



ORIGINAL ARTICLE

A constructive heuristic for time-dependent multi-depot vehicle routing problem with time-windows and heterogeneous fleet



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Abstract In this paper, we consider the time-dependent multi-depot vehicle routing problem. The objective is to minimize the total heterogeneous fleet cost assuming that the travel time between locations depends on the departure time. Also, hard time window constraints for the customers and limitation on maximum number of the vehicles in depots must be satisfied. The problem is formulated as a mixed integer programming model. A constructive heuristic procedure is proposed for the problem. Also, the efficiency of the proposed algorithm is evaluated on 180 test problems. The obtained computational results indicate that the procedure is capable to obtain a satisfying solution. © 2014 Production and hosting by Elsevier B.V. on behalf of King Saud University. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/3.0/>).

1. Introduction

The vehicle routing problem (VRP) is a combinatorial optimization problem seeking to service a number of customers with a fleet of vehicles in a distribution network. Since the introduction of vehicle routing problem (VRP) by Dantzig and Ramser (1959), developing real life variants of the VRP has gained increasing attention throughout the literature (El-Sherbeny, 2010). The vehicle routing problem with time windows

(VRPTW) is a well-studied version of VRP in which clients impose soft or hard time windows constraints. Due to NP-hardness of VRPTW, some meta-heuristics are developed to solve it such as genetic algorithms (Cheng and Wang, 2009; Ursani et al., 2011; Vidal et al., 2013), ant colony (Ding et al., 2012; Yu and Yang, 2011), Tabu search (Belhaiza et al., 2013; Ho and Haugland, 2004), simulated annealing (Baños et al., 2013; Deng et al., 2009; Kuo, 2010; Tavakkoli-Moghaddam et al., 2007, 2011) etc.

The time dependent vehicle routing problem with time windows (TDVRPTW) is a generalization of VRPTW which is introduced by Malandraki (1989). In this problem setting, it is assumed that the time taken to travel a route is a function of the departure time. The literature on solution methods for the TDVRPTW is scant. Hashimoto et al. (2008) generalized the VRPTW by allowing both traveling times and traveling costs to be time-dependent. Ichoua et al. (2003) proposed a parallel Tabu search for TDVRP with soft time windows.

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Soler et al. (2009) solved the TDVRPTW optimally by transforming the problem into an asymmetric capacitated vehicle routing problem, so it can be solved both optimally and heuristically with known codes. Balseiro et al. (2011) proposed an ant colony algorithm hybridized with insertion heuristics for TDVRPTW. Figliozzi (2010) proposed an iterative route construction and improvement algorithm to solve VRP with soft time windows. Also he recently proposed new replicable problems for time dependent routing problems with hard time windows with a new algorithm to solve the problem (Figliozzi, 2012). Kritzing et al. (2012) used efficient traffic data in order to develop a variable neighborhood search (VNS) to solve the problem. Recently, Nguyen et al. (2013) proposed a Tabu search meta-heuristic for the time-dependent multi-zone multi-trip vehicle routing problem with time windows.

The contribution of this paper is threefold: first, a mixed integer programming formulation is developed for the time-dependent multi-depot vehicle routing problem with heterogeneous fleet. In this problem, hard time windows and limitation on maximum number of vehicles in depots must be satisfied. The objective of this problem is to minimize the total heterogeneous fleet cost assuming that the travel time between locations depends on the departure time. This model is not considered in the past literature. Second, a constructive heuristic followed by three efficient local searches is developed for the problem. Finally, the effectiveness of the proposed method is analyzed.

The reminder of the paper is organized as follows: Section 2 describes the problem. In Section 3 we explain the proposed heuristic to obtain a satisfying solution for the problem. Computational results and performance evaluation are represented in Section 4. Finally, Section 5 contains the conclusions.

2. Problem description

Assume that a fleet of heterogeneous vehicles with different capacities and different travel costs is available to serve the transportation requests. Also, the travel time between locations is assumed as a function of departure time. Furthermore, the vehicles do not have to return to a central depot, although maximum number of vehicles in depots is restricted to prevent the aggregation of vehicles in some depots. All locations have to be visited within a specific time window. If the vehicle reaches one of these locations before the beginning of the time window, it has to wait.

The time-dependent multi-depot vehicle routing problem studied in this paper involves finding distinct feasible tours in order to minimize the total cost for operating the tours. A feasible tour of a vehicle is a journey starting from a depot and ending at the same or different depot, passing some customers such that time windows, capacity constraints and limitation on maximum number of vehicles in depots hold. In sequence, assume the problem represented in a graph $G = \{N, A\}$ where the set of nodes, N , represents locations (customers and depots) and the set of arcs, A , represents routes between locations. Also, there is no arc between depots. The set of nodes, N , are numbered from 1 to n , which $W = \{1, \dots, m\}$ are depots and $V = \{m + 1, \dots, n\}$ are customers. The fixed request of a customer i is denoted by q_i ; $i = m + 1, \dots, n$, while request of depots is assumed zero,

$q_i = 0$; $i = 1, \dots, m$. Number of vehicle types is assumed P . Number of available heterogeneous vehicles before routing at each depot is assumed to be arbitrarily large for each vehicle type, while maximum number of vehicles after routing at each depot is assumed Q . However, assuming that each vehicle serves only one customer, an upper bound on the number of vehicles at each depot is $P(n - m)$ at worst case. In doing so, we will have maximum $K = mP(n - m)$ vehicles in the network.

To take into account time dependency the time horizon is divided to U time intervals. In this study, a stepwise function of speed distribution is assigned to each arc and then the time distribution is obtained by integration (Fleischmann et al., 2004). In doing so, we can get continuous function of link travel time. This method ensures the network has FIFO property (Ichoua et al., 2003). A model with a FIFO property guarantees that if a vehicle leaves customer i to go to customer j at any time t , any identical vehicle with the same destination leaving customer i at a time $t + \varepsilon$, where $\varepsilon > 0$, will always arrive later. This is an intuitive and desirable property though it is not present in all models.

We have the following notations for the problem:

- N set of nodes of graph representing the locations, indexed by i and j
- A set of arcs of graph representing the potential routes between locations
- W set of depots $W \subset N$; indexed by i and $j = 1, \dots, m$
- V set of customers $V \subset N$, $W \cap V = \emptyset$, $W \cup V = N$; indexed by i and $j = m + 1, \dots, n$
- F set of heterogeneous vehicles, indexed by $k = 1, \dots, K$
- P number of vehicle types
- Q maximum number of vehicles in each depot after routing
- I set of time intervals, indexed by $u = 1, \dots, U$
- q_i request of customer i
- s_i service time of customer i
- $[a_i, b_i]$ time window of customer i
- C_k capacity of vehicle k
- cf_k fixed cost of vehicle k
- cv_k variable cost of vehicle k per unit traveled distance
- d_{ij} distance between locations i and j
- c_{ij}^k travel cost between locations i and j with vehicle k ;
- $c_{ij}^k = d_{ij} \times cv_k$
- t_{ij}^k travel time between locations i and j with vehicle k (time distribution which is a continuous function of departure time)
- T_u upper limit of u^{th} time interval
- Z objective function
- x_{ij}^{ku} 1; if vehicle k travel between locations i and j in time interval u
- 0; otherwise (binary decision variable)
- y_i^k service start time of customer i (real decision variable) served by vehicle k

Notice that, the decision variable y_i^k allows for waiting at customer i ; service start time may not necessarily be the same as arrival time. Using the above notation, the problem can be mathematically formulated as follows:

$$\min Z = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^K \sum_{u=1}^U c_{ij}^k x_{ij}^{ku} + \sum_{i=1}^m \sum_{j=m+1}^n \sum_{k=1}^K \sum_{u=1}^U cf_k x_{ij}^{ku} \quad (1)$$

Subject to:

$$\sum_{i=1}^n \sum_{k=1}^K \sum_{u=1}^U x_{ij}^{ku} = 1; \forall j = m+1, \dots, n \quad (2)$$

$$\sum_{j=1}^n \sum_{k=1}^K \sum_{u=1}^U x_{ij}^{ku} = 1; \forall i = m+1, \dots, n \quad (3)$$

$$\sum_{i=1}^m \sum_{j=m+1}^n \sum_{u=1}^U x_{ij}^{ku} \leq 1; \forall k = 1, \dots, K \quad (4)$$

$$\sum_{i=m+1}^n \sum_{j=1}^m \sum_{u=1}^U x_{ij}^{ku} \leq 1; \forall k = 1, \dots, K \quad (5)$$

$$\sum_{i=m+1}^n \sum_{k=1}^K \sum_{u=1}^U x_{ij}^{ku} \leq Q; \forall j = 1, \dots, m \quad (6)$$

$$\sum_{i=1}^n \sum_{u=1}^U x_{ir}^{ku} = \sum_{j=1}^n \sum_{u=1}^U x_{rj}^{ku}; \forall k = 1, \dots, K; \quad (7)$$

$$\forall r = m+1, \dots, n$$

$$a_i \sum_{j=1}^n \sum_{u=1}^U x_{ji}^{ku} \leq y_i^k \leq b_i \sum_{j=1}^n \sum_{u=1}^U x_{ji}^{ku}; \forall k = 1, \dots, K; \quad (8)$$

$$\forall i = m+1, \dots, n$$

$$y_i^k + s_i + t_{ij}^k(y_i^k + s_i) - y_j^k \leq M(1 - x_{ji}^{ku}); \forall k = 1, \dots, K; \quad (9)$$

$$\forall u = 1, \dots, U; \forall (i, j) \in A$$

$$y_i^k + s_i - T_u \leq M(1 - x_{ji}^{ku}); \forall k = 1, \dots, K; \quad (10)$$

$$\forall u = 1, \dots, U; \forall (i, j) \in A$$

$$y_i^k + s_i \geq T_{u-1} x_{ji}^{ku}; \forall k = 1, \dots, K; \forall u = 1, \dots, U; \quad (11)$$

$$\forall (i, j) \in A$$

$$\sum_{i=m+1}^n q_i \sum_{j=1}^n \sum_{u=1}^U x_{ij}^{ku} \leq C_k; \forall k = 1, \dots, K \quad (12)$$

$$x_{ij}^{ku} \in \{0, 1\}; y_i^k \in R; \forall k = 1, \dots, K; \quad (13)$$

$$\forall u = 1, \dots, U; \forall (i, j) \in A$$

The objective in Eq. (1) is to minimize the total cost of the heterogeneous fleet. Eqs. (2) and (3) force each customer to be visited exactly once. Eqs. (4) and (5) allow the vehicles to stay in the depots if not used. Eq. (6) guarantees the limitation on maximum number of vehicles in each depot after routing. Eq. (7) prevents the vehicles tour ending at customer's location. Eq. (8) imposes the time windows constraints. Eq. (9) guarantees that service start time must be feasible according to travel time between customers, where M is assumed a large constant. Eqs. (10) and (11) preserve the relation between service start times and time intervals. Eq. (12) stipulates that the maximum vehicle capacity cannot be violated. Eq. (13) specifies that the decision variables x_{ij}^{ku} are binary, while y_i^k are real. This formulation requires the definition of at most $U|A|K$ binary decision variables and of $K(n-m)$ real variables. Also, the

number of constraints of the formulation amounts to at most $(2+2K)(n-m)+3K+3U|A|K+m$.

3. Proposed heuristic

In this section, a constructive heuristic algorithm is designed to obtain a satisfying solution for the mentioned above problem. The proposed heuristic consists of five steps: construction of the sequence of customers *SOC*, assignment of vehicles, construction of the routes, determination of start and end depots, and local searches. In continuation the steps of the proposed algorithm are explained. Also, an example with $P = 2$ vehicle type (Lorry and Truck) is used to illustrate the proposed algorithm. In this example, we will assume 2 depots (1 and 2) and 4 customers (3, 4, 5 and 6). Time windows for the customers are considered as [7:00, 10:30], [6:10, 8:45], [11:35, 14:15] and [9:00, 12:20], relatively.

Step 1: Construction of the SOC

A solution is represented by a sequence of customers *SOC*, which determines the priority of customers' service. A solution is generated using a procedure, which builds a sequence by adding customers one by one. At each iteration, a customer list (*CL*) is calculated which is a set of customers that are not selected before. The procedure is initialized by identifying an unselected customer i with minimum a_i from the customer list. Then, customer i and all other customers who have overlapped time window with the customer i are filtered to restricted customer list *RCL*. We sort the restricted customer list in increasing order of time window length $b_i - a_i$. Finally, a customer in position j of *RCL* is selected with probability ρ_j as follows:

$$\rho_j = \frac{j}{|RCL|(|RCL|+1)/2}; \quad j = 1, \dots, |RCL| \quad (14)$$

where $|RCL|$ denotes the number of customers in *RCL*. According to relation (14), customer with closer and tighter time window will has higher priority for service. This procedure continues until a complete *SOC* is reached.

In our illustrative example, initially we have $CL = \{3, 4, 5, 6\}$. Customer 4 with minimum a_i is selected from *CL*. Customer 3 has overlapped time window with the customer 4. Therefore, customer 4 and customer 3 sorted in increasing order of time window lengths are considered as $RCL = \{4, 3\}$. Now, customers in position 1 (customer 4) and 2 (customer 3) will be selected with probabilities $\rho_1 = \frac{1}{3}$ and $\rho_2 = \frac{2}{3}$, respectively.

Step 2: Assignment of vehicles

To assign the vehicle type to the customers, a dynamic approach is proposed. Initially, each vehicle type p is assigned to the first customer in *SOC* with same probabilities $\rho_p = \frac{1}{P}$ ($p = 1, \dots, P$). Then, vehicle type p , ($p = 1, \dots, P$) is assigned to the next customers with probability ρ_p dynamically computed as follows:

$$\rho_p = \frac{P'}{P} \frac{\tau_p}{\sum_{p=1}^{P'} \tau_p} \quad (15)$$

where τ_p denotes remaining cumulative capacity of the previously assigned vehicle type p , and P' denotes the number of previously assigned vehicle types. This probability for vehicle types which are not assigned so far is assumed $\rho_p = \frac{1}{P}$. According to relation (15), the previously selected vehicle types with unused capacity will have higher probability for selection.

Also, this probability is an increasing function of remaining cumulative capacities.

In our example, consider that capacities of two vehicle types are 170 and 220, respectively. Also, assume the request of customers as $q_3 = 110$, $q_4 = 75$, $q_5 = 80$ and $q_6 = 95$. So, each vehicle type p will be assigned to the first customer in $SOC = \{4, 3, 6, 5\}$ (i.e., customer 4) with probability $\rho_p = \frac{1}{2}; p = 1, 2$. Assume that vehicle type 2 is selected and assigned to the customer 4. According to $q_4 = 75$, the remaining cumulative capacity of the vehicle type 2 is $\tau_2 = 220 - 75 = 145$. Also, number of the assigned vehicle types is $P' = 1$. Now, vehicle type 2, will be assigned to the next customer (i.e., customer 3) with probability $\rho_2 = \frac{1}{2} \times \frac{145}{145} = \frac{1}{2}$. However, vehicle type 1 which is not assigned so far will be selected with probability $\rho_1 = \frac{1}{2}$. Here, assume that vehicle type 1 is selected and assigned to the customer 3. Since $q_3 = 110$, we have remaining cumulative capacity of the vehicle type 1, $\tau_1 = 170 - 110 = 60$ and $P' = 2$. In continuation, vehicle types 1 and 2, will be assigned to the next customer (i.e., customer 6) with probabilities $\rho_1 = \frac{2}{2} \times \frac{60}{60+145} = \frac{12}{41}$ and $\rho_2 = \frac{2}{2} \times \frac{145}{60+145} = \frac{29}{41}$. In doing so, if we assume that vehicle type 1 is assigned to the customer 6 we will have $\tau_1 = 60 + 170 - 95 = 135$. Subsequently, there are probabilities $\rho_1 = \frac{2}{2} \times \frac{135}{135+145} = \frac{27}{56}$ and $\rho_2 = \frac{2}{2} \times \frac{145}{135+145} = \frac{29}{56}$ for the vehicle assignment to the last customer (i.e., customer 5).

Step 3: Construction of the route

To construct each route, procedure is initialized by selecting first unselected customer from SOC . If request of the selected customer is not greater than the capacity of the assigned vehicle, and time window and time interval constraints are satisfied, it is considered as first customer of the current route. Then, iteratively starting from the partial route assembled thus far, we select the first unselected customer from SOC which who has vehicle type same as the recently assembled customer. If capacity, time window and time interval constraints are satisfied, it is inserted to the partial route. This procedure continues until SOC is fully scanned and all customers are served so that all constraints are satisfied. In our example, assuming $SOC = \{4, 3, 6, 5\}$ with assigned vehicle types $\{2, 1, 1, 1\}$, we will have three routes. Route 1 contains customer 4, route 2 contains customers 3 and 6, while route 3 contains customer 5. Starting and ending depots of these routes are specified based on the cost of vehicles.

Step 4: Determination of start and end depots

After construction of the routes, starting and ending depots of the routes should be specified. Let S and L denote the first and last customer of the current route. Starting and ending depot of a route are devoted based on minimum routing cost according to Eqs. (16) and (17), respectively. In these relations, k is index of the assigned vehicle type to the current route. Also, maximum number of the vehicles after routing at each depot Q , may not be violated.

$$\min_{i=1}^m \{c_{is}^k + cf_k\} \quad (16)$$

$$\min_{j=1}^m \{c_{Lj}^k + cf_k\} \quad (17)$$

This procedure is continued until starting and ending depot of all routes are specified.

Step 5: Local searches

A neighborhood solution from the current solution is generated with the following three operators. Each operator is executed for predetermined number of runs. At each execution, if the neighbor solution is better than the current one, it replaces the current solution, else, next operator is applied. Proposed local searches are titled as vehicle type (VT), sequence of customers (SOC) or both (VT/ SOC).

- (i) LS1-VT: Select randomly a customer i and assign to it another vehicle type.
- (ii) LS2-SOC: Select randomly a customer i and exchange its position with next customer j in SOC in condition that their time intervals are overlapped.
- (iii) LS3-SOC/VT: Select randomly a customer i and exchange its vehicle type with vehicle type of the next customer j in SOC .

4. Computational results

In order to evaluate the proposed SA algorithm for the problem, a set of 180 problems was generated using the parameters given in Table 1. The indication $U[x, y]$ means that the value is generated based on uniform distribution on the interval $[x, y]$. Travel time between locations depends on time interval in which travel is started (a continuous function of the departure time).

For each combination of the parameters 10 problems were generated. Fixed and variable costs of the vehicles were generated in the following way: we assume that vehicle types are numbered in increasing order of their capacity. First, P random numbers were generated between 1 and 100. Subsequently, the numbers are sorted and assigned to the vehicle types in increasing order.

The proposed heuristic was coded in Borland C++ 5.02 and executed on a personal computer with an Intel Core2Dou, 2.5 GHz processor and 3 GB memory. The proposed heuristic is compared with the optimal solutions obtained by Lingo. The proposed heuristic is executed 10 times for each problem to obtain more reliable data. Also, each local search is executed n (number of locations) times. Results are summarized in Table 2 where the following notations are used:

- N.LG: Number of problems optimally solved by Lingo.
- ACT.LG: Average convergence time for Lingo (in seconds).

Table 1 The parameter structure for the test problems.

Control parameter	Value
Number of customers (depots)	9(3), 15(5), 23(7)
Number of vehicle types	2, 3, 4
Number of time intervals	2, 3
Maximum number of vehicles in depots after routing	5
Customer request	$U[1, 10]$
Service time (h)	$U[1, 3]$
Capacity of vehicles	$U[10, 50]$
Lower bound of time windows (h)	$U[0, 24]$
Upper bound of time windows (h)	$U[a_i, 24]$
Distance between locations	$U[1, 25]$
Speed of vehicles at time intervals (km/h)	$U[1, 5]$

Table 2 Computational results.

# Locations	# Vehicle types	#Problems	Lingo		Proposed heuristic					
					NS1		NS2		NS3	
			N.LG	ACT.LG	N.LS1	ARD (%)	N.LS2	ARD (%)	N.LS3	ARD (%)
12	2	20	20	11.43	176	0.04	181	0.01	196	0.02
12	3	20	20	23.60	172	0.06	186	0.01	189	0.03
12	4	20	20	56.92	177	0.05	185	0.02	191	0.03
20	2	20	17	267.37	174	0.10	181	0.07	197	0.12
20	3	20	15	395.11	172	0.13	183	0.09	194	0.23
20	4	20	11	562.48	168	0.16	190	0.14	199	0.26
30	2	20	–	–	158	0.33	179	0.27	200	0.50
30	3	20	–	–	166	0.48	186	0.36	200	0.73
30	4	20	–	–	151	0.59	188	0.41	200	0.88

N.LS_{*i*}: Number of runs for which the proposed heuristic with local search *i* gives better solution (or equal) than other local searches, *i* = 1, 2, 3.

ARD: Average relative deviation.

Relative deviation (*RD*) for each problem is obtained by the following formula:

$$RD = \frac{Z - Z^*}{Z^*} \quad (18)$$

where *Z* is the value of objective function obtained by the proposed algorithm and *Z*^{*} is the obtained solution by the Lingo or the best obtained solution by the proposed heuristic when Lingo is unable to solve.

From Table 2 it can be observed that when the number of locations is equal to 12, all 60 problems can be solved to optimality by Lingo within the allowed time limit (1000 s). Also, when the number of locations is equal to 20, 43 out of 60 problems can be solved by Lingo. The ARD percentages of the problems solved by Lingo are not high. This means that there is no significant difference between the solutions obtained by Lingo and the ones obtained by the proposed heuristic. Average CPU-time for Lingo indicates when the number of locations is increased; the complexity of the problem is increased too. However, Table 2 shows that when number of locations is 30, while the Lingo is unable to solve, there is a solution by the proposed heuristic with low ARD which shows that the proposed heuristic gives robust solutions. Also, Table 2 indicates that the proposed heuristic with local search 3 gives better solutions than two others, while the local search 2 relatively outranked measured by ARD.

5. Summary and conclusions

In this paper, we attempted to solve the multi-depot time dependent vehicle routing problem with heterogeneous fleet subject to hard time windows and limitation on the maximum number of the vehicles in depots. The objective of this problem is to minimize the total heterogeneous fleet cost assuming that the travel time between locations depends on the departure time. This problem has not been studied ever before. The problem was described with an integer programming model, and then a constructive heuristic method was proposed to solve it. Performance of the proposed algorithm was compared with the results of the Lingo based on 180 test problems. From the computation results, we could clearly see that the proposed

algorithm could efficiently obtain a satisfying solution. For further research, we recommend developing other competitive solution procedures for the problem.

References

- Balseiro, S.R., Loiseau, I., Ramonet, J., 2011. An Ant Colony algorithm hybridized with insertion heuristics for the time-dependent vehicle routing problems with time windows. *Comput. Oper. Res.* 38 (6), 954–966.
- Baños, R., Ortega, J., Gil, C., Fernández, A., Toro, F., 2013. A simulated annealing based parallel multi-objective approach to vehicle routing problems with time windows. *Expert Syst. Appl.* 40 (5), 1696–1707.
- Belhaiza, S., Hansen, P., Laporte, G., 2013. A hybrid variable neighborhood Tabu search heuristic for the vehicle routing problem with multiple time windows. *Comput. Oper. Res.* (in press), <http://dx.doi.org/10.1016/j.cor.2013.08.010>.
- Cheng, C.B., Wang, K.P., 2009. Solving a vehicle routing problem with time windows by a decomposition technique and a genetic algorithm. *Expert Syst. Appl.* 36 (4), 7758–7763.
- Dantzig, G.B., Ramser, J.H., 1959. The truck dispatching problem. *Manage. Sci.* 6 (1), 80–91.
- Deng, A.M., Mau, C., Zhou, Y.T., 2009. Optimizing research of an improved simulated annealing algorithm to soft time windows vehicle routing problem with pick-up and delivery. *Syst. Eng. Theory Pract.* 29 (5), 186–192.
- Ding, Q., Hu, X., Sun, L., Wang, Y., 2012. An improved ant colony optimization and its application to vehicle routing problem with time windows. *Neurocomputing* 98 (3), 101–107.
- El-Sherbeny, N.A., 2010. Vehicle routing with time windows: an overview of exact, heuristic and metaheuristic methods. *J. King Saud Univ. – Sci.* 22 (3), 123–131.
- Figliozi, M.A., 2010. An iterative route construction and improvement algorithm for the vehicle routing problem with soft time windows. *Transp. Res. Part C: Emerging Technol.* 18 (5), 668–679.
- Figliozi, M.A., 2012. The time dependent vehicle routing problem with time windows: benchmark problems, an efficient solution algorithm, and solution characteristics. *Transp. Res. Part E: Logist. Transp. Rev.* 48 (3), 616–636.
- Fleischmann, B., Gietz, M., Gnutzmann, S., 2004. Time-varying travel times in vehicle routing. *Transp. Sci.* 38 (2), 160–173.
- Hashimoto, H., Yagiura, M., Ibaraki, T., 2008. An iterated local search algorithm for the time-dependent vehicle routing problems with time windows. *Discrete Optim.* 5 (2), 434–456.
- Ho, S.C., Haugland, D., 2004. At Tabu search heuristic for the vehicle routing problems with time windows and split deliveries. *Comput. Oper. Res.* 31 (12), 1947–1964.
- Ichoua, S., Gendreau, M., Potvin, J.Y., 2003. Vehicle dispatching with time-dependent travel times. *Eur. J. Oper. Res.* 144 (2), 379–396.

- Kritzing, S., Doerner, K.F., Hartl, R.F., Kiechle, G.Ž., Stadler, H., Manohar, S.S., 2012. Using traffic Information for time dependent vehicle routing. *Procedia – Soc. Behav. Sci.* 39, 217–229.
- Kuo, Y., 2010. Using simulated annealing to minimize fuel consumption for the time-dependent vehicle routing problem. *Comput. Ind. Eng.* 59 (1), 157–165.
- Malandraki, C., 1989. Time dependent vehicle routing problems: formulations, solution algorithms and computations experiments (Ph.D. dissertation). Northwestern University, Evanston, Illinois.
- Nguyen, P.K., Crainic, T.G., Toulouse, M., 2013. A Tabu search for time-dependent multi-zone multi-trip vehicle routing problem with time windows. *Eur. J. Oper. Res.* 231 (1), 43–56.
- Soler, D., Albiach, J., Martínez, E., 2009. A way to optimally solve a time-dependent vehicle routing problem with time windows. *Oper. Res. Lett.* 37 (1), 37–42.
- Tavakkoli-Moghaddam, R., Gazanfari, M., Alinaghian, M., Salamatbakhsh, A., Norouzi, N., 2011. A new mathematical model for a competitive vehicle routing problems with time windows solved by simulated annealing. *J. Manuf. Syst.* 30 (2), 83–92.
- Tavakkoli-Moghaddam, R., Safaei, N., Kah, M.M.O., Rabbani, M., 2007. A new capacitated vehicle routing problem with split service for minimizing fleet cost by simulated annealing. *J. Franklin Inst.* 344 (5), 406–425.
- Ursani, Z., Essam, D., Cornforth, D., Stocker, R., 2011. Localized genetic algorithm for vehicle routing problem with time windows. *Appl. Soft Comput.* 11 (8), 5375–5390.
- Vidal, T., Crainic, T.G., Gendreau, M., Prins, C., 2013. A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time windows. *Comput. Oper. Res.* 40 (1), 475–489.
- Yu, B., Yang, Z.Z., 2011. An ant colony optimization model: the period vehicle routing problem with time windows. *Transp. Res. Part E: Logist. Transp. Rev.* 47 (2), 166–181.