CPE 381 VAH WORKED EXAMPLE 02 FALL 2024

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Q1 A current impulse $5\delta(t)$ is forced through a capacitor C. The voltage, $v_c(t)$ acerross is given by ....?

The voltage across capacitor is given by

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)dt = \frac{1}{C}\int_{-\infty}^{t} 5\delta(t)dt$$

$$= \frac{1}{C} 5u(t)$$

$$\left[\text{As} \quad \delta(t) = \frac{d}{dt}u(t) \right]$$

Q2. The value of the integral $\int_{-5}^{+6} e^{-2t}\delta(t-1)dt$ is given by

$$\int_{-5}^{+6} e^{-2t}\delta(t-1)dt = e^{-2t}\Big|_{t=1} = e^{-2\cdot 1} = e^{-2}$$

$$\left[\int_{t_1}^{t_2} x(t)\delta(t-t_0)\, dt = x(t_0) \right.$$

Q3. Calculate the value of the integral $\int_{-\infty}^{\infty}\delta(t)\cos\left(\frac{3t}{2}\right)dt$.

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)dt = f(0)$$

So $$\int_{-\infty}^{\infty}\delta(t)\cos\left(\frac{3t}{2}\right)dt = \cos\frac{(3\times 0)}{2} = 1$$

Q4. If a signal $f(t)$ has energy E, Calculate the energy of the signal $f(2t)$.

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt$$

$$E_2 = \int_{-\infty}^{\infty} |f(2t)|^2 dt$$

Let $2t = u$

then $2dt = du$

$$\Rightarrow E_2 = \int_{-\infty}^{\infty} |f(u)|^2 \frac{du}{2}$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} |f(u)|^2 du = E$$

$$= \frac{E}{2}.$$

Properties of Unit Impulse Functions:

1. $\int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt = f(t_0)$, $f(t)$ continuous at $t = t_0$

2. $\int_{-\infty}^{\infty} f(t - t_0)\,\delta(t)\,dt = f(-t_0)$, $f(t)$ continuous at $t = -t_0$

   Use variable substitution from 1 to prove it.

3. $f(t)\,\delta(t - t_0) = f(t_0)\,\delta(t - t_0)$, $f(t)$ is continuous at $t = t_0$

4. $\delta(t - t_0) = \frac{d}{dt} u(t - t_0)$

5. $u(t - t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)\,d\tau = \begin{cases} 1, & t > t_0 \\ 0, & t < t_0 \end{cases}$

⑥ $$\int_{-\infty}^{\infty} \delta(at - t_0)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta\left(t - \frac{t_0}{a}\right)dt$$

⑦ $$\delta[-t] = \delta[t]$$

Saw tooth Wave form and its significance

Pulse Width Modulation is a technique used to control the amount of power delivered to an electrical device by varying the width of the pulse in the pulse train.

Essentially, it involves switching the power on and off rapidly to control the effective voltage and current supplied to a device.

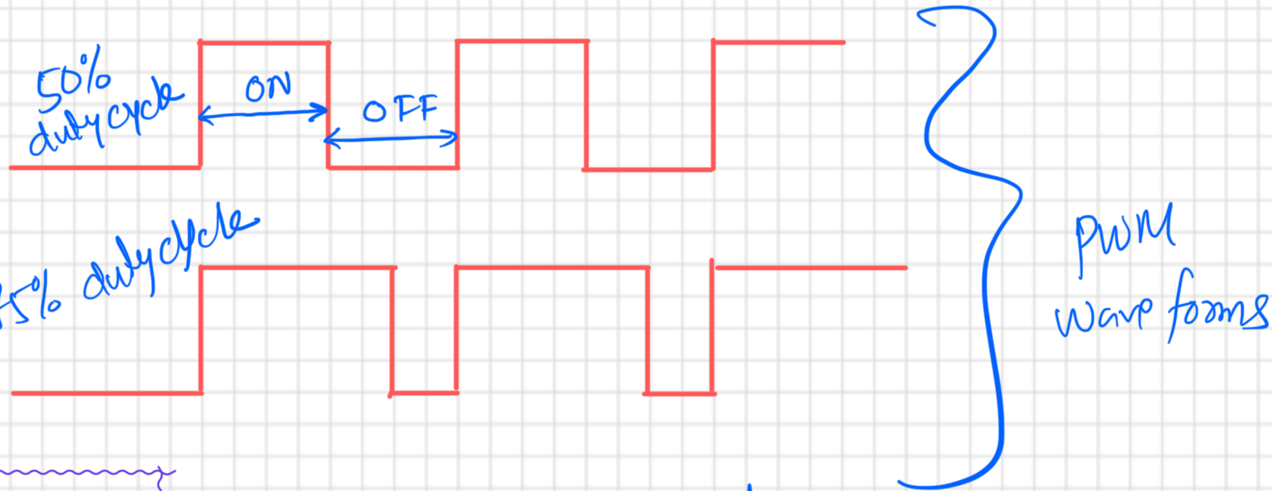
DUTY CYCLE: The duty cycle is the percentage of one period in which a signal is active.
A 100% duty cycle means the signal is always on, while 0% duty cycle means the signal is always off.
If a PWM Signal has a duty cycle of 50%, it means the signal is on for half time and off for the other half.

FREQUENCY This is the rate at which the PWM signal switches on and off.

Application ① MOTOR control: PWM is used to control the speed of the motor. By adjusting the duty cycle, you can control the amount of power delivered to the motor, thus controlling the speed.
In EV, PWM is used to control the speed of the motors driving the wheels. In lighting systems, PW is used to adjust the brightness of LED lights.

Sawtooth wave form is important in power system and power electronics due to its unique properties to be able to generate PWM signals.

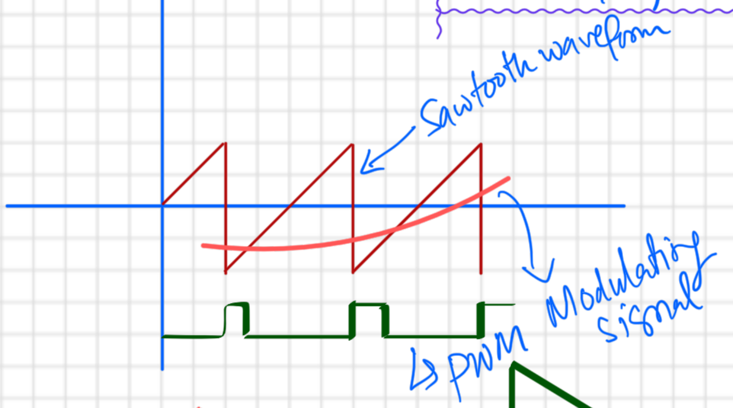
Sawtooth waveform serves as a reference signal that is compared with a modulating signal to produce the PWM signal.



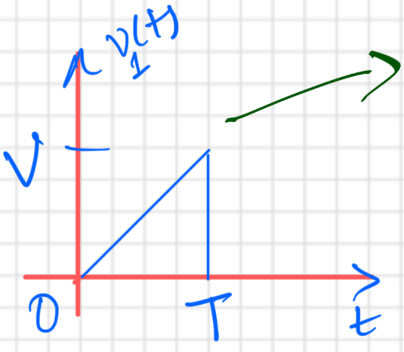
$$\text{Duty cycle} = \frac{\text{Pulse ON time}}{\text{Pulse Period}} \times 100\%$$

$$= \frac{T_{ON}}{T_{ON} + T_{OFF}} \times 100\%$$

$$\text{Period} = T_{ON} + T_{OFF}$$



Comparator



Sawtooth Sign

$$v_1(t) = \frac{V}{T} t, \quad 0 < t < T$$

$$= \frac{V}{T} t \left[u(t) - u(t-T) \right]$$

$$= \frac{V}{T} t \, \text{rect}\left[\left(t - \frac{T}{2}\right) / T\right]$$

Then

$$v_1(t-T) = \frac{V}{T}[t-T]\,[u(t-T) - u(t-2T)]$$

Then, saw tooth wave form can be written as

$$v(t) = \sum_{k=-\infty}^{\infty} v_1(t-kT)$$

which gives a waveform: