CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Continuous Signals

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Outline

1. Motivation
2. Operation on Signals
3. Basic Signals as Building Blocks

Motivation

Signals and Systems is ‘Grandfather’ of Data Science for Electrical and Computer Engineers

Classification of Signals

We care about the following properties when dealing with signals:

- Predictability: Random or Deterministic
- Variations of time and amplitude: continuous, discrete (time or x-axis) / quantized (amplitude or y-axis)
- Periodic/Aperiodic
- Finite energy/finite power; Infinite energy/Infinite power

Operation on Signals

Basic Mathematical Operations

- Addition: $x(t) + y(t)$
- Subtraction: $x(t) - y(t)$
- Constant multiplication: $kx(t)$ where k is a constant

Time-shift

- $x(t - \tau) \rightarrow$ Signal is delayed
- $x(t + \tau) \rightarrow$ Signal is advanced

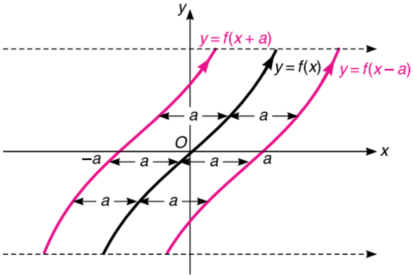
$f(x)$ transforms to $f(x - a)$

i.e., $f(x) \longrightarrow f(x - a)$; a is positive. Shift the graph of $f(x)$ through 'a' unit towards right

$f(x)$ transforms to $f(x + a)$.

i.e., $f(x) \longrightarrow f(x + a)$; a is positive. Shift the graph of $f(x)$ through 'a' units towards left.

Graphically it could be stated as



Time Reflection

- $x(t) \rightarrow x(-t)$: take mirror image along y-axis

The book doesn't specify whether to take the mirror image along the y-axis or not and it is confusing because the signal used in example 1.3.1 is symmetric with respect to both the x and y axes.

$f(x)$ transforms to $f(-x)$

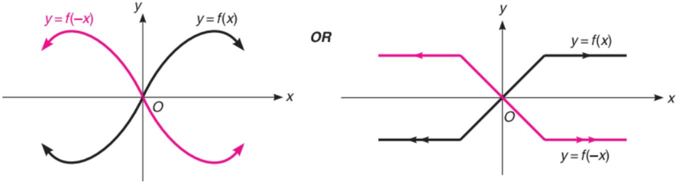
i.e., $$f(x) \longrightarrow f(-x)$$

To draw $y = f(-x)$, take the image of the curve $y = f(x)$ in y-axis as plane mirror.

OR

"Turn the graph of $f(x)$ by 180° about y-axis."

Graphically it is stated as;



Signal Stretching along y-axis

- $f(x) \rightarrow af(x); \quad a > 1:$ Stretch the graph of $f(x)$ a times along y-axis.
- $f(x) \rightarrow \frac{1}{a}f(x); \quad a > 1:$ Shrink the graph of $f(x)$ a times along y-axis.

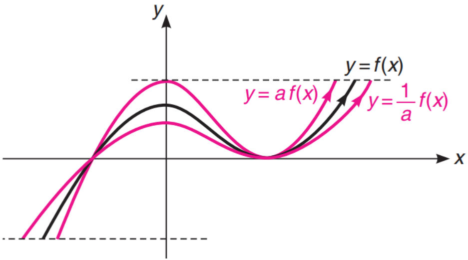
f(*x*) transforms to *a f*(*x*)

i.e., $$f(x) \longrightarrow af(x); \quad a > 1$$

Stretch the graph of f(x) '*a*' times along *y*-axis.

$$f(x) \longrightarrow \frac{1}{a}f(x); \; a > 1.$$

Shrink the graph of f(x) '*a*' times along *y*-axis.



Signal Stretching along x-axis

- $f(x) \rightarrow af(ax)$; $a > 1$: Stretch the graph of $f(x)$ a times along x-axis.
- $f(x) \rightarrow f\left(\frac{1}{a}x\right)$; $a > 1$: Shrink the graph of $f(x)$ a times along x-axis.

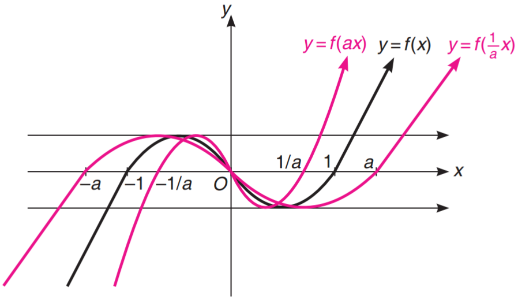
***f*(x) transforms to *f*(*ax*)**

i.e., $f(x) \longrightarrow f(ax); \quad a > 1$

Shrink (or contract) the graph of f(x) '*a*' times along *x*-axis.

again $f(x) \longrightarrow f\left(\frac{1}{a}x\right); \quad a > 1$

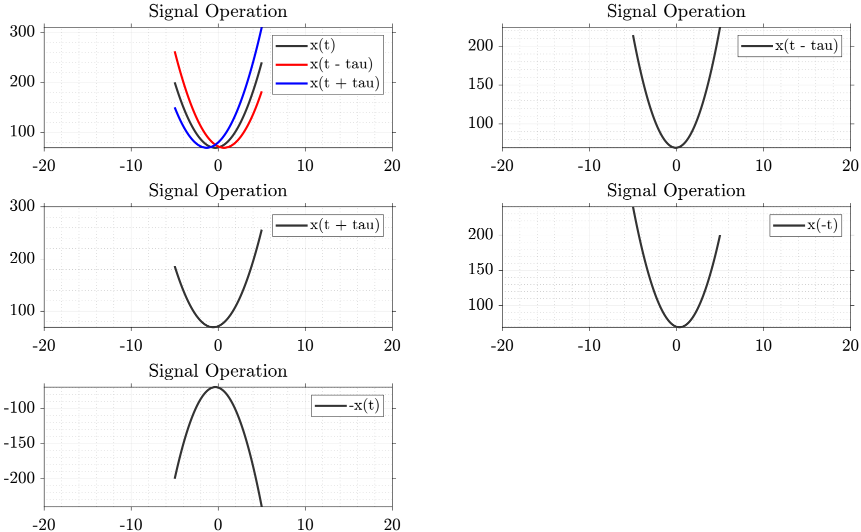
Stretch (or expand) the graph of f(x) '*a*' times along *x*-axis.



Example and MATLAB Code

$$x(t) = 6t^2 + 4t + 70$$

Code:
https://github.com/rahulbhadani/CPE381_FA24/blob/master/Code/signal_operation.m



Even and Odd Signals

- Even Signal: $x(t) = x(-t)$
- Odd Signal: $x(t) = -x(-t)$
- Any signal can be represented by the sum of even and odd signals
  $y(t) = y_e(t) + y_o(t)$

$$y_e(t) = 0.5[y(t) + y(-t)]$$

$$y_o(t) = 0.5[y(t) - y(-t)]$$

Periodic Signals

- Defined for all possible values of $t, -\infty < t < \infty$.
- There is the real value $T_0 \in \mathbb{R}^+$, called the fundamental frequency such that $x(t + kT_0) = x(t), k \in \mathbb{I}$.
- A constant signal is periodic of a non-definable fundamental period.
- A $\cos(\omega t + \theta)$, $\omega = 2\pi/T_0$, $\omega = 2, \theta = -\pi/2, A = 2$.

  What's the fundamental frequency, $1/T_0$?

Energy and Power of Signals

- **Energy:**

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- **Power:**

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

A signal is called finite-power if the signal power is finite.

Basic Signals as Building Blocks

Complex Exponentials

Consider $A = |A|e^{j\theta}$, $a = r + k\Omega_0$

- $x(t) = Ae^{at} = \dots$
- Real part $f(t) = \mathrm{Re}\{x(t)\}, = \dots$

  $-|A|e^{rt} \leq f(t)) \geq |A|e^{rt}$. $r < 0$, $f(t)$ is damped, $r > 0$, $f(t)$ grows.
- Imaginary part $g(t) = \mathrm{Im}\{x(t)\}, = \dots$

Sinusoids

A sinusoid of the general form:

$$A\cos(\Omega_0 t + \theta) = A\sin(\Omega_0 t + \theta + \pi/2), \quad -\infty < t < \infty$$

- A is the amplitude
- $\Omega_0 = 2\pi f_0$ is angular frequency in `rad/s`.
- θ is phase shift
- Fundamenta period T_0 is

$$T_0 = \frac{2\pi}{\Omega_0} = \frac{1}{f_0}$$