CPE 381: Fundamentals of Signals and Systems for Computer Engineers

01 Introduction and Preliminaries

Fall 2025

Rahul Bhadani

Outline

About Me

Rahul Bhadani
Assistant Professor
Electrical and Computer Engineering
The University of Alabama in Huntsville
Office: 217-H
Email: rahul.bhadani@uah.edu
Web: `https://rahulbhadani.github.io/`

Research Interests

Cyber-physical Systems, Intelligent Transportation, Connected-and-Autonomous Driving, Applied machine learning, Quantum Information Science

Course Logistics

Lecture:
M/W 11:20 AM - 12:40 PM

Location:
TBD

Prerequisites:

- EE 213 - Electrical Circuit Analysis I
- MA 238 – Applied Differential Equations

Office Hours:

- TBD

Instructor Email: rahul.bhadani@uah.edu

Textbooks

Required: *Signals and Systems using Matlab*. Luis F. Chaparro.
Elsevier, 3rd Edition, 2019.
ISBN: 978-0-12-814204-2, eBook ISBN: 9780128142059

Suggested Reading: *Linear Systems and Signals*, B. P. Lathi, Roger Green, Oxford University Press, 2017, 3rd Edition
Other Notable Textbooks:

- Schaum's Outline of Signals and Systems
- *Signals and Systems*. Haykin, Simon, and Barry Van Veen. John Wiley & Sons, 2007.
- Asadi, Farzin. Signals and Systems with MATLAB and Simulink. Springer, Dec 2023. ISBN: 9783031456220, 303145622X

Grading

Homework:	20%
Quizzes:	5%
Attendance/In-Class Participation:	15%
Mid-term Exam 1:	15%
Mid-term Exam 2:	15%
Final Exam:	30%

Grading Scale	
Percentage	**Grade**
90% - 100%	A
75% - 89%	B
60% - 74%	C
45% - 59%	D
0% - 44%	F

Homework Policy

- Each late submission will be penalized by 10% per day for up to 5 days maximum, thereafter, if later, one will receive 0 credit.
- Solution to homework will be posted 5 days after the due date.

Classwork

- Each lecture will be followed by in-class problem-solving that students will turn in the next lecture day. If you miss the lecture day (either the day it is handed to you, or the day you need to turn in), you will not receive any credit.
- There will be intermediate small tests to assess your skills based on lectures and the classwork. This portion will count towards your classwork credits.

Attendance Policy

- Must attend all lectures.
- Two unexcused absences permitted.
- No option to make up for classwork.

Exam Schedule

- **Mid Term 1:** September 29, Monday
- **Mid Term 2:** November 10, Monday
- **Final Exam:** December 12, Friday

Tentatative Topics

- Introduction, Mathematical Preliminaries
- Continuous and Discrete Signals
- Linear-time Invariant (LTI) Systems
- Laplace Transform
- Fourier Series for Frequency Analysis
- Fourier Transform
- Sampling Theory
- Discrete-Time Signals and Systems
- Z-transform
- Discrete Fourier Analysis

Refer to the syllabus for the detailed information on the syllabus.

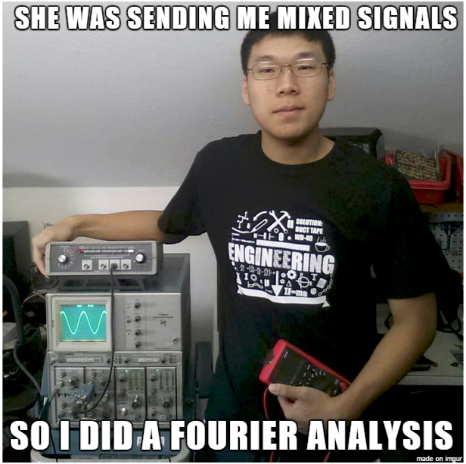
In-Class Activity

Introduce Yourself

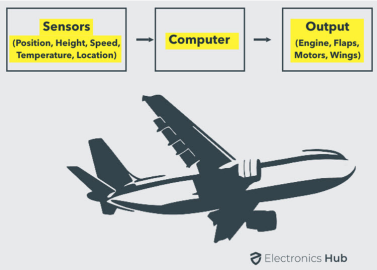
- Why Computer Engineering?
- Why do you want to take this course?
- What is your favorite Pokemon?

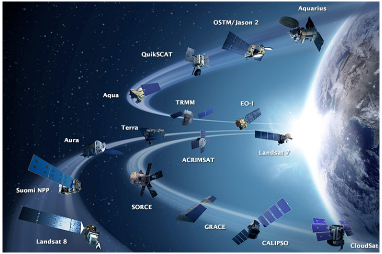
Motivation

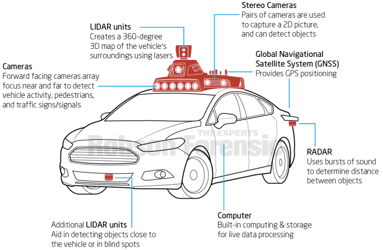
Why Study Signals and Systems



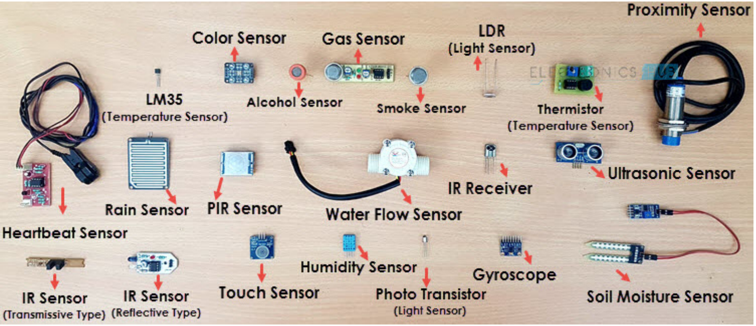
We are in digital era



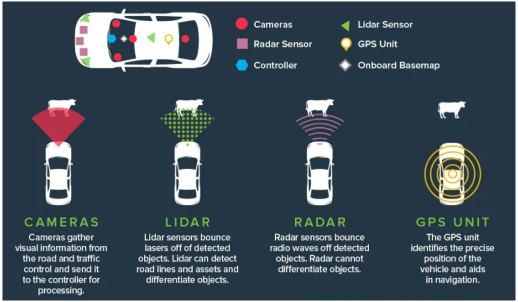




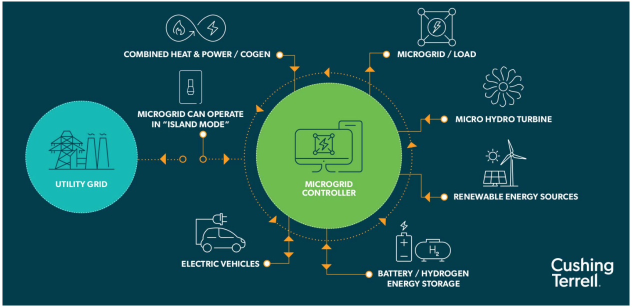
Sensors and Mobile Devices



Cyber-physical Systems



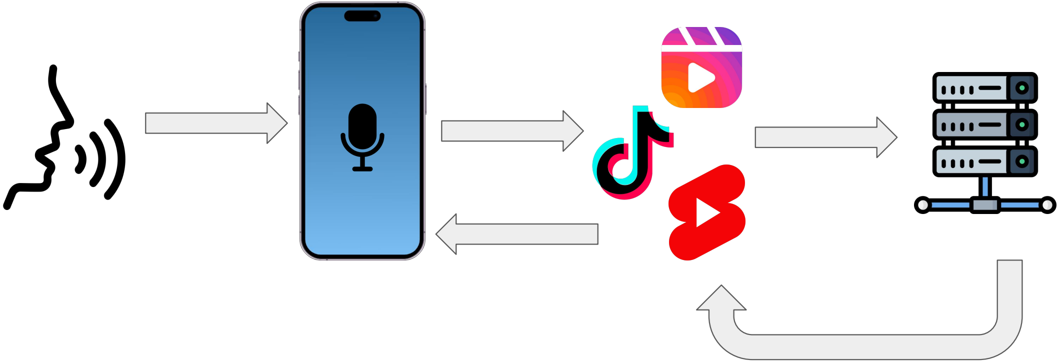
Connected and Autonomous Vehicles



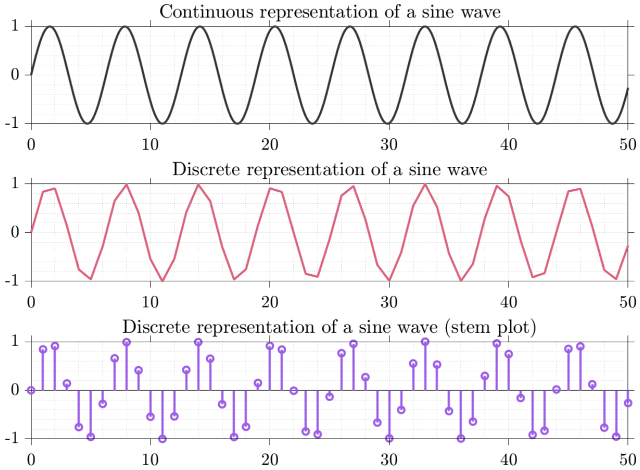
Microgrid

Signals in Nature

Everything in nature is analog and continuous. We use hardware/software interfaces to digitize them, extract information, make inferences, and send them back to the user.



Sampling continuous time signals



$x[n] = x(nT_s)$, T_s = sample time.

Code for the figure: `https://github.com/rahulbhadani/CPE381_FA25/blob/main/Code/sampled_sine_wave.m`

Inherent Discrete Time Signals

Nature may be continuous but human activities naturally lead to some discrete time signals.

Examples:

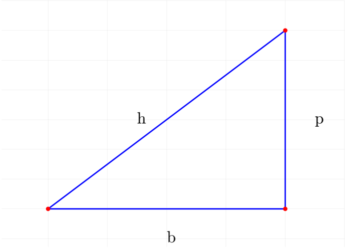
- Stock market closing data
- Adaptive cruise control states
- Thermostat setpoints

Mathematical Preliminaries

Trigonometry

- **Sine, Cosine, Tangent:**
  - $\sin(\theta) = \dfrac{\mathsf{p}}{\mathsf{h}}$
  - $\cos(\theta) = \dfrac{\mathsf{b}}{\mathsf{h}}$
  - $\tan(\theta) = \dfrac{\mathsf{p}}{\mathsf{b}}$
- **Pythagorean Identity:**

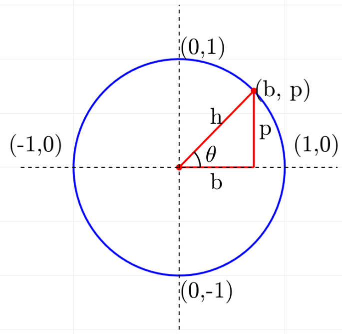
$$\sin^2(\theta) + \cos^2(\theta) = 1$$



- **Unit Circle:**

Trigonometric Identities

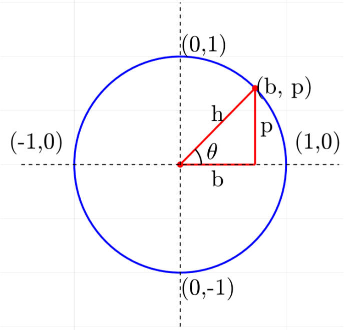
What is the value of h based on the diagram?



- $\cos\theta =$
- $\sin\theta =$
- $\sin(\alpha+\beta) =$
- $\cos(\alpha+\beta) =$
- $\sin^2(\theta) =$ in terms of $\cos(2\theta)$
- $\cos^2(\theta) =$ in terms of $\cos(2\theta)$
- $\sin 2\theta =$ in terms of θ
- $\cos 2\theta =$ in terms of θ
- $\sin(-\theta) =$
- $\cos(-\theta) =$

Trigonometric Identities

What is the value of h based on the diagram?



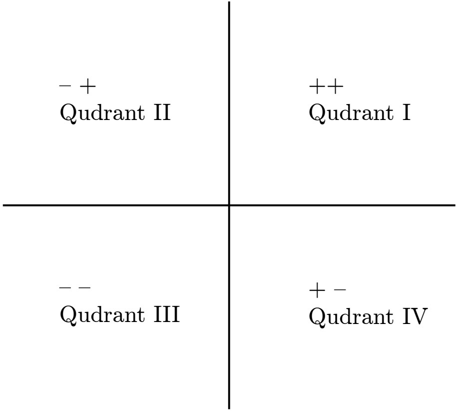
- $\tan(\alpha + \beta) =$
- $\tan(\alpha - \beta) =$
- $\sin 30^\circ =$
- $\sin 45^\circ =$
- $\sin 60^\circ =$

- $\cos 30^\circ =$
- $\cos 45^\circ =$
- $\cos = 60^\circ =$
- $\tan 30^\circ =$
- $\tan 45^\circ =$
- $\tan = 60^\circ =$

What's the range of $\sin\theta$ and $\cos\theta$?

Trigonometric Functions in Different Quadrant

Four Quadrants:



Considering a unit circle, in four different quadrants (x, y) signs are different, Hence the trigonometric ratios are also different in signs.

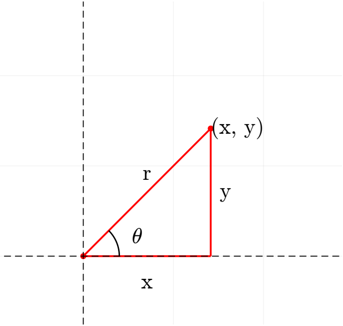
Quadrant	$\sin$	$\cos$	$\tan$
I	+	+	+
II	+	-	-
III	-	-	+
IV	-	+	-

Table: Signs of $\sin$, $\cos$, and $\tan$ in all four quadrants

Complex Number

$$z = x + iy$$

$$i^2 = -1$$



Two representations of complex numbers:

$$z = re^{i\theta}$$

Conjugate: $\bar{z} = re^{-i\theta}$

- $r = \sqrt{x^2 + y^2}$
- $\theta = \tan^{-1} \dfrac{y}{x}$

Euler's Identity:

- $e^{i\theta} = \cos\theta + i \sin\theta$
- $e^{-i\theta} =$
- $\cos\theta =$ (in terms of exponential)
- $\sin\theta =$ (in terms of exponential)

Complex Number

Properties of Conjugates

- $\overline{z+w} = \overline{z} + \overline{w}$
- $\overline{zw} = \overline{zw}$
- $\overline{z^n} = \overline{z}^n$
- $z\overline{z} = |z|^2$ **Prove it!**

De Moivre's Theorem If $z = r(\cos\theta + i\sin\theta)$ and n is a positive integer, then

$$z^n = [r(\cos\theta + i\sin\theta)^n] = r^n(\cos n\theta + i\sin n\theta)$$

Roots of a complex number $z = 0$ has n distinct roots:

$$w_k = r^{1/n}\left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right]$$

Complex Exponentials

We also need to give a meaning to the expression e^z when $z = x + iy$ is a complex number.
Based on the infinite series using Taylor's series expansion for e^x, we have

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots$$

Can you find out, what should be the value of e^{iy} where y is the real number?

Note: $\cos y = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \frac{y^6}{6!} + \cdots$ and $\sin y = y - \frac{y^3}{3!} + \frac{y^5}{5!} - \cdots$

Infinite Series

Sum of an Infinite Sequence

An infinite series is the sum of an infinite sequence of numbers:

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

The goal is to understand the meaning of such an infinite sum and develop methods to calculate it.

Partial Sums

The n-th partial sum is:

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

As n gets larger, the partial sums get closer to a limiting value.

Example 1

Consider the series:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

The partial sums are:

$$S_1 = 1$$

$$S_2 = 1 + \frac{1}{2} = \frac{3}{2}$$

$$S_3 = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$$

$$S_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} = \frac{2^n - 1}{2^{n-1}}$$

Convergence of the Series

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{2^n - 1}{2^{n-1}} = \lim_{n\to\infty} \left(\frac{2^n}{2^{n-1}} - \frac{1}{2^{n-1}} \right) = \lim_{n\to\infty} \left(2 - \frac{1}{2^{n-1}} \right) 2$$

Since the sequence of partial sums converges, the infinite series converges. That is to say:

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 2$$

Arithmetic Progression (AP)

An artihmetic series is of the form: $a, (a+d), (a+2d), \cdots (a+(n-1)d)$.
The sum can be written as:

$$S_n = a + (a+d) + (a+2d) + \cdots + (a+(n-1)d) = \frac{n}{2}[2a + (n-1)d]$$

$a =$ The first term, $d =$ common difference.

Geometric Series/Geometric Progression

A geometric series is of the form: $a, ar, ar^2, \cdots ar^n$.
The sum can be written as:

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^n + \cdots = \sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^n$$

$a =$ The first term, $r =$ common ratio.

Convergence of Geometric Series

If $|r| < 1$, the geometric series converges:

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

If $|r| \geq 1$, the series diverges.

Example 2

Consider the series:

$$\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$$

This is a geometric series with $a = 5$ and $r = -\frac{1}{4}$. Since $|r| < 1$, the series converges:

$$\sum_{n=1}^{\infty} 5\left(-\frac{1}{4}\right)^{n-1} = \frac{5}{1 - \left(-\frac{1}{4}\right)} = 4$$

Example 3: Telescoping Series

Find the sum of the telescoping series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

The partial sums are:

$$S_n = 1 - \frac{1}{n+1} \to 1 \text{ as } n \to \infty$$

Theorem

If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

Divergence Test

The series $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

Practice Problems

Determine whether the following series converge or diverge. In the case of convergence, give the sum of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{2^n}$
2. $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$
3. $\sum_{n=2}^{\infty} \frac{n^2}{n^2-1}$
4. $\sum_{n=1}^{\infty} \frac{n(n+2)}{(n+3)^2}$
5. $\sum_{n=1}^{\infty} \frac{1+2n}{3^n}$

Derivatives

Rate of change of a quantity with respect to another quantity.
In signals and systems, we are usually interested in the rate of change with respect to time.

$$f'(t) = \frac{df(t)}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

In computer implementation, h is Δt. Δt is the difference of timestamps between two consecutive samples of a signal.

Derivative Review

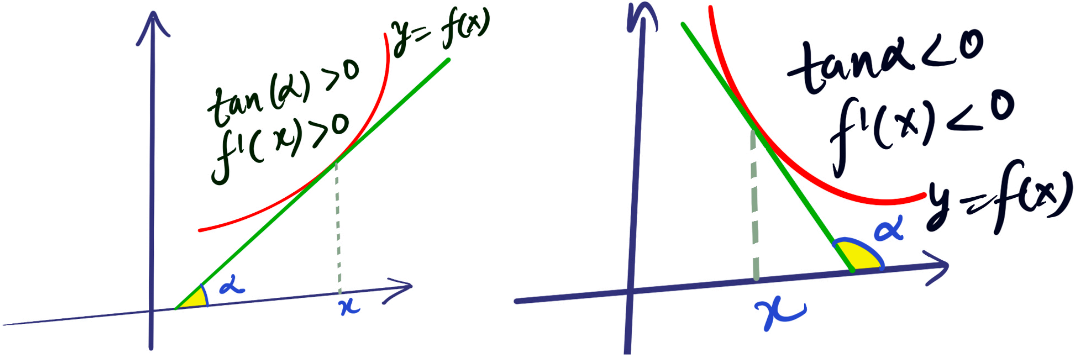
- $f'(x) = \dfrac{df(x)}{dx}$
- $f'(t) =$

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\sin(x)) = \cos(x)$
- $\frac{d}{dx}(\cos(x)) = -\sin(x)$
- $\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
- $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$

- $\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
- $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
- $\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{x^2+1}$
- $(F \cdot S)' = F \cdot S' + F' \cdot S$
- $\left(\frac{N}{D}\right)' = \frac{D \cdot N' - N \cdot D'}{D^2}$
- $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

Geometrical Interpretation of Derivatives

The derivative $f'(x)$ can be interpreted as the slope of the tangent line at the point (x, y) on the graph of the function $y = f(x)$.



Geometrical Interpretation of Derivatives

We can use the geometric interpretation
to help study the behavior and the graph of $f(x)$.

- $f'(x) > 0$ exactly when $f(x)$ is increasing.
- $f'(x) < 0$ exactly when $f(x)$ is decreasing.
- $f'(x) = 0$ exactly when $f(x)$ has a horizontal tangent.

In many applications, we want to find local maximum and local minimum values. In these applications, it makes sense to look at the locations on a graph where the slope is 0. In other words, we often use the following method to find all locations where the slope of the graph is 0.

- Find the formula for the derivative: $f'(x)$.
- Set this formula equal to 0 and solve for x.

Taylor Series

Function approximation:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

When $a = 0$, the series is called Maclaurin series.
Signal processing examples:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$
$$\cos(\omega t) = 1 - \frac{(\omega t)^2}{2!} + \frac{(\omega t)^4}{4!} - \cdots$$
$$\ln(1+t) = t - \frac{t^2}{2} + \frac{t^3}{3} - \cdots \quad (|t| < 1)$$

Taylor Series Applications

Small-angle approximation:
$\sin\theta \approx \theta - \frac{\theta^3}{6}$ for $\theta \ll 1$
Error at $\theta = 0.1$ rad: $|\text{Actual} - \text{Approx}| < 0.001\%$

Nonlinear system analysis:
$y(t) = e^{x(t)} \approx 1 + x(t) + \frac{x^2(t)}{2}$
When $x(t) = 0.2\cos(2\pi ft)$,
$y(t) \approx 1 + 0.2\cos(2\pi ft) + 0.02\cos^2(2\pi ft)$

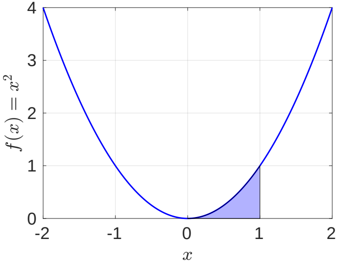
Integration

Integration is antiderivative but you can also understand it as an area under the curve. In other words, integration is the summation.

If $\dfrac{d}{dx}F(x) = f(x)$ then $\displaystyle\int f(x)dx = F(x)$.

Geometric Interpretation of Integral

Integral gives the area under a curve



$$\int_0^1 x^2 dx$$

Table of Integrals: Basic Form

- $\int x^n \, dx = \frac{1}{n+1} x^{n+1}$
- $\int \frac{1}{x} \, dx = \ln x$
- $\int u \, dv = uv - \int v \, du$
- $\int u(x) v'(x) \, dx = u(x) v(x) - \int v(x) u'(x) \, dx$

Table of Integrals: More Functions

- $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b)$
- $\int \frac{1}{(x+a)^2}\,dx = -\frac{1}{x+a}$
- $\int (x+a)^n\,dx = \frac{(x+a)^{n+1}}{n+1}, \quad n \neq -1$
- $\int x(x+a)^n\,dx = \frac{(x+a)^{n+1}(nx+x-a)}{(n+2)(n+1)}$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x$
- $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
- $\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln\left(a^2+x^2\right)$
- $\int \frac{x^2\,dx}{a^2+x^2} = x - a\tan^{-1}\left(\frac{x}{a}\right)$
- $\int \frac{x^3\,dx}{a^2+x^2} = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln\left(a^2+x^2\right)$
- $\int e^{ax} = \dfrac{1}{a}e^{ax}$
- $\int \sin x dx = -\cos x$
- $\int \cos x dx = \sin x$
- $\int \tan x dx = -\ln\cos x$

Differential Equations

Differential equations are simply equations that have derivatives in them.

Example:

- $\dfrac{dy}{dx} + 5y = 0$
- $y'' + 2y' + 10y = 0$
- $y'' + 3y' + 2y = 3\cos t$

The order of a differential equation is the order of the highest derivative in the equation. They are all examples of ‘ordinary differential equations’.

Solving Differential Equations

Consider

$$y' = 2y + 3$$

The Fundamental Theorem of Calculus implies $y(t) = \int y'(t)dt$, so we get

$$y(t) = 2\int y(t)dt + 3t + C$$

This is not the closed form and doesn't entirely solve to find a solution y. We look at some formula (and their derivation) using the rules of differentiation on how to get a closed-form solution.

Solving Differential Equations

The linear differential equation

$$y' = ay + b$$

with $a \neq 0, b$ has infinitely many solutions

$$y(t) = ce^{at} - \frac{b}{a}$$

Proof discussed in class.

Up Next

- Representation of Signals: Continuous and Discrete
- Continuous Signals
- Using MATLAB