Statistics Live Session

Jun 28, 2021

Q1 (a) Markov inequality

Suppose the average marks in an exam in a class of 100 students is 50. What is the maximum number of students who could have got more than 90 marks?

Q1 (b) Markov inequality

Suppose X is a continuous random variable uniformly distributed in [0,10]. Find an upper bound on P(X > 8) using Markov inequality. Compare with the actual probability.

Q2 (a) Markov inequality - Binomial distribution

Suppose a fair coin is tossed 200 times. Find an upper bound (using Markov's inequality) for the probability that more than 150 heads are seen.

Q2 (b) Markov inequality - Binomial

A biased coin with probability of heads equal to 1/3 is tossed two hundred times. Find an upper bound (using Markov's inequality) for the probability that more than 150 heads are seen.

Q3 (a) Markov inequality - Exponential

Suppose X is Exponential(4). Find an upper bound on P(X > 4) using Markov inequality. Compare with the actual probability.

Q3 (b) Markov inequality - Poisson

Suppose X is Poisson(4). Find an upper bound on P(X > 12) using Markov inequality.

Q3 (c) Markov inequality - Geometric

Suppose X is Geometric(1/4). Find an upper bound on $P(X > 8)$ using Markov inequality. Compare with the actual probability.

Q4 (a) Markov inequality - Sum of iid

Let X_1, X_2,..., X_5 be iid Uniform[0,100]. Let $X = X_1 + X_2 + ... + X_5$. Find an upper bound for $P(X>450)$ using Markov's inequality.

Q4 (b) Markov inequality - Sum of iid

Let X_1, X_2,..., X_{50} be iid X, where X has the following distribution:
$P(X = -3) = 0.1$, $P(X = 0) = 0.3$, $P(X = 0.5) = 0.1$, $P(X = 1) = 0.3$, $P(X = 2) = 0.2$

Let $S = X_1 + X_2 + ... + X_{50}$. Find an upper bound for $P(X>80)$ using Markov's inequality.

Q4 (c) Markov inequality - Average of iid

Let $X_1, X_2, \ldots, X_{100}$ be iid Beta(3,10). Let $Y = (X_1 + X_2 + \ldots + X_{100})/100$. Find an upper bound for $P(Y > 0.9)$ using Markov's inequality.

Q5 (a) Markov inequality - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let $X_i = 1$ if Bin *i* is empty and 0, otherwise. What is $P(X_i = 1)$? In other words, what is the probability that the *i*-th bin is empty? What is $E[X_i]$?

Q5 (b) Markov inequality - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let $X_i = 1$ if Bin i is empty and 0, otherwise. Let $X = X_1 + \ldots + X_{10}$ be the number of empty bins. What is $E[X]$? Can you comment on the distribution of X?

Q5 (c) Markov inequality - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let $X_i = 1$ if Bin i is empty and 0, otherwise. Let $X = X_1 + \ldots + X_{10}$ be the number of empty bins. Using Markov's inequality, find an upper bound for $P(X > 5)$.

Q6 (a) Chebyshev inequality - continuous uniform

Suppose X is a continuous random variable uniformly distributed in [-10,10]. Find an upper bound on P(|X| > 8) using Chebyshev inequality. Compare with the actual probability.

Q6 (b) Chebyshev inequality - discrete uniform

Suppose X is a discrete random variable uniformly distributed in {1,...,100}. Find a lower bound on P(X = 50 or 51) using Chebyshev inequality. Compare with the actual probability.

Q7 (a) Chebyshev inequality - Binomial

Suppose a fair coin is tossed 200 times. Find an upper bound (using Chebyshev inequality) for the probability that more than 150 heads or fewer than 50 heads are seen.

Q7 (b) Chebyshev inequality - Binomial

A biased coin with probability of heads equal to 1/3 is tossed two hundred times. Find an upper bound (using Chebyshev’s inequality) for the probability that more than 150 heads are seen.

Q8 (a) Chebyshev inequality - Normal

Suppose X is Normal(0, 2). Find an upper bound on P(|X| > 8) using Chebyshev inequality.

Q8 (b) Chebyshev inequality - Normal

Suppose X is Normal(100, 10). Find an upper bound on P(|X-100| > 80) using Chebyshev inequality.

Q9 (a) Chebyshev inequality - Exponential

Suppose X is Exponential(4). Find an upper bound on P(X > 4) using Chebyshev inequality. Compare with Markov.

Q9 (b) Chebyshev inequality - Poisson

Suppose X is Poisson(4). Find an upper bound on P(X > 12) using Chebyshev inequality. Compare with Markov.

Q9 (c) Chebyshev inequality - Geometric

Suppose X is Geometric(1/4). Find an upper bound on P(X > 8) using Chebyshev inequality. Compare with Markov.

Q10 (a) Chebyshev inequality - Sum of iid

Let $X_1, X_2, \ldots, X_5$ be iid Uniform[0,100]. Let $X = X_1 + X_2 + \ldots + X_5$. Find an upper bound for $P(|X - 250| > 200)$ using Chebyshev's inequality.

Q10 (b) Chebyshev inequality - Sum of iid

Let X_1, X_2,..., X_{50} be iid X, where X has the following distribution:
$P(X = -3) = 0.1$, $P(X = 0) = 0.3$, $P(X = 0.5) = 0.1$, $P(X = 1) = 0.3$, $P(X = 2) = 0.2$

Let $S = X_1 + X_2 + ... + X_{50}$. Find an upper bound for $P(X>80)$ using Chebyshev's inequality.

Q10 (c) Chebyshev inequality - Average of iid

Let $X_1, X_2, \ldots, X_{100}$ be iid Beta(3,10). Let $Y = (X_1 + X_2 + \ldots + X_{100})/100$. Find an upper bound for $P(Y > 0.9)$ using Chebyshev's inequality.

Q11 (a) Chebyshev inequality* - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let $X_i = 1$ if Bin *i* is empty and 0, otherwise. Let $X = X_1 + \ldots + X_{10}$ be the number of empty bins. Write an expanded form for $X^2 = (X_1 + \ldots + X_{10})^2$. Take expectations on both sides to find an expression for $E[X^2]$.

Q11 (b) Chebyshev inequality* - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let X_i = 1 if Bin *i* is empty and 0, otherwise. What is $E[X_i^2]$?

Q11 (c) Chebyshev inequality* - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let X_i = 1 if Bin *i* is empty and 0, otherwise. What is $E[X_1X_2]$? What is $E[X_iX_j]$ when *i* and *j* are not equal?

Q11 (d) Chebyshev inequality* - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let $X_i = 1$ if Bin i is empty and 0, otherwise. Let $X = X_1 + \ldots + X_{10}$ be the number of empty bins. What is $E[X^2]$? What is $Var(X)$? Using Chebyshev inequality, find an upper bound for $P(X > 5)$.

Q11 (e) Chebyshev inequality* - Sum of non iid

10 balls are thrown into 10 bins independently and uniformly at random.
Let $X_i = 1$ if Bin *i* is empty and 0, otherwise. Let $X = X_1 + \dots + X_{10}$ be the number of empty bins. Using Chebyshev inequality, find an upper bound for $P(X > 5)$.