

**IIT Madras**  
ONLINE DEGREE

**Mathematics for Data Science 2**  
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**Week-6 Tutorial 03**

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third example

$$V = M_{3 \times 3}(\mathbb{R})$$

$$W = \left\{ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \mid \begin{matrix} a_{11} + a_{12} + a_{13} = 0, \\ a_{21} + a_{22} + a_{23} = 0, \\ a_{31} + a_{32} + a_{33} = 0 \end{matrix} \right\}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

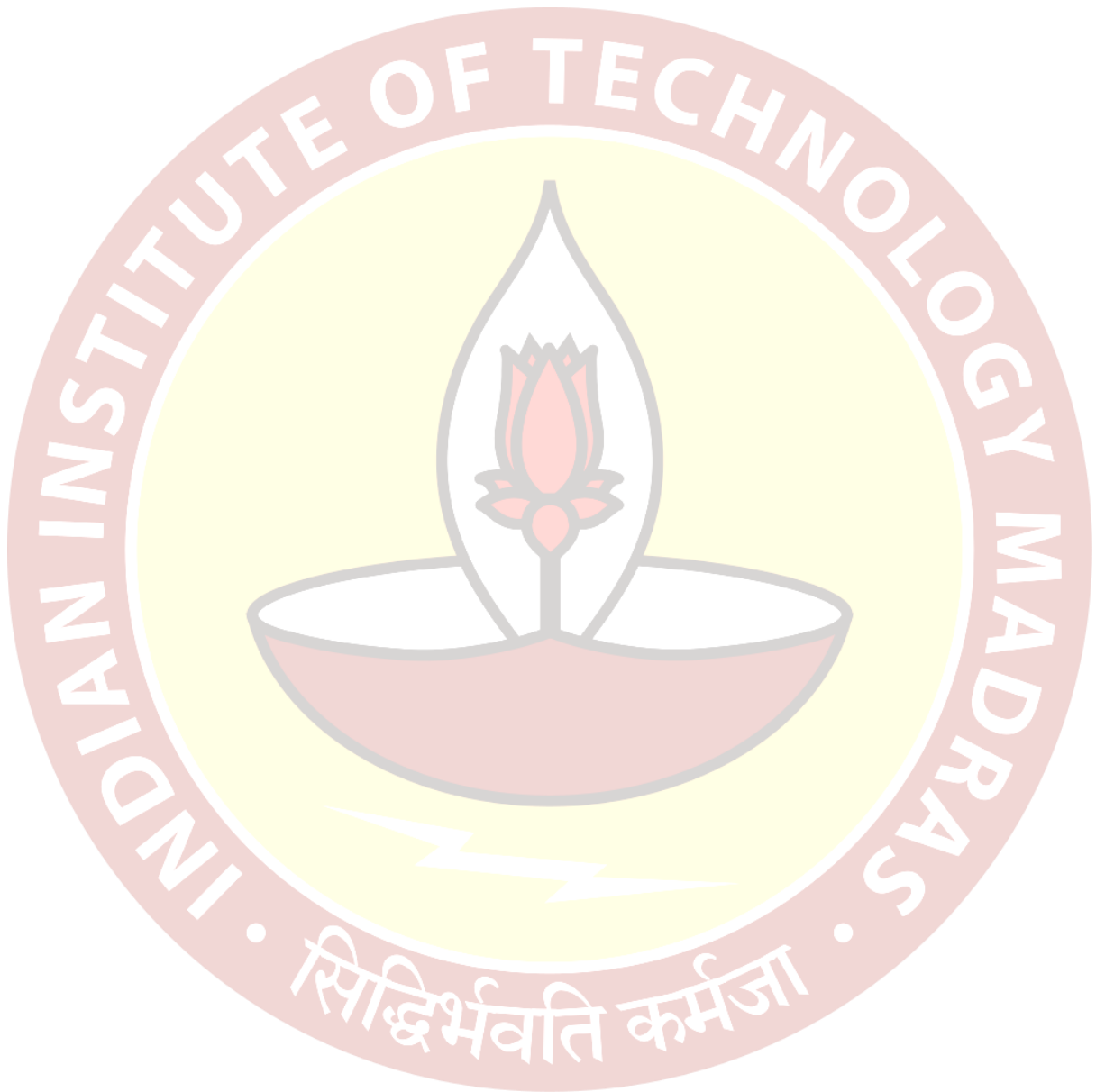
Now, let us discuss third example. The third example is, we are taking vector space as  $\mathbb{R}$ , collection of all  $3 \times 3$  matrices, which we use to say  $M_{3 \times 3}(\mathbb{R})$  and we are taking subset of all,  $3 \times 3$  matrices, a subset of all  $3 \times 3$  matrices is, this is the matrix with some condition, the condition is entries in a row sum up to 0. This is the first row we sum up to 0, this is the second row and entries sum up to 0 and this is the third sum up to, third row sum up to 0.

Now, we need to check is this subset is subspace of this vector space  $M_{3 \times 3}(\mathbb{R})$  vector space of all  $3 \times 3$  matrices. So, again we need to check all three criteria, if  $W$  follow that then  $W$  is a

subspace, so we need to check is 0 element in  $W$ , so we can think  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  So, if we see this, this all are sum up to 0 and follow this condition of  $W$ . So, it means 0, this element is in  $W$ .

$$W = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

And if we add with any general element of  $W$  with some, with this element zero, so if we add them, we will get the same element. So, it means this element is 0 element of  $W$  and which is in  $W$ . So, first condition followed by  $W$ .



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$$\textcircled{ii} \quad w_1 = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}, \quad w_2 = \begin{bmatrix} b_{11} \\ b_{12} \\ b_{13} \end{bmatrix}$$

$$w_1 + w_2 = \begin{bmatrix} a_{11} + b_{11} \\ a_{12} + b_{12} \\ a_{13} + b_{13} \end{bmatrix}$$

$$\textcircled{iii} \quad c \in \mathbb{R} \quad c.w_1 = \begin{bmatrix} ca_{11} \\ ca_{12} \\ ca_{13} \end{bmatrix} \quad c(a_{11} + a_{12} + a_{13}) = 0$$

Now, we need to check second condition. Second condition is saying this subset is closed under, should be closed under addition. So, again, we can take two element the  $w_1$  which is

$w_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$  and second element  $w_2$  is  $w_2 = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$  And these are the elements of  $w$ . So, if we add this is the condition on  $W$  that it is sum up to 0, this is sum up to 0 and this is sum up to 0 with the same with  $W_2$  these are all sum up to be 0, we have given these are.

Now if we add  $w_1 + w_2$  then we will get  $a_{11} + b_{11}$  this is the as usual matrix addition,  $a_{12} + b_{12}, a_{13} + b_{13}$  So, we will check for only one row. So, if we add these entries sum up, so and look again, rearrange the term, it will be sum of 0. So, it means addition is this  $W$  is closed under addition, and we need to check again the scalar multiplication.

So, again if we take any real number  $c$ , and if you multiply with this as a general element  $W$ ,  $w_1$  let us say  $w_1$  so  $c.w_1$  it will become  $ca_{11}$  as a scalar multiplication with a matrix we know that as usual will be the matrix multiplication with a scalar. So  $C$ , it will become  $ca_{12}$  and  $ca_{13}$  and all others are.

So, if we sum up with this, and if you take, if you sum this first row, this will be  $a_{11} + a_{12} + a_{13}$  And if we, as we said this is sum up to 0, it means, it becomes 0. So again, this third criteria is also satisfied with second and first criteria. So, it means  $W$  is subspace.