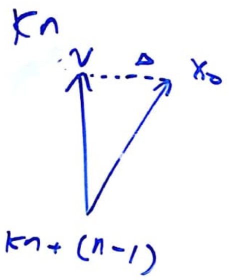


$\Delta[t] = \lambda_s^t \, \Delta[0]$

$\hat{v} = \vec{v} + \vec{\Delta}$

$\vec{x} \cdot \vec{d} =$ ___

$\sim \beta \vec{v}$

$\hat{v} \cdot \vec{d} =$ ___

$\vec{y}[i]$

T

$$\begin{bmatrix} \lambda_s \Delta[i] \\ \vdots \\ \Delta[i] \end{bmatrix} = \vec{\Delta}$$

$\lambda \lambda_s^k < 1$

$\Delta[i+n] = \lambda_s^n \Delta[i]$

$\log(\Delta[i+n]) = \log(\lambda_s^n) + \log(\Delta[i])$

$= n\log(\lambda_s) + \log(\Delta[i])$

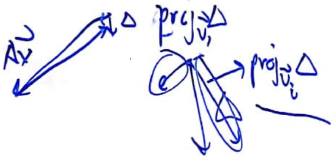
$$\boxed{-\log\left(\frac{1}{\lambda_s}\right)}$$

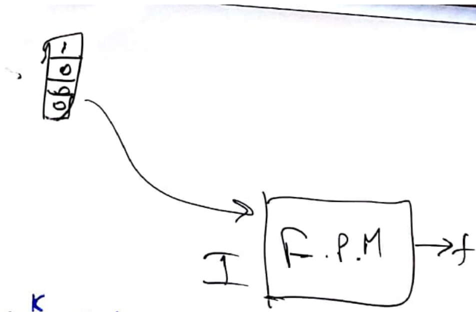
slip

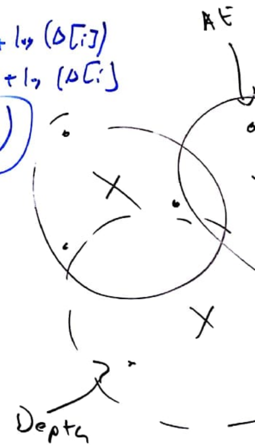
$\vec{\Delta}^2 = \lambda_s^2 (n-1) \Delta[i]^2 + (\Delta[i])^2$

$= \underbrace{(n-1)\lambda_s^2 \Delta[i]} + \underbrace{(\Delta[i])^2}_{\text{dominates}}$

$A\vec{x} - \lambda^{t-1}\vec{v} = \vec{x}[t+1]$







Let's say we know λ_1 and are trying to learn $\vec{v}_1$ 3/17/19

At time step t, the state is

$$\vec{x}(t) = \underbrace{A^t \vec{x}(0)}_{\text{unknown}} + \sum_{k=0}^{t} A^{t-k} \underbrace{\vec{u}(k)}_{\uparrow \text{ known}}$$

Because B matrix is I

In the euclidean basis $\vec{d}(t)$ where t indicates both a time and an index to differentiate, Note that $\vec{d}(i)$ are also orthonormal

$$\left(\{ \vec{d}(0), \vec{d}(1), \vec{d}(2), \dots \vec{d}(t) \} = \left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix}, \dots \begin{array}{c} \\ t\text{th} \end{array} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \right\} \right)$$

The eigenvector that corresponds to the largest eigenvalue is $\vec{v}_1 = \alpha_0 \vec{d}(0) + \alpha_1 \vec{d}(1) + \dots + \alpha_{n-1} \vec{d}(n-1)$

A can be decomposed as

$$A = \lambda_1 \vec{v}_1 \vec{v}_1^T + \sum_{k=2}^{N} \lambda_k \vec{v}_k \vec{v}_k^T$$

where $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_N\}$ is orthonormal

Assumptions to maybe make

① $A^t \approx \lambda_1^t \vec{v}_1 \vec{v}_1^T$ in the long run (assuming all other eigenvalues are stable)

In this case the "α_i" we get by dotting with $\vec{d}(i)$ are estimates, not the actual α_i. So label $\hat{\alpha}_1$ and note that we know this quantity.

↳ treat the extra stuff as a disturbance

↳ intermittent application of input

② Knowing vs. not knowing initial state? Magnitude? Assuming that $\vec{x}(0)$ is aligned w/ $\vec{v}_1$?

$\vec{x}(t) \cdot \vec{d}(0) = A^t \vec{x}(0) \cdot \vec{d}(0)$ ← assuming no inputs have been put in up to and before time t. Assume t large enough that our approximation holds true.

$$\vec{x}(t) \cdot \vec{d}(0) = \left(\left[\lambda_1^t \vec{v}_1 \vec{v}_1^T + \dots \right] \vec{x}(0) \right) \cdot \vec{d}(0)$$

$$\vec{x}(t) \cdot \vec{d}(0) = \left(\left[\lambda_1^t \vec{v}_1 \vec{v}_1^T + (\dots) \right] \vec{x}(0) \right) \cdot \vec{d}(0)$$

ditched all other terms

$$\approx \left(\lambda_1^t \vec{v}_1 \vec{v}_1^T \vec{x}(0) \right) \cdot \vec{d}(0)$$

Proceed by assuming we know $\vec{x}(0)$

$$\vec{x}(t)\cdot\vec{d}(0) \approx \lambda_1^t \vec{v}_1 \vec{v}_1^T \vec{x}(0)\cdot\vec{d}(0)$$

$$\approx \lambda_1^t \vec{v}_1^T \cdot |\vec{v}_1||\vec{x}(0)| \cos\theta_{\vec{v}_1,\vec{x}}\, \vec{d}(0)$$

unknown

known → normal | known → make 1 for convenience

not true, we watched state evolve ~~Don't know that t.~~

$$\approx \lambda_1^t \vec{v}_1^T \cos\theta_{\vec{v}_1,\vec{x}}\, \vec{d}(0)$$

$$\approx \lambda_1^t \hat{\alpha}_0 \cos\theta_{\vec{v}_1,\vec{x}}$$

known; unknown, also estimate because we ditched "noise terms" (↳ other λ_i); unknown

$$\frac{\vec{x}(t)\cdot\vec{d}(0)}{\lambda_1^t} \approx \hat{\alpha}_0 \cos\theta_{\vec{v}_1,\vec{x}}$$

How to deal w/ cosine unknown?

Proceed by assuming we don't

Proceed by assuming $\vec{x}(0)$ lies along $\vec{v}_1$, but we don't know $\vec{x}(0)$

↳ we know $\vec{x}(0)$ magnitude

$$\vec{x}(0) = \beta \vec{v}_1$$

known

$$\vec{x}(t) = A^t \vec{x}(0) = \lambda_1^t \vec{v}_1 \vec{v}_1^T \vec{x}(0)$$ (exact because $\vec{x}(0)$ lies along $\vec{v}_1$, orthogonal to all other $\vec{v}_i$)

$$\vec{x}(t) = \lambda_1^t \vec{v}_1 \beta$$

$$\vec{x}(t)\cdot\vec{d}_0 = \lambda_1^t \vec{v}_1^T \beta\, \vec{d}_0$$

$$\vec{x}(t)\cdot\vec{d}_0 = \lambda_1^t \beta \alpha_0$$ ⇒ we can find all α_i if we apply no inputs whatsoever

know this somehow (observation in subspace); know λ_1 and how many time steps; know this

Because we don't inject any other eigenvectors

Strange pattern of four drops

$$\frac{d}{dt}\vec{x} = A\vec{x}$$

$$A = \begin{bmatrix} 1 & 1 \\ z & z^* \end{bmatrix} \begin{bmatrix} re^{j\theta} & \\ & re^{-j\theta} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ z & z^* \end{bmatrix}^{-1}$$

$$z = \underline{m}e^{j\phi}$$

$$\vec{x}[k+1] = A\vec{x}[k]$$

$$\vec{x}[k+1] - \vec{x}[k] = (A-I)\vec{x}[k]$$

$$\Delta\vec{x}[k] =$$

Might not work for discrete case because velocity is innately continuous