

Homework 4 Problem 2 Solution

2. Periodic Signal Again

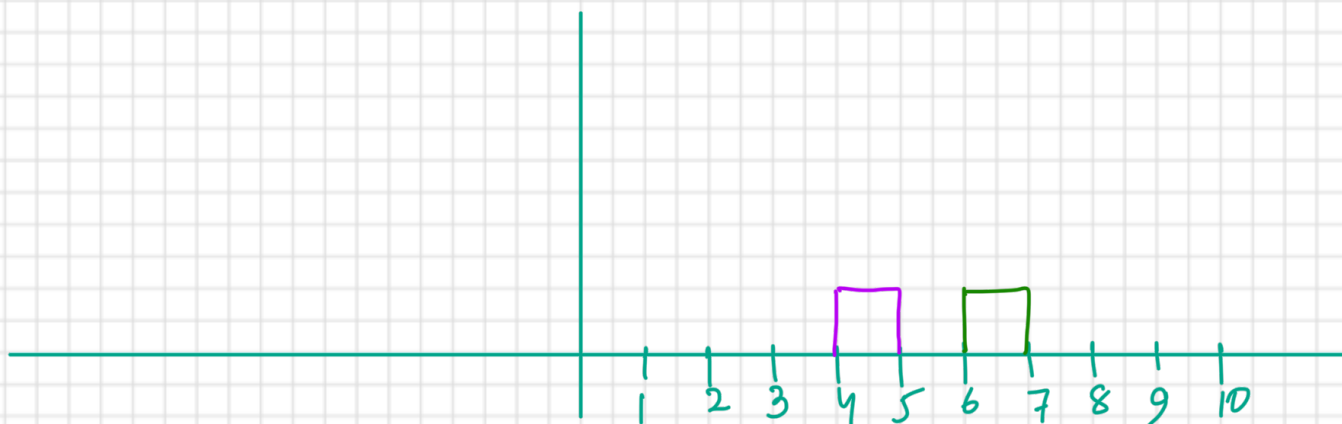
$$x(t) = \sum_{n=-\infty}^{\infty} u[(t-2n)] - u[(t-2n-1)]$$

① To sketch only one period, set $n=2$

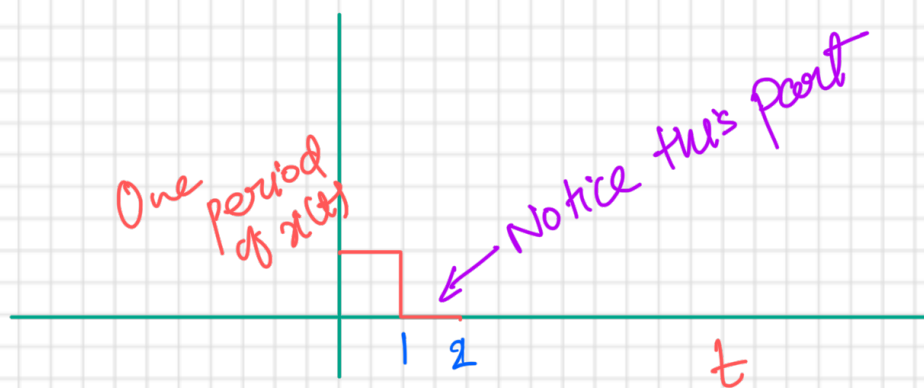
$$x_2(t) = u[t-4] - u[t-6]$$

Set $n=3$

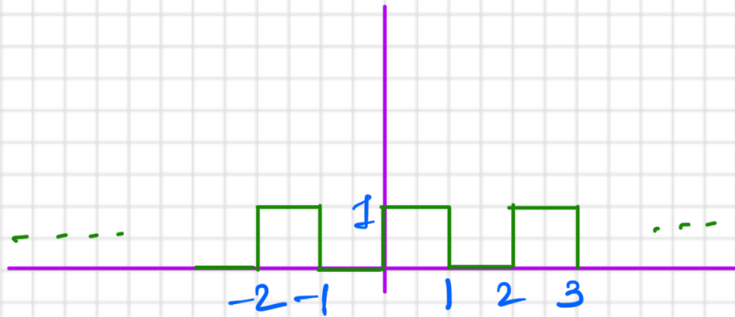
$$x_3(t) = u[t-6] - u[t-7]$$



So, we see that one period is



- ② From the graph, we see that its fundamental period is $T_0 = 2$ units. and $\Omega_0 = \frac{2\pi}{T_0} = \pi$ rad/unit and complete signal look like:



③ Fourier Series Coefficient

$$\begin{aligned}
 X_k &= \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\Omega_0 t} dt \\
 &= \frac{1}{2} \int_0^2 x(t) e^{-jk\pi t} dt \\
 &= \frac{1}{2} \int_0^1 1 \cdot e^{-jk\pi t} dt \quad \left\{ \begin{array}{l} \text{as from } 1 \text{ to } 2 \\ x(t) = 0 \\ \text{from } 0 \text{ to } 1 \\ x(t) = 1 \end{array} \right. \\
 &= \frac{1}{2} \left. \frac{e^{-jk\pi t}}{-jk\pi} \right|_0^1 \\
 &= \frac{1}{2} \frac{e^{-jk\pi} - 1}{-jk\pi} = \frac{(-1)^k - 1}{-2jk\pi}
 \end{aligned}$$

For even k $(-1)^k = 1$ so $X_k = \frac{1-1}{-2jk\pi} = 0$

For odd k $(-1)^k = -1$ so $X_k = \frac{-2}{-2jk\pi} = \frac{1}{jk\pi}$

For $k=0$

$$X_0 = \frac{1}{2} \int_0^1 1 \cdot dt = \frac{1}{2}(1-0) = \frac{1}{2}$$

Hence

$$X_k = \begin{cases} \frac{1}{2}, & k=0 \\ \frac{1}{j k \pi}, & k=2m+1, m \text{ is any integer} \\ 0, & k=2m, m \text{ is any integer.} \end{cases}$$