CPE 381 FINAL EXAM
CPE381

100 points

6 December 2023 8:00 am

Your only source of references for this exam is your own notebook, handwritten by you, calculator, and pen/pencil, eraser. No other booklets, additional paper, textbooks, or other materials may be referenced during this examination. You may ask for additional sheets from the examiner. **However, please use all the extra space provided in this exam paper.**

Read every question on this examination carefully. Although portions may seem familiar—do not assume that the information presented in this examination is duplicated from any examples (written or otherwise) that you may have seen before.

Total number of pages (including this page and three additional pages at the end for writing solution, excluding supplementary tables, if any): 11

NAME: _____

SCORE EARNED:

10	10	20	15	25	20	Total

***Note:*: Assume i and j are complex units, i.e.** $\sqrt{-1}$ for the entire exam.

1 Toss a Coin (10 points)

Indicate true (T) or false (F) for each of the below statements.

T F (a) A system can be represented by a differential equation.

T F (b) Anticausal signal is defined only on positive time-axis.

T F (c) Whenever there is a time-shift of a signal in the time domain, there is a change of phase in the frequency domain.

T F (d) z^{-2} is equivalent to two sample delay for discrete-time signals.

T F (e) If $x[n]$ is a two-sided sequence, and $X(z)$ converges for some values of z, then the region of convergence (ROC) is of the form $r_1 < |z| < r_2$ where r_1 and r_2 are the magnitudes of the two poles of $X(z)$.

T F (f) A system is said to be stable when all poles of its transfer function lay on the right half of the s-plane.

T F (g) An LTI System represented by its impulse response $h(t)$ is causal if $h(t) = 0$ for $t < 0$.

T F (h) Fourier transform is a special case of Laplace transform where $\sigma = 0$, given $s = \sigma + j\Omega$.

T F (i) The z-transform of the difference equation $y[n-3] + 4y[n+2] + x[n-4] = x[n]$ is $z^{-3}Y(z) + 4z^2Y(z) + z^{-4} = X(z)$.

T F (k) If W_N^{nk} is the twiddle factor used in the Fast-Fourier transform for N-point discrete-time signal, then $W_8^{nk} = W_8^{(n+8)}k$, where n, k are integer values such that $k, n = 0, 1, \cdots, N-1$.

2 Four-sided Dice (10 points)

(a) Consider a complex number $z = 6 + 5i$. The magnitude of the complex number $\frac{1}{z}$ is given by:

(A) $\frac{1}{\sqrt{61}}$

(B) $\frac{1}{61}$

(C) $\sqrt{61}$

(D) 61

(b) $e^{j\pi/4}$ equals

(A) 1

(B) $\sqrt{2} + j\sqrt{2}$

(C) $\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

(c) The sum of the infinite geometric progression $\sum_{n=0}^{\infty} 0.5^n$ is

(A) ∞

(B) 0

(C) 2

(D) $\frac{1}{2}$

(d) If z-transform of a discrete-time signal $x[n]$ is $\frac{1}{1-\alpha z^{-1}}$, then its discrete-time fourier transform (DTFT) is

(A) $\frac{1}{1-\alpha e^{j\omega}}$

X(B) $\frac{1}{1-\alpha e^{-j\omega}}$

(C) $\dfrac{1}{1+\alpha e^{-j\omega}}$

(D) $1-\alpha e^{-j\omega}$

(e) In an electric circuit, we are measuring a voltage $v_c(t)$ across a capacitor whose Laplace transform is given by $V_c(s) = \dfrac{1}{s} - \dfrac{1}{s+\frac{1}{RC}}$. Then the voltage $v_c(t)$ is given by

(A) $v_c(t) = (1-\frac{t}{RC})u(t)$

(B) $v_c(t) = (1-RC)u(t)$

(C) $v_c(t) = (1-e^{\frac{t}{RC}})u(t)$

(D) $v_c(t) = (1-e^{-\frac{t}{RC}})u(t)$

3 Artist and Equations (20 points)

3.1 Sketching

Sketch the graph for the following signals. Please label the x-axis and the y-axis appropriately. Unlabeled sketches of graphs will result in a penalty.

1. $a[n] = u[n] - u[n-5]$
2. $b[n] = 3(u[n] - u[n-3]) + \sum_{k=5}^{10} \delta[n-k]$
3. $c[n] = \sum_{k=1}^{5} k\delta[n-k]$
4. $d[n] = \sum_{k=1}^{4} \Big(u[n-k] - u[n-k-1]\Big)(k+1)$
5. $e(t) = \sin(2\pi t)u(t) + \sin(2\pi(t-0.5))u(t-0.5)$

3.2 Writing Equations

Represent each of the following graphs as mathematical equations using basic signals such as unit-step functions $u(t)$, unit-impulse function $\delta(t)$, ramp function functions $r(t)$, their discrete-equivalents, and commonly known functions such as sin, cos, etc.

1. $p(t)$

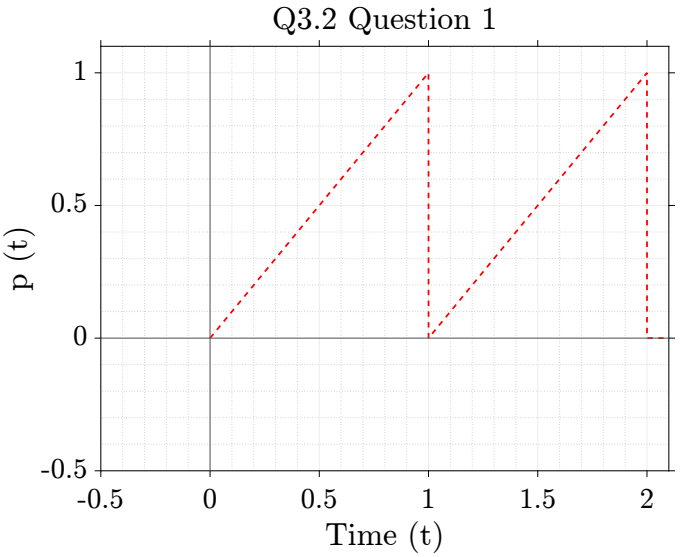

Figure 1: Question 3.2 (1)

2. $q(t)$

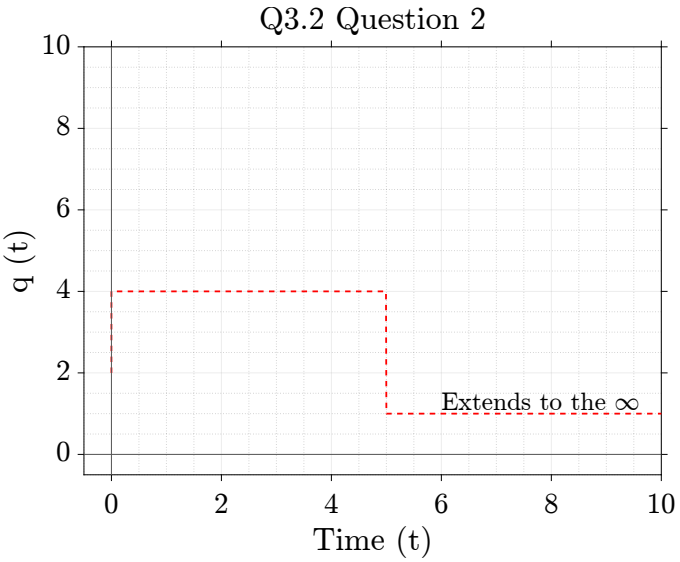

Figure 2: Question 3.2 (2)

3. $m(t)$

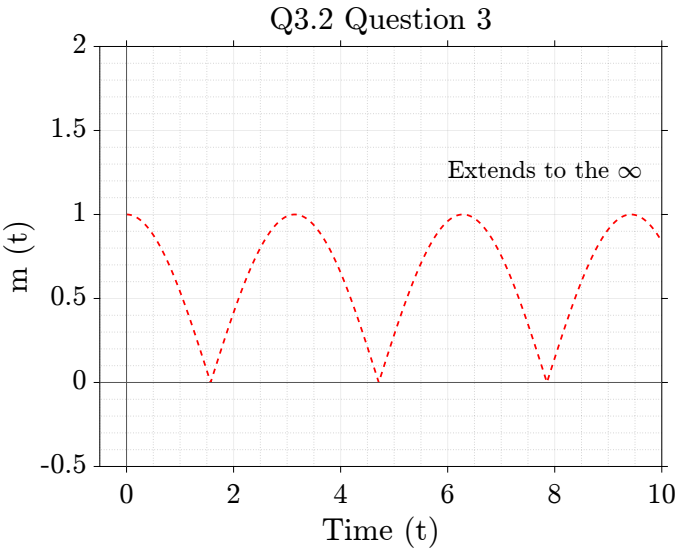

Figure 3: Question 3.2 (3)

4. $g[n]$

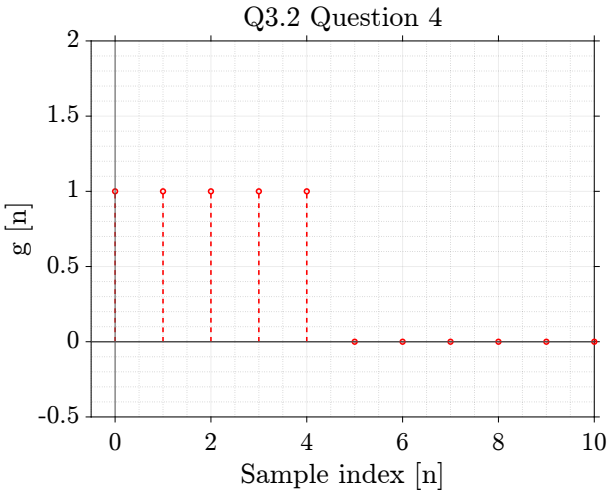

Figure 4: Question 3.2 (4)

5. $f[n]$

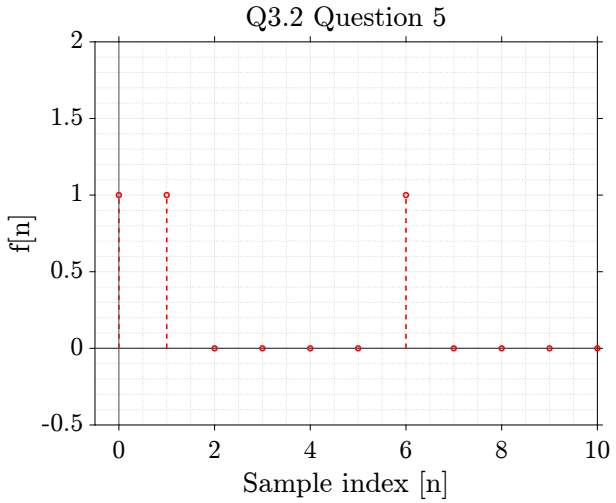

Figure 5: Question 3.2 (5)

4 Army of Transformers (15 points)

For each of the following, provide a specific transformation as being asked.

1. Determine the Laplace transform of

$$f(t) = \begin{cases} 1 & 0 \le t \le 4 \\ 3 & 4 \le t \le 5 \\ 0 & 5 \le t \le \infty \end{cases} \tag{1}$$

2. Determine the Inverse Laplace Transform of

$$X(s) = \frac{5s+13}{s(s^2+4s+13)} \tag{2}$$

   Hint: $s^2+4s+13 = (s+2-j3)(s+2+j3)$

3. Calculate the Fourier Series Coefficients of

$$y(t) = 0.5 + 4\cos(2\pi t) - 8\cos(4\pi t), \quad -\infty < t < \infty \tag{3}$$

4. Find the Fourier Transform of a rectangular pulse given by

$$p(t) = \begin{cases} 1, & |t| < a \\ 0, & |t| > a \end{cases} \tag{4}$$

   Bonus (2 points): Sketch $p(t)$. Label the graph properly.
   Bonus (2 points): Sketch the Fourier transform of $p(t)$. Label the graph properly.

5. Find the z-transform of

$$g[n] = \left(\frac{1}{2}\right)^n u[n-3] \tag{5}$$

Bonus (2 points): Specify the region of convergence for the z-transform of $g[n]$ shown above.

5 Not a TikTok Filter (6 + 6 + 6 + 7 = 25 points)

The transfer function of a filter is

$$H(s) = \frac{\sqrt{5}s}{s^2 + 2s + 2} \tag{6}$$

1. Find the poles and zeros of $H(s)$ and use this information to sketch the magnitude response $|H(j\Omega)|$ of the filter.

2. Find the impulse response $h(t)$ of the filter.

3. Indicate the magnitude response at frequencies 0, 1 and ∞.

4. Consider that the input to the filter comes from a signal generator, given by a biased $x(t) = B + \cos(\Omega t)$. Assume that the signal generator can generate all possible frequencies, for what frequency or frequencies Ω_0, we get the output $y(t) = \cos(\Omega_0 t + \theta)$ which means the DC component is filtered out. Determine the phase (or phases) θ_0 for that frequency (or frequencies).

6 Counting in Integers: Discrete-Time Systems (5 + 5 + 5 + 5 = 20 points)

Consider a causal discrete-time LTI system given by

$$y[n] - \tfrac{3}{4}y[n-1] + \tfrac{1}{8}y[n-2] = x[n] \tag{7}$$

where $x[n]$ and $y[n]$ are the input and output of the system, respectively.

1. Determine the system function $H(z)$.
2. Find the discrete-time Fourier transform (DTFT) $H(e^{j\omega})$ of the corresponding impulse response $h[n]$.
3. Find the integral $\int_{-\pi}^{\pi} H(e^{j\omega})d\omega$. *Hint: revisit the formula for inverse DTFT.*
4. Find the step-response $s[n]$ of the system. *Hint: remember impulse response $h[n]$ is given when the input $x[n] = \delta[n]$.*
5. **Bonus (5 points):** Determine the phase of $H(e^{j\omega})$.

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