Worked Example 10 CPE381 FA24 Instructor: Rahul Bhadani

① What is the magnitude of the following Fourier series of the following signal

$$y(t) = 3 + 2\sin(20t) + 8\cos(30t)$$

at frequency 10 unit?

$$y(t) = X_0 + 2\sum C_k \cos(k\Omega_0 t) + d_k \sin(k\Omega_0 t)$$

Fundamental time period of $\sin(20t)$ $\Omega_1 = 20$

$$T_1 = \frac{2\pi}{20} = \frac{\pi}{10}$$

$$\left[\text{L.C.M. of } \left(\frac{a}{b}, \frac{c}{d}\right) = \frac{LCM(a,c)}{GCD(b,d)} \right]$$

of $\cos(30t)$ $T_2 = \frac{2\pi}{30} = \frac{\pi}{15}$

$\Omega_2 = 30$

L.C.M. of $\frac{\pi}{10}$ and $\frac{\pi}{15}$ is $\frac{\pi}{5}$ as $\frac{\pi}{10} \times 2 = \frac{\pi}{5}$

GCD = Greatest common divisor

GCD of 10, 15 is $\frac{\pi}{15} \times 3 = \frac{\pi}{5}$

Divisor of 10 = 1, 2, 5, 10
Divisor of 15 = 1, 3, 5

5 is is greatest in both taken together.

So L.C.M of $\frac{\pi}{10}, \frac{\pi}{15}$ is $\frac{\pi}{5}$

$$\text{So } \Omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} \times 5 = 10$$

Now looking at

$$x(t) = 3 + 2\sin(20t) + 8(\cos 30t)$$

we see that none of the component has $\Omega = 10$

Hence, the magnitude of the Fourier series at

$\Omega = 10$ is zero.

What is the magnitude of the Fourier series at frequency 30?

Comparing $x(t) = 3 + 2\sin(20t) + 8\cos(30t)$

with $x(t) = X_0 + 2\sum_{k=1}^{\infty} [c_k \cos(k\Omega_0 t) + d_k \sin(k\Omega_0 t)]$

$2d_k = 8$ when $k\Omega_0 = 30$

Hence $d_k = 4$

Thus the magnitude is 4.

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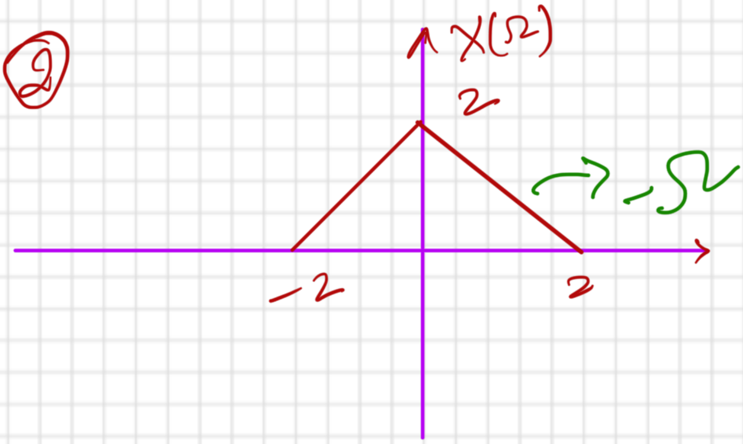
② 

Find $\int_{-\infty}^{\infty} |x(t)|^2 dt$

According to parseval's theorem,

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\Omega)|^2 d\Omega$$

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(\Omega)|^2 d\Omega$$

$$= \frac{1}{2\pi} 2 \int_0^2 |-\Omega|^2 d\Omega$$

$$= \frac{1}{\pi} \left. \frac{\Omega^3}{3} \right|_0^2$$

$$= \frac{1}{\pi} \times \frac{8}{3} = \frac{8}{\pi 3}.$$