Classwork 13: Fall 2024
CPE381

Instructor: Rahul Bhadani, The University of Alabama in Huntsville

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Total: 10 points
Some points to remember:

1 Fourier Series.

$x(t) = \cos 4t + \sin 6t$

1. Write down the fundamental period of $x(t)$. **(2 points)**
2. Compute its exponential Fourier series by first determining its Fourier coefficients. **(8 points)**

1. Fundamental period of $\cos 4t \Rightarrow T_a = \frac{2\pi}{4} = \frac{\pi}{2}$

$\Omega_a = 4$

of $\sin 6t \Rightarrow T_b = \frac{2\pi}{6} = \frac{\pi}{3}$

$\Omega_b = 6$

π is an intger multiple of $\frac{\pi}{2}$ as well as $\pi/3$

becauge $\frac{\pi}{2} \cdot 2 = \pi$ and $\frac{\pi}{3} \cdot 3 = \pi$

Hence the period for $x(t)$ is π, $T_0 = \pi$

$\Omega_0 = \frac{2\pi}{T_0}$

$= \frac{2\pi}{\pi} = 2$

$\Omega_0 = 2$

(2.) Hence

$$x(t) = \cos 4t + \sin 6t \quad , \Omega_0 = 2$$

$$= \cos 2\Omega_0 t + \sin 3\Omega_0 t$$

$$= \frac{1}{2}\left(e^{j2\Omega_0 t} + e^{-j2\Omega_0 t}\right) + \frac{1}{2j}\left(e^{j3\Omega_0 t} - e^{-j3\Omega_0 t}\right)$$

$$= -\frac{1}{2j} e^{-j3\Omega_0 t} + \frac{1}{2} e^{-j2\Omega_0 t} + \frac{1}{2} e^{+j2\Omega_0 t} + \frac{1}{2j} e^{+j3\Omega_0 t}$$

Comparing it with $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t}$

$$X_{-3} = -\frac{1}{2j}$$

$$X_{-2} = \frac{1}{2}$$

$$X_{+2} = \frac{1}{2}$$

$$X_{+3} = \frac{1}{2j}$$

and all other $X_k = 0$.