IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 5 Tutorial 4

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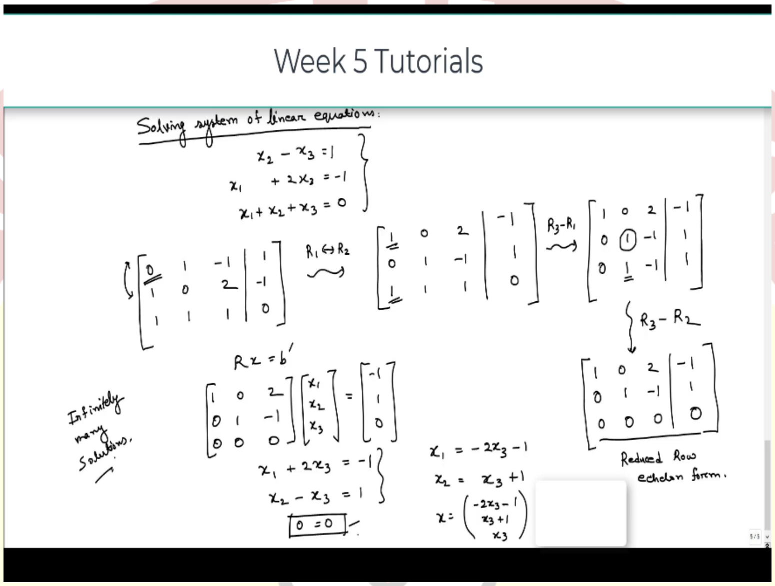

Hello. So, here we are considering another system of linear equation and we will try to solve it. So, at first, we will write the augmented matrix. So, here we are writing down the augmented matrix, so at first the coefficient matrix is $\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ this is the coefficient matrix and from the right-hand side of this equation, we are taking 1, -1, 0 and put a straight line in the between those two. So, this is our augmented matrix.

So, here we can see the first element, basically the element in the first one, first column, that is 0. So, that cannot be our pivot element. So, what we will do, we will interchange these two rows to make the first row begin with 1. So, what we are doing here? We are interchanging R_1 and R_2. So, we will get, 1, 0, 2; -1, 0, 1; -1, 1 and third row will remain same. So, this is our new matrix which we get by row operation by interchanging the first and the second row.

Now, we have to make all the element in the first column to be 0 except this pivot element. The second, in the second row, we already have 0 so we have to make this one to be 0, which is in the third row. So, what we have to do? We have to do $R_3 - R_1$. So, the first two row will remain same and the third row will become 0, 1, -1 and 1.

Now, in the second row, this is our pivot element which is already 1, so we have to make this element in the third row to be 0, so what we have to do? We have to make $R_3 - R_2$, so let us see what will happen. 1, 0, 2, -1 will be the first row which will remain same. The second row will also remain same and the third row will become 0, 0, 0. So, the third row is basically 0, row with all the entries to be 0. So, here this is our reduced row echelon form.

So, from this one, we can write $Rx = b'$ where R is 1, 0, 2; 0, 1 -1; 0, 0, 0, this is R, this is our x_1, x_2, x_3 and our b' will be -1, 1, 0. So, from the first row, we will get $x_1 + 2x_3 = -1$. And from the second row we will get $x_2 - x_3 = 1$ and the third row will give us 0 equal to 0. So, there is no absurdity as it was in the previous example where we can see that the equation, the system of linear equation had no solution, but here, 0 equal to 0, so here is no absurdity, this is actually true.

So, we have two equations now. So, from the first equation, we can write x_1 in terms of x_3 which is $-2x_3 - 1$ and from second one, we can write x_2 in terms of x_3 which is $x_2 = x_3 + 1$. And the last one is 0 equal to 0 which is absolutely possible, I mean which is exactly true, so nothing to do about that.

So, our solution $x = \begin{bmatrix} -2x_3 - 1 \\ x_3 + 1 \\ x_3 \end{bmatrix}$. Now, x_3 can take infinitely many values, that there are infinitely many real numbers. So, this system of equation has infinitely many solution, whatever value of x_3 you put here, you will get the value of x_1 and x_2 according to these equations. So, this system of linear equation which we have started doing has infinitely many solutions. So, let us see the geometric representation of this system of linear equations.

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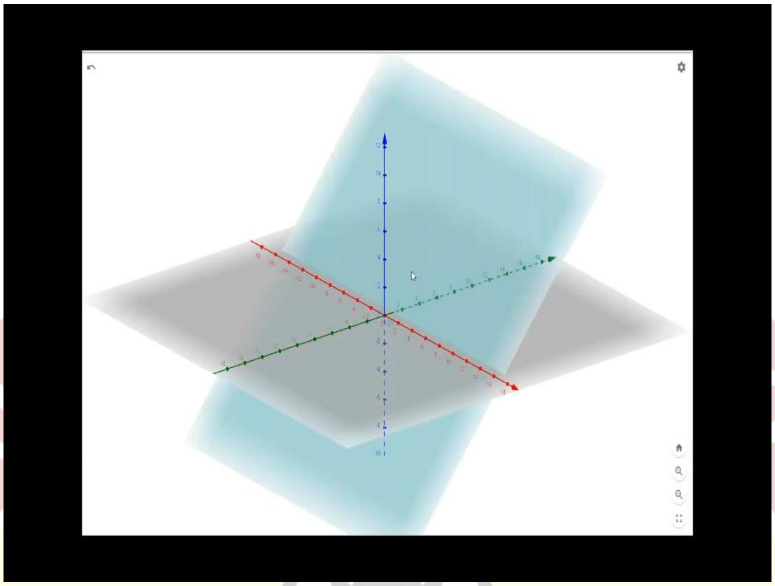

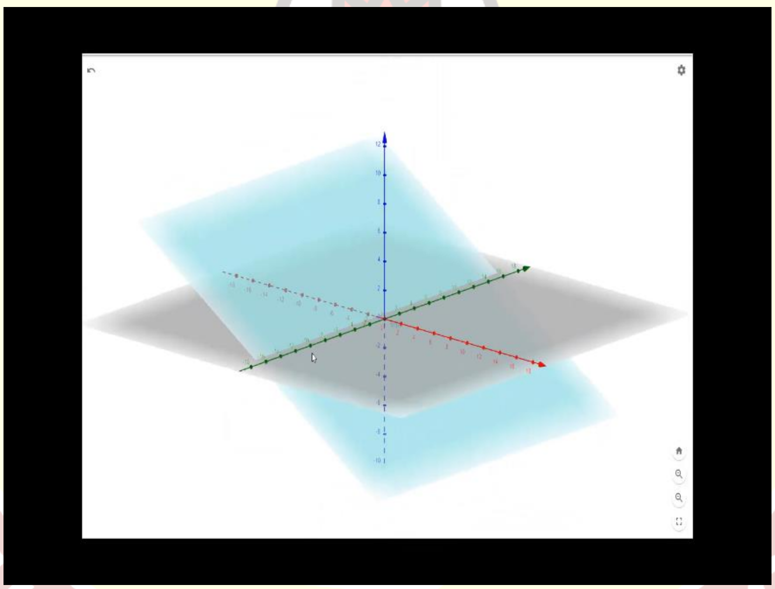

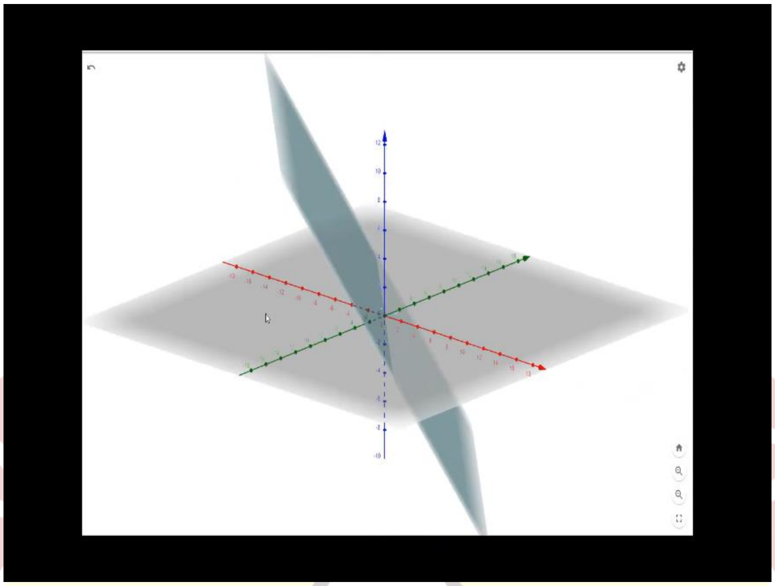

So, let us consider the first equation which was $x_2 - x_3 = 1$. So, the plane will look like this in our X, Y, Z plane. The second equation was $x_1 + 2x_3 = -1$, that will look like this plane. And the third equation was $x_1 + x_2 + x_3 = 0$, so this will look like this.

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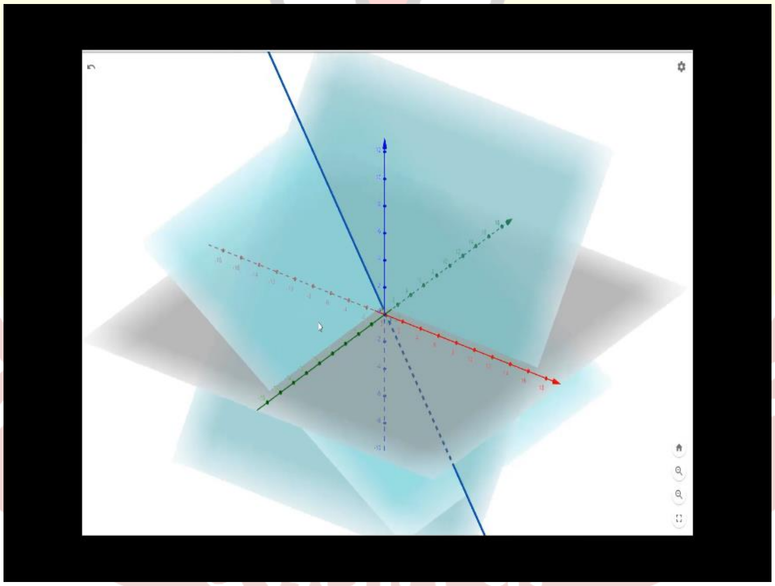

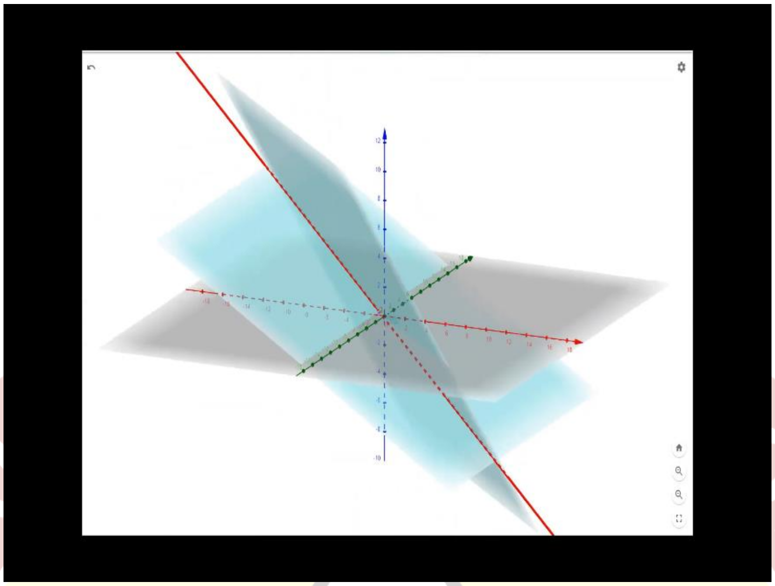

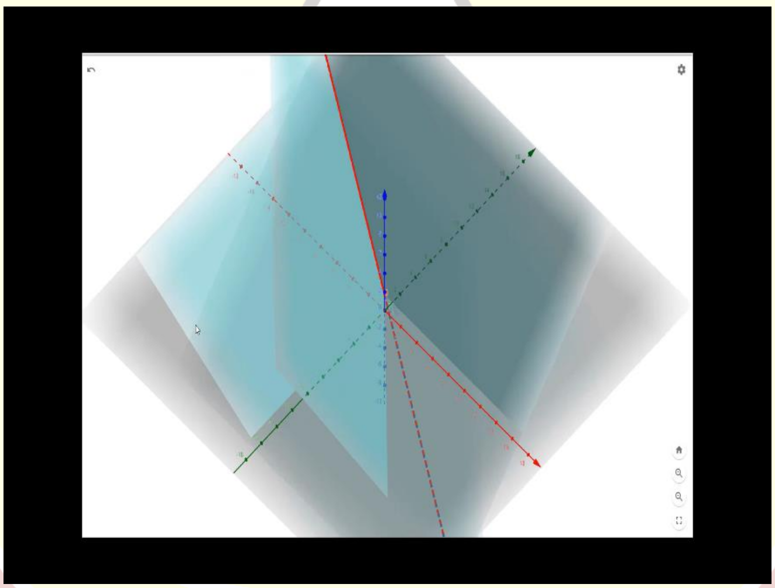

So, if we consider the first and second equation together, we will say that they will intersect at a straight line which is like this. Now, if we take equation-2 and equation-3, again they are intersecting at a straight line and we can see that this straight line is like this. So, if we consider the three equations together, we can see that the three equations actually passing through the same straight line.

So, there are infinitely many points on the straight line and these three planes are passing through all those points; basically, they are intersecting in this straight line so there are infinitely many solutions. Thank you.