CPE 381 WORKED EXAMPLE 04 FALL 2024 09/16/2024

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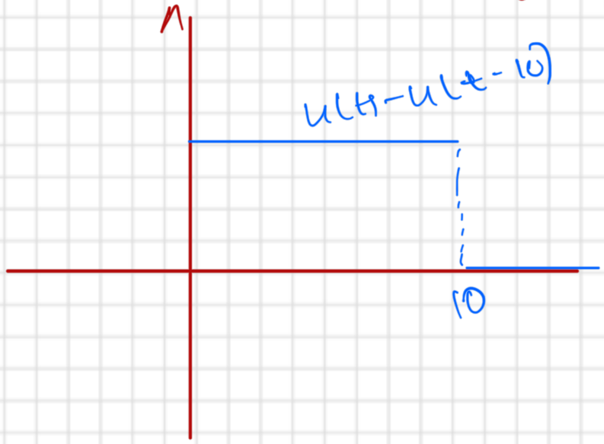
(1) Input : $x(t)$ Output: $x(t)f(t)$

Let $f(t) = u(t) - u(t-10)$, determine whether the system with input $x(t)$ and $y(t)$ is BIBO stable.

Solution

For bounded $x(t)$, i.e $|x(t)| < M < \infty$, the output is bound, as

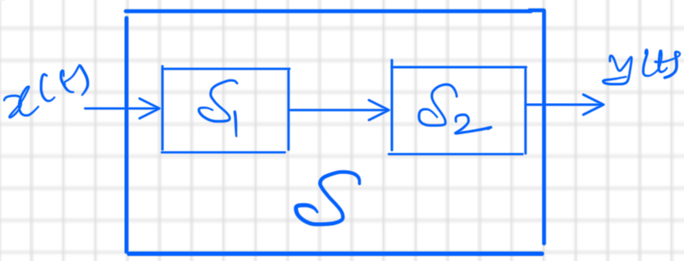


$f(t) = u(t) - u(t-10)$ is a constant signal

$y(t) = x(t)f(t)$ is bounded as well.

Hence, the System is BIBO Stable.

Q2. Consider a system composed of two subsysten: S_1 and S_2



Impulse response of S_1 is $h_1(t) = \delta(t) - 2e^{-t}u(t)$

" " " S_2 is $h_2(t) = e^{t}u(t)$

Is the System S BIBO stable.

Solution:

Composite system's impulse response is given by

$$h(t) = h_1(t) * h_2(t)$$

$$= e^{t}u(t) * [\delta(t) - 2e^{-t}u(t)]$$

Note: Convolution with $\delta(t)$ returns the original signal, i.e. $f(t) * \delta(t) = f(t)$

So

$$e^{t}u(t) * [\delta(t) - 2e^{-t}u(t)]$$

$$= e^{t}u(t) * \delta(t) - e^{t}u(t) * (2e^{-t}u(t))$$

$$= e^{t}u(t) - \underbrace{e^{t}u(t) * (2e^{-t}u(t))}$$

$$(f*g)(t) = \int_{-\infty}^{t} f(\tau)\, g(t-\tau)\, d\tau$$

$$= 2\int_{-\infty}^{t} e^{\tau}u(\tau)\, e^{-(t-\tau)}\, u(t-\tau)\, d\tau$$

$$= 2\int_{t}^{+\infty} e^{\tau}u(\tau)\, e^{-t}e^{+\tau}\, u(t-\tau)\, d\tau$$

$$= 2e^{-t}\int_{0}^{t} e^{2\tau}\, d\tau = 2e^{-t} \cdot \frac{e^{2\tau}}{2}\Big|_{0}^{t}$$

$$= \cancel{2}e^{-t}\frac{e^{2t}}{\cancel{2}} - e^{-t}$$

$$= e^{t} - e^{-t}$$

$$= e^{t}u(t) - e^{-t}u(t), \; t \geq 0$$

So $h(t) = e^{t}u(t) - (e^{t}u(t) - e^{-t}u(t))$

$$= e^{-t}u(t)$$

As $e^{-t}u(t)$ is absolutely integrable $\left(\int_{0}^{\infty} |e^{-t}u(t)|\, dt < \infty \right)$ the system is BIBO stable.