Equivalence and similarity of matrices

Sarang S. Sane

Equivalence of matrices

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$$QAP = \begin{bmatrix} I_{r\times r} & 0 \\ 0 & 0 \end{bmatrix} = Q'BP'$$

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- A is equivalent to itself $\quad A = I_{m\times m} A I_{n\times n}$
- A is equivalent to B implies B is equivalent to A. $\quad B = QAP \Rightarrow A = Q^{-1}BP^{-1}$
- A is equivalent to B and B to C implies A is equivalent to C.

$$B = QAP, \; C = Q'BP' \Rightarrow C = \underbrace{Q'Q}A\underbrace{PP'}$$

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Example

Consider the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^2$, defined as :

$$f(x, y, z) = (x + y, y + z).$$

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Consider two ordered bases for $\mathbb{R}^3$:

$\beta_1 = (1, 0, 0), (0, 1, 0), (0, 0, 1)$ and $\beta_2 = (1, 1, 0), (0, 1, 1), (0, 0, 1)$.

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Similarly, consider two ordered bases for $\mathbb{R}^2$:

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$f(1, 0, 0) = (1, 0)$,
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$f(0, 0, 1) = (0, 1).$ $\leftarrow$

Hence the matrix corresponding to f with respect to the bases β_1 and γ_1 is $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Example (contd.)

$$f(1,1,0) = (2,1) = 1(1,0) + 1(1,1),$$
$$f(0,1,1) = (1,2) = -1(1,0) + 2(1,1),$$
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Hence the matrix corresponding to f with respect to the bases β_2 and γ_2 is $B = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$.

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Choose $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

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$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

Choose $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$. Then

$$QAP = \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix} = B$$

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Consider a linear transformation $T : V \to W$, two ordered bases β_1 and β_2 for V, and two ordered bases γ_1 and γ_2 for W.

Let A be the matrix corresponding to T with respect to the bases β_1 and γ_1 and B be the matrix corresponding to T with respect to the bases β_2 and γ_2.

Then A is equivalent to B !

$P \to$ express the ordered basis β_2 in terms of the ordered basis β_1.

$Q \to$ express the ordered basis γ_1 in terms of γ_2.

Then $B = QAP$.

Similar matrices

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- A is similar to itself $P = I$: $A = I^{-1} A I$.
- A is similar to B implies B is similar to A.
- A is similar to B and B to C implies A is similar to C.

$B = P^{-1}AP \Rightarrow PBP^{-1} = A \Rightarrow A = (P^{-1})^{-1} B P^{-1}$

$B = P^{-1}AP$, $C = Q^{-1}BQ$

$\Rightarrow C = Q^{-1}(P^{-1}AP)Q = Q^{-1}P^{-1}APQ$

$= (PQ)^{-1}A(PQ)$

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Important properties of similar matrices

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- $$\begin{aligned} det(B) = det(P^{-1}AP) &= det(P^{-1})det(A)det(P) \\ &= \frac{1}{det(P)}det(A)det(P) = det(A). \end{aligned}$$

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- $$\begin{aligned} det(B) = det(P^{-1}AP) &= det(P^{-1})det(A)det(P) \\ &= \frac{1}{det(P)}det(A)det(P) = det(A). \end{aligned}$$
- Several other invariants of A and B are the same such as the characteristic polynomial, minimal polynomial and eigen values (with multiplicity).

An example of similar matrices

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Then we get :

$$\begin{aligned} f(1,0,0) = (-1,1,1) &= -1(1,0,0) + 1(0,1,0) + 1(0,0,1) \\ f(0,1,0) = (1,-1,1) &= 1(1,0,0) - 1(0,1,0) + 1(0,0,1) \\ f(0,0,1) = (1,1,-1) &= 1(1,0,0) + 1(0,1,0) - 1(0,0,1) \end{aligned}$$

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f(0,1,0) = (1,-1,1) &= 1(1,0,0) - 1(0,1,0) + 1(0,0,1) \leftarrow \\
f(0,0,1) = (1,1,-1) &= 1(1,0,0) + 1(0,1,0) - 1(0,0,1) \leftarrow
\end{aligned}
$$

Hence the matrix of f corresponding to the standard ordered basis is $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

Example (contd.)

Consider another ordered basis $\beta' = (1,1,1), (-1,1,0), (-1,0,1)$.

Example (contd.)

Consider another ordered basis $\beta' = (1,1,1), (-1,1,0), (-1,0,1)$.

Then we have the following:

$$\begin{aligned}
f(1,1,1) &= (1,1,1) &&= 1(1,1,1) + 0(-1,1,0) + 0(-1,0,1) \\
f(-1,1,0) &= (2,-2,0) &&= 0(1,1,1) - 2(-1,1,0) + 0(-1,0,1) \\
f(-1,0,1) &= (2,0,-2) &&= 0(1,1,1) + 0(-1,1,0) - 2(-1,0,1)
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Then we have the following:

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Hence the matrix of f corresponding to the ordered basis β' is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

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Let $P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Then $P^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}$.

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Then $P^{-1}AP = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

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$$\text{Then } P^{-1}AP = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -4/3 & 2/3 \\ 2/3 & 2/3 & -4/3 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

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Consider the ordered basis $(1,0),(1,1)$ for $\mathbb{R}^2$. Then we have the following:

$$f(1,0) = (2,0) = 2(1,0) + 0(1,1)$$
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Hence the matrix of f corresponding to this ordered basis is :

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

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Hence the matrix of f corresponding to this ordered basis is :

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$$\begin{aligned} \text{Then } P^{-1}AP &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}. = B. \end{aligned}$$

Hence the matrices $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ are similar.

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Let A be the matrix corresponding to T with respect to the basis β and B the matrix corresponding to T with respect to the basis γ.

Then A is similar to B ! $B = P^{-1}AP$

$P \to$ Express γ in terms of β.

$P^{-1} \to$ Express β " " " γ.

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Why do we care about similarity?

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Why do we care about similarity? Because under some basis, we hope that the corresponding matrix is a diagonal matrix which gives an easy geometric understanding of the linear transformation.

Thank you