INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 5 Tutorial 1

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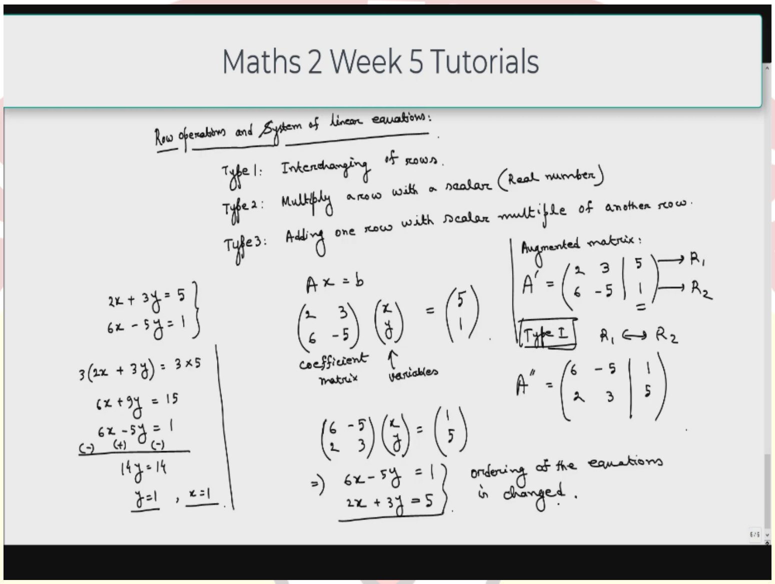

Hello. So, in this video we will see that how row operations affect the system of linear equations, affect the solution of system of linear equation. So, at first begin with a recollection of what all row operations we usually do.

So, there are mainly three types of row operation. So, type one is interchanging of rows. Type two, multiply a row with a scalar, here it is basically real number. And type three is adding one row with scalar multiple of other, another. So, to understand how the solution of a system of linear equation is affected by this row operation, so we can consider an example.

So, let us take an example of two equation with two variables x and y, the first equation is $2x + 3y = 5$ and the second equation is $6x - 5y = 1$. So, if you solve it, if you just multiply the first equation by 3 on both sides, you will get $6x + 9y = 15$ and the second equation, let it be remain as it is.

So, if we just subtract the second one from the first one, we will get $14y = 14$. So, $y = 1$. Now, if we substitute $y = 1$in any one of the equation you will get $x = 1$. So, $x = 1$ and $y = 1$will be a solution and that is the unique solution of this system of linear equation.

Now, let us see how this row operation affect this system. So, the augmented matrix for this system, basically the matrix representation of the system is like $A.x = b$ where a is the matrix $\begin{bmatrix} 2 & 3 \\ 6 & -5 \end{bmatrix}$this is the coefficient matrix x here is two variable $\begin{bmatrix} x \\ y \end{bmatrix}$ this is a column vector so these are variables and b is $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ so this is $A.x = b$, where A is the matrix is $\begin{bmatrix} 2 & 3 \\ 6 & -5 \end{bmatrix}$, this is the coefficient matrix, x here is two variable, $\begin{bmatrix} x \\ y \end{bmatrix}$, this is a column vector. So, these are variables. And b is $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$, so this is $A.x = b$.

So, the augmented matrix is, 2, 3, 6 - 5, 5, 1 and we generally put a straight line in between these two to separate the coefficient matrix and this b vector. So, this is our augmented matrix. So, this is the first row, this is the second row, so the type one is basically interchanging the row. So, if we interchange row one with row two, see what happens. So, this will be the new matrix.

So, let this be denoted by a prime augmented matrix, so our new a double prime will be like this. So, this is giving us 6 - 5, 2, 3 and here we have x y, it is 1, 5 so what we get is 6x - 5 y = 1 and 2x + 3 y = 5. So, these are basically the same system of linear equation but the order of the equation are just interchanged. So, ordering of the equations is changed. So, the type one of row operation is nothing but the changing of order of the equation in the system. So, it does not affect the solution of the system. Now, let us see the second type.

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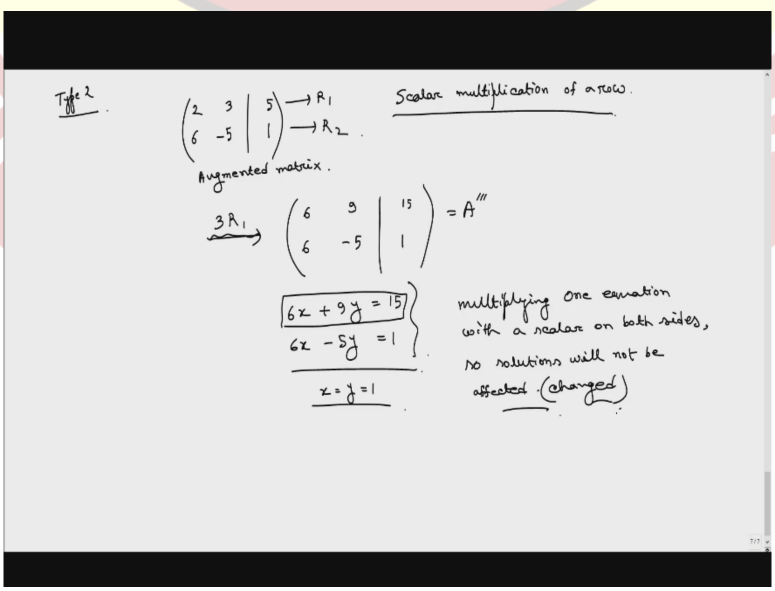

So, the type two row operation was scalar multiplication of a row, scalar multiplication of a row, basically a scalar is multiplied with a row. So, this is our row 1 this is our row 2 in the augmented matrix. So, this is the augmented matrix. So, if we, say, multiply the first row with 3 which we have actually done while we are solving the equation. So, 3 R_1, so what 3 R_1 gives us?

It gives us 6, 9, 15 and let R_2 be the same, so it will be like this. So, this is our new augmented matrix, say, A triple prime. So, the new equation from this we can get is $6x + 9y = 15$ and $6x - 5y = 16$. So, if we solve this then you can see that the same solution we will get $x = y = 1$, the same solution you will get.

So, this type two row operation will also not affect the solution of the system because we are just multiplying one equation with a scalar, so that will not change the solution of the system. So, this type 2 is basically multiplying one equation with a scalar on both sides. So, solution will not be affected or changed.

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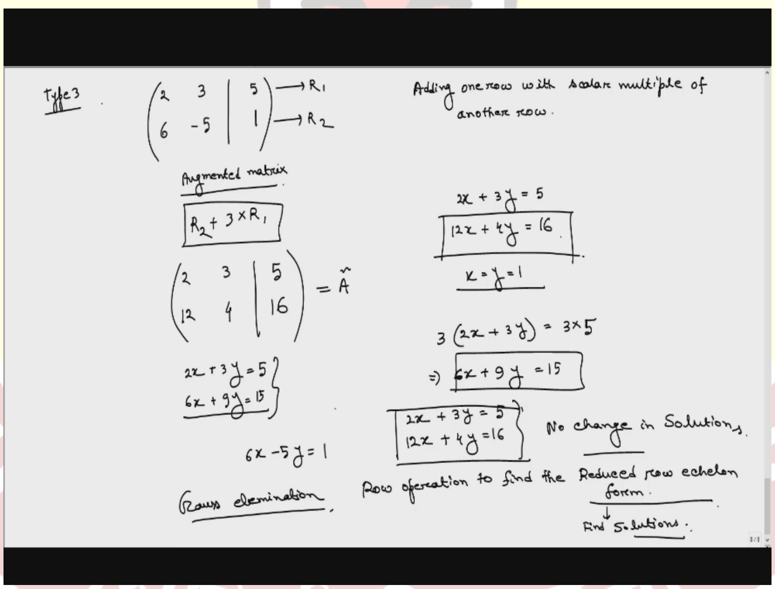

The type 3 row operation was adding one row with scalar multiple of another. So, what we are doing here? Suppose, I am multiplying 3 with R_1 and adding it with R_2, suppose I am doing this. So, what we will get here? The first row will remain unchanged, the second row will be 6 + 6, it will be 12 and 3 into R_1, so the 3 into 3 is 9, 9 + - 5, that will be 4 and 3 into 5 is 15 + 1, that will be 16.

So, this will be our new matrix, so let it write it as A tilde, just a notation here. So, from this matrix, we are getting $2x + 3y = 5$ and $12x + 4y = 16$. Now, again if we solve these two, then we will get x = y = 1. So, it will not change the solution of the system. Basically, what we are doing here?

So, we are multiplying the first equation with 3 on both side, so we will get a new equation that is this which we have seen that this will not change the solution of the system. So, we will get this, this will not change the solution of the system. So, this equation and we already have $2x + 3\,y = 5$, so this is the same system, nothing is changed here.

Now, we are adding this equation with the previous equation which is $6x - 5y = 1$. So, if we add these two, we will get $12x + 4y = 16$. So, these two equation, the first equation remains unchanged, these two equation form the same system and the solution will be the same. So, no change in solution again.

Hence, whatever row operation we do the solution of the system remains unchanged, we have seen in this example and that is true in general. That is why, in Gauss elimination, we are doing row operations to find the reduced row echelon form. And from the reduced row echelon form, we find the solution, from here we find the solution and that solution matches with the original equation, original solution of the equation. Thank you.