Q1. The transfer function (in s-domain) of a filter is

$$H(s) = \frac{\sqrt{5}s}{s^2+2s+2}.$$

(a) Find its poles and zeros. (5 points)

Poles are given by the roots of $s^2+2s+2=0$

$$s = \frac{-2 \pm \sqrt{2^2-4\times 2}}{2}$$

$$= \frac{-2\pm\sqrt{-4}}{2} = -1 \pm j\cdot 1$$

zero is given $\sqrt{5}s = 0$

$\Rightarrow s = 0.$

By looking at poles, we can say that it is at a band pass filter with center frequency around 1.

Its magnitude response is using vectors from the zero and poles to the point in $j\Omega$-axis.

(b) Find the magnitude $|H(j\Omega)|$. (5 points.)

Hint: ① set $s = j\Omega$

② For complex numbers z_1 and z_2 if $z = \frac{z_1}{z_2}$

$$|z| = \frac{|z_1|}{|z_2|}$$

and $|a+ib| = \sqrt{a^2+b^2}$

Set $\Omega = 0, 1, \infty$ to find $|H(0)|$, $|H(1)|$, $|H(\infty)|$

Solution

$$H(\Omega) = \frac{\sqrt{5}j\Omega}{(j\Omega)^2 + 2\cdot j\Omega + 2}$$

$$= \frac{\sqrt{5}j\Omega}{-\Omega^2 + 2j\Omega + 2}$$

$$= \frac{\sqrt{5}j\Omega}{(2-\Omega^2) + 2j\Omega}$$

$$|H(\Omega)| = \frac{|\sqrt{5}j\Omega|}{|(2-\Omega^2) + j2\Omega|}$$

$$= \frac{\sqrt{5}\Omega}{\sqrt{(2-\Omega^2)^2 + (2\Omega)^2}}$$

$$|H(0)| = \frac{\sqrt{5}\times 0}{\sqrt{(2-0^2)^2 + (2\cdot 0)^2}} = 0$$

$$|H(1)| = \frac{\sqrt{5}}{\sqrt{(2-1^2)^2 + (2\cdot 1)^2}} = \frac{\sqrt{5}}{\sqrt{1+4}} = \frac{\sqrt{5}}{\sqrt{5}} = 1$$

For $\Omega = \infty$

$$|H(\infty)| = \lim_{\Omega\to\infty} \frac{\sqrt{5}\Omega}{\sqrt{\left(\Omega^2\left(\frac{2}{\Omega^2} - 1\right)\right)^2 + 4\Omega^2}}$$

$$= \lim_{\Omega \to \infty} \frac{\sqrt{5}\,\Omega}{\sqrt{\Omega^4\left(\frac{2}{\Omega^2}-1\right)^2+4\Omega^2}}$$

$$= \lim_{\Omega \to \infty} \frac{\sqrt{5}\,\cancel{\Omega}}{\cancel{\Omega}\sqrt{\Omega^2\left(\frac{2}{\Omega^2}-1\right)^2+4}}$$

$$= \frac{1}{\infty(\cdots)+4} = 0.$$

Hence $|H(\infty)| = 0.$