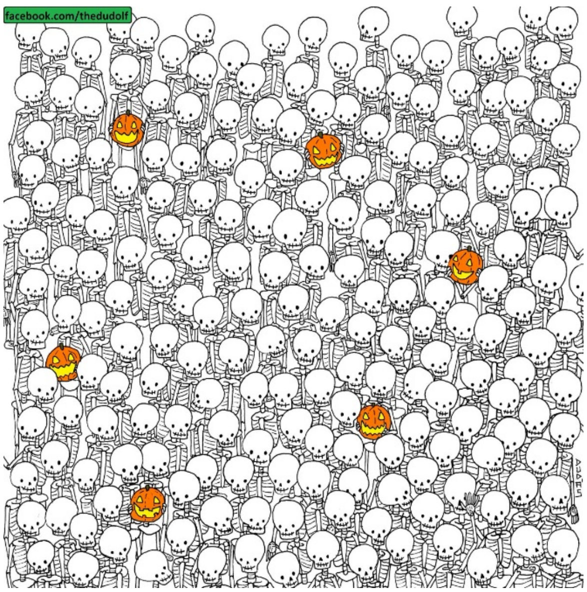
OPTI 512 Homework 7

1. In the spirit of Halloween, we are going to find a ghost using a matched filter. You will need Matlab or something equivalent to do this problem. The images below are called *Skeletons.jpg* and *Ghost.jpg*, and can be downloaded from the course website.



Do the following to find the ghost in a sea of skeletons:

a) Import the file Skeletons.jpg and convert it to gray scale. This should be a 1200 x 1200 pixel array. What is the mean value of the array? Subtract this mean value from the array. We will call this zero-mean skeleton array $s(x, y)$.

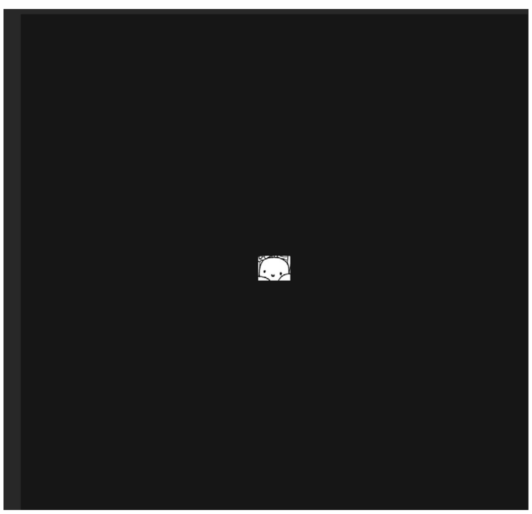
I just used the green channel from the image for the gray scale conversion. If the conversion was done differently, the means values might be slightly different. The mean value for the Skeletons gray scale image is 205.684.

b) Import the file Ghost.jpg and convert it to gray scale. This should be a 76 x 58 pixel array. What is the mean value of this array? Subtract this mean value from the array.

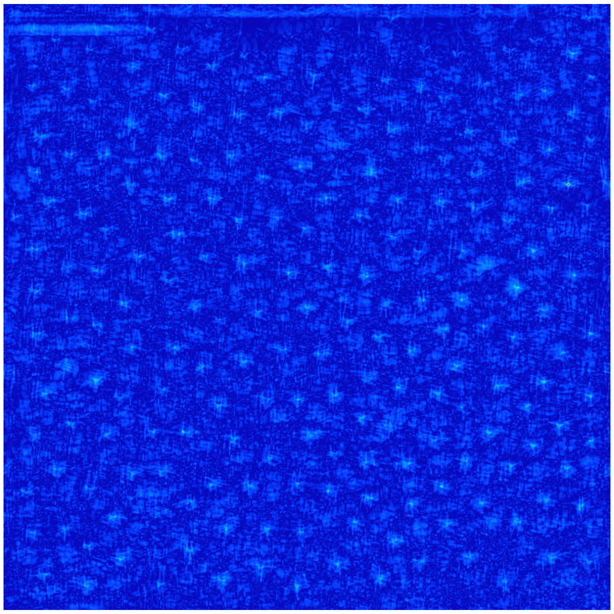
We will call this zero-mean ghost array $t(x, y)$ and use it as the template for the object we are trying to find in the skeleton array.

I again just used the green channel from the image for the gray scale conversion. If the conversion was done differently, the means values might be slightly different. The mean value for the Ghost gray scale image is 212.062.

c) Create an empty 1200 x 1200 pixel array and place the template $t(x, y)$ in the middle of the array We will call this array the ghost array $g(x, y)$. Display this array in your code to verify that you did it correctly. Save you ink. No need to put it in the assignment.

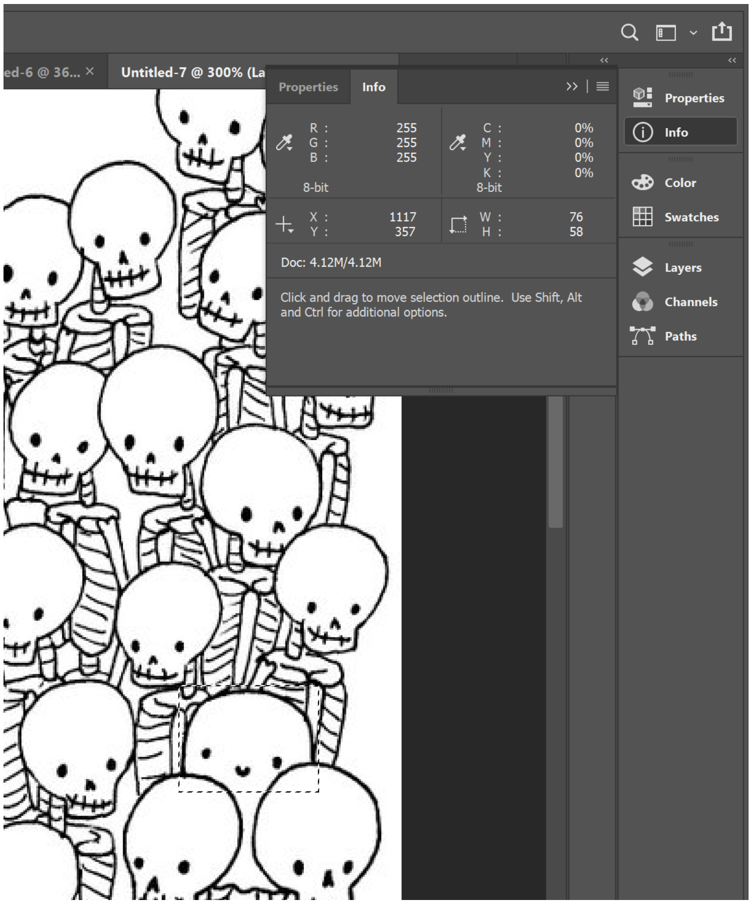


d) We now want to find the cross correlation $\gamma_{sg}(x, y)$ of the two arrays. This is more easily done in the Fourier domain. We know that $\mathcal{F}_{2D}\{\gamma_{sg}(x, y)\} = S(\xi, \eta)G^*(\xi, \eta)$, where $S(\xi, \eta) = \mathcal{F}_{2D}\{s(x, y)\}$ and $G(\xi, \eta) = \mathcal{F}_{2D}\{g(x, y)\}$. Knowing this, calculate the cross correlation as $\gamma_{sg}(x, y) = \mathcal{F}_{2D}^{-1}\{S(\xi, \eta)G^*(\xi, \eta)\}$. Plot $|\gamma_{sg}(x, y)|$.



e) What are the x, y coordinates of the maximum value of $\left|\gamma_{sg}(x, y)\right|$? Verify that the ghost is indeed located at this location in the original *Sketetons.jpg* image.

The maximum value of $\left|\gamma_{sg}(x, y)\right|$ *occurs at the point (1117, 357). Zooming into this position on the original image shows our ghost.*



2. State whether the following functions and *band-limited* or not. If they are *band-limited*, determine the Nyquist frequency.

   a) $rect\left(\frac{x}{3}\right)$

      The Fourier transform is $3sinc(3\xi)$ *and the sinc() function has non-zero values out to infinity, so the function is not band-limited.*

   b) $sinc\left(\frac{x}{3}\right)$

      The Fourier transform is $3rect(3\xi)$ *which is zero for* $|\xi| > \frac{1}{6}$, *so the function is band-limited and the Nyquist frequency* $N_\xi = \frac{1}{6}$.

   c) $Gaus(4x)$

The Fourier transform is $\frac{1}{4}Gaus\left(\frac{\xi}{4}\right)$ *which has non-zero values out to infinity, so the function is not band-limited.*

d) $tri\left(\frac{x}{2}\right)$

The Fourier transform is $2sinc^2(2\xi)$ *which has non-zero values out to infinity, so the function is not band-limited.*

e) $sinc^2\left(\frac{x}{2}\right)$

The Fourier transform is $2tri(2\xi)$ *which is zero for* $|\xi| > \frac{1}{2}$*, so the function is band-limited and the Nyquist frequency* $N_\xi = \frac{1}{2}$.

f) $\delta(x)$

The Fourier transform is 1 *which is non-zero values out to infinity, so the function is not band-limited.*

g) $cos(5\pi x)$

The Fourier transform is

$$\frac{1}{2}\left[\delta\left(\xi - \frac{5}{2}\right) + \delta\left(\xi + \frac{5}{2}\right)\right]$$

which is zero for $|\xi| > \frac{5}{2}$*, so the function is band-limited and the Nyquist frequency* $N_\xi = \frac{5}{2}$.

3. Download the files *amp.dat* and *phase.dat* from the course website. You will need Matlab or something equivalent to do this problem. These are binary files containing 1024 x 1024 consecutive double values corresponding to 1024 x 1024 arrays of amplitude values

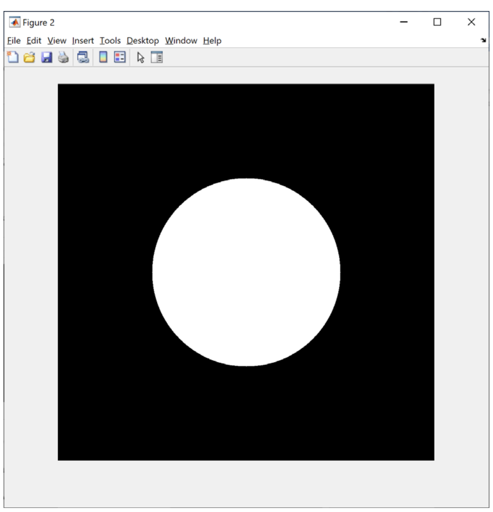
$A(x, y)$ and phase values $\Phi(x, y)$, respectively. For this problem, phase contrast techniques will be illustrated. Do the following:

a) Use the following Matlab code snippet below to aid in importing the data. You will need to edit the file path accordingly.

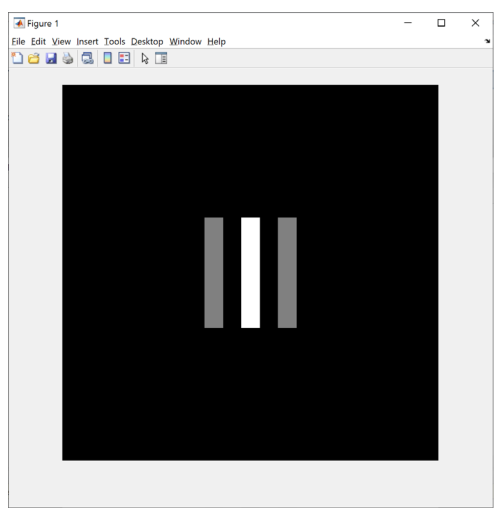
```
fid = fopen('C:\Users\jschw\Dropbox\Class\OPTI 512 Linear Systems, Fourier
Transforms\Homeworks\phase.dat','rb');
phase = fread(fid, [1024 1024], 'double');
imshow(phase,[])
fid = fopen('C:\Users\jschw\Dropbox\Class\OPTI 512 Linear Systems, Fourier
Transforms\Homeworks\amp.dat','rb');
amp = fread(fid, [1024 1024], 'double');
imshow(amp,[])
```

Plot the patterns $\mathrm{A}(x, y)$ and $\Phi(x, y)$.

$\mathrm{A}(x, y)$

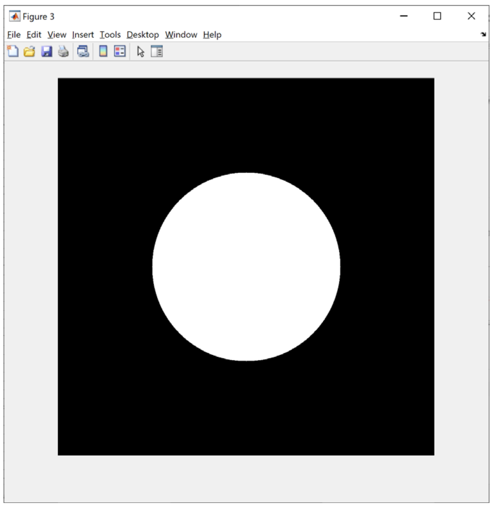


$\Phi(x, y)$



b) Create the complex array $f(x,y) = \mathrm{A}(x,y)exp[-i\Phi(x,y)]$. This will be the input to our phase contrast system. Sensors and our eyes can only record the squared magnitude of complex signals or $|f(x,y)|^2$. Plot $|f(x,y)|^2$ for this input.

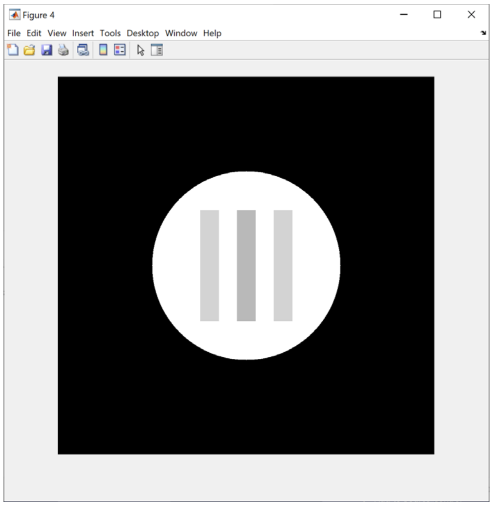
$|f(x,y)|^2 = \mathrm{A}(x,y)\ exp[-i\Phi(x,y)]\mathrm{A}(x,y)exp[i\Phi(x,y)] = A^2(x,y)$, *which means we can only see the squared modulus of the amplitude of the object.*



c) The transfer function for this system is

$$H(\xi,\eta) = \begin{cases} exp\left(i\frac{\pi}{2}\right) & for\ \xi = \eta = 0 \\ 1 & otherwise \end{cases},$$

so calculate $F(\xi,\eta)$ using the FFT routines in Matlab and then multiply $F(0{,}0)$ by $exp\left(i\frac{\pi}{2}\right)$ to create the output $G(\xi,\eta)$. Be careful with the shifting that occurs with the FFT and also note that Matlab starts its arrays with a value of 1 instead of 0. Finally, calculate output of the phase contrast system with the inverse FFT routines to get $g(x,y)$. Plot $|g(x,y)|^2$.



The point here is that by simply multiplying the point $F(0,0)$ *by* $exp\left(i\frac{\pi}{2}\right)$*, we were able to convert an input irradiance* $|f(x,y)|^2$ *where the phase is invisible into an output irradiance* $|g(x,y)|^2$ *where the phase pattern is visible.*