INDIAN INSTITUTE OF TECHNOLOGY MADRAS

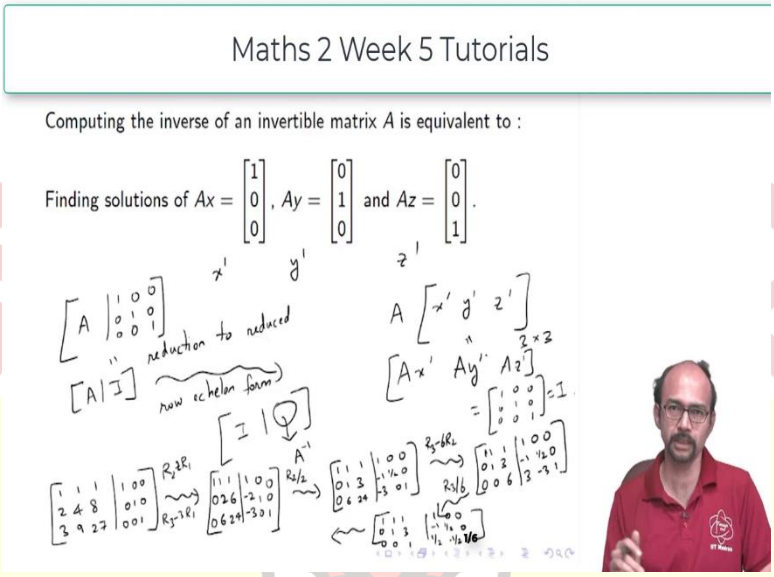
सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
Professor. Sarang S. Sane
Department of Mathematics
Indian Institute of Technology Madras
Week 05 - Tutorial 05

(Refer Slide Time: 0:14)



So, finally let us come to computing the inverse. So, Gaussian elimination can also help you in computing the inverse. How is this? Well, let us recall what we did in computing the inverse. So, the inverse, what is the inverse of a matrix? It is a matrix A in B such that $A * B$ is identity and $B * A$ is identity. If I can arrange the $A * B$ is identity, it kind of forces that $B * A$ is identity.

So, I just have to get a matrix B so that $A * B$ is identity. But how do I get $A * B$ is identity? So, let us do it for the 3 by 3 case. So, suppose A is a 3 by 3 matrix, so then, to say $A * B$ is identity, is the same as saying that A times the first column of B is 1 0 0. A times the second column of B is 0 1 0 and A times the third column of B is 0 0 1.

So, suppose I have solutions for $Ax = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $Ay = \begin{bmatrix}0\\1\\0\end{bmatrix}$ and $Az = \begin{bmatrix}0\\0\\1\end{bmatrix}$. So, suppose the solutions are given by x', y' and z' respectively. So, then I make my matrix $[x' \quad y' \quad z']$. Remember x', y' and z' are columns. So, this gives me a 3 by 3 matrix. And now, when I multiply this A on the left, what do I get? I get $[Ax' \quad Ay' \quad Az']$. But these are solutions to Ax is 1 0 0, Ay is 0 1 0 and Az is 0 0 1. So, this is exactly 1 0 0, this is exactly 0 1 0 and this is exactly 0 0 1 which is exactly the identity matrix.

So, if I solve this, if I solve these 3 equations, I can find a matrix B so that $A * B$ is identity and that will say that B is the inverse of A. Conversely, if I know $A * B$ is identity, then you can take the first column of B that will be solution of Ax is 1 0 0, the second column of B that will be your solution of Ay as 0 1 0 and the third column of B and that will be a solution of Az equals 0 0 1.

So, what is the procedure? So, well, you can already see that funding solutions is exactly what Gaussian elimination does for us. So, I can find this, but I want to show you the algorithm for doing this which some of you may have seen before. So, what you do is, instead of, so I want to find the solution of let us say $\left[\begin{array}{c|ccc} A & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right]$, so I write it like this.

So, instead of taking one column at a time, I take all 3 columns together and because the procedure is the same, I have to apply the elementary row operations which I am going to use to reduce A to the reduced row echelons form. And so, I take this all three and I put it like this. So, in other words, what I do is, I take a much bigger augmented matrix, I take A and I augment all these three columns.

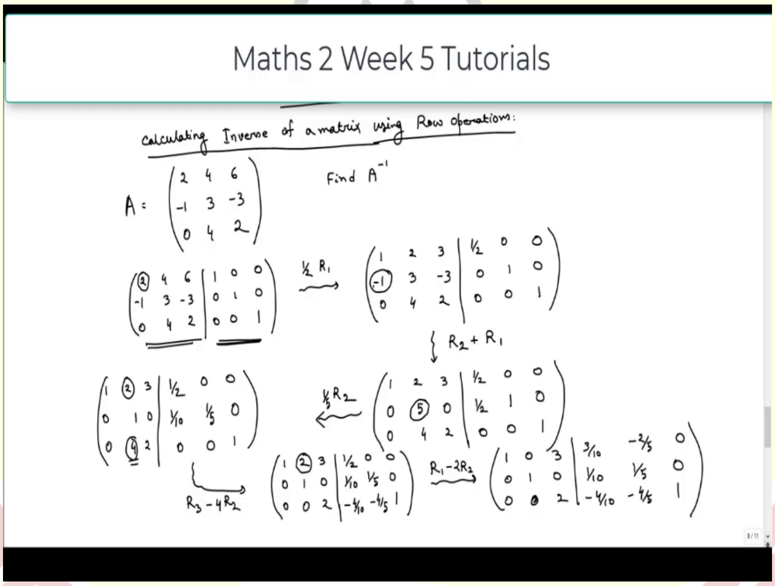
So, 1 0 0, 0 1 0 and 0 0 1. So, I essentially put $[A \quad | I]$ beside each other. Then I reduce so, row, let me say reduction to row, reduced row echelon form. And if I do this and I know that A is invertible, well if A is invertible, then the reduce to echelon form of A must be I and then whatever you get over here that is exactly A inverse. That is the procedure.

Let me maybe do a very fast example. So, maybe my matrix here is, so let us see $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}$. Let us reduce this to reduced row echelon form. Probably this is not going to give me a very good inverse. But let me do it anyway. So, how do I do this? of course augment the identity matrix. So, before I do this, I have to augment the identity matrix. So, let us do that $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 4 & 8 & 0 & 1 & 0 \\ 3 & 9 & 27 & 0 & 0 & 1 \end{array}\right]$.

So, I reduced this, so $R_2 - 2R_1$, $R_3 - 3R_1$, what do I get? So, I get $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 6 & -2 & 1 & 0 \\ 0 & 6 & 24 & -3 & 0 & 1 \end{array}\right]$. I continue this procedure. So, now I do $\frac{R_2}{2}$, so what do I get? So, $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & \frac{1}{2} & 0 \\ 0 & 6 & 24 & -3 & 0 & 1 \end{array}\right]$. I use my 1 to make the 6 entry 0, so that is $R_3 - 6R_2$. So, $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 6 & 3 & -3 & 1 \end{array}\right]$. I hope you understand what is happening, so I want to reduce this further. So, I will do $\frac{R_3}{6}$. Probably not optimised my space well. Nevertheless, so I have $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6} \end{array}\right]$, so this is already in row echelon form and maybe there is two more steps and you will get it into reduced echelon form.

And whatever matrix you have on the right-hand side, that 3 by 3 matrix, that will be the inverse of the original matrix. So, I hope the process is clear.

(Refer Slide Time: 8:51)



So, depending on the idea what sir has discussed till now, so we will take an example of a matrix and try to calculate the inverse of a matrix using Gaussian elimination or using row operation. So, let us take an example. Let us take this matrix $\begin{bmatrix} 2 & 4 & 6 \\ -1 & 3 & -3 \\ 0 & 4 & 2 \end{bmatrix}$. Let us consider this matrix and try to find its inverse. So, this is our A matrix. Question is to find A inverse.

So, we start with this $\left[\begin{array}{ccc|ccc} 2 & 4 & 6 & 1 & 0 & 0 \\ -1 & 3 & -3 & 0 & 1 & 0 \\ 0 & 4 & 2 & 0 & 0 & 1 \end{array}\right]$ and we are doing row and column operation and we will try the make this side as identity matrix, then whatever the matrix will be on this side, this 3 cross 3, that will be our inverse.

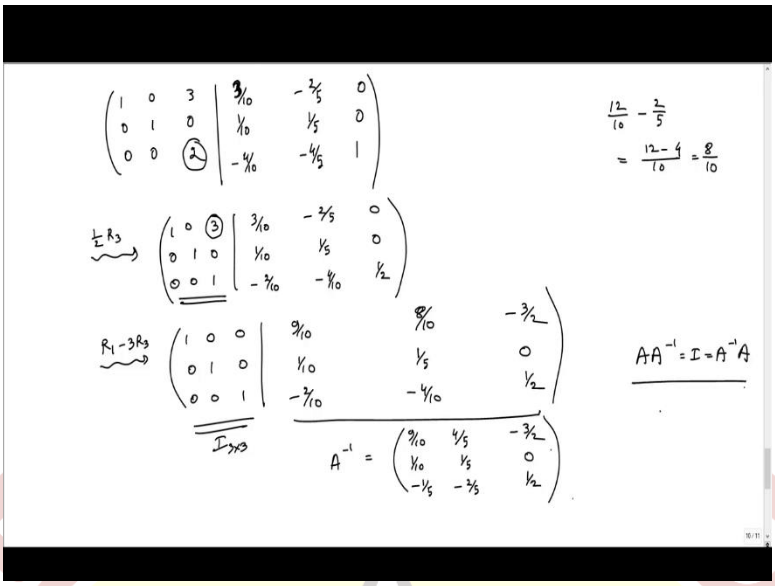
So, to begin with, at first there is 2 in the first element as first row and first column, so we have to multiply with half, so $\frac{R_1}{2}$, that will give us $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & \frac{1}{2} & 0 & 0 \\ -1 & 3 & -3 & 0 & 1 & 0 \\ 0 & 4 & 2 & 0 & 0 & 1 \end{array}\right]$. So, this is our first operation. This is what we get after the first operation. So, now we just, we want to make this one as 0, so we add $R_1 + R_2$.

So, we will get $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 5 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 4 & 2 & 0 & 0 & 1 \end{array}\right]$. Now what we have to do? We have to make this one as 1. So, we will do $\frac{R_2}{5}$, so that will give us $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{10} & \frac{1}{5} & 0 \\ 0 & 4 & 2 & 0 & 0 & 1 \end{array}\right]$.

Now what we have to do? We have to make this and this to be 0. So, to make this 4 to be 0, we have to do $R_3 - 4R_2$, then we will get this one as 0. So, let us write it. $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{10} & \frac{1}{5} & 0 \\ 0 & 0 & 2 & -\frac{4}{10} & -\frac{4}{5} & 1 \end{array}\right]$

Now we are making 4 into R2, so it will be 4 by 10, so it is minus 4 by 10, it is 4 by 5, so it will be minus 4 by 5 and 1. And to make this one as 0, we have to do $R_1 - 2R_2$,. So, what we have got here, so we will get $\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & \frac{3}{10} & -\frac{2}{5} & 0 \\ 0 & 1 & 0 & \frac{1}{10} & \frac{1}{5} & 0 \\ 0 & 0 & 2 & -\frac{4}{10} & -\frac{4}{5} & 1 \end{array}\right]$

(Refer Slide Time: 13:24)



So, now what we have to do? Here we have 2 in the third row and third column, so we have to make it 1, so basically, we will do $\frac{R_3}{2}$ so that will give us $\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & \frac{3}{10} & -\frac{2}{5} & 0 \\ 0 & 1 & 0 & \frac{1}{10} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{2}{10} & -\frac{4}{10} & \frac{1}{2} \end{array}\right]$. Now we have to make this one to be 0, so this is the last step basically, then we will get the identity matrix. So, identity matrix in this side.

So, we will, we have to do $R_1 - 3R_3$. So, it will give us $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{9}{10} & \frac{8}{10} & -\frac{3}{2} \\ 0 & 1 & 0 & \frac{1}{10} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{2}{10} & -\frac{4}{10} & \frac{1}{2} \end{array}\right]$.

So, the inverse, so in this side, we have identity matrix of order 3 and this is the 3 cross 3 matrix which is our A^{-1} inverse. So, this is $\begin{bmatrix} \frac{9}{10} & \frac{4}{5} & -\frac{3}{2} \\ \frac{1}{10} & \frac{1}{5} & 0 \\ -\frac{1}{5} & -\frac{2}{5} & \frac{1}{2} \end{bmatrix}$. So, this is our inverse. So, why can check that $AA^{-1} = I = A^{-1}A$. Thank you.