Classwork 09: Fall 2024
CPE381

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Total: 15 points

1 Tell me about Systems.

The steady-state solution of stable systems is due to simple poles in the $j\omega$ axis of the s-plane coming from the input. Suppose the transfer function of the system is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)^2 + 4} \tag{1}$$

1. Find the poles and zeros of $H(s)$and plot them on the s-plane. **(2 points)**
2. Find then the corresponding impulse response $h(t)$. **(2 points)**
3. Determine if the impulse response of this system is absolutely integrable so that the system is BIBO stable. **(2 points)**
4. Let the input $x(t) = u(t)$, the unit-step function, and the initial conditions be zero, find $y(t)$ and from it determine the steady state solution. **(4 points)**
5. Let the input $x(t) = tu(t)$ and the initial conditions be zero, find $y(t)$ and from it determine the steady-state response. What is the difference between this case and the previous one? **(2 points)**
6. To explain the behavior in the case above consider the following: Is the input $x(t) = tu(t)$ bounded? that is, is there some finite value M such that $|x(t)| < M$ for all times? So what would you expect the output to be knowing that the system is stable? **(3 points)**

1.1 Answers:

1. $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{1}{(s+1)^2 + 4} = \dfrac{Y(s)}{X(s)} = \dfrac{1}{(s+1)^2 + 2^2}$.
2. The impulse response of the system is of the form $h(t) = Ae^{-t}\cos(2t + \theta)u(t)$. Poles of $H(s)$ are $s = -1 \pm j2$.

3. As the real part of poles are negative, the impulse response will go to zero as $t \to \infty$. As $|\cos(2t+\theta)| \leq 1$, we can write:

$$\begin{aligned} |h(t)| &\leq |A|e^{-t}u(t) \\ \Rightarrow \int_{-\infty}^{\infty} |h(t)|dt &\leq |A| \int_{-\infty}^{\infty} e^{-t}dt = |A| \end{aligned} \tag{2}$$

The value of A can be found by writing

$$A = H(s)(s+1-j2) \tag{3}$$

and substituting for $s = 1 + j2$ which gives $A = \dfrac{1}{-1+j2+1+j2} = \dfrac{1}{j4} = 0.25e^{-j\pi/2}$ which is $< \infty$. Hence, the system is stable.

4. For $x(t) = u(t)$,

$$Y(s) = \frac{1}{s((s+1)^2+4)} = \frac{A}{s} + ... \tag{4}$$

where ... are the terms corresponding to the poles due to the system which are in the left-hand s-plane and so their inverse will go to zero.

Steady-state response can be determined used final value theorem, which means

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{1}{(s+1)^2+4} = \frac{1}{5} \tag{5}$$

Hence the steady-state response $y_{ss}(t) = \dfrac{1}{5} = 0.2$.

5. If $x(t) = tu(t)$, then $X(s) = \dfrac{1}{s^2}$, then

$$Y(s) = \frac{1}{s^2((s+1)^2+4)} \tag{6}$$

Complex planes are in the left-half plane.

The steady state is only due to the double pole $s = 0$, since the others would have a zero steady state (they are in the left hand s-plane)

The actual partial fraction expansion of $Y(s)$ is

$$Y(s) = \frac{a}{s^2} + \frac{b}{s} + \frac{c+(s+1)d}{(s+1)^2+4} \tag{7}$$

We only need to find the values of a and b to find the steady state, the other term will give zero in the steady state. But if $a \neq 0$, the steady-state response is of the form

$y_{ss} = [at + b]u(t) \to \infty$

Hence the steady-state response is infinite.

6. The input $x(t) = tu(t)$ is not bounded since there is no finite value M such that $|x(t)| < M < \infty$, as the signal keep growing. As the input is not bounded, the system will not be able to generate a bounded output even if the system is stable.