Assignment 1

Indian Institute of Information, Kanpur
Department of Computer Science
CS771 Introduction to Machine Learning
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Contents

1 Answer 1

As we have to map binary digits 0,1 to signs -1,+1,
We should make a function say, $m : \{0,1\} \rightarrow \{-1,+1\}$. Such that

$$m(x) = \begin{cases} -1, & \text{if } x = 1 \\ +1, & \text{if } x = 0 \end{cases}$$

and another function named $f : \{-1,+1\} \rightarrow \{0,1\}$ and it should be such that

$$f(x) = \begin{cases} 0, & \text{if } x = +1 \\ 1, & \text{if } x = -1 \end{cases}$$

So, for m the fuction is $m(x) = 1 - 2x$ and for f the function is $f(x) = \frac{1-sign(x)}{2}$.

And, we also observe that f is the inverse of m. Now, let's take an example to see that

$$XOR(b_1, b_2, \ldots, b_n) = f\left(\prod_{i=1}^{n} m(b_i)\right) \tag{1}$$

Assume $b_1 = 0, b_2 = 1, b_3 = 1$

$$\begin{aligned} XOR(0,1,1) &= f(m(0) * m(1) * m(1)) \\ &= f(1 * -1 * -1) && \because m(x) = 1 - 2x \\ &= f(1) \\ &= 0 && \because f(x) = \frac{1 - sign(x)}{2} \end{aligned} \tag{2}$$

Hence, we can implement XOR in this manner.

2 Answer 2

(To prove) To exploit the above result, first give a mathematical proof that for any real numbers (that could be positive, negative, zero) r_1, r_2, . . . ,r_n for any n $\in$ N , we always have

$$\prod_{i=1}^{n} sign(r_i) = sign \prod_{i=1}^{n} r_i$$

(Proof) To get the sign of a number we can use the following:

$$sign(r) = \frac{|r|}{r} \tag{3}$$

where,

$$sign(r) = \begin{cases} -1, & \text{if } r < 0 \\ 1, & \text{if } r > 0 \\ 0, & \text{if } r = 0 \end{cases}$$

Using above definition we can say that,

$$\frac{|r_1||r_2|...|r_n|}{r_1 r_2 ... r_n} = \frac{|r_1 r_2 ... r_n|}{r_1 r_2 ... r_n}$$

proving this will prove the theorem.

2.1 Case 1

For, $r_i \neq 0 \ \forall_{i=0}^{n} r_i$

$$\begin{aligned}
\text{L.H.S } &= \prod_{i=1}^{n} sign(r_1) \\
&= sign(r_1) * sign(r_2) * \cdots * sign(r_n) \\
&= \frac{|r_1|}{r_1} * \frac{|r_2|}{r_2} * \cdots * \frac{|r_n|}{r_n} \\
&= \frac{|r_1||r_2|...|r_n|}{r_1 r_2 ... r_n} \qquad (\because \text{ Product term amount to positive value})
\end{aligned} \tag{4}$$

$$sign \text{ of L.H.S depends on } (\frac{1}{r_1 r_2 ... r_n})$$

$$\begin{aligned}
\text{R.H.S } &= sign \prod_{i=1}^{n} r_i \\
&= sign[(r_1) * (r_2) * \cdots * (r_n)] \\
&= \frac{|r_1 * r_2 * \cdots * r_n|}{r_1 * r_2 * \cdots * r_n} \\
&= \frac{|r_1 r_2 \dots r_n|}{r_1 r_2 ... r_n}
\end{aligned} \tag{5}$$

Let $r_1 * r_2 * \cdots * r_n$ = x,
Then, we get from equation 3 $\frac{|x|}{x}$, and

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x > 0 \end{cases}$$

So, anyways numerator becomes positive. As it is a modular function. So,

$$sign \text{ of R.H.S depends on } (\frac{1}{r_1 r_2 \dots r_n}) \qquad (\because \text{ The numerator itself is positive})$$

L.H.S=R.H.S

2.2 Case 2

For, $r_i = 0 \ \forall_{i=0}^{n} r_i$

As $sign(0) = 0$. So, we don't care about other values and simply the whole value becomes zero for both L.H.S and R.H.S.

L.H.S=R.H.S
Hence Proved.

3 Answer 3

We have to prove that, we can map 9-Dimensional vector to D-Dimensional vector:

$$R^9 \to R^D$$

such that, $\forall (\tilde{u}, \tilde{v}, \tilde{w}), \exists w \in R^D$
Mathematically,

$$(\tilde{u}^T \tilde{x}).(\tilde{v}^T \tilde{x}).(\tilde{w}^T \tilde{x}) = w^T.\phi(\tilde{x}) \tag{6}$$

$$
\begin{aligned}
L.H.S &= (\tilde{u}^T\tilde{x}).(\tilde{v}^T\tilde{x}).(\tilde{w}^T\tilde{x}) \\
&= (\Sigma_{i=1}^{9}\tilde{u}_i\tilde{x}_i)(\Sigma_{j=1}^{9}\tilde{u}_j\tilde{x}_j)(\Sigma_{k=1}^{9}\tilde{u}_k\tilde{x}_k) \\
&= \Sigma_{i=1}^{9}\Sigma_{j=1}^{9}\Sigma_{k=1}^{9}\,\tilde{u}_i\tilde{u}_j\tilde{u}_k\tilde{x}_i\tilde{x}_j\tilde{x}_k
\end{aligned}
$$

We have to map from $\tilde{x} \to \phi(\tilde{x})$ such that $\tilde{x}$ is a 9-Dimensional and $\phi(\tilde{x})$ is 729 dimension.
So, we get

$\tilde{x} = (\tilde{x}_1\tilde{x}_2\ldots\tilde{x}_9) \to \phi(\tilde{x}) = (\tilde{x}_1\tilde{x}_1\tilde{x}_1, \tilde{x}_1\tilde{x}_1\tilde{x}_2, \ldots, \tilde{x}_1\tilde{x}_1\tilde{x}_9, \tilde{x}_1\tilde{x}_2\tilde{x}_1, \ldots, \tilde{x}_9\tilde{x}_9\tilde{x}_9)$

Now, we get

$$(\tilde{u}^T\tilde{x}).(\tilde{v}^T\tilde{y}).(\tilde{w}^T\tilde{z}) = w^T.\phi(\tilde{x})$$

where, w = $(\tilde{u}_1\tilde{v}_1\tilde{w}_1, \tilde{u}_1\tilde{v}_1\tilde{w}_2, \ldots, \tilde{u}_1\tilde{v}_1\tilde{w}_9, \tilde{u}_1\tilde{v}_2\tilde{w}_1, \ldots\ , \tilde{u}_9\tilde{v}_9\tilde{w}_9)$

and so w is of 729 dimension. So, we proved

$$(\tilde{u}^T\tilde{x}).(\tilde{v}^T\tilde{y}).(\tilde{w}^T\tilde{z}) = w^T.\phi(\tilde{x})$$

i.e we can map 9-Dimensional to 729 Dimensional vector.

where $\exists w \in R^D$ for any $(\tilde{u}, \tilde{v}, \tilde{w})$ such that $\forall \tilde{x} \in R^9$.

4 Answer 4

<Code Link>

5 Answer 5

5.1 Hyperparameters used in Code(Q4)

There are 3 Hyperparameters in the code submitted

1) Step Length or Learning Rate 'lr'
2) Correction Factor or lambda 'la'
3) Dynamic Epoch '_epoch'

5.2 Explanation for selection of Hyperparameters

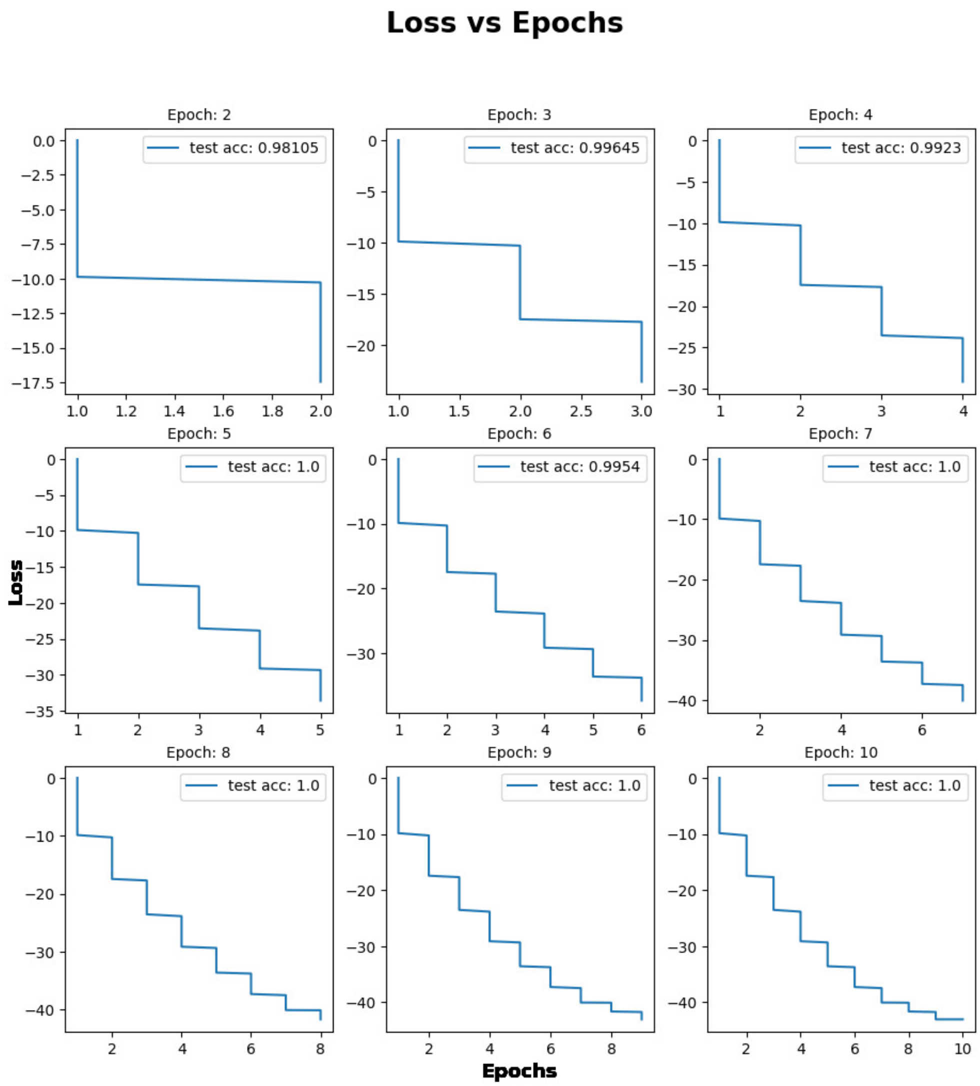
1) To obtain optimised values of Step Length and Correction Factor or Lambda, **grid search** has been used.

- Step length 'lr' was iterated in three different ranges of $[0.20e^{-01}\ to\ 0.29e^{-01}], [0.20e^{-02}\ to\ 0.29e^{-02}], [0.20e^{-03}\ to\ 0.29e^{-03}]$
- Correction Factor 'la' was iterated in three different ranges of $[0.20e^{-01}\ to\ 0.29e^{-01}], [0.20e^{-02}\ to\ 0.29e^{-02}], [0.20e^{-03}\ to\ 0.29e^{-03}]$
- Step length value of $0.23e^{-02}$ and Correction Factor value of $0.20e^{-03}$ have been obtained as optimised values from this grid search operation

2) To decide the total number of epochs '_epoch', we chose a dynamic epoch which varies based on number of input data points using ceil() function as

$$\_epochs = \lceil(\frac{epochs * 10000}{y.size})\rceil$$

- To arrive at ‘epochs’ value of 5, a curve was plotted between hinge loss and number of epochs for 10000 train datapoints, number of epochs varying from 2 to 10. It was found that at epochs 5, model reached maximum of 100 percent accuracy , then decreased in accuracy followed by a subsequent increase in accuracy. It was speculated that model started overfitting beyond epoch 5. Hence, 5 has been selected as ‘epochs’ value.
- Plot between Hinge Loss and Number of Epochs has been included below



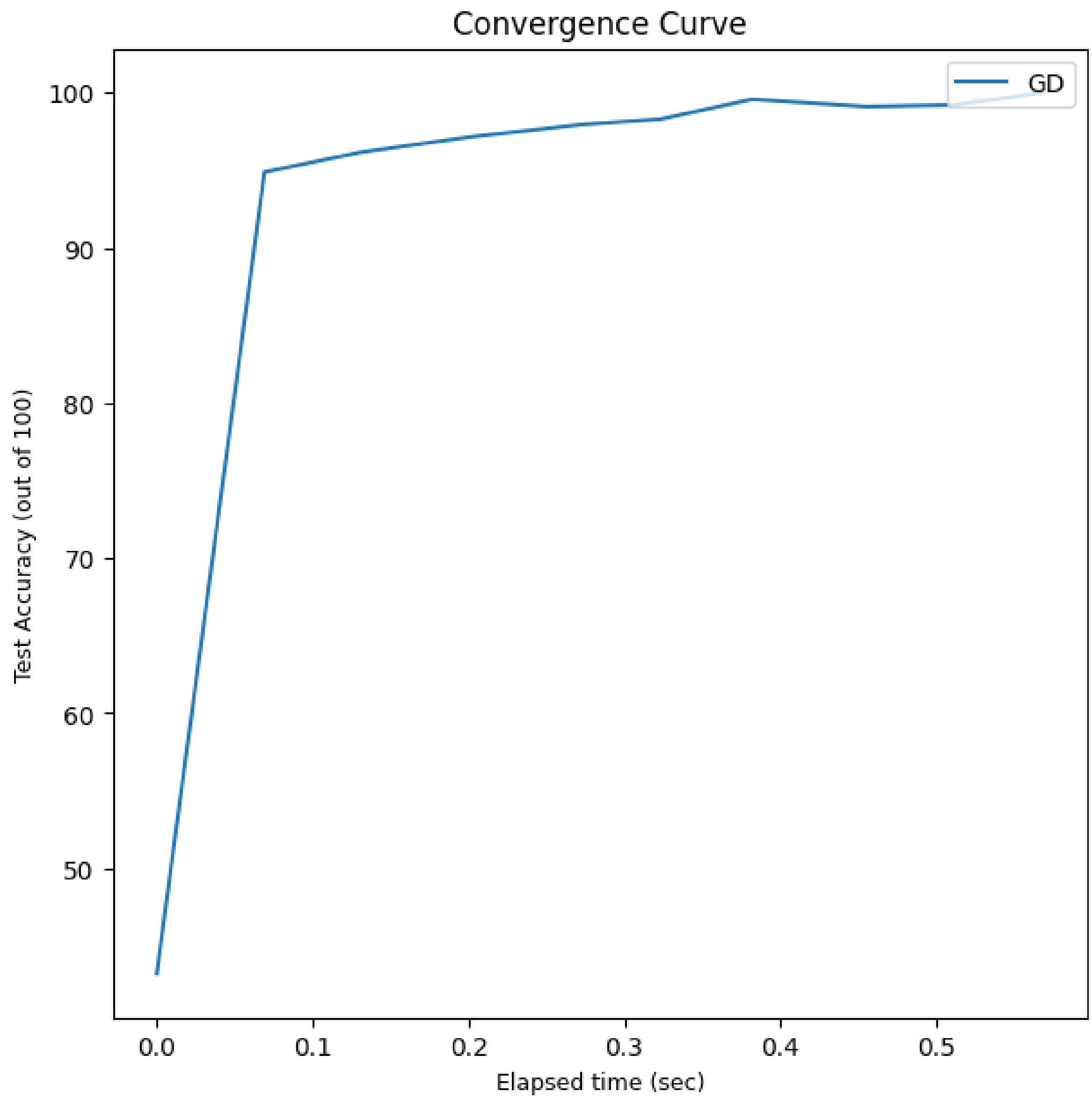
5.3 Explanation for initialization of weights and bias term

Both the Weights and bias term have been initialised with zero as it was found to be the fastest in converging to a optimised solution.

6 Answer 6

6.1 Convergence Curve

Convergence Curve between Time taken and test classificaton Accuracy has been included below. GD stands for Gradient Descent solver method



7 Citations, figures, tables, references

7.1 Figures

Acknowledgments and Disclosure of Funding

Use unnumbered first level headings for the acknowledgments. All acknowledgments go at the end of the paper before the list of references. Moreover, you are required to declare funding (financial activities supporting the submitted work) and competing interests (related financial activities outside the submitted work). More information about this disclosure can be found at: `https://neurips.cc/Conferences/2022/PaperInformation/FundingDisclosure`.

Do **not** include this section in the anonymized submission, only in the final paper. You can use the `ack` environment provided in the style file to autmoatically hide this section in the anonymized submission.

Figure 1: Sample figure caption.

References

References follow the acknowledgments. Use unnumbered first-level heading for the references. Any choice of citation style is acceptable as long as you are consistent. It is permissible to reduce the font size to `small` (9 point) when listing the references. Note that the Reference section does not count towards the page limit.

[1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D.S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems 7*, pp. 609–616. Cambridge, MA: MIT Press.

[2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GEneral NEural SImulation System.* New York: TELOS/Springer–Verlag.

[3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.

Checklist

The checklist follows the references. Please read the checklist guidelines carefully for information on how to answer these questions. For each question, change the default **[TODO]** to [Yes] , [No] , or [N/A] . You are strongly encouraged to include a **justification to your answer**, either by referencing the appropriate section of your paper or providing a brief inline description. For example:

- Did you include the license to the code and datasets? [Yes] See Section 2.
- Did you include the license to the code and datasets? [No] The code and the data are proprietary.
- Did you include the license to the code and datasets? [N/A]

Please do not modify the questions and only use the provided macros for your answers. Note that the Checklist section does not count towards the page limit. In your paper, please delete this instructions block and only keep the Checklist section heading above along with the questions/answers below.

1. Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? **[TODO]**
2. Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? **[TODO]**
3. Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? **[TODO]**
4. Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? **[TODO]**

A Appendix

Optionally include extra information (complete proofs, additional experiments and plots) in the appendix. This section will often be part of the supplemental material.