# Determinants (Part 3)

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- The $(i, j)$-th minor is the determinant of the submatrix formed by deleting the $i$-th row and $j$-th column.
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**Definition**

$det(A) = \Sigma_{j=1}^{n} (-1)^{1+j} a_{1j} M_{1j} = \Sigma_{i=1}^{n} a_{1j} C_{1j}$

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# Important properties and identities

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Property 2 : Switching two rows or columns changes the sign.

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Property 3' : Adding multiples of a column to another column leaves the determinant unchanged.

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1) The determinant of a matrix with a row or column of zeros is 0.
2) The determinant of a matrix in which one row (or column) is a linear combination of other rows (resp. columns) is 0.
3) Scalar multiplication of a row by a constant $t$ multiplies the determinant by $t$.
4) While computing the determinant, you can choose to compute it using expansion along a suitable row or column.

Thank you