Homework 1 Solution
CPE381

Canvas: `hw01`

Due: 01 September 2024, 11:59 PM

You are allowed to use a generative model-based AI tool for your assignment. However, you must submit an accompanied reflection report on how you use the AI tool, what was the query to the tool, and how it improved your understanding of the subject. You must also add your thoughts on how you would tackle the assignment if there was no such tool available. Failure to provide a reflection report for every single assignment where an AI tool was used may result in a penalty and subsequent actions will be taken in line with plagiarism policy.

Submission instruction:

Upload a PDF on Canvas with the format {firstname.lastname}_cpe381_hw01.pdf. I recommend using Latex to typeset your submissions.

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1 Let's face the truth. (20 points)

Prove:

1. $1 + e^{j\pi} = 0$.
2. $\sin(-\theta) = -\sin\theta$ using Euler's identity.

1.1 Answers:

1. Using Euler's Identity, we can write $e^{j\pi}$ as

$$e^{j\pi} = \cos\pi + j\sin\pi$$

   but $\cos\pi = -1$, and $\sin\pi = 0$, hence $e^{j\pi} = -1$.. Thus, $1 + e^{j\pi} = 0$.

2. Using Euler's identity,

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Replace θ by $-\theta$,

$$\Rightarrow \sin(-\theta) = \frac{e^{-j\theta} - e^{j\theta}}{2j}$$

Take the minus sign out of thenumerator from Right-hand side of the above equation

$$\Rightarrow \sin(-\theta) = -\frac{e^{j\theta} - e^{-j\theta}}{2j} = -\sin\theta \tag{1}$$

2 Tigonometry Fun (20 points)

1. Verify the following identities: $1 - \dfrac{\cos^2\theta}{1+\sin\theta} = \sin\theta$
2. Solve $\cos 2x - 3\sin x + 1 = 0$ for x.

2.1 Answers:

1. We know, $\sin^2\theta + \cos^2\theta = 1$

$$1 - \frac{1-\sin^2\theta}{1+\sin\theta} = 1 - \frac{(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta} = 1 - (1-\sin\theta) = \sin\theta \tag{2}$$

2.

$$\begin{aligned} \cos 2x - 3\sin x + 1 &= 0 \\ (1 - 2\sin^2 x) - 3\sin x + 1 &= 0 \\ -2\sin^2 x - 3\sin x + 2 &= 0 \\ 2\sin^2 x + 3\sin x - 2 &= 0 \\ (2\sin x - 1)(\sin x + 2) &= 0 \end{aligned}$$

$$\Rightarrow \sin x = \frac{1}{2} \text{ and } \sin x = -2$$

For $\sin x = \frac{1}{2}$, $x = 6$ and $\frac{5\pi}{6}$. For $\sin x = -2$, there is no solution as $\sin x \in [-1, 1]$.

3 It's Complicated (20 points)

1. Using Euler's formula evaluate the following complex numbers: (a) $z = e^{j2\pi}$ (b) $z = e^{j\pi}$
2. Using Euler's formula derive the following: $\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

3.1 Answers:

1. (a) $z = e^{j2\pi} = \cos(2\pi) + j\sin(2\pi) = 1 + 0 = 1$
   (b) $z = e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 + 0 = -1$

2.
$$\begin{aligned}\cos^2\theta = \left(\frac{e^{j\theta}+e^{-j\theta}}{2}\right)^2 &= \frac{e^{2j\theta}+e^{-2j\theta}+2e^{j\theta}e^{-j\theta}}{4} \\ &= \frac{e^{2j\theta}+e^{-2j\theta}}{4}+\frac{2}{4} = \frac{\cos(2\theta)}{2}+\frac{1}{2} = \tfrac{1}{2}(1+\cos(2\theta))\end{aligned}$$

4 It's all Derivatives (10 points)

1. Taylor's Formula is written as

$$f(x) \approx g_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

   If $f(x) = e^{\sin x}$, the Taylor's second degree $(n = 2)$ polynomial of initial point $c = 0$ is

$$g_2(x) = \cdots$$

2. Differentiate: (a) $f(x) = x(\ln x - 1)$, (b) $f(x) = \ln(\tan x)$.

4.1 Answers:

1. $g_2(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2}x^2$

$$f(x) = e^{\sin x} \Rightarrow f(0) = 1$$

$$f'(x) = e^{\sin x}\cos x \Rightarrow f'(0) = 1$$

$$f''(x) = e^{\sin x}\cos^2 x - e^{\sin x}\sin x \Rightarrow f''(0) = 1$$

   Hence, $g_2(x) = 1 + x + \dfrac{x^2}{2}$

5 Area under curve. (20 points)

1. Find the value $\int_0^{\pi} |\cos x| dx$.
2. Additionally, write a MATLAB script to create a plot of the function $|\cos x|$ and shade the region between $x = 0$ to $x = \pi$ with the green color. You may choose a curve color as a contrasting color. Use `trapz` function in MATLAB to calculate the area of $|\cos x|$ between $x = 0$ to $x = \pi$. Comment on the result with what you obtained from the absolute integration above.

5.1 Answers:

We know, graph of $y(x) = \cos x$ is between 0 to π is as given in Figure 1.

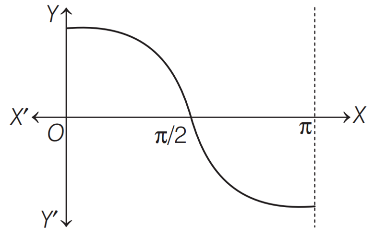

Figure 1: $\cos x$

$\therefore$ The graph of $y(x) = |\cos x|$ is as shown in Figure 2.

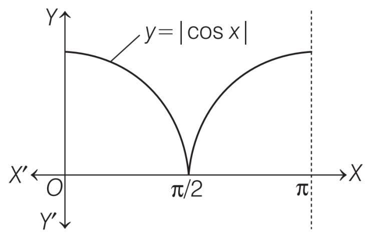

Figure 2: $|\cos x|$

Thus, effectively, taking absolute or mod divide the period by 2. Thus, we can write the integration given in the question as

$$I = \int_0^{\pi} |\cos x| dx = 2\int_0^{\pi/2} |\cos x| dx = 2\int_0^{\pi/2} \cos x dx = 2 \tag{3}$$

$\because |\cos x|$ is symmetric about $x = \frac{\pi}{2}$, and $\cos x \geq \forall\, x \in [0, \frac{\pi}{2}]$

```matlab
%(C) Rahul Bhadani
% CPE 381, Homework 1, Question 5.2

syms x
f = abs(cos (x));
% definite integral
int(f, 0, pi)

%% Numerical Integration using trapezoidal rule
x = 0:0.01:pi;
func = abs(cos(x));
trapz(x, func)
f = figure(1);
plot( x, func);
hold on;
area(x, func, 'FaceColor','g', 'EdgeColor','r');
```

```
grid on;
grid minor;
xlim([-0.5, 4.2]);
ylim([-0.5, 1.1]);
xL = xlim;
yL = ylim;
line([0 0], yL);  %x-axis
line(xL, [0 0]);  %y-axis

% Plot the line
set(gca, 'FontSize', 16);
set(gca, 'XColor', [0, 0, 0], 'YColor', [0, 0, 0], 'TickDir', 'out');
xaxis = get(gca, 'XAxis');
xaxis.TickLabelInterpreter = 'latex';
yaxis = get(gca, 'YAxis');
yaxis.TickLabelInterpreter = 'latex';
title('$|\cos(x)|$', 'Interpreter','latex');
exportgraphics(f, '../figures/FA24_CPE381_hw01_Q5.pdf');
```

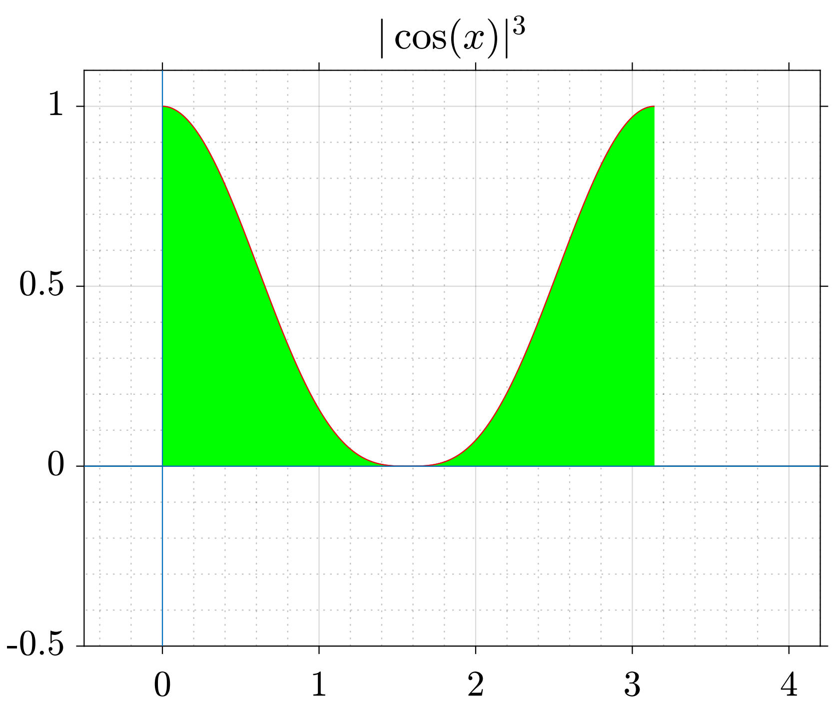

Figure 3: $|\cos x|^3$ with shaded area under the curve.

5.1.1 Commentary

Upon executing the code, we see that symbolic computation gives the exact value w, which matches the one obtained by solving the integration in the previous part. The numerical calculation using `trapz` gives the answer as 1.9984, when the time resolution is 0.01.

6 Initial value problem. (30 points)

Solve the differential equation $ty' + 3y = 0, \quad y(1) = 2$, assuming $t > 0$. In addition to doing to mathematically, write a MATLAB script to solve it programmatically.

6.1 Answers:

We write the equation in standard form as

$$\begin{aligned}
y' + 3\frac{y}{t} &= 0 \\
\Rightarrow \frac{dy}{dt} + 3\frac{y}{t} &= 0 \\
\Rightarrow \frac{dy}{dt} &= -3\frac{y}{t} \\
\Rightarrow \frac{dy}{y} &= -3\frac{dt}{t} \\
\Rightarrow \int \frac{dy}{y} &= \int -3\frac{dt}{t} \quad \text{Note: refer to integral tables.} \\
\ln y &= -3\ln t + C \\
\text{Taking anti-log both side,} & \\
\Rightarrow y &= e^{-3\ln t + C}, \quad C \text{ is the constant of integration} \\
\Rightarrow y &= e^{-3\ln t} e^{C} \\
\Rightarrow y &= e^{-3\ln t} \cdot A, \quad \text{Let } A = e^{C}\text{, another constant.} \\
\Rightarrow y &= Ae^{-3\ln t} \\
\Rightarrow y &= At^{-3}
\end{aligned} \tag{4}$$

Using initial condition $y(1) = 2$, $2 = A \cdot (1)^{-3}$, gives $A = 2$. Thus the complete solution is

$$y = 2t^{-3}.$$

```
%% (C) Rahul Bhadani
% % CPE 381, Homework 1, Question 5

 %% ty; + 3y = 0; y(1) = 2
 % solution y = 2/t^3
 syms y(t)
 eqn = diff(y, t) + 3*y/t ==0;
 cond = y(1) == 2;
 y(t) = dsolve(eqn, cond);
 y(t)
```