Classwork 08 UAH, CPE381, FA24 09/18/2024
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Q1. Find the Laplace transform of ramp signal $r(t)$ from the first principle. (5pts) Hint: use integration by parts $\int u\,dv = uv - \int v\,du$

Also, state ROC.

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$R(s) = \int_0^{\infty} t\, e^{-st}\, dt$$

$$\int u\,dv = uv - \int v\,du$$

$u = t \qquad dv = e^{-st}\,dt$

$du = dt \qquad v = -\frac{1}{s} e^{-st}$

$$= -\frac{t}{s} e^{-st}\Big|_0^{\infty} + \int_0^{\infty} \frac{1}{s} e^{-st}\, dt$$

$$= -\frac{t}{s} e^{-st}\Big|_0^{\infty} - \frac{1}{s^2} e^{-st}\Big|_0^{\infty}$$

$$= \underbrace{\lim_{t\to\infty} -\frac{t}{s} e^{-st}}_{=0 \text{ (apply L'Hopital Rule)}} + 0 - \underbrace{\lim_{t\to\infty} \frac{1}{s^2} e^{-st}}_{=0} + \frac{1}{s^2}$$

$$= \frac{1}{s^2}, \quad \text{ROC} \in \mathrm{Re}(s) > 0$$

[e^{-st} decays only if $s > 0$ for $t \to \infty$]

Q2. Given that $x(t) = e^{at}u(t)$ has Laplace transform $X(s) = \frac{1}{s-a}$

Find the Laplace transform of $y(t) = \sin(at)u(t)$

State ROC. (5pts)

[Hint: Use Euler's Identity. Don't need to use the first principle]

Solution

$$y(t) = \sin(at) = \frac{e^{jat} - e^{-jat}}{2j}$$

$$Y(s) = \frac{1}{2j}\left[\frac{1}{s-ja}\right] - \frac{1}{2j}\left[\frac{1}{s+ja}\right]$$

$$= \frac{1}{2j}\left\{\frac{\cancel{s}+ja-\cancel{s}+ja}{(s-ja)(s+ja)}\right\}$$

$$= \frac{\cancel{2j}a}{\cancel{2j}} \cdot \frac{1}{s^2+a^2} = \frac{a}{s^2+a^2}$$