INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science - 2
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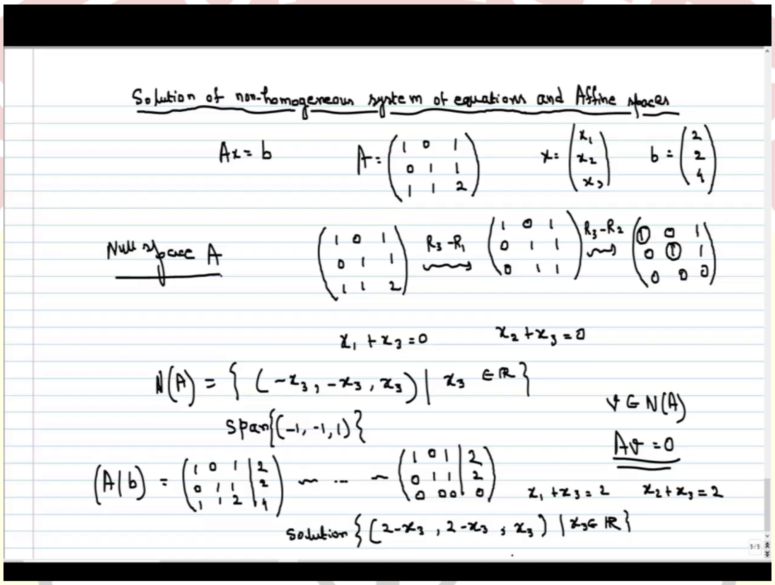

Hello, everyone. In this video, we will try to visualize how solution of non-homogeneous system of equations are related with affine spaces. So, let us consider a system of, non-homogeneous system of linear equation Ax = b, where A is given by the matrix, suppose, let us consider this matrix $A=\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 2\end{pmatrix}$ So, let, we consider this matrix as our A, the coefficient matrix and x is three variables. So, it is $x=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$ And let us consider $b=\begin{pmatrix}2\\ 2\\ 4\end{pmatrix}$

Now, if we just try to find the null space of A, what we will do? We will do the row reduction, row operations on A to find the reduced row matrix of A, reduced row echelon form of the matrix A. So, we start with the given matrix, $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ so if we do $R_3 - R_1$ we will get $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ Now, we will do $R_3 - R_2$ So, we will get $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

So, in the first two columns or first two rows, we have pivot elements, I mean, thus, these columns are non-zero columns, and these two has two rows are non-zeros. So, they are only one independent variable. And so if we write this, we will get $x_1 + x_3 = 0$ and $x_2 + x_3 = 0$ So, x_3 is our independent variable. So, we can write it as $(-x_3, -x_3, x_3)$ So, where x_3 is coming from a set of real number. So, this is our null space of A.

So, let us denote it as N(A). So, this is our null space of A. So, the basis of this space, so this is nothing but the spanning set of the vector Span{(-1,-1,1)}. So, (-1,-1,1) is a basis for this null space. So, that is what we got. So, basically, whenever we take any vector from the null space, if we compose it that is v, that v is from the null space of A, then Av= 0. So, these elements of null space are nothing but the solution of the homogeneous system of equation Ax = 0, but we have to find the solution for the system of equation Ax=b.

So, for finding the solution Ax = b, what we have to do? We use the Gaussian elimination method again. So, this is our augmented matrix and we do the row operation. So, let us write the augmented matrix first. This is $[A|B] = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1 \end{array}\right)$ So, if we do the same, similar row operations as we have done here, so at the end we will get back to $[A|B] = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$

So, at the end, we will get this reduced echelon form. And from here you can find that $x_1 + x_3 = 2$ and $x_2 + x_3 = 2$ And x_3 is our independent variable. So, the solution set is nothing but we can write it as $(2 - x_3, 2 - x_3, x_3)$ So, this is our solution set where x_3 comes from real number. So, all these vectors are in the solutions of Ax = b.

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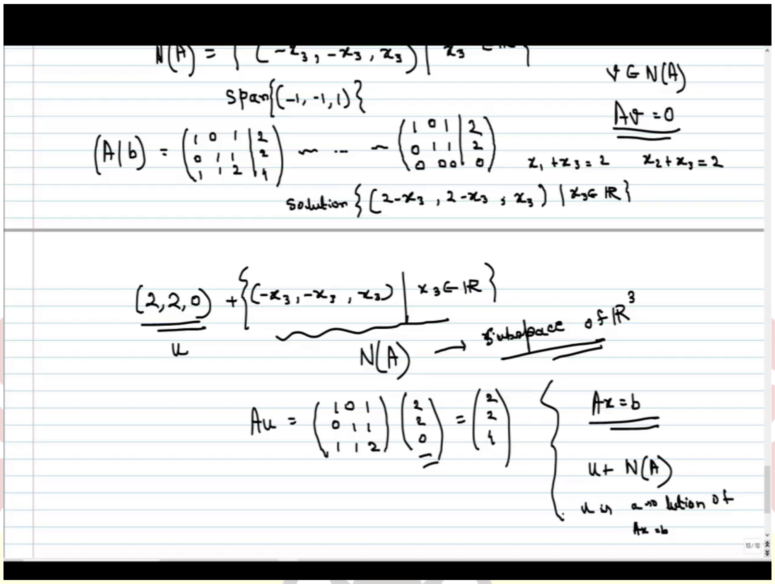

So, we can express it, so this is our solution we have written, so we can express it in terms of $(2,2,0)+\{(-x_3,-x_3,x_3)|x_3 \in \mathbb{R}\}$ So, this is our null space. This is nothing but the null space of A. And this we have got some vector which is let us denote it by u. And if we can calculate Au, so our A is the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ And if we multiply it with u, that is $Au=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ So, what we will get? We will get, $Au=\begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$

So, this u is a solution of this system of equation. So, u is one solution of the system of equation and this u plus this null space, this is the set of all solutions. This is the solution set of our system of equation. So, this is nothing but affine space. This N(A) is a vector space, N(A) is a subspace, rather we can say it is a subspace of R^3 here as we are considering within in R^3 So, N(A) is a one dimensional subspace as we can see here.

And u plus that subspace is giving us an affine space and this is nothing but the solution set of this system of linear equation, non-homogeneous system of linear equation Ax = b. So, solution set of this non-homogeneous system of linear equation is nothing but an affine space u +N(A), where u is a solution of Ax = b. So, this we can get from this example. Thank you.