CPE381 HW04 Problem3 Solution

Q3. Sketch the fourier spectra (i.e Magnitude spectrum and phase spectrum) for exponential Fourier series of $x(t)$ given by

$$x(t) = (2+j2)e^{-j3t} + j2e^{-jt} + 3 - 2e^{jt} + (2-j2)e^{j3t}$$

Solution:

$$x(t) = \sum_{k=-\infty}^{\infty} x_k e^{jk\Omega_0 t}$$

From the expression, we see that $\Omega_0 = 1$ rad/s

So

$X_{-3} = 2+j2$ $|X_{-3}| = \sqrt{2^2+2^2} = \sqrt{4+4} = 2\sqrt{2}$
                                        $= 2.828$

$X_{-1} = j2$ $|X_{-1}| = 2$

$X_0 = 3$ $|X_0| = 3$ $|X_2| = 0$

$X_1 = -j2$ $|X_1| = 2$ $|X_{-2}| = 0$

$X_3 = (2-j2)$ $|X_3| = 2.828$

$$\angle X_{-3} = \tan^{-1}\frac{2}{2} = 45° = \pi/4$$

$$\angle X_{-1} = \tan^{-1}\frac{1}{0} = \pi/2 \text{ or } 90°$$

$$\angle X_0 = \tan^{-1}\frac{3}{0} = 0° \text{ or } 0 \text{ radian}$$

$$\angle X_1 = \tan^{-1}(-1/0) = -\pi/2 \text{ or } -90°$$

$$\angle X_3 = \tan^{-1}(-2/2) = -45° \text{ or } -\pi/4$$

$$\angle X_2 = 0$$
$$\angle X_{-2} = 0$$

Magnitude Spectrum

Phase Spectrum

$\angle X_k$ in degrees.