

$$\vec{x}[t+1] = \lambda \vec{x}[t] + \omega[t]$$

$$\omega[t] = -\alpha (d^T[t] \times [t]) d[t]$$

$$\begin{aligned}
 & (\lambda - \alpha \gamma_1) (\lambda - \alpha \beta_1) \mathbf{v}_1 \mathbf{v}_1^T \\
 & + \sum_{i=2}^N \alpha^2 \beta_i^2 \gamma_i^2 \mathbf{v}_i \mathbf{v}_i^T \\
 & + \alpha^2 \sum_{i \neq j} \left(\begin{aligned} & \mathbf{v}_i \mathbf{v}_k^T \mathbf{v}_k \mathbf{v}_j \\ & \beta_i \beta_k \gamma_k \gamma_j \\ & \beta_i (\sum_k \beta_k \gamma_k) \gamma_j \end{aligned} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{x}[t+1] &= \lambda (\mathbf{v}^T \mathbf{x}[t]) \mathbf{v} + \mathbf{u}[t] \\
 \mathbf{u}[t] &= -\lambda (\mathbf{d}^T[t] \mathbf{x}[t]) \mathbf{d}[t] \\
 \mathbf{x}[t+1] &= \lambda (\mathbf{v}^T \mathbf{x}[t]) \mathbf{v} - \lambda (\mathbf{d}^T[t] \mathbf{x}[t]) \mathbf{d}[t] \\
 &= \lambda (\mathbf{v} \mathbf{v}^T - \mathbf{d} \mathbf{d}^T) \mathbf{x}[t] \\
 &\rightarrow \lambda (\hat{\mathbf{v}} (\hat{\mathbf{v}}^T \mathbf{x}[t]) - \alpha \mathbf{d}[t] \mathbf{d}^T[t] \mathbf{x}[t])
 \end{aligned}$$

$$\begin{aligned}
 \lambda \hat{\mathbf{v}}_1 \hat{\mathbf{v}}_1^T - \alpha \left(\sum \beta_i \hat{\mathbf{v}}_i \right) \left(\sum \beta_i \hat{\mathbf{v}}_i^T \right) \\
 \lambda \hat{\mathbf{v}}_1 \hat{\mathbf{v}}_1^T - \alpha \beta_1^2 \hat{\mathbf{v}}_1 \hat{\mathbf{v}}_1^T + \dots
 \end{aligned}$$

$$\begin{aligned}
 &= (\lambda - \alpha \beta_1^2) \hat{\mathbf{v}}_1 \hat{\mathbf{v}}_1^T + \\
 & \alpha \sum_{i \neq j} \beta_i \beta_j \hat{\mathbf{v}}_i \hat{\mathbf{v}}_j^T + \sum_{i=2}^N \alpha \beta_i^2 \hat{\mathbf{v}}_i \hat{\mathbf{v}}_i^T \\
 & \beta_i^2 \sim \frac{1}{N} \quad \alpha \leq N \text{ (average)}
 \end{aligned}$$