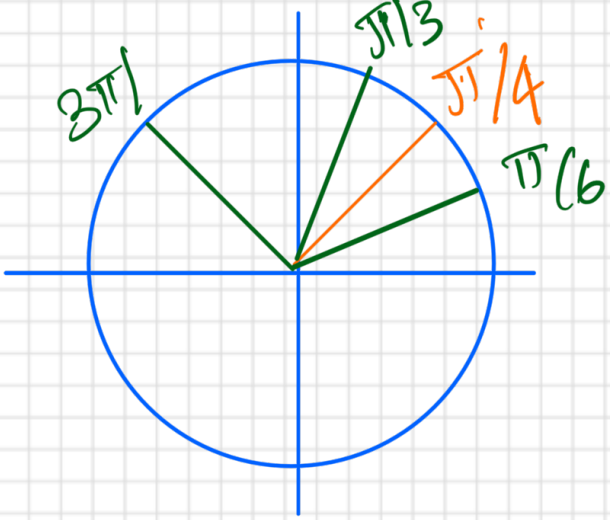
CPE 381 Classwork 02

Mathematical Preliminaries: Trigonometry

(1) Draw a circle and mark $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{3\pi}{4}$ and π. An Example is given below: (4pts)



(2) Convert $\frac{3\pi}{2}$ radian into degrees. (1pts)

(3) Find the value of θ : $4\sin^2\theta - 3 = 0$ (3pts)

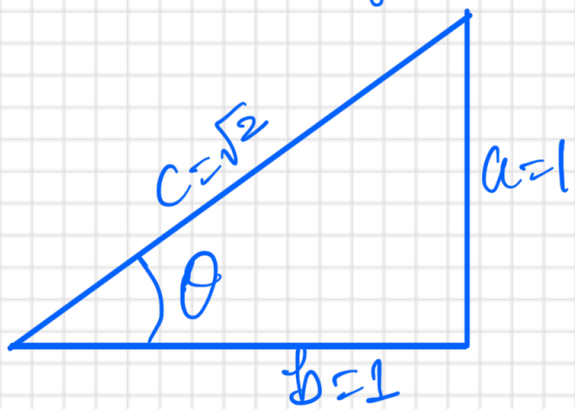
(4) Find the value of x: $2\sin^2 x - 3\sin x + 1 = 0$ (3pts)

(5) Prove: $(1-\sin^2(t))(1+\tan^2(t)) = 1$ (5pts)

(6) Prove $\frac{\sin^3(t)+\cos^3(t)}{\sin(t)+\cos(t)} = 1-\sin(t)\cos(t)$ (5pts)

(7) What is the value of $\sin\theta$ and $\cos\theta$, give $\tan\theta = \frac{4}{3}$? (2pts)

2 pts (8) Find the value of $\sin\theta$, $\cos\theta$ from the triangle



3 pts (9) Prove $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{12}$

2 pts (10) Find the value of x given $2\sin x = 1$

30 pts

Solution

(2) $\frac{3\pi}{2} \times \frac{180}{\pi} = 270^\circ$

(3) $4\sin^2\theta = 3$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

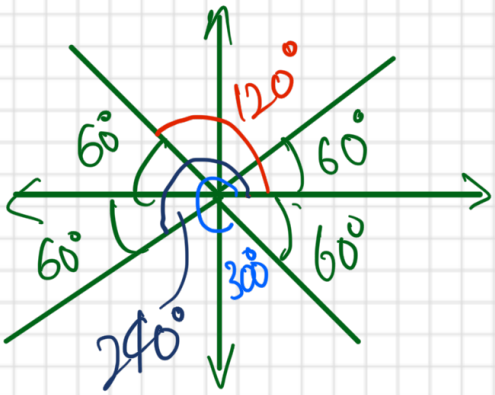
$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Sin is +ve in Quadrant I and II

$\theta = 60^\circ, 120^\circ$

Sin is -ve in Quadrant III and IV

$\theta = 240^\circ, 300^\circ$



④ $2\sin^2 x - 3\sin x + 1 = 0$

$2\sin^2 x - 2\sin x - \sin x + 1 = 0$

$2\sin x(\sin x - 1) - 1\cdot(\sin x - 1) = 0$

$(\sin x - 1)(2\sin x - 1) = 0$

$\sin x = 1/2$ or $\sin x = 1$

$\Rightarrow x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\Rightarrow x = \pi/2$

General Solution $x = \frac{\pi}{2} + 2\pi k$

$x = \frac{\pi}{6} + 2\pi k$

$x = \frac{5\pi}{6} + 2\pi k$

(5) $1-\sin^2(t) = \cos^2(t)$

$(1-\sin^2(t))(1+\tan^2(t)) = \cos^2(t) + \cos^2(t)\tan^2(t)$

$= \cos^2(t) + \frac{b^2}{h^2} \cdot \frac{p^2}{b^2}$

$= \cos^2(t) + \sin^2(t)$

$= 1$

(6) $\frac{\sin^3(t) + \cos^3(t)}{\sin(t) + \cos(t)}$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$= \frac{(\sin(t)+\cos(t))(\sin^2(t) - \sin(t)\cos(t) + \cos^2(t))}{(\sin(t)+\cos(t))}$

$= \sin^2(t) + \cos^2(t) - \sin(t)\cos(t)$

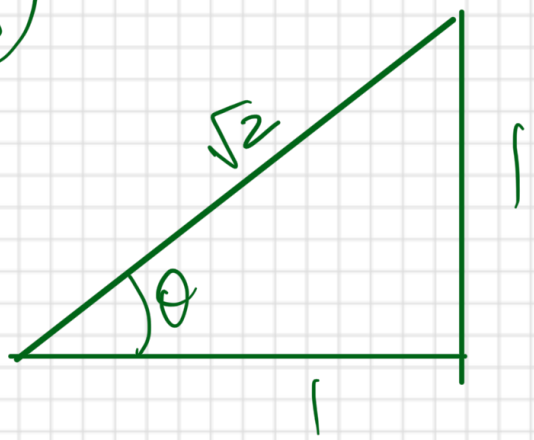
$= 1 - \sin(t)\cos(t)$

(7) $\tan\theta = \frac{4}{3} = \frac{P}{b}$

$h = \sqrt{3^2+4^2} = \sqrt{9+16} = 5$

$\cos\theta = \frac{3}{5}$ $\sin\theta = \frac{4}{5}$

(8)



$\sin\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

(9) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{4} - \frac{\pi}{6}$

$= \frac{3\pi - 2\pi}{12} = \frac{\pi}{12}$

(10) $2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$