



**IIT Madras**  
ONLINE DEGREE

**Mathematics for Data Science 2**  
**Professor Sarang S. Sane**  
**Department of Mathematics**  
**Indian Institute of Technology, Madras**  
**Week 10 - Tutorial 03**

(Refer Slide Time: 0:17)

calculating limit

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^3 + y^3}, & \text{if } x, y \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$$

Does  $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$  exist? (Does not exist).

$y = mx$

$\begin{matrix} y=x \\ y=0 \end{matrix}$	$\begin{matrix} m=1 \\ m=0 \end{matrix}$	$\begin{matrix} 1/2 \\ 0 \end{matrix}$	$\lim_{x \rightarrow 0} \frac{x^2(mx)}{m^3 x^3 + x^3} = \lim_{x \rightarrow 0} \frac{m}{m^3 + 1} = \frac{m}{m^3 + 1}$
--	--	--	---

Hello everyone, so in this video, we will try to calculate the limit of this function. And so, we want to see the  $\lim f(x, y)$  as  $(x, y) \rightarrow (0, 0)$ , that is when we are approaching towards the origin, whether these limits exist or not. So, our question is,  $f(x, y)$ ,  $\lim (x, y) \rightarrow (0, 0)$  exist?

So, here in this type of problem at first let us check what is the degree of denominator and numerator. So, the degree of the denominator, sorry degree of the numerator is 3, and the degree of the denominator is also 3. Now, in this kind of cases, we will generally try to approach towards the origin by this term  $y = mx$ .

So, we are trying, we are approaching the origin by any straight line. So, any straight line means  $y = mx$ . So, let us substitute  $y = mx$  and see what happened. So, if I, if we approach origin by this line, then as  $x \rightarrow 0$ ,  $y \rightarrow 0$ . So, when we substitute this will get  $\lim_{x \rightarrow 0}$  is a  $x^2$  and in place of  $y$ , we are substituting  $mx$ .

So, we will get  $m^3 x^3$  and  $x^3$ . So, the  $x^3$  will cancel up. So,  $\lim_{x \rightarrow 0} \frac{m}{m^3+1}$ . So, now this term is independent of  $x$ . So, the limit is  $\frac{m}{m^3+1}$ . Now, for different value of  $x$ , we will get different value of this limit, but for limit to exist, that has to be unique.

So, here we can clearly see that when we are approaching the origin, when we are approaching the origin from different direction. So, suppose this is my  $xy$  plane and we are approaching the origin from different straight lines, the limiting value will be different, it will depend on  $f$ , suppose we are approaching using  $y = 0$  that means when  $m = 0$ , then the limit is 0, but when we are approaching when  $y = x$  this straight line that is  $m = 1$ .

In that case, the limiting is  $\frac{m}{m^2+1}$  that is half, but when we are approaching via  $y = 0$  that is we are approaching by  $x$  axis then our limiting value and in this case  $m = 0$ . So, our limiting value is also 0 as you can see, as we have calculated it, so for different values of  $x$  you will get different limiting value. That is why the limit does not exist. So, for this function, this limit does not exist. Thank you.

