Homework 7
CPE381

Canvas: `hw07`

Due: 5th December November 2024, 11:59 PM
100 points

You are allowed to use a generative model-based AI tool for your assignment. However, you must submit an accompanied reflection report on how you use the AI tool, what was the query to the tool, and how it improved your understanding of the subject. You must also add your thoughts on how you would tackle the assignment if there was no such tool available. Failure to provide a reflection report for every single assignment where an AI tool was used may result in a penalty and subsequent actions will be taken in line with plagiarism policy.

Submission instruction:

Upload a .pdf on Canvas with the format {firstname.lastname}_cpe381_hw07.pdf. If there is a programming assignment, then you should include your source code along with your PDF files in a zip file firstname.lastname}_cpe381_hw07.zip. If a plot is being asked, your PDF file must also contain plots generated by your MATLAB code. Your submission must contain your name, and UAH Charger ID or the UAH email address. Please number your pages as well.

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1 Z-transform (10 points)

Find the z-transform of **(5 points each)**

1. $x[n] = -a^n u[-n-1]$
2. $x[n] = a^{-n} u[-n-1]$ where a is a constant, u represents discrete unit-step function.

2 ROC for Discrete-time System (10 points)

Find Z-transform $X(z)$ and sketch a pole-zero plot with ROC for each of the following sequences **(5 points each)**:

1. $x[n] = (\frac{1}{3})^n u[n] + (\frac{1}{2})^n u[-n-1]$
2. $x[n] = (\frac{1}{2})^n u[n] + (\frac{1}{3})^n u[-n-1]$

3 Inverse Z-transform (5 points)

Find the inverse z-transform of $X(z) = \dfrac{2}{2z^2 - 3z + 1}$, $|z| > 1$.

4 Difference Equation (10 points)

1. For the following difference equation, and associated input and initial conditions, determine the output $y[n]$:

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

with $x[n] = \left(\frac{1}{3}\right)^n$

and initial condition $y[-1] = 1$.

5 Fourier Coefficients of a Discrete-Time Signal (15 points)

Consider a sequence $x[n] = \sum_{-\infty}^{\infty} \delta[n - 4k]$.

1. Sketch $x[n]$. You only need to sketch a portion of $x[n]$. **(5 points)**
2. Find the Fourier coefficient of X_k of $x[n]$. **(5 points)**
3. From the Fourier coefficients found in the previous section plot $|X_k|$ against k using MATLAB. **(5 points)**.

6 DTFT (10 points)

1. Find the discrete-time Fourier transform of $x[n] = -a^n u[-n-1]$ where a is a real number. **Hint:** Start with finding z-transform first. **(5 points)**
2. Find the discrete-time Fourier transform of the rectangular pulse sequence

$$x[n] = u[n] - u[n-N]$$

. **(5 points)**

7 Causal LTI Discrete-time System (10 points)

A causal discrete-time LTI system is described by

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n]$$

where $x[n]$ and $y[n]$ are the input and the output of the system, respectively.

1. Determine the frequency response $H(\omega)$ of the system. **Hint: Find the discrete transfer function of the system first. (5 points)**
2. Find the impulse response $h[n]$ of the system. **(5 points)**

8 A Little More on DTFT (20 points)

From DTFT and Inverse DTFT of $x[n] = 0.5^{|n|}$,

1. Determine the sum

$$\sum_{-\infty}^{\infty} 0.5^{|n|}$$

. **(5 points)**

2. Find the integral

$$\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$$

Hint: Think about the formula for calculating the inverse DTFT. **(10 points)**

3. Find the phase of $X(e^{j\omega})$.

Hint: A phase of a complex number $a + jb$ is $\tan^{-1}\left(\frac{b}{a}\right)$. **(5 points)**

9 Decimation-in-Time FFT (10 points)

Consider the decimation-in-time FFT algorithm for $N = 4$. Give the equation to compute the four DFT values $X[k]$, $k = 0, 1, \cdots, 3$, in the matrix form using original input signals $x[n]$ and twiddle factor W.