# Statistics 2 Live Session

July 8, 2021

# MGF, WLLN, CLT

If X takes value x1 with prob $p_1$, x2 with prob $p_2$ etc,

$$\text{MGF of X, } E[e^{tX}] = p_1 e^{t\,x1} + p_2 e^{t\,x2} + \dots$$

iid samples: $X_1, X_2, \dots, X_n$ iid X. Let $T = X_1 + \dots + X_n$

$$MGF(T) = MGF(X)^n$$

WLLN: $P(\,|\,T/n - E[X]\,| > t\,) \leq Var(X) / (n\,t^2)$

CLT: $T \cong Normal(\,n\,E[X],\ n\,Var(X)\,)$, or $T/n \cong Normal(\,E[X],\ Var(X)\,/\,n\,)$

# Q1 (a) MGF discrete

Let X be a r.v. such that P(X = -1) = 1/4, P(X = 0) = 1/2, P(X = 1) = 1/4. What is the MGF of X?

# Q1 (b) MGF discrete

Let X be a r.v. such that P(X = -1) = 1/4, P(X = 0) = 1/2, P(X = 1) = 1/4. Let $X_1$ and $X_2$ be iid X. What are the MGF and PMF of $X_1+X_2$?

# Q1 (c) MGF discrete

Let X be a r.v. such that P(X = -1) = 1/4, P(X = 0) = 1/2, P(X = 1) = 1/4. Let $X_1$ and $X_2$ be iid X. What are the MGF and PMF of $X_1+2X_2$?

# Q2 (a) Expected value of sample mean

Let X be a r.v. such that $P(X = 1) = 1/2$, $P(X = 2) = 1/3$, $P(X = 3) = 1/6$. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $T = X_1 + \ldots + X_n$. What is $E[T]$? What is $E[T/n]$?

# Q2 (b) Variance of sample mean

Let X be a r.v. such that $P(X = 1) = 1/2$, $P(X = 2) = 1/3$, $P(X = 3) = 1/6$. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $T = X_1 + \ldots + X_n$. What is $Var[T]$? What is $Var[T/n]$?

# Q3 (a) Second moment

Let X be a r.v. such that $P(X = -1) = 1/6$, $P(X = 0) = 2/3$, $P(X = 1) = 1/6$. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $M_2 = X_1^2 + \ldots + X_n^2$. What is $E[M_2]$? What is $E[M_2 / n]$?

# Q3 (a) Second moment

Let X be a r.v. such that $P(X = -1) = 1/6$, $P(X = 0) = 2/3$, $P(X = 1) = 1/6$. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $M_2 = X_1^2 + \ldots + X_n^2$. What is $Var[M_2]$? What is $Var[M_2 / n]$?

# Q4 (a) Frequencies

Let X be a r.v. such that P(X = 1) = 1/6, P(X = 2) = 1/6, P(X = 3) = 1/3, P(X = 4) = 1/3. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $F_i$ = number of 'i' in the samples. What is $E[F_1]$? What is $E[F_3]$?

# Q4 (b) Frequencies

Let X be a r.v. such that $P(X = 1) = 1/6$, $P(X = 2) = 1/6$, $P(X = 3) = 1/3$, $P(X = 4) = 1/3$. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $F_i$ = number of 'i' in the samples. What is $Var[F_1]$? What is $Var[F_3]$?

# Q5 (a) WLLN

Let X be a r.v. such that $P(X = 1) = 1/2$, $P(X = 2) = 1/3$, $P(X = 3) = 1/6$. Let $X_1$, $X_2,..., X_n$ be iid X. Let $T = X_1 + … + X_n$. Using WLLN, find an upper bound for $P( T > 2n )$.

# Q5 (b) CLT

Let X be a r.v. such that P(X = 1) = 1/2, P(X = 2) = 1/3, P(X = 3) = 1/6. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $T = X_1 + \ldots + X_n$. Using CLT, find an estimate for P( T > 2n ).

# Q6 (a) WLLN

Let X be a continuous r.v. uniform in [-1,1]. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let T = $X_1$ + … + $X_n$. Using WLLN, find an upper bound for P( |T| > n/2).

# Q6 (b) CLT

Let X be a continuous r.v. uniform in [-1,1]. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let T = $X_1$ + … + $X_n$. Using CLT, find an estimate for P( |T| > n/2).

# Q7 (a) Frequencies

Let X be a r.v. such that P(X = 1) = 1/6, P(X = 2) = 1/6, P(X = 3) = 1/3, P(X = 4) = 1/3. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $F_i$ = number of 'i' in the samples. Using WLLN, find an upper bound for P( | $F_1$ - n/6 | > 10 sqrt(n) ).

# Q7 (b) Frequencies

Let X be a r.v. such that $P(X = 1) = 1/6$, $P(X = 2) = 1/6$, $P(X = 3) = 1/3$, $P(X = 4) = 1/3$. Let $X_1$, $X_2$,..., $X_n$ be iid X. Let $F_i$ = number of 'i' in the samples. Using CLT, find an estimate for $P( | F_3 - n/3 | > 10 \sqrt{n} )$.

# Q8 (a) Estimation

Consider the following samples from Bernoulli(p):

1, 0, 0, 0, 1, 0, 0, 0, 0, 1

Find the sample mean. Find the MM estimate for p.

# Q8 (b) Estimation

Consider the following samples from Bernoulli(p):

1, 0, 0, 0, 1, 0, 0, 0, 0, 1

What is the likelihood function? Find the ML estimate for p.

# Q9 (a) Estimation

Consider the following samples from the discrete distribution P(X=1) = t / 3, P(X=2) = t / 6, P(X=3) = 1 - t / 2:

1, 2, 1, 3, 2, 3, 2, 1, 1, 2

Find the sample mean. Find the MM estimate for t.

# Q9 (b) Estimation

Consider the following samples from the discrete distribution P(X=1) = t / 3, P(X=2) = t / 6, P(X=3) = 1 - t / 2:

1, 2, 1, 3, 2, 3, 2, 1, 1, 2

Find the likelihood function. Find the ML estimate for t.

# Q10 (a) Estimation

Consider the following samples from the Geometric(p) distribution:

4, 5, 7, 3, 6, 5, 4, 5

Find the sample mean. Find the MM estimate for p.

# Q10 (b) Estimation

Consider the following samples from the Geometric(p) distribution:

4, 5, 7, 3, 6, 5, 4, 5

Find the likelihood function. Find the ML estimate for p.

# Q11 (a) Estimation

Consider the following samples from the Beta(2,b) distribution:

0.86, 0.76, 0.08, 0.24, 0.66

Find the sample mean. Find the MM estimate for b.

# Q11 (b) Estimation

Consider the following samples from the Gamma(a, 5) distribution:

0.52, 0.25, 0.33, 0.87, 0.42

Find the sample mean. Find the MM estimate for a.

# Q12 (a) Estimation*

Consider the following samples from the distribution with PDF $(ax+1)/(2a+2)$, for $0 < x < 2$:

0.1, 0.5, 0.2, 0.4, 1.1

Find the sample mean. Find the MM estimate for 'a'.

# Q12 (b) Estimation*

Consider the following samples from the distribution with
PDF (ax+1)/(2a+2), for 0 < x < 2:

0.1, 0.5

Find the likelihood function. Find the ML estimate for 'a'.