# Assignment 2

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CS771 Introduction to Machine Learning
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# 1 Answer 1

As we have to map binary digits 0,1 to signs -1,+1,
We should make a function say, $m : \{0,1\} \rightarrow \{-1,+1\}$. Such that

$$m(x) = \begin{cases} -1, & \text{if } x = 1 \\ +1, & \text{if } x = 0 \end{cases}$$

and another function named $f : \{-1,+1\} \rightarrow \{0,1\}$ and it should be such that

$$f(x) = \begin{cases} 0, & \text{if } x = +1 \\ 1, & \text{if } x = -1 \end{cases}$$

So, for $m$ the fuction is $m(x) = 1 - 2x$ and for $f$ the function is $f(x) = \frac{1-sign(x)}{2}$.

And, we also observe that $f$ is the inverse of $m$. Now, let's take an example to see that

$$XOR(b_1, b_2, \ldots, b_n) = f\left(\prod_{i=1}^{n} m(b_i)\right) \tag{1}$$

Assume $b_1 = 0, b_2 = 1, b_3 = 1$

$$\begin{aligned} XOR(0,1,1) &= f(m(0) * m(1) * m(1)) \\ &= f(1 * -1 * -1) && \because m(x) = 1 - 2x \\ &= f(1) \\ &= 0 && \because f(x) = \frac{1 - sign(x)}{2} \end{aligned} \tag{2}$$

Hence, we can implement XOR in this manner.

# 2 Answer 2

**(To prove)** To exploit the above result, first give a mathematical proof that for any real numbers (that could be positive, negative, zero) $r_1$, $r_2$, . . . ,$r_n$ for any n $\in$ N , we always have

$$\prod_{i=1}^{n} sign(r_i) = sign \prod_{i=1}^{n} r_i$$

**(Proof)** To get the sign of a number we can use the following:

$$sign(r) = \frac{|r|}{r} \tag{3}$$

where,

$$sign(r) = \begin{cases} -1, & \text{if } r < 0 \\ 1, & \text{if } r > 0 \\ 0, & \text{if } r = 0 \end{cases}$$

Using above definition we can say that,

$$\frac{|r_1||r_2|...|r_n|}{r_1 r_2 ... r_n} = \frac{|r_1 r_2 ... r_n|}{r_1 r_2 ... r_n}$$

proving this will prove the theorem.

## 2.1 Case 1

For, $r_i \neq 0 \; \forall_{i=0}^{n} r_i$

$$\begin{aligned}
\text{L.H.S } &= \prod_{i=1}^{n} sign(r_1) \\
&= sign(r_1) * sign(r_2) * \cdots * sign(r_n) \\
&= \frac{|r_1|}{r_1} * \frac{|r_2|}{r_2} * \cdots * \frac{|r_n|}{r_n} \\
&= \frac{|r_1||r_2|...|r_n|}{r_1 r_2 ... r_n} \qquad (\because \text{Product term amount to positive value})
\end{aligned} \tag{4}$$

$$sign \text{ of L.H.S depends on } (\frac{1}{r_1 r_2 ... r_n})$$

$$\begin{aligned}
\text{R.H.S } &= sign \prod_{i=1}^{n} r_i \\
&= sign[(r_1) * (r_2) * \cdots * (r_n)] \\
&= \frac{|r_1 * r_2 * \cdots * r_n|}{r_1 * r_2 * \cdots * r_n} \\
&= \frac{|r_1 r_2 \ldots r_n|}{r_1 r_2 ... r_n} \qquad (\because \text{The numerator itself is positive})
\end{aligned} \tag{5}$$

Let $r_1 * r_2 * \cdots * r_n$ = x,
Then, we get from equation 3 $\frac{|x|}{x}$, and

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x > 0 \end{cases}$$

So, anyways numerator becomes positive. As it is a modular function. So,

$$sign \text{ of R.H.S depends on } (\frac{1}{r_1 r_2 \ldots r_n})$$

L.H.S=R.H.S

## 2.2 Case 2

For, $r_i = 0 \; \forall_{i=0}^{n} r_i$

As $sign(0) = 0$. So, we don't care about other values and simply the whole value becomes zero for both L.H.S and R.H.S.

L.H.S=R.H.S
Hence Proved.

# 3 Answer 3

We have to prove that, we can map 9-Dimensional vector to D-Dimensional vector:

$$R^9 \rightarrow R^D$$

such that, $\forall (\tilde{u}, \tilde{v}, \tilde{w}), \exists w \in R^D$
Mathematically,

$$(\tilde{u}^T \tilde{x}).(\tilde{v}^T \tilde{x}).(\tilde{w}^T \tilde{x}) = w^T.\phi(\tilde{x}) \tag{6}$$

$$\begin{aligned} L.H.S &= (\tilde{u}^T\tilde{x}).(\tilde{v}^T\tilde{x}).(\tilde{w}^T\tilde{x}) \\ &= (\Sigma_{i=1}^{9}\tilde{u}_i\tilde{x}_i)(\Sigma_{j=1}^{9}\tilde{u}_j\tilde{x}_j)(\Sigma_{k=1}^{9}\tilde{u}_k\tilde{x}_k) \\ &= \Sigma_{i=1}^{9}\Sigma_{j=1}^{9}\Sigma_{k=1}^{9}\,\tilde{u}_i\tilde{u}_j\tilde{u}_k\tilde{x}_i\tilde{x}_j\tilde{x}_k \end{aligned}$$

We have to map from $\tilde{x} \to \phi(\tilde{x})$ such that $\tilde{x}$ is a 9-Dimensional and $\phi(\tilde{x})$ is 729 dimension.
So, we get

$\tilde{x} = (\tilde{x}_1\tilde{x}_2\ldots\tilde{x}_9) \to \phi(\tilde{x}) = (\tilde{x}_1\tilde{x}_1\tilde{x}_1, \tilde{x}_1\tilde{x}_1\tilde{x}_2, \ldots, \tilde{x}_1\tilde{x}_1\tilde{x}_9, \tilde{x}_1\tilde{x}_2\tilde{x}_1, \ldots, \tilde{x}_9\tilde{x}_9\tilde{x}_9)$

Now, we get

$$(\tilde{u}^T\tilde{x}).(\tilde{v}^T\tilde{y}).(\tilde{w}^T\tilde{z}) = w^T.\phi(\tilde{x})$$

where, w = $(\tilde{u}_1\tilde{v}_1\tilde{w}_1, \tilde{u}_1\tilde{v}_1\tilde{w}_2, \ldots, \tilde{u}_1\tilde{v}_1\tilde{w}_9, \tilde{u}_1\tilde{v}_2\tilde{w}_1, \ldots\ , \tilde{u}_9\tilde{v}_9\tilde{w}_9)$

and so w is of 729 dimension. So, we proved

$$(\tilde{u}^T\tilde{x}).(\tilde{v}^T\tilde{y}).(\tilde{w}^T\tilde{z}) = w^T.\phi(\tilde{x})$$

i.e we can map 9-Dimensional to 729 Dimensional vector.

where $\exists w \in R^D$ for any $(\tilde{u}, \tilde{v}, \tilde{w})$ such that $\forall \tilde{x} \in R^9$.

# 4 Answer 4

<Code Link> : https://github.com/phanisai97/mlass1/raw/main/submit.zip

# 5 Answer 5

## 5.1 Hyperparameters used in Code(Q4)

There are 3 Hyperparameters in the code submitted

1) Step Length or Learning Rate 'lr'
2) Correction Factor or lambda 'la'
3) Dynamic Epoch '_epoch'

## 5.2 Explanation for selection of Hyperparameters

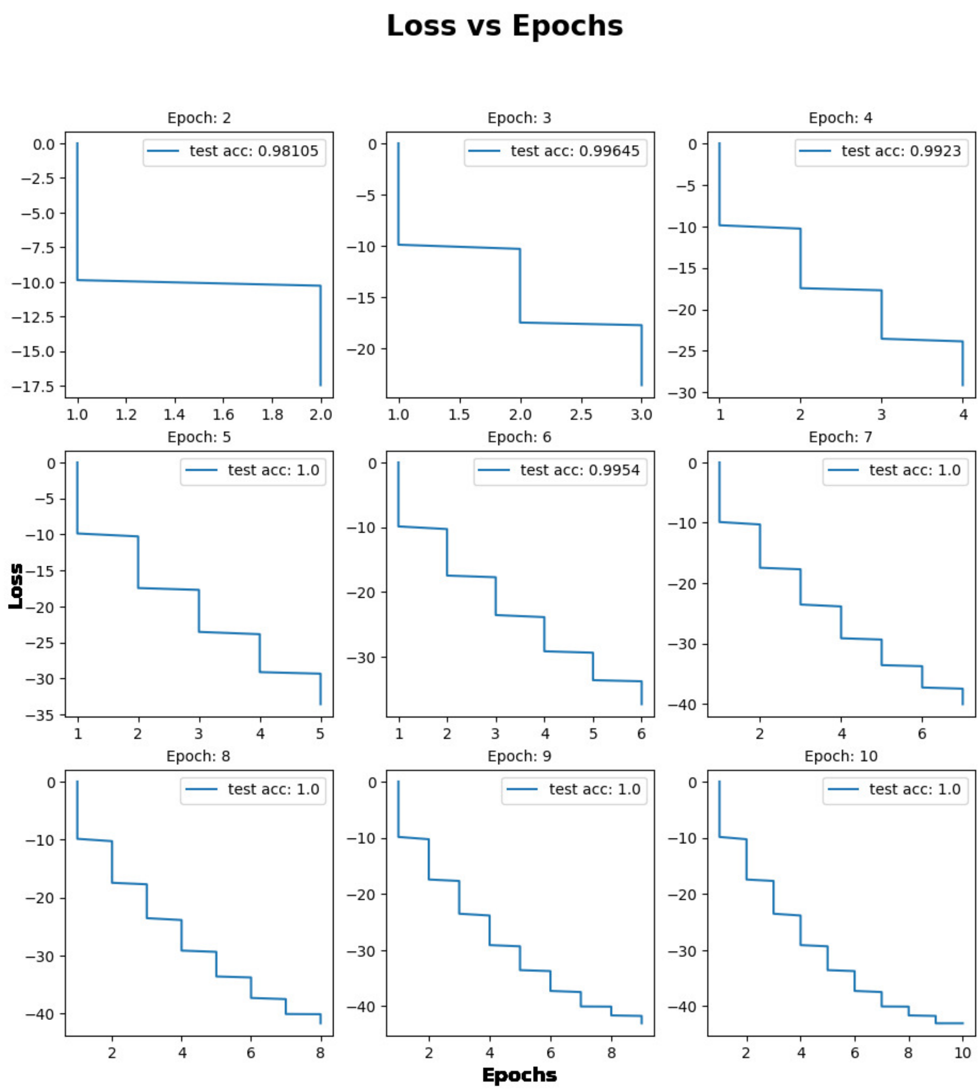
1) To obtain optimised values of Step Length and Correction Factor or Lambda, **grid search** has been used.
- Step length 'lr' was iterated in three different ranges of $[0.20e^{-01}\ to\ 0.29e^{-01}], [0.20e^{-02}\ to\ 0.29e^{-02}], [0.20e^{-03}\ to\ 0.29e^{-03}]$
- Correction Factor 'la' was iterated in three different ranges of $[0.20e^{-01}\ to\ 0.29e^{-01}], [0.20e^{-02}\ to\ 0.29e^{-02}], [0.20e^{-03}\ to\ 0.29e^{-03}]$
- Step length value of $0.23e^{-02}$ and Correction Factor value of $0.20e^{-03}$ have been obtained as optimised values from this grid search operation
2) To decide the total number of epochs '_epoch', we chose a dynamic epoch which varies based on number of input data points using ceil() function as

$$\_epochs = \lceil(\frac{epochs * 10000}{y.size})\rceil$$

10000 is the size of the train data set with which optimum epochs has been obtained. The epochs gets scaled with the number of train data points , as per the formula.

- To arrive at optimum 'epochs' value of 5, a curve was plotted between hinge loss and number of epochs for 10000 train data points, number of epochs varying from 2 to 10. It was found that at epochs 5, model reached maximum of 100 percent accuracy , then decreased in accuracy followed by a subsequent increase in accuracy. It was speculated that model started over fitting beyond epoch 5. Hence, 5 has been selected as 'epochs' value.

- Plot between Hinge Loss and Number of Epochs has been included below



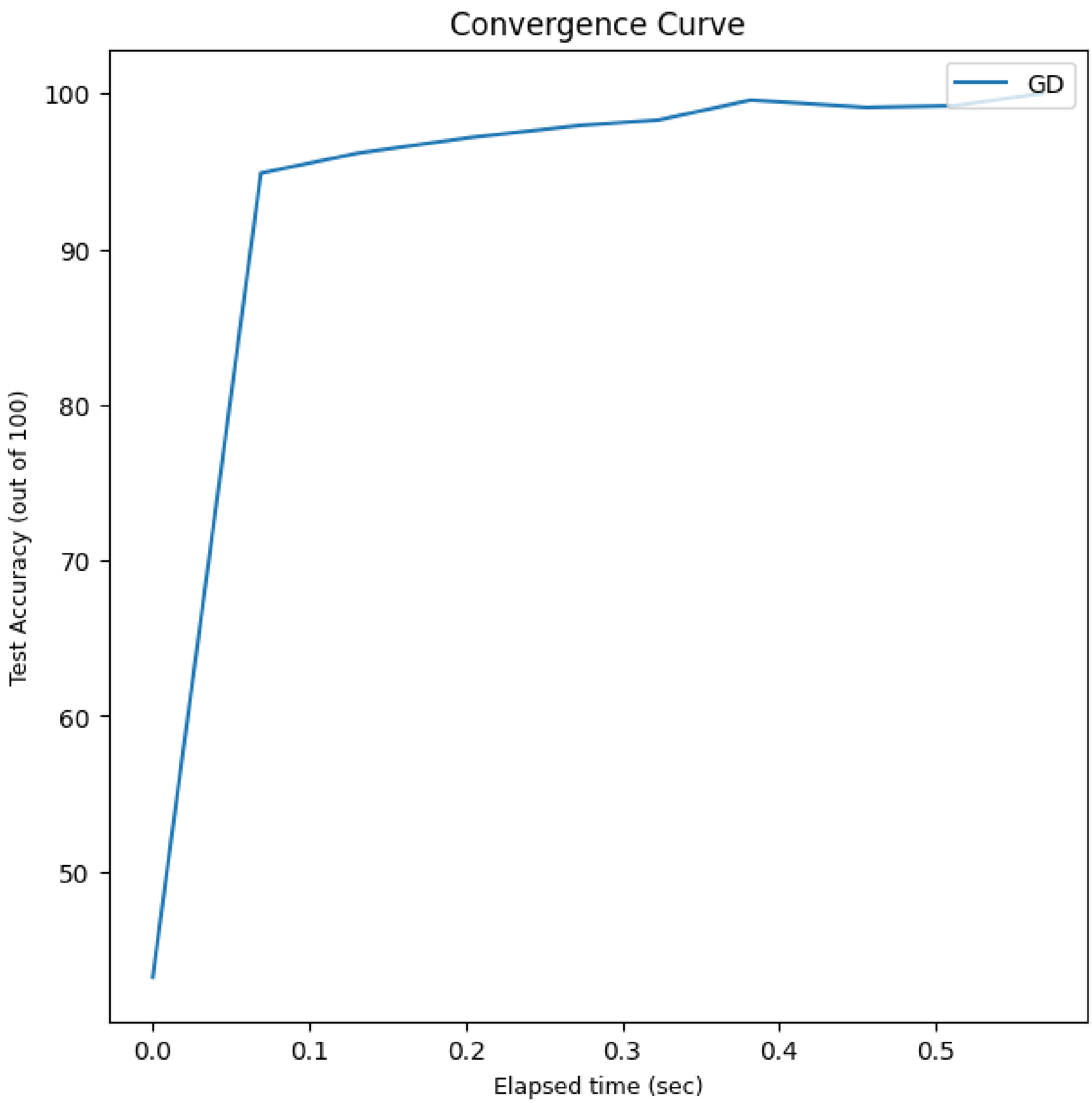

### 5.3 Explanation for initialization of weights and bias term

Both the Weights and bias term have been initialised with zero as it was found to be the fastest in converging to a optimised solution.

# 6 Answer 6

## 6.1 Convergence Curve

Convergence Curve between Time taken and test classificaton Accuracy has been included below.
GD stands for Gradient Descent solver method




# 7 References

[1] https://programmathically.com/understanding-hinge-loss-and-the-svm-cost-function/

[2] https://web.cse.iitk.ac.in/users/purushot/courses/ml/2022-23-a/discussion.html