CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Continuous-Time Systems

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Announcement

- Quiz 01, based on Chapter 01 Continuous-Time Signals from the Textbook. Available from September 05, 12:01 AM to September 07, 11:59 PM. 30 Questions, 45 Minutes.

Outline

1. **Linear Time-Invariant (LTI) Continuous-Time Systems**

System Thinking

System is an abstraction. Think of a device or a process as a system – something that transforms one signal into another.

$$x(t) \Longrightarrow \boxed{\mathcal{S}} \Longrightarrow y(t) = \mathcal{S}[x(t)]$$

Continuous-time Systems

When inputs and outputs are continuous, the system is continuous.

Linear Time-Invariant (LTI) Continuous-Time Systems

Linear Time-Invariant (LTI) Continuous-Time Systems

For most analyses at system-level, we care about the following properties:

- Linearity
- Time-invariance
- Causality
- Stability

Linearity

$$\mathcal{S}[a \cdot x(t) + b \cdot y(t)] = \mathcal{S}[a \cdot x(t)] + \mathcal{S}[b \cdot y(t)] = a \cdot \mathcal{S}[x(t)] + b \cdot \mathcal{S}[y(t)]$$

Time-invariance

If

$$x(t) \Rightarrow y(t) = \mathcal{S}[x(t)],$$

then, the time-invariance says:

$$x(t \mp \tau) \Rightarrow y(t \mp \tau) = \mathcal{S}[x(t \mp \tau)].$$

Linear-time Invariant (LTI) Systems

A system that satisfies both linearity and time-invariance is an LTI system.

Class Problem 1

Consider a system $g(t) = \mathcal{S}[f(t)] = a[f(t)]^2 - b[f(t)]$

- Is this system linear?
- Is this system time-invariant?

Class Problem 2

Consider

$$g(y) = \mathcal{S}[f(t)] = \int_{-\infty}^{\infty} f(t)\mathrm{rect}(y-t)dt$$

- Is this system linear?
- Is this system time-invariant?

Static Systems vs Dynamic Systems

In static systems, output depends on the current input but not on the historical input or output

In dynamic systems, output depends on current input, past inputs, and past outputs.

Differential equations are used to represent dynamic systems (literature also refers to them as dynamical systems).

Representation of Dynamic Systems

Dynamic systems are represented by ordinary differential equations (ODE) with constant coefficients:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) =$$
$$b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \cdots + b_1 \frac{dx(t)}{dt} + b_0 x(t)$$

with n initial conditions, $x(t) = 0$ for $t < 0$.

This equation represents a linear ordinary differential equation with constant coefficients, where $y(t)$ is the output, $x(t)$ is the input, and a_i and b_i are the constant coefficients.

Response of a Dynamic Systems

Assessing the output of the system:

- Consider output due to input only, considering initial conditions to be zero. **(zero-state response)**.
- Consider output due to initial conditions only, considering inputs to be zero. **(zero-input response)**.
- Complete response is the sum of the above two.