Classwork 10: Fall 2024
CPE381

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Total: 10 points
Some points to remember:

1 System Response.

5 points each.

1. Solve the second-order linear differential equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + x(t)$$

   with initial condition $y(0^-) = 2$, and $y'(0^-) = 1$, and $x(t) = e^{-4t}u(t)$.

   The output $y(t)$ is called the **complete response**.

2. By setting the initial condition to zero, we get the zero state response (ZSR). By assuming only the initial condition, and setting input $x(t)$ to zero, we get the zero input response (ZIR).

   For the previous question, assume the initial condition to be zero, and calculate the output. It will be your zero state response (ZSR), y_{zsr}. Zero input response (ZIR) can be $y_{zir} = y(t) - y_{zsr}$.

1.1 Solution:

Part 1:

$$\frac{dy(t)}{dt} \xrightarrow{\mathcal{L}} sY(s) - y(0^-) = sY(s) - 2$$

$$\frac{d^2y(t)}{dt^2} \xrightarrow{\mathcal{L}} s^2Y(s) - sy(0^-) - y'(0^-) = s^2Y(s) - 2s - 1$$

$$X(s) = \frac{1}{s+4}$$

$$\frac{dx(t)}{dt} \xrightarrow{\mathcal{L}} sX(s) - x(0^-) = \frac{s}{s+4} - 0 = \frac{s}{s+4}$$

Hence, the Laplace transform of the differential equation is

$$s^2Y(s) - 2s - 1 + 5sY(s) - 10 + 6Y(s) = \frac{s}{s+4} + \frac{1}{s+4}$$

Collecting all the like term and solving, we get

$$Y(s) = \frac{2s^2 + 20s + 45}{(s+4)(s^2+5s+6)} = \frac{2s^2 + 20s + 45}{(s+4)(s+2)(s+3)} = \frac{A}{(s+4)} + \frac{B}{s+3} + \frac{C}{s+2}$$

Using Partial fraction, $A = -3/2$, $B = -3$, $C = 13/2$
Hence,

$$Y(s) = \frac{-3/2}{(s+4)} + \frac{-3}{s+3} + \frac{13/2}{s+2}$$

Hence, its inverse Laplace Transform is

$$y(t) = \left(\frac{13}{2}e^{-2t} - 3e^{-3t} - \frac{3}{2}e^{-4t}\right)u(t)$$

Part 2:
Setting, the initial condition to zero,

$$s^2Y(s) + 5sY(s) + 6Y(s) = \frac{s}{s+4} + \frac{1}{s+4}$$

which leads to

$$Y(s) = \frac{s+4}{(s+4)(s^2+5s+6)} = \frac{s+4}{(s+4)(s+2)(s+3)}$$

Using partial fractions

$$Y(s) = \frac{-1/2}{s+2} + \frac{2}{s+3} + \frac{-3/2}{s+4}$$

Hence,
$y_{zsr}(t) = -\frac{1}{2}e^{-2t} + 2e^{-3t} - \frac{3}{2}e^{-4t}$
zsr subscript denotes that the initial condition is set to 0.
And, $y_{zir}(t) = y(t) - y_{zsr}$.