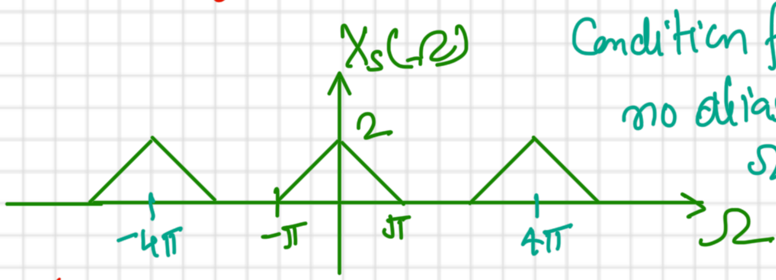
Classwork 17, UAH, FA24, CPE381, Instructor: Rahul Bhadani

Date: 10/23/2024, Name of the student:

Q. A signal $x(t)$ is sampled with no aliasing using an ideal sampler. The spectrum is given below:



Condition for no aliasing
$\Omega_s \geq 2\Omega_{max}$

(a) Determine the sampling period T_s used. (5 points)

Check diagram to see what is Ω_s.

Solution. From diagram $\Omega_s = 4\pi$

Then $T_s = \frac{2\pi}{\Omega_s} = \frac{\cancel{2\pi}}{\cancel{4\pi}} = 0.5$ s/samples

(b) Determine the signal $x(t)$.

Hint ① Spectrum $X(\Omega) = r(\Omega+\pi) - 2r(\Omega) + r(\Omega-\pi)$

② Apply duality property to first determine the Fourier transform of a similar time-domain signal. (5 points)

Solution

Let $z(t) = r(t+\pi) - 2r(t) + r(t-\pi)$ is a similar time domain signal.

then $Z(\Omega)\Big|_{s=j\Omega} = \frac{e^{s\pi} - 2 + e^{-s\pi}}{s^2}\Big|_{s=j\Omega}$

$= 2 \cdot \frac{1-\cos(\Omega)}{\Omega^2} = \frac{4\sin^2(\pi\Omega/2)}{\Omega^2}$

$$= \left(\frac{\sin(\pi\Omega/2)}{\Omega/2}\right)^2$$

then by duality

$$z(t) \Longleftrightarrow 2\pi\, Z(-\Omega)$$
$$= 2\pi\, Z(-\Omega)$$

thus

$$x(t) = \frac{Z(t)}{2\pi} = \frac{1}{2\pi}\left(\frac{\sin(\pi t/2)}{t/2}\right)^2$$