

3A) To express $(\tilde{u}^T \tilde{x}) (\tilde{v} \cdot \tilde{x}) (\tilde{w}^T \tilde{x})$.

$$\tilde{u}^T \tilde{x} = \sum_{i=1}^n u_i x_i$$

$$\tilde{v}^T \tilde{x} = \sum_{j=1}^n v_j x_j$$

$$\tilde{w}^T \tilde{x} = \sum_{k=1}^n w_k x_k$$

$$(\tilde{u}^T \tilde{x}) (\tilde{v} \cdot \tilde{x}) (\tilde{w} \cdot \tilde{x}) = \left(\sum_{i=1}^n u_i x_i \right) \cdot \left(\sum_{j=1}^n v_j x_j \right) \cdot \left(\sum_{k=1}^n w_k x_k \right)$$

$$= \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n u_i v_j w_k (x_i x_j x_k)$$

$$= \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n w_{ijk} \phi_{ijk}(\tilde{x})$$

where $w_{ijk} = u_i v_j w_k$ $\phi_{ijk}(\tilde{x}) = x_i x_j x_k$

$$= W^T \phi(\tilde{x})$$

$$W = [w_{ijk}] = \begin{bmatrix} u_1 v_1 w_1 & u_1 v_1 w_2 & \dots & u_1 v_1 w_n \\ u_1 v_2 w_1 & u_1 v_2 w_2 & \dots & u_1 v_2 w_n \\ \vdots & \vdots & \ddots & \vdots \\ u_n v_n w_1 & u_n v_n w_2 & \dots & u_n v_n w_n \end{bmatrix}$$

$$\phi(\tilde{x}) = \begin{bmatrix} x_1 x_1 x_1 & x_1 x_1 x_2 & \dots & x_n x_n x_n \end{bmatrix}$$