Classwork 22
FA24
UAH
CPE 381
Instructor: Rahul Bhadani

Name of the Student:

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Total Marks: 10 points

① Determine the z-transform of $x[n] = \left(\frac{1}{2}\right)^n u[n-3]$, and write down its ROC. (5 point).

Solution:

$$X(z) = \sum_{n=-\infty}^{n=\infty} x[n] z^{-n} = \sum_{n=3}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=1}^{\infty} \left(\frac{z^{-1}}{2}\right)^n$$

Let $m = n-3$, then $\Rightarrow n = m+3$

$$X(z) = \sum_{m=0}^{\infty} \left(\frac{z^{-1}}{2}\right)^{m+3}$$

$$= \frac{(z^{-1}/2)^3}{1-(z^{-1}/2)}$$

$$= \frac{1}{8z^2(z-\frac{1}{2})}$$

$$\left\{ S_\infty = a + ar + ar^2 + \cdots = \frac{a}{1-r} \right.$$

Here $a = \left(\frac{z^{-1}}{2}\right)^3$

$r = \frac{z^{-1}}{2}$

ROC: $|r| < 1$

$$\left|\frac{z^{-1}}{2}\right| < 1$$

$$\Rightarrow \left|\frac{1}{2z}\right| < 1$$

$$\Rightarrow |z| > \frac{1}{2}$$

② If a discrete-time signal $x[n]$ has discre transfer function $X(z)$

then for a signal $x[n-A]$, its transfer function is $z^{-A}X(z) + x[-1]z^{-A+1}$

for an integer A.

For for causal systems, $x[-1]=0$

and thus $x[n-A] \longleftrightarrow z^{-A}X(z)$

Using this identity, find the discrete transfer function of the system given by

$$y[n] = y[n-1] - y[n-2] + x[n]$$

where $y[n]$ is the output and $x[n]$ is the input, consider the system as causal. (5 points)

Remember;

transfer function $H(z) = \frac{Y(z)}{X(z)}$.

Solution:

$$y[n] \hookrightarrow Y(z)$$

$$y[n-1] \hookrightarrow z^{-1}Y(z)$$

$$y[n-2] \hookrightarrow z^{-2}Y(z)$$

$$x[n] \hookrightarrow X(z)$$

Hence taking z-transform of the given system

$$Y(z) = z^{-1}Y(z) - z^{-2}Y(z) + X(z)$$

$$\Rightarrow \quad Y(z) - z^{-1}Y(z) + z^{-2}Y(z) = X(z)$$

$$Y(z)\left(1-z^{-1}+z^{-2}\right) = X(z)$$

$$\Rightarrow \quad \frac{Y(z)}{X(z)} = \frac{1}{1-z^{-1}+z^{-2}}$$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-z^{-1}+z^{-2}}$$

$$\Rightarrow \quad \boxed{H(z) = \frac{1}{1-z^{-1}+z^{-2}}}$$