IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 10 - Tutorial 05

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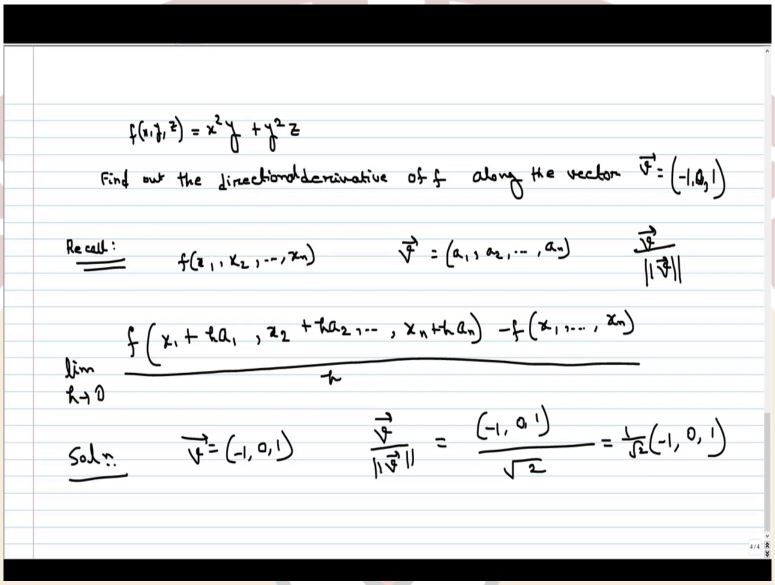

Hello everyone, so in this video, we will try to find out the directional derivative of this function f (x, y, z) , so, this is a function of 3 variables, and it is a scalar value function along this vector (-1,0,1).

So, let us recall the definition of directional derivative. Suppose, we have a function of n variables, scalar valued function of n variables, f(x1, x2,...... xn) and we have to calculate the directional derivative along some vector v, which is given as(a1, a2,...... an).

Now, if v is not a unit vector what we have to do? We have to normalize it first. So, we have to do this calculation $\frac{v}{\|v\|}$, which you can do so that the norm of this vector will become 1. Now, to make the computation easier here, we are assuming that one of the vector which we have started with is actually a unit vector.

So, let us assume that this v is a unit vector, so what we have to do to calculate the directional derivative, we have to see the increment of this function along this vector. So, basically, we have to take, (x1 + ha1, x2+ha2………,xn+han). Basically, we are adding this (x1,x2…..xn) +h(v), we are finding this functional value at that point. We have to subtract the function of value at (x1,x2,.....,xn), and then we have to divide it with h and we have to calculate the limit of this function as h $\rightarrow$ 0. So, this is the definition of directional derivative along this vector. So, here in the given examples, if I try to solve this problem, our v = (-1,0,1), so, let us normalize it first.

So, we have to do this thing first, give a norm of v, so v = (-1,0,1) and ||v|| = $\frac{1}{\sqrt{2}}$. So, the normalized vector, the unit vector along this direction is $\frac{1}{\sqrt{2}}$ (-1,0,1) . So, this is our unit vector. Now, what we have to do is to apply this definition.

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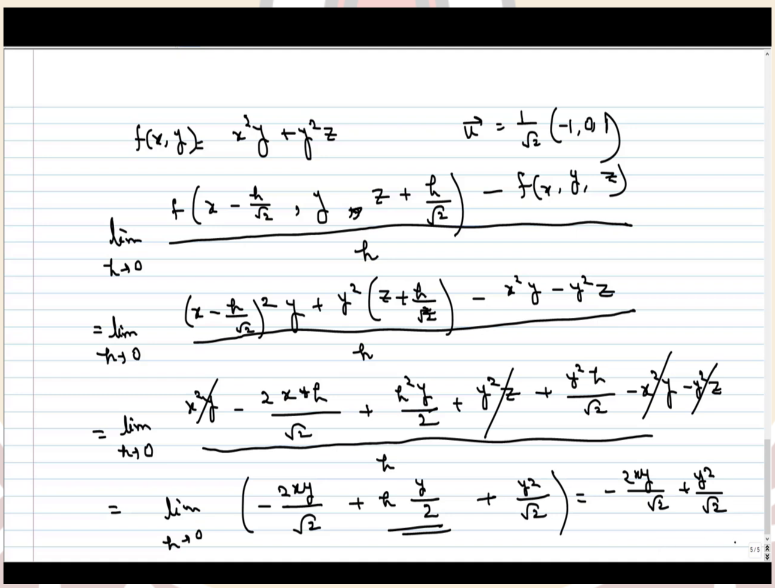

So, let us apply this definition. So, $f(x,y)=x^2y+y^2z$. And the unit vector which we got here is $\frac{1}{\sqrt{2}}(-1,0,1)$. So, now, we apply the definition here, so, we have to calculate the

$$\lim_{h\to 0}\frac{f\left(x-\frac{h}{\sqrt{2}},y,z+\frac{h}{\sqrt{2}}\right)-f(x,y,z)}{h}$$

$$=\lim_{h\to 0}\frac{\left(x-\frac{h}{\sqrt{2}}\right)^2 y+y^2\left(z+\frac{h}{\sqrt{2}}\right)-\left(x^2y+y^2z\right)}{h}$$

$$=\lim_{h\to 0}\frac{\left(x^2y-\frac{2xyh}{\sqrt{2}}+\frac{h^2y}{2}\right)+y^2z+\frac{y^2h}{\sqrt{2}}-\left(x^2y+y^2z\right)}{h}$$

$$=\lim_{h\to 0}\left(-\frac{2xy}{\sqrt{2}}+h\left(\frac{y}{2}\right)+\frac{y^2}{\sqrt{2}}\right)=-\frac{2xy}{\sqrt{2}}+\frac{y^2}{\sqrt{2}}$$

So, this is the directional derivative, along the vector which we want to calculate.

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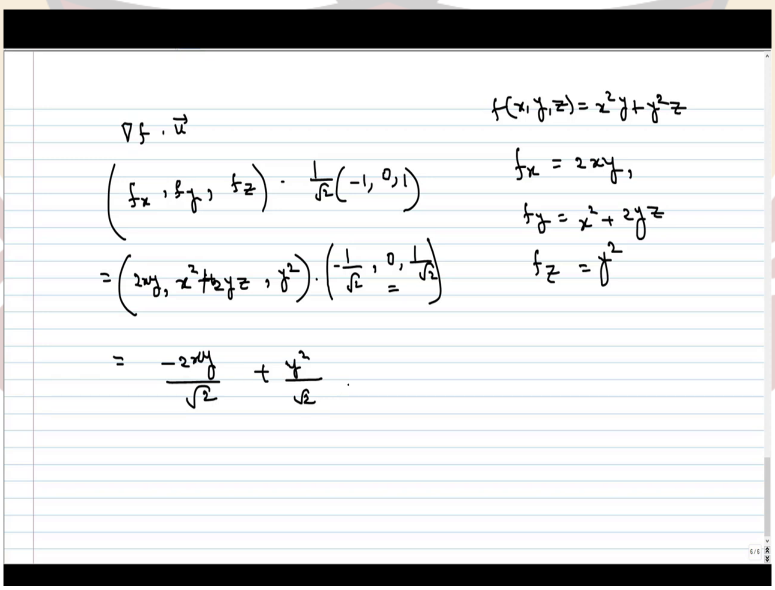

Now, there is another theorem which is saying that, if we calculate the grad of this f, and if we take the dot product with the unit vector along which direction, we want to calculate the directional derivative, then also we will get the same results. So, the grad of f means we have to calculate f_x, f_y and f_z. Basically, these are vector responses of the partial derivative along each axis, and here are unit vectors, which you have already calculated, this is $\frac{1}{\sqrt{2}}(-1,0,1)$.

So, if we calculate the partial derivative, recall what was our function, our function was x^2y+y^2z. So, what was our f_x, $f_x = 2xy$. What was our f_y? $f_y = x^2 + 2yz$ and what was f_z, $f_z = y^2$.

So, if I write this down here, $(2xy, x^2 + 2yz, y^2)$, if I take dot product with $\frac{1}{\sqrt{2}}(-1,0,1)$.

So, if I calculate it, I will get, $-\frac{2xy}{\sqrt{2}}+\frac{y^2}{\sqrt{2}}$. So, it is exactly the same thing that we have calculated above. Exactly the same thing as what we have calculated here. So, here definitely, we can calculate the directional derivative using this method and also we can verify that when in we are using the definition we are getting the same directional derivative. Thank you.