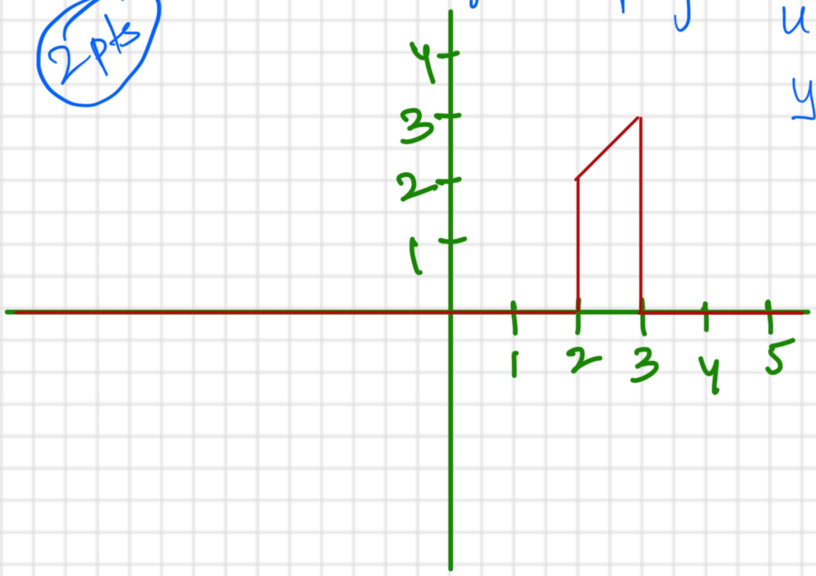
CPE 381 Classwork 06 FALL 2024 UAH

Instructor: Rahul Bhadani 09/11/2024 | Total: 20 pts

① (2 pts) Write the compact form for the following signal shown below using ramp function $r(t)$ and unit-step function $u(t)$.



You may need to apply appropriate transformation to ramp or unit-step signal.

Answer:

$$r(t)[u(t-2) - u(t-3)]$$

② (4 pts) Check if the signal $x(t) = e^{-at}u(t)$, $a > 0$ is an energy signal.

Note: a signal is energy signal if its energy is finite.

Answer:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |e^{-at}u(t)|^2 dt$$

$$= \int_0^{\infty} e^{-2at} dt = \frac{1}{2a} < \infty$$

③ (5 pts) Calculate the power of the sign $x(t) = A\cos(\Omega_0 t + \theta)$

Hint: first state the time period of $x(t)$.

Power of the signal $P = \frac{1}{T}\int_0^T |x(t)|^2 dt$

Answer:

$x(t)$ is a sinusoidal signal with period $T = \frac{2\pi}{\Omega_0}$

$$\left[P = \int_{-T/2}^{T/2} |x(t)|^2 dt = \int_0^T |x(t)|^2 dt \right]$$

Hence power $P = \frac{1}{T}\int_0^T |x(t)|^2 dt$

$$= \frac{\Omega_0}{2\pi} \int_0^{2\pi/\Omega_0} A^2 \cos^2(\Omega_0 t + \theta)\, dt$$

$$= \frac{A^2 \Omega_0}{2\pi} \int_0^{2\pi/\Omega_0} \frac{1}{2}\left[1 + \cos(2\Omega_0 t + 2\theta)\right] dt$$

$$= \frac{A^2 \Omega_0}{2\pi} \left[\frac{1}{2} t + \frac{1}{2\Omega_0} \sin(2\Omega_0 t + 2\theta)\right]\Bigg|_0^{\frac{2\pi}{\Omega_0}}$$

$$= \frac{A^2 \Omega_0}{2\pi} \left[\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\pi}{\Omega_0} + \frac{1}{2\Omega_0} \sin\left(2\cancel{\Omega_0} \times \frac{2\pi}{\cancel{\Omega_0}} + 2\theta\right) - \frac{1}{2} \times 0 - \frac{1}{2\Omega_0} \sin(0 + 2\theta)\right]$$

$$= \frac{A^2 \Omega_0}{2\pi} \left[\frac{\pi}{\Omega_0} + \cancel{\frac{1}{2\Omega_0} \sin(4\pi + 2\theta)} - \cancel{\frac{1}{2\Omega_0} \sin(2\theta)}\right]$$

$$\left(\sin(2\theta + 4\pi) = \sin 2\theta\right)$$

$$= \frac{A^2 \cancel{\Omega_0}}{2\cancel{\pi}} \cdot \frac{\cancel{\pi}}{\cancel{\Omega_0}} = \frac{A^2}{2} < \infty$$

④ Evaluate $\int_{-1}^{1} (3t^2 + 1)\, \delta(t)\, dt$

(2 pts)

Solution:

$$\int_{-1}^{1} (3t^2 + 1)\, \delta(t)\, dt = (3t^2 + 1)\Big|_{t=0} = 1$$

⑤ Input $x(t) = u(t)$

(5 pts) Impulse response $h(t) = e^{-at} u(t)$, $a > 0$.

Compute $y(t)$.

Use the convolution integration $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$

Note that $x(\tau)$ and $h(t-\tau)$ overlap between $\tau = 0$ and $\tau = t$.

Hint: sketch $x(t)$ and $h(t)$, and consider various shifted version of $h(t)$.

Solution

$$y(t) = \int_{\infty}^{\infty} u(\tau)\, e^{-a(t-\tau)}\, d\tau$$

$$= \int_0^t e^{-a(t-\tau)}\, d\tau$$

$$= e^{-at} \int_0^t e^{a\tau}\, d\tau = e^{-at} \frac{1}{a}(e^{at} - 1)$$

$$= \frac{1}{a}(1 - e^{-at})$$

(6) (2pts) Is a system described by input-output relationship

$$y(t) = x(at)$$

time-invariant?

Solution:

Let delayed input is $x_1(t) = x(t-T)$

Then $y_1(t) = x_1(at)$

$$= x(at - T)$$

$$= x\left(a\left(t - \frac{T}{a}\right)\right)$$

$$= y\left(t - \frac{T}{a}\right)$$

Since T-delayed input produces $\frac{T}{a}$- delayed output, the system is time-varying.

Alternatively,

for delayed input $x(t-T)$ we get output as

$$y(t) = x(a(t-T))$$

$$= x(at - Ta)$$

but for time invariant system $y(t)$ should be $x(at-T)$