Classwork 03 : Calculus CPE381

(1) Using the first principle differentiate the function $f(x) = e^{2x}$ with respect to x.

Solution:

$$f(x) = e^{2x} \qquad f(x+h) = e^{2(x+h)}$$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{2(x+h)} - e^{2x}}{h} = \lim_{h\to 0} \frac{e^{2x}(e^{2h}-1)}{h}$$

$$= e^{2x}\left(\lim_{h\to 0} \frac{e^{2h}-1}{2h}\right) \times 2 = 2e^{2x}$$

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

$$\left[\because \lim_{x\to 0} \frac{e^{x}-1}{x} = 1\right]$$

(2) If $y = \sin x + e^x$, find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x + e^x)$$

$$= \frac{d}{dx}\sin x + \frac{d}{dx}e^x$$

$$= \cos x + e^x$$

(3) If $y = x^2 + \sin^{-1}x + \log_e x$, find dy/dx

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin^{-1}x) + \frac{d}{dx}(\log_e x)$$

$$= 2x^{2-1} + \frac{1}{\sqrt{1-x^2}} + \frac{1}{x}$$

$$= 2x + \frac{1}{\sqrt{1-x^2}} + \frac{1}{x}$$

④ If $y = e^x \sin x$ find $\frac{dy}{dx}$

Hint if $y = u(x)v(x)$

$$\frac{dy}{dx} = \left\{\frac{d}{dx} u(x)\right\} v(x) + u(x) \left\{\frac{d}{dx} v(x)\right\}$$

Solution

$$\frac{dy}{dx} = \left\{\frac{d}{dx}(e^x)\right\} \cdot \sin x + e^x \cdot \left\{\frac{d}{dx}(\sin x)\right\}$$

$$= e^x \cdot \sin x + e^x \cos x = e^x(\sin x + \cos x)$$

⑤ If $y = \frac{x}{x^2+1}$, find $\frac{dy}{dx}$

Hint: use the quotient rule as

If $y = \frac{u(x)}{v(x)}$, then

$$\frac{dy}{dx} = \frac{\left\{\frac{d}{dx} u(x)\right\} v(x) - \left\{\frac{d}{dx} v(x)\right\} u(x)}{\{v(x)\}^2}$$

Solution:

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}(x) - x\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1) \cdot 1 - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(1+x^2)^2}$$

6. Evaluate $\int \frac{(x+1)}{x^3+x^2-6x}\,dx$

Solution:

$$\frac{x+1}{x^3+x^2-6x} = \frac{x+1}{x(x^2+x-6)}$$

$$= \frac{x+1}{x(x^2+3x-2x-6)} = \frac{x+1}{x(x(x+3)-2(x+3))}$$

$$= \frac{x+1}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$$

$$A(x-2)(x+3) + B\,x(x+3) + C\,x(x+2) = x+1$$

Equating coefficients on the both sides

$$A = -1/6 \quad B = 3/10 \quad C = -2/15$$

Thus, $$\int \frac{x+1}{x^3+x^2-6x}\,dx = -\frac{1}{6}\int \frac{1}{x}\,dx + \frac{3}{10}\int \frac{1}{x-2}\,dx - \frac{2}{15}\int \frac{1}{x+3}\,dx$$

$$= -\frac{1}{6}\ln x + \frac{3}{10}\ln|x-2| - \frac{2}{15}\ln|x+3| + C$$

⑦ Find the indefinite integral of $f(x) = 3x^2 + 4x - 2$

Solution:

$$\int f(x)dx = \int 3x^2\,dx + \int 4x\,dx - \int 2dx$$

$$= \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{4}^{2}x^2}{\cancel{2}} - 2x + C$$

$$= x^3 + 2x^2 - 2x + C$$

⑧ Find $\int x \sin x \, dx$

Hint: Use integration parts: $\int u dv = uv - \int v du$

Solution

$\Rightarrow$ $u = x$ $\quad du = dx$

$dv = \sin x$ $\quad v = \int dv = -\cos x$

$$\int x \sin x \, dx = -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + C$$

(9) Solve the differential Equation

$$y' = 3x^2$$

Solution

$$\frac{dy}{dx} = 3x^2$$

$$\Rightarrow dy = 3x^2 dx$$

$$\Rightarrow \int dy = \int 3x^2 dx$$

$$\Rightarrow y = x^3$$

(10) Solve the equation:

$$(1+y^2)\, y' = \frac{3}{x}$$

Solution:

separate the variable:

$$(1+y^2)\frac{dy}{dx} = \frac{3}{x}$$

$$\Rightarrow (1+y^2)\, dy = \frac{3}{x} dx$$

$$\Rightarrow \int dy + \int y^2 dy = 3\int \frac{dx}{x}$$

$$\Rightarrow y + \frac{y^3}{3} = 3\ln(x) + \ln c$$

$$\Rightarrow y + \frac{y^3}{3} = \ln cx^3 \Rightarrow e^{y + y^3/3} = cx^3$$