

**IIT Madras**  
ONLINE DEGREE

**Mathematics for Data Science 2**  
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**Week-6 Tutorial 4**

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Dependent & Independent Vectors

Three vectors  $\left\{ \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} \right\} \in \mathbb{R}^3$

$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ -3 & -1 & 3 \end{bmatrix}$

$\det(A) = 1(3-1) + 1(3-3) - 1(-1+3)$   
 $= 2 + 0 - 2$   
 $= 0$

$x+y+z=0$   
 $x+2y+2z=0$   
 $-1+2-3=0$

Hello guys, in this video we will see vectors, linearly independent and dependent vector. So, suppose we have given three vector in  $\mathbb{R}^3$ , which is  $(1, 1, -3)$ , second vector is  $(-1, 1, -1)$  and third vector is  $(-1, -1, 3)$ , these are the vector from vector space  $\mathbb{R}^3$ , which is formed vector space over  $\mathbb{R}$ .

Now, we will check that these three vectors are linearly dependent or independent. So, we know that a method we will form a matrix using by writing these vectors in column wise and form a matrix, which are here we can see this, and we will check if determinant of this matrix is 0, then we will say these vectors are linearly dependent.

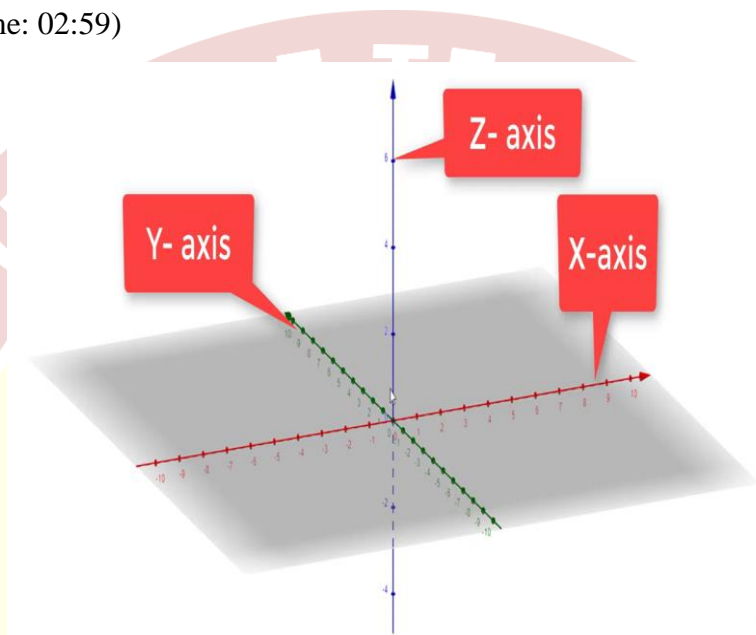
Now, this is the matrix  $A$ . And if we calculate the determinant of this matrix along row, first row, so we will get  $\det(A) = 1(3 - 1) + 1(3 - 3) - 1(-1 + 3)$ . Now, if you calculate these, we will get 0, it means, the determinant of matrix is 0. So, it means these vectors are linearly dependent.

Now, if we see clearly these coordinates, suppose this is  $x$ , this is  $y$  and this is  $z$ . This is  $x$ , if we add with  $y$  coordinate this  $y$  with  $2y + z$  it means it will become  $1 + 2 - 3$ , which is

actually 0. Again, here we will check with the same pattern, means  $-1 + 2 - 1 = 0$  which is also 0 for this vector. And for this vector, if you check that  $-1 - 2$ , say for second  $-2 + 3$ , this is also makes 0.

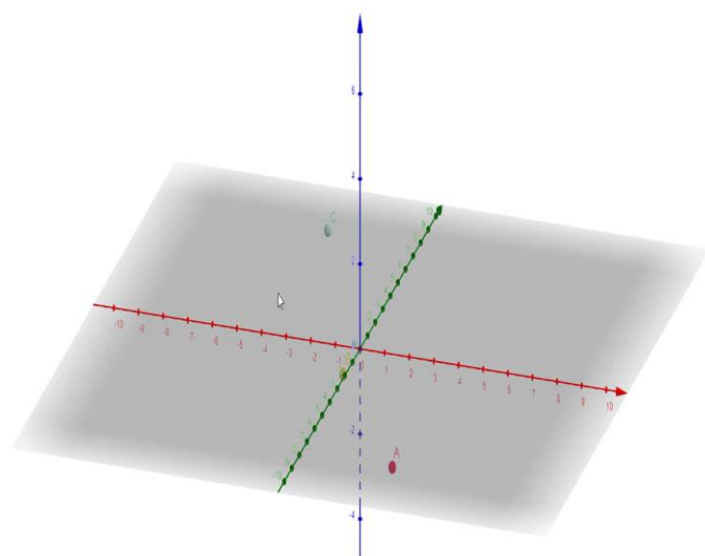
So, if you see clearly this three vector follow this best term,  $x + 2y + z = 0$  is equal to 0, and if in coordinate system in  $\mathbb{R}^3$ , this is a plane.

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Now, let us see it geometrically. So, as we have given three vectors  $(1, 1, -3)$ ,  $(-1, 1, -1)$  and  $(-1, -1, 3)$ , so these vectors are in  $\mathbb{R}^3$ . So, here red line represent  $x$  - axis, green line represent  $y$  - axis and the blue line represent  $z$  - axis.

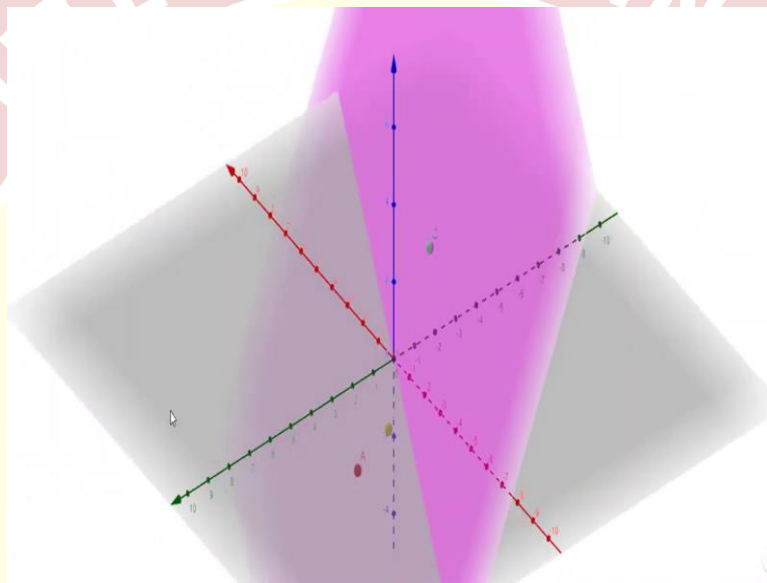
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And where the arrow shows that side is a positive side of the axis, and without arrow side shows negative side of the axis and 0 is in between them. Now, our first vector is  $(1, 1, -3)$ . So, here we can see a is actually, the first vector  $(1, 1, -3)$ , this is the first vector. Similarly, second vector is  $(-1, 1, -1)$ . So, which is a yellow, so this is our second vector.

And third vector is  $(-1, -1, 3)$ , so both  $x$  – coordinate and  $y$  – coordinate are  $-1$ . So and  $z$  – coordinate is  $+3$ , so you can see here that it will be upward to the 0. Now, we have seen that vectors follow a pattern, which is  $x + 2y + z = 0$  and this is represent a plane.

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So, if we draw this plane that is like this. So, here we can see that these three points lie, these three vectors lie in the same plane. We can here see that. So this is the mean. If three vectors linearly dependent, then surely, they will lie in a plane.