IIT Madras

ONLINE DEGREE

Mathematics for Data Sciences 2
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Week 11 - Tutorial 1

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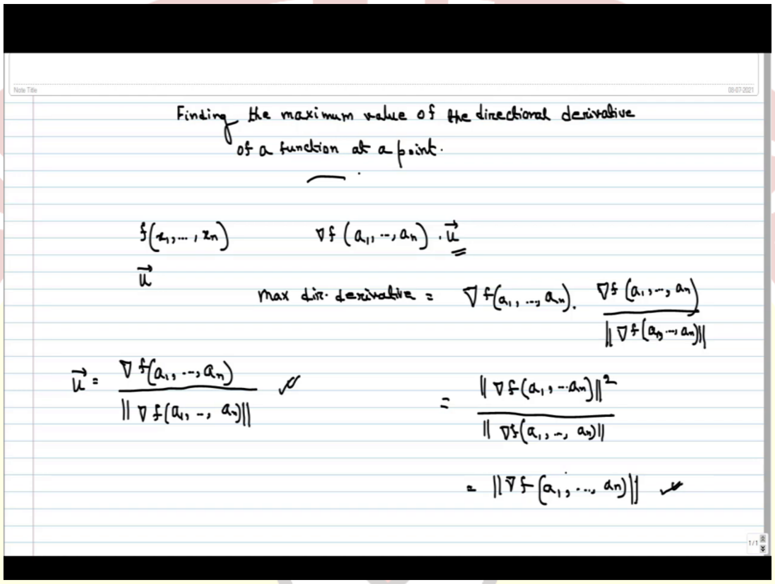

Hello everyone. So, in this video we will try to find the maximum value of the directional derivative of a function at some given point. So, the maximum value of the directional derivative occurs when the grad of that function, the gradient of that function and the unit vector point in the same direction.

So, what does it mean? Suppose, I have given a function $f(x_1, x_2, \dots, x_n)$ so this is a function of n variable, let us assume this is a scalar valued function. Now, what we have to do, so for calculating the directional derivative what we generally do, we calculate the gradient of it at some point.

So, suppose the point is given as $a_1, a_2, \dots, a_n$ so we will calculate the gradient at that point and some vector is given at the direction of which we want to calculate the directional derivative. Suppose, u is the given vector and suppose u is the unit given vector, so we will calculate ∇f at that point with dot product with u, so u is the vector, unit vector given at that direction. So, given that, which is given and at that direction we are calculating the directional derivative.

Now, it will be maximum when this unit vector will be at the direction of this gradient vector. So, it will be, the directional derivative will be maximum, so I can write the maximum directional derivative, it is basically when this gradient vector, so this is the gradient vector, the unit vector is along the same direction, so we will calculate the unit vector along this gradient vector along this direction of this gradient vector. So, what we have to do? We have to divide it by its norm.

So, we have to normalize it to get the unit vector along that direction, so, and so this is my unit vector, this $\frac{\nabla f(a_1, a_2, ..., a_n)}{\|\nabla f(a_1, a_2, ..., a_n)\|}$ at that point and we have to take the dot product with gradient vector. So, this will give us the maximum directional derivative at some point, at that given point. So, this is, what this is, this is basically the numerator we have got, norm of gradient vector at that point, square of the norm and in the denominator what we got, at the denominator we got this norm of gradient vector only i.e., $\frac{\|\nabla f(a_1, a_2, ..., a_n)\|^2}{\|\nabla f(a_1, a_2, ..., a_n)\|}$

So, this one we can say now and we will get the norm of gradient vector at that point $\|\nabla f(a_1, a_2, ..., a_n)\|$ so this is the maximum directional derivative at the given point. And the direction at which we are calculating this directional derivative that u, here u is nothing but the unit vector along the direction of the gradient. So, this is $u = \frac{\nabla f(a_1, a_2, ..., a_n)}{\|\nabla f(a_1, a_2, ..., a_n)\|}$ so this is the direction at which we are calculating the directional derivative, so at this direction the reaction derivative will be the maximum and the value will be norm of gradient vector at that point.

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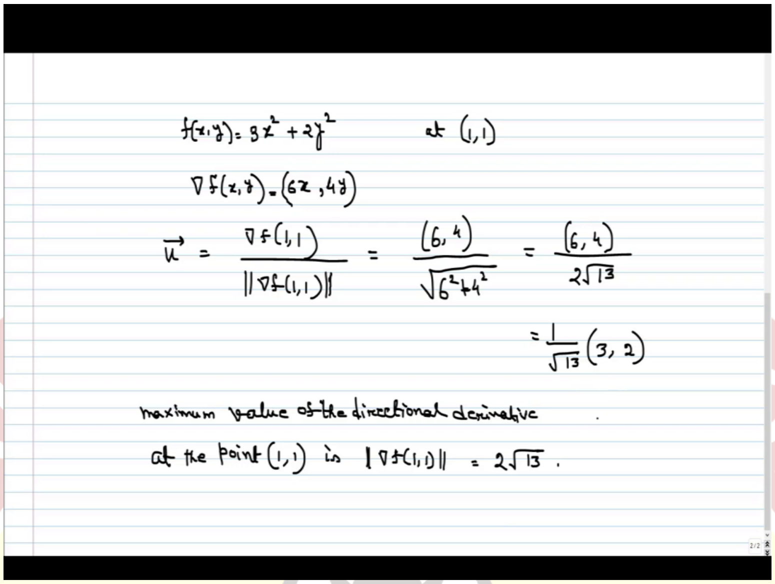

So, let us try to figure take an example and see this. So, let us consider the function $f(x,y)=3x^2+2y^2$ so let us calculate the ∇f first, so grad of $\nabla f(x,y)=(6x,4y)$ so this is basically (fx, fy), so partial derivative with respect to x it will be the first coordinate our partial derivative with respect to y it will be the second coordinate, so this is (6x, 4y).

Now, we want to find the direction at which this directional derivative will be maximum. So, the direction will be, suppose I am denoting it by u, so this is $\vec{u}=\frac{\nabla f(x,y)}{\|\nabla f(x,y)\|}$ So, suppose I want to find this at the point (1, 1) so the point is given. So, what should I do here, I should replace this x and y by (1, 1).

So, I will replace this x and y by (1, 1). So, the unit vector along which we will calculate the direction derivative so that the directional derivative will be maximum that is this vector. So, what is $\nabla f(1,1)$ this is (6, 4) and if we calculate the norm of it, it will be $\sqrt{6^2+4^2}$

So, this is $\frac{(6,4)}{2\sqrt{13}}$ so these 2 get cancel up, I will get $\frac{1}{\sqrt{13}}(3,2)$ So, along this direction the directional derivative will be maximum and what will be the value of the maximum directional derivative. This is basically the norm of the grad at that point.

So, the maximum value of the directional derivative at the point (1,1) is basically $\|\nabla f(1,1)\|$ So, we have already calculated the norm that is $2\sqrt{13}$ so this is the maximum value of the directional derivative and it is at the direction 1 $\frac{1}{\sqrt{13}}(3,2)$ along this direction. Thank you.