

PRINT your student ID: \_\_\_\_\_

How would we have  
to enforce not knowingabout  $\hat{e}_1, \hat{e}_2$   
w/ respect  
to  $\hat{b}$ ?

Extra page for scratchwork.

Work on this page will NOT be graded

$$\vec{v}$$

$$\hat{\vec{v}}$$

$$\vec{x} \cdot \vec{d} = |\vec{x}| \cos \phi$$

state mag.

(mostly in  $\vec{v}$ )

$$\vec{v} \cdot \vec{d} = \cos \phi$$

$$\hat{\vec{v}} \cdot \vec{d} = \cos(\phi + \theta)$$

$$\frac{\vec{x} \cdot \vec{d}}{\hat{\vec{v}} \cdot \vec{d}} = \frac{|\vec{x}| \cos \phi}{\cos(\phi + \theta)} = |\vec{x}|$$

$$\vec{x}_{k+1} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \vec{x}_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$\lambda_2 x_2[k] + k \frac{\cos \phi}{\cos(\phi + \theta)} |\vec{x}|$$

$$u = k |\hat{\vec{x}}|$$

$$u = k x_1$$

$$x_1[k+1] = (\lambda_1 + k) x_1[k]$$

$$x_1[k+1] = \left( \lambda_1 + k \frac{\cos \phi}{\cos(\phi + \theta)} \right) x_1[k]$$

(just see what happens  
stay concrete)

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Setup 1

$$\vec{X}(t+1) = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} \vec{X}(t) + \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} u(t)$$

$\lambda_1, \dots, \lambda_n$  unknown.

$\vec{b}$

Setup 2

$$\vec{X}(t+1) = P \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} P^{-1} \vec{X}(t) + \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} u(t)$$

Setup becomes equivalent when we randomize  $\vec{b}$  above  $P$

$$A = P \Lambda P^{-1}$$

$P$  orthogonal.

$P$  is unknown,  $\lambda_1, \dots, \lambda_n$  unknown.

But  $\lambda_1 \gg \lambda_2$

$\uparrow$

0

$\lambda_i$  not necessarily stable ( $|\lambda_i| \neq 1$ )

Other assumptions

- knowledge of  $\vec{x}(0)$
- noise  $w[k]$

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(a) (4 points) Express  $\bar{y}[0]$  in terms of  $\bar{x}[0]$ .

$$\bar{X}(t+1) = \begin{bmatrix} \lambda_1 & \dots & 0 \\ 0 & \dots & \lambda_n \end{bmatrix} \bar{X}(t) + I_{n \times n} \cdot \bar{u}(t)$$

① Don't observe  $\bar{x}$   
 $\bar{x} \cdot \bar{d}$   
 $\uparrow$  random.

$\lambda_1 \gg \lambda_2 > \lambda_3 \dots$   
 $e, 0$   
 $\uparrow$

ordered version.

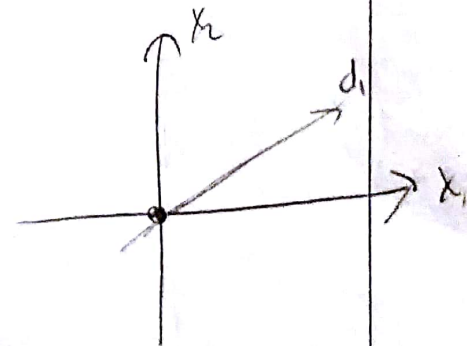
Unordered version.  $\exists$  one  $\lambda_i \gg \lambda_j \quad \forall j \neq i$

this case better understood

(b) (6 points) Express  $\bar{x}[1]$  and  $\bar{y}[1]$  in terms of  $\bar{x}[0]$ .

But I don't know i

How to answer Rahul's Q  
 of "how many observ.  
 needed?"  
 $\hookrightarrow$  consult lit.



Rahul's not knowing  $\vec{v} \Rightarrow \lambda \vec{v} \vec{v}^T$   
 $\Rightarrow$  compressed sensing

OMP? Sparsity, leverage this?

OMP when we're projecting

$\Downarrow$   
 Solve easier problem w/ known  $\bar{x}$   
 $n$  is sufficient

cross terms  
 as unknowns  
 $\Downarrow$   
 why not just  
 solve for  $n^2$   
 entries of  $A$

See if robust PCA sticks

$\leftarrow$  Lookup: every  $A$  under conditions  
 can be decomposed into a low rank  
 and sparse matrices (additively)