

# CPE 381: Fundamentals of Signals and Systems for Computer Engineers

01 Introduction and Preliminaries

Fall 2025

**Rahul Bhadani**

# Outline

**1. Course Logistics**

**2. Motivation**

**3. Mathematical Preliminaries**

# About Me

## Rahul Bhadani

Assistant Professor

Electrical and Computer Engineering

The University of Alabama in Huntsville

Office: 217-H

Email: [rahul.bhadani@uah.edu](mailto:rahul.bhadani@uah.edu)

Web: <https://rahulbhadani.github.io/>



## Research Interests

Cyber-physical Systems, Intelligent Transportation, Connected-and-Autonomous Driving, Applied machine learning, Quantum Information Science

# Course Logistics

**Lecture:**

M/W 11:20 AM - 12:40 PM

**Location:**

ENG 240

**Prerequisites:**

- ⚡ EE 213 - Electrical Circuit Analysis I
- ⚡ MA 238 – Applied Differential Equations

**Office Hours:**

⚡ Tuesday: 2:00 PM - 4:00 PM

⚡ Wednesday: 12:45 PM - 2:00 PM

**Instructor Email:** rahul.bhadani@uah.edu

# Textbooks

Required: *Signals and Systems using Matlab*. Luis F. Chaparro.

Elsevier, 3rd Edition, 2019.

ISBN: 978-0-12-814204-2, eBook ISBN: 9780128142059

Suggested Reading: *Linear Systems and Signals*, B. P. Lathi, Roger Green, Oxford University Press, 2017, 3rd Edition

Other Notable Textbooks:

⚡ Schaum's Outline of Signals and Systems

⚡ *Signals and Systems*. Haykin, Simon, and Barry Van Veen. John Wiley & Sons, 2007.

⚡ Asadi, Farzin. Signals and Systems with MATLAB and Simulink. Springer, Dec 2023.

ISBN: 9783031456220, 303145622X

# Grading

|   |     |
|---|-----|
| <b>Homework:</b>                          | 25% |
| <b>Quizzes:</b>                           | 5%  |
| <b>Attendance/In-Class Participation:</b> | 10% |
| <b>Mid-term Exam 1:</b>                   | 15% |
| <b>Mid-term Exam 2:</b>                   | 15% |
| <b>Final Exam:</b>                        | 30% |

| Grading Scale |       |
|---------------|-------|
| Percentage    | Grade |
| 90% - 100%    | A     |
| 75% - 89%     | B     |
| 60% - 74%     | C     |
| 45% - 59%     | D     |
| 0% - 44%      | F     |

# Exam Schedule

- ⚡ **Mid Term 1:** September 30, Monday
- ⚡ **Mid Term 2:** November 06, Wednesday
- ⚡ **Final Exam:** December 13, Friday

# Tentative Topics

- ⚡ Introduction, Mathematical Preliminaries
- ⚡ Continuous and Discrete Signals
- ⚡ Linear-time Invariant (LTI) Systems
- ⚡ Laplace Transform
- ⚡ Fourier Series for Frequency Analysis
- ⚡ Fourier Transform
- ⚡ Sampling Theory
- ⚡ Discrete-Time Signals and Systems
- ⚡ Z-transform
- ⚡ Discrete Fourier Analysis

Refer to the syllabus for the detailed information on the syllabus.

# In-Class Activity

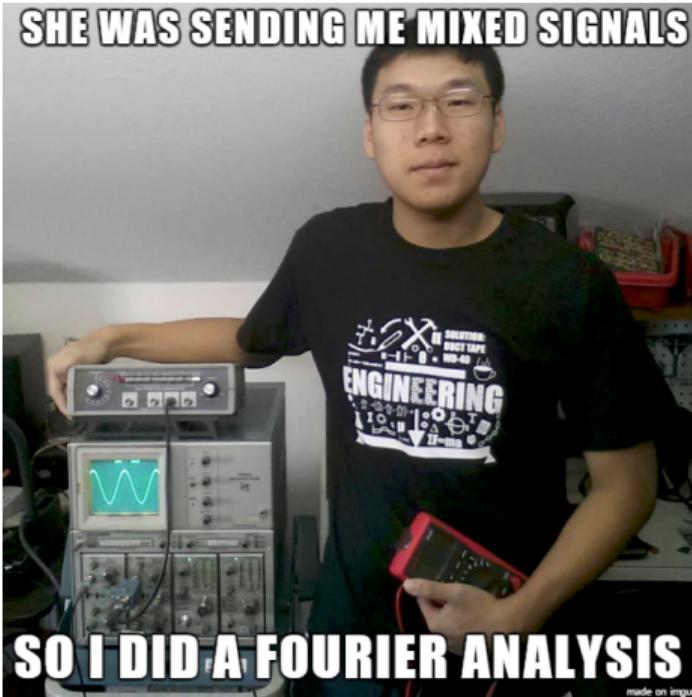
## Introduce Yourself

- ⚡ Why Computer Engineering?
- ⚡ Why do you want to take this course?
- ⚡ One fun fact about you that's not on social media

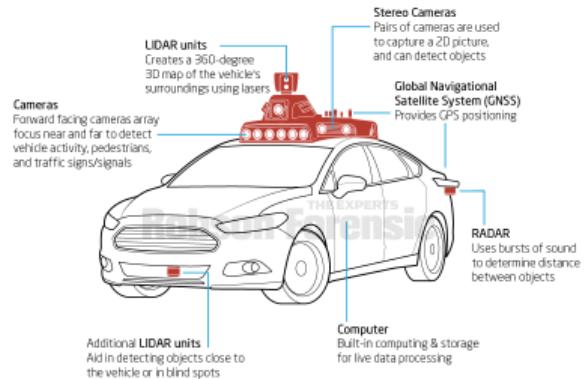
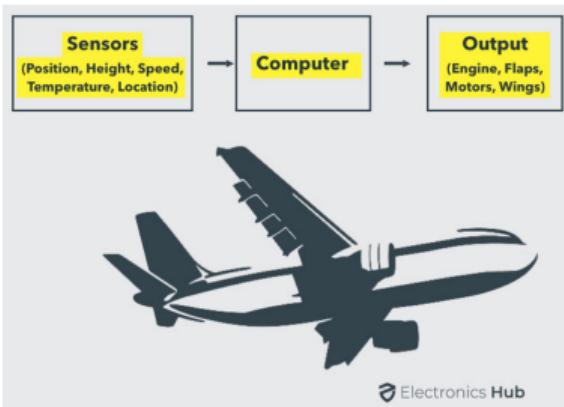


# Motivation

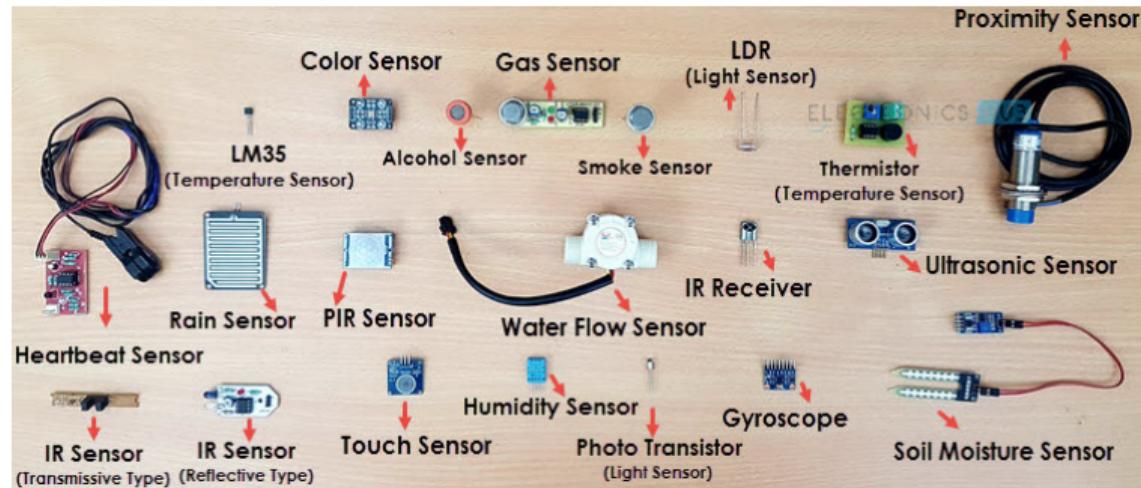
# Why Study Signals and Systems



# We are in digital era



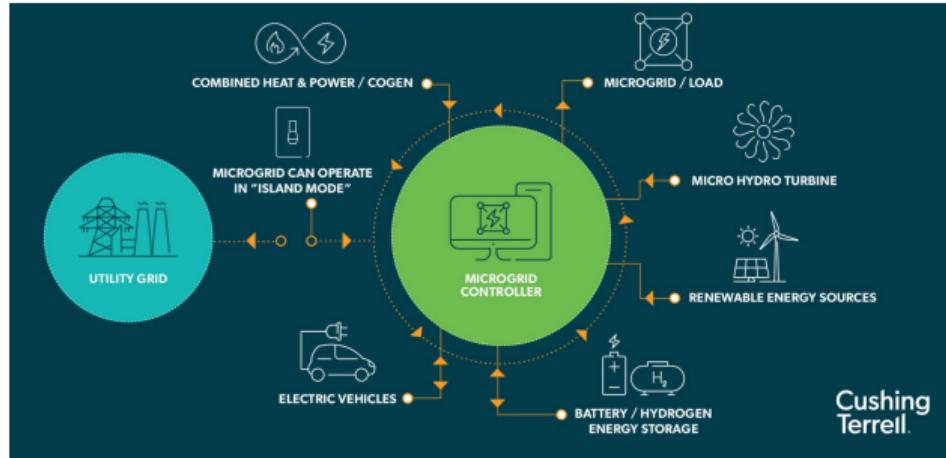
# Sensors and Mobile Devices



# Cyber-physical Systems



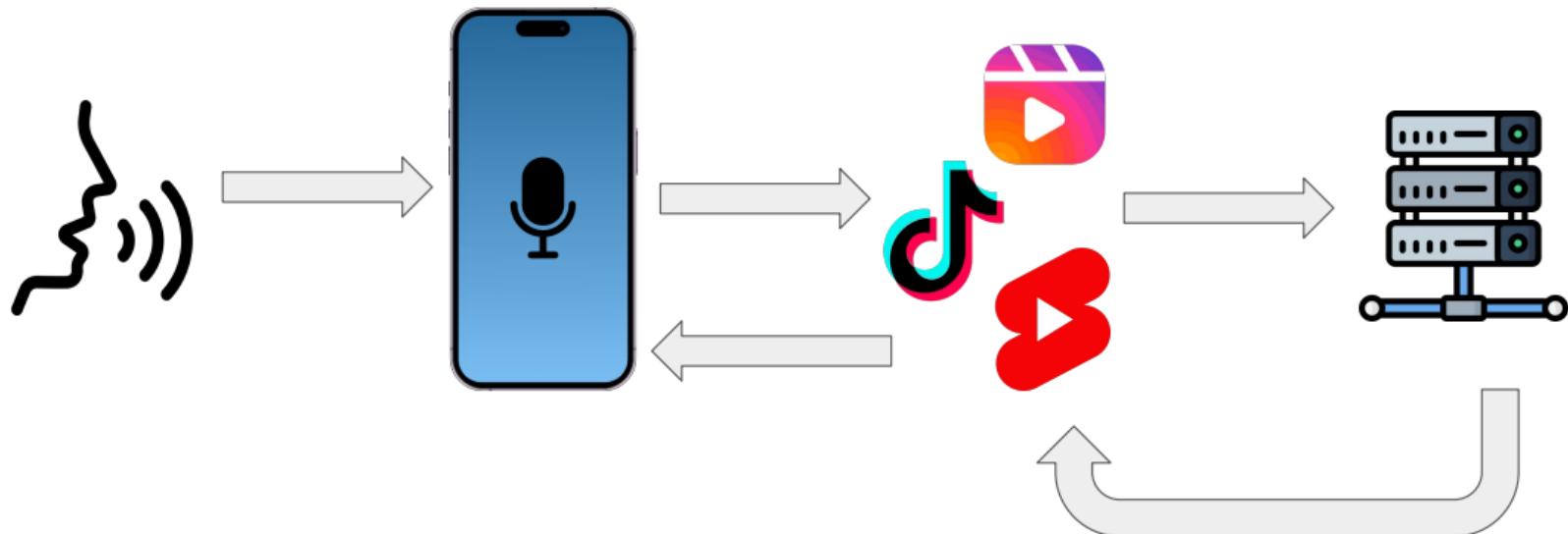
Connected and Autonomous Vehicles



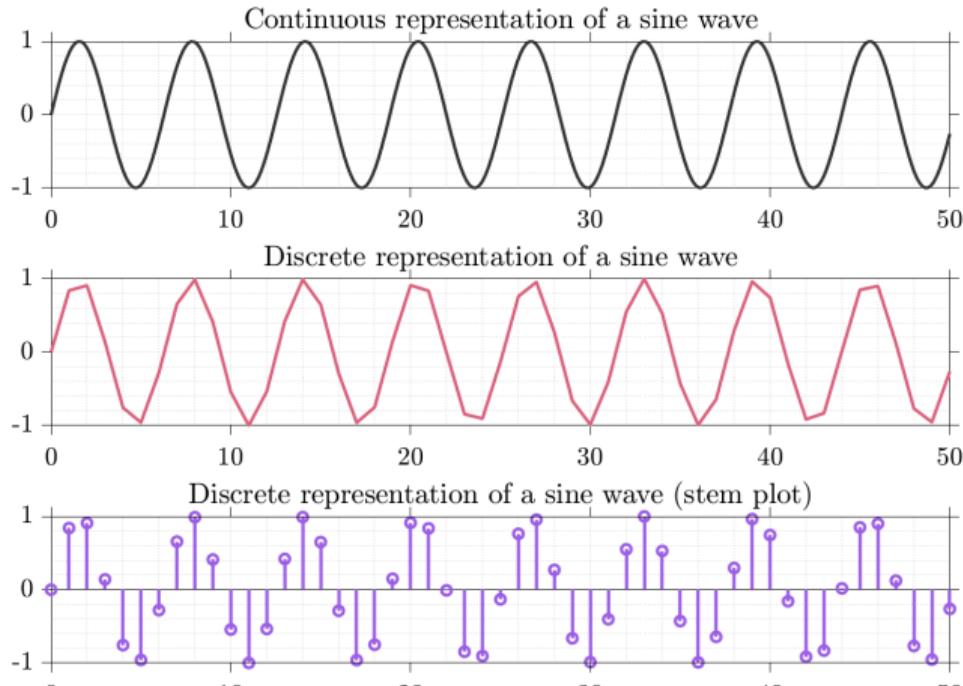
Microgrid

# Signals in Nature

Everything in nature is analog and continuous. We use hardware/software interfaces to digitize them, extract information, make inferences, and send them back to the user.



# Sampling continuous time signals



Code for the figure: [https://github.com/rahulbhadani/CPE381\\_FA25/blob/main/Code/sampled\\_sine\\_wave.m](https://github.com/rahulbhadani/CPE381_FA25/blob/main/Code/sampled_sine_wave.m)

# Inherent Discrete Time Signals

**Nature may be continuous but human activities naturally lead to some discrete time signals.**

Examples:

- ⚡ Stock market closing data
- ⚡ Adaptive cruise control states
- ⚡ Thermostat setpoints



# Mathematical Preliminaries

# Trigonometry

## ⚡ Sine, Cosine, Tangent:

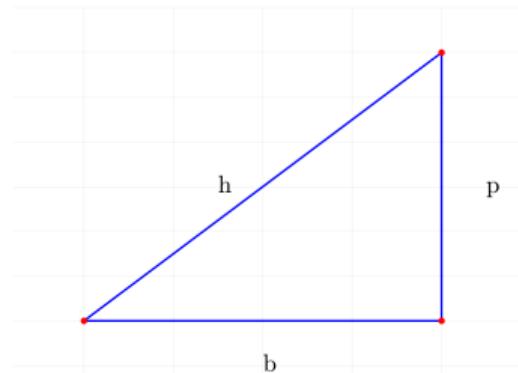
$$-\sin(\theta) = \frac{p}{h}$$

$$-\cos(\theta) = \frac{b}{h}$$

$$-\tan(\theta) = \frac{p}{b}$$

## ⚡ Pythagorean Identity:

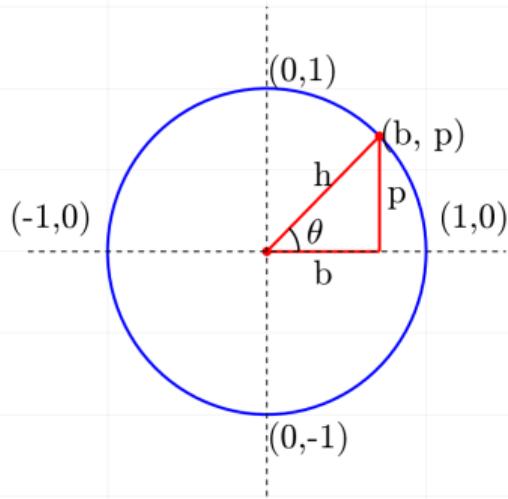
$$\sin^2(\theta) + \cos^2(\theta) = 1$$



## ⚡ Unit Circle:

# Trigonometric Identities

What is the value of  $h$  based on the diagram?



⚡  $\cos \theta =$

⚡  $\sin \theta =$

⚡  $\sin(\alpha + \beta) =$

⚡  $\cos(\alpha + \beta) =$

⚡  $\sin^2(\theta) =$  in terms of  $\cos(2\theta)$

⚡  $\cos^2(\theta) =$  in terms of  $\cos(2\theta)$

⚡  $\sin 2\theta =$  in terms of  $\theta$

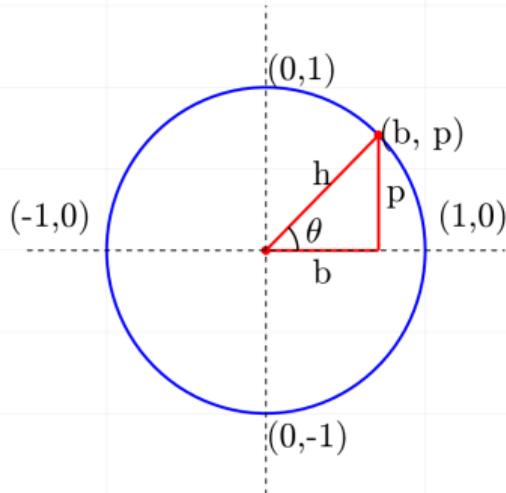
⚡  $\cos 2\theta =$  in terms of  $\theta$

⚡  $\sin(-\theta) =$

⚡  $\cos(-\theta) =$

# Trigonometric Identities

What is the value of  $h$  based on the diagram?



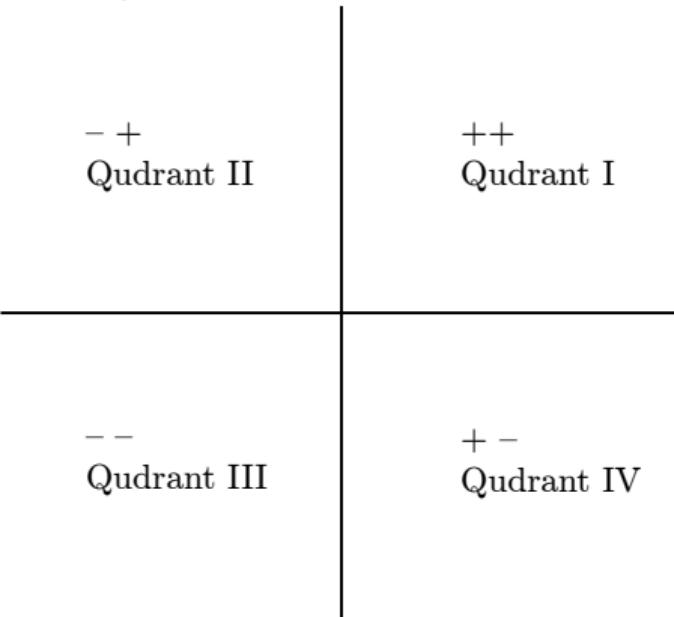
- ⚡  $\tan(\alpha + \beta) =$
- ⚡  $\tan(\alpha - \beta) =$
- ⚡  $\sin 30^\circ =$
- ⚡  $\sin 45^\circ =$
- ⚡  $\sin 60^\circ =$

- ⚡  $\cos 30^\circ =$
- ⚡  $\cos 45^\circ =$
- ⚡  $\cos 60^\circ =$
- ⚡  $\tan 30^\circ =$
- ⚡  $\tan 45^\circ =$
- ⚡  $\tan 60^\circ =$

**What's the range of  $\sin \theta$  and  $\cos \theta$ ?**

# Trigonometric Functions in Different Quadrant

Four Quadrants:



Considering a unit circle, in four different quadrants ( $x, y$ ) signs are different, Hence the trigonometric ratios are also different in signs.

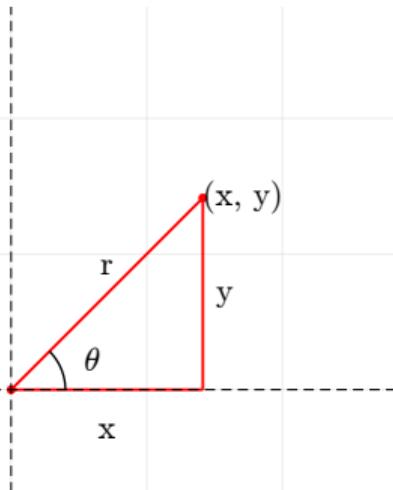
| Quadrant | sin | cos | tan |
|----------|-----|-----|-----|
| I        | +   | +   | +   |
| II       | +   | -   | -   |
| III      | -   | -   | +   |
| IV       | -   | +   | -   |

Table: Signs of sin, cos, and tan in all four quadrants

# Complex Number

$$z = x + iy$$

$$i^2 = -1$$



Two representations of complex numbers:

$$z = re^{i\theta}$$

$$\text{Conjugate: } \bar{z} = re^{-i\theta}$$

$$\blacksquare r = \sqrt{x^2 + y^2}$$

$$\blacksquare \theta = \tan^{-1} \frac{y}{x}$$

Euler's Identity:

$$\blacksquare e^{i\theta} = \cos \theta + i \sin \theta$$

$$\blacksquare e^{-i\theta} =$$

$$\blacksquare \cos \theta =$$

(in terms of exponential)

$$\blacksquare \sin \theta =$$

(in terms of exponential)

# Complex Number

## Properties of Conjugates

- ⚡  $\overline{z+w} = \bar{z} + \bar{w}$
- ⚡  $\overline{zw} = \bar{z}\bar{w}$
- ⚡  $\overline{z^n} = \bar{z}^n$
- ⚡  $z\bar{z} = |z|^2$  **Prove it!**

**De Moivre's Theorem** If  $z = r(\cos \theta + i \sin \theta)$  and  $n$  is a positive integer, then

$$z^n = [r(\cos \theta + i \sin \theta)^n] = r^n(\cos n\theta + i \sin n\theta)$$

**Roots of a complex number**  $z = 0$  has  $n$  distinct roots:

$$w_k = r^{1/n} \left[ \cos \left( \frac{\theta + 2k\pi}{n} \right) + i \sin \left( \frac{\theta + 2k\pi}{n} \right) \right]$$

# Complex Exponentials

We also need to give a meaning to the expression  $e^z$  when  $z = x + iy$  is a complex number.

Based on the infinite series using Taylor's series expansion for  $e^x$ , we have

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$$

Can you find out, what should be the value of  $e^{iy}$  where  $y$  is the real number?

**Note:**  $\cos y = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \frac{y^6}{6!} + \dots$  and  $\sin y = y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$

# Derivatives

**Rate of change of a quantity with respect to another quantity.**

**In signals and systems, we are usually interested in the rate of change with respect to time.**

$$f'(t) = \frac{df(t)}{dt} = \lim_{h \rightarrow 0} \frac{f(t + h) - f(t)}{h}$$

**In computer implementation,  $h$  is  $\Delta t$ .  $\Delta t$  is the difference of timestamps between two consecutive samples of a signal.**

# Derivative Review

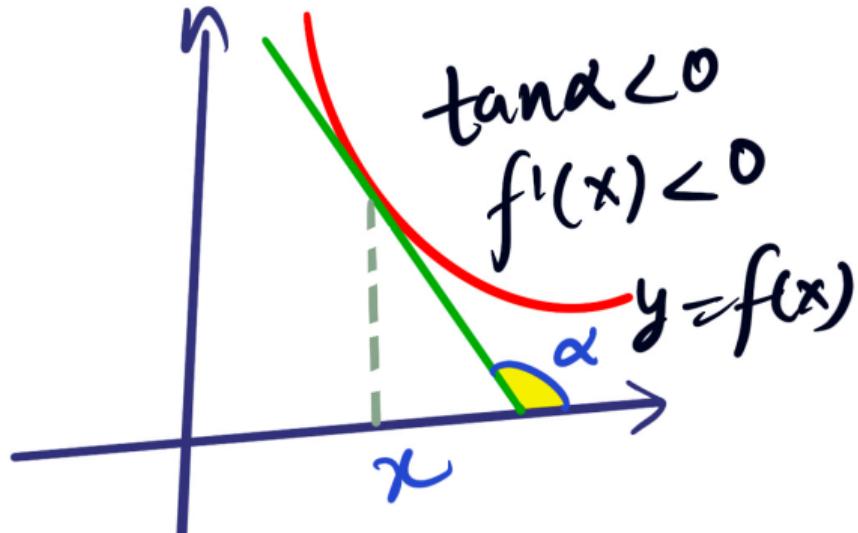
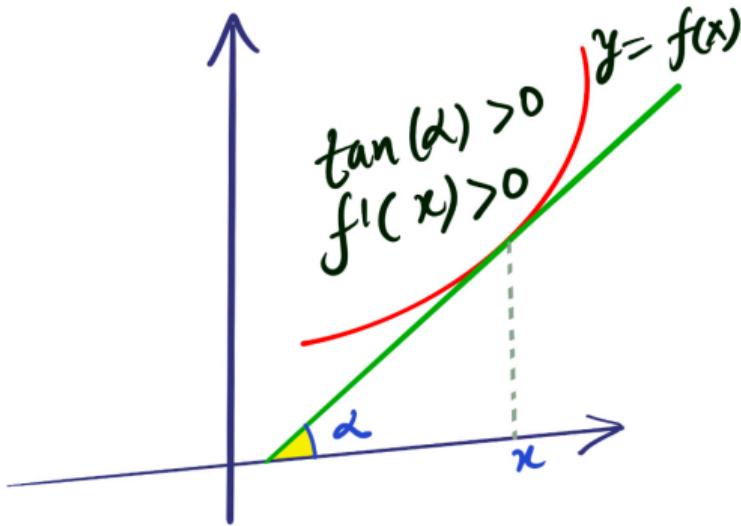
$$\begin{array}{l} \textcolor{red}{\checkmark} f'(x) = \frac{df(x)}{dx} \\ \textcolor{red}{\checkmark} f'(t) = \end{array}$$

$$\begin{array}{l} \textcolor{red}{\checkmark} \frac{d}{dx}(x^n) = nx^{n-1} \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\ln(x)) = \frac{1}{x} \\ \textcolor{red}{\checkmark} \frac{d}{dx}(e^x) = e^x \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\sin(x)) = \cos(x) \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\cos(x)) = -\sin(x) \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\tan(x)) = \sec^2(x) \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\cot(x)) = -\csc^2(x) \end{array}$$

$$\begin{array}{l} \textcolor{red}{\checkmark} \frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}} \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\sec(x)) = \sec(x)\tan(x) \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x) \\ \textcolor{red}{\checkmark} \frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{x^2+1} \\ \textcolor{red}{\checkmark} (F \cdot S)' = F \cdot S' + F' \cdot S \\ \textcolor{red}{\checkmark} \left(\frac{N}{D}\right)' = \frac{D \cdot N' - N \cdot D'}{D^2} \\ \textcolor{red}{\checkmark} [f(g(x))]' = f'(g(x)) \cdot g'(x) \end{array}$$

# Geometrical Interpretation of Derivatives

The derivative  $f'(x)$  can be interpreted as the slope of the tangent line at the point  $(x, y)$  on the graph of the function  $y = f(x)$ .



# Geometrical Interpretation of Derivatives

We can use the geometric interpretation to help study the behavior and the graph of  $f(x)$ .

- ⚡  $f'(x) > 0$  exactly when  $f(x)$  is increasing.
  - ⚡  $f'(x) < 0$  exactly when  $f(x)$  is decreasing.
  - ⚡  $f'(x) = 0$  exactly when  $f(x)$  has a horizontal tangent.
- 

In many applications, we want to find local maximum and local minimum values. In these applications, it makes sense to look at the locations on a graph where the slope is 0. In other words, we often use the following method to find all locations where the slope of the graph is 0.

- ⚡ Find the formula for the derivative:  $f'(x)$ .
- ⚡ Set this formula equal to 0 and solve for  $x$ .

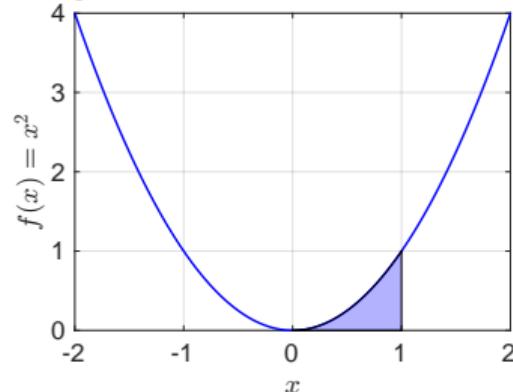
# Integration

**Integration is antiderivative but you can also understand it as an area under the curve.  
In other words, integration is the summation.**

If  $\frac{d}{dx}F(x) = f(x)$  then  $\int f(x)dx = F(x).$

# Geometric Interpretation of Integral

Integral gives the area under a curve



$$\int_0^1 x^2 dx$$

# Table of Integrals: Basic Form

$$\int x^n dx = \frac{1}{n+1}x^{n+1}$$

$$\int \frac{1}{x} dx = \ln x$$

$$\int u dv = uv - \int v du$$

$$\int u(x)v'(x) dx = u(x)v(x) - \int v(x)u'(x) dx$$

# Table of Integrals: More Functions

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax + b)$$

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, \quad n \neq -1$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}(nx+x-a)}{(n+2)(n+1)}$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$$

$$\int \frac{x dx}{a^2+x^2} = \frac{1}{2} \ln(a^2 + x^2)$$

$$\int \frac{x^2 dx}{a^2+x^2} = x - a \tan^{-1} \left( \frac{x}{a} \right)$$

$$\int \frac{x^3 dx}{a^2+x^2} = \frac{1}{2}x^2 - \frac{1}{2}a^2 \ln(a^2 + x^2)$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

$$\int \tan x dx = -\ln \cos x$$

# Differential Equations

**Differential equations are simply equations that have derivatives in them.**

Example:

⚡  $\frac{dy}{dx} + 5y = 0$

⚡  $y'' + 2y' + 10y = 0$

⚡  $y'' + 3y' + 2y = 3 \cos t$

The order of a differential equation is the order of the highest derivative in the equation. They are all examples of 'ordinary differential equations'.

# Solving Differential Equations

Consider

$$y' = 2y + 3$$

The Fundamental Theorem of Calculus implies  $y(t) = \int y'(t)dt$ , so we get

$$y(t) = 2 \int y(t)dt + 3t + C$$

This is not the closed form and doesn't entirely solve to find a solution  $y$ . We look at some formula (and their derivation) using the rules of differentiation on how to get a closed-form solution.

# Solving Differential Equations

The linear differential equation

$$y' = ay + b$$

with  $a \neq 0, b$  has infinitely many solutions

$$y(t) = ce^{at} - \frac{b}{a}$$

**Proof discussed in class.**

# Up Next

- ⚡ Representation of Signals: Continuous and Discrete
- ⚡ Continuous Signals
- ⚡ Using MATLAB