Review : Maths 1

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Some topics from Maths 1

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- Straight lines

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- Quadratic equations

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- Polynomials
- Functions
- Exponentials and logarithms

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What is a function of one variable?

$f : D \longrightarrow \mathbb{R}$ where $D \subseteq \mathbb{R}$. $f(x)$ where $x \in \mathbb{R}$.

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$$f : X \to Y$$

Domain : X Codomain : Y Range : $\{f(x) \mid x \in X\}$

Linear functions

$$f : \mathbb{R} \to \mathbb{R} \qquad f(x) = mx + c \qquad m, c \in \mathbb{R}.$$

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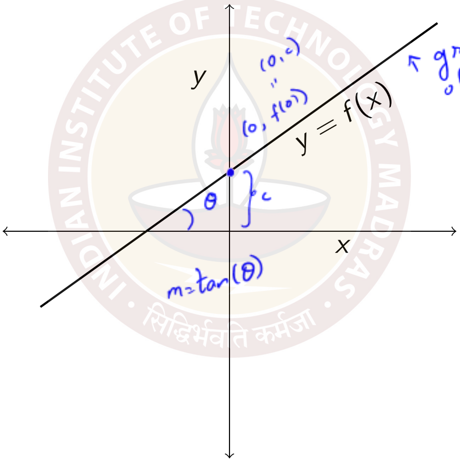
$$f : \mathbb{R} \to \mathbb{R} \qquad f(x) = \overset{\text{slope}}{m}x + c \qquad m, c \in \mathbb{R}.$$



graph of the function

$(x, f(x))$

$y = f(x)$

Quadratic functions

$$f : \mathbb{R} \to \mathbb{R} \qquad f(x) = a(x-b)^2 + c \qquad a, b, c \in \mathbb{R}.$$

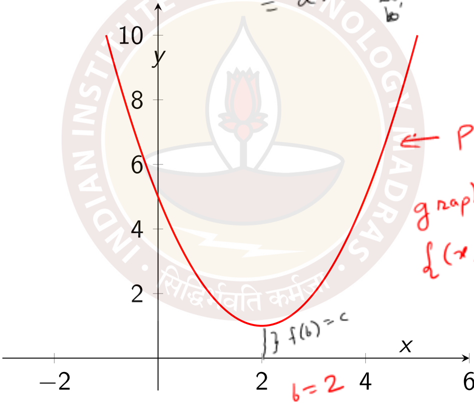
Quadratic functions

>0

$$f : \mathbb{R} \to \mathbb{R} \qquad f(x) = a(x-b)^2 + c \qquad a, b, c \in \mathbb{R}.$$

$= a(x^2 - 2bx + b^2) + c$

$= ax^2 \underbrace{- 2ab}_{b'}\, x + \underbrace{ab^2 + c}_{c'} = ax^2 + b'x + c'.$



← parabola

graph of $f(x)$

$\{(x, f(x)) \mid x \in \mathbb{R}\}$

Polynomial functions

$$f : \mathbb{R} \to \mathbb{R} \qquad f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \quad ; \quad a_i \in \mathbb{R}.$$

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Example : Graph of $f(x) = x^3 - 4x$

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Example : Graph of $f(x) = x^3 - 4x$

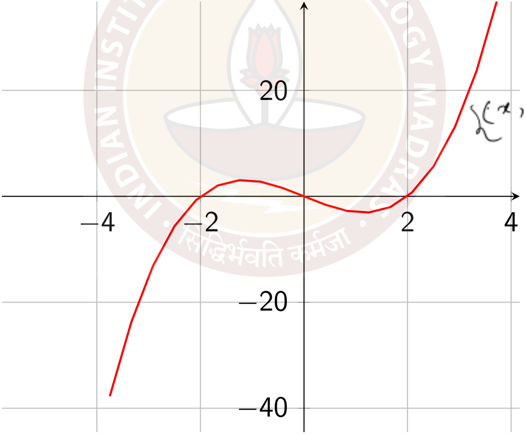
$f(x) = 0$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x-2)(x+2) = 0$

$f(x) = 0$ when exactly $x = -2, 0, 2$.



$\{(x, f(x)) \mid x \in \mathbb{R}\}$

$= \{(x, x^3 - 4x) \mid x \in \mathbb{R}\}$

The exponential and logarithmic functions

$g,\ f : \mathbb{R} \to \mathbb{R}$ $\boxed{f(x) = a^{x}}$ $g(x) = log_a(x)$.

$2^2 = 4$.

$(0.5)^2 = 0.25$

$\pi^2 = \pi \times \pi$.

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$(0.5)^{-2} = \frac{1}{(0.5)^2} = \frac{1}{1/4} = 4$.

$\pi^{-2} = \frac{1}{\pi^2}$.

2^{π}

$2^{\sqrt{2}}$

$$\boxed{\begin{array}{l} 2^{1/3} = \sqrt[3]{2} \\ (0.5)^{5/6} \end{array}}$$

$2^{\sqrt{2}}$

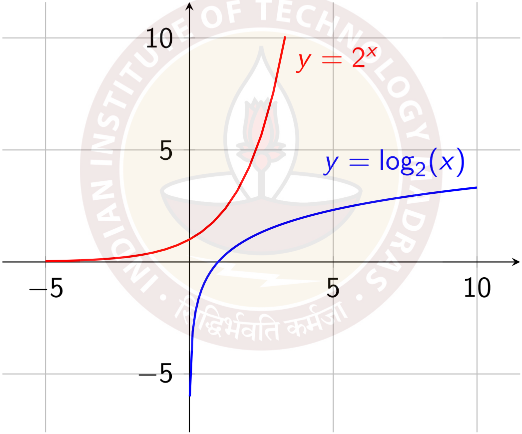
Take a rational no. q which approximates $\sqrt{2}$ as closely as we desire.

2^q

The exponential and logarithmic functions

$f : \mathbb{R} \to \mathbb{R}$ $\qquad f(x) = a^x$ $\qquad g(x) = log_a(x).$

$a > 0.$



$\log_2(2^x) = x.$

Monotonicity of functions

A function $f : \mathbb{R} \to \mathbb{R}$ is said to be monotone increasing if $x_1 \leq x_2$ implies $f(x_1) \leq f(x_2)$.

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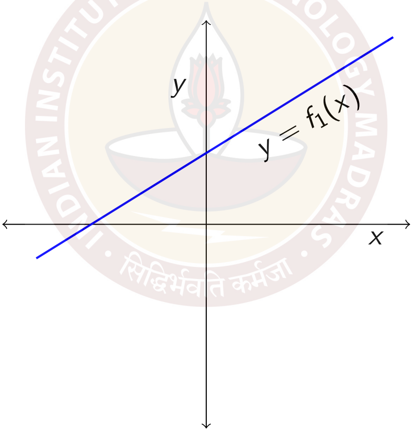

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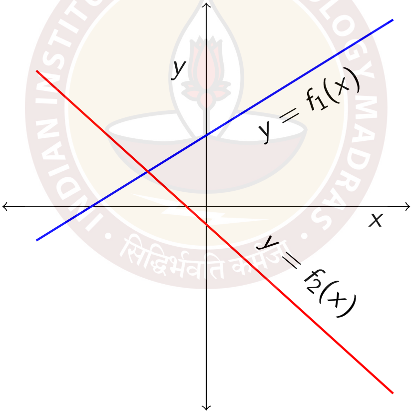
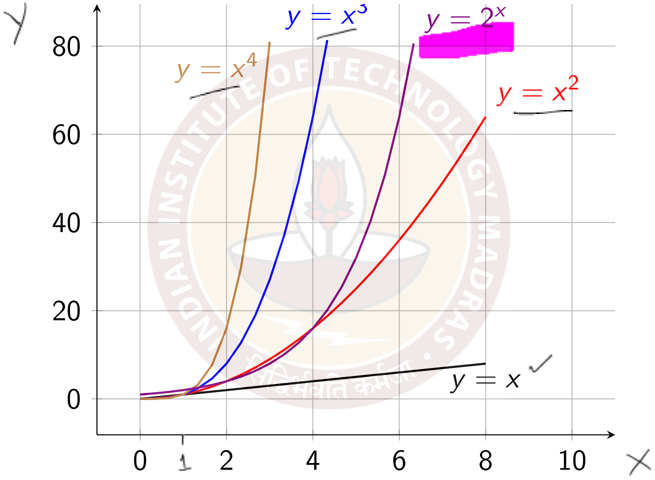
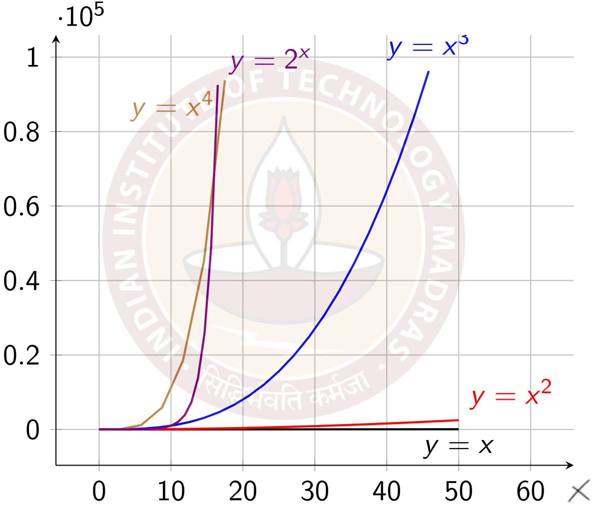


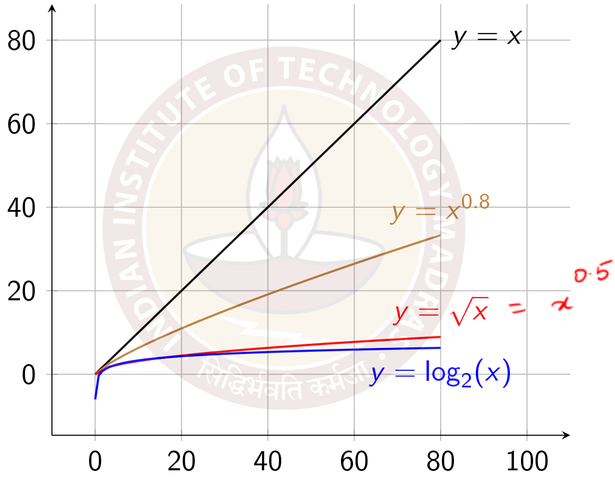
Comparing various functions : fast growth, close to 0



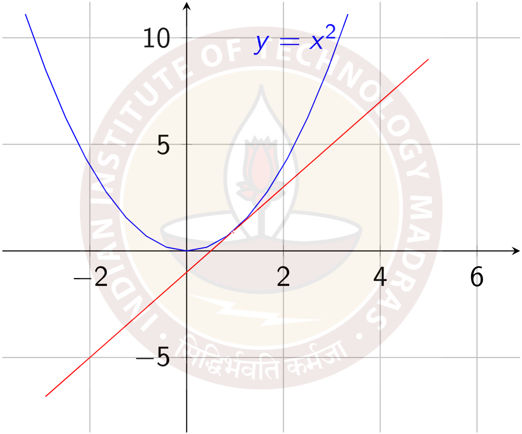
Comparing various functions : fast growth



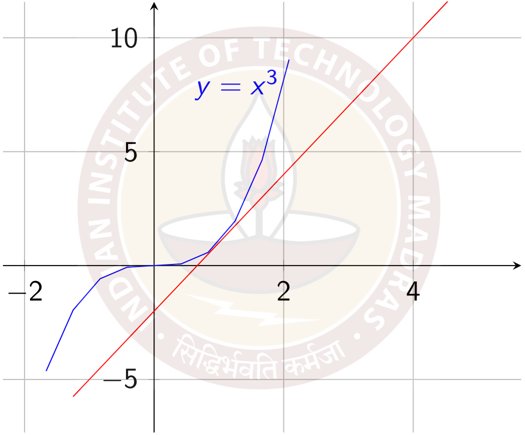
Comparing various functions : slow growth



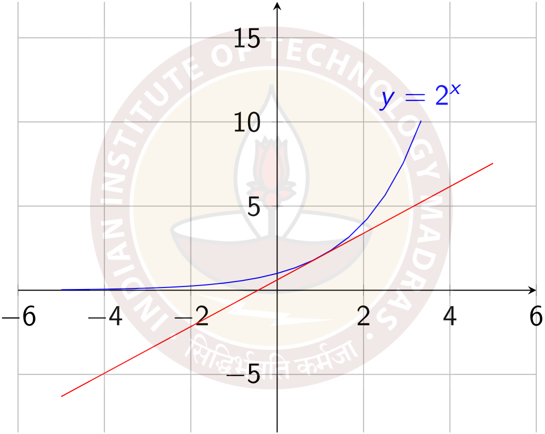
Tangent lines : Example 1



Tangent lines : Example 2



Tangent lines : Example 3



Thank you