CS1231 Assignment #1
AY2017/18 Semester 1
Deadline: 5pm on Thursday, 21 September 2017

Instructions:

This is a **graded** assignment worth 10% of your final grade. Please work on it **by yourself** (not in a group or in collaboration with anybody), and submit your answers by the deadline stated above. Answer all questions. A handwritten submission is fine; there is no need to use Word or Latex to typeset. However, please write legibly. Also, state **your name, matric number, and tutorial group** at the top of the first page. Staple all pages. Missing pages will cause you to lose marks.

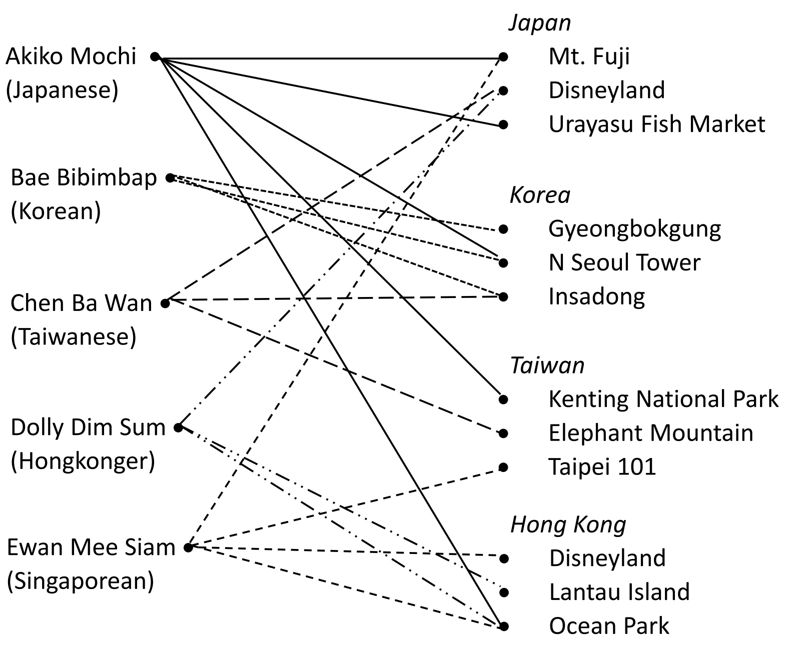
Late submissions will not be graded.

How to submit:

Drop your submission into a slot labeled "CS1231 Assignment" at the Undergraduate Studies Office at COM1-02-19.

Question 1. (5 marks)

The diagram below shows the places of interest five visitors visited. Each country or region has three places of interest. Every person is resident in the country or region of their nationality. A line joining a person to a place of interest indicates that the person visited that place of interest. Answer the following questions below with either "true" or "false".



(a) (1 marks) There is a visitor who visited all the places of interest in some country or region.

(b) (1 marks) Every visitor visited at least two places of interest in some country or region.

(c) (1 marks) Some of the places of interest in every country or region were visited only by non-residents of that country.

(d) (1 marks) Every visitor visited some places of interest that was also visited by another visitor.

(e) (1 marks) The place of interest visited by the most visitors is located in the country or region visited by the most visitors.

Question 2. (5 marks)

Prove the following argument, using Theorem 2.1.1 (Epp) and other theorems and laws covered in lectures, such as implication law, existential instantiation, universal instantiation, etc. Cite the law used for every step. Please leave a blank line between steps so that your Tutor can write comments if necessary.

1. $\exists x(P(x) \wedge \sim Q(x))$
2. $\forall x(P(x) \rightarrow R(x))$
3. $\therefore \exists x(R(x) \wedge \sim Q(x))$

To help you, here is a small example on how you may write your solution.

Example: Prove the following argument:

1. $\forall x, TakeCS1231(x) \rightarrow NUSStudent(x)$
2. $TakeCS1231(\text{John})$
3. $\therefore NUSStudent(\text{John})$

Solution for the above example:

1. $\forall x, TakeCS1231(x) \rightarrow NUSStudent(x)$ [Premise]
2. $TakeCS1231(\text{John}) \rightarrow NUSStudent(\text{John})$ [Universal instantiation on (1)]
3. $TakeCS1231(\text{John})$ [Premise]
4. $\therefore NUSStudent(\text{John})$ [Modus Ponens on (2) and (3)]

Question 3. (5 marks)

Given some positive integer a, prove by Mathematical Induction that $(a+1)^{n+1} - an - (a+1)$ is divisible by a^2, for all $n \in \mathbb{N}$. Do not use the Binomial Theorem.

Question 4. (5 marks)

A positive integer is said to be *wonderful* iff the number of its decimal digits (ie. in base 10) equals the number of its distinct prime factors. For example, 21 is wonderful because it has 2 decimal digits and 2 distinct prime factors; but 25 is not wonderful.

(a) (1 marks) Find the smallest three-digit number that is **not** wonderful.

(b) (1 marks) Find the largest four-digit number that is wonderful.

(c) (3 marks) Give a proof sketch (ie. an informal proof) for the fact that wonderful numbers are upper-bounded; that is, there exists a maximum wonderful number.

Hint: Let x be a wonderful number with n digits. Then $10^n > x$. From this, derive an inequality involving n, and argue why this implies that n is upper-bounded.

—— **END** ——