Homework 5
CPE381

Canvas: `hw05`

Due: 1st November 2024, 11:59 PM
100 points

You are allowed to use a generative model-based AI tool for your assignment. However, you must submit an accompanied reflection report on how you use the AI tool, what was the query to the tool, and how it improved your understanding of the subject. You must also add your thoughts on how you would tackle the assignment if there was no such tool available. Failure to provide a reflection report for every single assignment where an AI tool was used may result in a penalty and subsequent actions will be taken in line with plagiarism policy.

Submission instruction:

Upload a .pdf on Canvas with the format {firstname.lastname}_cpe381_hw05.pdf. If there is a programming assignment, then you should include your source code along with your PDF files in a zip file firstname.lastname}_cpe381_hw05.zip. If a plot is being asked, your PDF file must also contain plots generated by your MATLAB code. Your submission must contain your name, and UAH Charger ID or the UAH email address. Please number your pages as well.

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1 Fourier Transform (20 points)

The rectangular pulse is defined by

$$x(t) = \begin{cases} 1, & |t| \leq |a| \\ 0, & |t| > |a| \end{cases}$$

1. Write a MATLAB program to plot the signal $x(t)$. **(5 points)**.
2. Find the Fourier transform $X(\Omega)$ analytically (i.e. mathematically) of $x(t)$. **(10 points)**.
3. Write a MATLAB program to plot the magnitude spectrum $|X(\Omega)|$ vs Ω. **(5 points)**.

2 Duality (10 points)

Use the duality property to find the Fourier transform of the sinc signal:

$$x(t) = 20\frac{\sin(0.5t)}{0.5t} = 20\mathbf{sinc}(0.5t), \quad -\infty < t < \infty.$$

3 Filter Design (35 points)

Consider a sine wave as $x(t) = \sin(t)$.

1. What's the fundamental period of $x(t)$? **(2 points)**.

2. Write a MATLAB function to plot $x(t)$. Using the sampling frequency `fs` as 100 Hz. **(2 points)**.

3. Compute its Fourier Transform numerically using MATLAB using `fft` function. Do not use symbolic computation. Check the lecture slides, and course GitHub repo for an example. Plot the magnitude spectrum as a function frequency. **(10 points)**

4. Create a sine wave $\eta(t)$ of a frequency ten times that of $x(t)$ with an amplitude 10% of the amplitude of $x(t)$. Add $\eta(t)$ to the original sine wave so that $y(t) = \eta(t) + x(t)$. Plot $y(t)$. **(2 points).**

5. Compute the Fourier transform of $y(t)$ using `fft` function and plot its Magnitude spectrum $|Y(\Omega)|$ as a function of Ω. Explain in two sentences how the magnitude spectrum of $y(t)$ differs from that of $x(t)$. **(4 points)**.

6. Now, we will design a filter to remove high-frequency noise from $y(t)$. We will design a low-pass filter using `butter` function (that implements Butterworth filter). Choose the filter order of 4 and a cutoff frequency of $5Hz$.

   An example syntax for `butter` is:

   ```
   [b, a] = butter(order, cutoff_freq / (fs/2), 'low');
   ```

   `fs` is the sampling frequency.

   Write down filter coefficients b, and a. **(5 points)**.

7. Now, we will use the filter to clean the noisy sine wave $y(t)$. The syntax is as follows

   ```
   filtered_sine = filter(b, a, noisy_sine);
   ```

   Plot the filtered sine wave. **(5 points)**.

8. Report the mean squared error between the original signal $x(t)$ and the filtered sine wave. **(5 points)**.

4 Filtering a Real Signal (35 points)

1. Download an acceleration data from https://github.com/rahulbhadani/CPE381_FA24/blob/master/Data/accel.csv, and read into MATLAB. The unit of the acceleration is m/s^2. Plot the signal as a function of time. **(5 points)**

2. Perform Fourier transform on the data using `fft` function. **(5 points)**. Note that the signal is not uniformly sampled. Hence, you need to determine the average sampling frequency from the timestamp provided in the data file. **(10 points)**

3. Design a low pass filter using `butter` function with cutoff frequency $2Hz$ and choose the order as 4. Write down filter coefficients b, and a. **(5 points)**.

4. Use the filter coefficients with `filter` function to smoothened-out the acceleration data. Plot the acceleration data along with the smoothened-out acceleration data on the same plot, and also create another plot between 130 - 133 seconds to zoom out and visualize the smoothened-out acceleration signal. **(5 points)**

5. Plot the Magnitude and Phase spectrum of the Fourier Transform of the origin acceleration data and smoothened-out acceleration on the same plot for the comparison. (Magnitude and Phase spectrum will be on separate plots) **(5 points)**