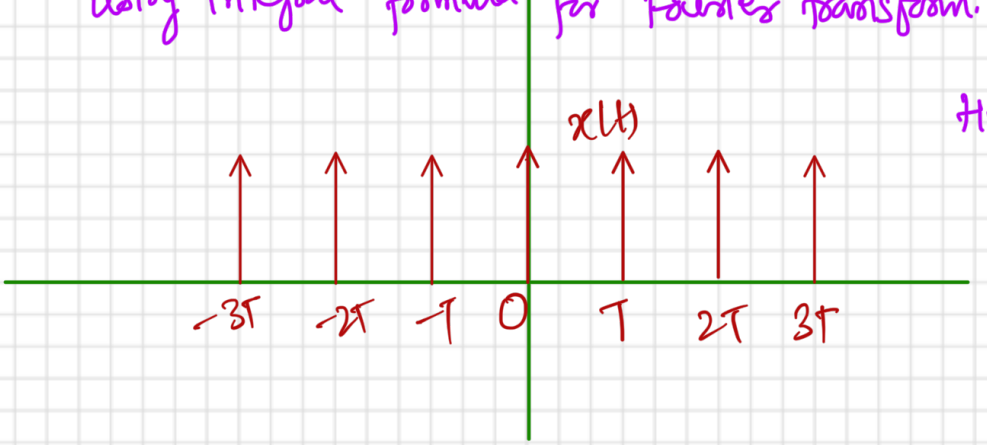
10/14/2024

① Compute the fourier transform of a truncated Dirac delta function $x(t) = \sum_{n=-3}^{3} \delta_c(t-nT)$ given below:

using integral formula for Fourier transform. (5 points)



Hint: Take summation Σ out of the integral $\int$.

Solution

$$X(\Omega) = \sum_{n=-3}^{3} \int_{-\infty}^{\infty} \delta_c(t-nT)\, e^{-j\Omega nt}\, dt$$

$$= \sum_{n=-3}^{n=3} e^{-j\Omega nT}$$

Q2. Using Laplace transform, find the fourier transform of

$$x(t) = A[u(t) - u(t-b)].$$

Solution:

$$X(s) = A\left[\frac{1}{s} - \frac{1}{s} e^{-sb}\right]$$

$$X(\Omega) = X(s)\Big|_{s=j\Omega} = A\left[\frac{1}{j\Omega} - \frac{1}{j\Omega} e^{-j\Omega b}\right]$$

with ROC as entire s-plane. Hence we can use Laplace transform.

$$= \frac{A}{j\Omega}\left[1 - e^{-j\Omega b}\right]$$

$$= A e^{-j\Omega b/2} \frac{\left[e^{j\Omega b/2} - e^{-j\Omega b}\right]}{j\Omega\left(\frac{b}{2}\right) \cdot \frac{2}{b}}$$

$$= A e^{-j\Omega b/2} \frac{\sin\left(\frac{\Omega b}{2}\right)}{\frac{\Omega b}{2}} \cdot \frac{b}{2}$$

$$= A e^{-j\Omega b/2} \cdot \frac{b}{2} \operatorname{sinc}\left(\frac{\Omega b}{2}\right)$$