CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Continuous-Time Signals

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Announcement

- Homework 01 Due September 01 11:59 PM
- Quiz 01, based on Chapter 01 Continuous-Time Signals from the Textbook. Available from September 05, 12:01 AM to September 07, 11:59 PM. 30 Questions, 45 Minutes.
- Office hour: 08/28 Aug Wednesday, 1 PM - 3:30 PM.
- No class on September 02, 2024: Labor Day, University Closed.

Outline

1. Motivation
2. Operation on Signals
3. Basic Signals as Building Blocks
4. Modulation and Windowing

Motivation

Signals and Systems is ‘Grandfather’ of Data Science for Electrical and Computer Engineers

Classification of Signals

We care about the following properties when dealing with signals:

- Predictability: Random or Deterministic
- Variations of time and amplitude: continuous, discrete (time or x-axis) / quantized (amplitude or y-axis)
- Periodic/Aperiodic
- Finite energy/finite power; Infinite energy/Infinite power

Operation on Signals

Basic Mathematical Operations

- Addition: $x(t) + y(t)$
- Subtraction: $x(t) - y(t)$
- Constant multiplication: $kx(t)$ where k is a constant

Time-shift

- $x(t-\tau) \rightarrow$ Signal is delayed
- $x(t+\tau) \rightarrow$ Signal is advanced

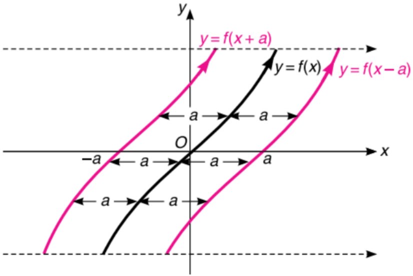
$f(x)$ transforms to $f(x-a)$

i.e., $f(x) \longrightarrow f(x-a)$; a is positive. Shift the graph of $f(x)$ through 'a' unit towards right

$f(x)$ transforms to $f(x+a)$.

i.e., $f(x) \longrightarrow f(x+a)$; a is positive. Shift the graph of $f(x)$ through 'a' units towards left.

Graphically it could be stated as



Time Reflection

- $x(t) \to x(-t)$: take mirror image **along the y-axis**

Note: The book doesn't specify whether to take the mirror image along the y-axis or not and it is confusing because the signal used in example 1.3.1 is symmetric with respect to both the x and y

***f*(*x*) transforms to *f*(−*x*)**

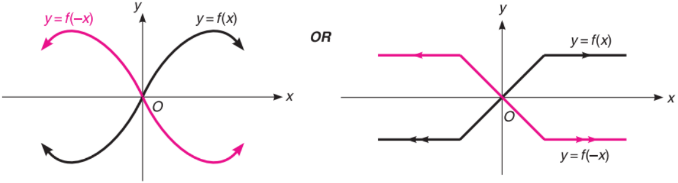
i.e., $f(x) \longrightarrow f(-x)$

To draw $y = f(-x)$, take the image of the curve $y = f(x)$ in y-axis as plane mirror.

OR

"Turn the graph of $f(x)$ by 180° about y-axis."

Graphically it is stated as;



axes.

Signal Stretching along y-axis

- $f(x) \rightarrow af(x); \quad a > 1$: Stretch the graph of $f(x)$ 'a' times along y-axis.
- $f(x) \rightarrow \frac{1}{a}f(x); \quad a > 1$: Shrink the graph of $f(x)$ 'a' times along y-axis.

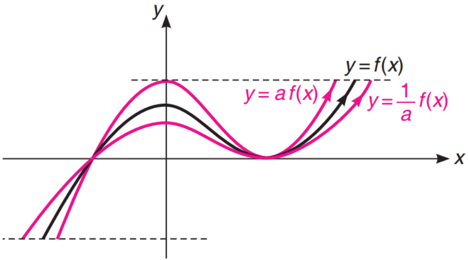
***f*(*x*) transforms to *a f*(*x*)**

i.e., $$f(x) \longrightarrow af(x); \quad a > 1$$

Stretch the graph of f(x) '*a*' times along *y*-axis.

$$f(x) \longrightarrow \frac{1}{a} f(x); \ a > 1.$$

Shrink the graph of f(x) '*a*' times along *y*-axis.



Signal Stretching along x-axis

- $f(x) \rightarrow af(ax); \quad a > 1$: Stretch the graph of $f(x)$ 'a' times along x-axis.
- $f(x) \rightarrow f\left(\frac{1}{a}x\right); \quad a > 1$: Shrink the graph of $f(x)$ 'a' times along x-axis.

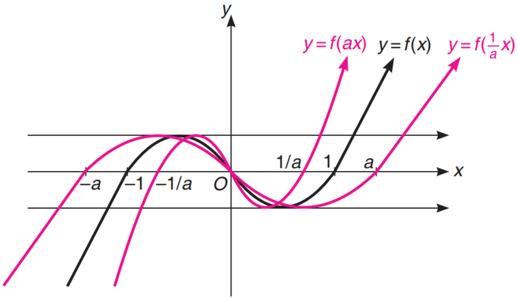
***f*(x) transforms to *f*(*ax*)**

i.e., $\quad f(x) \longrightarrow f(ax); \quad a > 1$

Shrink (or contract) the graph of f(x) '*a*' times along *x*-axis.

again $\quad f(x) \longrightarrow f\left(\frac{1}{a}x\right); \quad a > 1$

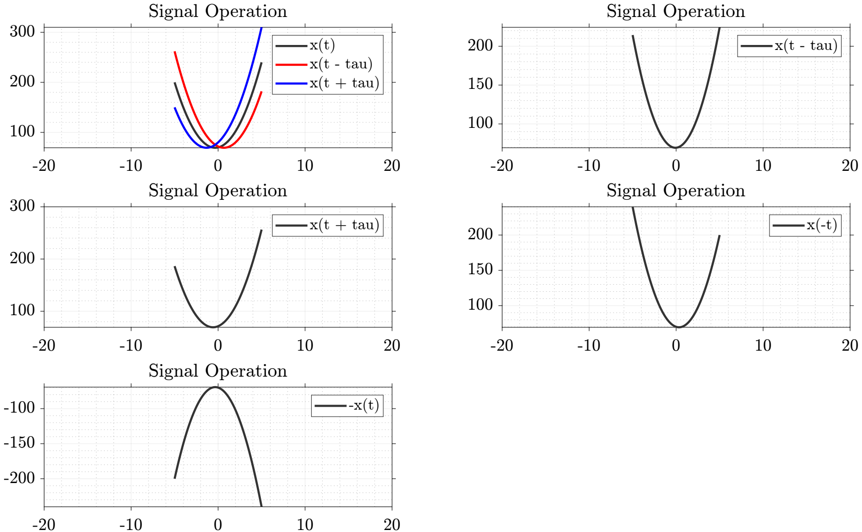
Stretch (or expand) the graph of f(x) '*a*' times along *x*-axis.



Example and MATLAB Code

$$x(t) = 6t^2 + 4t + 70$$

Code: https://github.com/rahulbhadani/CPE381_FA24/blob/master/Code/signal_operation.m



Even and Odd Signals

- Even Signal: $x(t) = x(-t)$
- Odd Signal: $x(t) = -x(-t)$
- Any signal can be represented by the sum of even and odd signals
  $y(t) = y_e(t) + y_o(t)$

$$y_e(t) = 0.5[y(t) + y(-t)]$$

$$y_o(t) = 0.5[y(t) - y(-t)]$$

Periodic Signals

- Defined for all possible values of $t, -\infty < t < \infty$.
- There is the real value $T_0 \in \mathbb{R}^+$, called the fundamental frequency such that $x(t + kT_0) = x(t), k \in \mathbb{I}$.
- A constant signal is periodic of a non-definable fundamental period.
- A $\cos(\omega t + \theta)$, $\omega = 2\pi/T_0$, $\omega = 2, \theta = -\pi/2, A = 2$.

  What's the fundamental frequency, $1/T_0$?

Energy and Power of Signals

What's the instantaneous power of a resistor?

- **Energy:**

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- **Power:**

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

A signal is called finite power if the signal power is finite.

Basic Signals as Building Blocks

Complex Exponentials

Consider $A = |A|e^{j\theta}$, $a = r + k\Omega_0$

- $x(t) = Ae^{at} = \ldots$
- Real part $f(t) = \mathrm{Re}\{x(t)\}, = \ldots$

  $-|A|e^{rt} \leq f(t)) \geq |A|e^{rt}$. $r < 0$, $f(t)$ is damped, $r > 0$, $f(t)$ grows.
- Imaginary part $g(t) = \mathrm{Im}\{x(t)\}, = \ldots$

Sinusoids

A sinusoid of the general form:

$$A\cos(\Omega_0 t + \theta) = A\sin(\Omega_0 t + \theta + \pi/2), \quad -\infty < t < \infty$$

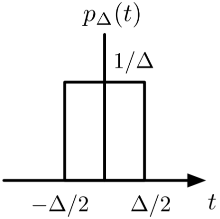
- A is the amplitude
- $\Omega_0 = 2\pi f_0$ is angular frequency in `rad/s`.
- θ is phase shift
- Fundamental period T_0 is

$$T_0 = \frac{2\pi}{\Omega_0} = \frac{1}{f_0}$$

Rectangular pulse and Unit impulse

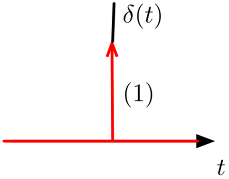
- A rectangular pulse of duration Δ and unit area:

$$p_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & -\Delta/2 \leq t \leq \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$



- Unit Impulse:

$$\delta(t) = \lim_{\Delta \to 0} p_\Delta(t)$$



- **Calculate**

$$\int_{-\infty}^{t} p_\Delta(t)$$

Unit Step

- Integration of rectangular pulse:

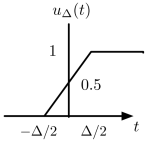
$$u_\Delta(t) = \int_{-\infty}^{t} p_\Delta(t) = \begin{cases} 1, & t \geq \dfrac{\Delta}{2} \\ \frac{1}{\Delta}(t + \frac{\Delta}{2}), & \dfrac{\Delta}{2} \leq t \leq \dfrac{\Delta}{2} \\ 0, & t < -\dfrac{\Delta}{2} \end{cases}$$

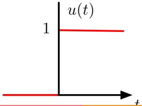
- Limit case:

$$\lim_{\Delta \to 0} u_\Delta(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

- A common case is to ignore $t = 0$ case, which gives us unit step function as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



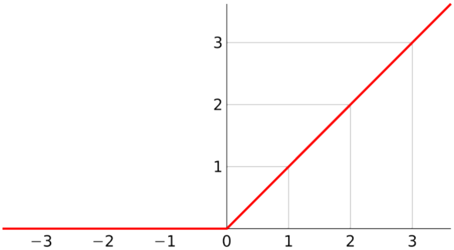


Ramp Signal

The ramp signal is $r(t) = tu(t)$

- The relation between the ramp, the unit step, and the unit impulse:

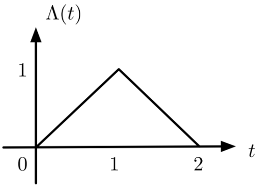
$$\frac{dr(t)}{dt} = u(t)$$
$$\frac{d^2r(t)}{dt^2} = \delta(t)$$

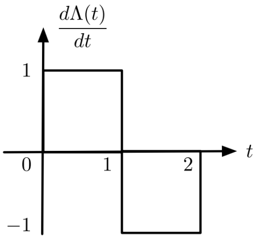


Triangular Pulse

The triangular pulse is

$$\Lambda(t) = \begin{cases} t, & 0 \le t \le 1 \\ -t+2, & 1 < t \le 2 \\ 0, & \text{otherwise} \end{cases}$$





- $\Lambda(t)$ can also be written as:

$$\Lambda(t) = r(t) - 2r(t-1) + r(t-2)$$

Triangular Pulse as Ramp Functions I

Let's verify this by evaluating $\Lambda(t)$ at different intervals:

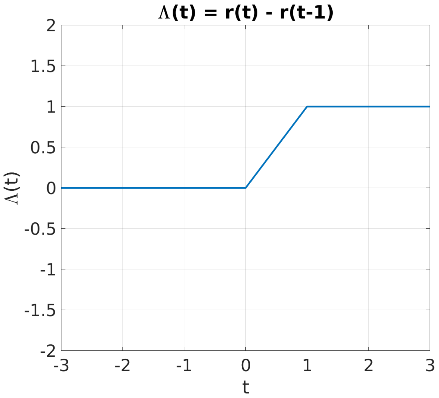
First, find out the first part:

$$\Lambda_1(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

which can be written as:

can be written using the ramp function $r(t)$ as:

$$\Lambda_1(t) = r(t) - r(t-1)$$



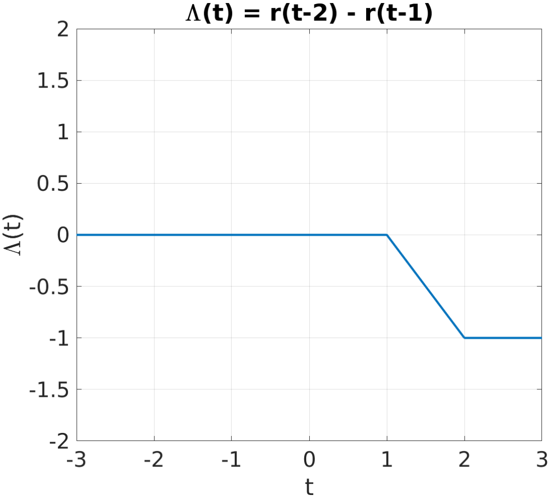
Triangular Pulse as Ramp Functions II

Second part:

$$\Lambda_1(t) = \begin{cases} -t+2, & 1 < t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

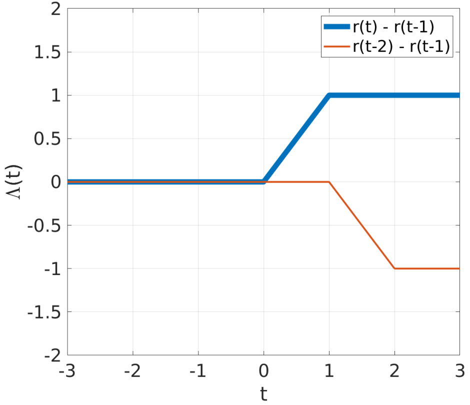
which can be written as:
can be written using the ramp function $r(t)$ as:

$$\Lambda_1(t) = r(t-2) - r(t-1)$$



Triangular Pulse as Ramp Functions III

Putting together



Sifting Property

The product of $f(t)$ and $\delta(t)$ gives zero everywhere except at the origin where we get an impulse of area $f(0)$, that is, $f(t)\delta(t) = f(0)\delta(t)$.
Hence,

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = \int_{-\infty}^{\infty} f(0)\delta dt = f(0)\int_{-\infty}^{\infty} \delta(t)dt = f(0)$$

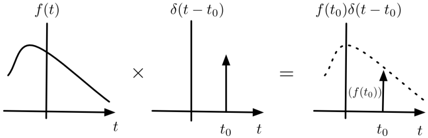
This is called **Sifting Property**.
If we delay or advance the $\delta(t)$ function in the integrated, the result is that all values of $f(t)$ are sifted out except for the value corresponding to the location of the delta function, that is,

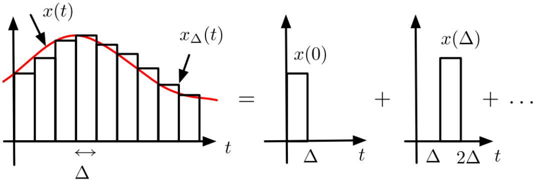
$$\int_{-\infty}^{\infty} f(t)\delta(t-\tau)dt = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)dt = f(\tau)\int_{-\infty}^{\infty} \delta(t-\tau)dt = f(\tau) \quad \text{for any } \tau$$

Generic Representation of Signals

Hence, if we do integration in terms of variable τ, we get a generic representation of signals in terms of impulse and shifted impulse.

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$





- Approximation of $x(t)$:

$$x_\Delta(t) = \sum_{-\infty}^{\infty} x_\Delta(t-k\Delta) = \sum_{-\infty}^{\infty} x(k\Delta)p_\Delta(t-k\Delta)\Delta$$

In the limit as $\Delta \to 0$ these pulses become impulses, separated by an infinitesimal value: $\lim_{\Delta\to 0} x_\Delta(t) \to x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$

Modulation and Windowing

Modulation

Multiplication by a complex exponential shifts the frequency of the original signal.

Definition

Superimposing a low-frequency signal on a high-frequency carrier signal is called **Modulation**.

Example:

Consider an exponential signal $x(t) = e^{j\Omega_0 t}$ of frequency Ω_0. If we multiply an exponential $e^{j\phi t}$ with $x(t)$, then:

$$x(t)e^{j\phi t} = e^{j(\Omega_0+\phi)t} = \cos((\Omega_0 + \phi)t) + j\sin((\Omega_0 + \phi)t)$$

$\phi > 0$: the frequency of new exponential is greater than Ω_0, otherwise lower.

Various Types of Modulation

$A(t)cos(\Omega(t)t + \theta(t))$

- $A(t)$ changes: Amplitude Modulation
- $\Omega(t)$ changes: Frequency Modulation
- $\theta(t)$ changes: Phase Modulation

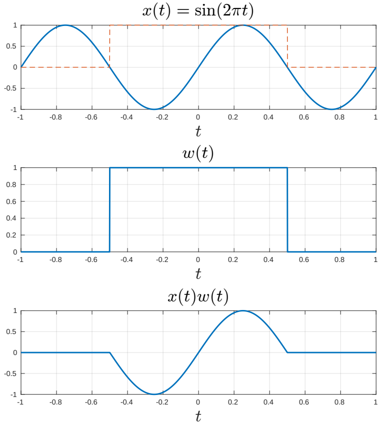
Windowing

For a window signal $w(t)$, the time-windowed signal
$x(t)w(t)$ displays $x(t)$ within the support of $w(t)$.

Example:

$x(t) = \sin(2\pi t)$

$$w(t) = \begin{cases} 1 & \text{if } -0.5 \leq t \leq 0.5 \\ 0 & \text{otherwise} \end{cases}$$



Code for the graph:
https://github.com/rahulbhadani/CPE381_FA24/blob/master/Code/windowing.m

Classwork

Classwork

Classwork

Classwork

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Up Next

- Continuous-time Systems
  - Linear-Time Invariance
  - Static vs Dynamic Systems
  - Convolutional Integral
  - BIBO Stability