WORKED EXAMPLE 06 CPE381 UAH 09/23/2024

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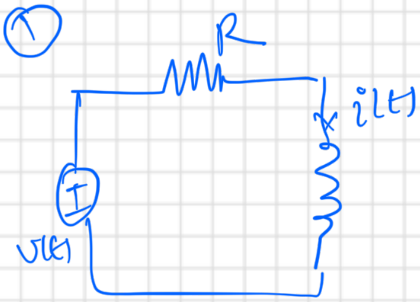
①



$R = 1\,\Omega$

$L = 1\,H$

Equation for the circuit is

$$\frac{di(t)}{dt} + i(t) = v(t)$$

Taking Laplace transform,

$$sI(s) - i(0^-) + I(s) = V(s)$$

$$I(s) = \frac{V(s) + i(0^-)}{s+1}$$

Exam format

① True / False

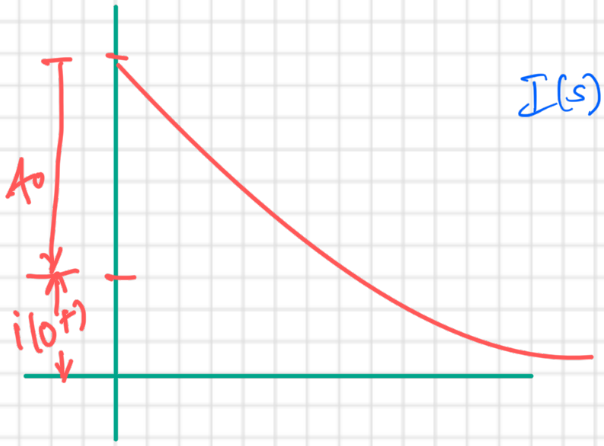
② Multiple Choice

③ Numerical questions

Some questions will have graphs.

④ 1-page Cheatsheet One Sided

Tables will be provided



Let the voltage source in Fig be an impulse function with

$$v(t) = A_0 \delta(t)$$

Then $V(s) = A$, and for $t>0$, the current $i(t)$ is

$$I(s) = \frac{A_0 + i(0^-)}{s+1}$$

$$i(t) = (A_0 + i(0))\, e^{-t}$$

② Continuous-time systems and Transfer function

The general equations for n^{th} order LTI model is

$$\sum_{k=0}^{n} a_k \frac{d^k y(t)}{dt^k} = \sum_{p=0}^{m} b_p \frac{d^p x(t)}{dt^k}$$

$x(t)$: Input

$y(t)$ = Output

as $$\mathcal{L}\left[\frac{d^k f(t)}{dt^k}\right] = s^k F(s) - s^{k-1} f(0^+) - \cdots - f^{(k-1)}(0^+)$$

we will ignore initial conditions as a system with non-zero initial conditions are not linear, but in this course we only talk about linear system.

So $$\mathcal{L}\left[\frac{d^k f(t)}{dt^k}\right] = s^k F(s)$$

Hence,

$$\sum_{k=0}^{n} a_k s^k Y(s) = \sum_{p=0}^{m} b_p s^p X(s)$$

$$\Rightarrow \left[a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0\right] Y(s)$$
$$= \left[b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0\right] X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

Hence, the transfer function is a rational function.

In terms of poles (Roots of denominator) and zeros (Roots of numerator)

$$H(s) = \frac{K(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$K =$ Constant

(3) $x(t) = u(t) - 2u(t-1) + u(t-2)$

$$Y(s) = \frac{(s+2)(1-e^{-s})^2}{s(s+1)^2}$$

Find the impulse response of the system.

Solu^n

$$X(s) = \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s} = \frac{1}{s}(1-e^{-s})^2$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{(s+2)\cancel{(1-e^{-s})^2}}{\cancel{s}(s+1)^2} \times \frac{\cancel{s}}{\cancel{(1-e^{-s})^2}}$$

$$= \frac{s+2}{(s+1)^2}$$

Using partial fraction techniques

$$\frac{s+2}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2} = \frac{A(s+1)+B}{(s+1)^2} = \frac{As+A+B}{(s+1)^2}$$

$A=1, B=1$

$$\Rightarrow H(s) = \frac{1}{(s+1)} + \frac{1}{(s+1)^2}$$

(4) The Laplace transform of the output of an LTI system is

$$Z(s) = \frac{1}{s((s+2)^2+1)}$$

what would be the steady state response $z_{ss}(t)$?

Solution

Complex root goes to zero in the steady state.
Hence, the steady state value is given by

$$\lim_{t\to\infty} z(t) = sZ(s) = \frac{1}{(0+2)^2+1} = \frac{1}{5}$$ is the steady-state value.

(5) Find the inverse Laplace transform of

$$Z(s) = \frac{s^2+6s+7}{s^2+3s+2}, \quad Re(s) > -2$$

Solution

We perform long-division,

$$\begin{array}{r|l} & 1 \\ s^2+3s+2 & s^2+6s+7 \\ & \underline{-\;s^2+3s+2} \\ & \quad 3s+5 \end{array}$$

$$Z(s) = 1 + \frac{3s+5}{s^2+3s+2} = 1 + \frac{3s+5}{(s+1)(s+2)}$$

$$= 1 + \frac{A}{s+1} + \frac{B}{s+2}$$

Use Partial Fraction to solve

$$A = 2, \quad B = 1$$

$$Z(s) = 1 + \frac{2}{s+1} + \frac{1}{s+2}$$

As $Re(s) > -2$.

Hence $z(t)$ is the right handed signal.

However, the inverse of Laplace transform

$\frac{2}{s+1}$ won't exist

Hence $z(t) = \delta(t) + e^{-2t}u(t)$

(6) $H(s) = \frac{e^s}{s+1}$, $Re(s)$ (s) > -1

Check the causality of the system.

The ROC is to the right of the rightmost pole.

therefore impulse response must be one-sided.

and

$$e^{-t}u(t) \longleftrightarrow \frac{1}{s+1} \quad Re(s) > -1$$

Using the time shift property

$$e^{-(t+1)}u(t+1) \longleftrightarrow \frac{e^{s}}{s+1}, Re(s) > -1$$

thus the impulse response is

$$h(t) = e^{-(t+1)}u(t+1)$$

which is non zero for $-1 < t < 0$.

Hence the system is non-causal.