IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 10 - Tutorial 04

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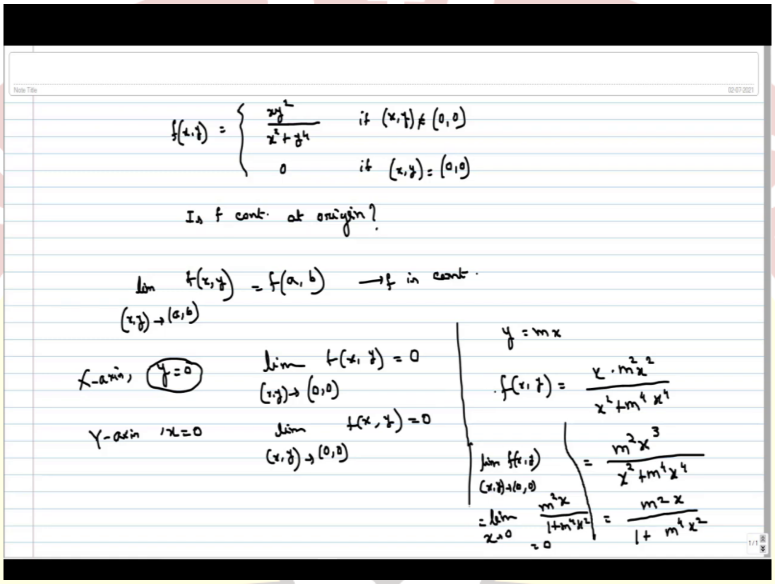

Hello everyone, so in this video we will try to see that this function which is a two variable function, scalar function, whether it is continuous at origin or not. So, the function is defined part wise. So, at origin it is 0 and other than that it is basically it is $\frac{xy^2}{x^2+y^4}$

Now, it is continuous at origin if the limit at origin exist, and the value is same as the value at the origin. So, if it is continuous, then we have to show that xy tending to (a, b) suppose, I am checking the continuity at (a, b). So, if limit of the function at (a, b) is saying as $\lim_{(x,y)\to(a,b)} f(x,y)=f(a,b)$ so, then we can say that f is continuous.

So, at first state we have to check whether the limit exist at the origin or not. So, what happened if we approach via x axis, that means if y = 0, so, this is my x axis. So, if y = 0 we can see the functional value is 0. So, $\lim_{(x,y)\to(0,0)} f(x,y)$ at the origin this f (x, y) is basically 0.

But when we see along the Y axis, then our x is 0. And in that case also this limiting value you can check that this is 0, because the function value is 0 when x is 0 as you can see, and as y approaches to 0 we are getting closer to 0. So, does it imply that it is continuous, no, because we have only checked for two straight line.

Now, if we check for any arbitrary straight-line y = mx, let us see what happened. For y = mx if we substitute this then our f (x, y) is nothing but $\frac{xm^2x^2}{x^2+m^4x^4}$ So, it is $\frac{m^2x^3}{x^2+m^4x^4}$

So, this x square will cancel, cancel out from both the numerator and numerator, so, I am getting $\frac{m^2x}{1+m^4x^2}$ So, as (x, y) tending to (0, 0). that is we are tending to origin along this straight line, then limit f(x, y) as (x, y) both tending to (0, 0), this is nothing but limit x tending to 0, this function which we have got, $\frac{m^2x}{1+m^4x^2}$

So, let me separate this out. Yeah. So, as x tending to 0, the value of this function is 0. So, along any straight line we have seen that this function is going towards 0, but still we cannot conclude that this function is going towards 0, because there are infinitely many curves which, by which we can approach origin. So, is there any curve by which we are getting any other limiting value.

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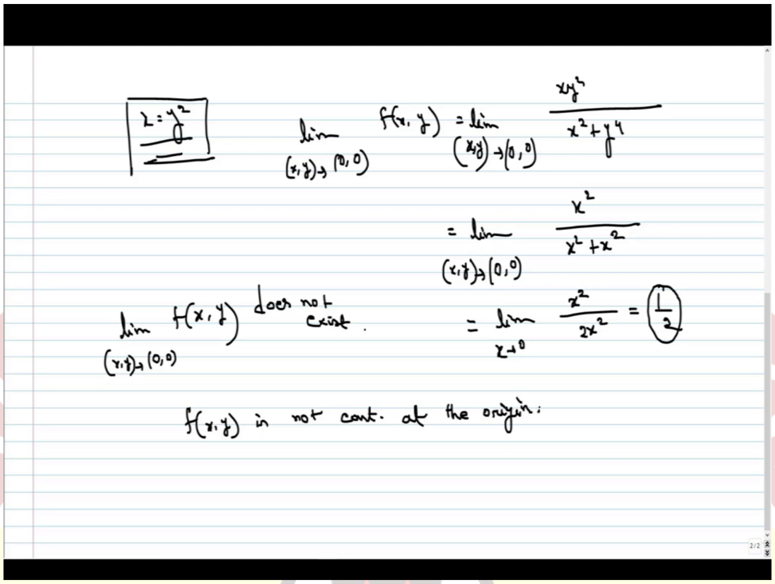

So, let us try one curve that is $x = y^2$ let us try this curve. Now, for this curve, now as (x, y) tending to origin that is (0, 0). So, this is my f (x, y). So, let us write the functional value here. So, our functional value was given to be $\frac{x y^2}{x^2 + y^4}$ this was given.

So, now, I am approaching through the curve $x = y^2$ So, I am just writing (x, y) tending to (0, 0). Now, $x = y^2$ square that means, in the numerator I get x square and, in the denominator, I get x square plus x square. So, this is nothing but limit x tending to 0, $\frac{x^2}{2x^2}$ So, I am getting half. So, this limiting value is half which is not 0.

So, when we are approaching via any straight line, we have seen the limiting value is 0, but when we are taking this curve, so this is a parabola. So, when we are approaching this parabola, we can see that the limiting value is half. So, the limiting value does not matches for any curve. So, we can conclude that the limit of this function does not exist. So, we meet (x, y) tending to (0, 0), f (x, y) does not exist. So, that means, it is not, so f (x, y) is not continuous at the origin. Thank you.