

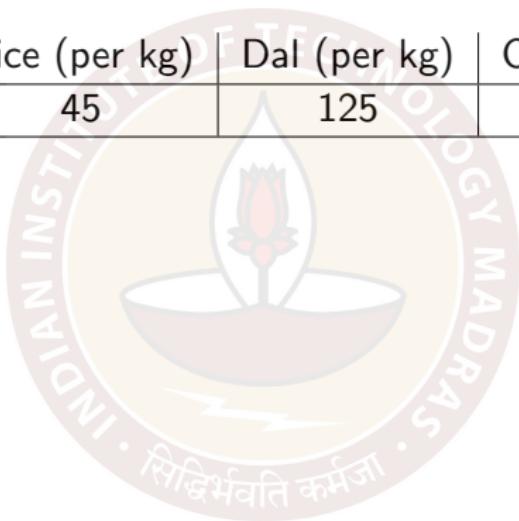
What is a linear mapping

Sarang S. Sane

Grocery shop example

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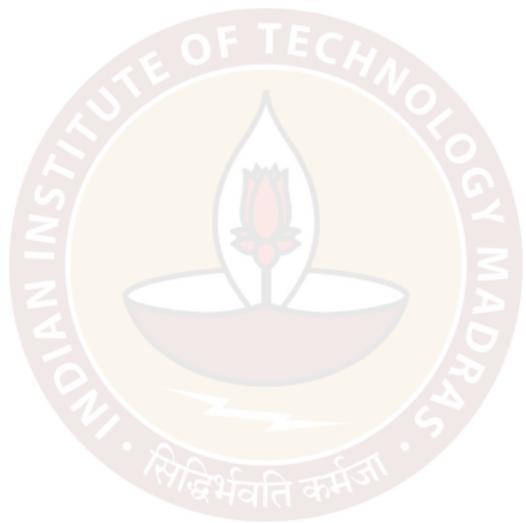
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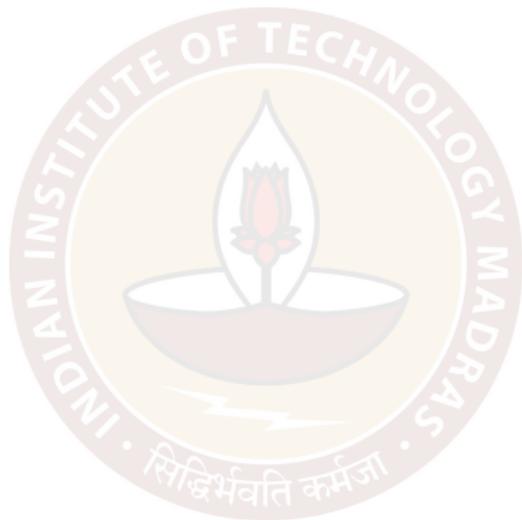
$$x_1 \times 45 + x_2 \times 125 + x_3 \times 150 = 45x_1 + 125x_2 + 150x_3.$$

Expressions and linear combinations



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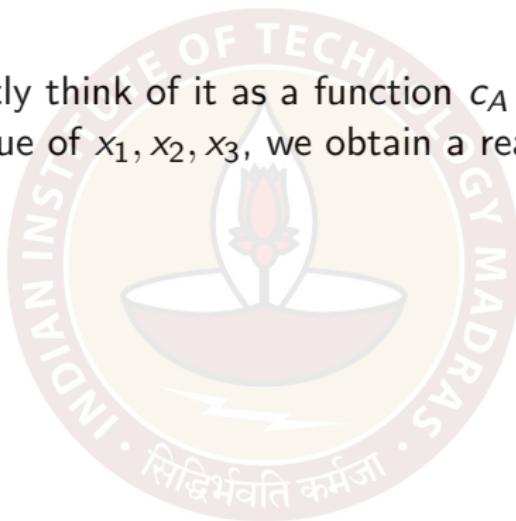
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Since c_A is a linear combination of x_1, x_2, x_3 (with coefficients 45, 125, 150), it is an example of a linear function.

Recall that linear combinations can also be expressed in terms of matrix multiplication e.g.

$$c_A(x_1, x_2, x_3) = 45x_1 + 125x_2 + 150x_3 = [45 \quad 125 \quad 150] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Example : Cost linearity

A caterer gets an order from Office 1 in Malgudi on Monday for 30 tiffins prepared in some prescribed way and buys 20 kg. of rice, 10 kg. of dal and 4 litres of oil from Shop A for the purpose.



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Note that the required quantities of rice, dal and oil for preparing the 15 tiffins for office 1 on Wednesday will be half of the amounts on Monday i.e. 10 kg. of rice, 5 kg. of dal and 2 litres of oil.

Cost linearity (contd.)

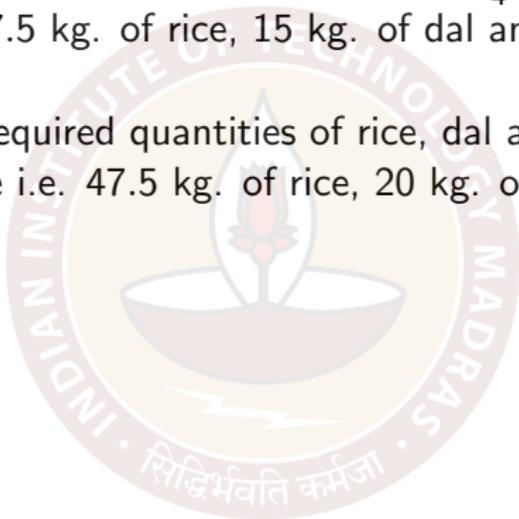
Similarly, the required quantities of rice, dal and oil for preparing the 50 tiffins for office 2 on Wednesday will be $\frac{5}{4}$ times the amounts on Tuesday i.e. 37.5 kg. of rice, 15 kg. of dal and 2.5 litres of oil.



Cost linearity (contd.)

Similarly, the required quantities of rice, dal and oil for preparing the 50 tiffins for office 2 on Wednesday will be $\frac{5}{4}$ times the amounts on Tuesday i.e. 37.5 kg. of rice, 15 kg. of dal and 2.5 litres of oil.

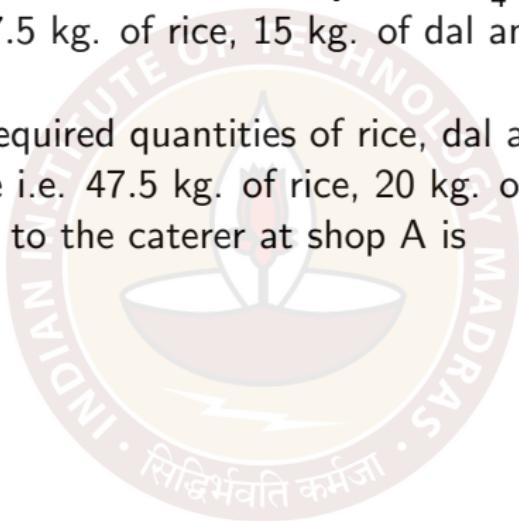
Hence, the total required quantities of rice, dal and oil on Wednesday will be i.e. 47.5 kg. of rice, 20 kg. of dal and 4.5 litres of oil.



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$$45 \times 47.5 + 125 \times 15 + 150 \times 2.5 = 4387.5 \text{ rupees.}$$

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Cost to the caterer for office 1 on Monday : 2750 rupees

Cost to the caterer for office 2 on Tuesday : 3150 rupees

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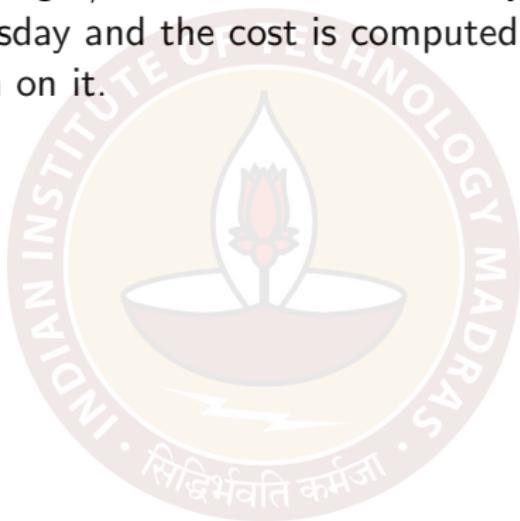
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Cost to the caterer on Wednesday :

$$\frac{1}{2} \times 2750 + \frac{5}{4} \times 3150 = 5312.5 \text{ rupees.}$$

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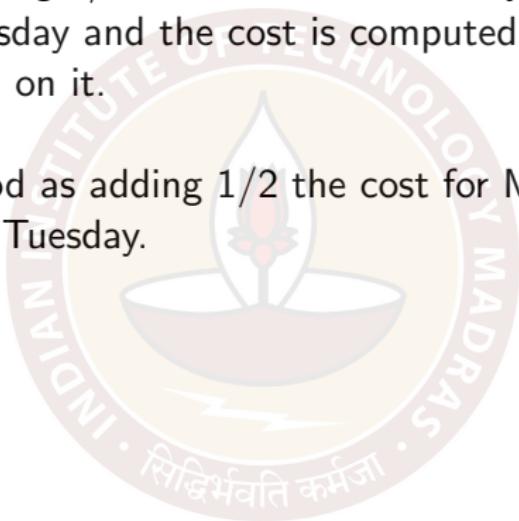
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The second method as adding $1/2$ the cost for Monday and $5/4$ times the cost for Tuesday.

The following table summarizes this data : adding $1/2$ the first row and $5/4$ times the second row yields the third row.

	Kgs. of rice	Kgs. of dal	Litres of oil	Cost c_A
Monday	20	10	4	2750
Tuesday	30	12	2	3150
Wednesday	47.5	20	4.5	5312.5

Why does the second method work?

$$\begin{aligned}
 & \text{Total Cost on Wed.} \\
 &= 45 \times \frac{\text{rice}}{\text{regd. on W.}} + 125 \times \frac{\text{dal}}{\text{regd. on W.}} + 150 \times \frac{\text{oil}}{\text{regd. on W.}} \\
 &= 45 \times \left(\frac{1}{2} \times \frac{\text{rice}}{\text{on M}} + \frac{5}{4} \times \frac{\text{rice}}{\text{on T}} \right) + 125 \times \left(\frac{1}{2} \times \frac{\text{dal}}{\text{on M}} + \frac{5}{4} \times \frac{\text{dal}}{\text{on T}} \right) \\
 &\quad + 150 \times \left(\frac{1}{2} \times \frac{\text{oil}}{\text{on M}} + \frac{5}{4} \times \frac{\text{oil}}{\text{on T}} \right) \\
 &= \frac{1}{2} \times \left\{ \left(45 \times \frac{\text{rice}}{\text{on M}} \right) + \left(125 \times \frac{\text{dal}}{\text{on M}} \right) + \left(150 \times \frac{\text{oil}}{\text{on M}} \right) \right\} \\
 &\quad + \frac{5}{4} \times \left\{ \left(45 \times \frac{\text{rice}}{\text{on T}} \right) + \left(125 \times \frac{\text{dal}}{\text{on T}} \right) + \left(150 \times \frac{\text{oil}}{\text{on T}} \right) \right\} \\
 &= \frac{1}{2} \times \text{wst on Monday} + \frac{5}{4} \times \text{wst on Tuesday}.
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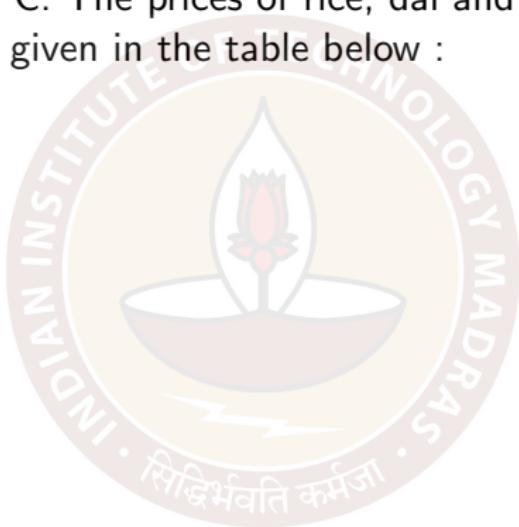
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Write the **expression** for the total cost of buying x_1 kg. of rice, x_2 kg. of dal and x_3 kg. of oil for each of the three shops and try to compare them.

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Example(contd.) : Linear mappings

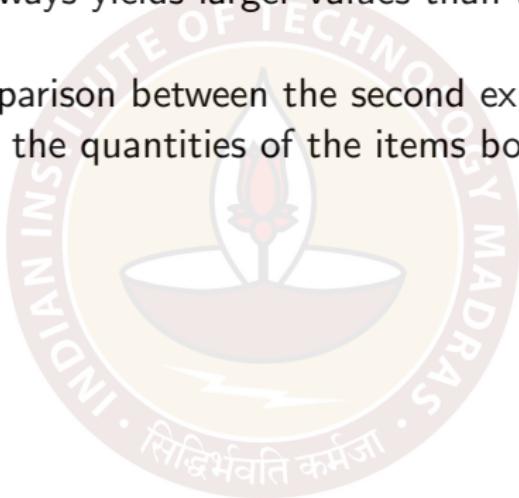
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A natural way to make this comparison would be to create a vector of costs i.e. $(c_A(x_1, x_2, x_3), c_B(x_1, x_2, x_3), c_C(x_1, x_2, x_3))$.

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We can then think of the cost vector as a function c from \mathbb{R}^3 to \mathbb{R}^3 by setting these expressions as the coordinates in \mathbb{R}^3 i.e.

$$c(x_1, x_2, x_3) = (c_A(x_1, x_2, x_3), c_B(x_1, x_2, x_3), c_C(x_1, x_2, x_3)) = (45x_1 + 125x_2 + 150x_3, 40x_1 + 120x_2 + 170x_3, 50x_1 + 130x_2 + 160x_3).$$

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$$\begin{bmatrix} 45 & 125 & 150 \\ 40 & 120 & 170 \\ 50 & 130 & 160 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 515 \\ 540 \\ 550 \end{bmatrix}.$$

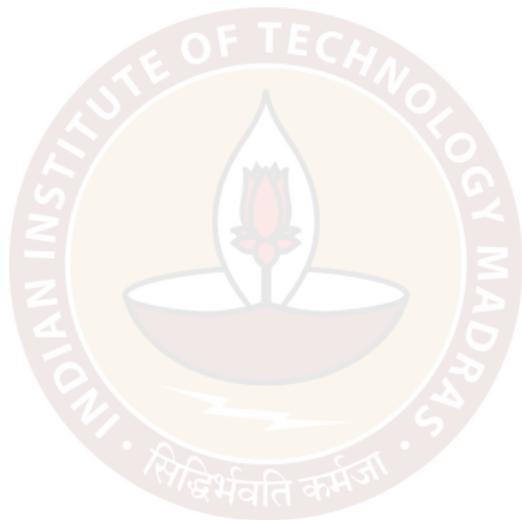
Linearity

As in the case of the function c_A , the property of "linearity" of costs can now be extracted from the matrix form for the cost function c .

$$\begin{aligned} c & \left(\frac{1}{2} \times \frac{\text{rice}}{m} + \frac{5}{4} \times \frac{\text{onion}}{T}, \frac{1}{2} \times \frac{\text{dal}}{m} + \frac{5}{4} \times \frac{\text{onion}}{T}, \frac{1}{2} \times \frac{\text{oil}}{m} + \frac{5}{4} \times \frac{\text{oil}}{T} \right) \\ & = (c_A(\text{rice}), c_A(\text{dal}), c_A(\text{oil})) \cdot M + (c_A(\text{onion}), c_A(\text{onion}), c_A(\text{onion})) \cdot T \\ & = (\frac{1}{2} \times c_A(\text{rice}) + \frac{5}{4} \times c_A(\text{onion}), \frac{1}{2} \times c_A(\text{dal}) + \frac{5}{4} \times c_A(\text{onion}), \frac{1}{2} \times c_A(\text{oil}) + \frac{5}{4} \times c_A(\text{onion})) \\ & = \frac{1}{2} \times c(\text{rice}) + \frac{5}{4} \times c(\text{onion}) + \frac{1}{2} \times c(\text{dal}) + \frac{5}{4} \times c(\text{onion}) + \frac{1}{2} \times c(\text{oil}) + \frac{5}{4} \times c(\text{onion}) \\ & = c(\alpha(x_1, x_2, x_3) + y_1, y_2, y_3). \\ c(\alpha(x_1, x_2, x_3) + y_1, y_2, y_3) & = \alpha \left[\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right] + \left[\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right] \end{aligned}$$

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where the coefficients a_{ij} s are real numbers (scalars). A linear mapping can be thought of as a collection of linear combinations.

We can write the expressions on the RHS in matrix form as Ax

$$\text{where } A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}.$$

Linearity of linear mappings

It follows that a linear mapping satisfies linearity, i.e. for any $c \in \mathbb{R}$ (scalar)

$$f(x_1 + cy_1, x_2 + cy_2, \dots, x_n + cy_n) = f(x_1, x_2, \dots, x_n) + cf(y_1, y_2, \dots, y_n).$$

$$\begin{aligned} f(x_1 + cy_1, x_2 + cy_2, \dots, x_n + cy_n) &= A \begin{bmatrix} x_1 + cy_1 \\ x_2 + cy_2 \\ \vdots \\ x_n + cy_n \end{bmatrix} \\ &= A \left\{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + c \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\} = A \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + cA \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \\ &= f(x_1, \dots, x_n) + cf(y_1, \dots, y_n). \end{aligned}$$

Thank you

