$x[1] = T(A x[0] + \vec{u}[0])$

$A = VDV^T$

$Ax[0] = VDV^T x[0]$

We have

$$E(A\vec{x}[0]) = E\left(\sum_i \lambda_i (\vec{x}[0]^T \vec{v}_i)\vec{v}_i\right)$$
$$= \sum_i E(\lambda_i) E((\vec{x}[0]^T \vec{v}_i)\vec{v}_i),$$

since λ_i and $(\vec{x}[0]^T \vec{v}_i)\vec{v}_i$ are independent for each i. Clearly,

$$E(\lambda_i) = \hat{\lambda}_i.$$

Since $\vec{x}[0]$ is constrained to be uniformly distributed over a circle centered at $y[0]\vec{d}[0]$, we have that

$$E((\vec{x}[0]^T \vec{v}_i)\vec{v}_i) = \int_{\vec{v}} E((\vec{x}[0]^T \vec{v}_i)\vec{v}_i \mid \vec{v}_i = \vec{v}) f_{\vec{v}_i}(\vec{v})\, d\vec{v}_i$$
$$= \int_{\vec{v}} y[0]\vec{d}[0]\vec{v}^T \vec{v} f_{\vec{v}_i}(\vec{v})\, d\vec{v}$$
$$= y[0]\vec{d}[0] \int_{\vec{v}} \vec{v}\vec{v}^T f_{\vec{v}_i}(\vec{v}) d\vec{v}$$
$$= y[0]\vec{d}[0] E(\vec{v}_i \vec{v}_i^T).$$

$\|x[1]\|$

Therefore, we have

$x(1) = Ax(0) + u[0]$

$$E(A\vec{x}[0]) = \sum_i \hat{\lambda}_i y[0]\vec{d}[0] \underbrace{E(\vec{v}_i \vec{v}_i^T)}_{= \frac{1}{N} I}$$
$$= \frac{1}{N} \sum_i \hat{\lambda}_i \vec{y}[0]\vec{d}[0],$$

so our optimal control should be

$$\vec{u}[0] = -\left(\frac{1}{N}\sum_i \hat{\lambda}_i\right) \vec{y}[0]\vec{d}[0].$$

After applying this control, we can look at the expected value of the magnitude of $\vec{x}[1]$. We find that

$d[0]^T x[0] = y[0]$

$d^T[1]\, x[1]$

→ spherical symmetry

$E(x^n]$

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Extra page for scratchwork.
Work on this page will NOT be graded.

$y[t] = \vec{d}[t]^T \vec{x}[t]$

Λ

$V \Lambda V^T \vec{x}$

$\downarrow$

$\Lambda \vec{z}$

$y[t] = \vec{d}[t]^T V z[t]$

$\vec{v}_1$

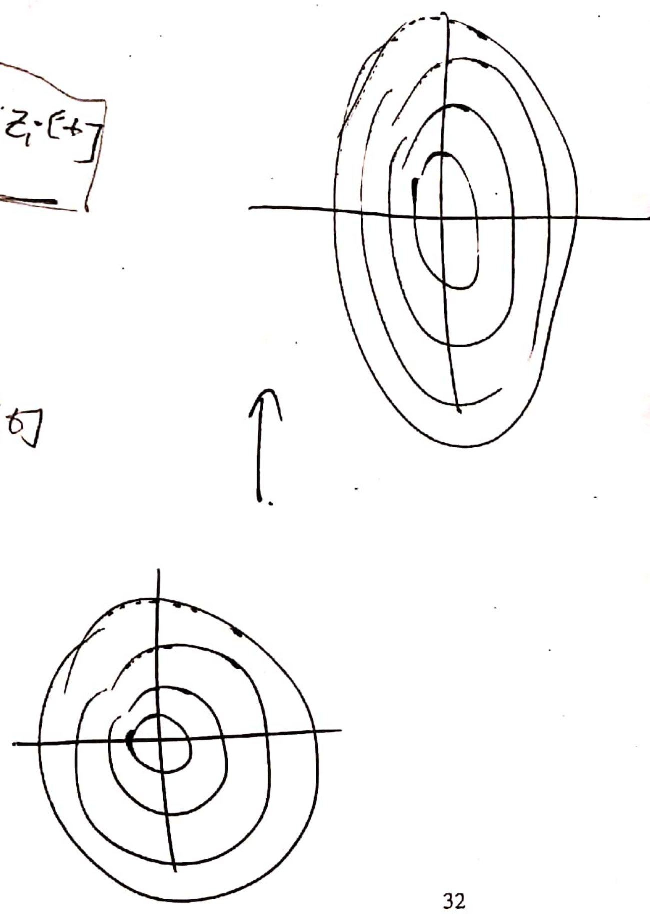
unknow

$\vec{d}[t] = \sum_{i=1}^{n} \alpha_i \vec{v}_i \rightarrow \vec{d}^T[t] \vec{v}_1$

$y[t] = \boxed{\alpha_1 z_1[t]} + \boxed{\overset{\alpha_1}{\sum_{i=2}^{n} \alpha_i z_i[t]}}$

$z_1[t+1] = \lambda_1 z[t] + \tilde{u}[t]$

$\vec{v}_1^T \vec{u}[t]$



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$E[\vec{x}_0^T \cdot \vec{v}_1]$

$= E[[y_0 \vec{d}_0 + \vec{\alpha} \cdot \vec{d}_0^{\perp}] \cdot \vec{v}_1]$.

$= y_0 \vec{d}_0 \cdot E[\vec{v}_1] + E[\vec{\alpha} \cdot \boxed{\vec{d}_0^{\perp} \cdot \vec{v}_1}]$

$x[t] = a\,x[t-1] + u[t]$.

$y[t] = c \cdot x[t]$.

$c \in [+1, -1]$.

$$x(t+1) = a \cdot x(t) + u(t).$$

$$y(0) = c_0 \cdot x(0). \qquad c = +1, -1$$

$$\cancel{x(0)} \quad y(0) = 2$$

$$u(0) = 1$$

$$x(1) = a \cdot x(0) + 1$$

$$y(1) = c_1 \cdot x(1) = c_1 \cdot a \cdot x(0) + \underbrace{c_1 \cdot u(0)}_{= \mathrm{sgn}(c_1)}$$

$$\mathrm{sgn}(c_0 c_1)\, y(0)$$