Week-4
Mathematics for Data Science - 2
Vectors and Matrices
Assignment

1 Multiple Choice Questions (MCQ)

1. Match the matrices in the column A with the properties of those in column B. (MCQ)

	Matrix (Column A)		Properties of matrix (Column B)
a)	$\begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 7 \end{bmatrix}$	i)	has determinant 0
b)	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	ii)	is a scalar matrix
c)	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	iii)	is a lower triangular matrix but not a diagonal matrix
d)	$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$	iv)	is a diagonal matrix but not a scalar matrix

Table: W4AT1

- ○ Option 1: a) → i), b) → ii), c) → iii), d) → iv)
- ○ Option 2: a) → ii), b) → i), c) → iv), d) → iii)

○ Option 3: a) → iii), b) → iv), c) → i), d) → ii)

○ **Option 4:** a) → iii), b) → i), c) → iv), d) → ii)

Solⁿ: a) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 7 \end{bmatrix}$ → all the elements above the diagonal are 0. Hence it is a lower triangular matrix. There are non-zero elements below the diagonal too. Hence it is not a diagonal matrix.

So, (a) → (iii)

b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Hence the determinant is 0.

So, (b) → (i)

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ → The non-zero elements are only on the diagonal, moreover, they are not the same.

So, it is a diagonal matrix, but not a scalar matrix. So, (c) → (iv)

d) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ → It is a scalar matrix.

So, (d) → (ii)

Answer: option (4) (a) → (iii), (b) → (i), (c) → (iv), (d) → (ii).

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2. Match the systems of linear equations in Column A with their number of solutions in column B and their geometric representation in Column C.

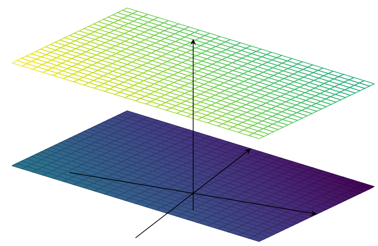
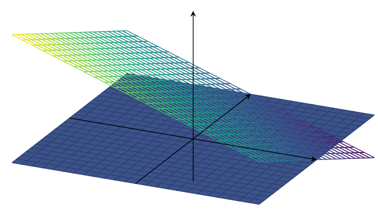
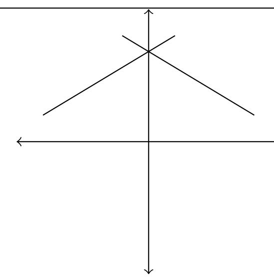
	System of linear equations (Column A)		Number of solutions (Column B)		Geometric representations (Column C)
i)	$x + y = 3$, $x - y = -3$	a)	Infinite solutions	1)	
ii)	$x + y + z = 1$, $x + y + z = 7$	b)	Unique solution	2)	
iii)	$z = 0$, $x + y + z = 1$	c)	No solution	3)	

Table: W4AT2

- ◯ Option 1: i) $\to$ a) $\to$ 3); ii) $\to$ c) $\to$ 1); iii) $\to$ b) $\to$ 2)
- ◯ **Option 2:** i) $\to$ b) $\to$ 3); ii) $\to$ c) $\to$ 1); iii) $\to$ a) $\to$ 2)
- ◯ Option 3: i) $\to$ b) $\to$ 3); ii) $\to$ c) $\to$ 2); iii) $\to$ a) $\to$ 1)
- ◯ Option 4: i) $\to$ a) $\to$ 3); ii) $\to$ c) $\to$ 1); iii) $\to$ b) $\to$ 2)

Soln (i) $\left.\begin{array}{l} x+y=3 \\ x-y=-3 \end{array}\right\}$ This system of linear equations involves only two variable.

Add the 1st one with the 2nd, to get: $2x = 0$
$\Rightarrow x = 0$

Substituting $x=0$ in the 1st one, we get: $y = 3$

Hence, there is a unique solution.

In $\mathbb{R}^2$ both the equations represent straight lines, and they intersect at the point $(0, 3)$ which is on Y-axis.

So, i) $\longrightarrow$ b) $\longrightarrow$ 3)

(ii) $\left.\begin{array}{l} x+y+z=1 \\ x+y+z=7 \end{array}\right\}$ If there exists some solution (a,b,c) of this system of linear equations then the point (a,b,c) should lie on both the planes.

Which gives us $a+b+c=1$ and $a+b+c=7$
$\Rightarrow 1 = 7$ which is absurd.

Hence, there cannot exist any such point.
So, the system of linear equations has no soln.

Observe that: the above system of linear equations involve three variables. They represent planes on the co-ordinate system.

$ax+by+cz=d$ represents a plane on the co-ordinate system.

let $\left.\begin{array}{l} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \end{array}\right\}$ be the two linear equations.

Both of them represent planes.

Now if, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} \neq \dfrac{d_1}{d_2}$

then the planes represented by them must be parallel (distinct) to each other.

ii) $\longrightarrow$ c) $\longrightarrow$ 1)

iii) $\left.\begin{array}{l} z = 0 \\ x+y+z=1 \end{array}\right\}$ $z=0$ denotes the xy-plane.
$x+y+z=1$ denotes a plane on the coordinate system.

Substituting $z=0$ in the second equation we get,

$$x+y=1$$
$$\Rightarrow y = 1-x.$$

Hence any point of the form $(a, 1-a, 0)$ will satisfy both the equations.

So, the system of linear equations has infinitely many solutions.

iii) $\longrightarrow$ a) $\longrightarrow$ 2)

Answer: Option 2 : i) $\rightarrow$ b) $\rightarrow$ 3), ii) $\rightarrow$ c) $\rightarrow$ 1), iii) $\rightarrow$ a) $\rightarrow$ 2)

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2 Multiple Select Questions (MSQ):

3. Let $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}$. Which of the following options are true for a matrix A, such that $AB = C$? (MSQ)

- ◯ Such a matrix does not exist.
- ◯ There is a unique matrix A satisfying this property.
- ◯ **There are infinitely many such matrices.**
- ◯ **A should be a 2×3 matrix.**
- ◯ A should be a 3×2 matrix.

Soln: Let A be a $m \times n$ matrix and B be a $n \times p$ matrix.

AB must be a $m \times p$ matrix.

Here, B is a 3×2 matrix, i.e., $n = 3$, $p = 2$

$C = AB$ is a 2×2 matrix, i.e., $m = 2$, $p = 2$

Hence, A must be a 2×3 matrix.

Let us take an arbitrary 2×3 matrix A and try to see the conditions on the elements of A for which $AB = C$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} - a_{13} & a_{12} \\ a_{21} - a_{23} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix} = C$$

comparing the elements we get,

$a_{11} - a_{13} = 0$ $\Rightarrow a_{11} = a_{13}$	$a_{12} = 0$
$a_{21} - a_{23} = -1$ $\Rightarrow a_{21} = a_{23} - 1$	$a_{22} = 2$

Hence, $A = \begin{bmatrix} a_{11} & 0 & a_{11} \\ a_{23}-1 & 2 & a_{23} \end{bmatrix}$

where, a_{11} and a_{23} can take any arbitrary real number.

So, there are infinitely many such matrices A, such that $AB = C$

<u>Answer:</u> option 3: There are infinitely many such matrices.

option 4: A should be a 2×3 matrix.

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4. Let A be a 2×2 real matrix and let $trace(A)$ denote the sum of the elements in the diagonal of A. Which of the following is true? (MSQ)

   - ○ $det(A - cI)$ is a polynomial in c of degree 1.
   - ○ $\boldsymbol{det(A - cI)}$ **is a polynomial in** $\boldsymbol{c}$ **of degree 2.**
   - ○ $\boldsymbol{det(A - cI) = c^2 - trace(A)c + det(A)}$
   - ○ $det(A - cI) = c^2 + trace(A)c - det(A)$
   - ○ $det(A - cI) = trace(A)c - det(A)$
   - ○ $det(A - cI) = -trace(A)c + det(A)$

Soln. Let us choose an arbitrary 2×2 real matrix A as follows:

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$trace(A) = p + s \quad , \quad det(A) = ps - qr$$

$$A - cI = \begin{bmatrix} p & q \\ r & s \end{bmatrix} - c\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} - \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} p-c & q \\ r & s-c \end{bmatrix}$$

$$det(A - cI) = (p-c)(s-c) - qr$$

$$= ps - cs - cp + c^2 - qr$$

$$= c^2 - c(p+s) + (ps - qr)$$

$$= c^2 - trace(A) \cdot c + det\,A$$

Answer: Option 2: $det(A - cI)$ is a polynomial in c of degree 2.

Option 3: $det(A - cI) = c^2 - trace(A) \cdot c + det(A)$

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5. Suppose there are two types of oranges and two types of bananas available in the market. Suppose 1 kg of each type of orange costs ₹50 and 1 kg of each type of banana costs ₹40. Gargi bought x kg of first type of each fruit, orange and banana, and y kg of second type of each fruit, orange and banana. She paid ₹250 for oranges and ₹200 for bananas. Which of the following options are correct with respect to the given information? (MSQ)

   ○ **Option 1:** The matrix representation to find x and y can be

   $$\begin{bmatrix} 50 & 50 \\ 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 250 \\ 200 \end{bmatrix}$$

   ○ Option 2: The matrix representation to find x and y can be

   $$\begin{bmatrix} 50 & 40 \\ 50 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 250 \\ 200 \end{bmatrix}$$

   ○ **Option 3:** The matrix representation to find x and y can be

   $$\begin{bmatrix} 40 & 40 \\ 50 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 250 \end{bmatrix}$$

   ○ **Option 4:** x can be 2 and y can be 3.

   ○ **Option 5:** There are infinitely many real values possible for x and y.

   ○ Option 6: There are only finitely many real values possible for x and y.

   ○ **Option 7:** There are only finitely many natural numbers possible for x and y.

Soln.

Orange type 1 cost: 50

Orange type 2 cost: 50

Banana type 1 cost: 40

Banana type 2 cost: 40

Gargi bought

orange type 1 : x kg

Banana type 1 : x kg

orange type 2 : y kg

Banana type 2 : y kg

Gargi paid :

For oranges ₹ 250

For Bananas ₹ 200

Hence, $50x + 50y = 250 \longrightarrow (1)$

$40x + 40y = 200 \longrightarrow (2)$

It can be represented by

$$\begin{bmatrix} 50 & 50 \\ 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 250 \\ 200 \end{bmatrix}$$

If we interchange the order of the equations we get,

$$40x + 40y = 200$$
$$50x + 50y = 250.$$

It can be represented by

$$\begin{bmatrix} 40 & 40 \\ 50 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 250 \end{bmatrix}$$

From (1) we get, $50(x + y) = 250$

$\Rightarrow x + y = 5$

From (2) we get, $40(x + y) = 200$

$\Rightarrow x + y = 5.$

Hence, the solution of the system of linear equations is $(a, 5-a)$, where a can take any arbitrary real number.

But in this context, both of them have to be positive.

$a \geq 0, \quad 5 - a \geq 0$

$\Rightarrow 5 \geq a$

$\Rightarrow \underline{0 \leq a \leq 5}$

Hence, a can be any arbitrary real number in between 0 and 5.

Clearly, $x = 2$, $y = 3$ can be a solution.

clearly, there are infinitely many real values possible for x and y.

As the solutions are of the form $(a,\ 5-a)$ and $0 \leq a \leq 5$,

there are finitely many natural numbers possible as solutions.

those are,

$$\left.\begin{array}{l} x=0,\ y=5 \\ x=1,\ y=4 \\ x=2,\ y=3 \\ x=3,\ y=2 \\ x=4,\ y=1 \\ x=5,\ y=0 \end{array}\right\}$$

Answer: option 1: The matrix representation to find x and y can be,

$$\begin{bmatrix} 50 & 50 \\ 40 & 40 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 250 \\ 200 \end{bmatrix}$$

option 3: The matrix representation to find x and y can be

$$\begin{bmatrix} 40 & 40 \\ 50 & 50 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 250 \end{bmatrix}$$

option 4: x can be 2, y can be 3

option 5: there are infinitely many real values possible for x and y.

option 7: There are only finitely many natural numbers possible for x and y.

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3 Numerical Answer Type (NAT):

6. Suppose $det(3A) = n \times det(A)$ for any 3×3 real matrix A. What is the value of n? (NAT) [Answer: 27]

Soln. If any real number c is multiplied with a row of a $p \times p$ matrix then the determinant of the new matrix will be c times the determinant of the earlier matrix.

Now, cA means c is multiplied with all the elements of matrix A.

If A is a $p \times p$ matrix, then there are p rows.

Hence, $\det(cA) = c^p \det(A)$

Here, $c = 3$ and $p = 3$, so, $\det(3A) = 3^3 \det(A) = 27 \det(A)$

Answer: $n = 27$

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7. Suppose $A = \begin{bmatrix} 2019 & 100 & 2119 \\ 2020 & 200 & 2220 \\ 2021 & 300 & 2321 \end{bmatrix}$. What will be the value of $det(A)$?

(NAT) [Answer: 0]

Soln:

$$\begin{vmatrix} 2019 & 100 & 2119 \\ 2020 & 200 & 2220 \\ 2021 & 300 & 2321 \end{vmatrix} \xrightarrow{R_3 - R_2} \begin{vmatrix} 2019 & 100 & 2119 \\ 2020 & 200 & 2220 \\ 1 & 100 & 101 \end{vmatrix}$$

$$\Big\downarrow R_2 - R_1$$

$$\begin{vmatrix} 2019 & 100 & 2119 \\ 1 & 100 & 101 \\ 1 & 100 & 101 \end{vmatrix}$$

Recall: Row operation: adding scalar multiple of one row with other does not change the determinant of a matrix.

As the two rows of the matrix becomes identical, the determinant will be 0.

Answer: $\det(A) = 0$

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4 Comprehension Type Question:

Suppose there are three families F_1, F_2, F_3 living in different cities and they pay ₹x_1, ₹x_2, ₹x_3 per unit respectively for electric consumption in each month. In January 2021, the electric consumption by F_1, F_2, and F_3 are 30 units, 20 units, and 25 units, respectively. In February 2021, it is 20 units, 35 units, and 25 units, respectively. In March 2021, it is 20 units, 10 units, and 15 units, respectively. The total amount paid by the three families together for the electricity consumption in January, February, and March are ₹670, ₹730, and ₹400 respectively.

Answer the following questions using this given data.

8. If we want to find x_1, x_2, x_3 by solving a system of linear equations represented by the matrix form $Ax = b$, where $x = (x_1, x_2, x_3)^T$, then which of the following options is correct? (MCQ)

○ Option A : $A = \begin{bmatrix} 30 & 20 & 20 \\ 20 & 35 & 10 \\ 25 & 25 & 15 \end{bmatrix}$ and $b = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$

○ **Option B :** $A = \begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix}$ and $b = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$

○ Option C : $A = \begin{bmatrix} 30 & 35 & 15 \\ 20 & 20 & 25 \\ 20 & 10 & 25 \end{bmatrix}$ and $b = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$

○ Option D : $A = \begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix}$ and $b = \begin{bmatrix} 400 \\ 730 \\ 670 \end{bmatrix}$

Solution :

Month	electric consumption			Total payment by F_1, F_2, F_3 together
	F_1	F_2	F_3	
In Jan 2021	30	20	25	670
In Feb 2021	20	35	25	730
In March 2021	20	10	15	400.

F_1 pays ₹ x_1 per unit

F_2 pays ₹ x_2 per unit

F_3 pays ₹ x_3 per unit.

Hence we have,

$$\left.\begin{aligned} 30x_1 + 20x_2 + 25x_3 &= 670 \\ 20x_1 + 35x_2 + 25x_3 &= 730 \\ 20x_1 + 10x_2 + 15x_3 &= 400 \end{aligned}\right\}$$

Matrix representation:

$$\begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$$

$$Ax = b.$$

Answer: option B: $A = \begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix}$ and $b = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$

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9. Which of the following is the possible solution of $Ax = b$, where $x = (x_1, x_2, x_3)^T$? (MCQ)

○ Option A: $x = \begin{bmatrix} 8 \\ 9 \\ 10 \end{bmatrix}$

○ Option B: $x = \begin{bmatrix} 9 \\ 8 \\ 10 \end{bmatrix}$

○ **Option C:** $x = \begin{bmatrix} 9 \\ 10 \\ 8 \end{bmatrix}$

○ Option D: $x = \begin{bmatrix} 8 \\ 10 \\ 9 \end{bmatrix}$

Soln We have the matrix representation of the system of linear equations.

$$\begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 670 \\ 730 \\ 400 \end{bmatrix}$$

The system of linear equations is as follows:

$$30x_1 + 20x_2 + 25x_3 = 670 \quad \text{---} (1)$$
$$20x_1 + 35x_2 + 25x_3 = 730 \quad \text{---} (2)$$
$$20x_1 + 10x_2 + 15x_3 = 400 \quad \text{---} (3)$$

(2) − (1) gives: $-10x_1 + 15x_2 = 60$

$$\Rightarrow -2x_1 + 3x_2 = 12 \quad \text{---} (4)$$

6 × (2) − 10 × (3) gives:

$$\begin{array}{rrrr} 120x_1 & + 210x_2 & + 150x_3 & = 4380 \\ 200x_1 & + 100x_2 & + 150x_3 & = 4000 \\ (-) & (-) & (-) & (-) \\ \hline -80x_1 & + 110x_2 & & = 380 \end{array}$$

$$\Rightarrow -8x_1 + 11x_2 = 38 \quad \text{---} (5)$$

$4 \times (4) - (5)$ gives:

$$
\begin{array}{rcl}
-8x_1 + 12x_2 &=& 48 \\
-8x_1 + 11x_2 &=& 38 \\
(+) \quad (-) & & (-) \\
\hline
x_2 &=& 10
\end{array}
$$

Substituting the value of x_2 in (4) we get,

$$-2x_1 + 3(10) = 12$$

$$\Rightarrow -2x_1 + 30 = 12$$

$$\Rightarrow -2x_1 = -18$$

$$\Rightarrow x_1 = 9$$

Substituting the values of x_1 ad x_2 in (1) we get,

$$30(9) + 20(10) + 25x_3 = 670$$

$$\Rightarrow 270 + 200 + 25x_3 = 670$$

$$\Rightarrow 470 + 25x_3 = 670$$

$$\Rightarrow 25x_3 = 670 - 470 = 200$$

$$\Rightarrow x_3 = 8 .$$

Hence, $x_1 = 9, \quad x_2 = 10, \quad x_3 = 8 .$

<u>Answer:</u> $x = \begin{bmatrix} 9 \\ 10 \\ 8 \end{bmatrix}$

10. Which of the following is(are) correct? (MSQ)

○ **Option A:**

$$det(A) = 30 \times det\left(\begin{bmatrix} 35 & 25 \\ 10 & 15 \end{bmatrix}\right) - 20 \times det\left(\begin{bmatrix} 20 & 25 \\ 20 & 15 \end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix} 20 & 35 \\ 20 & 10 \end{bmatrix}\right)$$

○ **Option B:**

$$det(A) = 30 \times det\left(\begin{bmatrix} 35 & 25 \\ 10 & 15 \end{bmatrix}\right) + 20 \times det\left(\begin{bmatrix} 25 & 20 \\ 15 & 20 \end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix} 20 & 35 \\ 20 & 10 \end{bmatrix}\right)$$

○ **Option C:**

$$det(A) = -20 \times det\left(\begin{bmatrix} 20 & 25 \\ 10 & 15 \end{bmatrix}\right) + 35 \times det\left(\begin{bmatrix} 30 & 25 \\ 20 & 15 \end{bmatrix}\right) - 25 \times det\left(\begin{bmatrix} 30 & 20 \\ 20 & 10 \end{bmatrix}\right)$$

○ Option D:

$$det(A) = 20 \times det\left(\begin{bmatrix} 20 & 25 \\ 10 & 15 \end{bmatrix}\right) - 35 \times det\left(\begin{bmatrix} 30 & 25 \\ 20 & 15 \end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix} 30 & 20 \\ 20 & 10 \end{bmatrix}\right)$$

Soln:

$$A = \begin{bmatrix} 30 & 20 & 25 \\ 20 & 35 & 25 \\ 20 & 10 & 15 \end{bmatrix}$$

$$det(A) = 30 \times det\left(\begin{bmatrix} 35 & 25 \\ 10 & 15 \end{bmatrix}\right) - 20 \times det\left(\begin{bmatrix} 20 & 25 \\ 20 & 15 \end{bmatrix}\right) + 25 \times det\left(\begin{bmatrix} 20 & 35 \\ 20 & 10 \end{bmatrix}\right)$$

$$= 30\,(35 \times 15 - 25 \times 10) - 20\,(20 \times 15 - 25 \times 20) + 25\,(20 \times 10 - 35 \times 20)$$

$$= 30\,(525 - 250) - 20\,(300 - 500) + 25\,(200 - 700)$$

$$= 30\,(275) - 20\,(-200) + 25\,(-500)$$

$$= 8250 + 4000 - 12500$$

$$= 12250 - 12500 = \underline{\underline{-250}}\,.$$

$$det(A) = 30 \times det\left(\begin{bmatrix} 35 & 25 \\ 10 & 15 \end{bmatrix}\right) - 20 \times \underbrace{det\left(\begin{bmatrix} 20 & 25 \\ 20 & 15 \end{bmatrix}\right)}_{\text{Interchanging the column will change the sign of the determinant.}} + 25 \times det\left(\begin{bmatrix} 20 & 35 \\ 20 & 10 \end{bmatrix}\right)$$

Hence, $det\left(\begin{bmatrix} 20 & 25 \\ 20 & 15 \end{bmatrix}\right) = -det\left(\begin{bmatrix} 25 & 20 \\ 15 & 20 \end{bmatrix}\right)$

which gives us the expression in option B.

Option C:

$$-20 \times \det\left(\begin{bmatrix} 20 & 25 \\ 10 & 15 \end{bmatrix}\right) + 35 \times \det\left(\begin{bmatrix} 30 & 25 \\ 20 & 15 \end{bmatrix}\right) - 25 \times \det\left(\begin{bmatrix} 30 & 20 \\ 20 & 10 \end{bmatrix}\right)$$

$$= -20\,(20 \times 15 - 25 \times 10) + 35\,(30 \times 15 - 25 \times 20) - 25\,(30 \times 10 - 20 \times 20)$$

$$= -20\,(300 - 250) + 35\,(450 - 500) - 25\,(300 - 400)$$

$$= -20\,(50) + 35\,(-50) - 25\,(-100)$$

$$= -1000 - 1750 + 2500$$

$$= -2750 + 2500 = -250 = \det(A)$$

Option D:

$$20 \times \det\left(\begin{bmatrix} 20 & 25 \\ 10 & 15 \end{bmatrix}\right) - 35 \times \det\left(\begin{bmatrix} 30 & 25 \\ 20 & 15 \end{bmatrix}\right) + 25 \times \det\left(\begin{bmatrix} 30 & 20 \\ 20 & 10 \end{bmatrix}\right)$$

$$= 20\,(300 - 250) - 35\,(450 - 500) + 25\,(300 - 400)$$

$$= 20\,(50) - 35\,(-50) + 25\,(-100)$$

$$= 1000 + 1750 - 2500$$

$$= 2750 - 2500 = 250 \neq \det(A)$$

Answer: Option A, Option B, and Option C are correct.