CPE 381 FA 2024 Classwork 11 10/02/2024.

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Q1. Find the fourier Coefficients for $x(t) = B + A\sin(\Omega_0 t + \theta)$ and hence write the fourier series representation in terms of exponentials.

Solution:

$$\sin(\Omega_0 t + \theta) = \frac{1}{2j}\left(e^{j(\Omega_0 t+\theta)} - e^{-j(\Omega_0 t+\theta)}\right)$$

$$= -\frac{1}{2j} e^{-j\Omega_0 t} e^{-j\theta} + \frac{1}{2j} e^{j\Omega_0 t} e^{j\theta}$$

Hence

$$x(t) = B - \frac{1}{2j} e^{-j\Omega_0 t} e^{-j\theta} + \frac{1}{2j} e^{j\Omega_0 t} e^{j\theta}$$

is the fourier series representation.

Comparing it with

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t}$$

$$X_0 = B$$

$$X_{-1} = -\frac{1}{2j} e^{-j\theta} \qquad X_1 = \frac{1}{2j} e^{j\theta}$$

$$X_k = 0 \text{ for } k \neq \{-1, 0, 1\}$$

Q2. $x(t) = \cos\left(2t + \frac{\pi}{4}\right)$. Find the fourier series representation by finding the fourier series coefficients. First, write down the value of fundamental frequency Ω_0.

$\Omega_0 = 2$

$$x(t) = \cos\left(2t + \frac{\pi}{4}\right) = \frac{1}{2}\left[e^{j(2t+\frac{\pi}{4})} + e^{-j(2t+\frac{\pi}{4})}\right]$$

$$= \underbrace{\frac{1}{2} e^{j(2)\cdot 1\cdot t} \cdot e^{j\pi/4}}_{k=+1} + \underbrace{\frac{1}{2} e^{-j(2)\cdot(-1)\cdot t} \cdot e^{-j\pi/4}}_{k=-1}$$

Hence

$$x_{-1} = \frac{1}{2} e^{-j\pi/4} = \frac{1}{2}\left(\frac{1-j}{\sqrt{2}}\right)$$

$$x_{+1} = \frac{1}{2} e^{j\pi/4} = \frac{1}{2}\left(\frac{1+j}{\sqrt{2}}\right)$$

$$x_k = 0 \text{ for } k \neq \{1, -1\}$$