Matrix Condition Number

a. Explain the notion of condition number of a matrix.

The condition number of a function shows how the output value of the function can change for a small change in the input argument.it can be used to measure how sensitive a function is to changes or errors in the input, and how much error in the output results from an error in the input.

$$K(A) \triangleq \|A\| \cdot \|A^{-1}\|$$

Provided A is non singular

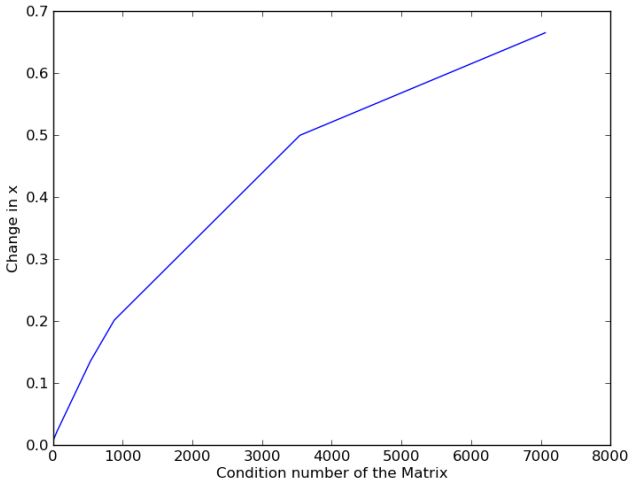
$$\frac{\|\Delta x\|}{\|x\|} \leq K(A) \cdot \frac{\|\Delta b\|}{\|b\|}$$

In Ax=b here K(A) is the condition number of matrix A.

When the condition number K(A) becomes large, the system is regarded as being ill conditioned. Matrices with condition numbers near 1 are said to be well-conditioned.

b. How does it affect the accuracy of a numerical computation?

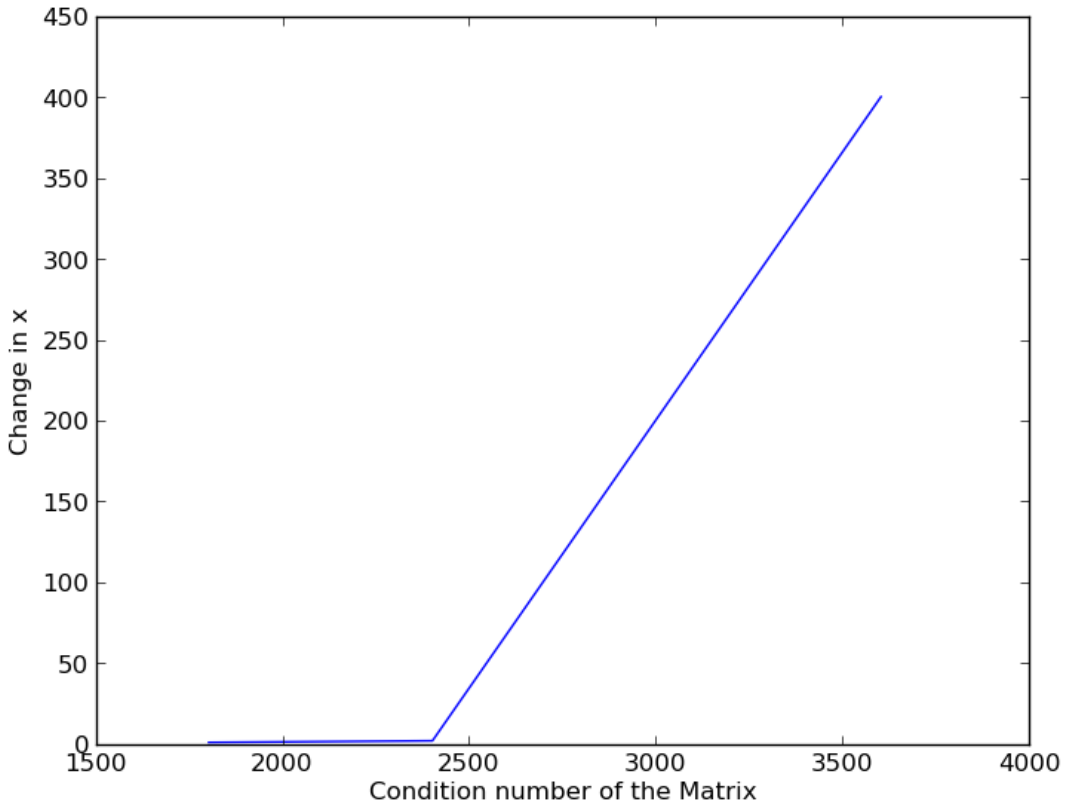
When the condition number K(A) becomes large, the system is regarded as being ill-conditioned. For a system with condition number K(A) , expect a loss of roughly $log_{10} K(A)$ decimal places in the accuracy of the solution.



c. Solve the following problem

Plot1

Plot 2


450
400
350
300
250
200
150
100
50
0
Change in x
1500
2000
2500
3000
3500
4000
Condition number of the Matrix