IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 9 Tutorial 3

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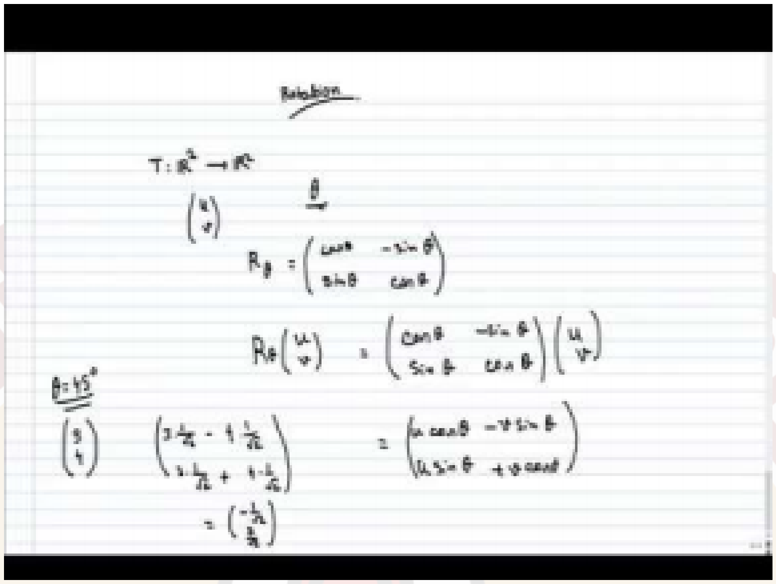

Hello everyone, so in this video we will take an example of an orthogonal linear transformation. So, basically in the lectures sir has given one example as rotation, so where T is a map from we are considering T: $\mathbb{R}^2 \to \mathbb{R}^2$. Rather you can say that these are linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$. The vector space of 2 dimension. So, if we have a vector (u,v). So, if you have this vector $\begin{pmatrix} u \\ v \end{pmatrix}$ and if we rotate this vector by angle θ, then the matrix corresponding to this rotational transformation if we denote it by $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

So, this is the first corresponding matrix.

So, if we have any vector (u,v) or rather we have written it as a column vector $\begin{pmatrix} u \\ v \end{pmatrix}$, then after rotation by angle θ the new vector will be R_θ x $\begin{pmatrix} u \\ v \end{pmatrix}$. Basically we apply this transformation on this vector so we get $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u\cos\theta - v\sin\theta \\ u\sin\theta + v\cos\theta \end{pmatrix}$

So, this will be the image of the vector in a two dimensional plane. So, suppose we have, $\theta = 45^0$, then we know the value of $\cos 45^0$ and $\sin 45^0$. So, let me write it down as an example, if $\theta = 45^0$ and our vector, our given vector is (3,4), then if we rotate this vector by 45^0 what will we get?, we will get

$$\begin{pmatrix} 3\cos45^\circ - 4\sin45^\circ \\ 3\sin45^\circ + 4\sin45^\circ \end{pmatrix} = \begin{pmatrix} 3\left(\frac{1}{\sqrt{2}}\right) - 4\left(\frac{1}{\sqrt{2}}\right) \\ 3\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{\sqrt{2}}\right) \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{7}{\sqrt{2}} \end{pmatrix}$$

So, this will be the vector after rotating this (3,4) by 45^0. Now let us try to visualize this example in GeoGebra.

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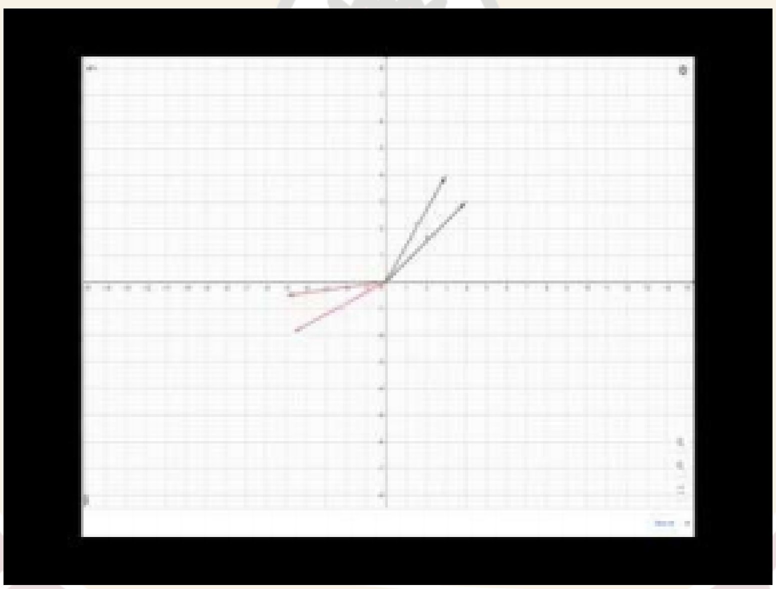

So, this is our vector (3,4) and after rotating this we will get some new vectors, so now if I rotate it, these are the new vector spaces. So, depending on the degree of rotation you will get different vectors as you can see in this diagram.

Now suppose we take another vector V which I have taken here as $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$. So, these is some angle between u and v and now if we see one rotation of this thing, this new vector v we will get some new vector v'. Now if we see the rotation for different angles you will see that the angle between u' and v' will always be the same and that is exactly the angle between u and v.

So, the angle between u and v does not change due to this rotation transformation and that is basically the main characteristic of orthogonal transformation, because in orthogonal transformation we know that the inner product of u and v will be the same as the inner product of T_u and T_v. So, here we can observe this case in this animation. Thank you.