Worked Example 15 10/30/2024 FA24 CPE 381

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(1) Find the impulse response $h[n]$ for each of the following causal LIT discrete-time system

(a) $y[n] + 2y[n-1] = x[n] + x[n-1]$

(b) $y[n] - \frac{1}{2}y[n-2] = 2x[n] - x[n-2]$

and indicate whether each system is FIR (finite impulse response) or IIR (Infinite Impulse Response)

Solution:

(a) Setting $x[n] = \delta[n]$ gives $y[n] = h[n]$ where $h[n]$ is the impulse response.

Hence

$$h[n] = -2h[n-1] + \delta[n] + \delta[n-1]$$

Since the system is causal, $h[-1] = 0$

$$h[0] = -2h[-1] + \delta[0] + \delta[-1] = \delta[0] = 1$$

$$h[1] = -2h[0] + \delta[1] + \delta[0] = -2 + 1 = -1$$

$$h[2] = -2h[1] + \delta[2] + \delta[1] = -2(-1) = 2$$

$$h[3] = -2h[2] + \delta[3] + \delta[2] = -2(2) = -2^2$$

$\vdots$ we see a pattern here

$$h[n] = -2h[n-1] + \delta[n] + \delta[n-1] = (-1)^n 2^{n-1}$$

As we see that, the system's impulse response doesn't become zero past a certain point but continues indefinitely. Hence the system is infinite impulse response (IIR) system.

(b) Similarly,

$$h[n] = \frac{1}{2} h[n-2] + 2\delta[n] - \delta[n-2]$$

$$h[-2] = h[-1] = 0$$ (as the system is causal)

So

$$h[0] = \frac{1}{2} h[-2] + 2\delta[0] - \delta[-2] = 2\delta[0] = 2$$

$$h[1] = \frac{1}{2} h[-1] + 2\delta[1] - \delta[-1] = 0$$

$$h[2] = \frac{1}{2} h[0] + 2\delta[2] - \delta[0] = \frac{1}{2}(2) - 1 = 0$$

$$h[3] = \frac{1}{2} h[1] + 2\delta[3] - \delta[1] = 0$$

$$h[4] = \frac{1}{2} h[2] + 2\delta[4] - \delta[2] = 0 + 2\cdot 0 - 0 = 0$$

Hence $h[n] = 2\delta[n]$

Since $h[n]$ goes to zero eventually and has only one term, it is a finite impulse response (FIR) system.