Functions of one variable

Sarang S. Sane

Trigonometric functions : the sine function

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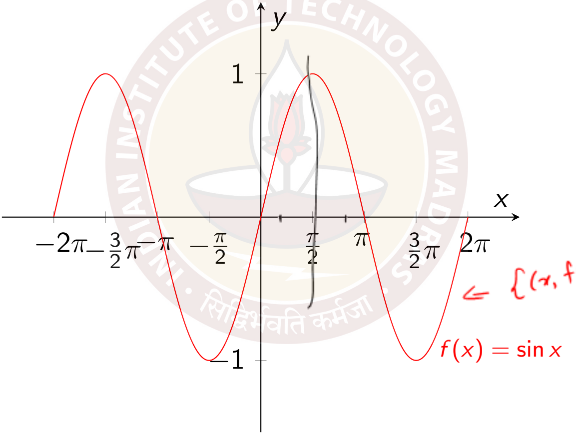
$0 \leq x \leq \pi/2$

$f : \mathbb{R} \to \mathbb{R} \qquad f(x) = \sin x$

$\dfrac{|opp|}{|hyp|} = \sin(x)$



$\leftarrow \{(x, f(x)) \mid x \in \mathbb{R}\}$

Trigonometric functions : the cosine function

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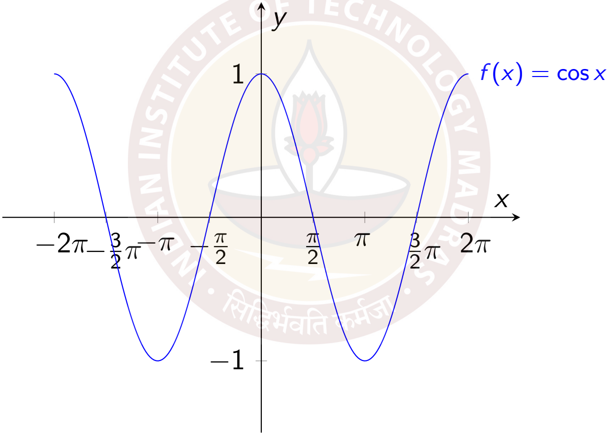

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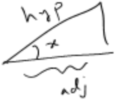
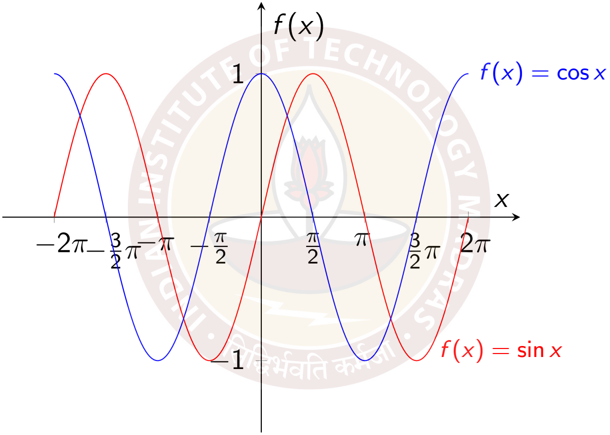


$\cos(x) = \frac{|adj|}{|hyp|}$.

Comparing the *sin* and *cos* functions



Trigonometric functions : the tangent function

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$\mathbb{R} \setminus \{ \frac{n\pi}{2} \mid n \text{ runs over odd integers} \}$.

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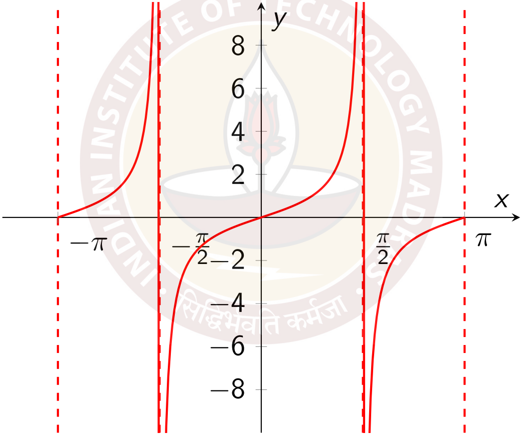

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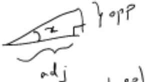
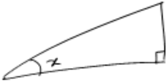


$\tan(x) = \frac{|opp|}{|adj|}$



Trigonometric functions : other functions and identities

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iv) If $g(x) \neq 0$, $x \in D$, the quotient f/g is defined on D by
$(f/g)(x) = f(x)/g(x)$, $x \in D$.

$h(x) = \dfrac{x}{x^2+1}$

$\mathbb{R} \to \mathbb{R}$

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Then $g \circ f(x) = \sqrt{x^2 + 1}$.

$Range(f) = \{y \in \mathbb{R} \mid y = x^2+1 \text{ for some } x\}$
$= [1, \infty) \subseteq Domain(g) = [0, \infty).$

Thank you