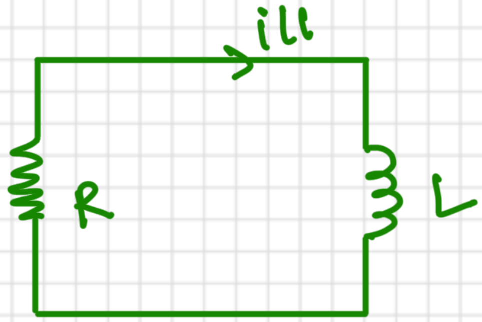
FA2024
CPE381

Complex Exponentials and Sinusoids



$$L\frac{di(t)}{dt} + Ri(t) = 0$$

$$\Rightarrow \frac{di(t)}{dt} = -\frac{R}{L}\, i(t)$$

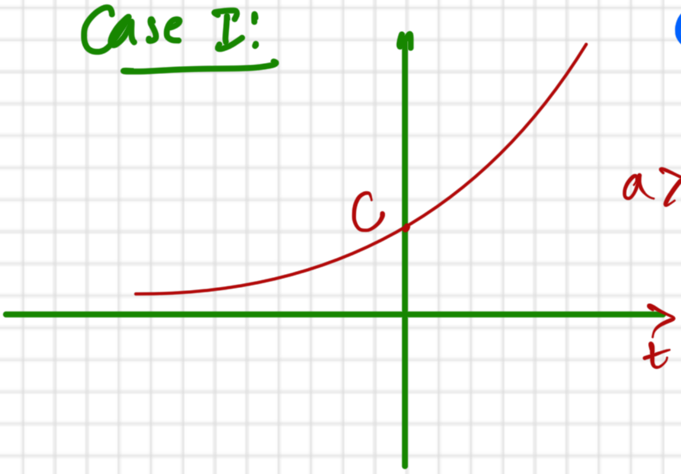
$$i(t) = i(0)\, e^{-Rt/L}$$

So, in general,

$$x(t) = Ce^{at}$$

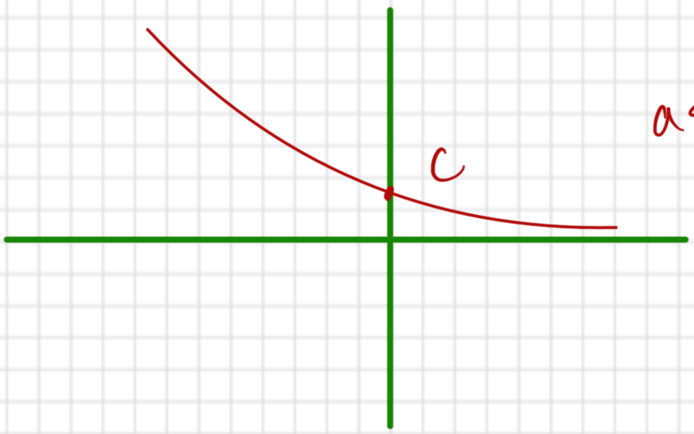
a = Time Constant

Case I: C, a, are Real.



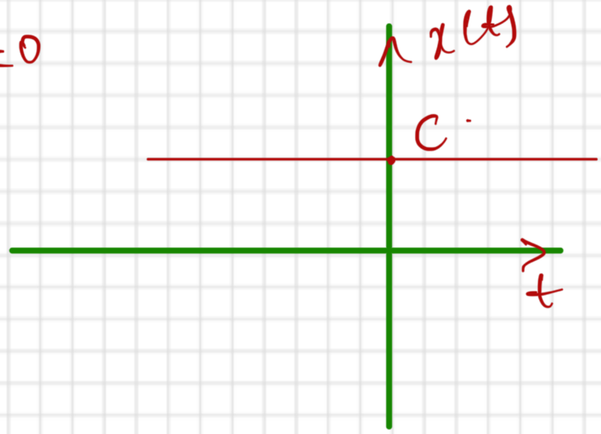
$a > 0$

$$x(t) = Ce^{at}\Big|_{t=0} = C$$



$a < 0$

$a = 0$

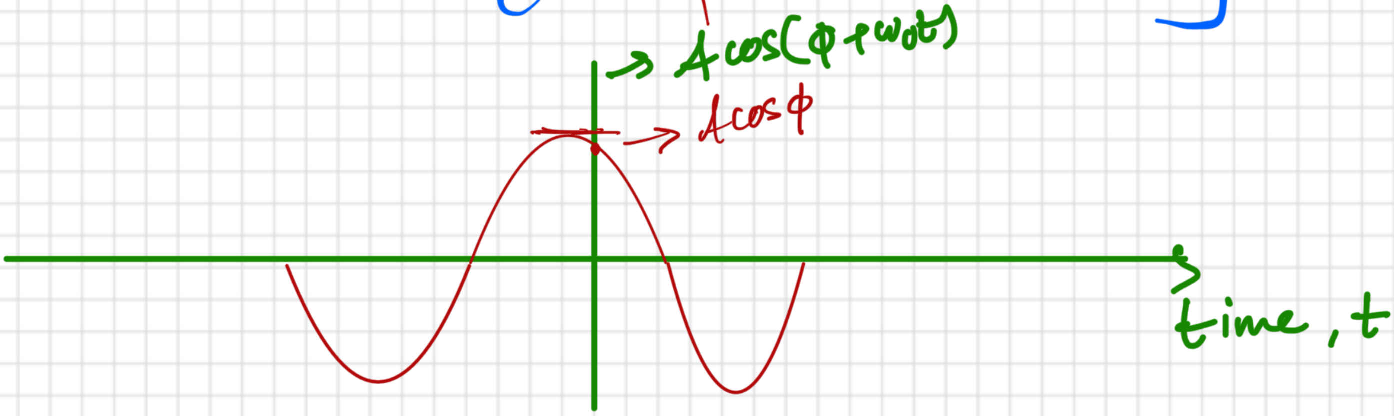


Case II: $C \in \mathbb{C}$ (Compl) a is imaginary.

$a = j\omega_0$ $\qquad$ $C = Ae^{j\phi}$

$$x(t) = Ce^{at} = Ce^{j\omega_0 t}$$
$$= Ae^{j\phi} \cdot e^{j\omega_0 t} = Ae^{j(\phi + \omega_0 t)}$$

$$x(t) = A\left[\cos(\phi + \omega_0 t) + j\sin(\phi + \omega_0 t)\right]$$



Case III: Both C and a are complex

$$C = Ae^{j\phi}$$

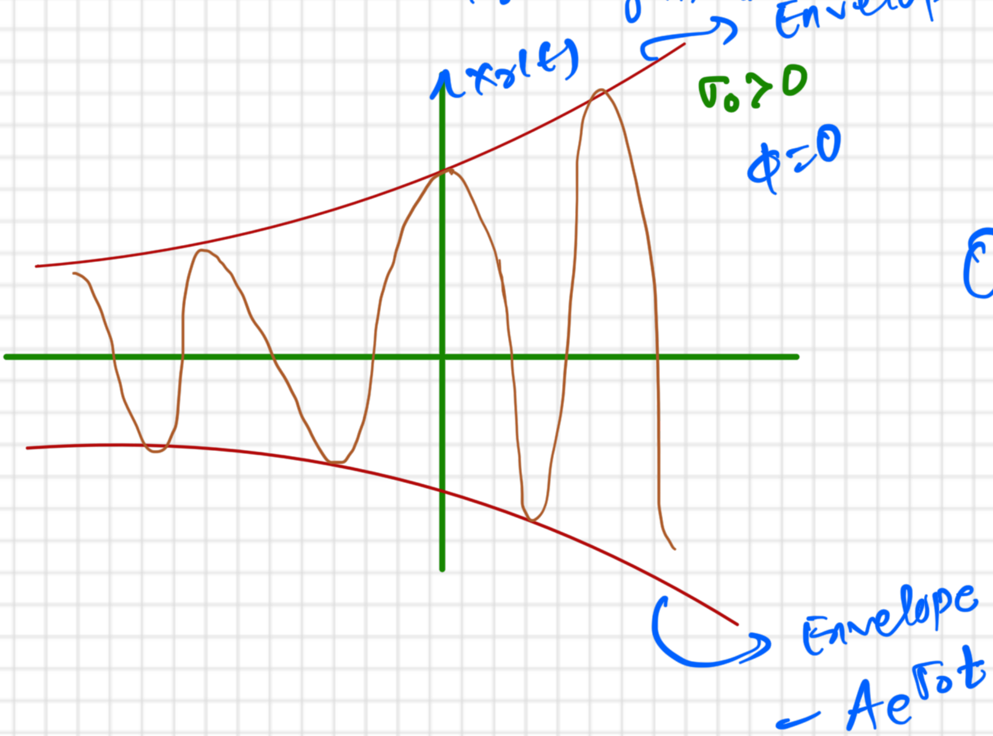
$$a = \sigma_0 + j\omega_0$$

$$x(t) = Ae^{j\phi}e^{(\sigma_0 + j\omega_0)t}$$

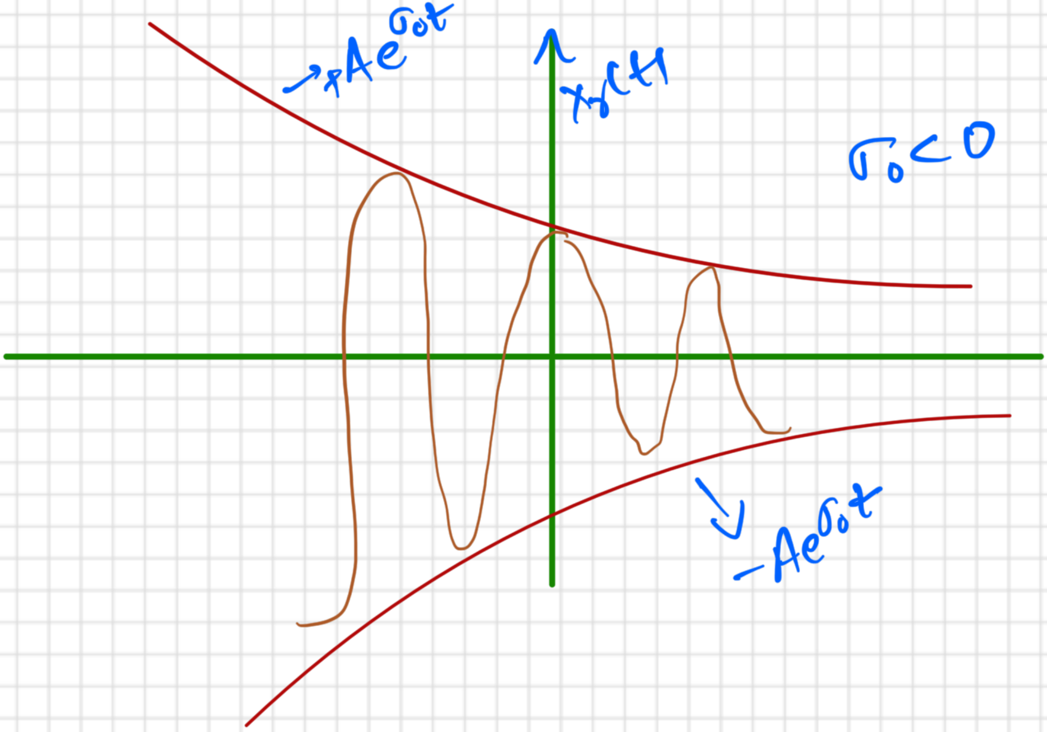
$$= \underbrace{Ae^{\sigma_0 t}\cos(\omega_0 t + \phi)}_{x_r(t)} + j\underbrace{Ae^{\sigma_0 t}\sin(\omega_0 t + \phi)}_{x_i(t)}$$

$$= x_r(t) + j x_i(t)$$

Envelope $Ae^{\sigma_0 t}$



Oscillation will amplify.



Damped Oscillation