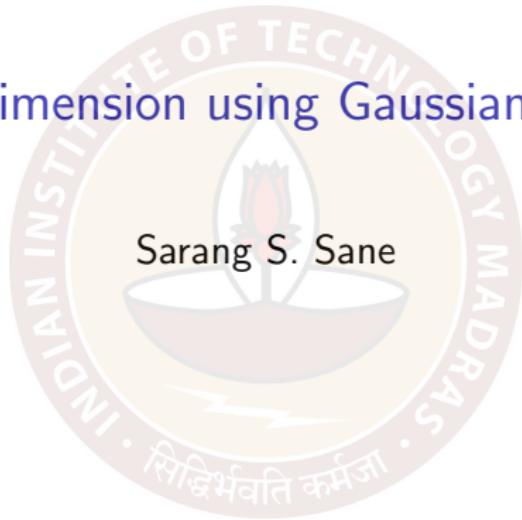


# Rank and dimension using Gaussian elimination

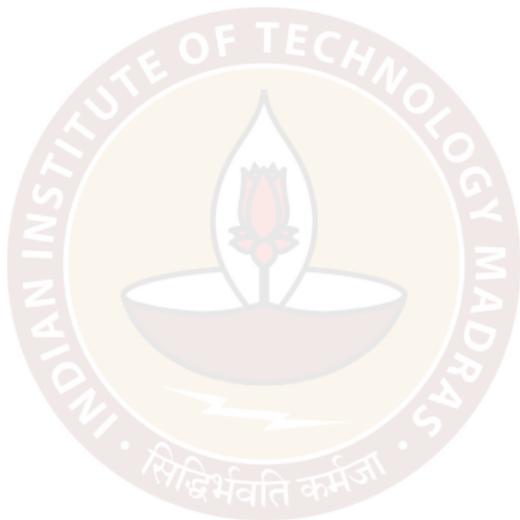


# Finding dimension and basis with a given spanning set



# Finding dimension and basis with a given spanning set

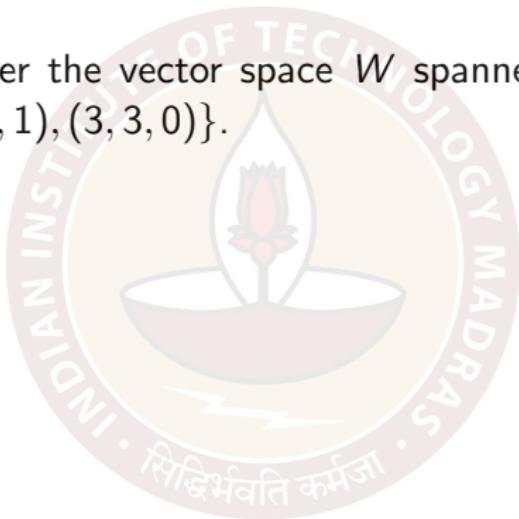
Consider a vector space  $W$  spanned by a set  $S$ .



## Finding dimension and basis with a given spanning set

Consider a vector space  $W$  spanned by a set  $S$ .

e.g. let us consider the vector space  $W$  spanned by the set  $S = \{(1, 0, 1), (-2, -3, 1), (3, 3, 0)\}$ .



## Finding dimension and basis with a given spanning set

Consider a vector space  $W$  spanned by a set  $S$ .

e.g. let us consider the vector space  $W$  spanned by the set  $S = \{(1, 0, 1), (-2, -3, 1), (3, 3, 0)\}$ .

We will use the following steps to find the dimension and a basis for  $W$  and carry out the steps for our example.

## Finding dimension and basis with a given spanning set

Consider a vector space  $W$  spanned by a set  $S$ .

e.g. let us consider the vector space  $W$  spanned by the set  $S = \{(1, 0, 1), (-2, -3, 1), (3, 3, 0)\}$ .

We will use the following steps to find the dimension and a basis for  $W$  and carry out the steps for our example.

- ▶ Form a matrix with the vectors in the spanning set as the rows.

## Finding dimension and basis with a given spanning set

Consider a vector space  $W$  spanned by a set  $S$ .

e.g. let us consider the vector space  $W$  spanned by the set  $S = \{(1, 0, 1), (-2, -3, 1), (3, 3, 0)\}$ .

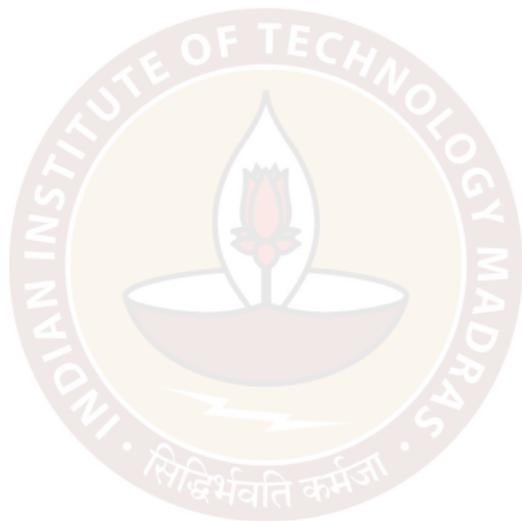
We will use the following steps to find the dimension and a basis for  $W$  and carry out the steps for our example.

- ▶ Form a matrix with the vectors in the spanning set as the rows.

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -3 & 1 \\ 3 & 3 & 0 \end{bmatrix}$$

## Finding dimension and basis (Contd.)

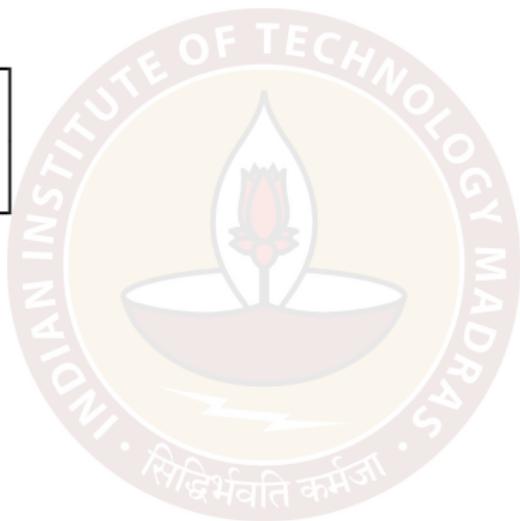
- ▶ Reduce to a matrix in the row echelon form.



## Finding dimension and basis (Contd.)

- ▶ Reduce to a matrix in the row echelon form.

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -3 & 1 \\ 3 & 3 & 0 \end{bmatrix}$$



## Finding dimension and basis (Contd.)

- ▶ Reduce to a matrix in the row echelon form.

$$\left[ \begin{array}{ccc} 1 & 0 & 1 \\ -2 & -3 & 1 \\ 3 & 3 & 0 \end{array} \right] \xrightarrow{R_2+2R_1} \left[ \begin{array}{ccc} 1 & 0 & 1 \\ 0 & -3 & 3 \\ 3 & 3 & 0 \end{array} \right]$$

## Finding dimension and basis (Contd.)

- ▶ Reduce to a matrix in the row echelon form.

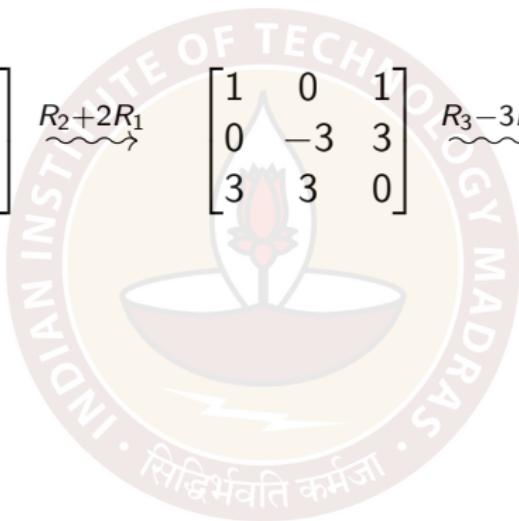
$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -3 & 1 \\ 3 & 3 & 0 \end{bmatrix}$$

$\xrightarrow{R_2+2R_1}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 3 \\ 3 & 3 & 0 \end{bmatrix}$$

$\xrightarrow{R_3-3R_1}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}$$



## Finding dimension and basis (Contd.)

- ▶ Reduce to a matrix in the row echelon form.

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -3 & 1 \\ 3 & 3 & 0 \end{bmatrix}$$

$\xrightarrow{R_2+2R_1}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 3 \\ 3 & 3 & 0 \end{bmatrix}$$

$\xrightarrow{R_3-3R_1}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}$$

$\left\{ \begin{array}{l} -R_2/3 \\ \downarrow \end{array} \right.$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 3 & -3 \end{bmatrix}$$

## Finding dimension and basis (Contd.)

- ▶ Reduce to a matrix in the row echelon form.

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -3 & 1 \\ 3 & 3 & 0 \end{bmatrix}$$

$\xrightarrow{R_2+2R_1}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 3 \\ 3 & 3 & 0 \end{bmatrix}$$

$\xrightarrow{R_3-3R_1}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}$$

$\left\{ \begin{array}{l} -R_2/3 \\ \downarrow \end{array} \right.$

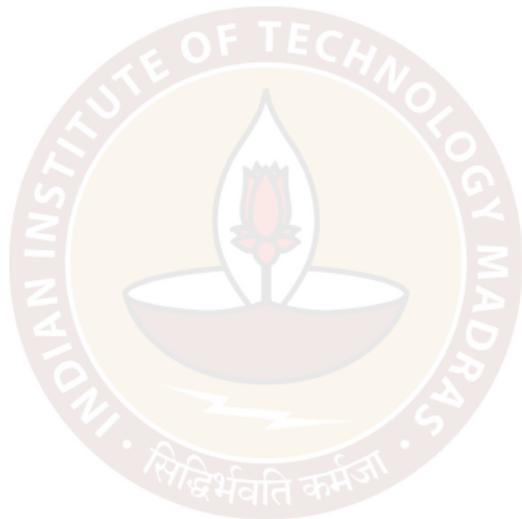
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\xleftarrow{R_3-3R_2}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 3 & -3 \end{bmatrix}$$

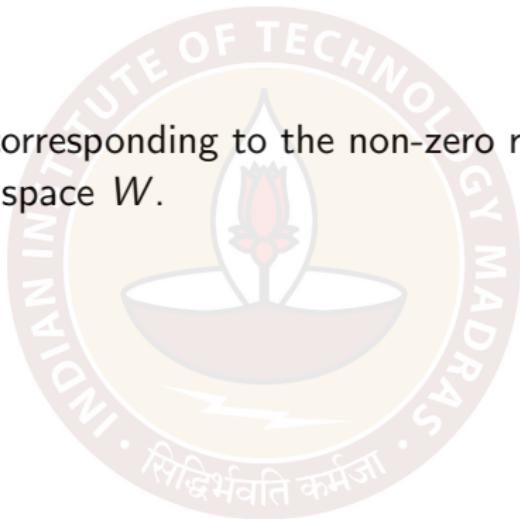
## Finding dimension and basis (Contd.)

- The number of non-zero rows is the dimension of the vector space  $W$ .



## Finding dimension and basis (Contd.)

- ▶ The number of non-zero rows is the dimension of the vector space  $W$ .
- ▶ The vectors corresponding to the non-zero rows form the basis of the vector space  $W$ .



## Finding dimension and basis (Contd.)

- ▶ The number of non-zero rows is the dimension of the vector space  $W$ .
- ▶ The vectors corresponding to the non-zero rows form the basis of the vector space  $W$ .

In the example, the final matrix is

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}.$$

## Finding dimension and basis (Contd.)

- ▶ The number of non-zero rows is the dimension of the vector space  $W$ .
- ▶ The vectors corresponding to the non-zero rows form the basis of the vector space  $W$ .

In the example, the final matrix is  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ .

Hence, dimension of the vector space spanned by  $\{(1, 0, 1), (-2, -3, 1), (3, 3, 0)\}$  is 2

## Finding dimension and basis (Contd.)

- ▶ The number of non-zero rows is the dimension of the vector space  $W$ .
- ▶ The vectors corresponding to the non-zero rows form the basis of the vector space  $W$ .

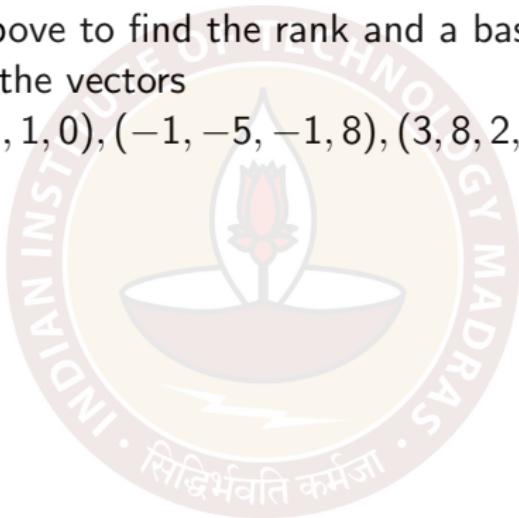
In the example, the final matrix is  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ .

Hence, dimension of the vector space spanned by  $\{(1, 0, 1), (-2, -3, 1), (3, 3, 0)\}$  is 2 and a basis is given by  $(1, 0, 1), (0, 1, -1)$ .

## Example in $\mathbb{R}^4$

Apply the steps above to find the rank and a basis of the vector space spanned by the vectors

$$\{(1, -2, 0, 4), (3, 1, 1, 0), (-1, -5, -1, 8), (3, 8, 2, -12)\}.$$



## Example in $\mathbb{R}^4$

Apply the steps above to find the rank and a basis of the vector space spanned by the vectors

$$\{(1, -2, 0, 4), (3, 1, 1, 0), (-1, -5, -1, 8), (3, 8, 2, -12)\}.$$

We construct the matrix with rows corresponding to the vectors in the spanning set :

## Example in $\mathbb{R}^4$

Apply the steps above to find the rank and a basis of the vector space spanned by the vectors

$$\{(1, -2, 0, 4), (3, 1, 1, 0), (-1, -5, -1, 8), (3, 8, 2, -12)\}.$$

We construct the matrix with rows corresponding to the vectors in the spanning set :

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

## Example contd.

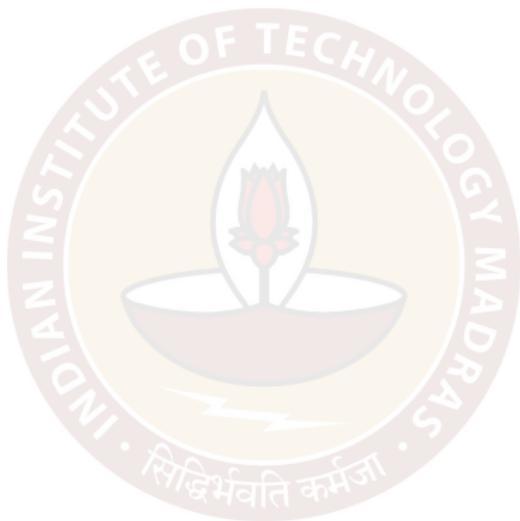
Apply row reduction :



## Example contd.

Apply row reduction :

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$



## Example contd.

Apply row reduction :

$$\left[ \begin{array}{cccc} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{array} \right] \xrightarrow{R_2 - 3R_1} \left[ \begin{array}{cccc} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & -7 & -1 & 12 \\ 0 & 14 & 2 & -24 \end{array} \right]$$

Hence the dimension of this vector space is 2 and  $\{(1, -2, 0, 4), (0, 1, 1/7, 12/7)\}$  is a basis.

## Example contd.

Apply row reduction :

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix} \xrightarrow{R_2 - 3R_1}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & 7 & 1 & 12 \\ 0 & 14 & 2 & 24 \end{bmatrix} \xrightarrow{R_3 + R_1}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & -7 & -1 & 12 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

Hence the dimension of this vector space is 2 and  $\{(1, -2, 0, 4), (0, 1, 1/7, 12/7)\}$  is a basis.

## Example contd.

Apply row reduction :

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

$$R_2 - 3R_1$$

$$\left[ \begin{array}{cccc} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & -7 & -1 & 12 \\ 0 & 14 & 2 & -24 \end{array} \right]$$

$$R_3 + R_1$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & -7 & -1 & 12 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

$$\left. \begin{array}{c} \\ \\ \\ \end{array} \right\} R_4 - 3R_1$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & -7 & -1 & 12 \\ 0 & 14 & 2 & -24 \end{bmatrix}$$

Hence the dimension of this vector space is 2 and  $\{(1, -2, 0, 4), (0, 1, 1/7, 12/7)\}$  is a basis.

## Example contd.

Apply row reduction :

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix} \xrightarrow{R_2 - 3R_1}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & 7 & 1 & 12 \\ 0 & 14 & 2 & -24 \end{bmatrix} \xrightarrow{R_3 + R_1}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & -7 & -1 & 12 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

$$\left. \begin{array}{c} \\ \\ \\ \end{array} \right\} R_4 - 3R_1$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 1 & 1/7 & 12/7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 + 7R_2} \xrightarrow{R_4 - 14R_3}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 1 & 1/7 & 12/7 \\ 0 & -7 & -1 & 12 \\ 0 & 14 & 2 & -24 \end{bmatrix} \xrightarrow{R_2 / 7} \xrightarrow{R_3 + 7R_2}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & 12 \\ 0 & -7 & -1 & 12 \\ 0 & 14 & 2 & -24 \end{bmatrix}$$

Hence the dimension of this vector space is 2 and  $\{(1, -2, 0, 4), (0, 1, 1/7, 12/7)\}$  is a basis.

# Thank you

