ECE368: Probabilistic Reasoning
Lab 2: Bayesian Linear Regression

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You should hand in: 1) A scanned .pdf version of this sheet with your answers (file size should be under 2 MB); 2) four figures for Question 2 and three figures for Question 4 in the .pdf format; and 3) one Python file regression.py that contains your code. All these files should be uploaded to Quercus.

1. Express the posterior distribution $p(\mathbf{a}|x_1, z_1, \ldots, x_N, z_N)$ using $\sigma^2, \beta, x_1, z_1, x_2, z_2, \ldots, x_N, z_N$. (**1 pt**)

$$\text{let } z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \quad \text{and let } x = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

$$\boxed{p(a|z_1, z_1, \ldots, x_n, z_n) = N(\mu, \Sigma)} \text{ where } \mu = \left(x^T x + \frac{\sigma^2}{\beta} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1} x^T z$$

$$\Sigma = \left(x^T x + \frac{\sigma^2}{\beta} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1} \sigma^2$$

2. Let $\sigma^2 = 0.1$ and $\beta = 1$. Draw four contour plots corresponding to the distributions $p(\mathbf{a})$, $p(\mathbf{a}|x_1, z_1)$, $p(\mathbf{a}|x_1, z_1, \ldots, x_5, z_5)$, and $p(\mathbf{a}|x_1, z_1, \ldots, x_{100}, z_{100})$. In all contour plots, the x-axis represents a_0, and the y-axis represents a_1. Please save the figures with names **prior.pdf, posterior1.pdf, posterior5.pdf, posterior100.pdf**, respectively. (**1.5 pt**)

3. Suppose that there is a new input x, for which we want to predict the corresponding target value z. Write down the distribution of the prediction z, i.e, $p(z|x, x_1, z_1, \ldots, x_N, z_N)$. (**1 pt**)

$$x = X \text{ from above }, \quad z = z \text{ from above}, \quad \mu = \mu \text{ from above}, \quad \Sigma = \Sigma \text{ from above}.$$

$$\text{inp} = \begin{bmatrix} 1 & x \end{bmatrix} ; \quad \mu' = \text{inp} \times \mu$$

$$\Sigma' = \text{inp} \times (\mu \times \text{inp}^T) + \sigma^2$$

$$\boxed{p(z|x, x_1, z_1 \ldots x_n, z_n) = N(\mu', \Sigma')}$$

4. Let $\sigma^2 = 0.1$ and $\beta = 1$. Given a set of new inputs $\{-4, -3.8, \ldots, 3.8, 4\}$, plot three figures, whose x-axis is the input and y-axis is the prediction, corresponding to three cases:

(a) The predictions are based on one training sample, i.e., based on $p(z|x, x_1, z_1)$.

(b) The predictions are based on 5 training samples, i.e., based on $p(z|x, x_1, z_1, \ldots, x_5, z_5)$.

(c) The predictions are based on 100 training samples, i.e., based on $p(z|x, x_1, z_1, \ldots, x_{100}, z_{100})$.

The range of each figure is set as $[-4, 4] \times [-4, 4]$. Each figure should contain the following three components: 1) the new inputs and the corresponding predicted targets; 2) a vertical interval at each predicted target, indicating the range within one standard deviation; 3) the training sample(s) that are used for the prediction. Use plt.errorbar for 1) and 2); use plt.scatter for 3). Please save the figures with names **predict1.pdf, predict5.pdf, predict100.pdf**, respectively. (**1.5 pt**)