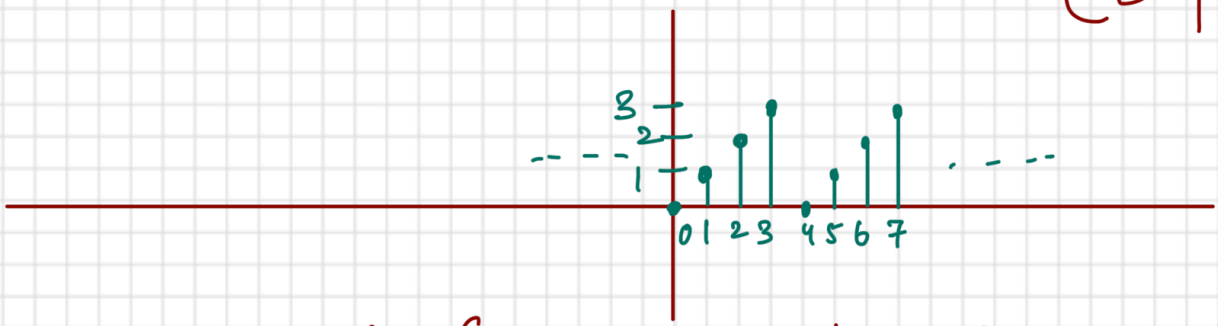
Classwork 24
FA24
UAH
CPE 381
Instructor: Rahul Bhadani

Name of the Student:

11/20/2024

Total Marks: 10 points

① Determine the Fourier coefficients for the periodic sequence $x[n]$ shown below: (5 points)



Note: Discrete fourier representation of a periodic sequence $x[n]$ is given by

$$x[n] = \sum_{k=0}^{N_0-1} c_k e^{jk\omega_0 n}$$

$\omega_0 = \frac{2\pi}{N_0}$

N_0 = fundamental period.

where c_k is the fourier coefficient given by

$$c_k = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-jk\omega_0 n}$$

Solution

$N_0 = 4$, $\omega_0 = \frac{2\pi}{4}$

$e^{-j\omega_0} = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2} = -j$

So $e^{-j\omega_0 n} = (-j)^n$

$$c_0 = \frac{1}{4} \sum_{n=0}^{3} x[n] \cdot (-j)^{0\cdot n} = \frac{1}{4}(0+1+2+3) = 3/2$$

$$c_1 = \frac{1}{4} \sum_{n=0}^{3} x[n] (-j)^{1\cdot n} = \frac{1}{4}(0 - j\cdot 1 - 2 + j\cdot 3) = -\frac{1}{2} + j\frac{1}{2}$$

$$C_2 = \frac{1}{4}\sum_{n=0}^{3} x[n](-j)^{2\cdot n} = \frac{1}{4}(0-1+2-3) = -\frac{1}{2}$$

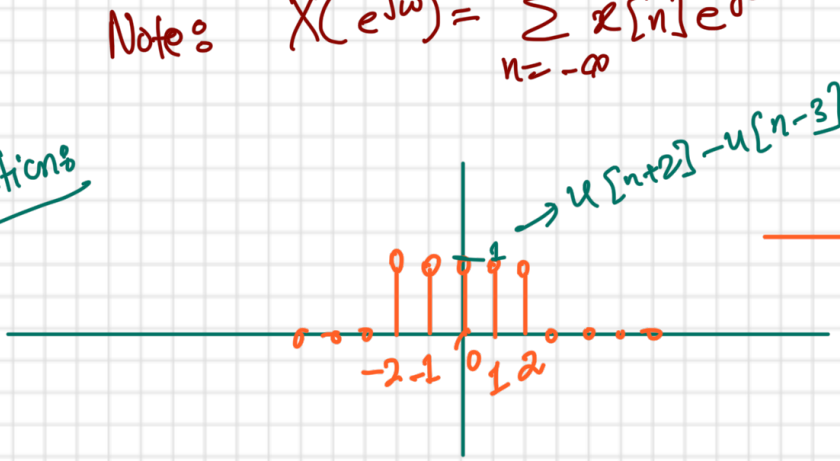
$$C_3 = \frac{1}{4}\sum_{n=0}^{3} x[n](-j)^{3n} = \frac{1}{4}(0+j\cdot 1-2-j\cdot 3) = -\frac{1}{2}-j\cdot\frac{1}{2}$$

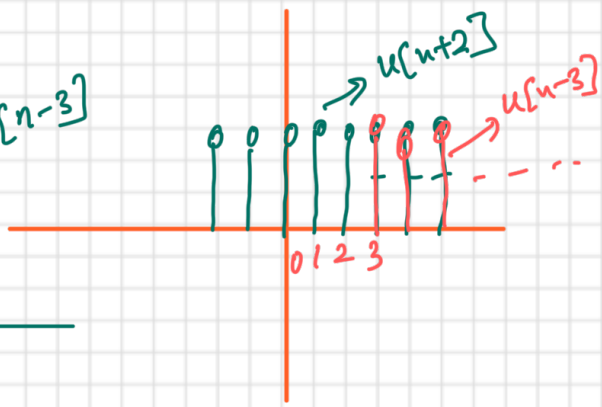
(2) What is DTFT $X(e^{j\omega})$ of $x[n] = u[n+2] - u[n-3]$

Hint: Sketch the signal. (5 points)

Note: $X(e^{j\omega}) = \sum_{n=-\infty}^{n=+\infty} x[n]e^{-j\omega n}$

Solutions:





Then

$$X(e^{j\omega}) = \sum_{n=-2}^{+2} x[n]e^{-j\omega n}$$

$$= e^{-j\omega\cdot(-2)} + e^{-j\omega(-1)} + e^{-j\omega\cdot 0} + e^{-j\omega\cdot 1} + e^{-j\omega\cdot 2}$$

$$= e^{+j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$$

$$= \boxed{1 + \cos(\omega) + \cos(2\omega).}$$

$$\left\{ \frac{e^{j\omega}+e^{-j\omega}}{2} = \cos(\omega) \right.$$