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Mathematics for Data Sciences 2
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Partial Derivatives

Hello, and welcome to the Maths 2 component of the Online BSc Program on Data Science and Programming. In this video, we are going to study the notion of partial derivatives. So, we have seen functions of several variables. And now partial derivatives corresponds to the notion of the rate of change of a function of several variables, according to one variable, so with respect to one variable, so we will study this in this video.

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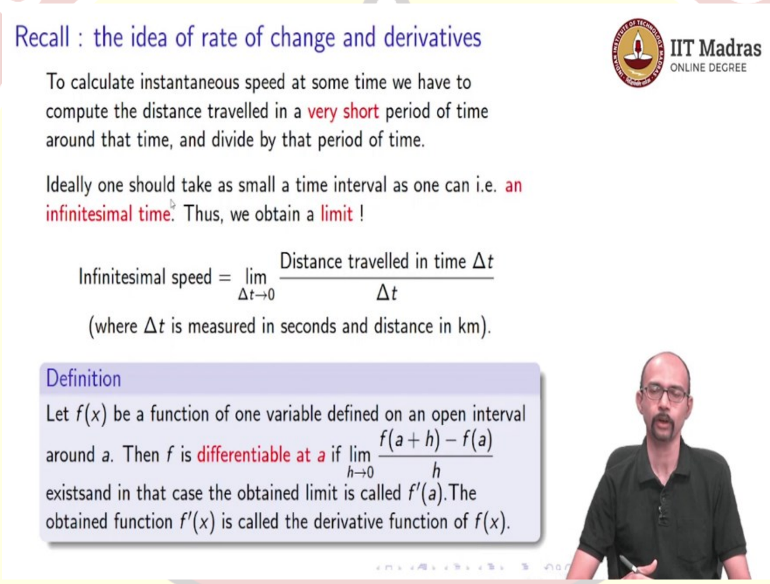

So, let us recall first the idea of the rate of change and derivatives from one variable calculus. So, to calculate instantaneous speed at some time, we have to compute the distance traveled in a very short period of time and /that by the time period and that should be a close approximation of the instantaneous speed.

So, this was something that we studied in when we introduced the notion of derivatives in one variable calculus. In particular, we had this example of a truck traveling from Punjab to Tamil Nadu. And based on that example, we extrapolated what the correct definition should be.

So, ideally, one should take this time interval as small as possible, that was the main content of that example. And we move towards this idea of an infinitesimal time. And what is an infinitesimal time, that is just a limit. So based on these, we reach the following conclusion that infinitesimal speed is the limit as Δt tends to 0 where Δt is the length of the time interval, distance traveled in that time $\frac{\Delta d}{\Delta t}$.

So, of course, here we have to start talking about units. So, if we want kilometers per second, we want to measure it in kilometers per second, you have to measure delta t in seconds, and the distance in kilometers. So, this led us to the general notion of a derivative, which captured the rate of change. So let f(x) be a function of one variable defined on an open interval around a. Then f is differentiable at a, if $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists, and in that case, the obtained limit is called f'(a).

And then out of this f', we made a function. So, if we do this for each, a, which is in the domain of f, and for those ones for which it exists, this limit exists, we define the derivative function f'. So, you collect together all those points a, for which f'(a) make sense, and then you define this function f' from that set to R. So, we want to do something similar now for functions of several variables. So, the obtained function f'(x) is called the derivative function of f(x). And we want to do the same thing now for functions of several variables with respect to each variable.

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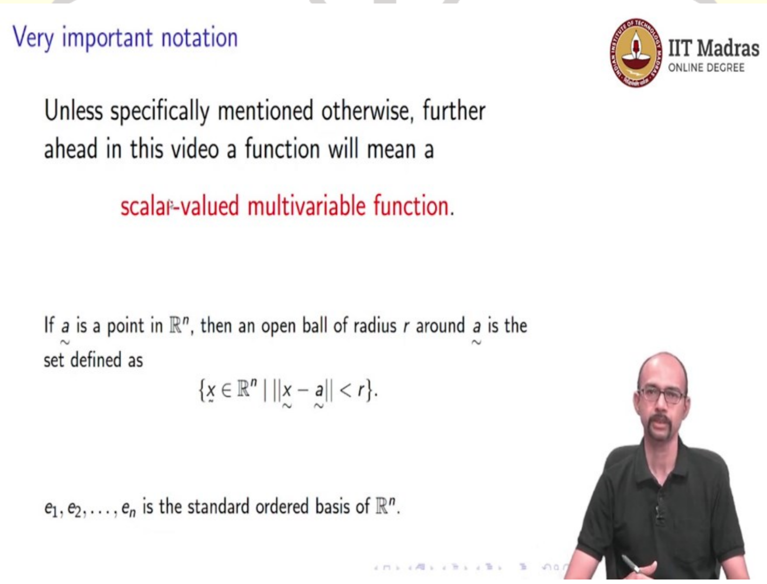

So, let us fix some notation before we go ahead. So, from hereon, in this video, at least, and possibly in several of the next videos, unless specifically mentioned, otherwise, a function will mean a scalar-valued multivariable function. So, we have seen earlier the difference between scalar-valued and vector-valued multivariable functions. So, scalar-valued multivariable functions are those such that you have the domain, which is a domain in Rn, where n is greater than 1, and the range is in R, so the codomain is R.

One more thing that we should know, if $\underset{\sim}{a}$ is a point in Rn, then an open ball of radius r around $\underset{\sim}{a}$ is the set defined as $\underset{\sim}{x}$ in Rn, such that the distance between $\underset{\sim}{x}$ and $\underset{\sim}{a}$ is less than R. So, if you think of this in $\mathbb{R}^2$R2, what this means is, you have a disc of radius r around the point a. So that means, you draw a circle of radius r around that point $\underset{\sim}{a}$, and then it is all those points that are within that circle.

So, and the final thing that one has to remember is, that $e_1, e_2, e_3, \dots e_n$ is the standard ordered basis of Rn. So, this was something we studied in the linear algebra part. So, we will have some use for this for these standard basis vectors in what is coming next.

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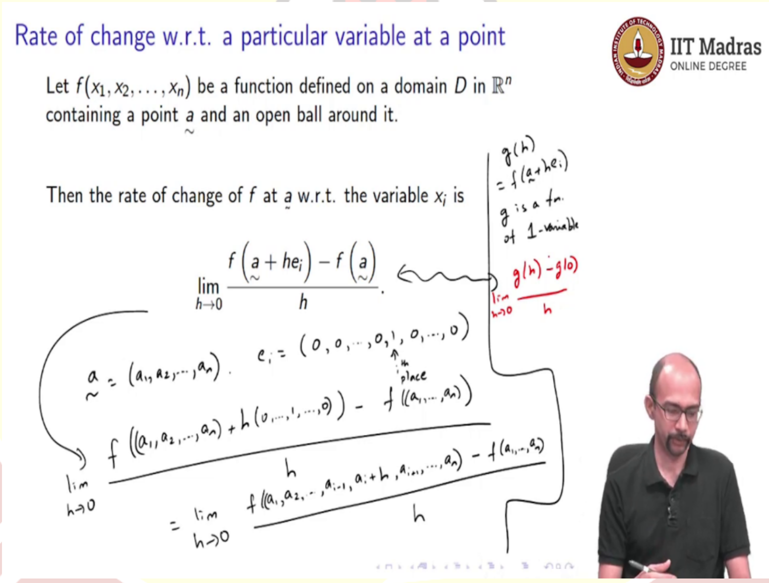

Fine, so let us talk about the rate of change with respect to a particular variable at a point. So, what we want to do is, we want to ask, suppose I have a function of several variables, $f(x_1, x_2, x_3, \dots x_n)$ and I want to ask, well, how does this function behave at the point some fixed point $\underset{\sim}{a}$ when x_1 varies? So, we keep everything else fixed, and just very x_1, and ask how does it behave? How is the behavior of the function around this point? And in particular, we want to ask, how does the function change? Is it, does it change fast? Does it change slowly?

So, this is sort of similar to saying what is the instantaneous speed as you change x_1, so we can do this with any particular variable. And that is what we will study now. So, let

$f(x_1, x_2, x_3, \dots x_n)$ be a function defined on a domain D in $\mathbb{R}^n$ containing a point $\underset{\sim}{a}$ and an ball around it. So, when we say open ball around it that means for some positive radius. Then the rate of change of f at $f(\underset{\sim}{a})$with respect to the variable Xi, and this is important that it is with respect to the variable x_i is $\lim_{h\to 0} \frac{f(\underset{\sim}{a}+he_i)-f(\underset{\sim}{a})}{h}$.

So, this is very similar to the notion of the derivative that we saw for these single variable functions. And in fact, I am going to sort of in a second say that it is actually the same thing. There is an underlying single variable function, and that is what I will described now. So, let us explicitly work out what this means. So, if $\underset{\sim}{a}$ has coordinates $a_1, a_2, a_3, \dots a_n$we know that ei means the standard basis vector with zeros everywhere except in the ith place. So, this is the ieth place, and you have zeros everywhere. So, let us unravel what this definition means.

So, what this is saying is that you have $f(a_1, a_2, a_3, \dots a_n + h) * 0$f of a1, a2, an + h × 0 all the way till the ith 1, which is 1 and then all zeros again, - f of a1, a2, an. And then this /h, and then limit as h tends to 0. So, if you work out what this is, this is exactly f of a1, so we can add these two vectors. So, $a_1, a_2, a_3, \dots a_n$ + h × ei. So, there will be no change in the other coordinates. So, all those remain the same up to i - 1 and then in the ith one, you have ai + h and then you go all the way up to an. And then this - again, $f(a_1, a_2, a_3, \dots a_n)$ /h.

So, as you can see, really what is happening is that only the ieth coordinate is changing, everything else is remaining the same. So, what we are really saying is, let us define the new function, let us maybe call it a g. And what is this new function? This new function is g of h is equal to $f(\underset{\sim}{a} + he_i)$+ h × ei. Then what we are asking. And now, remember, that this is a function of one variable. So, g is a function of one variable. This is an important point, g is a function of one variable. Because h is just a number, it is a real number.

So, this h is a real number, everything else in here they are vectors. So, $\underset{\sim}{a}$is a vector ei is a vector, but h is a real number. So, you are doing h × e_i, so scalar × that vector ei. So, g is a function of h, which means g is a function of one variable. And now, what this limit translates to is g of h, because that is exactly what this is. – g(0) /h limit as h tends to 0, g of 0 is exactly f($\underset{\sim}{a}$), which is what we are subtracting from g(h). So, this is computing the derivative of the function g at the point 0 that is what is happening.

So, really, this limit is or this rate of change with respect to a variable x_i is saying, let us forget all the other variables, let us concentrate only on x_i and let us vary x_i and treat that alone as a function and see what happens, and then compute its derivative because rate of change is derivative of that function, so rate of change with respect to that variable is the derivative of g. So, I hope this is clear. We will see pictures of this in a few minutes.

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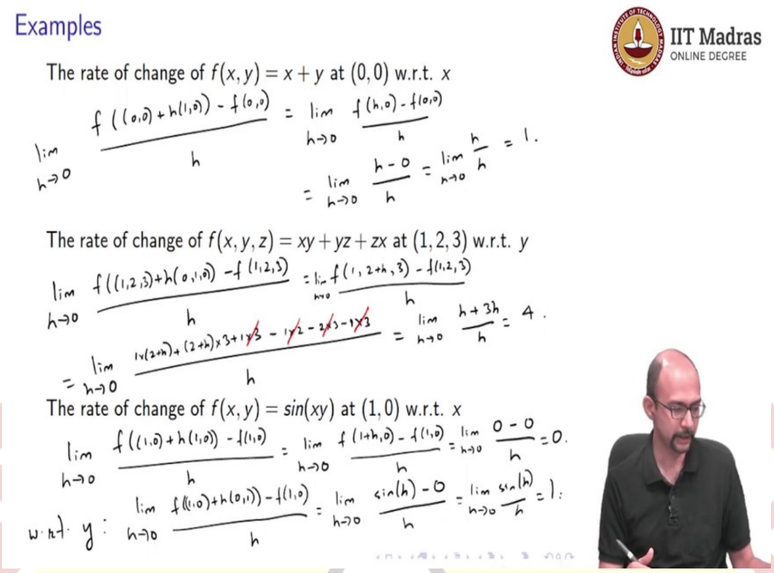

So, let us do a couple of computations before we see the pictures. So, the rate of change of f(x), y, x , y is x + y at 0, 0 with respect to x, let us compute this. So, what do I want to do? I want to compute with respect to x, so I should take e1. So, this is at 0, 0, f(0), 0 + h × 1, 0 - f(0), 0 /h limit h tends to 0. And what is that? That is exactly a f of, so if I add these, this is f of h , 0 - f(0) , 0 /h.

Well, f of h , 0 is h + 0, so that is h, f(0) , 0 is 0 then /h. So, this is a very familiar limit now. So, this is going to be 1, so this is a function 1, so this limit will be 1, as h tends to 0. So, this is, this will reduce down to our good old limits that we have seen earlier in calculus, single variable calculus.

Let us do another example. So, the rate of change of f(x), y, z at 1, 2, 3 with respect to y. So, let us compute this. So, this is limit h tends to 0, f(1), 2, 3 + h × this is with respect to y. So, 0 , 1 , 0 - f(1), 2, 3 /h. So, what that means is, you get f(1) , limit h tends to 0, f(1) , 2 + h ,, 3 - f(1) , 2 , 3 /h, let us find out what those values are.

So, the f(1) , 2 + h , 3 is 1 × 2 + h, + 2 + h× × 3, + 1 × 3, and then - 1 × 2, - 2 × 3 - I have put a bracket unnecessarily - 1 × 3. This is what the expression in the numerator is, and I should put my limit /h. So, this gives us, so 1 × 3 and 1 × 3 cancel, and then you have 1 × 2 + h. So, the 1 × 2 cancels, and the 2 × 3 cancels and then what you are left with is 1 × h, which is h, + 3 × h, /h, so this is going to be 4. So, 4 h by h, which is 4, and then the limit no longer matters. So, this gives us 4, and this computation is important, keep track of what happened here.

Let us do this final example. So, the rate of change of f(x), y is sine of x, y at 1 , 0 with respect to x. So, let us see what that is. So, we have limit h tends to 0 sine of, my bad, let us write down the expression first. So, f(1) , 0, which is the point + h × it is with respect to x. So, this is h × 1 , 0 - f(1) , 0 /h. So, let us see what this evaluates to.

So, the first term gives us f(1) + h , 0, and then - f(1) , 0 /h. So now, if we substitute the equations, the function sine of x, y, so this is sine of 1 + h × 0, so this is sine of 0, which is just 0. And then this is again sine of 1 × 0 again, 0 and /h, so this is going to be 0. So, this is going to be the limit with respect to x.

Suppose, now I want this with respect to y. Just for practice, let us see what it is with respect to y. So, I can do the same thing, except now I have 1 , 0, + h × 0 , 1, then f(1) , 0 /h. So, I will move directly to the last step where we get limit h tends to 0. So, f(1) , h, so which is sine of h, and then - 0 and then /h, this is a very familiar limit. So, limit h tends to 0 sine of h by h, and we know, this is 1.

So, the point here is that all of these reduced to very nice limits that we have computed earlier, we have calculated earlier in one variable calculus. So, this is not something very different from what we have seen earlier. So, now let us look at the pictures for some these functions, and see if we can interpret to what we are doing.

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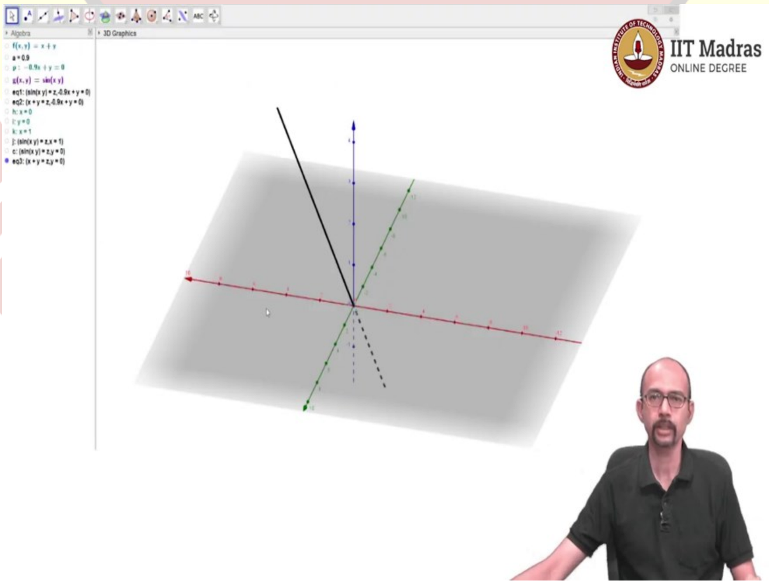

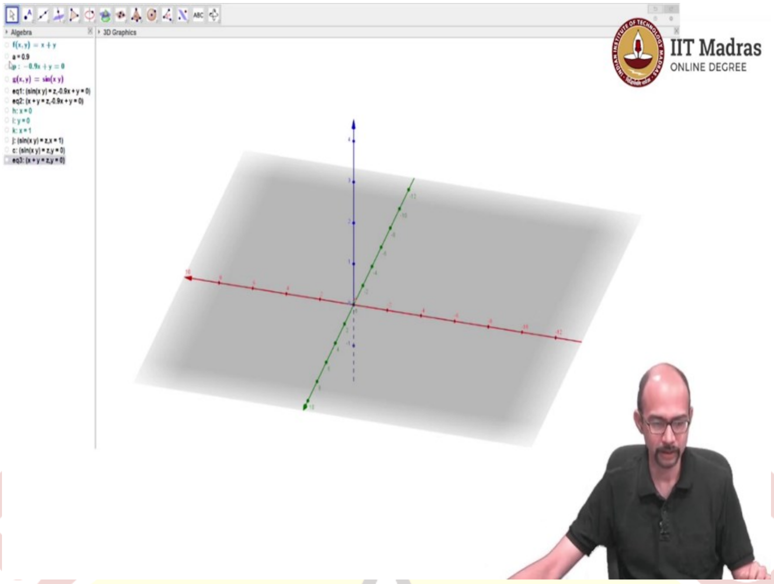

Algebra
3D Graphics
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a = 0.9
p : −0.9x + y = 0
g(x, y) = sin(x y)
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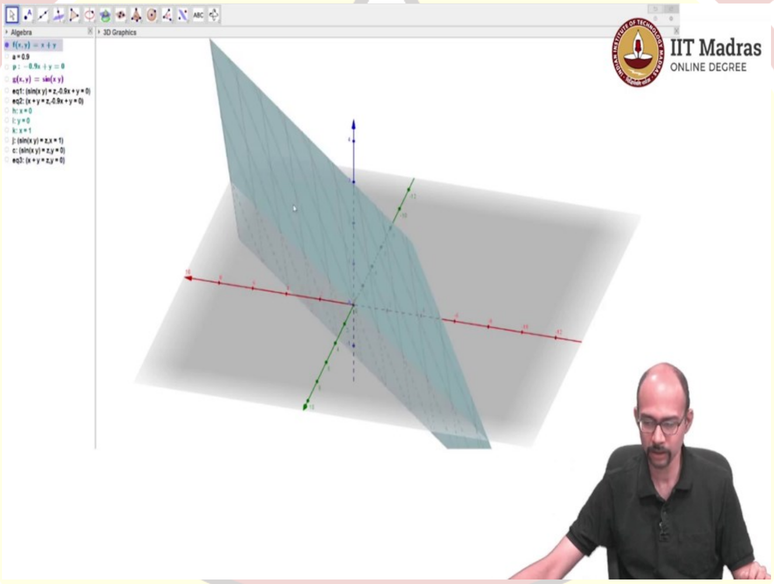

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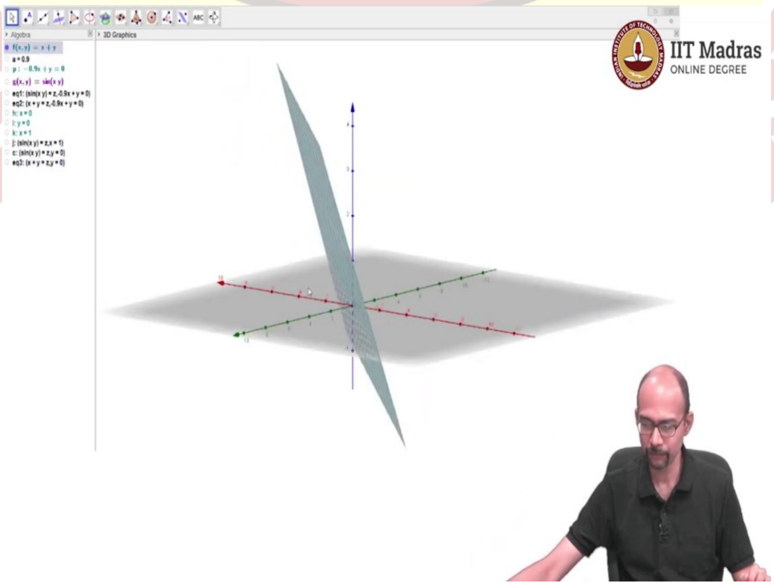

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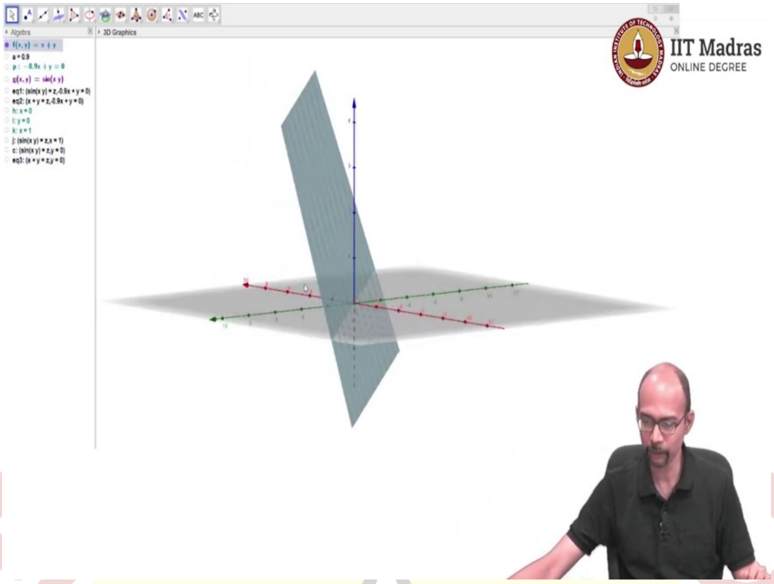
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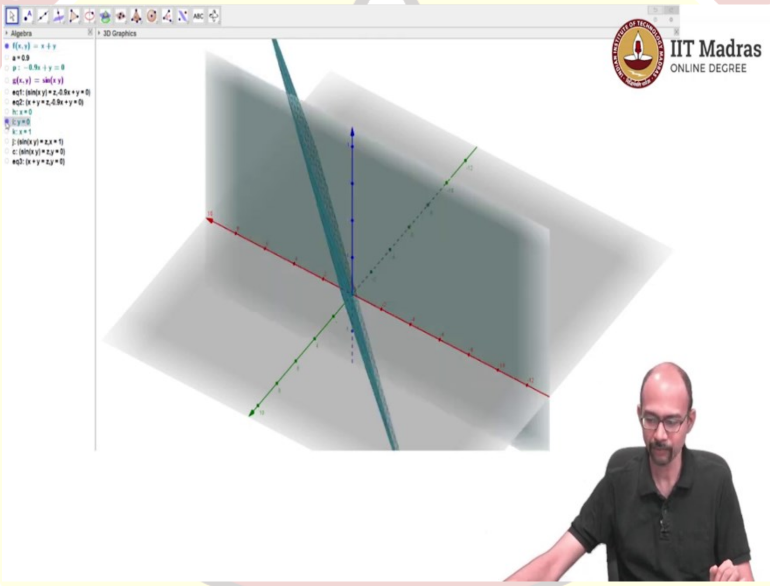
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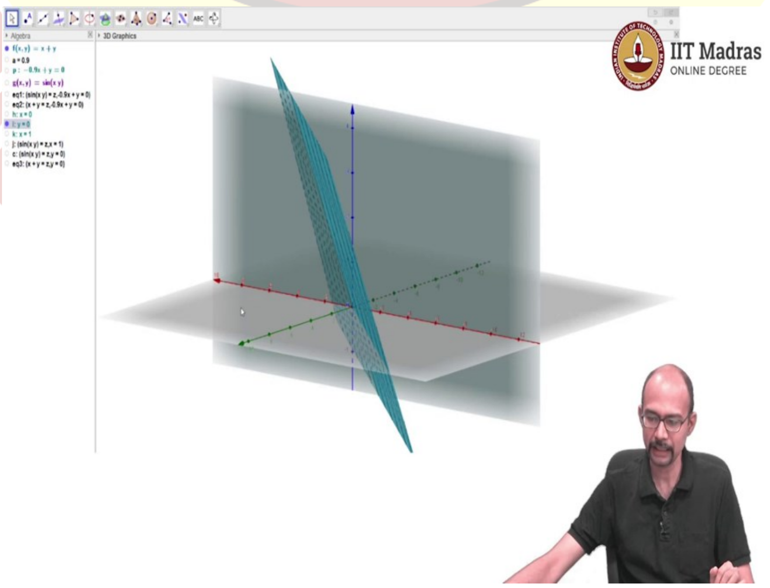
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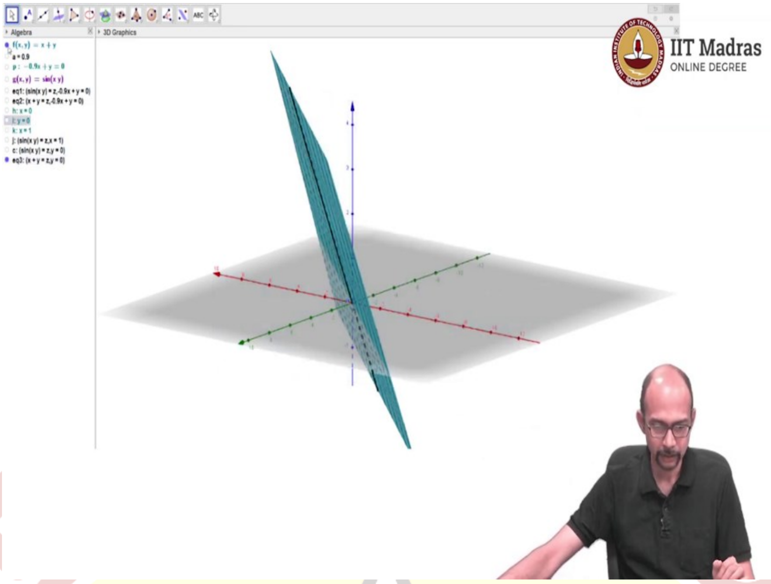
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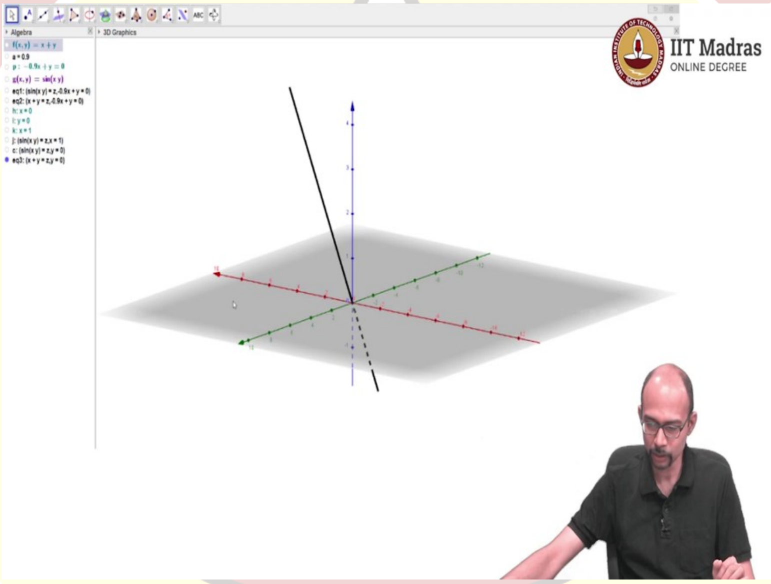
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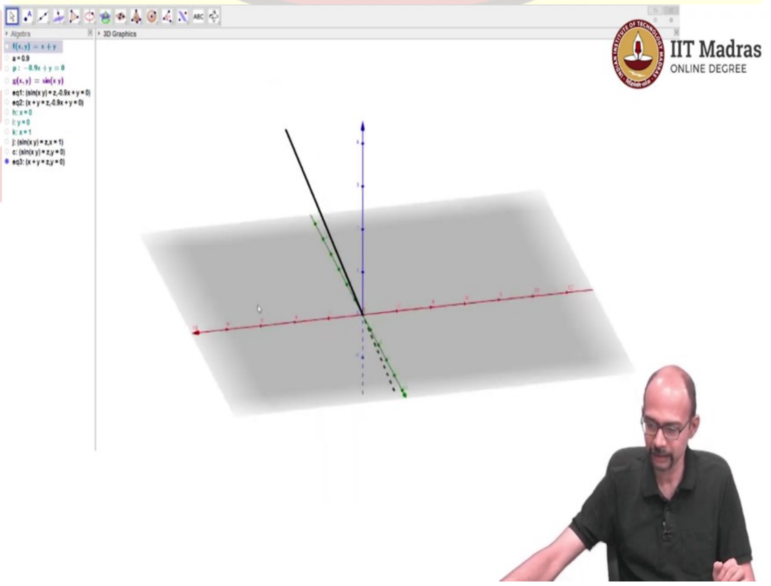
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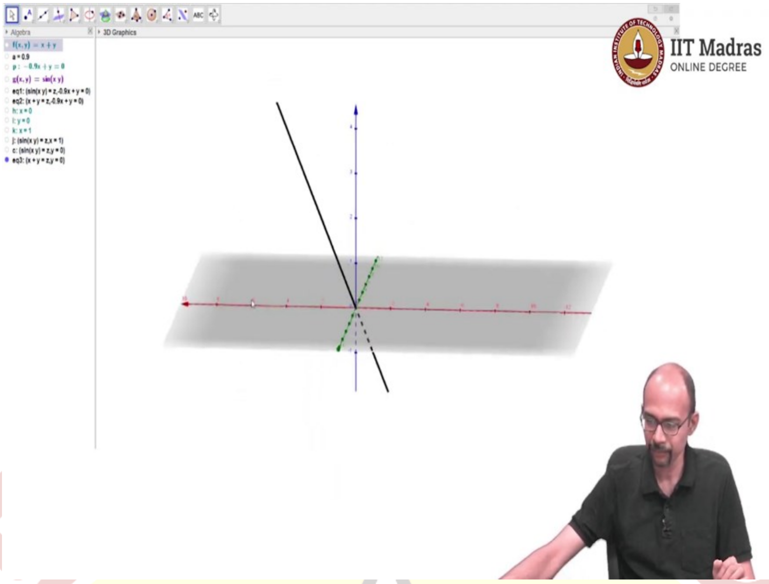

So, let us look at the function, f(x), y is x + y. And we were computing the limit or the rate of change of this function at 0, 0 with respect to the variable x. So, for this, let us look at the function, first of all, how does this function look like. So, this is a graph of that function, f(x), y is x + y.

So, the red line is your x axis, the green line is your y axis. And in terms of what we discussed, when we have to do this with respect to x, we want to study the rate of change with respect to x, that means we are really at the point 0, 0 that means we are interested in asking that along the x axis, that is where y is constant, and x is changing, we want to ask on top of that, how does this function behave?

So, of course, we can substitute y is 0 and then we know that the function g that we talked about g of x is just x because you get x + 0. And in terms of pictures, what that means is you take this graph of f(x), y, and then you look at the plane y, 0 that is the part above the x axis and then you look at the intersection, and that is exactly the graph of the function that you obtain when you restrict this function only to the x axis.

So, what we get in that case is this line that we have here. So, if I now take out these two, you can see this and this is indeed the line. And this is a graph of the function, g of x is equal to x. And so, we are asking what is the derivative of this, of this function? That is what that limit compute it at 0.

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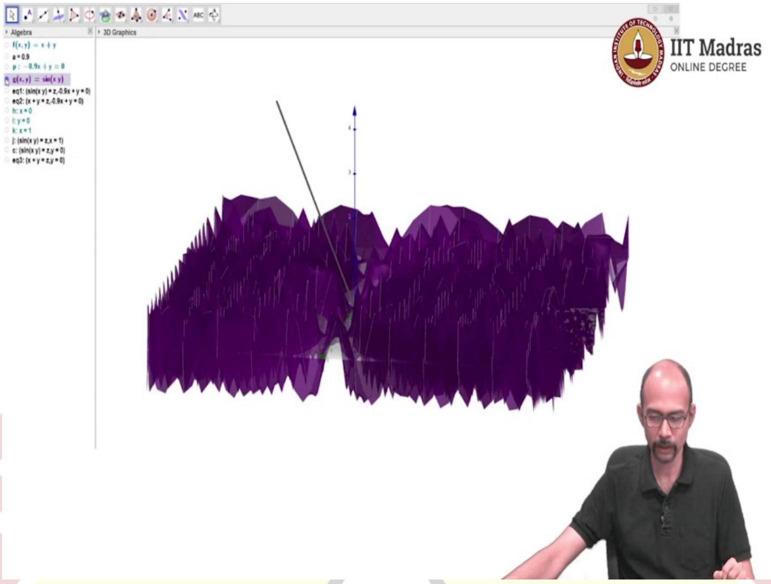

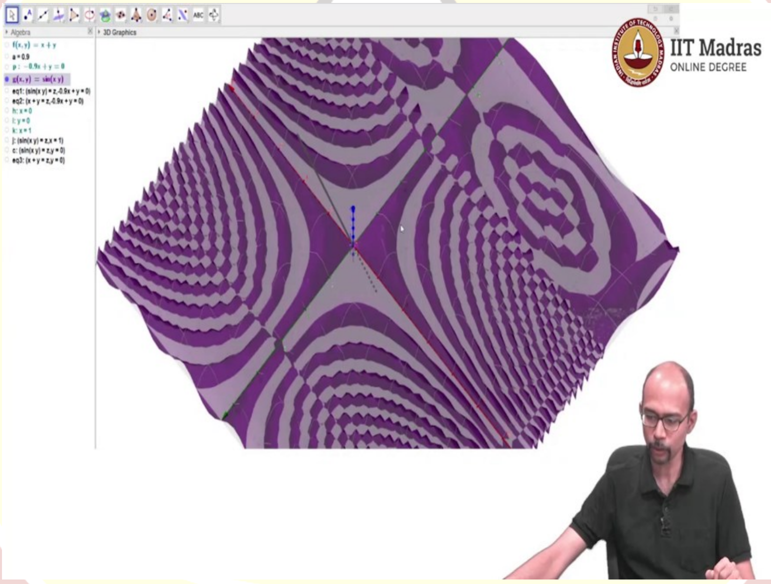

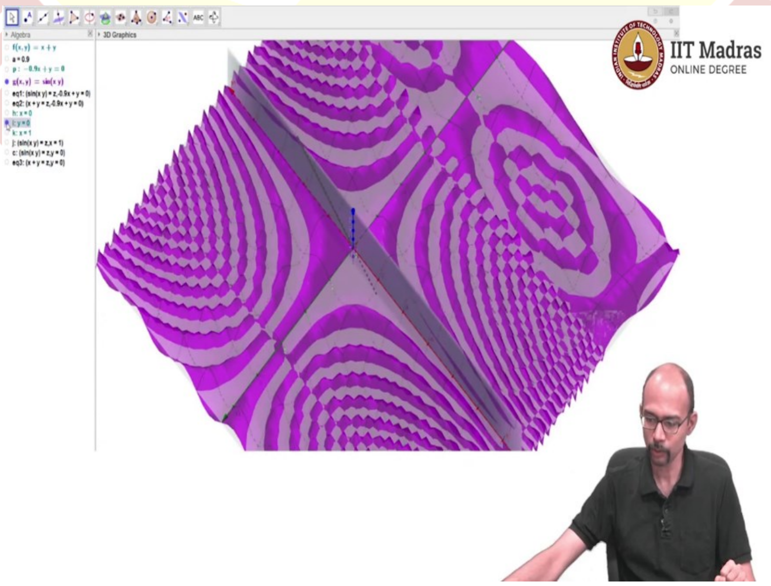

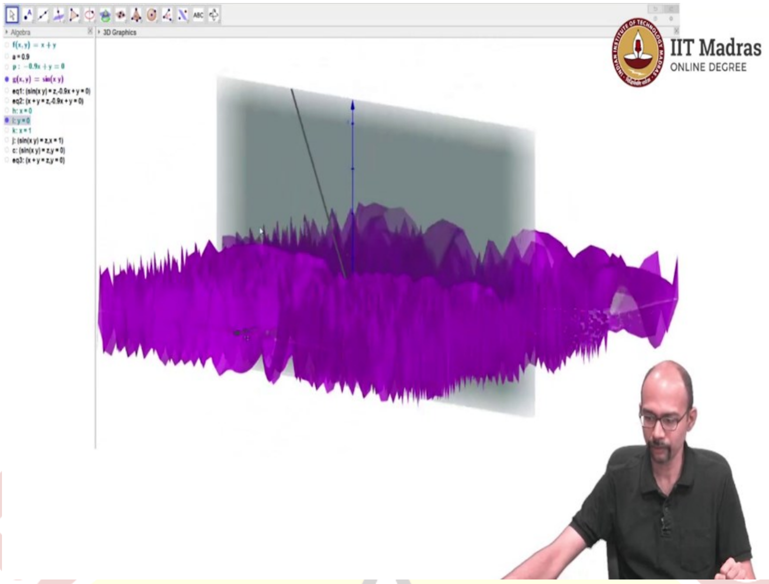
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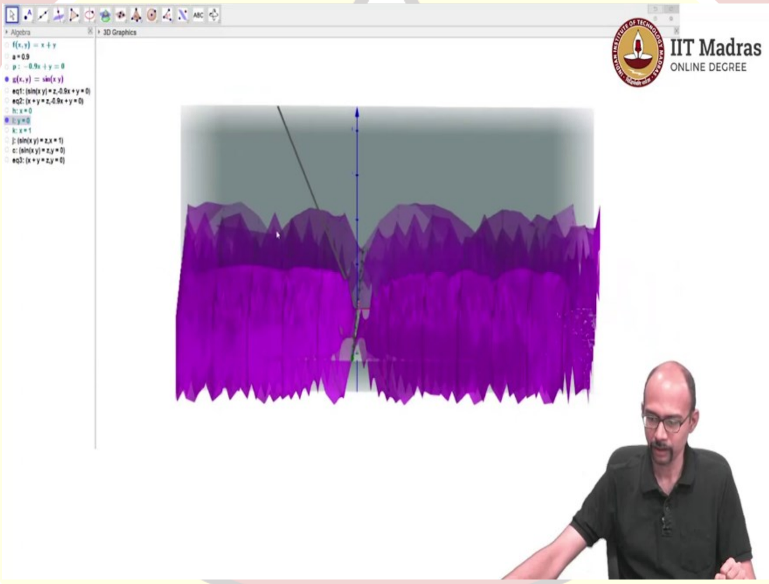
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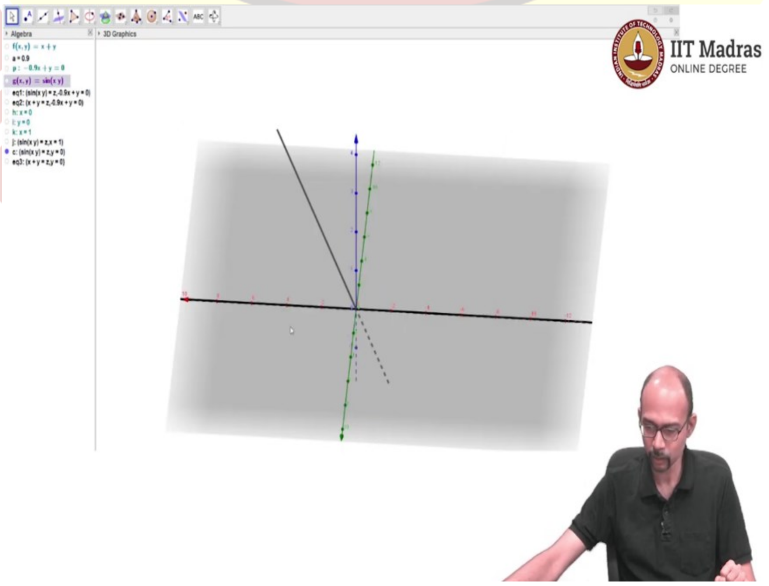
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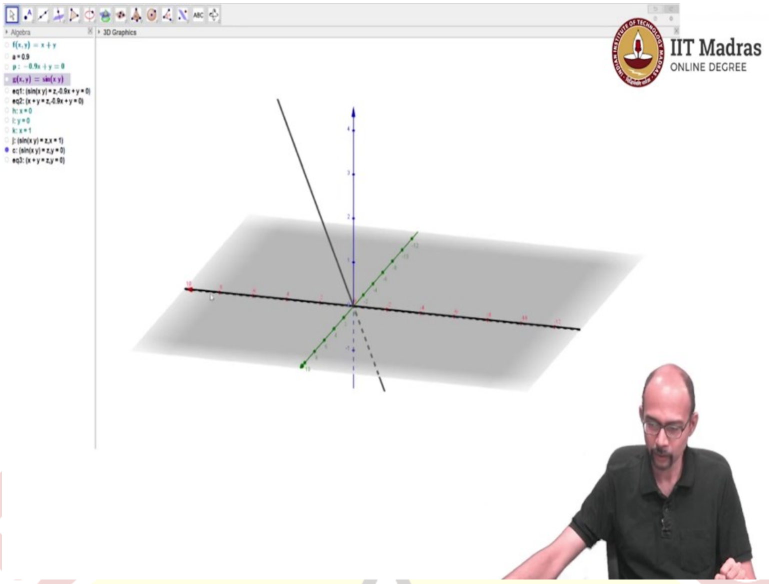

So, similarly, we did one more example. That was the example of the function sine of x, y. And so, let us look at the graph of that function. This is a really interesting graph. We wanted to look at the point 1 , 0, and we computed the rates of changes with respect to x and with respect to y. So, with respect to x, that means I have to look at the plane Y0. If you look at the plane Y0, this is what you get, and we should intersect those. So, if we intersect those, here is what we get.

So let me knock out these two, and you will immediately see what we get. Well, what we got actually is this line. Why is that? Because for Y0, the function actually is the 0 function, so that is why we get a just the graph of the 0 function. And at 1 , 0, which is over here the derivative is 0 as a result. Instead, now, let us look at the what happens when we compute this with respect to y. So, when we do it with respect to y.

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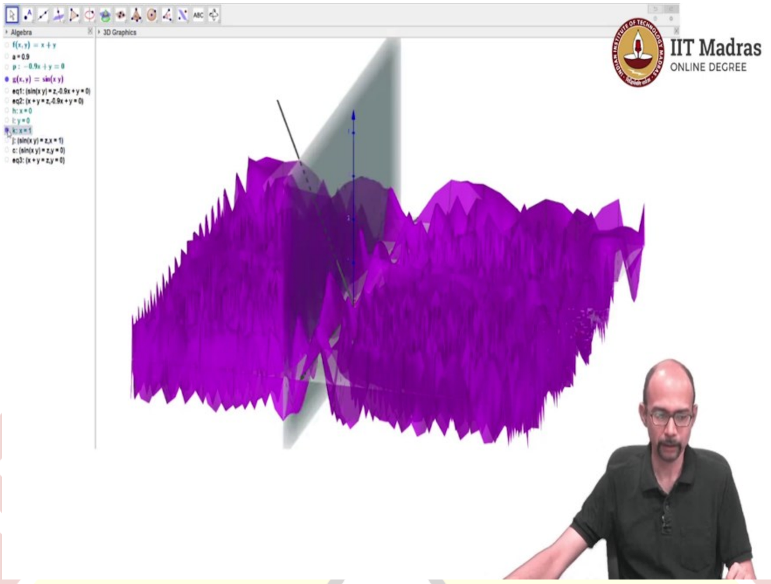

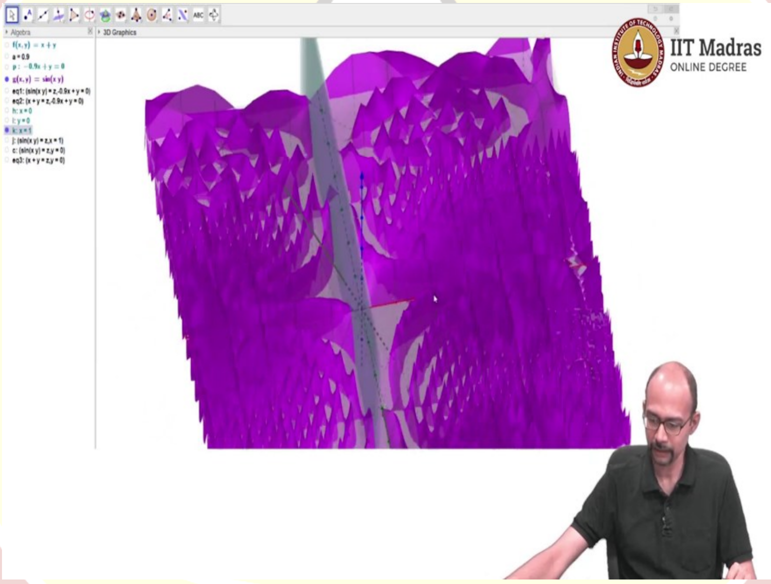

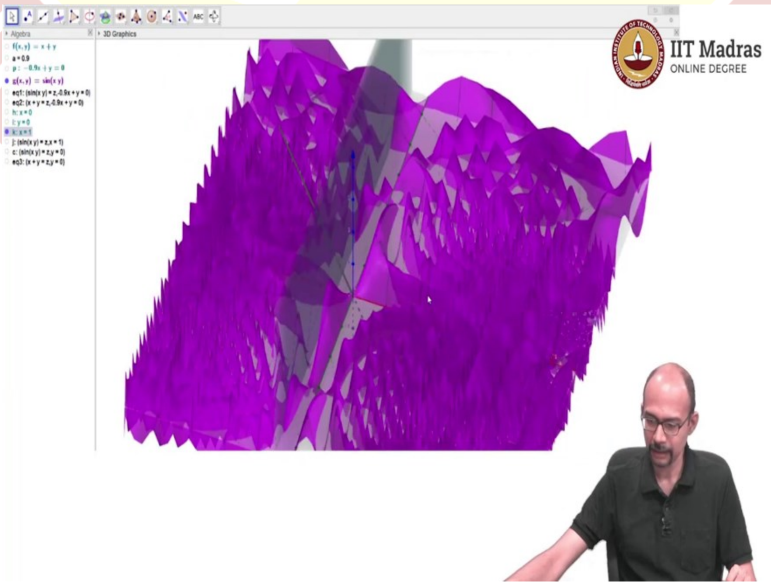

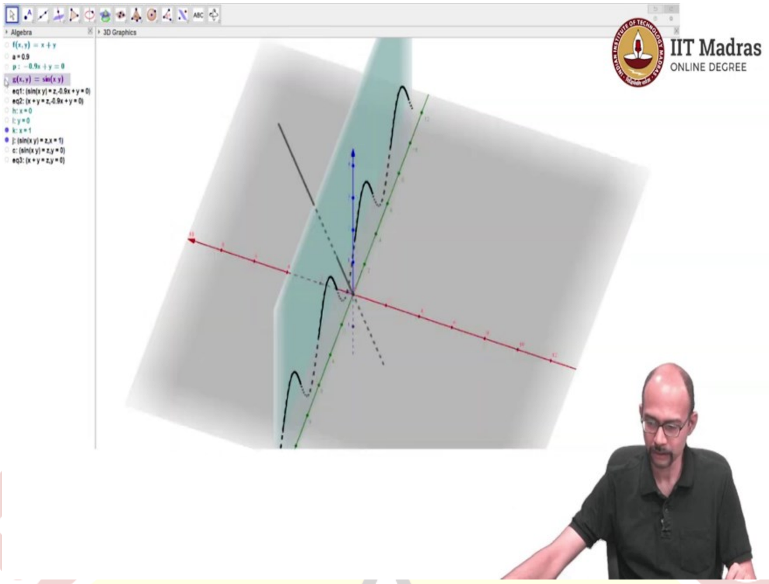
Algebra
3D Graphics
f(x, y) = x + y
a = 0.9
p : −0.9x + y = 0
g(x, y) = sin(x y)
eq1: (sin(x y) = z,-0.9x + y = 0)
eq2: (x + y = z,-0.9x + y = 0)
h: x = 0
l: y = 0
k: x = 1
j: (sin(x y) = z,x = 1)
c: (sin(x y) = z,y = 0)
eq3: (x + y = z,y = 0)
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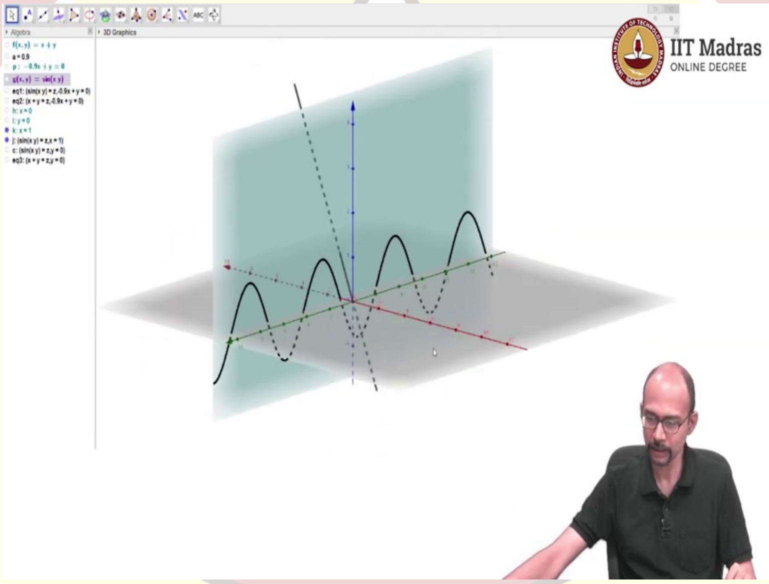
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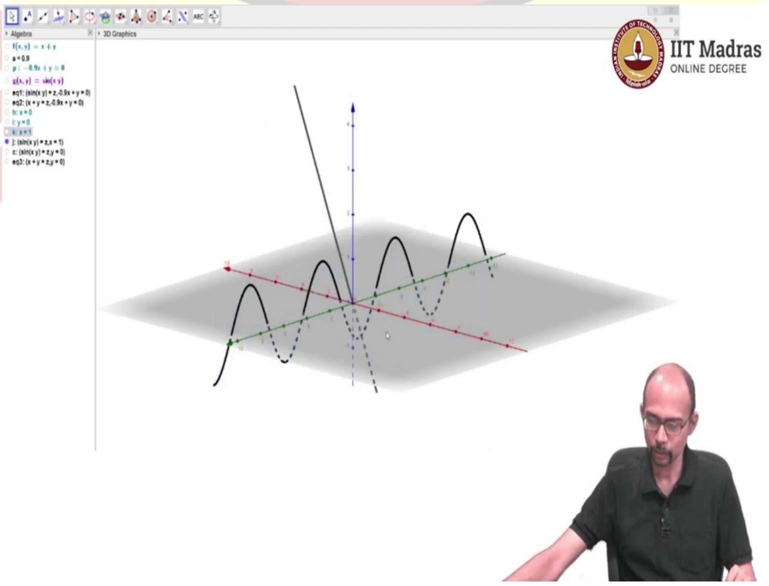
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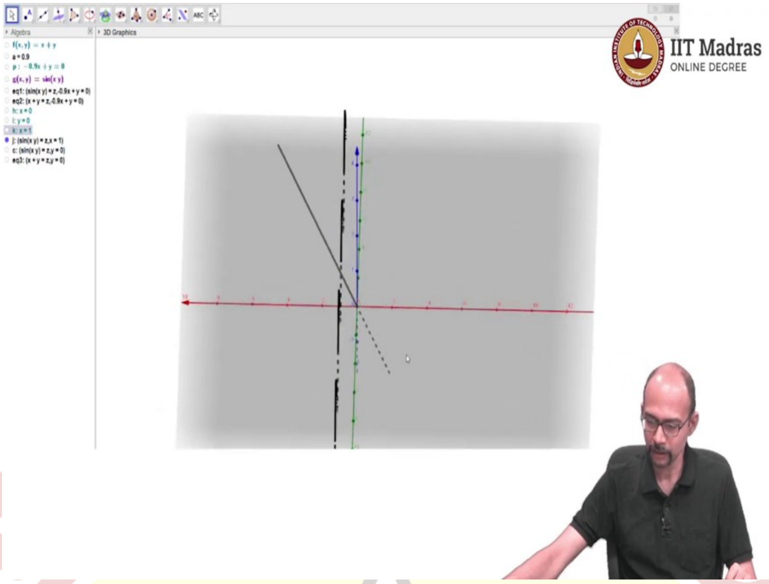

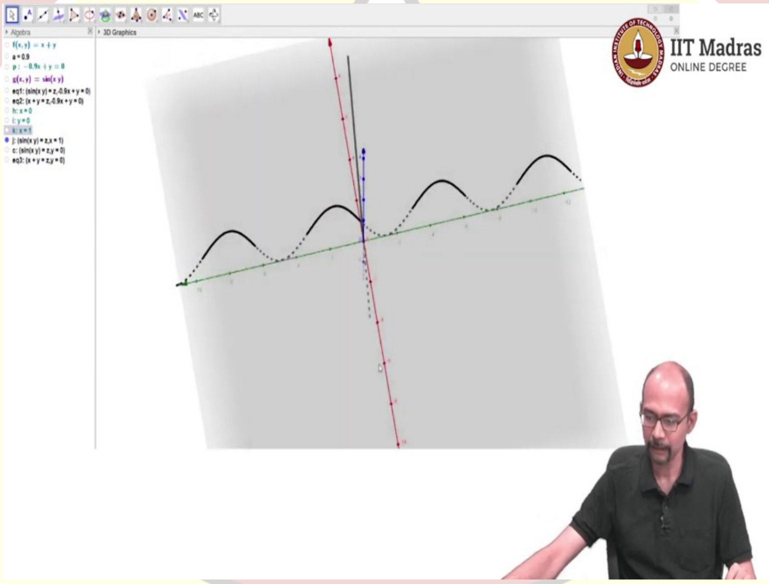

Well, let us look back at, look at our function again. So, this is a graph of the function. So, when we want to do this at y with respect to y, then we should intersect this with the plane x is 1. So, here is the plane x is 1. And so now, when we intersect, here is what we get.

So, let me remove the plane x or let me remove the graph first, and now you can see on that plane, we have this very nice function, this is actually the sine function. So, on the plane, x is 1, this is actually the sine function and so we are computing the derivative with respect to the sine function at the point 0, and indeed, that is 1, that is something we know. So, this is a, this is what is really happening, this is what we mean by rate of change graphically.

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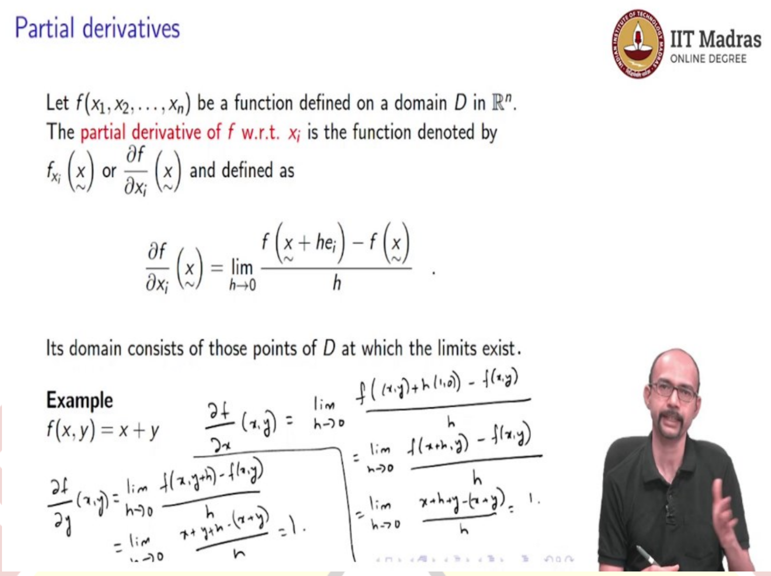

So, now that we have understood the rate of change, both in terms of the algebra where we actually computed the limits, and in terms of the geometry, where we saw those pictures, let us continue and talk about partial derivatives. So, partial derivatives are basically the function that you obtained by taking these limits at various points.

So, let $f(x_1, x_2, x_{3\ldots..}\, x_n)$ be a function defined on the domain D in Rn. The partial derivative of f with respect to Xi is the function denoted by f_{x_i}, or $\frac{\partial f}{\partial x_i}$ So, this, this symbol, which looks like a weirdly shaped d is called del, So, del f by del Xi. And what is the definition $\frac{\partial f}{\partial x_i}$ at the $\underset{\sim}{x}$ is $\lim_{h \to 0} \frac{f(\underset{\sim}{x}+he_i)-f(\underset{\sim}{x})}{h}$.

Remember that we are doing this with respect to the ith variable. So, we have to do $f(\underset{\sim}{x} + he_i)$, which means we have to do $f(x_1, x_2, x_{3\ldots..}\, x_i + h)$, again, x_{i+1} up to x_n - $f(x_1, x_2, x_{3\ldots..}\, x_n)$ /h and then take the limit as h tends to 0. So, it is like we are we are fixing all the other variables, and we are allowing the ith variable to change.

So, this is a definition of the partial derivative, this is a function. And what that means is this is a scalar-valued multivariable function because it takes as input something from Rn, and it produces an output some number, which is this limit. Now, of course, it may happen that this limit does not exist everywhere. So, the domain of these functions will consist of those points of D at which the limits do exist.

So, let us do this example. So, for this example, I want to compute what is the partial derivative with respect to x and the partial derivative with respect to y. So, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. So, let us start by asking what is your definition. So, if I want to compute $\frac{\partial f(x,y)}{\partial x}$ del f del x of x , y, by definition, this is limit h tends to 0 f(x) , y + h × 1 , 0. Why 1 , 0 because this is with respect to x - f(x) , y /h.

Well, if I compute this limit, so, this is $x + h, y - f(x), y/h$. And now, it is an easy computation where you put in the values, the expressions for the function, so, x + h + y - x + y /h, and it is clear that this is h by h, which is 1. So, this is the derivative, the partial derivative of the function f(x), y is x + y with respect to x. So, this is the function 1, so, the partial derivative of f with respect to x is 1 for this particular function.

By symmetry, I think you can see that for y also you will get the same thing but just for the sake of completeness, let me compute this. So, this is the limit h tends to 0 and I am going to skip to the last step now. So, this is $x, y + h - f(x), y$. figure out why that is the case. And now if you substitute in the expression expressions, again, you get $x + y + h - x + y/h$ and this is going to be 1.

So, both the partial derivatives for this function are 1. And you can see what we are doing here. We are really holding each variable constant for which we are not computing the partial derivative. And for the one with respect to which we are computing it, we are adding an h then evaluating the function, subtracting out the $f(x_1, x_2, x_{3\ldots\ldots}\, x_n)$and then taking dividing by h and taking the limit.

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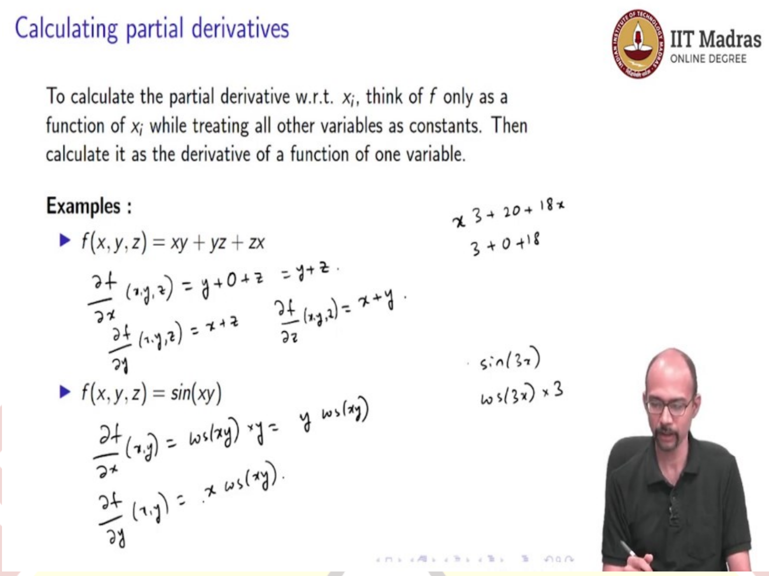

So, if your functions are nice, then there is a very nice way of doing this. So, to calculate the partial derivative with respect to x_i, think of f only as a function of x_i while reading all the other variables as constants, because that is really what we were doing here. Then calculated as the derivative of a function of one variable, this is this is exactly the game plan.

So, let us do these two examples. And now I am going to do it really fast, no more limits. So, $\frac{\partial f(x,y,z)}{\partial x}$. So, I am going to think of y, z as constants. So, suppose instead of x, y + y, z + z, x suppose I had x × 3 + 20 + 18 × x, what would you have computed this derivative as? You would have said, the derivative of this part is 3, derivative of this, this part is 0, and derivative of this part of 18. This is exactly what you should do here. So here, treat y as a constant.

So, what you get is the derivative with respect to x for the term x × y is y, the derivative for the term y × z is 0, because y and z are being treated as constants. The derivative for the term z × x is z, because that is like constant × x, what is that constant, z, so this is y + z. Similarly, if you do del f del y, you will get x + z and $\frac{\partial f(x,y,z)}{\partial z}$ is $x + y$. And I will suggest that you do the limits and see that this is exactly what you get. Let us do this example of f(x), y is sine of x, y.

So, we did this when add the point 1 , 0, so let us now do it for any point. So, if you do it for any point what you get is, $\frac{\partial f}{\partial x}$off x , y, so this is like a composition of functions. So, this is sine of, so suppose this was like sine of 3x, what would you have computed the limit as, sorry, the derivative as you would have computed it as cosine of 3x multiplied by 3, this is exactly what you should do here. So, this is cosine of x, y, multiplied by y. So, I will write it as y × cosine of x, y.

And similarly, if you take del f del y, this is x × cosine of x, y. So, you do your limits, and check that this is exactly what you get. And you will see that this is partial derivatives are actually quite easy. It is very much like your usual one variable, calculus derivate.

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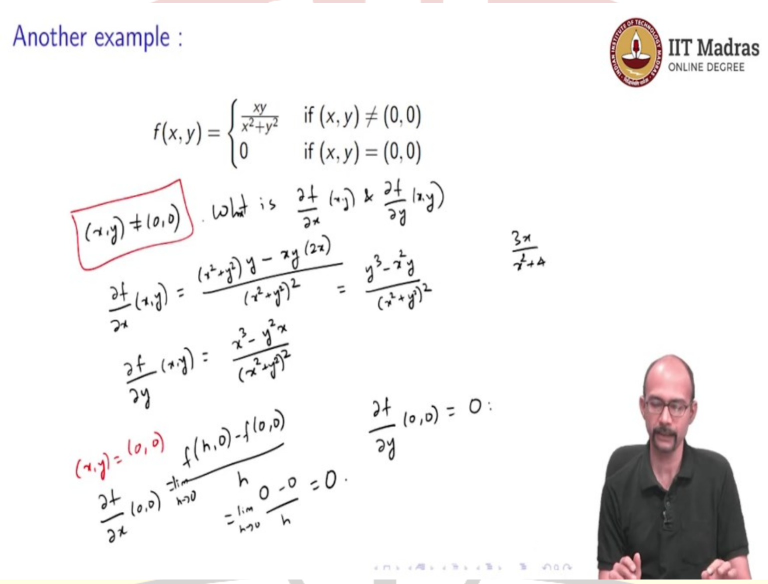

I will end with one example where, suppose your function is not defined in a nice way, meaning it is piecewise defined like this one, so in that case, you might have to actually compute those limits, because there, you cannot use the previous rules that we have from one variable calculus. So, even in one variable calculus, remember that if your function was not some nice function, then you had to actually compute the limits. So, this is what is happening here.

So, first, suppose x, y is not 0, 0. So, I want to compute x, y is not 0, 0 what is the $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$. So, $\frac{\partial f(x,y)}{\partial x}$ for such a point is just the usual way. So, this is a suppose you had a function like 3x by $x^2 + 4$, think about how you would have computed the derivative and do it according to that. So, this is going to be the U /V rule. So, this is $x^2 + y^2$ × derivative of x, y with respect to x, that is y - x, y × are derivative of $x^2 + y^2$, which is 2x.

Remember that I am doing, when I am saying derivative of x squared + y squared, I am doing it with respect to x with the idea that y is a constant, and then divide it by the denominator square, and you can probably simplify this. And if, so you have x squared y + $y^3 - 2x^2$ y, so $y^3 - x^2 y / x^2 + y^2$. And then similarly, either by symmetry or if you compute it you are going to get the same thing for the same type of expression for $\frac{\partial f}{\partial y}$. And this is both of these are in the case where x, y is not the point 0, 0.

So here, I can compute limits with impurity, no problem, because I can apply my U /V rule. At 0, 0 I have to be careful because now the definition is very specific. So, let us see what happens to x, y at 0, 0. So, suppose x, y 0, 0 what is del f, del x at 0, 0 $\frac{\partial f}{\partial y}$ at 0, 0? So, to do that, I have to compute this by from first principles by hand. So, this is f or h , 0 - f(0) , 0 /h limit as h tends to 0.

So, I am doing this from first principles, but I am doing it slightly faster. So, f of h , 0 what is that? Well, if h is, remember that h is nonzero, so, if you substitute h , 0 in this expression, well you get 0 in the numerator, - 0 and then /h. How did I get the second 0 because f(0) , 0 is defined to be 0 at 0, 0. So, this is just 0 and the same by symmetry or if you compute this, you will get that this is also 0. So, for the point 0, 0 you have to be careful and do it by hand for the other point, we are good to go.

So, in this video, we have talked about the partial derivatives. So, basically, those study the rate of change of the function with respect to a particular variable, that means when we are looking at the in the direction of the x axis or the y axis. So, of course, if your point is not on the x or the y axis, you have to move your axes parallely.

So, if your point is, as coordinates even a to an, we are looking at the, and you want to say, I want to do it with respect to x_1, so you have to keep all the other things constant. So, you have to look at the line, which is parallel to the x axis, which is given by x_2 is a_2, x_3 is a_3 all the way up to x_n as a_n so of course, in two or three variables, this is easily done. And so, you look at the function there, which becomes a function of one variable and then you take your usual derivative and that is exactly how you compute the partial derivative. Thank you.