

# Automatic Differentiation

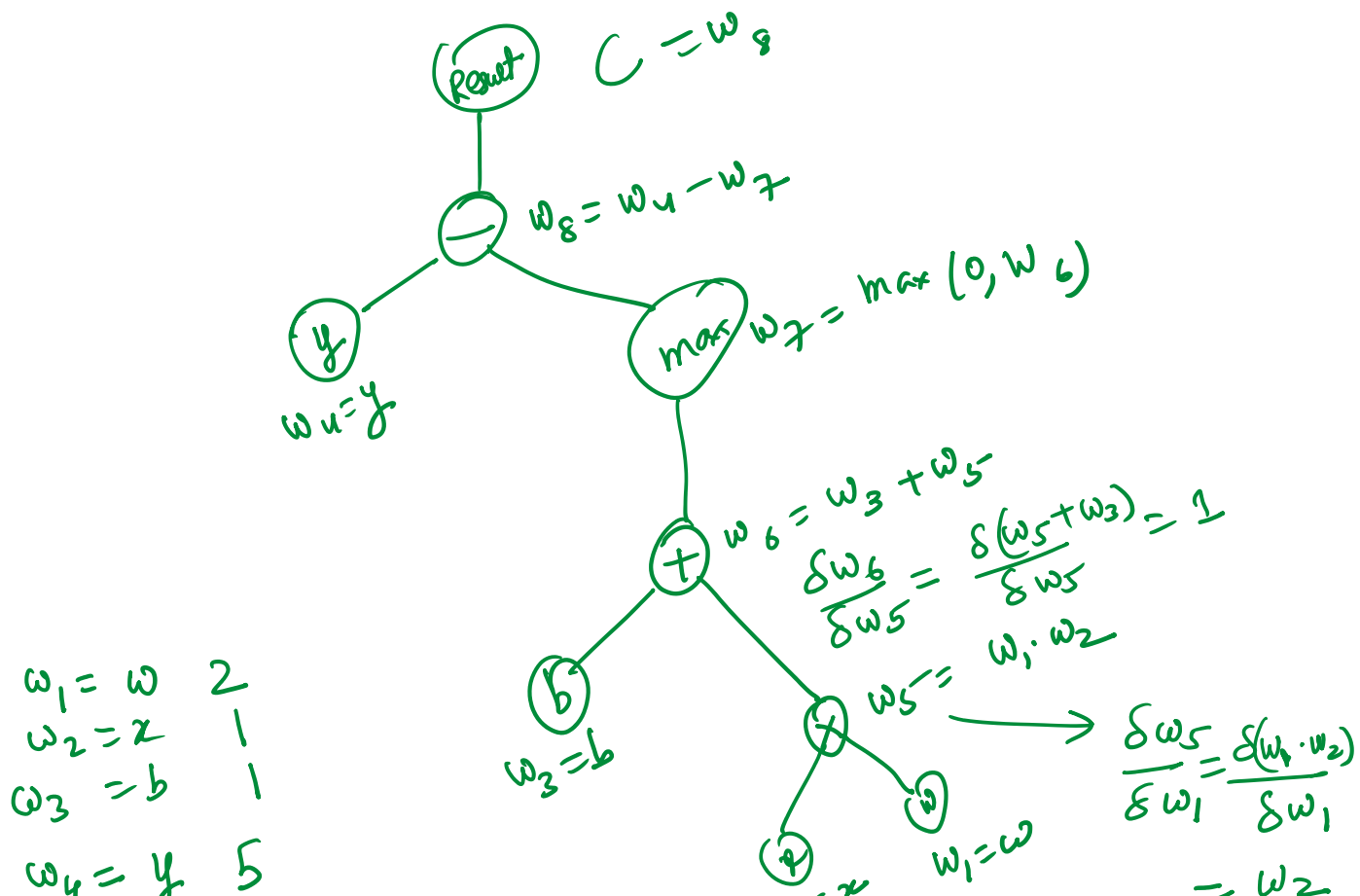
Thursday, February 8, 2024 12:43 PM

$$C(y, wx+b) = y - \max(0, wx+b)$$

Automatic differentiation will only calculate the partial derivative of an expression on a certain point

$$y=5 \quad w=2 \quad x=1 \quad b=1$$

we construct an expression graph



$w_5$

$$w_4 = 4 \quad 5$$

$$w_5 = w_1 \cdot w_2 = 2$$

$$w_6 = w_5 + w_3 = 3$$

$$w_7 = \max(0, w_6) = 3$$

$$w_8 = w_4 - w_7 = -2$$

$$C = w_8 = -2$$

$$w_2 = x$$

$$w_1 = w$$

$$0 \cdot w_1 \quad \delta w_1 = w_2$$

$$\frac{\delta w_5}{\delta w_1} = \frac{\delta(w_1 w_2)}{\delta w_1} = w_2$$

$$\frac{\delta w_5}{\delta w_2} = \frac{\delta(w_1 w_2)}{\delta w_2} = w_1$$

$$\frac{\delta w_6}{\delta w_5} = \frac{\delta(w_5 + w_3)}{\delta w_5} = 1$$

$$\frac{\delta w_7}{\delta w_6} = \frac{\delta \max(0, w_6)}{\delta w_6} = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$$

$$\frac{\delta w_8}{\delta w_7} = \frac{\delta(w_4 - w_7)}{\delta w_7} = -1$$

$$\frac{\delta w_8}{\delta w_4} = \frac{\delta(w_4 - w_7)}{\delta w_4} = 1$$

$$\frac{\delta C}{\delta w_8} = 1$$

$$\frac{\delta C}{\delta w_1} = \frac{\delta C}{\delta w_8} \times \frac{\delta w_8}{\delta w_7} \times \frac{\delta w_7}{\delta w_6} \times \frac{\delta w_6}{\delta w_5} \times \frac{\delta w_5}{\delta w_1}$$

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