Calculus

CPE 381 FA24 08/28

Derivative from the first principle

$x(t)$ timeseries

$f(x) = x^2$

function

$$\frac{d f(x)}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + h^2 + 2xh - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+2x)}{\cancel{h}} = \lim_{h\to 0} (h+2x)$$

$$= 2x$$

$$\because \lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\frac{\cancel{1} + x + \frac{x^2}{2!} - \cancel{1}}{\cancel{x}}$$

Q6.

$$\int \frac{x+1}{x^3+x^2-6x}$$

$$\frac{x+1}{x^3+x^2-6x} = \frac{x+1}{x(x^2+x-6)}$$

$$= \frac{x+1}{x(x^2+3x-2x-6)} = \frac{x+1}{x(x(x+3)-2(x+3))}$$

$$= \frac{x+1}{x(x+3)(x-2)}$$

$$\frac{x+1+0\cdot x^2}{x(x+3)(x-2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$= \frac{A(x-2)(x+3) + Bx(x+3) + Cx(x-2)}{x(x-2)(x+3)}$$

$$= \frac{A(x^2+3x-2x-6) + B(x^2+3x) + C(x^2-2x)}{x(x-2)(x+3)}$$

$$\begin{cases} A+B+C=0 \\ Ax+3Bx-2Cx = x \\ A+3B-2C=1 \end{cases}$$

$6A=-1 \quad A=-1/6$

$A+3B+2(A+B)=1$

$1/6+3B+2\cdot\frac{1}{6}+2B=1$

$B = 3/10$

$C = -2/15$

$$\int \frac{1}{x}\,dx = |\ln x|$$

$$\int \frac{1}{x+2}\,dx = |\ln(x+2)|$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$