Homework 3
CPE381

Canvas: `hw03`

Due: 27 September 2024, 11:59 PM

You are allowed to use a generative model-based AI tool for your assignment. However, you must submit an accompanied reflection report on how you use the AI tool, what was the query to the tool, and how it improved your understanding of the subject. You must also add your thoughts on how you would tackle the assignment if there was no such tool available. Failure to provide a reflection report for every single assignment where an AI tool was used may result in a penalty and subsequent actions will be taken in line with plagiarism policy.

Submission instruction:

Upload a .pdf on Canvas with the format {firstname.lastname}_cpe381_hw03.pdf. If there is a programming assignment, then you should include your source code along with your PDF files in a zip file firstname.lastname}_cpe381_hw03.zip. Your submission must contain your name, and UAH Charger ID or the UAH email address. Please number your pages as well.

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1 Autobots, transform and roll out! (10 points)

Find the Laplace transform of the following and specify their region of convergence (ROC):

1. $x(t) = -e^{-at}u(-t)$. Use the first principle. where $u(t)$ is the unit-step signal.
2. $x(t) = e^{at}u(-t)$. Use the first principle.
3. $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$. Use the Laplace transform table.
4. $x(t) = e^{-3t}u(t) + e^{2t}u(-t)$. Use the Laplace transform table.
5. $x(t) = 5e^{-0.3t}u(t-2)$ Use the Laplace transform table.

2 Time is mysterious. (10 points)

1. The unilateral Laplace transform of $f(t)$ is $\dfrac{1}{s^2+s+1}$. Calculate the Laplace transform of $tf(t)$.
2. The transfer function of a system is given by $H(s) = \dfrac{1}{s^2(s-2)}$. Find out the impulse response of the system if $u(t)$ is used to denote the unit-step signal.

3 Are you the Riddler? (20 points)

1. A system is described by the following differential equation, where $u(t)$ is the input to the system and $y(t)$ is the output of the system

$$y'(t) + 5y(t) = u(t) \tag{1}$$

When $y(0^-) = 1$ and $u(t)$ is a unit step function, find $y(t)$ using the Laplace transform method.

2. A continuous-time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t) \tag{2}$$

Assuming zero initial condition, find out the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$.

4 Ready steady, go! (10 points)

1. For the system shown below, $x(t) = \sin(t)u(t)$, find out the response $y(t)$ in the steady-

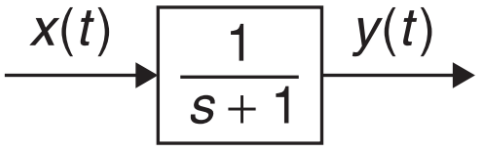

Figure 1: Question 4.1

state.

Note that here, the impulse response $h(t)$ is shown in $s-$domain in the diagram. The impulse response in $s-$domain is called transfer function.

5 Journey to the *s*-verse (30 points)

Consider a ramp signal $r(t)$ as shown in Figure 2 by the dashed line. The x-axis denotes the time-axis and the y-axis denotes the signal value.

1. Write the equation of the ramp signal in terms of time t, and the unit-step signal $u(t)$. **(3 points)**

2. Derive the Laplace transform $R(s)$ of the ramp signal from the definition of the Laplace transform integral, (i.e. do not use Laplace Transform Table). **(9 points)** *Hint: Use integration by parts* $\int f(t)g(t)'dt = f(t)g(t) - \int f'(t)g(t)dt$.

3. What is the region of convergence (ROC) for the Laplace transform $R(s)$ to exist? **(3 points)**

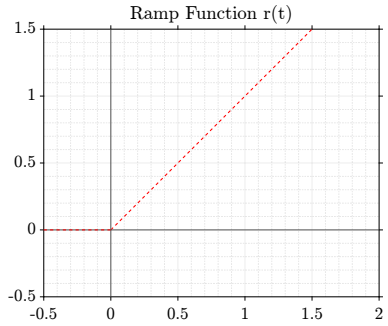

Figure 2: Q3: Ramp Signal $r(t)$

4. Consider a frequency shift e^{-2s} in s-domain applied to the ramp signal $r(t)$. Using the properties of the Laplace transform, write down the resulting signal in terms of time t and the unit-step signal $u(t)$. You may use the provided tables to facilitate your answer. **(6 points)**

5. The ramp signal is time-differentiated and used as an input to a system represented by the transfer function $H(s) = \dfrac{1}{s+2}$. What's the output signal $y(t)$ in the time domain? **(6 points)**

6. Find $\lim_{t\to\infty} y(t)$. **(3 points)**

6 Systems (10 points)

Consider a causal LTI continuous system described by the differential equation

$$y''(t) + 3y'(t) + 2y(t) = 2x'(t) - x(t) \tag{3}$$

1. Using Laplace transform technique, determine the zero-input response $y_{zi}(t)$ if $y'(0^-) = 2$ and $y(0^-) = -3$. **(3 points)**

2. Using Laplace transform technique, determine the zero-state response y_{zs} if the input is $x(t) = u(t)$. **(2 points)**

3. Find the complete response of the system for $y'(0^-) = 2$ and $y(0^-) = -3$ if the input is $x(t) = u(t)$. **(3 points)**

4. Is the system stable with the $x(t) = u(t)$? Provide your argument. **(2 points)**

7 Laplace Transform in MATLAB (10 points)

Use Matlab symbolic computation to find the Laplace transform of a real exponential

$$x(t) = 5e^{-2t}\cos(8t)u(t)$$

Plot the signal and the poles and zeros of their Laplace transform.
Repeat the analysis and plot the results for

$$x(t) = 5e^{-4t}\cos(8t)u(t)$$

Discuss the changes in the s plane and describe their effect on function in the time domain.