Homework 4
CPE381

Canvas: `hw04`

Due: 20th October 2024, 11:59 PM
100 points

You are allowed to use a generative model-based AI tool for your assignment. However, you must submit an accompanied reflection report on how you use the AI tool, what was the query to the tool, and how it improved your understanding of the subject. You must also add your thoughts on how you would tackle the assignment if there was no such tool available. Failure to provide a reflection report for every single assignment where an AI tool was used may result in a penalty and subsequent actions will be taken in line with plagiarism policy.

Submission instruction:

Upload a .pdf on Canvas with the format {firstname.lastname}_cpe381_hw04.pdf. If there is a programming assignment, then you should include your source code along with your PDF files in a zip file firstname.lastname}_cpe381_hw04.zip. If a plot is being asked, your PDF file must also contain plots generated by your MATLAB code. Your submission must contain your name, and UAH Charger ID or the UAH email address. Please number your pages as well.

See the end for the solution to the last problem.

1 Expand (10 points)

Find the Fourier coefficients for the given signal using the integral method **(5 points each)**:

1. $x(t) = \cos(2\pi t) + \cos(3\pi t)$. Hint: find the fundamental period of $x(t)$ first.
2. $x(t) = |\cos(2\pi f_0 t)|$. See the textbook example 4.19.

Show your work.

2 Periodic Signal Again (20 points)

Given the periodic signal $x(t) = \sum_{n=-\infty}^{\infty}[u(t-2n) - u(t-2n-1)]$,

1. Sketch only one period of the signal $x(t)$. Let's call it $x_1(t)$. **(5 points).**
2. Find its fundamental period. **(5 points)**.
3. Find the Fourier coefficients for complex exponential Fourier series. **(10 points)**.

3 Spectra (10 points)

Sketch the exponential Fourier spectra (i.e. Magnitude Spectrum and Phase spectrum) for the exponential Fourier series of $x(t)$ given by

$$x(t) = (2 + j2)e^{-j3t} + j2e^{-jt} + 3 - j2e^{jt} + (2 - j2)e^{j3t}$$

4 Graph Detective (20 points))

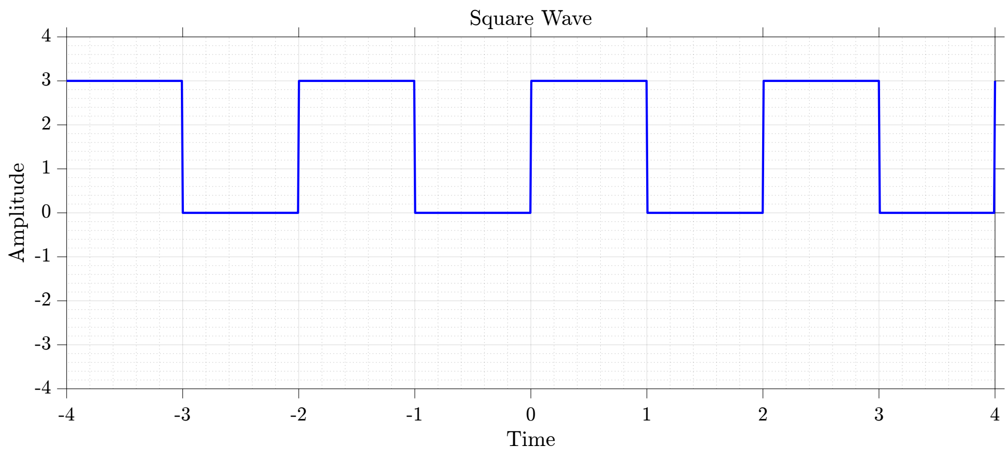

Figure 1: Square Wave $x(t)$

For the square wave given in Figure 3,

1. What is the Time period of the square wave? **(5 points)**
2. Compute the Fourier series coefficients for the complex exponential Fourier series as well as trigonometric Fourier series representation using the integral method. Refer to page 255 of the textbook and review the lecture recording if you are unsure of how to do that. Remember each term in the Fourier series represents a harmonic.

   Write down the expression for X_k, c_k, and d_k. **(15 points)**.

5 Harmonics in MATLAB (40 points)

1. Write a MATLAB script to reproduce the square wave shown in Figure 3.**(10 points)**
2. From the previous question, you will obtain the formula for X_k. As you see other than the DC term X_0, k varies from 1 to ∞.

   Using your MATLAB program, approximate the square wave $x(t)$ using the first 10 terms from your trigonometric Fourier series **(10 points)**.
3. Now add 20 terms, and then 50 terms. What do you observe? Explain. **(10 points)**.
4. Plot the magnitude spectrum, i.e. $|X_k|$ vs the index k **(5 points)**.
5. Plot the phase spectrum, i.e. $\angle X_k$ vs the index k **(5 points)**.

5.1 Answers:

```
close all;
% Define the parameters
T = 2;  % Period of the square wave
N = 100;  % Number of Fourier series terms
t = linspace(-2*T, 2*T, 1000);  % Time array
Amplitude = 3.0;

% Define the square wave function
squarewave = @(t) Amplitude.*(mod(t, T) < T/2);

% Plot the results
f = figure;
f.Position = [517 687 1464 551];
plot(t, squarewave(t), 'b', 'LineWidth',2);
hold on;
xlabel('Time (s)');
title('Square Wave','Interpreter', 'latex');
%legend('Square Wave', 'Fourier Series Approximation');
set(gca, 'XColor', [0, 0, 0], 'YColor', [0, 0, 0], 'TickDir', 'out');
xaxis = get(gca, 'XAxis');
xaxis.TickLabelInterpreter = 'latex';
yaxis = get(gca, 'YAxis');
yaxis.TickLabelInterpreter = 'latex';
xlabel('Time','Interpreter', 'latex');
ylabel('Amplitude','Interpreter', 'latex');
set(gca, 'FontSize', 18);
ylim([-4,4])
grid on;
grid minor;
exportgraphics(f, '../figures/CPE381_FA24_HW04_Q5_Square_Wave.pdf');


Omega  = (2.*pi)./T;


k_end = 10;
x_t_10 = zeros(size(t));
for t_index = 1:length(t)
    for k = -k_end/2:1:k_end/2
      x_t_10(t_index) = x_t_10(t_index) + X_k(k).*exp(1j.*k.*Omega.*t(t_index));
    end
end

k_end = 20;
x_t_20 = zeros(size(t));
for t_index = 1:length(t)
```

```
    for k = -k_end/2:1:k_end/2
      x_t_20(t_index) = x_t_20(t_index) + X_k(k).*exp(1j.*k.*Omega.*t(t_index));
    end
end
k_end = 50;
x_t_50 = zeros(size(t));
for t_index = 1:length(t)
    for k = -k_end/2:1:k_end/2
      x_t_50(t_index) = x_t_50(t_index) + X_k(k).*exp(1j.*k.*Omega.*t(t_index));
    end
end


f = figure;
f.Position = [517 687 1464 551];
plot(t, squarewave(t), 'b', 'LineWidth',2, ...
    'DisplayName', 'Original Signal');
hold on;
plot(t, x_t_10, 'r', 'LineWidth',2, 'DisplayName','10 Harmonics');
plot(t, x_t_20, 'g', 'LineWidth',2, 'DisplayName','20 Harmonics');
plot(t, x_t_50, 'k', 'LineWidth',2, 'DisplayName','50 Harmonics');
hold on;
xlabel('Time (s)');
title('Square Wave','Interpreter', 'latex');
%legend('Square Wave', 'Fourier Series Approximation');
set(gca, 'XColor', [0, 0, 0], 'YColor', [0, 0, 0], 'TickDir', 'out');
xaxis = get(gca, 'XAxis');
xaxis.TickLabelInterpreter = 'latex';
yaxis = get(gca, 'YAxis');
yaxis.TickLabelInterpreter = 'latex';
xlabel('Time','Interpreter', 'latex');
ylabel('Amplitude','Interpreter', 'latex');
set(gca, 'FontSize', 18);
ylim([-4,4])
grid on;
legend;
grid minor;

exportgraphics(f, '../figures/CPE381_FA24_HW04_Q5_Harmonics.pdf');


% Magnitude Spectrum
k = -50:1:50;
Xk = X_k(k);

f = figure;
f.Position = [517 687 1464 551];
subplot(2,1,1)
```

```
stem(k, abs(Xk), 'b', 'LineWidth',2);
hold on;
xlabel('Time (s)');
title('Magnitude Spectrum','Interpreter', 'latex');
%legend('Square Wave', 'Fourier Series Approximation');
set(gca, 'XColor', [0, 0, 0], 'YColor', [0, 0, 0], 'TickDir', 'out');
xaxis = get(gca, 'XAxis');
xaxis.TickLabelInterpreter = 'latex';
yaxis = get(gca, 'YAxis');
yaxis.TickLabelInterpreter = 'latex';
xlabel('k','Interpreter', 'latex');
ylabel('$|X_k|$','Interpreter', 'latex');
set(gca, 'FontSize', 18);
ylim([-1.5,1.5])
grid on;
grid minor;

% Phase spectrum

subplot(2,1,2)
stem(k, angle(Xk), 'b', 'LineWidth',2);
hold on;
xlabel('Time (s)');
title('Phase Spectrum','Interpreter', 'latex');
%legend('Square Wave', 'Fourier Series Approximation');
set(gca, 'XColor', [0, 0, 0], 'YColor', [0, 0, 0], 'TickDir', 'out');
xaxis = get(gca, 'XAxis');
xaxis.TickLabelInterpreter = 'latex';
yaxis = get(gca, 'YAxis');
yaxis.TickLabelInterpreter = 'latex';
xlabel('k','Interpreter', 'latex');
ylabel('$\angle X_k$','Interpreter', 'latex');
set(gca, 'FontSize', 18);
ylim([-2.5,2.5])
grid on;
grid minor;

exportgraphics(f, '../figures/CPE381_FA24_HW04_Q5_Spectrum.pdf');

function result = X_k(k)
    Amplitude = 3.0;
    result = zeros(size(k));

    % Handle k == 0 separately
    idx = (k == 0);
    result(idx) = Amplitude/2;

    % Calculate for non-zero k
```

```
    idx = (k ~= 0);
    result(idx) = (Amplitude/2) .* (1 - (-1).^k(idx)) ./ (1j .* k(idx) .* pi);
end
```

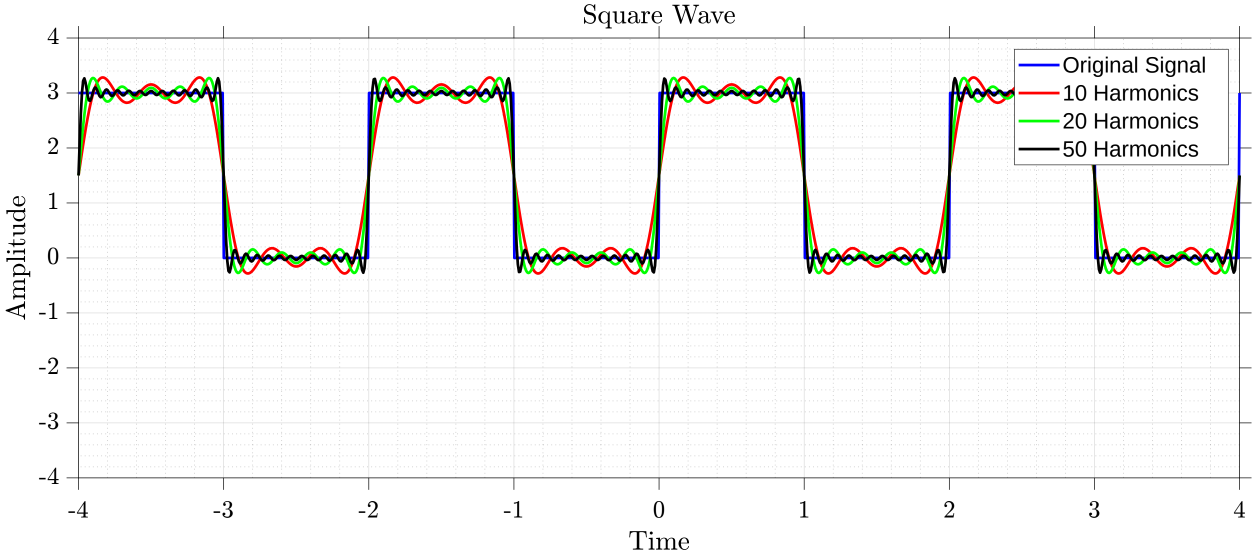

Figure 2: Harmonics

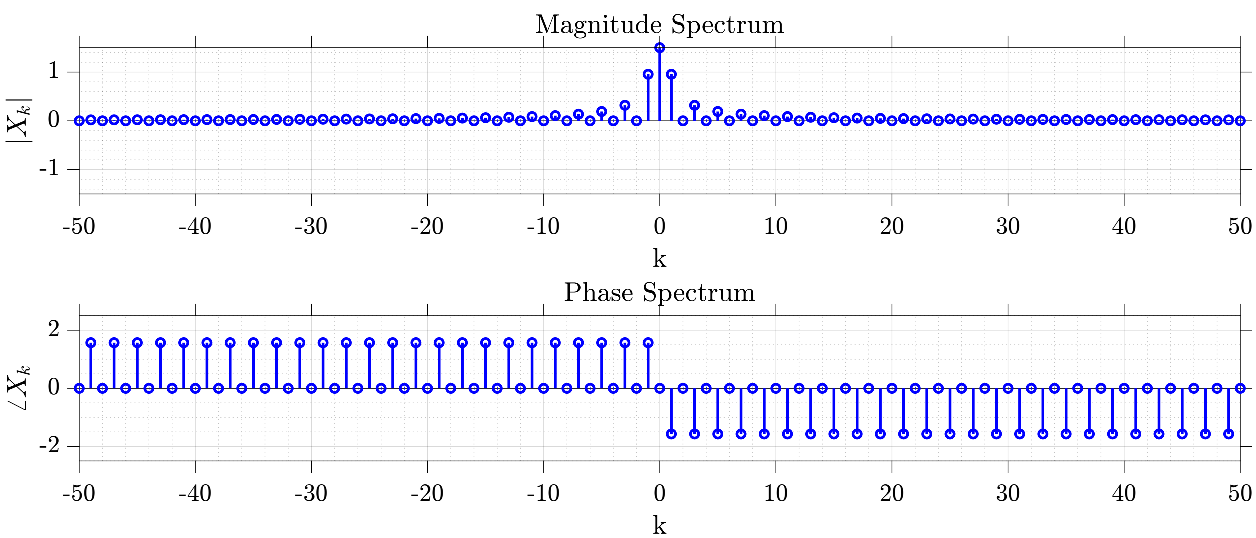

Figure 3: Harmonics

We observe that as you add more and more harmonics, the signal approximates to the true square waves. However, you will notice that there will be some kind of ripple effect there.