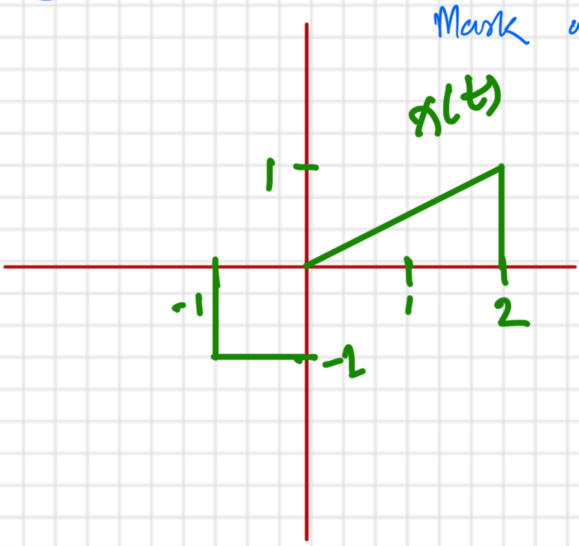
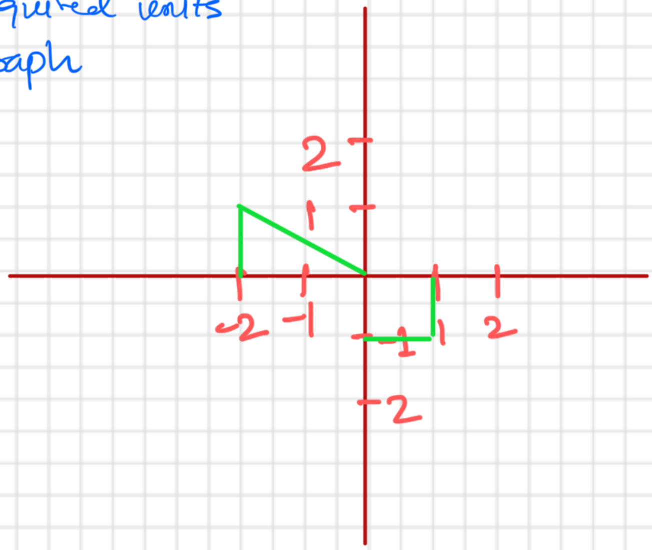
Classwork 05 CPE381 FA2024

Total: 20 pts

① Given $x(t)$, draw $x(-t)$ (4 pts)

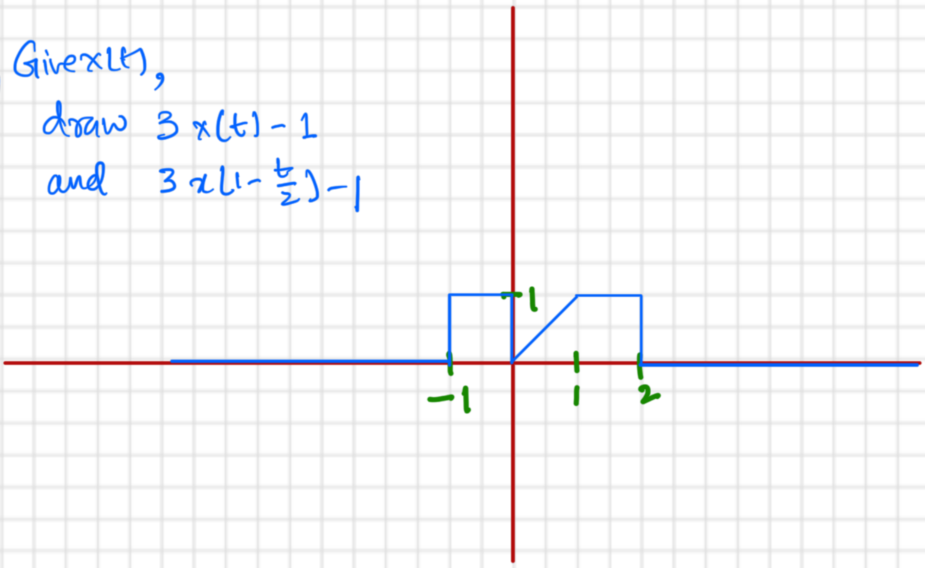
Mark axes with required units as in the graph for $x(t)$

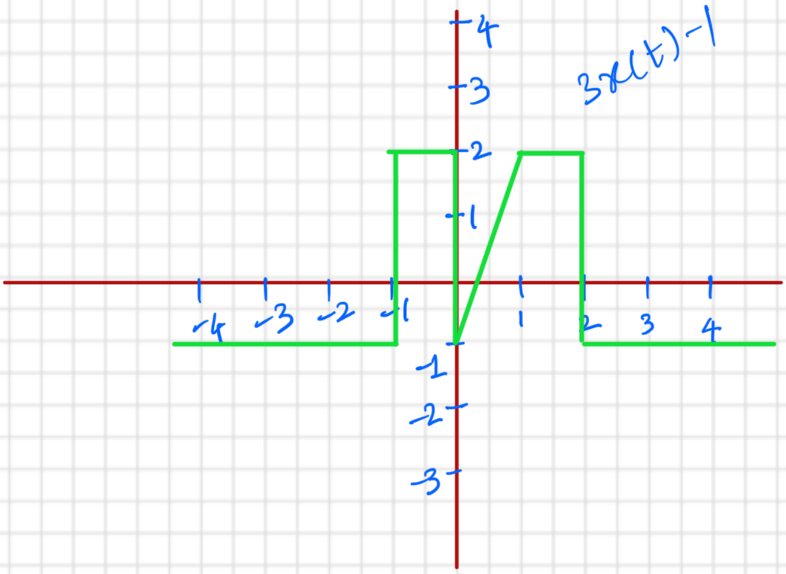


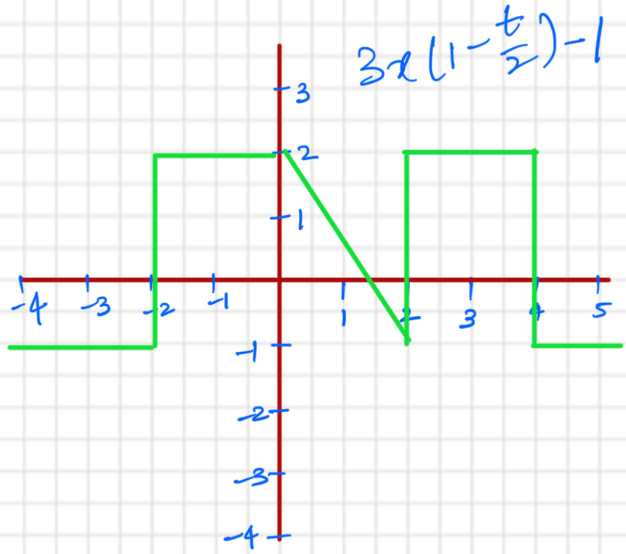


② Given $x(t)$, draw $3x(t) - 1$ and $3x(1 - \frac{t}{2}) - 1$

(4pts)



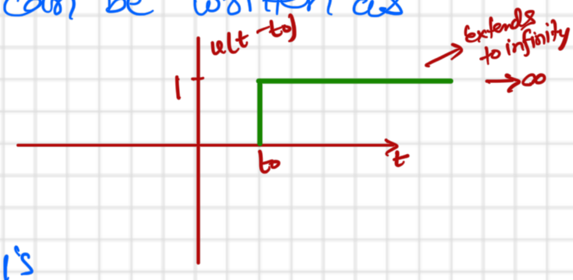




③ A general unit step function can be written as

(4pts)

$$u(t-t_0) = \begin{cases} 1 & t > t_0 \\ 0 & t < t_0 \end{cases}$$



A mathematically useful function is unit rectangular pulse with time period T as shown below:

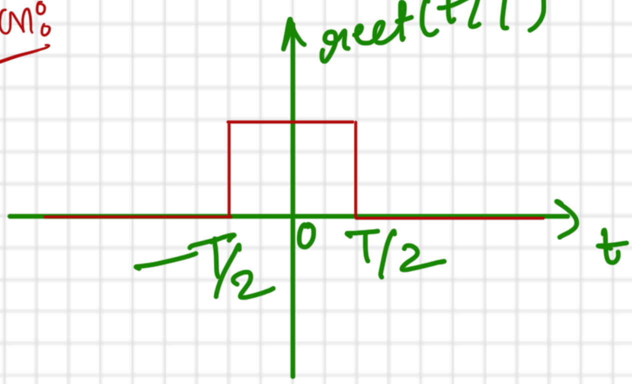
$$\text{rect}(t/T) = \begin{cases} 1, & -T/2 < t < T/2 \\ 0, & \text{otherwise} \end{cases}$$

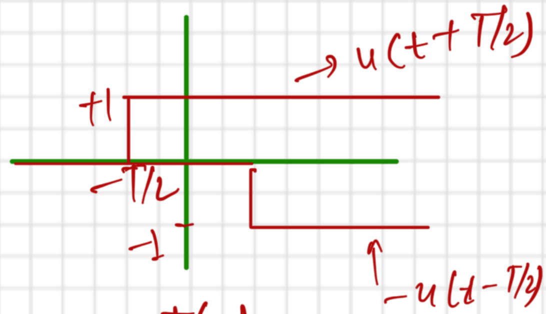
ⓐ Sketch the rect (t/T).

ⓑ Write down rect (t/T) usin $u(t-t_0)$ where you can choose t_0 based on the time period T.

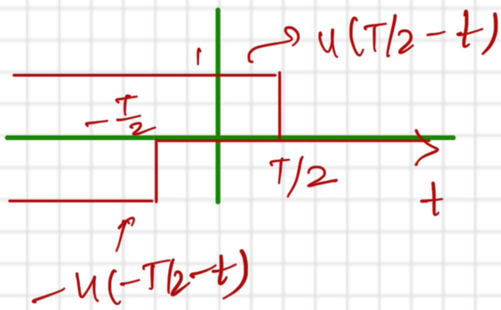
Note : more than one answer is possible.

Solution:





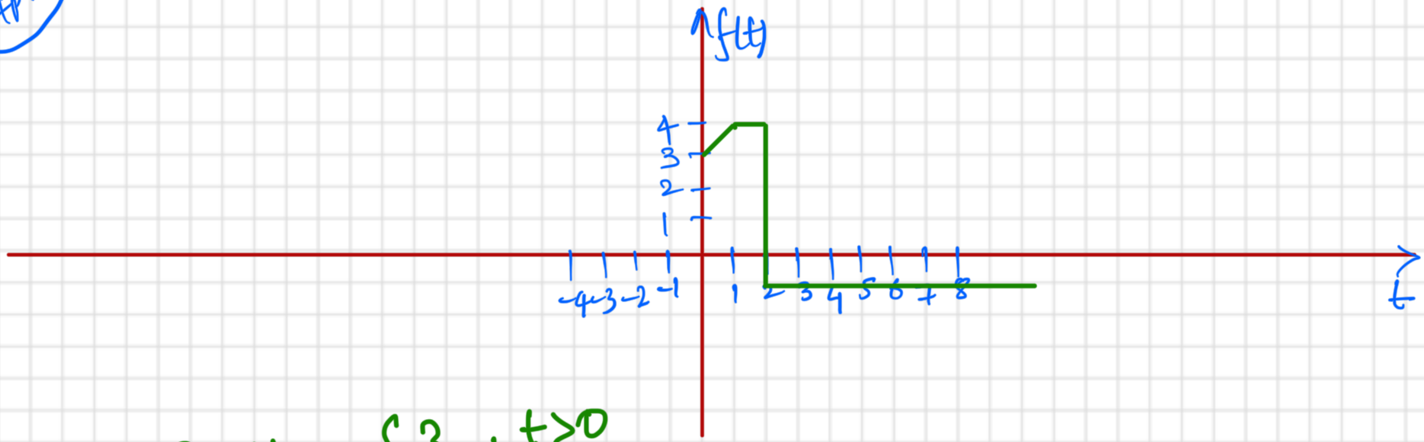
$$\text{rect}(t/T) = \begin{cases} u(t+T/2) - u(t-T/2) \\ u(T/2-t) - u(-T/2-t) \end{cases}$$



④ Plot the signal wave form for

(4pts)

$$f(t) = 3u(t) + t\,u(t) - [t-1]\,u(t-1) - 5u(t-2)$$



$$3u(t) = \begin{cases} 3, & t>0 \\ 0 & t<0 \end{cases}$$

$$t\,u(t) = \begin{cases} t, & t>0 \\ 0, & t<0 \end{cases}$$

$$(t-1)\,u(t-1) = \begin{cases} t-1 & , t>1 \\ 0, & t<1 \end{cases}$$

$$5u(t-2) = \begin{cases} 5, & t>2 \\ 0, & t<2 \end{cases}$$

$t<0$, $f(t) = 0$

$0<t<1$, $f(t) = 3+t$

$1<t<2$, $f(t) = 4$

$2<t$, $f(t) = -1$

⑤ Find $\int_{-\infty}^{+\infty} \delta(bt-a)\cos^2(t-c)\,dt$. Hint: Use a change of variable.

(4pts)

Solution:

$$\delta(bt-a) = \frac{1}{|b|}\delta\left(t-\frac{a}{b}\right)$$

then integral is $\int_{-\infty}^{\infty} \frac{1}{|b|}\delta\left(t-\frac{a}{b}\right)\cos^2(t-c)\,dt$

Now, we apply the property of the Dirac delta function:

$$\int_{-\infty}^{\infty} \frac{1}{|b|} \delta\left(t-\frac{a}{b}\right) \cos^2(t-c)\,dt = \frac{1}{|b|} \cos^2\left(\frac{a}{b}-c\right)$$

$$b \neq 0$$

We applied the following properties to solve the integration:

① Scaling Property:

$$\delta(bt-a) = \frac{1}{|b|} \delta\left(t-\frac{a}{b}\right)$$

② Sifting property : $\int_{-\infty}^{\infty} \delta(x-a) f(x)\,dx = f(a)$
