Homework 6
CPE381

Canvas: `hw06`

Due: 4th November 2024, 11:59 PM
100 points

You are allowed to use a generative model-based AI tool for your assignment. However, you must submit an accompanied reflection report on how you use the AI tool, what was the query to the tool, and how it improved your understanding of the subject. You must also add your thoughts on how you would tackle the assignment if there was no such tool available. Failure to provide a reflection report for every single assignment where an AI tool was used may result in a penalty and subsequent actions will be taken in line with plagiarism policy.

Submission instruction:

Upload a .pdf on Canvas with the format {firstname.lastname}_cpe381_hw06.pdf. If there is a programming assignment, then you should include your source code along with your PDF files in a zip file firstname.lastname}_cpe381_hw06.zip. If a plot is being asked, your PDF file must also contain plots generated by your MATLAB code. Your submission must contain your name, and UAH Charger ID or the UAH email address. Please number your pages as well.

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1 Bandlimited (10 points)

State whether the following functions and band-limited or not. If they are band-limited, determine the Nyquist frequency **(5 points each(**.

1. $\left(\dfrac{\mathbf{rect}}{3}\right)$
   where

$$\left(\frac{\mathbf{rect}}{a}\right) = \begin{cases} 1, & |t| \leq \dfrac{a}{2}, \\ 0, & \text{otherwise} \end{cases}$$

   for a positive number a.

   Hint: sketch the function.

2. $\operatorname{sinc}\left(\dfrac{x}{3}\right)$

2 Nyquist Sampling Rate Condition (25 points)

1. Consider the signal $x(t) = 2\cos(2\pi t + \pi/4)$, $0 - \infty < t < \infty$, determine if it is bandlimited. **(5 points)**

2. Use $T_s = 0.1, 0.2, 0.3$, and 2 seconds/sample as sampling periods, and for each of these write down their sampled version of the signal $x_s(t)$, and then find out whether the Nyquist sampling rate condition is satisfied and if the sampled signal looks like the original signal or not. **(10 points)**

3. Write a MATLAB code to plot continuous signal $x(t)$. Vary time from $t = 0$ to $t = 3.0, s$ Choose the sampling time of $T_s = 0.10.2, 0.3$, and 2 seconds/sample and create 4 subplots in a single plot, showing original $x(t)$ and corresponding sampled signal $x_s(t)$ for each of the four $T_s s$. Make sure you plot the sampled signal using a stem plot.

   (10 points)

3 Draw Again (10 points)

For the discrete-time signals,

$$x[n] = \begin{cases} 1, & n = -1, 0, 1 \\ 0.5, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

Sketch and label properly the following signals:

1. $x[n-1]$, $x[-n]$, $x[2-n]$. **(3 points)**

2. The even components $x_e[n]$ of $x[n]$. **(3.5 points)**

3. The odd components $x_o[n]$ of $x[n]$. **(3.5 points)**

4 Impulse Response (20 points)

1. Determine the impulse response $h[n]$ of an LTI system represented by the difference equation

   $$y[n] = -0.5y[n-1] + x[n]$$

   where $x[n]$ is the input , $y[n]$ is the output and the initial conditions are zero. **(5 points)**

2. Find $y[n]$ analytically when the input is

   $$x[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

3. Compute $y[n]$ using `for` loop by implementing in MATLAB and plot for n = 1:20. Use analytical solution obtained from the previous part of this question to plot separately and compare your result. **(10 points)**

5 Z-transform (10 points)

Find the z-transform of **(5 points each)**

1. $x[n] = -a^n u[-n-1]$
2. $x[n] = a^{-n} u[-n-1]$ where a is a constant, u represents discrete unit-step function.

6 ROC for Discrete-time System (10 points)

Find Z-transform $X(z)$ and sketch a pole-zero plot with ROC for each of the following sequences **(5 points each)**:

1. $x[n] = (\frac{1}{3})^n u[n] + (\frac{1}{2})^n u[-n-1]$
2. $x[n] = (\frac{1}{2})^n u[n] + (\frac{1}{3})^n u[-n-1]$

7 Inverse Z-transform (5 points)

Find the inverse z-transform of $X(z) = \dfrac{2}{2z^2 - 3z + 1}$, $|z| > 1$.

8 Difference Equation (10 points)

1. For the following difference equation, and associated input and initial conditions, determine the output $y[n]$:

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

with $x[n] = \left(\dfrac{1}{3}\right)^n$

and initial condition $y[-1] = 1$.