Classwork 23
FA24
UAH
CPE 381
Instructor: Rahul Bhadani

Name of the Student:

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Total Marks: 10 points

① Transfer function to difference equation:

Given the transfer function $H(z) = \frac{1+z^{-1}}{2(1-z^{-1})}$ write down equivalent difference equation assuming input as $x[n]$ and the output as $y[n]$. Assume all initial conditions to be zero, and the system to be causal. (5 points)

Hint: Use time-shift property of z-transform.

Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{2(1-z^{-1})}$$

$$\Rightarrow 2(1-z^{-1})Y(z) = (1+z^{-1})X(z)$$

$$\Rightarrow Y(z) - z^{-1}Y(z) = \frac{1}{2}X(z) + \frac{1}{2}z^{-1}X(z)$$

Take inverse z-transform, such that

$$y[n] \hookleftarrow Y(z) \qquad x[n] \hookleftarrow X(z)$$

and

$$y[n-A] \hookleftarrow z^{-A}Y(z)$$

we can write

$$y[n] - y[n-1] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1]$$

$$\Rightarrow y[n] = y[n-1] + \frac{1}{2}x[n] + \frac{1}{2}x[n-1].$$

(2) Find the inverse z-bans form, i.e $x[n]$ for different values of n:

$$X(z) = \frac{z}{2z^2 - 3z + 1}, \quad |z| < \frac{1}{2}$$

Note: You'll not get a closed form solution. (5 points)

First, based on ROC: $\{|z| < \frac{1}{2}\}$, tell me if it is a left handed sequence or a right handed sequence.

Second, you need to use long divisicn method to find $x[n]$ for differeft values n.

You must write down the definition of z-bansform first.

Solution:

$$X(z) = \frac{z}{2z^2 - 3z + 1}$$

$$2z^2 - 3z + 1 \;\overline{)\; z + 2z^2 + 7z^3 \;\cdots}$$

$$z - 3z^2 + 2z^3$$

$$3z^2 - 2z^3$$

$$6z^4 - 9z^3 + 3z^2$$

$$-6z^4 + 7z^3$$

$$14z^5 - 21z^4 + 7z^3$$

$$-14z^5 + 15z^4 + \cdots$$

Thus comparing the quotient with

$$X(z) = \sum x[n] z^{-n}$$

we only see positive powers of z. thus

$x[0] = 0 \quad x[-1] = 1 \quad x[-2] = 3 \quad x[-3] = 7 \quad \cdots$