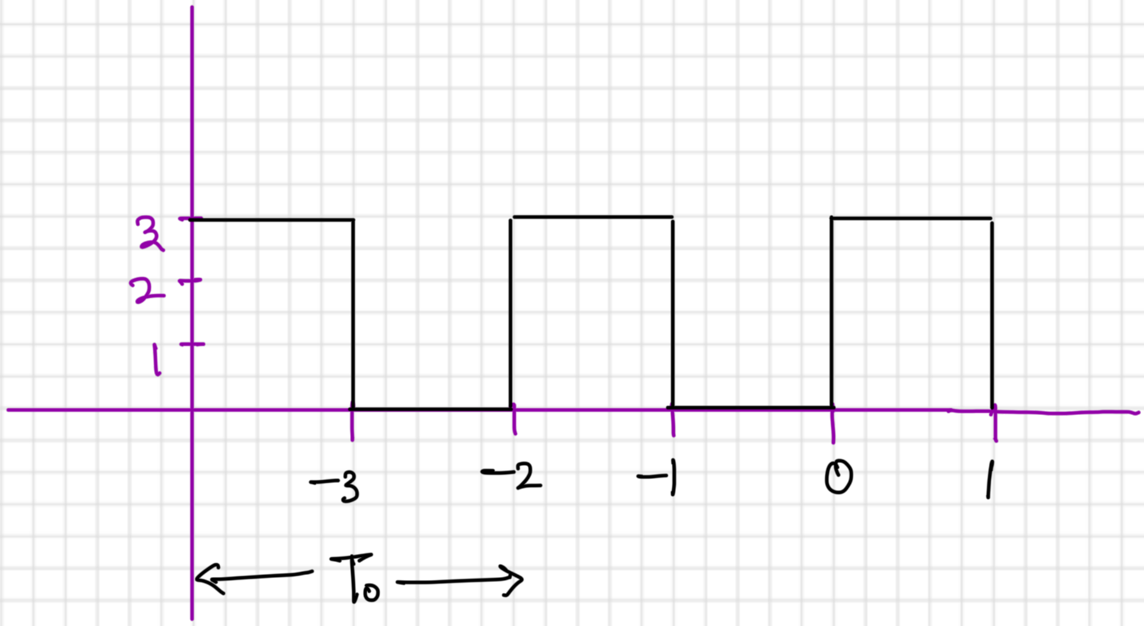
CPE381 Homework 4 Problem 4 Solution



① Clearly, the time period is 2 units.

$T_0 = 2$

$\Omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi$

② Exponential Fourier Series:

$$X_k = \frac{1}{T_0}\int_{t_0}^{T_0} x(t)\, e^{-jk\Omega_0 t}\, dt$$

$t_0 = 0 \quad T_0 = 2$

$$X_k = \frac{1}{2}\int_0^2 x(t)\, e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\int_0^1 3 \cdot e^{-jk\pi t}\, dt$$

$$= \frac{3}{2}\left.\frac{e^{-jk\pi t}}{-jk\pi}\right|_0^1 = \frac{3}{2}\left[\frac{e^{-jk\pi} - 1}{-jk\pi}\right]$$

$$= \frac{3}{2}\left[\frac{(-1)^k - 1}{-jk\pi}\right]$$

$$= \frac{3}{2}\left[\frac{1 - (-1)^k}{jk\pi}\right]$$

$e^{-jk\pi}$
$= \cos k\pi - j\sin k\pi$
$= (-1)^k$

$$X_k = \frac{3}{2}\left[\frac{1-(-1)^k}{jk\pi}\right] \quad , k \neq 0$$

$$X_0 = \frac{3}{2}\int_0^1 1 \cdot dt = \frac{3}{2}[1-0] = 3/2$$

For trigonometric fourier series:

$$x(t) = c_0 + 2\sum_{k=1}^{\infty}\left[c_k \cos(k\Omega_0 t) + d_k \cos(k\Omega_0 t)\right]$$

$$\text{and } c_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\cos(k\Omega_0 t)dt$$

$$\Omega_0 = \pi \qquad T_0 = 2$$

$$= \frac{1}{2}\int_0^2 x(t)\cos(k\Omega_0 t) = \frac{1}{2}\int_0^1 3 \cdot \cos(k\pi t)dt$$

$$\int \cos(\alpha t)dt = \frac{1}{\alpha}\sin(\alpha t) + c$$

$$= \frac{3}{2}\frac{1}{k\pi}\sin(k\pi t)\Big|_0^1$$

$$= \frac{3}{2}\frac{1}{k\pi}\left[\sin(k\cdot\pi\cdot 1) - \sin(k\cdot\pi\cdot 0)\right]$$

$$= \frac{3}{2}\frac{1}{k\pi}[0-0] = 0$$

$$c_k = 0, \quad k = 0, 1, 2 \ldots$$

$$d_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\sin(k\Omega_0 t)dt = \frac{1}{2}\int_0^2 x(t)\sin(k\pi t)dt$$

$$= \frac{1}{2}\int_0^1 3\sin(k\pi t)dt = -\frac{3}{2}\frac{1}{k\pi}\cos(k\pi t)\Big|_0^1$$

$$= -\frac{3}{2}\frac{1}{k\pi}\left[\cos(k\pi) - \cos(0)\right]$$

$$d_k = -\frac{3}{2}\frac{1}{k\pi}\left[(-1)^k - 1\right]$$

$\cos k\pi$
$= (-1)^k$

$$= 3\left[\frac{1-(-1)^k}{2k\pi}\right] \qquad k = 1, 2 \ldots$$

For $k=0$,

$$d_0 = \frac{1}{2}\int_0^1 3 \cdot \sin(\pi t)\,dt = -\frac{3}{2}\frac{1}{\pi}\cos(\pi t)\Big|_0^1$$

$$= -\frac{3}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{3}{2\pi}\left[-1-1\right] = +\frac{\cancel{2}}{2\pi}$$

$$= \frac{3}{\pi}$$

$$d_0 = \frac{3}{\pi}$$