Limits for sequences

Sarang S. Sane

Example : The limit of a sequence

Consider the sequence of numbers $2 - \frac{1}{n}$ as n increases.

More examples : Limits of sequences

The sequence $2 - \frac{1}{n^2}$ as n increases :

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The sequence $2 - \frac{1}{(1+log(n))^{1.1}}$ as n increases :

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Some examples of applying the rules

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$\frac{-1}{1}, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \frac{-1}{5},$

$c_n = \frac{(-1)^n}{n}$

$\underbrace{\frac{1}{n}}_{b_n} \to 0, \quad \underbrace{\frac{-1}{n}}_{a_n} \to 0$

$a_n \le c_n \le b_n \to 0$

$a_n \to 0, \quad c_n \to 0$

2. $\left\{\frac{\overset{a_n}{\frac{1}{ln(1+n)}} + \overset{b_n}{\frac{5n^2}{1+n^2}}}{\left(1+\frac{1}{n}\right)^{2n}}\right\}$ converges to $\frac{5}{e^2}$

$\ln(1+n) \to \infty$ (diverges to ∞)

$\frac{1}{\ln(1+n)} \to 0$

$\left(1+\frac{1}{n}\right)^{2n} = \{\underbrace{(1+1/n)^n}_{\to e}\}^2 \to e^2$

$\frac{5n^2}{1+n^2} = \frac{5}{1/n^2+1} \to \frac{5}{1} = 5$

$1+\frac{1}{n^2} \to 1$

Thank you