Multivariable functions : visualization

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4. Trigonometric funtions
5. The exponential function

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Examples :

1. Linear functions $f(x) = ax + b$
2. Polynomial functions $f(x) = x^2 + x + 1$
3. Rational functions $f(x) = \frac{x}{x^2+1}$
4. Trigonometric funtions $\sin(x), \cos(x), \tan(x), \ldots$
5. The exponential function e^x
6. The logarithm function $\log(x)$
7. (Arithmetic) combinations or compositions $\log(x^2+1)$

$e^x + e^{-x}$ $e^{\sin(x)}$

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1. Linear transformations

   $\mathbb{R}^n \to \mathbb{R}$

   $T(x_1, \dots, x_n) = a_1x_1 + a_2x_2 + \dots + a_nx_n$.

2. Polynomial functions

   $f(x_1, \dots, x_n) = x_1^2 + x_2^2 + \dots + x_n^2$

   $g(x_1, \dots, x_n) = x_1x_2\dots x_n + x_1^2x_2^3x_3^4 - x_1^5x_3^6$.

3. (Arithmetic) combinations or compositions

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$$f(x,y,z) = (x^2+y^2,\ y^2+z^2,\ z^2+x^2).$$

$$\mathbb{R}^3 \to \mathbb{R}^3.$$

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Further, if we want to refer to an element in D without bothering about the coordinates, we will use $\underset{\sim}{x} \in D$.

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- $f(x, y, z) = (sin(x)cos(y), tan(y + z), ln(x^2 + y^2 + z^2), e^{xyz})$

$D \subseteq \mathbb{R}^3$ $\qquad \mathbb{R}^4$.

$f : D \to \mathbb{R}^4$.

$$f(x,y) = \begin{cases} 1 & \text{when } 0 \le x, y \le 1. \\ 0 & \text{o.w.} \end{cases}$$

$$f(x,y) = \begin{cases} xy & (x,y) \in D \\ 0 & \text{o.w.} \end{cases}$$

Arithmetic operations on multivariable functions

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iv) If $m = 1$, and $g(\underset{\sim}{x}) \neq 0$, $\underset{\sim}{x} \in D$, the quotient f/g is defined on D by $(f/g)(\underset{\sim}{x}) = f(\underset{\sim}{x})/g(\underset{\sim}{x})$, $\underset{\sim}{x} \in D$.

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Thus, we obtain a multivariable function $g \circ f : D \to \mathbb{R}^p$ called the composition of f and g defined as $g \circ f(\underset{\sim}{x}) = g(f(\underset{\sim}{x}))$, $\underset{\sim}{x} \in D$.

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Example : $f(x, y) = x^2 + y^2$ is a function from $\mathbb{R}^2$ to $\mathbb{R}$. $g(x) = \sqrt{x}$ is a function from $E = \{x \in \mathbb{R} \mid x \geq 0\}$ to $\mathbb{R}$.

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3. Conics in $\mathbb{R}^2$ $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad xy = 1.$ $\quad x^2 + y^2 = a^2 \quad (a\cos\theta, a\sin\theta)$
4. The helix in $\mathbb{R}^3$: $s(t) = (cos(t), sin(t), t)$
5. The subset $\{(x, y) \mid y^2 = x^3\}$ of $\mathbb{R}^2$.

Thank you