INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 9 Tutorial 2

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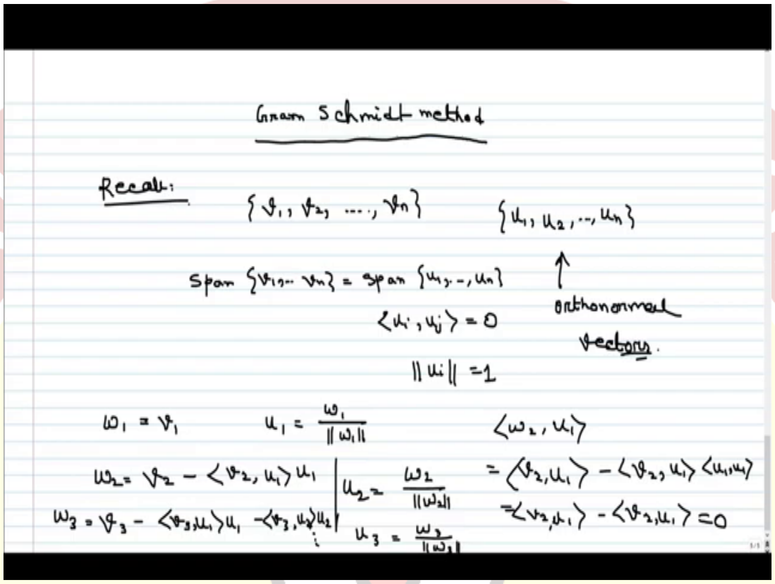

Hello everyone, so in this video we will use Gram Schmidt method to transform our basis to an orthogonal basis, corresponding to that. So, first let us recall the Gram Schmidt method first. So, we will start with a set of given vectors which are linearly independent. So, let the vector v called as $\{v_1, v_2 \ldots, v_n\}$ Now this is, these are linearly independent set of vectors. Now we want to transform it to the set of vectors which are orthogonal to each other, I mean which are pairwise orthogonal to each other or mutually orthogonal to each other, and further we want to transform it with orthonormal. So, we have to divide all those vector with their norm.

So, we want to make orthonormal set of vectors $\{u_1, u_2, \ldots\ldots, u_n\}$ such that the span of both these set will be same. So, such that the span of this $\{v_1, v_2 \ldots, v_n\}$ is same as span of $\{u_1, u_2, \ldots\ldots, u_n\}$ and what are the more properties we have here? So, $\langle u_i, u_j \rangle$ the inner product $\langle u_i, u_j \rangle = 0$ whatever the inner product we are considering. So, in general, in practice we are considering dot product as the inner product so $\langle u_i, u_j \rangle = 0$ and moreover the norm of each vector, norm of each u_i is 1. So, this will be an orthonormal set of vectors.

So, what we do here, so for the first vector we consider w_1 the new vector and the first vector is remains same, so we consider w_1 to be our v_1 Now we will have to make it orthonormal so our $u_1 = \frac{w_1}{\| w1 \|}$ So, we basically normalize this vector. So, the norm of $\| u_1 \| = 1$ Now for the second vector w_2 we do, we take v_2 and subtract the inner product of v_2 and u_1 and we multiply it with u_1 and subtract this from it so this is our second vector $w_2 = v_2 - \langle v_2 - v_1 \rangle$

Now if we calculate $\langle w_2, u_1 \rangle$ what we will get? So, we will get $\langle v2, u1 \rangle - \langle v2, u1 \rangle \langle u1, u1 \rangle$ now the norm of u_1 is 1, we know so this inner product we $\langle v_1, u_1 \rangle$ is 1. So, we get $\langle v2, u1 \rangle - \langle v2, u1 \rangle$ and that is 0. So, we get a vector which is orthogonal to u_1 Now we have to make it normalized, so our new u_2 is $\frac{w_2}{\| w_2 \|}$ so this will normalize the vector.

Now for u_3 what we have to do, for u_3 we have to take v_3 and we have to subtract this $\langle v_3, u_1 \rangle u_1 - \langle v_3, u_2 \rangle u_2$ So, for each step we have to do the same process. So, we will be using the vectors so which we already get and we will have to use those vectors to calculate the new vector.

So, our u_3 will be $\frac{w_3}{\| w_3 \|}$ So, this is the procedure. Now let us take a set of linearly developed vector and try to calculate the corresponding orthonormal vectors, orthonormal set of vectors.

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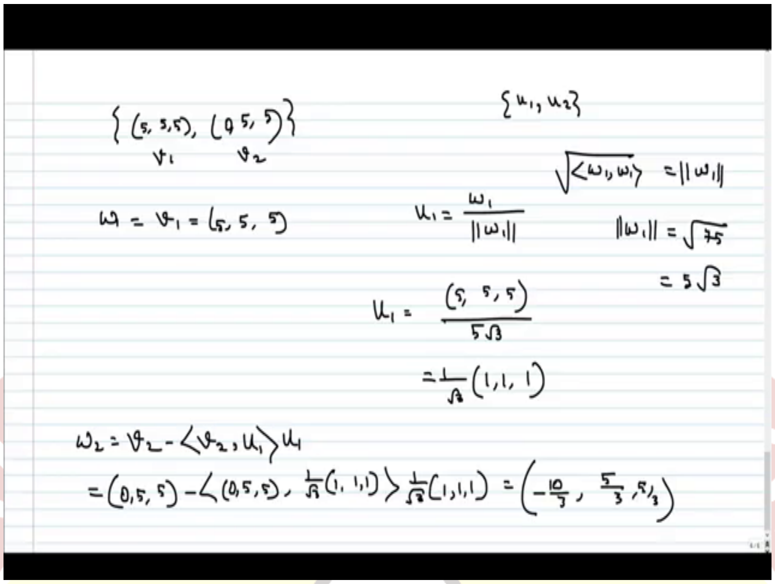

So, at first I am considering the set {(5, 5, 5), (0, 5, 5)} so these are my set of linearly independent vector so this is my v_1 and this is my v_2 and I want to find u_1 and u_2 such that they are orthogonal to each other and more over they are orthonormal. So, for the first vector w_1 is same as v_1 so it is (5, 5, 5) and u_1 is $\frac{w_1}{\| w_1\|}$

Now $\| w_1\| = \sqrt{\langle w_1, w_1\rangle}$ So, this is our norm of w_1 So, if you calculate this, if you take this inner product and we are considering it as your dot product, so our norm of w_1 nothing but square root of 75. So, this is $5\sqrt{3}$ as it is 25 into 3, so it is $5\sqrt{3}$ So, if I want to calculate u1 this is $\frac{(5,5,5)}{5\sqrt{3}}$ so I get $\frac{1}{\sqrt{3}}(1,1,1)$

So, basically it is $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ I can take this $\frac{1}{\sqrt{3}}$ out and I can write this as $\frac{1}{\sqrt{3}}(1,1,1)$ Now we have to calculate the second vector, so second vector is w_2 so this is $w_2 = v_2 - \langle v_2, u_1\rangle u_1$ So, if you calculate it, so our v_2 is (0, 5, 5) and I have to calculate the inner product of (0, 5, 5) with $\frac{1}{\sqrt{3}}(1,1,1)$ therefore, $w_2 = (0,5,5) - \langle (0,5,5), \frac{1}{\sqrt{3}}(1,1,1)\rangle \frac{1}{\sqrt{3}}(1,1,1)$

So, if you do the calculation you will get (- 10/3, 5/3, 5/3). So, this is our new vector w_2 and you can check that this w_2 is basically orthogonal to u_1 Now we have to normalize it, we have to calculate the norm of w_2 and divide w_2 with it.

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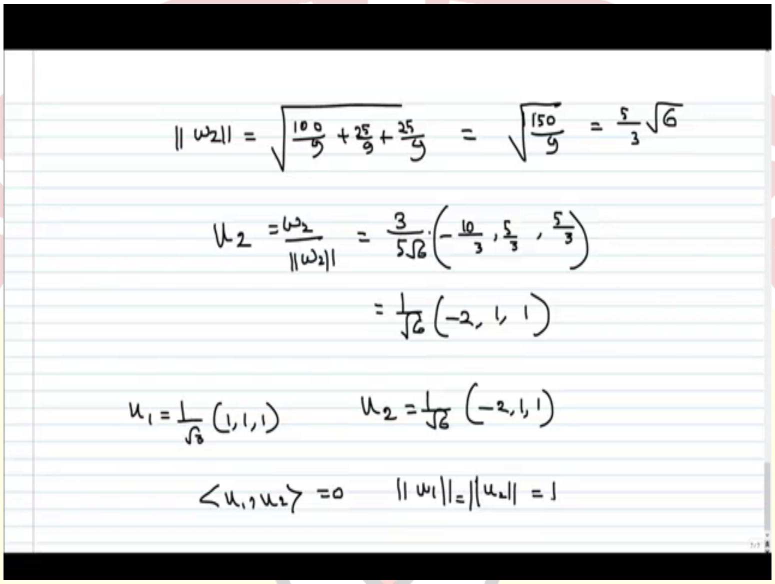

So, we have to calculate the norm of w_2 first, so norm of w_2 if I calculate it I will get this $\| w_2\| = \sqrt{\frac{100}{9}+\frac{25}{9}+\frac{25}{9}}$ So if we calculate it we will get $\sqrt{\frac{50}{9}}$ so it will give me $\frac{5}{3}\sqrt{6}$ and so the new vector u_2 is nothing but $\frac{w_2}{\| w_2\|}$ We have already calculated w_2 and norm of it, so if we do this calculation we will get $u_2 = \frac{w_2}{\| w_2\|} = \frac{3}{5\sqrt{6}}(\frac{-10}{3}, \frac{5}{3}, \frac{5}{3})$

So, you can take 5/3 out and it will cancel up. So, we will get $\frac{1}{\sqrt{6}}(-2,1,1)$ So, our set of vectors, our $u_1 = \frac{1}{\sqrt{3}}(1,1,1)$ and our u_2 is $\frac{1}{\sqrt{6}}(-2,1,1)$ Now if we calculate this $\langle u_1, u_2\rangle$ you will get 0 and if you calculate the norm of both the vectors you will get the norm is 1. So, we really get a orthonormal set of vectors corresponding our v_1, v_2 from which we have started.

So, this is the method which is called the Gram Schmidt method and by this algorithm we can transform any given set of linearly independent set, independent vectors, we can transfer into an orthonormal set of vectors.

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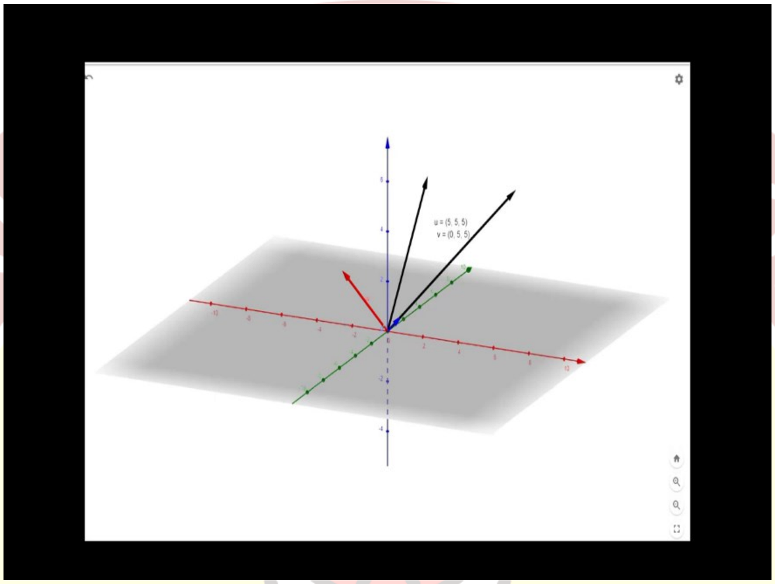

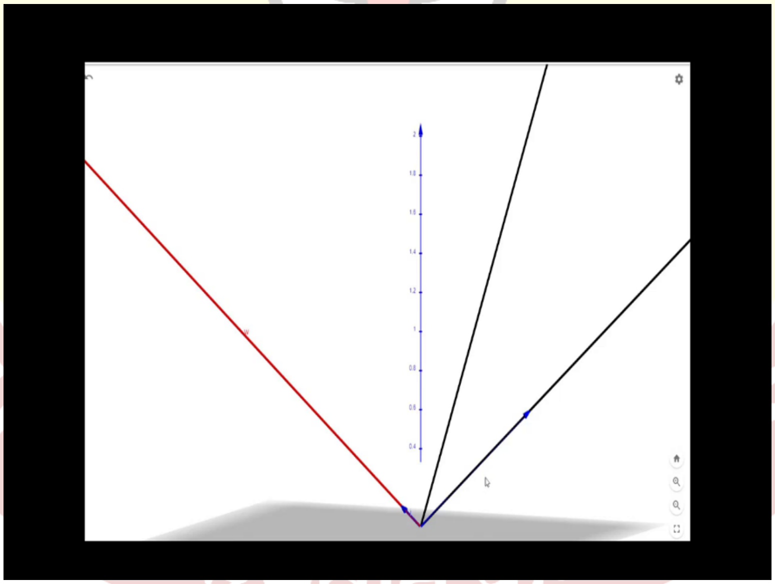

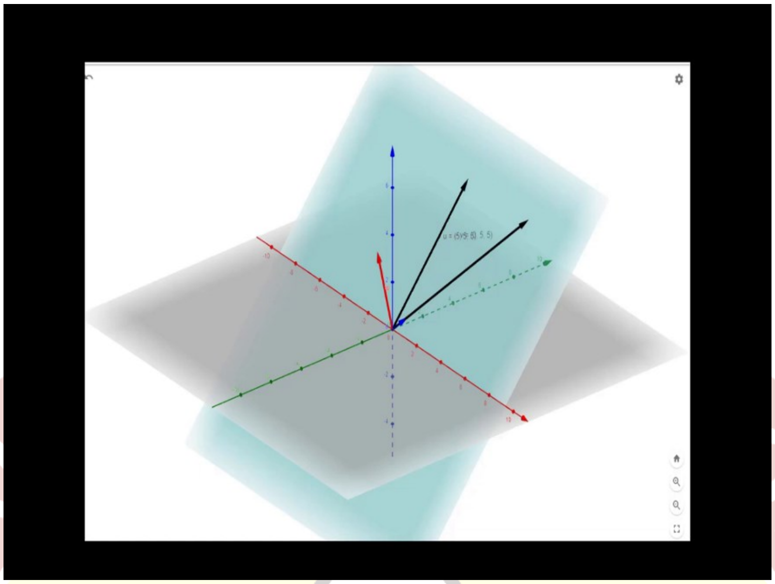

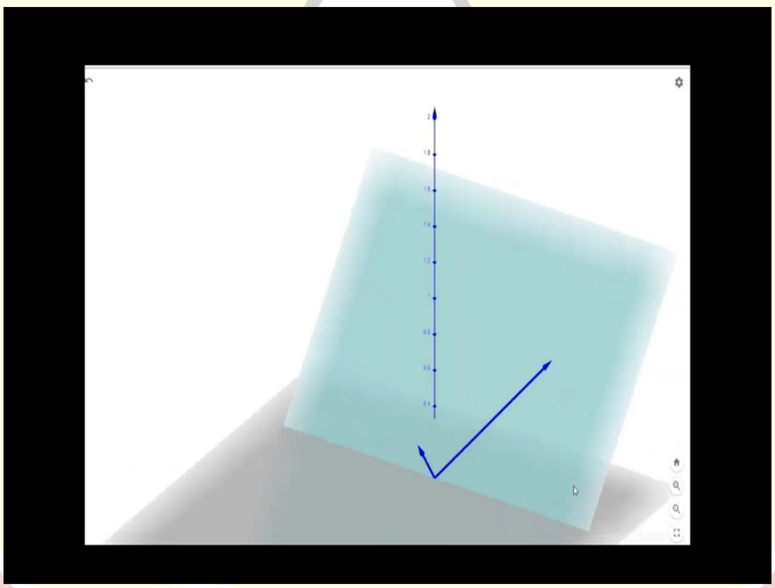

Now let us try to see this in Geo Gebra. So, we have started we a vector (5, 5, 5) and (0, 5, 5). So, these are two vectors as we can see here. Now what we have done at first, so we have normalized the first vector so it will give me a unit vector, so you can see the unit vector here. So, if we zoom this is your unit vector along the first vector. So, let us zoom out again.

Now we have to find a vector which is orthogonal to this unit vector or rather the first vector. So, we get this orthogonal vector, so this is the orthogonal vector we have calculated. Now what we have to do, we have to normalize this vector so again we will normalize this vector and we will get a unit vector along that direction.

So, if we again zoom in you can see there is a vector. Now these two normal vectors, normalized vectors are basically orthogonal to each other. So, if you see the plain passing through these vectors, so this is the plain passing through these vectors so span of all these vectors are basically the same plain, so this is the vector space generated by all these vectors.

So, the Gram Schmidt method, by Gram Schmidt method we basically found a new set of vectors which are orthonormal to each other, but the spanning set remains the same. So, for the new orthonormal set of vectors here, so if we only see those vectors only, so we can see that if we delete all these other vectors so these two are the orthonormal set of vectors, which you get and this is the spanning set which is the same as the original one. Thank you.