Class work 12. 10/07/2024 CPE 381 FA24 Instructor: Rahul Bhadani

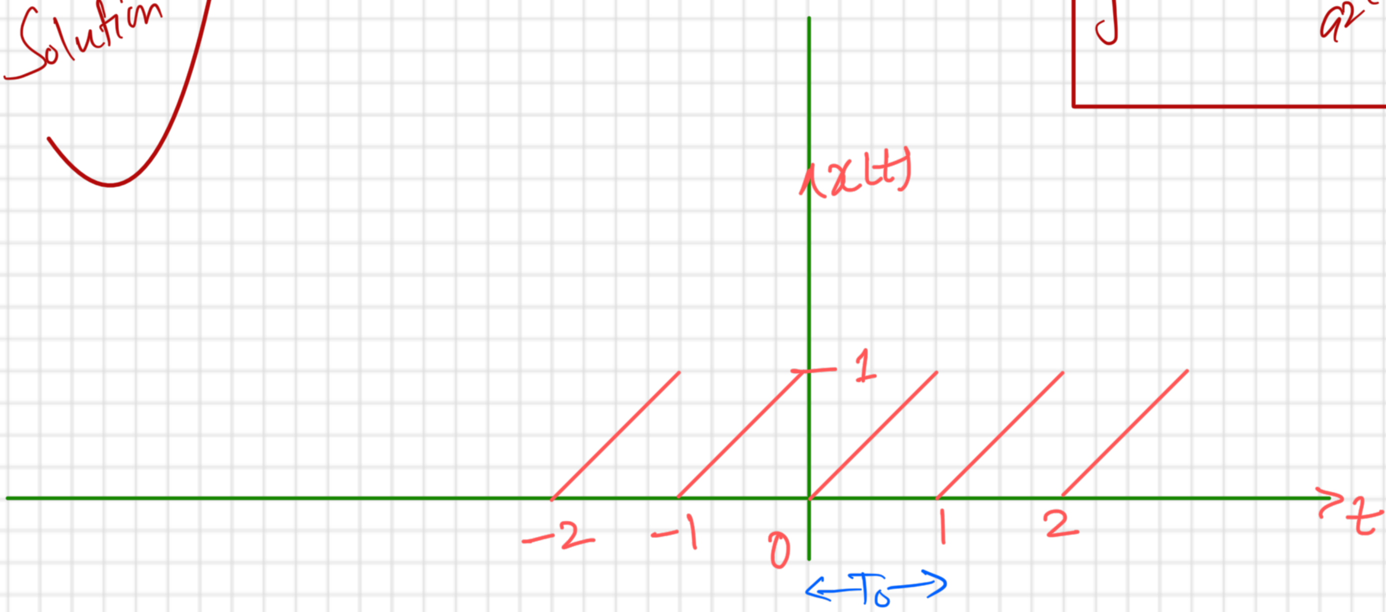
Q. Consider a period signal $x(t)$ with a frequency $\Omega_0 = 2\pi$ has period

$$x_1(t) = t\,[u(t) - u(t-1)]$$

1. Plot $x(t)$, and indicate its fundamental period T_0.
2. Compute the Fourier series coefficients of $x(t)$ using its integral definition.

$$\boxed{\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax-1)}$$

Solution ✓



$$X_k = \frac{1}{T_0}\int_0^1 t\, e^{-j2\pi k t}\, dt$$

$$\int t e^{-j2\pi k t} dt = \frac{e^{-j2\pi k t}}{(-j2\pi k t)^2}(-j2\pi k t - 1)$$

So $$X_k = \frac{1}{T_0} \left. \frac{e^{-j2\pi k t}}{(-j2\pi k t)^2}(-j2\pi k t - 1) \right|_{t=0}^{1}$$

$$= \frac{j\,2\pi k + 1}{4\pi^2 k^2} - \frac{1}{4\pi^2 k^2} = \frac{j}{2\pi k}, \quad k \neq 0$$

for $k=0$,

$$x_0 = \frac{1}{T_0}\int_0^1 t\,dt = \frac{t^2}{2}\Bigg|_{t=0}^{1} = 0.5$$

Hence,

$$x(t) = 0.5 + \sum_{k=-\infty}^{\infty} \frac{j}{2\pi k} e^{jkt}$$