**Problem 2: Implement K-SVD algorithm for dictionary learning and sparse coding. K-SVD algorithm learns a set of basis vectors that represents the data, similar to PCA. However, unlike PCA coefficients, K-SVD aims at finding sparse coefficients.**

**Your input is a set of $d$ dimensional vectors and you are supposed to learn the dictionary or basis that defines the data. When $d = 2$, show the dictionary elements by plotting.**

**Given a set of data $X = [x_1, x_2, \ldots, ..x_n]$, $x_i \in \mathbb{R}^d$, dictionary learning can be formulated as**

$$\hat{\alpha_i} = \arg\min_{\alpha_i, D} \sum_{i=1}^{i=n} ||x_i - D\ \alpha_i||_2 \quad \textbf{s.t.} \quad ||\alpha_i||_0 < k_0 \tag{1}$$

**where $D = [D_1, D_2, \ldots, D_L] \in \mathbb{R}^{d \times L} (L >> d)$ is known as dictionary or basis, $||.||_0$ denote $l_0$-norm which indicates the number of non-zero entries in a vector and $k_0$ denote the sparsity threshold ($k_0 < d$).**

**Please refer to [1] for optimizing the above problem. It invloves solving for two unknowns $\alpha_i$ and $D$. K-SVD is an iterative algorithm that repeats the two basic steps.**

- **Step1: (Sparse coding) In this step, fix $D$ and obtain $\alpha_i$ for every $x_i$,**

$$\hat{\alpha_i} = \arg\min_{\alpha_i} \sum_{i=1}^{i=n} ||x_i - D\ \alpha_i||_2 \quad \textbf{s.t.} \quad ||\alpha_i||_0 < k_0 \tag{2}$$

  **You need not implement this step. We will provide the matlab implementation to obtain $\alpha_i$. Call the matlab function (OMP.m) as follows.**

  **[alpha] = OMP(D,X,$k_0$);**

- **Step2 : (dictionary Update) Fix $\alpha_i$ obtained in the previous step and obtain $D_i$ for $i = 1, 2, \ldots, L$**

$$\hat{D} = \arg\min_{\alpha_i} \sum_{i=1}^{i=n} ||x_i - D\ \alpha_i||_2 \quad \textbf{s.t.} \quad ||\alpha_i||_0 < k_0 \tag{3}$$

  **Refer to [1] for the details of this step.**

  **Iterate the steps 1 and 2 till the convergence is reached.**

**Plot the error $\sum_{i=1}^{i=n} ||x_i - D\ \alpha_i||_2$ as a function of iteration. For $d = 2$ and $k_0 = 1$, plot the given data points along with obtained dictionary elements.**

**For $d = 2$ and $k_0 = 1$, plot the given input data points along with dictionary elements.**

**Reference:**

*1. M. Aharon, M. Elad, A.M. Bruckstein, K-SVD: An algorithm for designing of overcomplete dictionaries for sparse representation, IEEE Trans. on Signal Processing, 2006.*