

Minkowski Sum - An Example*

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1 DEFINITION

The Minkowski sum of the two sets \mathbf{A} and \mathbf{B} is a set with the sum of all elements from \mathbf{A} and all elements of \mathbf{B} , denoted by $\mathbf{A} \oplus \mathbf{B}$ is written as

$$\mathbf{A} \oplus \mathbf{B} = \{a + b \mid a \in \mathbf{A}, b \in \mathbf{B}\} \quad (1)$$

2 EXAMPLE

Let \mathbf{A} is a rectangle $[0, 2] \times [0, 1]$ or $\{(0, 0), (0, 1), (2, 1), (2, 0)\}$ and \mathbf{B} be a line segment given by two point $\{(0, 0), (2, 3)\}$. Then the Minkowski sum is given by

$$\begin{aligned} (0, 0) + (0, 0) &= (0, 0) \\ (0, 1) + (0, 0) &= (0, 1) \\ (2, 1) + (0, 0) &= (2, 1) \\ (2, 0) + (0, 0) &= (2, 0) \\ (0, 0) + (2, 3) &= (2, 3) \\ (0, 1) + (2, 3) &= (2, 4) \\ (2, 1) + (2, 3) &= (4, 4) \\ (2, 0) + (2, 3) &= (4, 3) \end{aligned} \quad (2)$$

*Last updated on 21 May, 2019. This is an improvised example written to improve my understanding. If you find any mistake in this article, please write to me at rahulbhadani@email.arizona.edu

However, we see that after joining all points obtained in (2), points $(2, 1)$ and $(2, 3)$ are internal points of the resulting polygon. Hence, the complete Minkowski sum of the sets **A** and **B** is given by $\{(0, 0), (0, 1), (2, 0), (2, 4), (4, 4), (4, 3)\}$ which is a hexagon.

This has been illustrated in

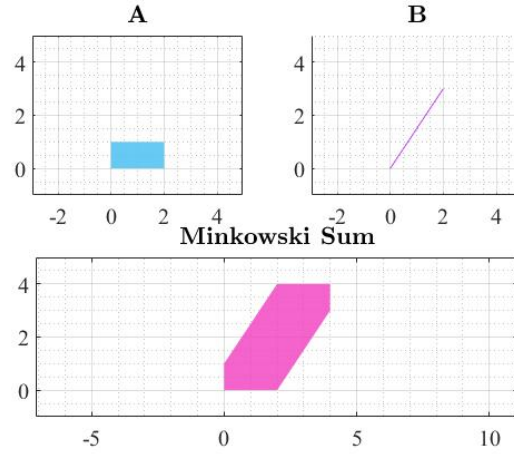


Figure 1: Minkowski Sum Example.

Note: This is one of the simplest I have discussed to illustrate Minkowski sum. However, a deeper discussion on Minkowski sum can be found at https://doc.cgal.org/latest/Minkowski_sum_2/index.html.