CPE 381 Worked Example 01 FA2024

1. Discuss continuous-time signal, analog signal, discrete-time signal, multi-level signal, digital signal.

2. Characterize the sinusoidal signal $x(t) = \sqrt{2}\cos(\pi t/2 + \pi/4)$, $-\infty < t < \infty$

(a)

→ Amplitude of signal is $\sqrt{2}$.

→ Signal is deterministic

→ Support of the signal is from $-\infty < t < \infty$

→ Frequency is $\Omega = \frac{\pi}{2}$ rad/s, phase is $\frac{\pi}{4}$

(b) Express the signal $y(t) = (1+j)e^{j\pi t/2}$, $0 \le t \le 10$ in terms of $x(t)$.

$$y(t) = (1+j)e^{j\pi t/2}$$

$$(1+j) = \sqrt{2}e^{j\pi/4}$$

$$\Rightarrow y(t) = \sqrt{2}e^{j\pi/4}e^{j\pi t/2} = \sqrt{2}e^{j(\pi t/2+\pi/4)}$$

$$= \sqrt{2}\left[\cos(\pi t/2+\pi/4) + j\sin(\pi t/2+\pi/4)\right]$$

$$\mathrm{Re}(y(t)) = \sqrt{2}\cos(\pi t/2 + \pi/4) = x(t) \quad \text{(Given)}, \ 0 \le t \le 10$$

$$\mathrm{Im}(y(t)) = \sqrt{2}\sin(\pi t/2 + \pi/4)$$

$$= \sqrt{2}\cos\left(\frac{\pi}{2} - \left(\frac{\pi t}{2} + \frac{\pi}{4}\right)\right)$$

$$= \sqrt{2}\cos\left(\frac{\pi}{2}(1-t) + \frac{\pi}{4}\right)$$

$$= x(t-1)$$

$$\left[\begin{array}{l} \sin\theta = \cos\left(\frac{\pi}{2}-\theta\right) \\ \cos\theta = \sin\left(\frac{\pi}{2}-\theta\right) \end{array}\right]$$

Hence, $y(t) = x(t) + j\,x(t-1)$, $0 \le t \le 10$

③ Consider $x(t) = A\cos(\Omega_0 t + \theta)$, $-\infty < t < \infty$
Determine the fundamental period of the signal, and indicate for what frequency Ω_0 the fundamental period of $x(t)$ is not defined.

Answer:

The analog frequency is $\Omega_0 = \frac{2\pi}{T_0}$

so $T_0 = \frac{2\pi}{\Omega_0}$

whenever $\Omega_0 > 0$, these sinusoids are periodic

Example

$$x(t) = 2\cos(2t - \pi/2), \quad -\infty < t < \infty$$

its fundamental period is $\Omega_0 = 2 = 2\pi f_0$ rad/s.

$$f_0 = \frac{1}{\pi} = \frac{1}{T_0}$$

$$T_0 = \pi$$

and

$$\Omega_0 T_0 = 2T_0 = 2\pi$$

$$x(t + NT_0) = 2\cos(2(t + NT_0) - \pi/2)$$

$$= 2\cos(2t + 2\pi N - \pi/2)$$

$$= 2\cos(2t - \pi/2) = x(t)$$

$$N \in \mathbb{Z}$$

④ Consider a signal $x(t) = e^{-at}$, $a > 0$, for $t \geq 0$, zero otherwise.
Find its energy and power.

Answer

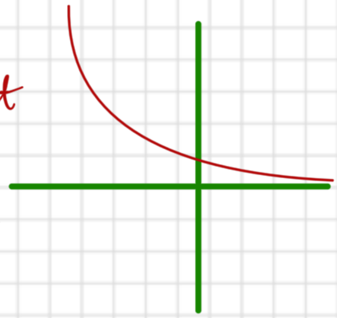
① $x(t) = e^{-at}$ is a periodic.

Energy, $E_x = \int_0^{\infty} e^{-2at}\,dt = \frac{1}{2a} < \infty$

Power $P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |e^{-at}|^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left|e^{-2at}\right| dt \qquad \left(\text{as } |e^{-x}| = e^{-x}\right)$$



$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} e^{-2at}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left.\frac{e^{-2at}}{-2a}\right|_{-T}^{T}$$

$$= \lim_{T\to\infty} \underbrace{\frac{1}{2T}}_{=0}\left(-\underbrace{\frac{e^{-2aT}}{2a}}_{=0} + \underbrace{\frac{e^{+2aT}}{2a}}_{=0}\right)$$

$$= 0$$

(5) Find the energy and Power of $y(t) = (1+j)e^{j\pi t/2}, \ 0\le t\le 10$

$$E_y = \int_0^{10} \left|(1+j)e^{j\pi t/2}\right|^2 dt$$

$$= 2\int_0^{10} dt = 20 < \infty$$

$$\left[\begin{array}{l} (1+j) = \sqrt{2}\,e^{j\pi/4} \\ (1+j)\,e^{j\pi t/2} = \sqrt{2}\,e^{j(\pi t + \pi/4)} \\ |e^{j\theta}| = 1 \end{array}\right]$$

Tip: A finite energy signal has zero Power.

$$P_x = \lim_{T\to\infty} \frac{E_x}{2T} = 0$$