

# CPE 381: Fundamentals of Signals and Systems for Computer Engineers

03 Continuous-Time Signals

Fall 2025

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# Announcement

- ⚡ Homework 01 Due September 01 11:59 PM
- ⚡ Quiz 01, based on Chapter 01 Continuous-Time Signals from the Textbook. Available from September 05, 12:01 AM to September 07, 11:59 PM. 30 Questions, 45 Minutes.
- ⚡ Office hour: 08/28 Aug Wednesday, 1 PM - 3:30 PM.
- ⚡ No class on September 02, 2024: Labor Day, University Closed.

# Outline

## 1. Motivation

## 2. Operation on Signals

## 3. Basic Signals as Building Blocks

## 4. Modulation and Windowing



# Motivation

# Signals and Systems is 'Grandfather' of Data Science for Electrical and Computer Engineers

# Classification of Signals

We care about the following properties when dealing with signals:

- ⚡ Predictability: Random or Deterministic
- ⚡ Variations of time and amplitude: continuous, discrete (time or x-axis) / quantized (amplitude or y-axis)
- ⚡ Periodic/Aperiodic
- ⚡ Finite energy/finite power; Infinite energy/Infinite power



# Operation on Signals

# Basic Mathematical Operations

- ⚡ Addition:  $x(t) + y(t)$
- ⚡ Subtraction:  $x(t) - y(t)$
- ⚡ Constant multiplication:  $kx(t)$  where  $k$  is a constant



# Time-shift

⚡  $x(t - \tau) \rightarrow$  Signal is delayed

⚡  $x(t + \tau) \rightarrow$  Signal is advanced

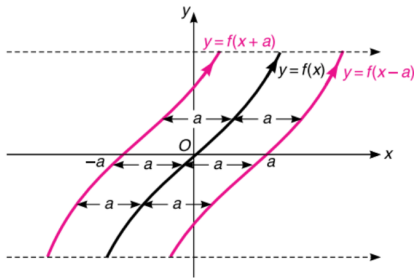
$f(x)$  transforms to  $f(x - a)$

i.e.,  $f(x) \longrightarrow f(x - a)$ ;  $a$  is positive. Shift the graph of  $f(x)$  through ' $a$ ' unit towards right

$f(x)$  transforms to  $f(x + a)$ .

i.e.,  $f(x) \longrightarrow f(x + a)$ ;  $a$  is positive. Shift the graph of  $f(x)$  through ' $a$ ' units towards left.

Graphically it could be stated as



# Time Reflection

Note: The book doesn't specify whether to take the mirror image along the y-axis or not and it is confusing because the signal used in example 1.3.1 is symmetric with respect to both the x and y axes.

$f(x)$  transforms to  $f(-x)$

i.e.,

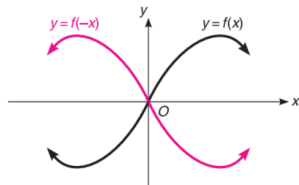
$$f(x) \longrightarrow f(-x)$$

To draw  $y = f(-x)$ , take the image of the curve  $y = f(x)$  in y-axis as plane mirror.

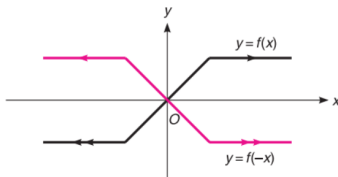
OR

“Turn the graph of  $f(x)$  by  $180^\circ$  about y-axis.”

Graphically it is stated as;



OR



⚡  $x(t) \rightarrow x(-t)$  : take mirror image **along the y-axis**

# Signal Stretching along $y$ -axis

⚡  $f(x) \rightarrow af(x)$ ;  $a > 1$  : Stretch the graph of  $f(x)$  ' $a$ ' times along  $y$ -axis.

⚡  $f(x) \rightarrow \frac{1}{a}f(x)$ ;  $a > 1$  : Shrink the graph of  $f(x)$  ' $a$ ' times along  $y$ -axis.

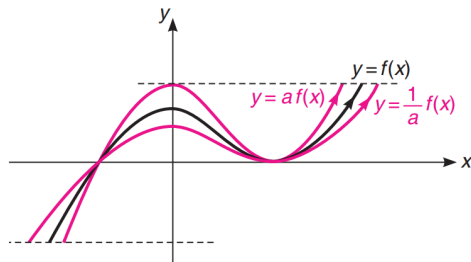
$f(x)$  transforms to  $a f(x)$

i.e.,  $f(x) \longrightarrow af(x)$ ;  $a > 1$

**Stretch** the graph of  $f(x)$  ' $a$ ' times along  $y$ -axis.

$$f(x) \longrightarrow \frac{1}{a} f(x); a > 1.$$

**Shrink** the graph of  $f(x)$  ' $a$ ' times along  $y$ -axis.



# Signal Stretching along x-axis

⚡  $f(x) \rightarrow af(ax)$ ;  $a > 1$  : Stretch the graph of  $f(x)$  'a' times along x-axis.

⚡  $f(x) \rightarrow f\left(\frac{1}{a}x\right)$ ;  $a > 1$  : Shrink the graph of  $f(x)$  'a' times along x-axis.

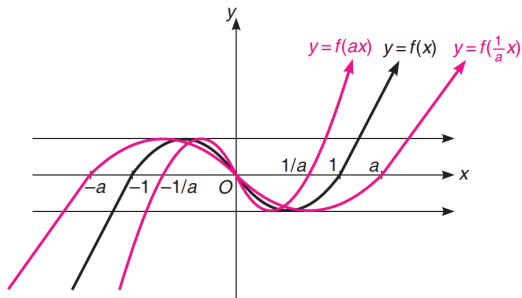
$f(x)$  transforms to  $f(ax)$

i.e.,  $f(x) \longrightarrow f(ax)$ ;  $a > 1$

**Shrink** (or contract) the graph of  $f(x)$  'a' times along x-axis.

again  $f(x) \longrightarrow f\left(\frac{1}{a}x\right)$ ;  $a > 1$

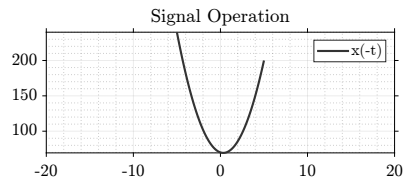
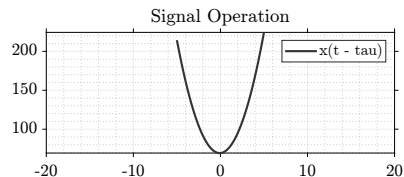
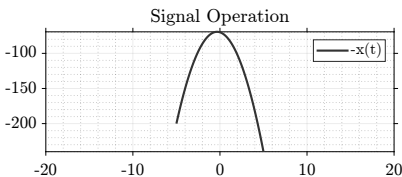
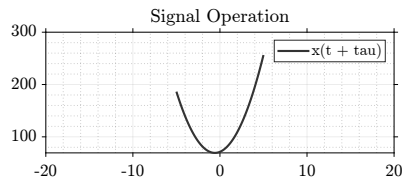
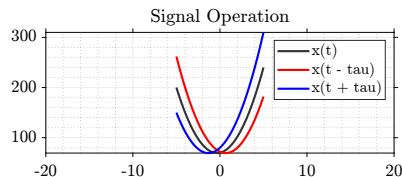
**Stretch** (or expand) the graph of  $f(x)$  'a' times along x-axis.



# Example and MATLAB Code

$$x(t) = 6t^2 + 4t + 70$$

Code: [https://github.com/rahulbhadani/CPE381\\_FA25/blob/main/Code/signal\\_operation.m](https://github.com/rahulbhadani/CPE381_FA25/blob/main/Code/signal_operation.m)



# Even and Odd Signals

- ⚡ Even Signal:  $x(t) = x(-t)$
- ⚡ Odd Signal:  $x(t) = -x(-t)$
- ⚡ Any signal can be represented by the sum of even and odd signals  
 $y(t) = y_e(t) + y_o(t)$

$$y_e(t) = 0.5[y(t) + y(-t)]$$

$$y_o(t) = 0.5[y(t) - y(-t)]$$

# Periodic Signals

- ⚡ Defined for all possible values of  $t$ ,  $-\infty < t < \infty$ .
- ⚡ There is the real value  $T_0 \in \mathbb{R}^+$ , called the fundamental frequency such that  $x(t + kT_0) = x(t)$ ,  $k \in \mathbb{I}$ .
- ⚡ A constant signal is periodic of a non-definable fundamental period.
- ⚡  $A \cos(\omega t + \theta)$ ,  $\omega = 2\pi/T_0$ ,  $\omega = 2$ ,  $\theta = -\pi/2$ ,  $A = 2$ .

**What's the fundamental frequency,  $1/T_0$ ?**

# Energy and Power of Signals

**What's the instantaneous power of a resistor?**

**⚡ Energy:**

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

**⚡ Power:**

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

**A signal is called finite power if the signal power is finite.**





# Basic Signals as Building Blocks

# Complex Exponentials

Consider  $A = |A|e^{j\theta}$ ,  $a = r + j\Omega_0$

⚡  $x(t) = Ae^{at} = \dots$

⚡ Real part  $f(t) = \operatorname{Re}\{x(t)\}, = \dots$

$-|A|e^{rt} \leq f(t) \leq |A|e^{rt}$ .  $r < 0$ ,  $f(t)$  is damped,  $r > 0$ ,  $f(t)$  grows.

⚡ Imaginary part  $g(t) = \operatorname{Im}\{x(t)\}, = \dots$

# Sinusoids

A sinusoid of the general form:

$$A \cos(\Omega_0 t + \theta) = A \sin(\Omega_0 t + \theta + \pi/2), \quad -\infty < t < \infty$$

- ⚡  $A$  is the amplitude
- ⚡  $\Omega_0 = 2\pi f_0$  is angular frequency in rad/s.
- ⚡  $\theta$  is phase shift
- ⚡ Fundamental period  $T_0$  is

$$T_0 = \frac{2\pi}{\Omega_0} = \frac{1}{f_0}$$

# Rectangular pulse and Unit impulse

⚡ A rectangular pulse of duration  $\Delta$  and unit area:

$$p_{\Delta}(t) = \begin{cases} \frac{1}{\Delta}, & -\Delta/2 \leq t \leq \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$

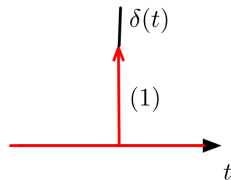
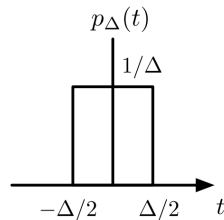
⚡ Unit Impulse:

$$\delta(t) = \lim_{\Delta \rightarrow 0} p_{\Delta}(t)$$

⚡ Calculate

$$\int_{-\infty}^t p_{\Delta}(t) dt$$

**Hint: there are three possible cases.**



# Unit Step

⚡ Integration of rectangular pulse:

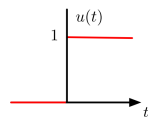
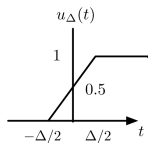
$$u_{\Delta}(t) = \int_{-\infty}^t p_{\Delta}(t) = \begin{cases} 1, & t \geq \frac{\Delta}{2} \\ \frac{1}{\Delta}(t + \frac{\Delta}{2}), & \frac{\Delta}{2} \leq t \leq \frac{\Delta}{2} \\ 0, & t < -\frac{\Delta}{2} \end{cases}$$

⚡ Limit case:

$$\lim_{\Delta \rightarrow 0} u_{\Delta}(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

⚡ A common case is to ignore  $t = 0$  case, which gives us unit step function as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



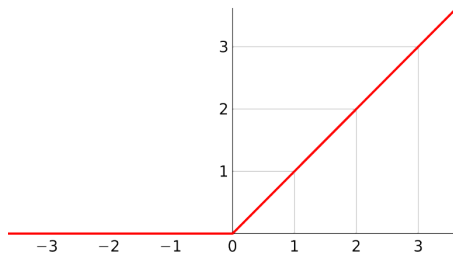
# Ramp Signal

The ramp signal is  $r(t) = tu(t)$

⚡ The relation between the ramp, the unit step, and the unit impulse:

$$\frac{dr(t)}{dt} = u(t)$$

$$\frac{d^2r(t)}{dt^2} = \delta(t)$$



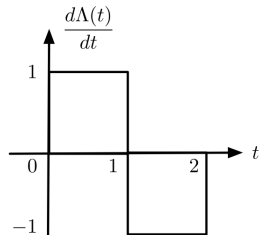
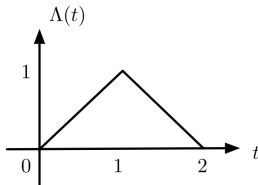
# Triangular Pulse

The triangular pulse is

$$\Lambda(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ -t + 2, & 1 < t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

⚡  $\Lambda(t)$  can also be written as:

$$\Lambda(t) = r(t) - 2r(t - 1) + r(t - 2)$$



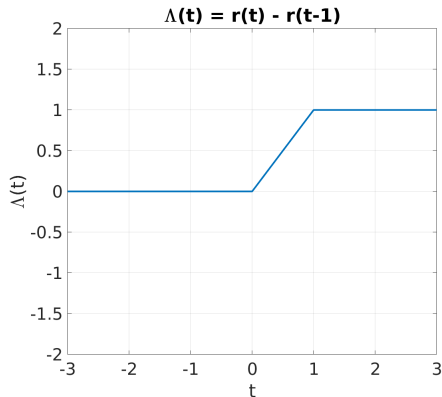
# Triangular Pulse as Ramp Functions I

Let's verify this by evaluating  $\Lambda(t)$  at different intervals:  
First, find out the first part:

$$\Lambda_1(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

which can be written as:  
can be written using the ramp function  $r(t)$   
as:

$$\Lambda_1(t) = r(t) - r(t - 1)$$





# Triangular Pulse as Ramp Functions II

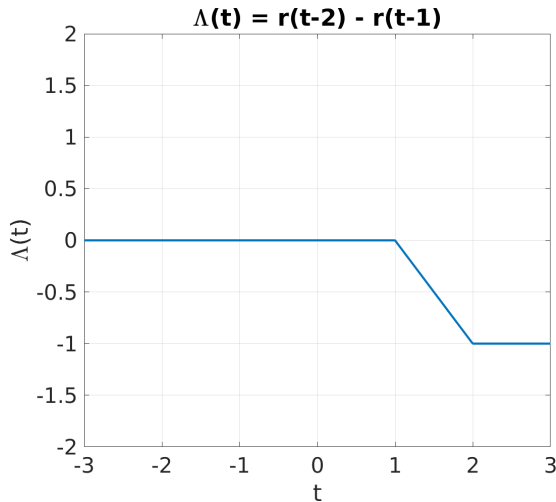
Second part:

$$\Lambda_1(t) = \begin{cases} -t + 2, & 1 < t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

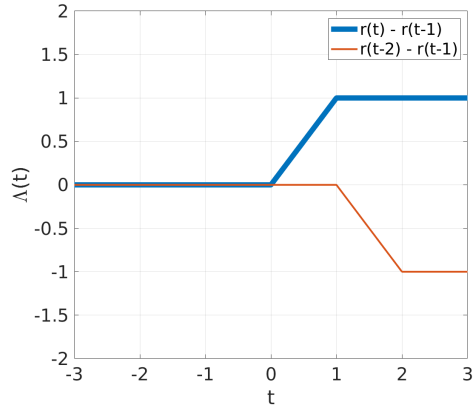
which can be written as:  
can be written using the ramp function  $r(t)$   
as:

$$\Lambda_1(t) = r(t - 2) - r(t - 1)$$

Putting together



# Triangular Pulse as Ramp Functions III



# Sifting Property

The product of  $f(t)$  and  $\delta(t)$  gives zero everywhere except at the origin where we get an impulse of area  $f(0)$ , that is,  $f(t)\delta(t) = f(0)\delta(t)$ .

Hence,

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = \int_{-\infty}^{\infty} f(0)\delta(t)dt = f(0) \int_{-\infty}^{\infty} \delta(t)dt = f(0)$$

This is called **Sifting Property**.

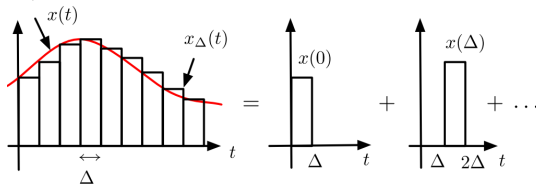
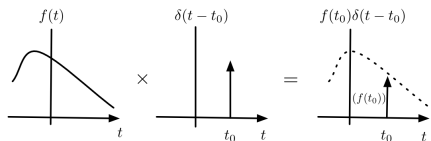
If we delay or advance the  $\delta(t)$  function in the integrated, the result is that all values of  $f(t)$  are sifted out except for the value corresponding to the location of the delta function, that is,

$$\int_{-\infty}^{\infty} f(t)\delta(t - \tau)dt = \int_{-\infty}^{\infty} f(\tau)\delta(t - \tau)dt = f(\tau) \int_{-\infty}^{\infty} \delta(t - \tau)dt = f(\tau) \quad \text{for any } \tau$$

# Generic Representation of Signals

Hence, if we do integration in terms of variable  $\tau$ , we get a generic representation of signals in terms of impulse and shifted impulse.

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$



⚡ Approximation of  $x(t)$ :

$$x_{\Delta}(t) = \sum_{-\infty}^{\infty} x_{\Delta}(t - k\Delta) = \sum_{-\infty}^{\infty} x(k\Delta) p_{\Delta}(t - k\Delta) \Delta$$

In the limit as  $\Delta \rightarrow 0$  these pulses become impulses, separated by an infinitesimal value:

$$\lim_{\Delta \rightarrow 0} x_{\Delta}(t) \rightarrow x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$



# Modulation and Windowing

# Modulation

Multiplication by a complex exponential shifts the frequency of the original signal.

## Definition

Superimposing a low-frequency signal on a high-frequency carrier signal is called **Modulation**.

### Example:

Consider an exponential signal  $x(t) = e^{j\Omega_0 t}$  of frequency  $\Omega_0$ . If we multiply an exponential  $e^{j\phi t}$  with  $x(t)$ , then:

$$x(t)e^{j\phi t} = e^{j(\Omega_0 + \phi)t} = \cos((\Omega_0 + \phi)t) + j \sin((\Omega_0 + \phi)t)$$

$\phi > 0$  : the frequency of new exponential is greater than  $\Omega_0$ , otherwise lower.

# Various Types of Modulation

$$A(t)\cos(\Omega(t)t + \theta(t))$$

⚡  $A(t)$  changes: Amplitude Modulation

⚡  $\Omega(t)$  changes: Frequency Modulation

⚡  $\theta(t)$  changes: Phase Modulation

# Windowing

For a window signal  $w(t)$ , the time-windowed signal  $x(t)w(t)$  displays  $x(t)$  within the support of  $w(t)$ .

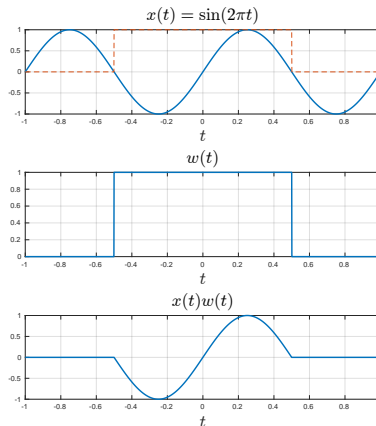
**Example:**

$$x(t) = \sin(2\pi t)$$

$$w(t) = \begin{cases} 1 & \text{if } -0.5 \leq t \leq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Code for the graph:

[https://github.com/rahulbhadani/CPE381\\_FA25/blob/main/Code/windowing.m](https://github.com/rahulbhadani/CPE381_FA25/blob/main/Code/windowing.m)





# Classwork

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# Classwork

# Up Next

## ⚡ Continuous-time Systems

- Linear-Time Invariance
- Static vs Dynamic Systems
- Convolutional Integral
- BIBO Stability