CPE 381, FA24, UAH Worked Example 3 Instructor: Rahul Bhadani
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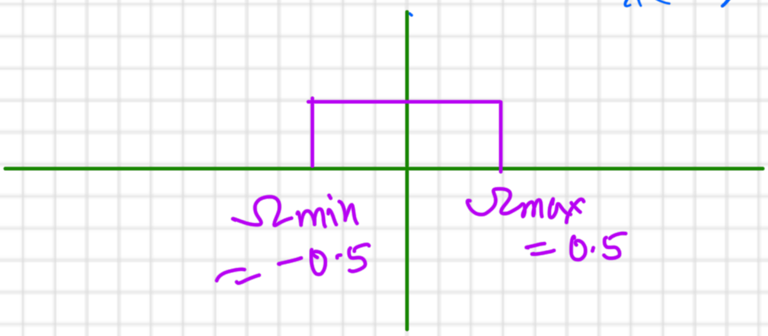
(1) Consider a signal $x(t) = \dfrac{2\sin(0.5t)}{t}$.

(a) Is $x(t)$ band-limited? If so, indicate its maximum frequency Ω_{max}. [5 points]

Hint: write the fourier transform of $x(t)$.

Solution, the fourier transform of $x(t)$ is

$$X(\Omega) = 2\pi\left[u(\Omega+0.5) - u(\Omega-0.5)\right]$$



Clearly $\Omega_{max} = 0.5$ rad/s.

Hence $x(t)$ is band-limited.

(b) Suppose that $T_s = 2\pi$, how does Ω_s relate to the Nyquist frequency $2\Omega_{max}$? Explain. What is the sampled signal $x(nT_s)$ equal to? [5 points]

Hint: 1) First write down the Nyquist Sampling Criteria.
2) Check if Ω_{max} from part (a) and $T_s = 2\pi$ satisfies Nyquist Criteria.

Solution)

According to the Nyquist Sampling Criteria, we have:

$$\Omega_s = \frac{2\pi}{T_s} \geq 2\Omega_{max},$$

or the sampling period:

$$T_s \leq \frac{\pi}{\Omega_{max}} = 2\pi$$

The given value satisfies the Nyquist Sampling criteria, so

we can sample the signal without aliasing.
And the given sampling period is the Nyquist sampling period.

and $x(nT_s) = \dfrac{\sin(0.5\ 2\pi n)}{0.5\, n\, 2\pi} = \dfrac{\sin(\pi n)}{\pi n}$

which is 1 for $n=0$ and 0 for any other value of n.

(c) Determine the spectrum of the sampled signal $X_s(\Omega)$ when $T_s = 2\pi$ and indicate how to reconstruct the original signal from the sampled signal.

Answer
The spectrum of the sampled signal $x_s(t)$ for $T_s = 2\pi$ ($\Omega_s = 1$) is

$$X_s(\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\Omega - k\,\Omega_s) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(\Omega - k) = 1$$

Passing this signal through the ideal low-pass filter with amplitude 2π, and cut off frequency $\frac{\Omega_s}{2} = \frac{1}{2}$, the reconstructed signal, the output of this filter, is the inverse F.T. of a pulse in the frequency, i.e., a sinc function that coincides with the original signal.