(1,0,0),(0,1,0),(0,0,1)\}$ So, this is the standard ordered basis. So, we have to express, β_2 in terms of β_1 So, this is the matrix which we get in this process, is called as change of basis matrix.

So, let us try to write (1, 0, 1) in terms of standard ordered basis so which is nothing but $1(1,0,0)+0(0,1,0)+1(0,0,1)$ So, similarly for the other two vectors we can write this expression, so this is nothing but the matrix which we get by taking the vectors of β_2 as the columns of this matrix. So, the matrix we get is the column of this, we replace the columns with the vectors in β_2 So, this is the matrix we get, this is the change of basis matrix and we denote it by P.

Now, if we calculate PB we will get $PB=\begin{pmatrix}1&0&1\\1&1&2\\2&1&1\end{pmatrix}$ So, this is our PB. And if we calculate A into P so this is the matrix P into B and A into P will give us the same matrix $AB=\begin{pmatrix}1&0&1\\1&1&2\\2&1&1\end{pmatrix}$
I want you to verify this two, these two matrices PB and AP. So, what we get here? We get PB = AP. So, $B=P^{-1}AP.$ So, we can verify that this A and B are really a similar matrix, but P is constructed by expressing β_2 in terms of β_1

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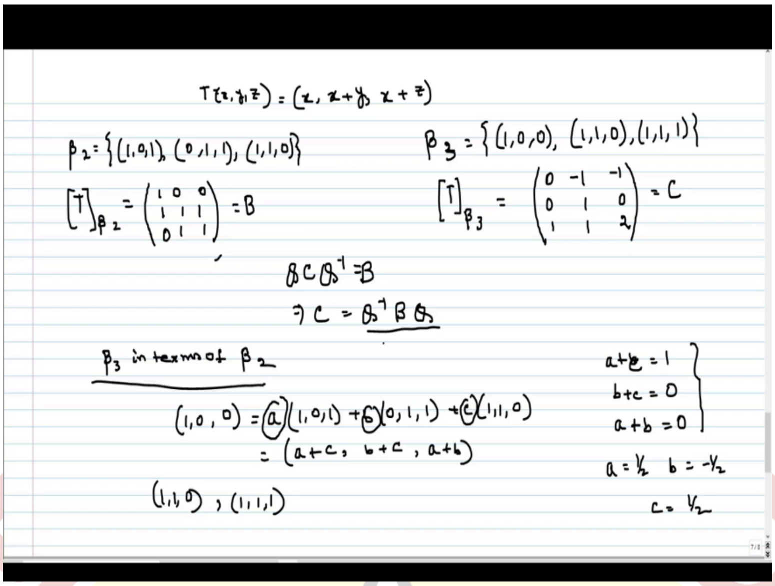

Now, in the same example we have not changed the linear transformation and we have also taken β_2 the basis as {(1, 0, 1), (0, 1, 1), (1, 1, 0)} as earlier. So, this beta, B matrix we have calculated earlier. Similarly, if we take the another basis $\beta_3 = \{(1,0,0),(1,1,0),(1,1,1)\}$ you can calculate the matrix representation, you can verify it that it will be C.

Now, in this case we have also got two matrices B and C by just changing the basis. So, this B and C will be similar and we want to find some matrix Q such that $QCQ^{-1} = B$ That means, $C = Q^{-1}BQ$ We want to find this Q. Now, how we can find this Q? We have to express the vectors in β_3 in terms of the vectors in β_2 We have to find this matrix and that will be our Q.

So, let us try to write the first vector (1, 0, 0) in terms of the vectors of β_2 So, that is nothing but $(1,0,0) = a(1,0,1) + b(0,1,1) + c(1,1,0)$ So, what we get here? We get $(a+c, b+c, a+b)$ So, our a+b will be 1; b+c will be 0 and a plus, sorry the first one is a plus c, a +c will be 1, and the last one is a + b that is 0. So, we got, we get 3 equations and if we solve it we will get $a = \frac{1}{2}, b = \frac{-1}{2}, c = \frac{1}{2}$

So, similarly we can express the other two vectors which are (1, 1, 0) and (1, 1, 1) in terms of vectors in β_2 and if we write all these things in the columns, as the coefficients as the columns of a matrix, then we get the matrix Q.

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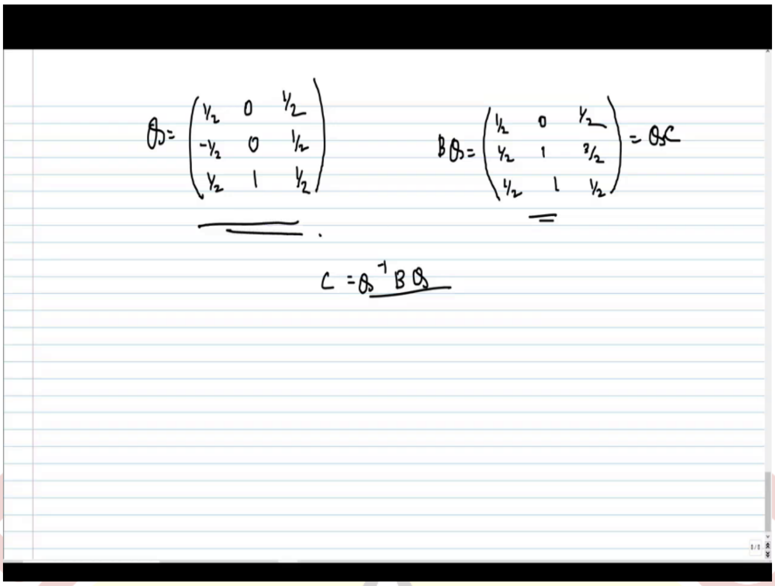

And I want all of you to verify that this is nothing but (1/2, -1/2, 1/2) which we have already checked and the next one will be (0, 0, 1) and the last one will be (1/2, 1/2, 1/2). So, this is our matrix $Q = \begin{pmatrix} 1/2 & 0 & 1/2 \\ -1/2 & 0 & 1/2 \\ 1/2 & 1 & 1/2 \end{pmatrix}$ Now if you calculate BQ, you will get $Q = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 1/2 & 1 & 3/2 \\ 1/2 & 1 & 1/2 \end{pmatrix}$ So, this is the matrix BQ and which will be same as QC. So, if you take product of Q and C and B and Q, you will get the same matrix. So, we will get $C = Q^{-1}BQ$ where Q is nothing but the change of basis matrix.

So, how we can get Q? We just express all the vectors in β_3 in terms of the vectors in β_2 I will write this coefficient, this a, b and c for each case. So, write a, b and c as the column of the matrix of Q. So, that is how we can calculate Q and you can verify that two matrices are similar matrix or not. So, whenever there is a change of basis and we get two representation of a linear transformation with respect to that basis, then those two matrices are similar matrices. Thank you.