

IIT Madras
ONLINE DEGREE

Mathematics for Data Science 2
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Week 11 - Tutorial 06

Hello everyone. So, in this video let us try to take another example and try to find this critical point.

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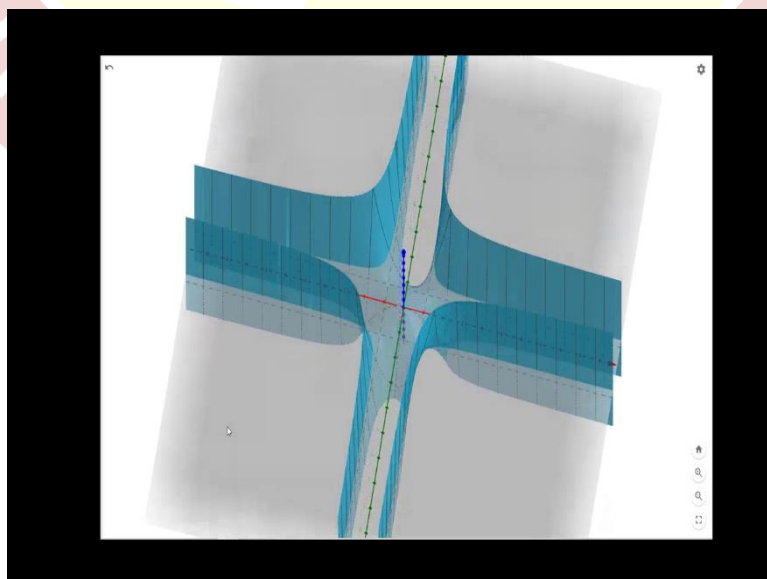
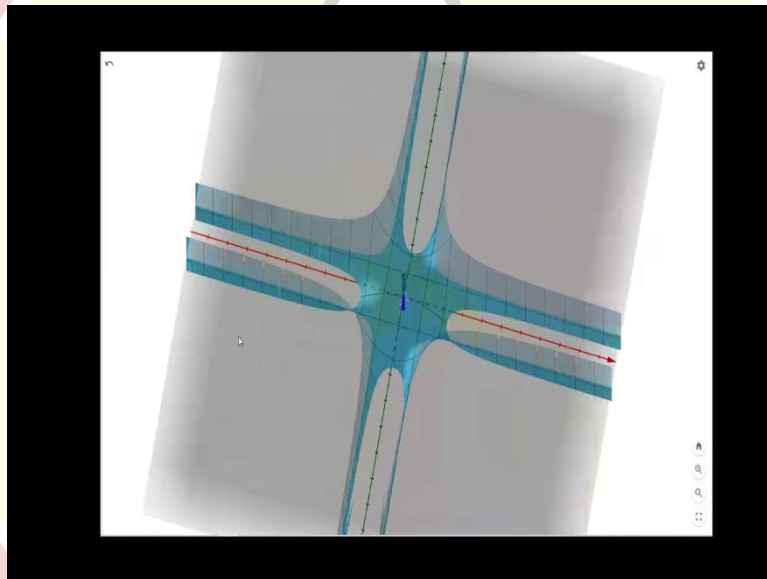
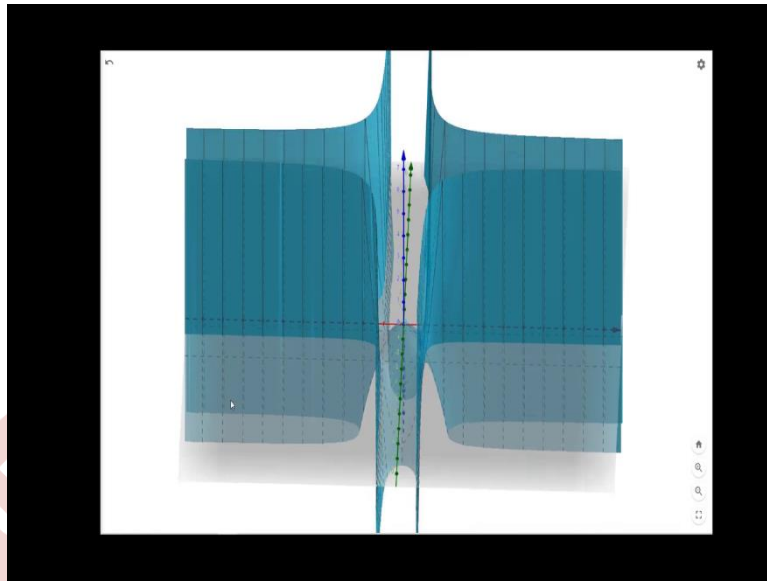
$$\begin{aligned}f(x, y) &= x^2y^2 - x^2 - y^2 \\f_x(x, y) &= 2xy^2 - 2x = 0 \\f_y(x, y) &= 2x^2y - 2y = 0 \\2x(y^2 - 1) &= 0 \\2y(x^2 - 1) &= 0 \\(0, 0) &\text{ maximal} \\(1, 1), (1, -1), (-1, 1), (-1, -1) &\text{ Saddle points.}\end{aligned}$$

So, let us take this example $f(x, y) = x^2y^2 - x^2 - y^2$. So, what we have to do, we have to find the f_x and f_y first. So, $f_x(x, y) = 2xy^2 - 2x$, $f_y(x, y) = 2x^2y - 2y$. Now, if I equate both of them to be 0, 0, so I will get two equations. The first equation will give me $2x(y^2 - 1) = 0$, $2y(x^2 - 1) = 0$.

So, I get these two equations and I had to find all those points which satisfy these two equations simultaneously. So, if you solve this, you will get $(0, 0)$ is one of the points as you can see directly, if I put 0 in x and y , in place of x and y , these equations are satisfied and all the other points are $(0, 0)$, $(1, 1)$, $(1, -1)$, $(-1, 1)$, $(-1, -1)$. So, there are total 5 critical points as you can see.

Now, we will go to Geogebra to find out whether these points are maxima-minima, saddle point. So, when we go to Geogebra, we will find out that this $(0, 0)$ is a local maxima, this will give us a local maxima and all are, these are the 4 points these are basically saddle points. So, let us go to Geogebra and verify this.

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Now, let us try to visualize the graph of the function $f(x, y) = x^2y^2 - x^2 - y^2$. So, this is the graph of the function. So, we can clearly see, so if I invert it, you can see that it is also passing through origin because when x and y both are 0, the function value is also 0.

So, at origin it is giving a local maxima and you can see there are the 4 points $(1, 1)$, $(1, -1)$, $(-1, 1)$, $(-1, -1)$, these 4 points are on this surface and that will be the saddle points. Thank you.

