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Mathematics for Data Sciences 2
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Directional Derivatives

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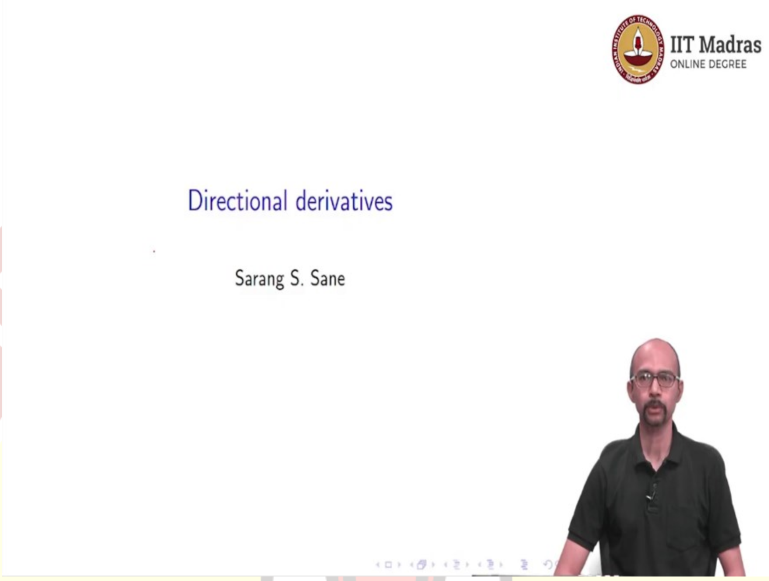

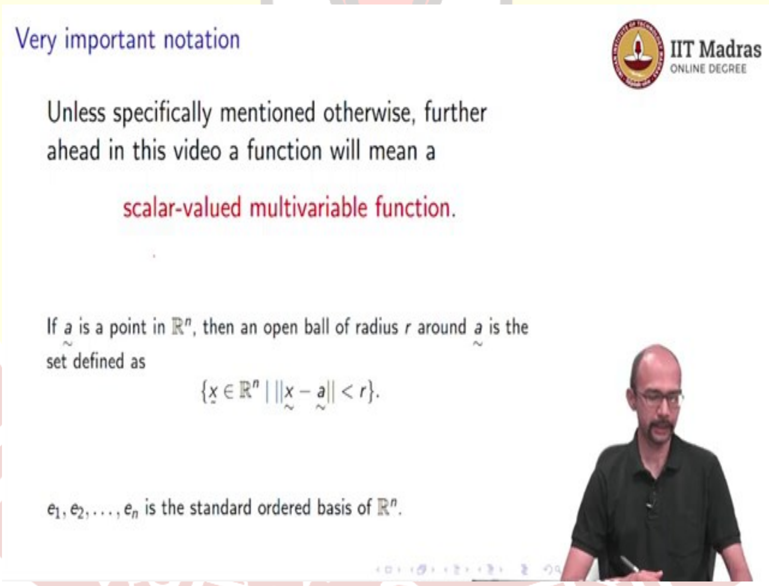

Hello, and welcome to the Maths 2 comp1nt of the Online BSc program on Data Science and Programming. In this video we are going to talk about directional derivatives. So, we have already seen the notion of partial derivatives. Let us recall first some of the notations that we menti1d in the partial derivatives video.

So, unless specifically menti1d otherwise, in this video also a function will mean a scalar-valued multivariable function. If $\binom{a}{\sim}$ is a point in $\mathbb{R}^n$, then open ball of radius is r, around $a\ tilde$ is the set consisting of those points, such that the distance of the point x from a < R.

And finally, even $e_1, e_2, e_3, \dots e_n$ is the standard ordered bases in $\mathbb{R}^n$. So, we made use of the standard ordered bases in Rn in order to define the partial derivatives.

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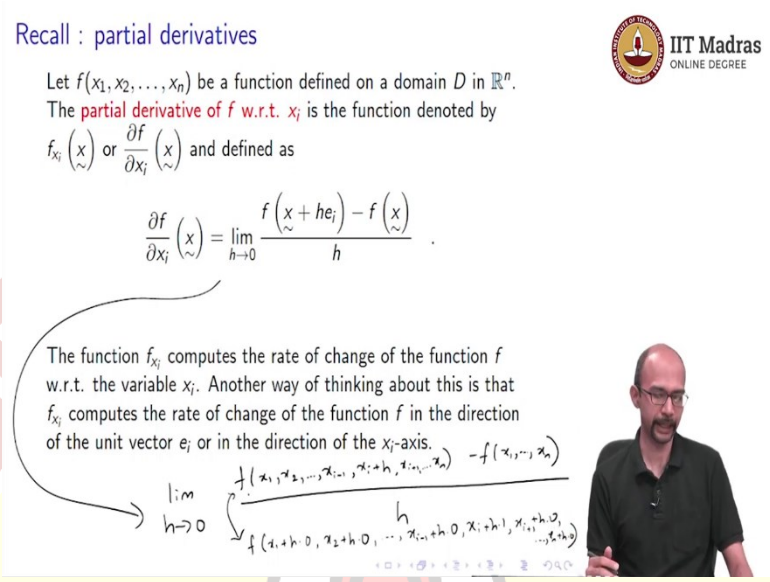

Let us recall the definition. So, if f of $x_1, x_2, x_3, \dots x_n$ is a function defined on some domain D in $\mathbb{R}^n$, the partial derivative of f with respect to the variable x_i is the function denoted $/f_{x_i}$, or $\frac{\partial f}{\partial x_i}$. And how do we define it, $\frac{\partial f}{\partial x_i}(\underset{\sim}{x})$ is the function, which is obtained as $\lim_{h \to 0} \frac{f(\underset{\sim}{x} + he_i) - f(\underset{\sim}{x})}{h}$.

And what did this thing compute? So, the function $f(f_{x_i})$, which is the partial derivative with respect to x_i computes the rate of change of the function f with respect to the variable x_i. This is what it computed. So, essentially what we did was, we said, well, your function is defined on the entire domain D, but now, let us restrict it only on the axes corresponding to x_i.

So, for example, it is a function of 2 variables X and Y, when you want to compute this partial derivative function at a point A, you move your say with respect to X. So, you move your X axis parallelly. So, you look at the parallel line to the X axis passing through the point A, you restrict your function to that, just that line and then you see what happens.

So, it becomes a function of 1 variable, you compute the rate of change at that point with respect to what is happening on that line. So, you can think of the rate of change of the function X, f with respect to x_i, which is a partial derivative. You can think of this as it computes the rate of change of the function f in the direction of the unit vector e_i and e_i is showing up in the expression as well. So, this is the rate of change in that direction, meaning on that line.

So, which line, the line containing ei, but which is. So, e_i, remember as a vector, and now we are moving that vector to start from the point A. So, in the direction of the Xi axis, so the direction is the direction of the Xi axis. So, which means that you are moving this Xi axis parallely, so that it passes through your point a tilde.

To spell it out explicitly, let us recall that we can write this expression as $\lim_{h \to 0} \frac{f(x_1, x_2, x_3, \ldots x_{i-1}, x_{i+h}, \ldots) - f(x_1, x_2, x_3, \ldots x_n)}{h}$.

So, the only change happens in the ith coordinate, all the other coordinates remain the same. This is how this function is, and we can rewrite this as f of so this part e can rewrite as $f(x_{1+h}) \times 0$, $f(x_{2+h}) \times 0$and so on. $f(x_{i-h}) \times 0$, $f(x_{1+h}) \times 1$ and then $f(x_{i+1+h}) \times 0$and then all the way up to $f(x_{n+h}) \times 0$. That is what, this is that is how we get this expression $f(x_1, x_2, x_3, \ldots x_{i-1}, x_{i+h}, \ldots x_n)$.

So now, we can ask, instead of thinking about the rate of change in the direction of 1 of the axes, what if I want to think of the rate of change in some other direction? Yeah, after all, there are many directions. In the X, Y plane, well, you have the X axis, you have the Y axis, and then there are all these other directions. So, you could ask, what about the rate of change in 1 of those directions?

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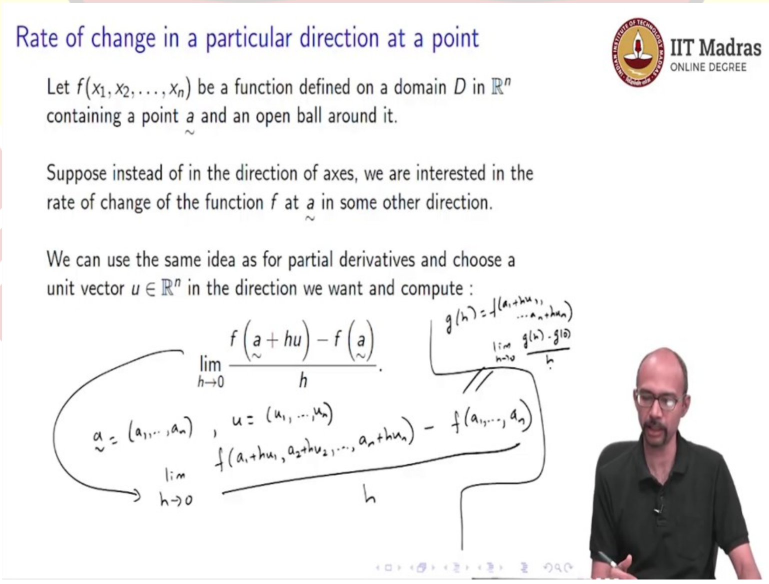

So, let us talk about the rate of change at a point in a particular direction. So, let $f(x_1, x_2, x_3, \ldots x_{i-1}, x_{i+h}, \ldots x_n)$. be a function defined on a domain D in Rn containing a point a tilde and an open ball around it. Suppose instead of the direction of the axis, we are interested in the rate of change of the function in some other direction. So, we can use the same idea as we have for partial derivatives. So, we choose a unit vector.

So, remember that we had chosen the vectors $e_1, e_2, e_3, \dots e_n$ these are remember, that these are unit vectors. So, these have what we call norm 1. Recall this from the linear algebra part, so these have norm 1 or these have length 1. So instead of$e_1, e_2, e_3, \dots e_n$, if you have some other direction, not a direction for the axis then we can take a unit vector in that direction and then we can compute this number $\lim_{h\to 0}\frac{f(\underset{\sim}{a}+hu)-f(\underset{\sim}{a})}{h}$.

So, when we want the direction to be the direction of the axis, that time we use e_i, if you it is some other direction, you choose a unit vector in that direction and then you do the same thing that you did earlier. And if you agree that what the previous expression computed the rate of change in the direction of the axis, so on the lines parallel to the axis, then this will compute the rate of change in the direction of the line passing through this vector u.

So, you take that line passing the vector u, move it parallel, so that it passes through A that will give you some line passing through a, and in the direction of the vector u, and then you restrict your function to that line, so it becomes a function of 1 variable and then we know how to compute the derivative that is what this is.

So, to spell this out explicitly what we are saying is, if $\underset{\sim}{a}$ is $a_1, a_2, a_3, \dots a_n$, those are the coordinates of $\underset{\sim}{a}$, u has coordinates $u_1, u_2, u_3, \dots u_n$ then this what is this expression, this expression is exactly $\lim_{h\to 0}\frac{f((a_1+h)\times u_1.(a_2+h)\times u_2,(a_n+h)\times u_n))-f(x_1,x_2,x_3,\dots x_n)}{h}$.

So, you think of this entire thing as a function of h, and you take its limit. So, how is this a function of 1 variable? So, you think of g of h as this $f(a_1 + h\times u_1)$all the way up to $f(a_n + h\times u_1)$. And then what this translates to is, this translates to a $\lim_{h\to 0}\frac{g(h)-g(0)}{h}$, so this is exactly the derivative of this function g at 0. So, this will, this is the rate of change in that direction as a result. So, this is the same idea that we saw for partial derivatives.

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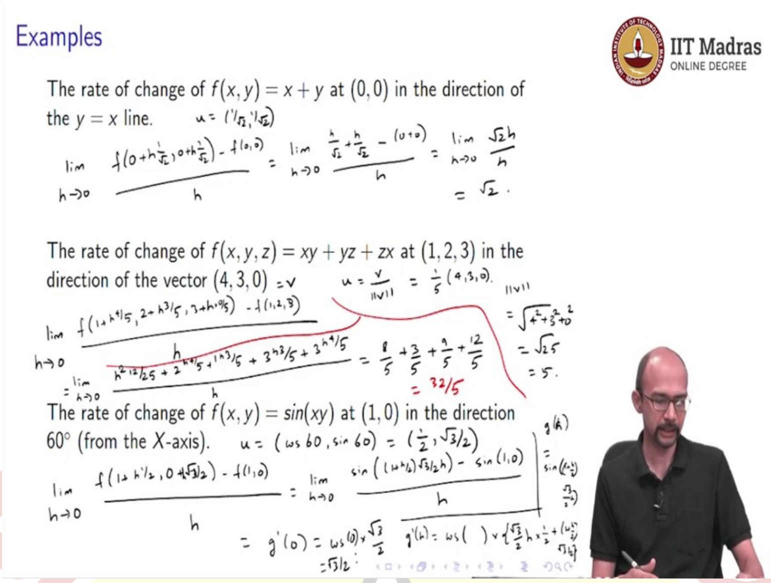

Fine, let us do a couple of examples to be clear about what we are saying. So, let us find the rate of change of f(x) y, which is x + y at 0, 0 in the direction of the y = x line. So, we will apply the definition. So, first of all, we need to understand what is the unit vector. So, for y = x, the unit vector you can check is $1/\sqrt{2}$, $1/\sqrt{2}$.

So, if you prefer in terms of angles, the y = x line makes an angle 45 degrees with respect to the x axis and so, unit vector is going to be cosine 45, sin 45. So, this is exactly what we have. So, now what we want to compute is limit h tends to 0, f of $0 + h \times 1/\sqrt{2}$, , $0 + h \times 1/\sqrt{2}$, - f of 0, 0 /h. So, let us evaluate what this is. So, this is f of $h/\sqrt{2}$, $h/\sqrt{2}$, which is $h/\sqrt{2} + h/\sqrt{2} - 0 + 0$ /h.

So, this is $2 \times h/\sqrt{2}$, so that is $\sqrt{2} \times h$ in the numerator /h, which is exactly, $\sqrt{2}$. So that is the rate of change of this function in the direction of the y = x line. Fine. Let us now do the second 1, so this is not at the origin. So, the rate of change of f(x), y, z which is x, y + y, z + z, x at 1,2,3 in the direction of the vector 4,3,0. So now this vector is given to us, but this is not a unit vector, so we have to first convert it into a unit vector.

So, let us see what the unit vector here is. So, if we call this vector V, so as we know unit vector in the same direction is given /dividing /its norm. So, dividing /its length. So, if we do that, we get 4 , 3 , 0 /norm of V, so let us compute what is norm of V. So, norm of V is $\sqrt{(4^2 + 3^2 + 0^2)}$. This is very conveniently 16 + 9, which is 25, so $\sqrt{25}$ which is 5. So, this is $1/5 \times 4$, 3 , 0, so this is a unit vector u.

So, now what do we have to compute, we want to compute limit h tends to 0, f of $1 + h \times 4/5$ to $+ h \times 3/5$, and $3 + h \times 0/5$ - f of 1,2,3 /h. Let us evaluate what this is. So, $\lim_{h \to 0}$. So, if you substitute these, 1 + h, 4 h /5, 2 , + 3h /5, and 3 into this expression and also substitute 1,2,3 in this expression you will see there is a fair amount of cancellation.

And so, the terms that are left are of, are going to be h^2 × 12 /25, + 2 × h × 4 /5, + 1 × h × 3 /5. So, this is coming from your first x, y term, this is the expression. And then for the other 2 terms you probably have only 1 term each. So, three × h × 3 /5, + 3 × h × 4 /5. And the other terms cancel with f of 1 , 2 , 3.

So, I will suggest that you do this. So, and then /h. Fine. So, what do we get? So, the h cancels from the numerator in the denominator and then in the numerator, the first term, you have an h^2, so that limit will be 0, and the others will survive. So, what you will get is 2 × 4 /5, so 8 /5, + 3 /5, + 9 /5, + 12 /5. And that is what the answer is. So, the answer is going to be 8 + 3,11 + 9, 20 + 12, 32 /5, so this is what the answer will be.

So, I hope this this was a slightly more tedious calculation, but not difficult at all. I will suggest if you did not follow it, as I was doing it, please do it yourself. And finally, here is an example where we specify the direction, so the direction is 60 degrees from the X axis so it is on the x, y plane. So, let us see what is u here. So u here is going to be, so if the angle is Ɵ, then the u will be cosine Ɵ sinƟ.

So here you have cosine of 60, sin of 60. So, if I remember my sins and cosines, correctly, this is half-end √3 /2. And so, now we can plug this in and see what we get. So, the limit h tends to 0, f of 1 + h × half, , 0 + √3 /2 × so h × √3 /2, - f of 1 , 0 /h. So, what is that, limit h tends to 0 sin of 1 + h /2 × 0 + √3 /2 × h - sin of 1 , 0 /h.

So, now we can use our trigonometric formula and expand this and get the answer. And, so maybe an easier way of doing this is to actually use the function g that we defined. So here, g of h is sign of this guy, 1 + h √h /2 √3 /2h. And what this thing is, is really g’ at 0. So, let us evaluate what is g’ of h here. So here, if we evaluate g’ of h, we get, so g’ of h is a cosine of the same thing, multiplied /1 +, so √3 /2h ×, this is 1 + h /2, so × half + 1 + h /2 × √3 /2. This is using the composition root.

And so now if I want g’ of 0, this is going to be y substitute h is 0 in this expression. So, a cosine of the same thing. So, 1 + 0 /2 × √3 /2 × 0, so that gives us just cosine of 0. And then × when I substitute here, the first term does not continue because h is 0. And the second term contributes a √3 /2, so cosine of 0 × √3 /2, so that is just √3 /2. So, that is the rate of change of this function in the direction of 60 degrees from the x axis. So, I hope it is clear how we are computing this, and that these limits are tedious, but not difficult.

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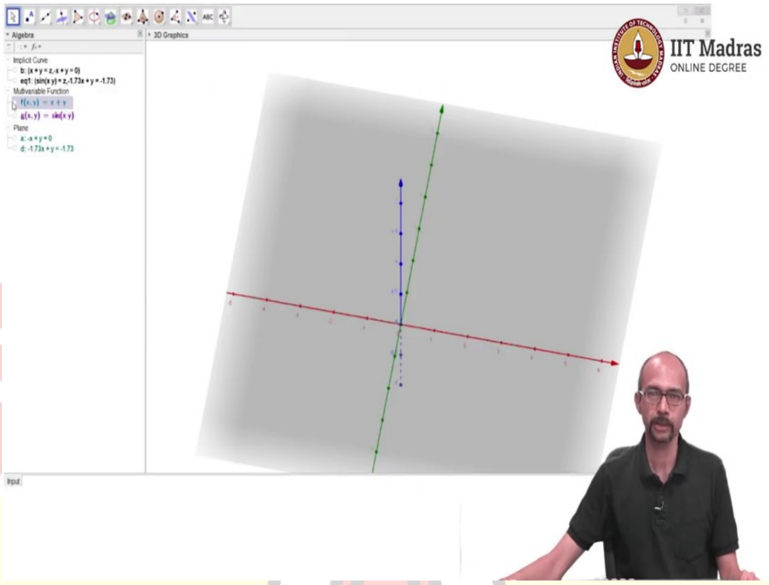

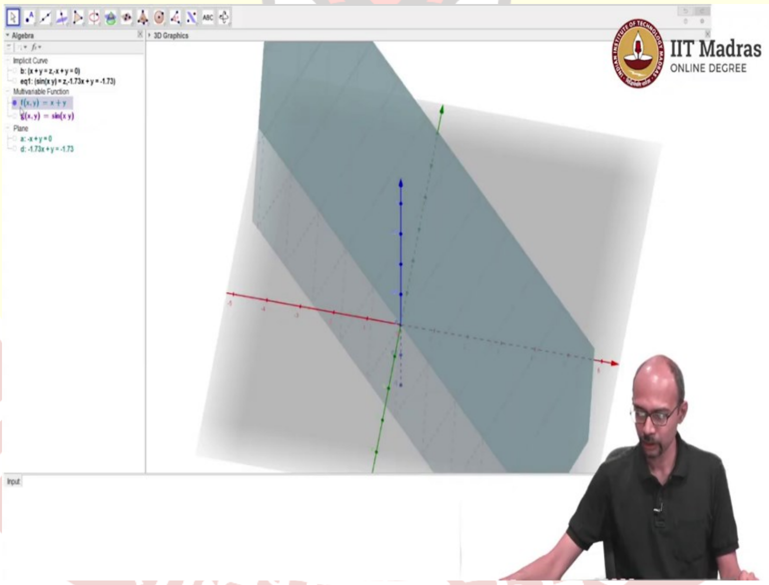

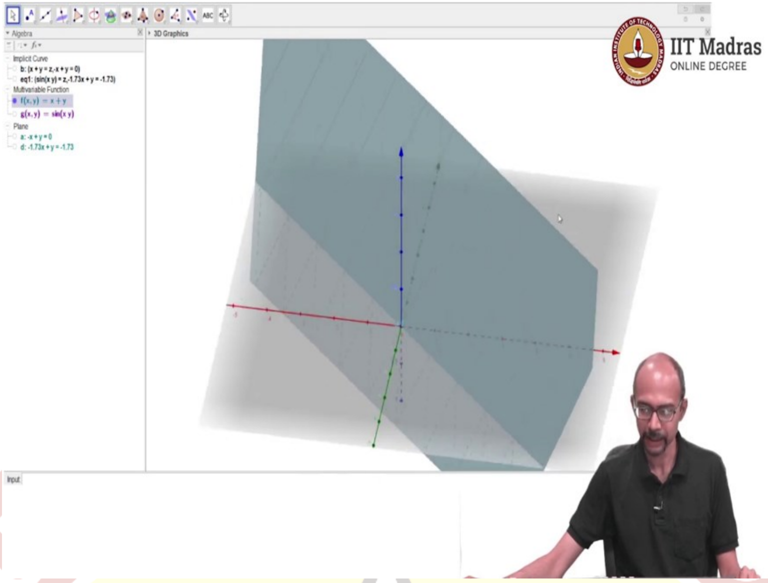
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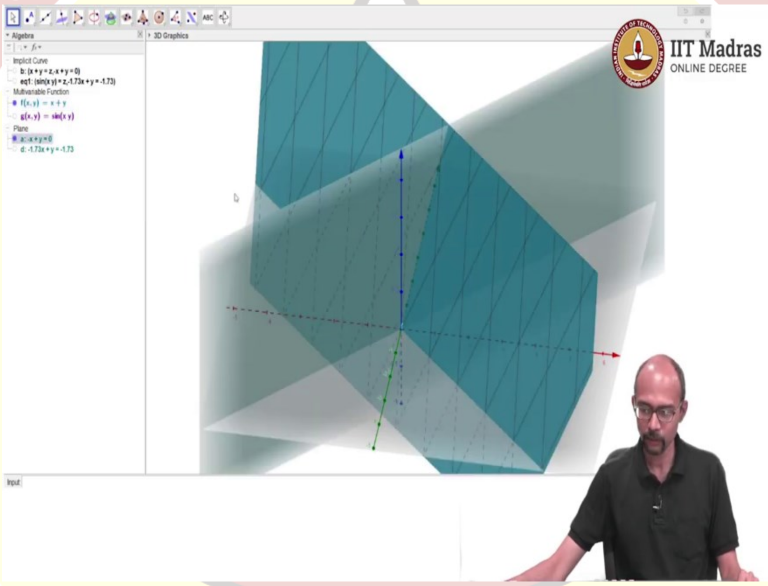
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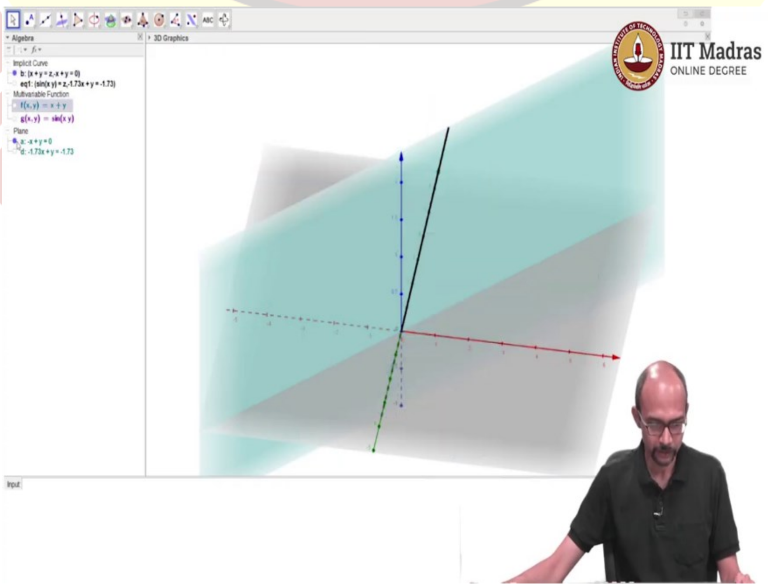
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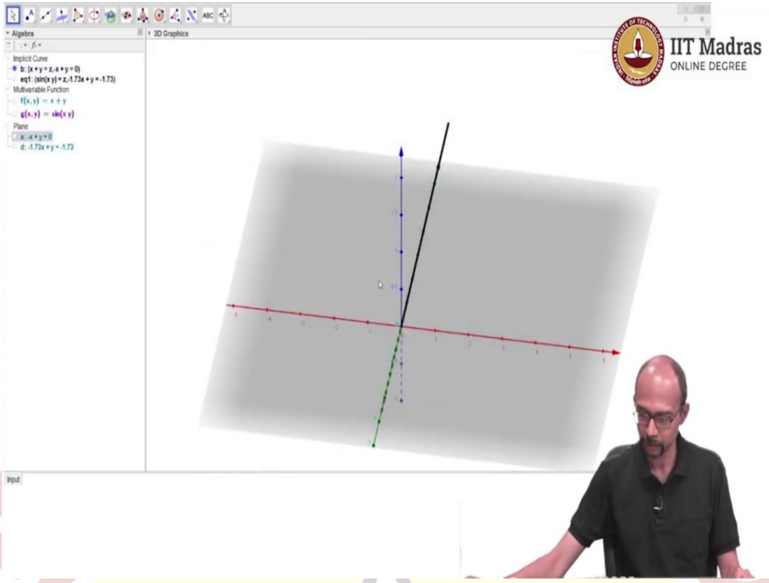
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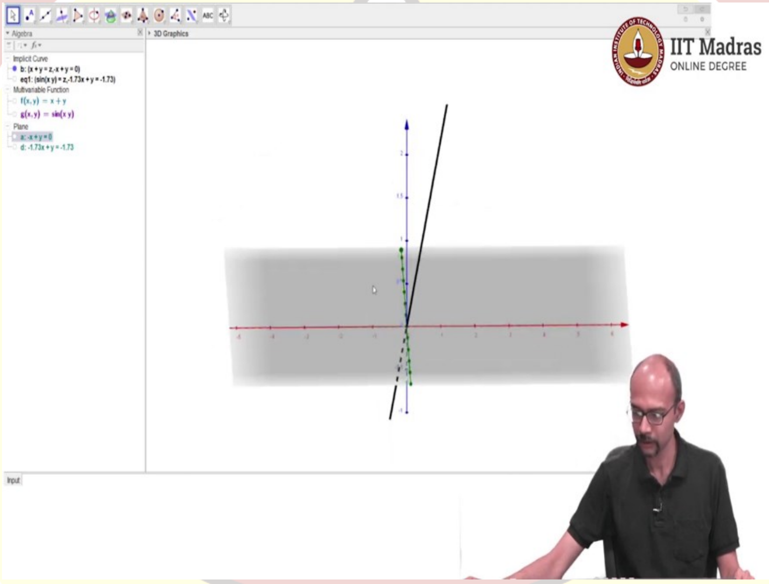
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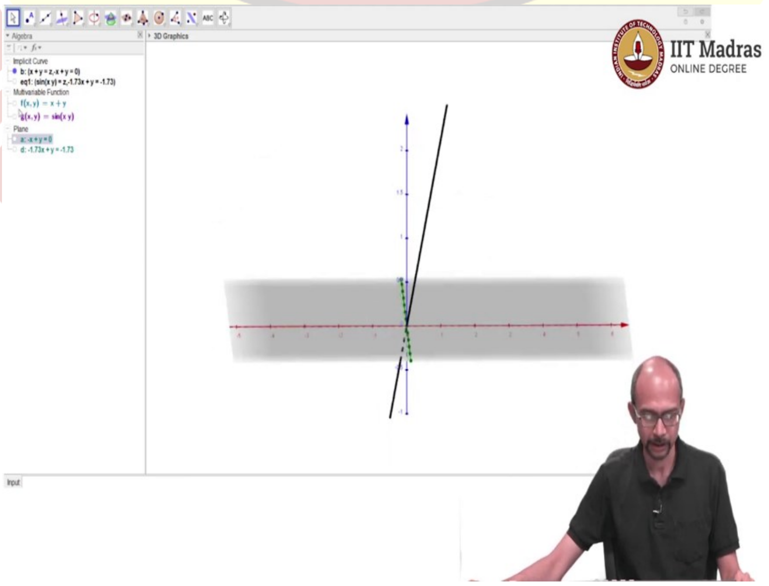
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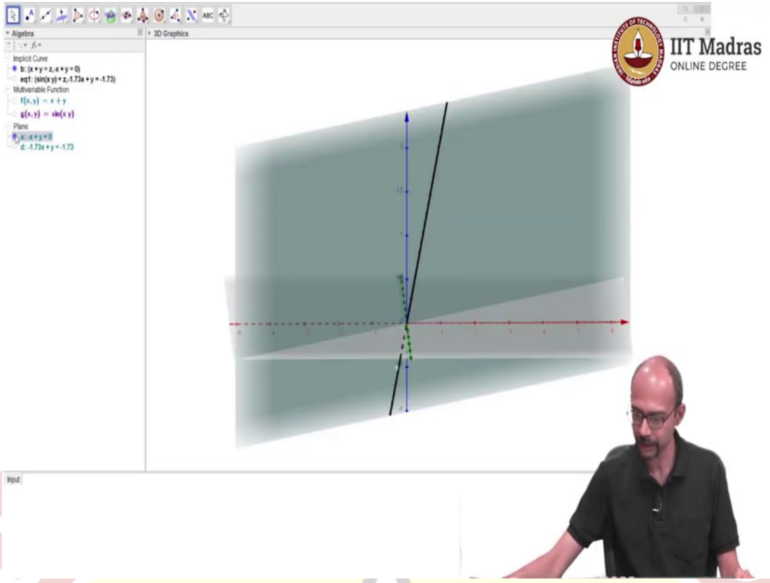

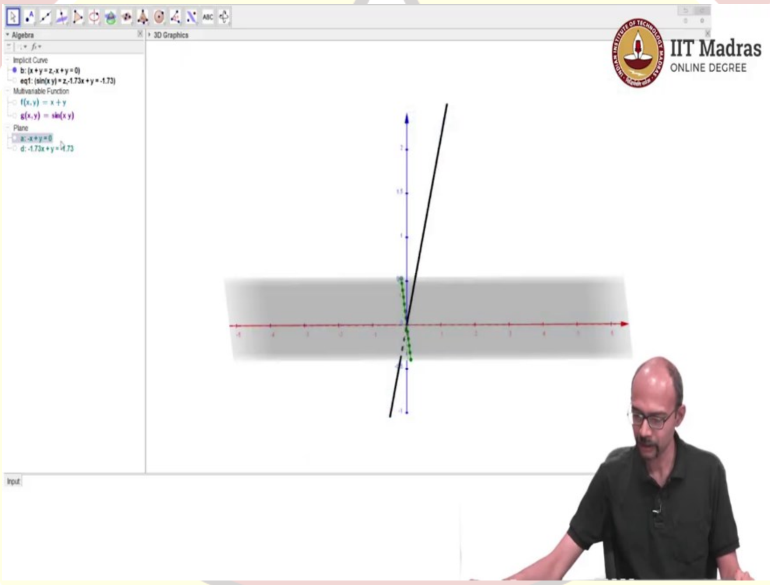

So, let us look at the graphical representation of some of these examples in GeoGebra, and try to see what the directional derivative at these points is computing for us. So, let us look at the first example, which was f(x), y is x + y. So, here is the graph of f(x), y is x + y. So, the red line is the X axis, the green line is the Y axis, and here is how that graph looks like.

So, it is a plane, as we have seen before as well. And we want to look at the line Y = X on the x, y plane and look at the corresponding plane on top. And then once we intersect the graph with that plane, that will tell us how the function behaves or looks like above that line. So, here is the line Y equals X extended into the plane above it.

So, this is the intersection of 2 planes. And so, if we intersect them, we get this line. So, if I remove the graph and the plane, here is how the picture looks like. So, just and once again, draw the plane. So, here is the plane on which that line is. And once I remove it this is how the graph of the function on the line or above the line Y = X looks like. So, what is the directional derivative telling us? It is telling us the slope of this line.

So, if you take the angle of this line with the line Y = X, it is telling us something about the angle there, so tan of that angle, that is exactly what the directional derivative is computing. And you can try and compute it and see that what we have d1 is exactly that computation.

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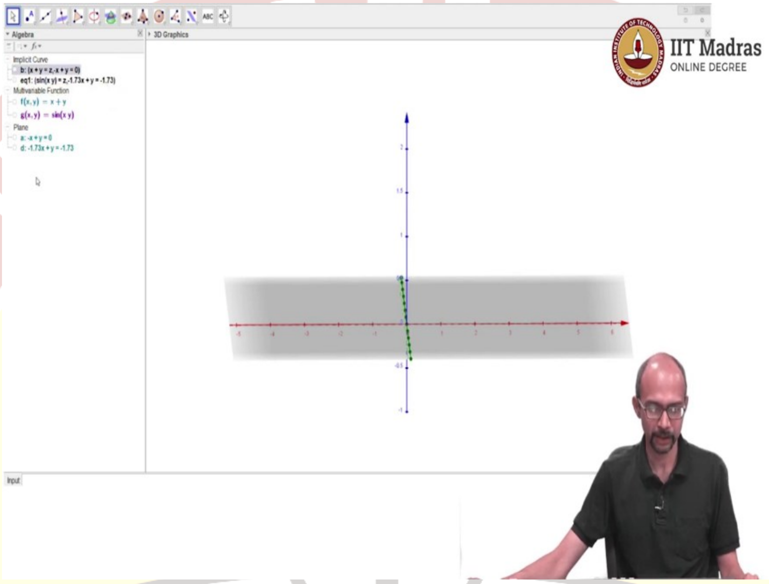

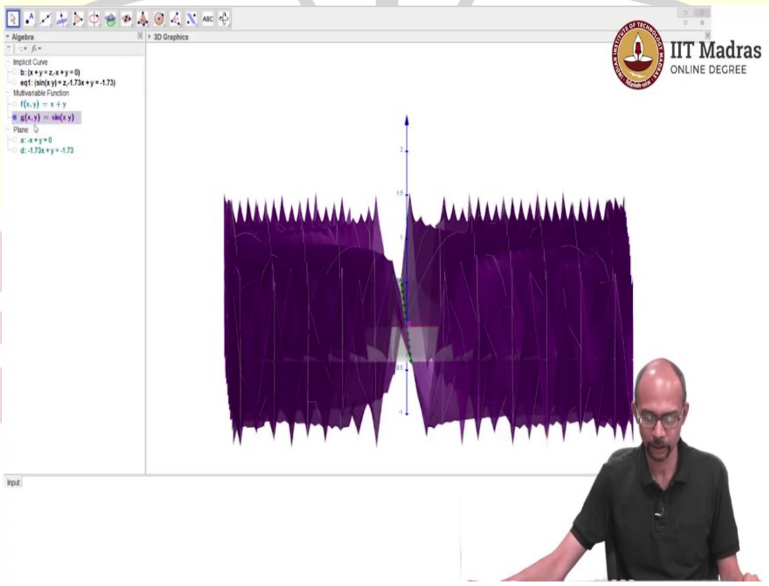

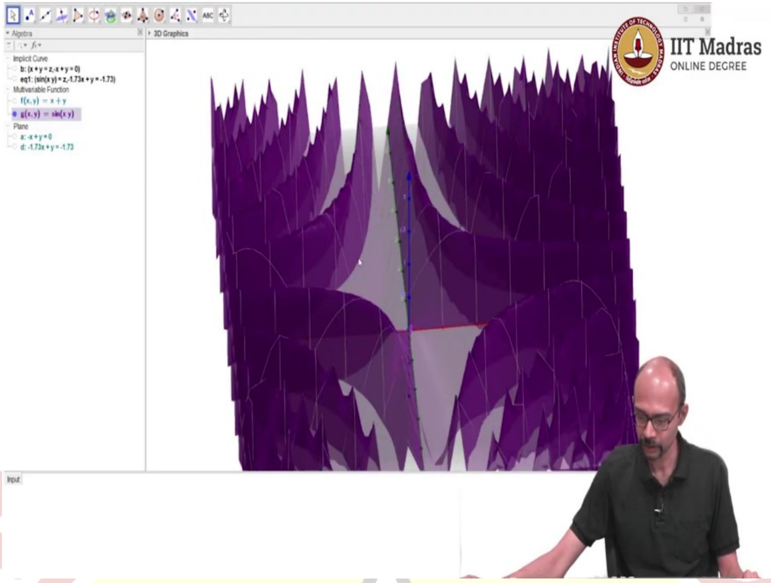
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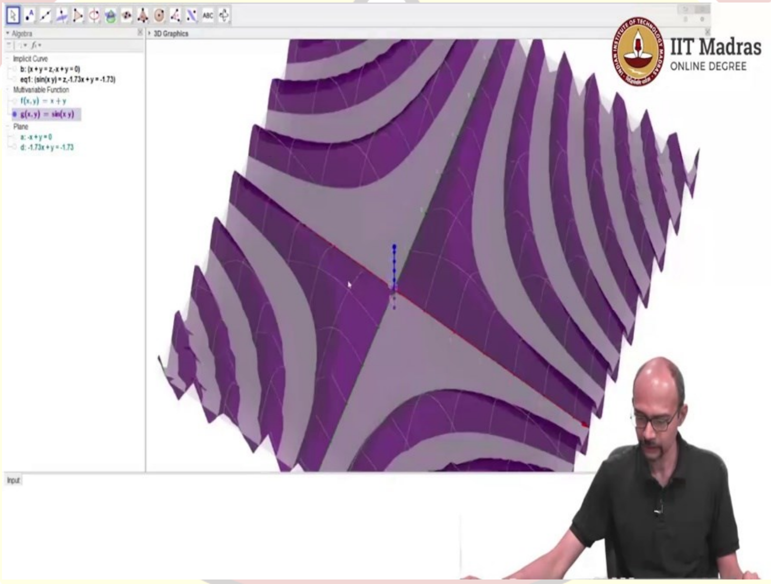
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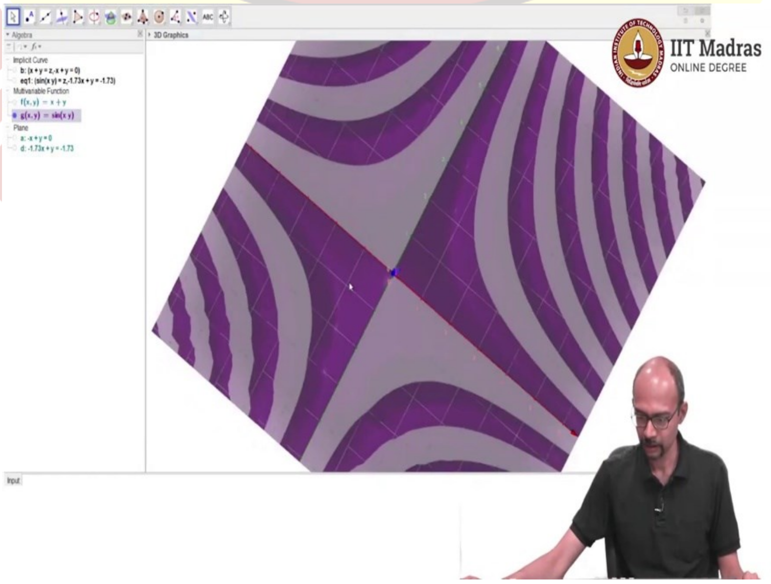
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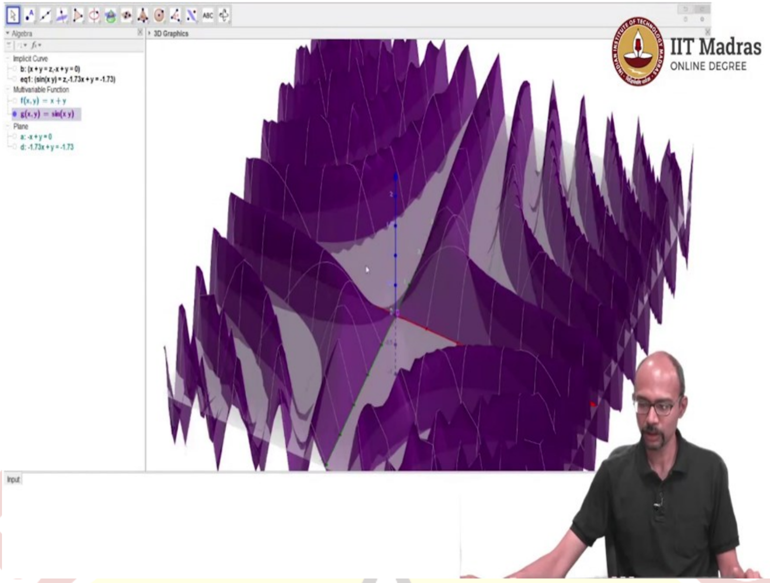
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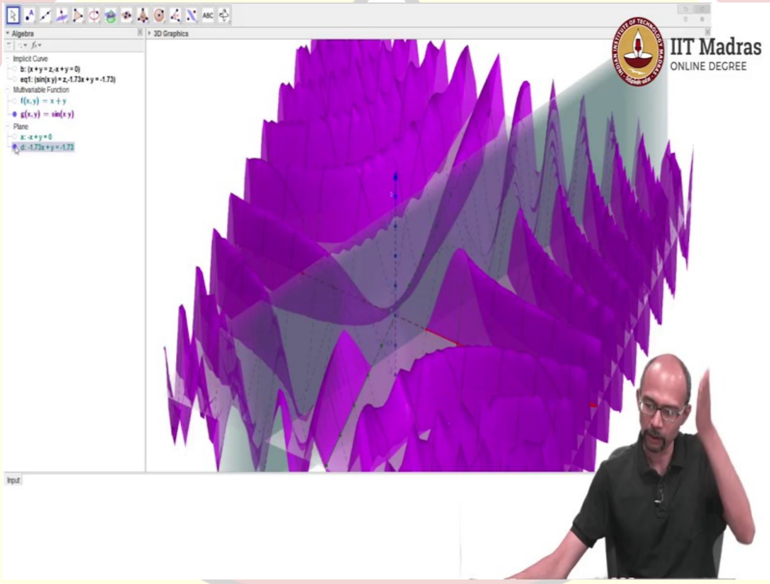
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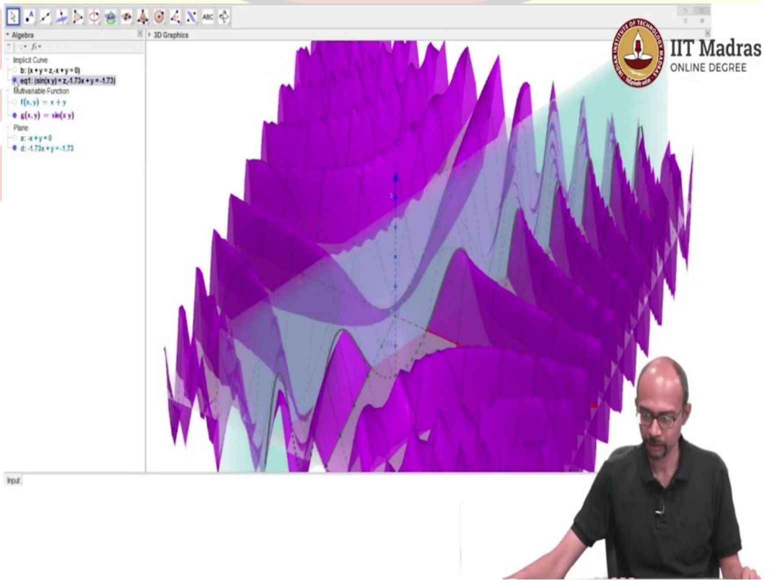
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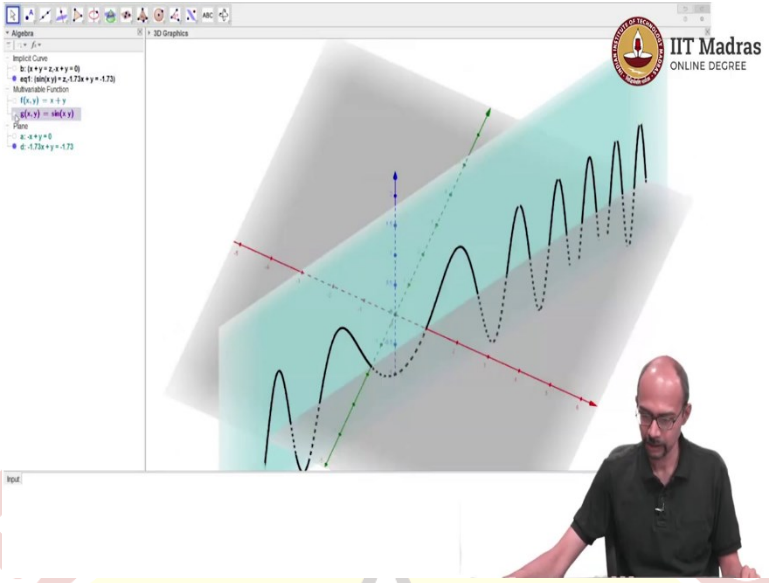
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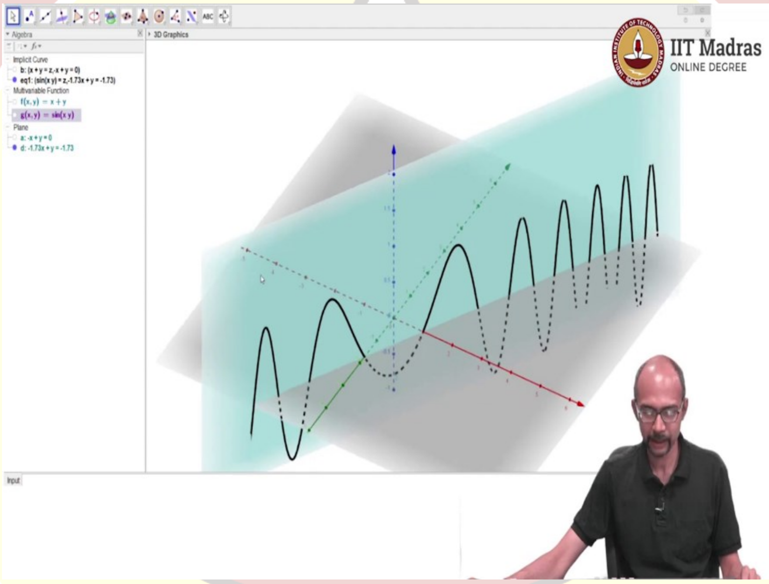
Algebra
3D Graphics
Implicit Curve
b: (x + y = z, -x + y = 0)
eq1: (sin(x y) = z, -1.73x + y = -1.73)
Multivariable Function
f(x, y) = x + y
g(x, y) = sin(x y)
Plane
a: -x + y = 0
d: -1.73x + y = -1.73
Input:
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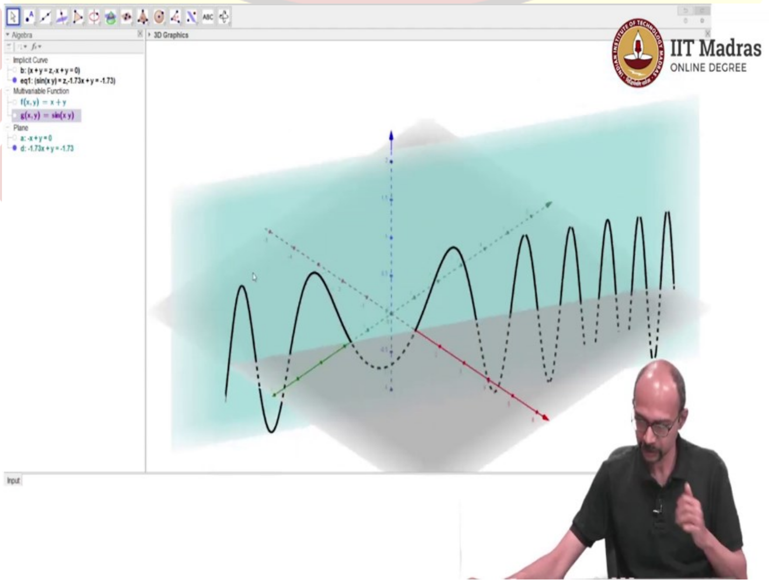
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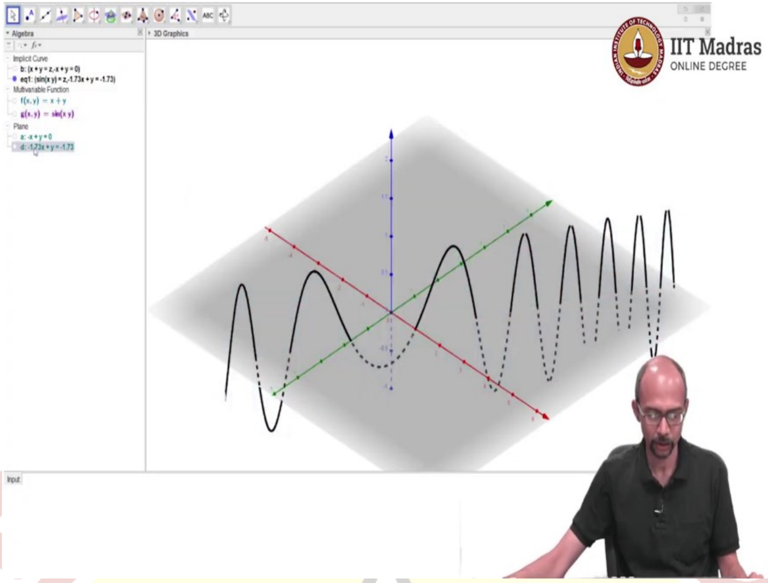

Let us look at the, the other example, in three dimensions, which was the function sin of x, y. So, here is how the function sin of x, y looks like. So, we have seen this function before. So, this is a very beautiful looking function. And now, we wanted to look at the point 1 , 0, and we computed the directional derivative at that point with in the direction where the angle made was 60 degrees with the X axis.

So, if you compute the equation of that line, it will give you this line here, and then we want to look at the plane above that. So that is the plane above that. And when we intersect these 2, that is exactly what happens on that, above that line. So, above that line, that is what happens.

So, here is what happens if we intersect those 2. And if I now remove the graph, you can see this is what it looks like. So, this is like the' function, except that the oscillations keep getting faster, as you move away from the axis, as the X or the Y values, increasing in magnitude, as you would expect.

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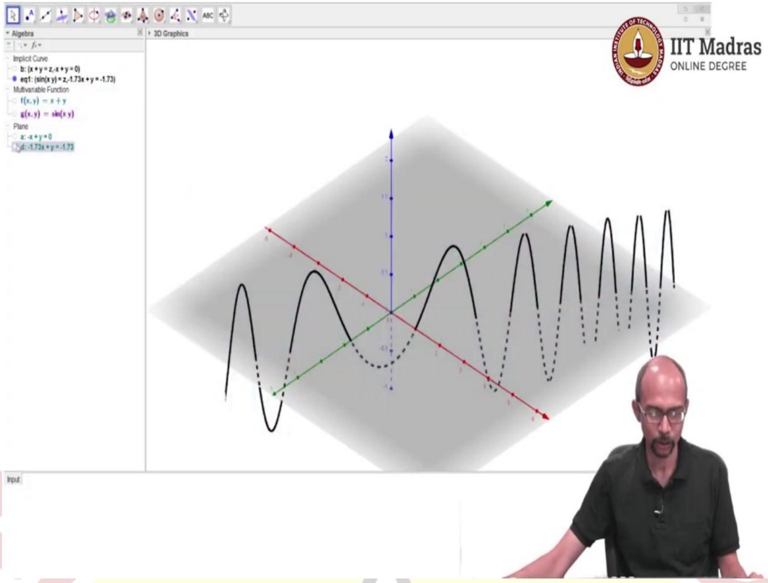
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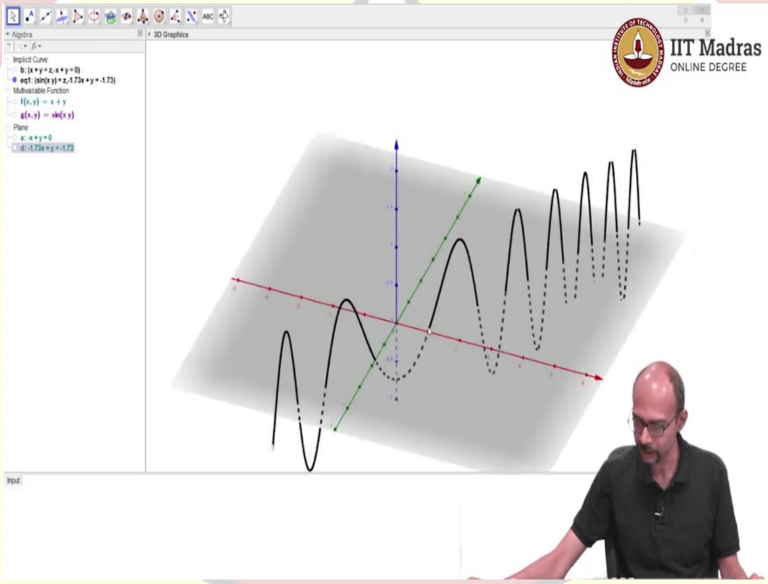
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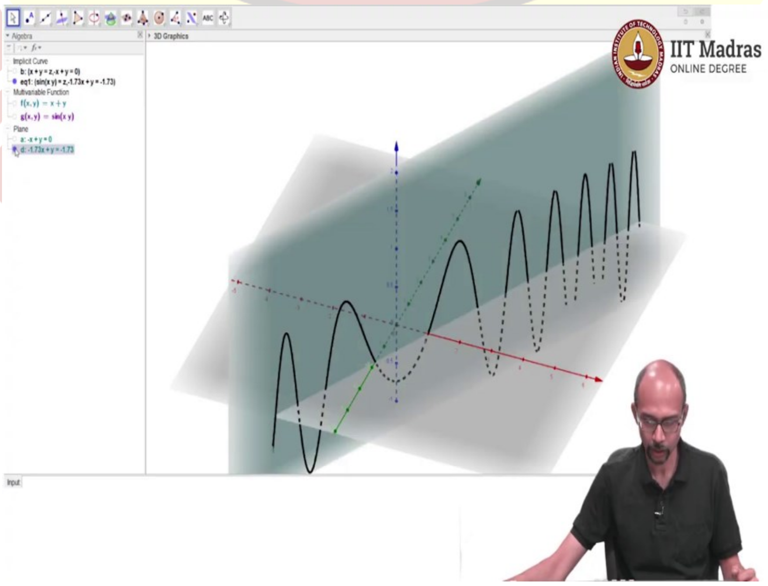
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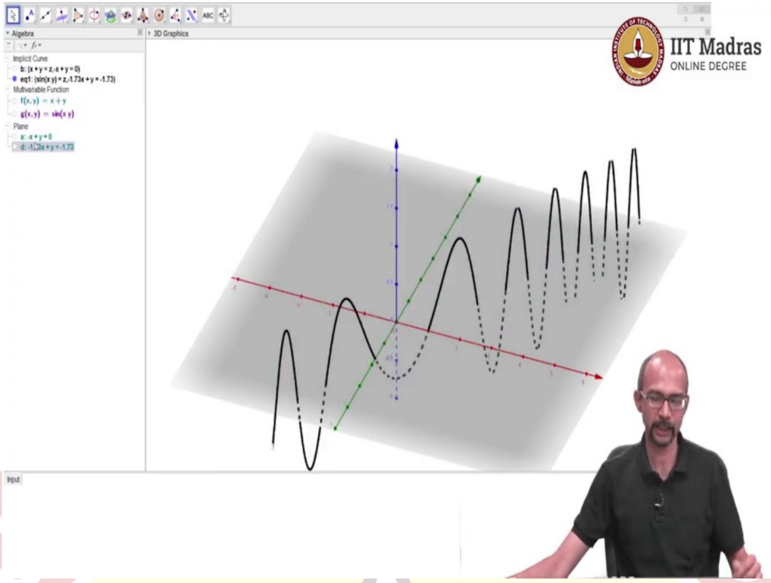

So, we want to compute, the question that we had or the thing that we computed was, what happens at 1 , 0. So, this is the point 1 , 0 over here. And you can see that at that point this is how that curve looks like. And what we are computing is the, if you draw the derivative at that point, the slope of the derivative with respect to this line that we just saw on that plane, so that that we just erased. So, on this plane, what is the slope of that line? And that is exactly what the directional derivative computes. Fine.

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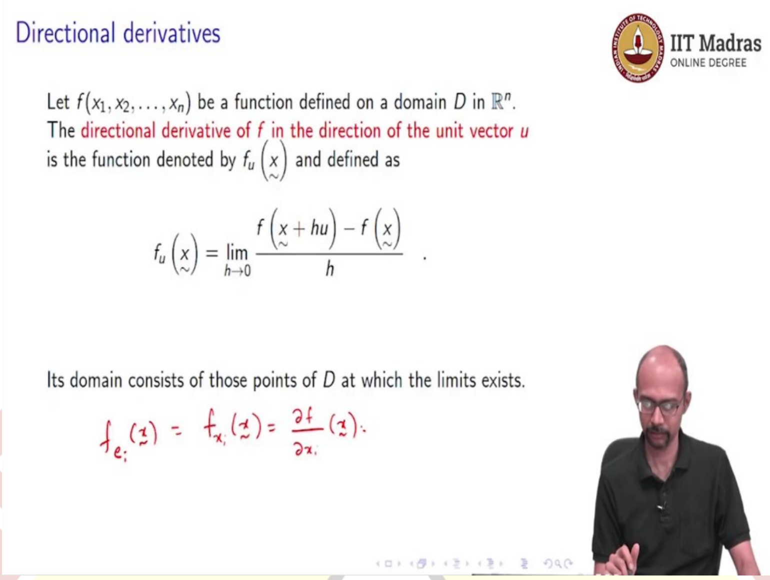

So, let us now define what is the directional derivative. So, this is exactly on the same lines, as we did for the partial derivative. So, let f(x)1, X2, Xn be a function defined in a domain D in Rn, the directional derivative of f in the direction of the unit vector u is the function. So earlier, we are computing limits at a particular point, now we do it as the point varies. So, this is a function, and the important part here is the notation.

So, the notation is f subscript u. So earlier, we had f subscript Xi. So here, instead, we have f subscript u, where what is u? It is that unit vector in whose direction we are computing this partial directional derivative. So, f subscript u, and the definition is exactly the same. So, instead of a tilde, because we are treating it as a function, we do this at $\underset{\sim}{x}$.

And the domain of this function is exactly those points of D at which the limit exists. So, if the limit does not exist, we do not talk about directional derivatives at that point and the limit exists, you define the function over there. So, I just want to make a small remark here, but an important 1.

So, if you take the unit vector ei, so you can define the directional derivative in the direction of the unit vector ei, that is exactly our partial derivative. This is exactly what partial derivative. So, this is a general version of the partial derivative.

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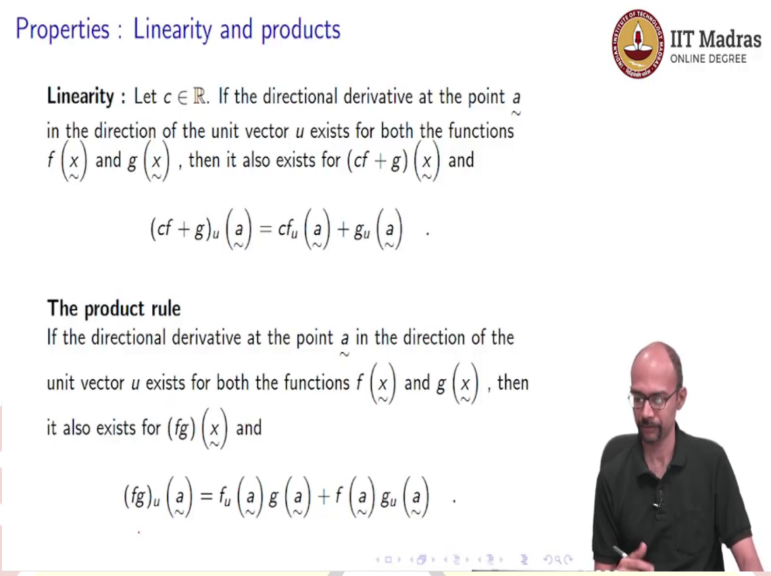

So, let us quickly go through some properties. We saw these in the examples when we did the partial derivatives, but these hold generally for directional derivatives, and let us just put them down clearly. So, 1 is linearity. So, if you have constant C and you have 2 functions for which the directional derivative at a point a, in the direction of the unit vector, u exists, then it also exists for the function $cf + g$.

And, and to compute it, you can just do linearity which means cf + g, the directional derivative is c × directional derivative of f + the directional derivative of g. Remember that u has to be the same, you cannot choose different directions and hope to get the same thing. So, that is linearity.

So, in particular, it follows from here that if you have f + g, then the additional derivative is the sum of the directional derivatives. If you have f - g, it is the difference. And if you just multiply /a constant that constant comes out. Then you have the product rule, we did all these for derivatives, and now we are repeating these for directional derivative. When I say derivatives, I mean from 1 variable calculus.

So, if the directional derivative at the point a in the direction of the unit vector u exist for both the functions f and g, then it also exists for the product, and it is given /the product rule exactly the same as we had for 1 variable calculus. So, f × g directional derivative at u, in the direction of the unit vector u is f in the directional derivative of f in the direction u at a × g(a) + f(a) × the directional derivative of g in the direction of u at a.

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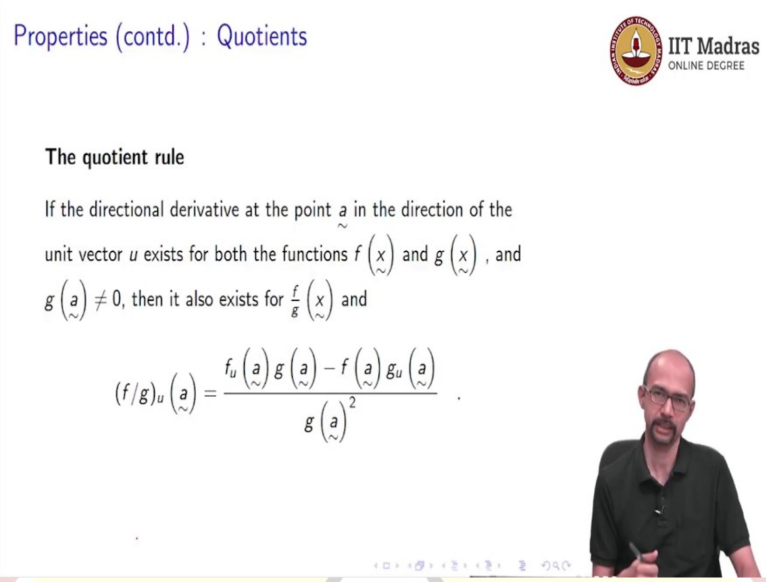

And finally, we have the quotient rule. So, again, with the same hypothesis, but here we need that the denominator g at a is not 0. In that case, the directional derivative at a, in the direction of the unit vector u, also exists, and it is given /this formula, directional derivative of f in the direction u, at a × g(a) - f(a) × g, directional derivative of g in the direction of the unit vector u evaluated at a /g(a)^2. So, we have seen the exact same formula in 1 variable calculus. So, these are useful properties, and they will help us evaluate.

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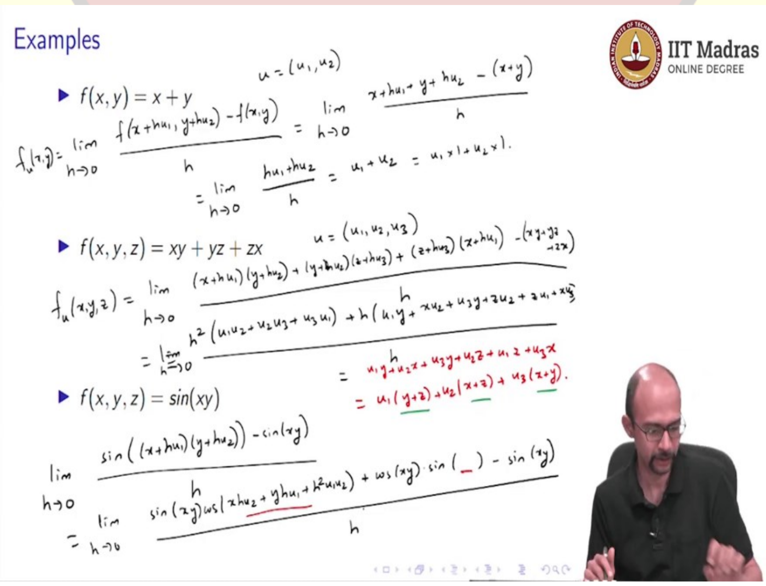

Let us do a couple of directional derivatives for arbitrary directions. I am going to assume an arbitrary vector. So, here u is u1, u2, and let compute what happens. So, limit h tends to 0. So, I want f of fu, x, y, so this is f(x) + h × u1, y + h × u2 - f(x), y /h. So, limit h tends to 0, x + h × u1 + y + h × u2 - x + y /h.

And the x and the y cancel out, so we are left with h × u1 + h × u2 /h, which is u1 + u2. And I will remind you that in the video on partial derivatives, we computed that the partial derivative of this function with respect to both x and y was 1 and 1 respectively. And I can write this as u1 × 1 + u2 × 1. For some strange reason, I am going to do that.

So, how about this, this is probably much more complicated. So, here are our unit vectors u1, u2, u3. So, if I try to compute this, what I will get is x + h × u1 × y + h × u2 + y + h × u2, × z + h × u3 + z + h × u3 × x + h × u1 - x, y + y, z + z, x this is a numerator /h.

So, now if I cancel out all the terms only in x, y and z which does happen because of the term on the right, that you are subtracting out, what we are left with is all these terms with the h. And then there is the some h^2 terms, so when you divide /h, so let us write that down. So, h^2× u1, u2 + u2, u3 + u3, u1 and then + h × u1 × y + x × u2 + u3 × y + z × u2 + z × 1 + x × u3.

So, you can check this is what you get, then divide that /h, take the limit as h tends to 0, and then what we have here is whatever you have in the bracket here. So, this is u1, y + u2x + u3y + u2z + u1z + u3x. And if I rewrite that, I get u1 × y + z, + u2 × x + z + u3 × x + y. And I will ask you to go back and check what were the partial derivatives for this function or you can check it right away, and see if these terms here. Whether these are things you can identify.

Finally, let us come to sin of x, y. This is probably the expression is going to get too messy, so I would not be able to complete this. So, sin of x + h × u1 × y + h × u2 - sin of x, y /h. So, I have to use my trigonometry. So, this is sin of x, y × cosine of some huge thing so x × h, u2 + y × hu1 + x^2× u1, u2 -, sorry, + cosine of x, y × sin of the same thing that I have here in the cos. So, this thing comes here and then - of - sin x, y /h.

So, this is a limit that I claim is totally doable. You have solved limits of this type, so I will suggest that you sit down and work this limit out, and see what you get. And once you can write down that limit, check out what that limit is, and what the partial derivatives for this function are, and what relation they have. Fine.

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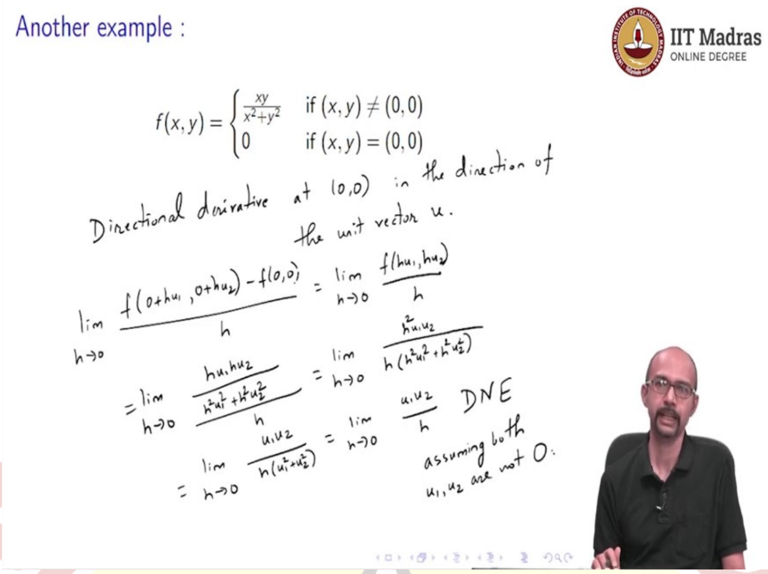

So, I will end with this example of f(x), y is x, y /$x^2 + y^2$, if x, y is not the origin and x, y is 0 if x, y is 0, 0. And I want to compute these directional derivative at 0, 0 specifically. For the other points it is a tedious calculation, but doable, but at 0, 0 there is something specific that we want to see in the direction of the unit vector u. So, what do I have, my definition, this is limit h tends to 0, f of 0 + h × u1, 0 + h × u2 - f of 0, 0 /h.

Well, let us substitute in the values. So, this is f of hu1 , hu2, - f of 0, 0, which is 0, so I will drop this /h. This is limit h tends to 0, h × u1, h × u2 /h^2 u1 ^{2} + h^2 u2 ^{2}, the whole thing /h. So, if we know how quotients work, you will be able to see that this is just hu1, u2 /h × h^2 u1 ^{2} + h^2u2 ^{2}, sorry, there is an h^2 here. So, this is h^2 u1, u2 /h ×, h^2 u1 ^{2}+ h^2 u2 s^2.

So, the h^2cancels with h^2 from denominator. And what this gives us is that you have u1, u2 /h × u1 ^{2} + u2 ^{2}. This is actually a unit vector, so the denominator u1 ^{2} + u2 ^{2} is 1, although that is irrelevant for what is coming. So, this is u1, u2 /h. So, this is what you get. And now u1 and u2 are, of course, fixed I meaning they have nothing to do with it. And in the denominator, you have an h, which means that this limit does not exist.

So, the directional derivative in the direction u does not exist, unless, of course, 1 of u1 or u2 is 0, assuming both u1 and u2 are not 0. What happens if 1 of them is 0? Well, that is exactly the case where you have the partial derivatives, and we checked in the partial derivatives video that the partial derivatives are both actually 0. So, the partial derivatives here do exist, both of them are zero, but other than the partial derivatives, no other directional derivative exists.

So, the only directions where the rate of change is defined is in the direction of the x axis or the y axis for no other direction is it defined. So, in this video, we have seen the notion of the directional derivative. This is a generalization of the partial derivative, where you compute the rate of change for any direction instead of just the directions parallel to the x axis or the y axis, you compute in any direction. The definition is almost analogous, and the calculations are slightly more tedious.

Although, when we saw examples, we saw that there was something interesting happening. So, there is some background here, and there is a phenomenon that we have to uncover, that this phenomenon unfortunately, depends, as we saw based on the last example, on some conditions, so we have to talk about something called continuity. So, once we talk about continuity, we will come back and we will show an easy way of computing the directional derivatives once we know what are the partial derivatives. Thank you.